

Question Paper

Conducted by the Central Board of Secondary Education (CBSE)



Time Allowed: 3 Hours

Maximum Marks: 100

General Instructions

- (i) This question paper contains **29 questions**. All questions are compulsory.
- (ii) Section A: Questions 1 to 4 carry 1 mark each.
- (iii) Section B: Questions 5 to 12 carry 2 marks each.
- (iv) Section C: Questions 13 to 23 carry 4 marks each.
- (v) Section D: Questions 24 to 29 carry 6 marks each.
- (vi) Attempt one alternative where an internal choice is provided. Follow the instructions within each question.
- (vii) Use of calculators is not allowed.

Section A

1. Find the angle between the line $\vec{r} = (2\hat{i} - \hat{j} + 3\hat{k}) + \lambda(3\hat{i} - \hat{j} + 2\hat{k})$ and the plane $\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = 3$.

OR

Find the co-ordinates of the point, where the line $\frac{x-2}{1} = \frac{y-3}{-3} = \frac{z+1}{5}$ cuts the yz-plane.

2. If $y = 5e^{7x} + 6e^{-7x}$, show that $\frac{d^2y}{dx^2} = 49y$.

3. If A is a square matrix of order 2 and $|A| = 4$, then find the value of $|2 \cdot AA'|$, where A' is the transpose of matrix A .

4. Find the order of the differential equation of the family of circles of radius 3 units.

Section B

5. Find the value of $(x - y)$ from the matrix equation

$$2 \begin{bmatrix} x & 5 \\ 7 & y-3 \end{bmatrix} + \begin{bmatrix} -3 & -4 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 6 \\ 15 & 14 \end{bmatrix}.$$

6. Solve the following differential equation :

$$(y + 3x^2) \frac{dx}{dy} = x.$$

7. Find :

$$\int e^x (\cos x - \sin x) \operatorname{cosec}^2 x \, dx.$$

8. Using vectors, prove that the points $(2, -1, 3)$, $(3, -5, 1)$ and $(-1, 11, 9)$ are collinear.

OR

For any two vectors $\vec{a}$ and $\vec{b}$, prove that $(\vec{a} \times \vec{b})^2 = \vec{a}^2 \vec{b}^2 - (\vec{a} \cdot \vec{b})^2$.

9. Find :

$$\int \frac{x-1}{(x-2)(x-3)} \, dx.$$

OR

Integrate $\frac{e^x}{\sqrt{5-4e^x-e^{2x}}}$ with respect to x .

10. If $P(A) = 0.6$, $P(B) = 0.5$ and $P(B | A) = 0.4$, find $P(A \cup B)$ and $P(A | B)$.

11. If an operation $*$ on the set of integers $\mathbb{Z}$ is defined by $a * b = 2a^2 + b$, then find (i) whether it is a binary or not, and (ii) if a binary, then is it commutative or not.

12. Four cards are drawn one by one with replacement from a well-shuffled deck of playing cards. Find the probability that at least three cards are of diamonds.

OR

The probability of two students A and B coming to school on time are $\frac{2}{7}$ and $\frac{4}{7}$, respectively. Assuming that the events 'A coming on time' and 'B coming on time' are independent, find the probability of only one of them coming to school on time.

Section C

13. If $x^p y^q = (x + y)^{p+q}$, prove that $\frac{dy}{dx} = \frac{y}{x}$ and $\frac{d^2y}{dx^2} = 0$.

14. Find :

$$\int (\sin x \cdot \sin 2x \cdot \sin 3x) dx.$$

15. Differentiate $\tan^{-1}\left(\frac{3x - x^3}{1 - 3x^2}\right)$, $|x| < \frac{1}{\sqrt{3}}$, w.r.t. $\tan^{-1}\left(\frac{x}{\sqrt{1 - x^2}}\right)$.

OR

If $\sqrt{1 - x^2} + \sqrt{1 - y^2} = a(x - y)$, $|x| < 1$, $|y| < 1$, show that $\frac{dy}{dx} = \sqrt{\frac{1 - y^2}{1 - x^2}}$.

16. Find the particular solution of the differential equation :

$$(1 + e^{2x}) dy + (1 + y^2) e^x dx = 0, \text{ given that } y(0) = 1.$$

OR

Find the particular solution of the differential equation :

$$x \frac{dy}{dx} \sin\left(\frac{y}{x}\right) + x - y \sin\left(\frac{y}{x}\right) = 0, \text{ given that } y(1) = \frac{\pi}{2}.$$

17. Prove that the relation R in the set $A = \{1, 2, 3, 4, 5, 6, 7\}$ given by $R = \{(a, b) : |a - b| \text{ is even}\}$ is an equivalence relation.

OR

Show that the function f in $A = \mathbb{R} - \left\{\frac{2}{3}\right\}$ defined as $f(x) = \frac{4x + 3}{6x - 4}$ is one-one and onto. Hence, find f^{-1} .

18. Find whether the function $f(x) = \cos\left(2x + \frac{\pi}{4}\right)$ is increasing or decreasing in the interval $\frac{3\pi}{8} < x < \frac{5\pi}{8}$.

19. Find the equation of the plane passing through the point $(-1, 3, 2)$ and perpendicular to the planes $x + 2y + 3z = 5$ and $3x + 3y + z = 0$.

20. Prove that :

$$\sin^{-1} \frac{4}{5} + \tan^{-1} \frac{5}{12} + \cos^{-1} \frac{63}{65} = \frac{\pi}{2}.$$

21. Evaluate :

$$\int_1^5 (|x - 1| + |x - 2| + |x - 4|) dx.$$

22. Using vectors, find the value of x such that the four points $A(x, 5, -1)$, $B(3, 2, 1)$, $C(4, 5, 5)$ and $D(4, 2, -2)$ are coplanar.

23. If x, y, z are different and $\Delta =$

$$\begin{vmatrix} x & x^2 & x^3 - 1 \\ y & y^2 & y^3 - 1 \\ z & z^2 & z^3 - 1 \end{vmatrix} = 0, \text{ then using properties of determinants, show that } xyz = 1.$$

Section D

24. Using integration, find the area of the following region :

$$\{(x, y) : x^2 + y^2 \leq 16a^2 \text{ and } y^2 \leq 6ax\}.$$

OR

Using integration, find the area of triangle ABC bounded by the lines $4x - y + 5 = 0$, $x + y - 5 = 0$ and $x - 4y + 5 = 0$.

25. Find the vector equation of the line passing through $(2, 1, -1)$ and parallel to the line $\vec{r} = (\hat{i} + \hat{j}) + \lambda(2\hat{i} - \hat{j} + \hat{k})$. Also, find the distance between these two lines.

OR

Find the coordinates of the foot Q of the perpendicular drawn from the point $P(1, 3, 4)$ to the plane $2x - y + z + 3 = 0$. Find the distance PQ and the image of P treating the plane as a mirror.

26. A company manufactures two types of novelty souvenirs made of plywood. Souvenirs of type A require 5 minutes each for cutting and 10 minutes each for assembling. Souvenirs of type B require 8 minutes each for cutting and 8 minutes each for assembling. There are 3 hours 20 minutes available for cutting and 4 hours for assembling. The profit for type A souvenirs is Rs. 100 each and for type B souvenirs, profit is Rs. 120 each. How many souvenirs of each type should the company manufacture in order to maximise the profit? Formulate the problem as a LPP and then solve it graphically.

27. In answering a question on a multiple choice questions test with four choices in each question, out of which only one is correct, a student either guesses or copies or knows the answer. The probability that he makes a guess is $\frac{1}{4}$ and the probability that he copies is also $\frac{1}{4}$. The probability that the answer is correct, given that he copied it is $\frac{3}{4}$. Find the probability that he knows the answer to the question, given that he correctly answered it.

28. An isosceles triangle of vertical angle 2θ is inscribed in a circle of radius a . Show that the area of the triangle is maximum when $\theta = \frac{\pi}{6}$.

29. Using elementary row transformations, find the inverse of the matrix $\begin{bmatrix} 3 & 0 & -1 \\ 2 & 3 & 0 \\ 0 & 4 & 1 \end{bmatrix}$.

OR

Using matrices, solve the following system of linear equations :

$$2x + 3y + 10z = 4$$

$$4x - 6y + 5z = 1$$

$$6x + 9y - 20z = 2.$$

- End of Question Paper -