CBSE Class 12 Physics Question Paper 2019 official pdf-Set 55/1/1

Question Paper with Solutions

Conducted by the Central Board of Secondary Education (CBSE)

Time Allowed: 3 hours **Maximum Marks: 70**

General Instructions

(i) All questions are compulsory. There are **27** questions in all.

(ii) This question paper has **four** sections : Section **A**, Section **B**, Section **C** and Section **D**.

(iii) Section **A** contains **five** questions of **one** mark each, Section **B** contains **seven** questions of **two** marks each, Section **C** contains **twelve** questions of **three** marks each, and Section **D** contains **three** questions of **five** marks each.

(iv) There is no overall choice. However, an internal choice(s) has been provided in **two** questions of **one** mark, **two** questions of **two** marks, **four** questions of **three** marks and **three** questions of **five** marks weightage. You have to attempt only **one** of the choices in such questions.

You may use the following values of physical constants wherever necessary :

$c = 3 \times 10^8 \text{ m/s}$

$h = 6.63 \times 10^{-34} \text{ J s}$

$e = 1.6 \times 10^{-19} \text{ C}$

$\mu_0 = 4\pi \times 10^{-7} \text{ T m A}^{-1}$

$\varepsilon_0 = 8.854 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$

$\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$

Mass of electron $(m_e) = 9.1 \times 10^{-31} \text{ kg}$

Mass of neutron $= 1.675 \times 10^{-27} \text{ kg}$

Mass of proton $= 1.673 \times 10^{-27} \text{ kg}$

Avogadro's number $= 6.023 \times 10^{23}$ per gram mole

Boltzmann constant $= 1.38 \times 10^{-23} \text{ J K}^{-1}$

Section A (5 marks)

1. Draw the pattern of electric field lines, when a point charge $-Q$ is kept near an uncharged conducting plate. [1 mark]

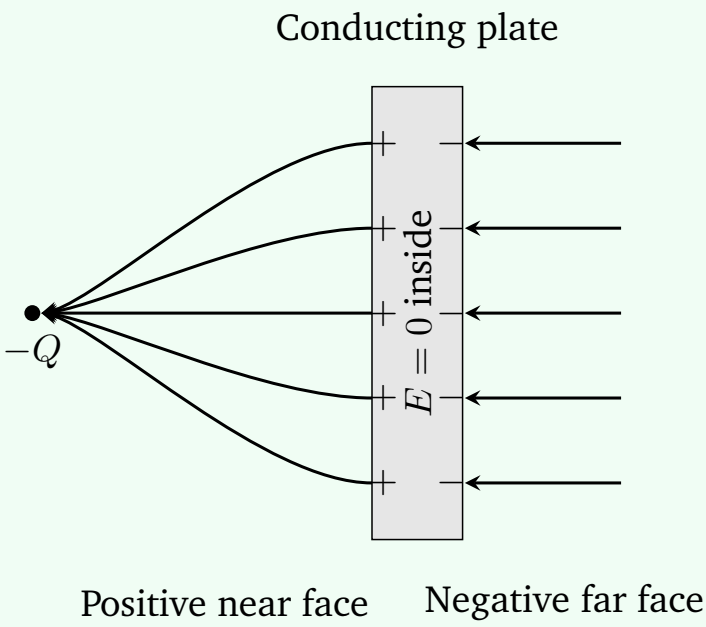
Solution:

Concept:

- A conductor in electrostatic equilibrium has zero electric field inside its material; the external field is perpendicular to its surface.
- Electric field lines originate on positive charge and terminate on negative charge. An isolated uncharged plate acquires equal and opposite induced charges.

Step 1: Locate the induced charges The nearby charge $-Q$ repels free electrons towards the far face of the plate. The near face therefore becomes positively charged and the far face negatively charged, while the plate's net charge remains zero.

Step 2: Draw the field direction Draw lines leaving the positive near face normally and curving towards $-Q$, with arrowheads directed towards the point charge. On the far side, lines enter the negative face normally. No field line passes through the conducting material. The sketch shows representative lines near both faces.



Representative electric field lines near an isolated neutral conducting plate

Step 3: Conclusion The field lines from the induced positive face terminate on $-Q$; they meet the conducting surface at right angles, and the field inside the plate is zero.

Quick Tip to Solve: For a negative external charge, put positive induced charge on the nearer face and point the field arrows towards the negative charge.

2. How does the mobility of electrons in a conductor change, if the potential difference applied across the conductor is doubled, keeping the length and temperature of the conductor constant? [1 mark]

Solution:

Concept:

- Electron mobility is the magnitude of drift velocity per unit electric field: $\mu = v_d/E$.
- In the usual ohmic regime, $v_d = eE\tau/m_e$, so $\mu = e\tau/m_e$, where τ is the mean relaxation time.

Step 1: Compare the electric fields For a uniform conductor of fixed length L, the field magnitude is $E = V/L$. Doubling the potential difference therefore doubles the electric field: $E' = 2E$.

Step 2: Compare the drift velocities At unchanged temperature the relaxation time remains the same. Hence the drift speed also doubles, $v_d' = 2v_d$, and its ratio to the field is

$$\mu' = \frac{v_d'}{E'} = \frac{2v_d}{2E} = \mu.$$

Step 3: Conclusion The mobility of the electrons remains unchanged when the applied potential difference is doubled at constant temperature.

Quick Tip to Solve: Distinguish drift speed from mobility: at fixed temperature voltage changes the speed and field by the same factor.

3. Define the term "threshold frequency", in the context of photoelectric emission. [1 mark]

Solution:

Concept:

- Each incident photon has energy $h\nu$, where h is Planck's constant and ν is the radiation frequency.
- The work function ϕ is the minimum energy needed to liberate an electron from a given material's surface.

Step 1: Apply the minimum energy condition Photoelectric emission begins only when the photon can supply at least the work function. At the limiting frequency ν_0, Einstein's equation gives $h\nu_0 = \phi$, with zero maximum kinetic energy for the emitted electrons.

Step 2: Identify what happens below the limit If $\nu < \nu_0$, each photon has insufficient energy to eject an electron. Increasing the intensity only increases the number of such photons and cannot cause ordinary photoelectric emission below the threshold.

Step 3: Conclusion Threshold frequency is the minimum frequency of incident radiation required to cause photoelectric emission from a given material: $\nu_0 = \phi/h$.

Quick Tip to Solve: Define the threshold using frequency, not intensity; it is fixed by the work function of the emitting surface.

OR

3. Define the term "Intensity" in photon picture of electromagnetic radiation. [1 mark]

Solution:

Concept:

- Intensity is the radiant energy incident per unit time per unit area perpendicular to propagation; its SI unit is W m^{-2}.
- In the photon picture, monochromatic radiation consists of photons each carrying energy $h\nu$, where ν is the frequency.

Step 1: Count the incident photons If N photons of frequency ν cross a perpendicular area A in time Δt, their total energy is $Nh\nu$. Define the photon flux density as $n = N/(A\Delta t)$.

Step 2: Express the energy flux Dividing this energy by the area and time gives

$$I = \frac{Nh\nu}{A\Delta t} = nh\nu.$$

Thus, at fixed frequency, a higher intensity means more photons arriving per unit area per unit time, while the energy of each photon remains unchanged.

Step 3: Conclusion In the photon picture, intensity is the photon energy arriving per unit area per unit time: $I = nh\nu$, with n the incident photon flux density.

Quick Tip to Solve: Multiply photons per unit area per second by the energy of one photon; photon count alone is not the energy intensity.

4. What is the speed of light in a denser medium of polarising angle $30°$? [1 mark]

Solution:

Concept:

- Brewster's law is $\tan i_p = n_2/n_1$, where i_p is the polarising angle in the incident medium and n_1, n_2 are the incident and refracting indices.
- For incidence from air, $n_1 \simeq 1$ and the speed in the second medium is $v = c/n_2$, with $c = 3 \times 10^8$ m s^{-1}.

Step 1: Substitute the printed angle Using the usual air-to-medium convention, the printed angle gives

$$n_2 = \tan 30° = \frac{1}{\sqrt{3}}, \qquad v = \frac{c}{\tan 30°} = 3\sqrt{3} \times 10^8 \text{ m s}^{-1}.$$

This is the formal numerical result of substituting the data as printed.

Step 2: Check the meaning of denser A medium optically denser than air must have $n_2 > 1$ and hence an air-side polarising angle exceeding $45°$. The printed $30°$ contradicts that description. If instead the angle is measured for incidence from the denser medium into air, Brewster's law gives

$n = 1/\tan 30^\circ = \sqrt{3}$ and $v = c/\sqrt{3} = 1.73 \times 10^8$ m s^{-1}. That is a different incidence convention, which the question does not specify.

Step 3: Conclusion With the standard air-incidence convention, the formal answer is 5.20×10^8 m s^{-1}, but it is inconsistent with the stated denser medium. The value 1.73×10^8 m s^{-1} applies only to incidence from the denser medium into air.

Quick Tip to Solve: Always write $\tan i_p = n_2/n_1$ first; a Brewster angle below 45° corresponds to passage into the optically rarer medium.

5. In sky wave mode of propagation, why is the frequency range of transmitting signals restricted to less than 30 MHz? [1 mark]

Solution:

Concept:

- Sky waves reach distant receivers after the ionosphere bends the transmitted radio waves back towards the Earth's surface.
- Whether a wave returns depends on its frequency, the ionospheric electron density and the angle of incidence; very high frequencies are insufficiently refracted.

Step 1: Identify the required return path In sky-wave communication the ionosphere must redirect the signal back to the Earth. Its ionised layers provide the effective reflection through refraction that permits communication beyond the direct line of sight.

Step 2: Examine higher frequencies For frequencies above approximately 30 MHz, under usual conditions, the ionosphere does not bend the waves enough to return them to Earth. They penetrate the ionosphere and escape into space, so a reliable sky-wave return path is lost.

Step 3: Conclusion The transmitting frequency is normally restricted to below 30 MHz because higher-frequency waves pass through the ionosphere instead of being returned to Earth.

Quick Tip to Solve: Link the frequency limit to the loss of ionospheric return: waves above the usual limit escape into space.

OR

5. On what factors does the range of coverage in ground wave propagation depend? [1 mark]

Solution:

Concept:

- Ground waves travel along the Earth's surface and lose energy through absorption in the ground.
- Useful coverage ends when the received signal falls below the strength needed for reliable reception.

Step 1: Identify the signal factors The range depends on the power of the transmitter and the frequency of the transmitted wave. Greater transmitted power permits reception farther away; higher frequencies generally suffer greater ground-wave attenuation and give a shorter range.

Step 2: Identify the ground factor The electrical conductivity of the ground also affects energy loss. A better conducting surface causes less attenuation, so the wave can cover a greater distance under otherwise comparable conditions.

Step 3: Conclusion Ground-wave coverage depends mainly on transmitter power, signal frequency and the conductivity or nature of the ground.

Quick Tip to Solve: Recall one source factor (power), one wave factor (frequency) and one surface factor (ground conductivity).

Section B (14 marks)

6. Two bulbs are rated (P_1, V) and (P_2, V). If they are connected (i) in series and (ii) in parallel across a supply V, find the power dissipated in the two combinations in terms of P_1 and P_2. [2 marks]

Solution:

Concept:

- A bulb rated at power P and voltage V has resistance $R = V^2/P$, using the usual constant-resistance model for this calculation.
- Across the same supply voltage, the total power of a combination is $P_{\text{total}} = V^2/R_{\text{eq}}$, where R_{eq} is its equivalent resistance.

Step 1: Express the bulb resistances Both ratings refer to the same voltage V. The individual resistances are therefore

$$R_1 = \frac{V^2}{P_1}, \qquad R_2 = \frac{V^2}{P_2}.$$

These resistances, rather than the rated powers, must first be combined according to the connection.

Step 2: Calculate the series power In series the resistances add, and the supply voltage is shared by the bulbs:

$$R_{\text{s}} = V^2 \left(\frac{1}{P_1} + \frac{1}{P_2} \right).$$

Applying the power relation to the complete combination gives

$$P_{\text{s}} = \frac{V^2}{R_{\text{s}}} = \frac{P_1 P_2}{P_1 + P_2}.$$

Step 3: Calculate the parallel power In parallel each bulb receives its rated voltage V, so each dissipates its own rated power. Equivalently,

$$\frac{1}{R_{\text{p}}} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{P_1 + P_2}{V^2}, \qquad P_{\text{p}} = \frac{V^2}{R_{\text{p}}} = P_1 + P_2.$$

Step 4: Conclusion The total powers are $\boxed{P_{\text{s}} = P_1 P_2/(P_1 + P_2)}$ in series and $\boxed{P_{\text{p}} = P_1 + P_2}$ in parallel.

Quick Tip to Solve: For bulbs with the same rated voltage, add rated powers directly only when every bulb receives that voltage in parallel.

7. Calculate the radius of curvature of an equi-concave lens of refractive index 1.5, when it is kept in a medium of refractive index 1.4, to have a power of -5 D. [2 marks]

Solution:

Concept:

- For a thin lens in a surrounding medium, the lens maker's relation is $P = (n_\ell/n_m - 1)(1/R_1 - 1/R_2)$, with P in dioptres and the radii in metres.
- An equi-concave lens has equal radius magnitudes R. For light travelling from left to right, the Cartesian signs are $R_1 = -R$ and $R_2 = +R$.

Step 1: Use the relative refractive index Refraction depends on the index of the lens relative to its surroundings, so use the ratio of the given indices:

$$\frac{n_\ell}{n_m} = \frac{1.5}{1.4} = \frac{15}{14}, \qquad \frac{n_\ell}{n_m} - 1 = \frac{1}{14}.$$

The lens is still optically denser than its surroundings and hence remains diverging in the equi-concave shape.

Step 2: Substitute the signed radii and power Inserting $R_1 = -R$, $R_2 = +R$ and $P = -5$ D gives

$$-5 = \frac{1}{14}\left(-\frac{1}{R} - \frac{1}{R}\right) = -\frac{1}{7R}.$$

Thus $35R = 1$, when R is measured in metres. The negative power is consistent with divergence; it does not make the radius magnitude negative.

Step 3: Conclusion Each surface has radius of curvature of magnitude $\boxed{R = 1/35\,\text{m} \simeq 2.86\,\text{cm}}$. The signed radii are $R_1 = -2.86$ cm and $R_2 = +2.86$ cm.

Quick Tip to Solve: Replace the lens index by the relative index before using the lens maker's formula, and keep all radii in metres for power in dioptres.

OR

7. An equilateral glass prism has a refractive index 1.6 in air. Calculate the angle of minimum deviation of the prism, when kept in a medium of refractive index $\frac{4\sqrt{2}}{5}$. [2 marks]

Solution:

Concept:

- The relevant refractive index for an immersed prism is $\mu = n_{\text{prism}}/n_{\text{medium}}$.
- At minimum deviation the ray path is symmetric: $r_1 = r_2 = A/2$, $i = e$ and $\delta_m = 2i - A$, where A is the prism angle and i is the angle of incidence.

Step 1: Calculate the relative refractive index The prism is equilateral, so its refracting angle is

$A = 60°$. Its index relative to the surrounding medium is

$$\mu = \frac{1.6}{4\sqrt{2}/5} = \frac{8/5}{4\sqrt{2}/5} = \frac{2}{\sqrt{2}} = \sqrt{2}.$$

The quoted value 1.6 alone applies to a prism in air and must not be used directly after immersion.

Step 2: Apply the minimum-deviation condition At minimum deviation, $r_1 = A/2 = 30°$. Snell's law at entry gives

$$\sin i = \mu \sin r_1 = \sqrt{2} \sin 30° = \frac{1}{\sqrt{2}}.$$

The physical incidence angle lies between $0°$ and $90°$, hence $i = 45°$. Symmetry also gives emergence angle $e = 45°$; the total deviation is therefore obtained from $i + e - A$.

Step 3: Conclusion The angle of minimum deviation is $\boxed{\delta_m = 2(45°) - 60° = 30°}$.

Quick Tip to Solve: For an equilateral prism at minimum deviation, start with the internal angle $30°$, then apply Snell's law using the relative index.

8. An α-particle and a proton of the same kinetic energy are in turn allowed to pass through a magnetic field $\vec{B}$, acting normal to the direction of motion of the particles. Calculate the ratio of radii of the circular paths described by them. [2 marks]

Solution:

Concept:

- For velocity perpendicular to a uniform magnetic field, the magnetic force supplies centripetal force: $|q|vB = mv^2/r$, where m, q, v and r are mass, charge, speed and path radius.
- In the non-relativistic treatment, kinetic energy $K = mv^2/2$. Thus the radius is $r = \sqrt{2mK}/(|q|B)$.

Step 1: Express the radius for equal kinetic energies Equating the magnetic and centripetal forces first gives $r = mv/(|q|B)$. On substituting $v = \sqrt{2K/m}$,

$$r = \frac{\sqrt{2mK}}{|q|B}.$$

Since both particles enter the same magnetic field with the same kinetic energy, their radii are proportional to $\sqrt{m}/|q|$, rather than to $m/|q|$.

Step 2: Insert the mass and charge ratios An alpha particle has approximately four times the proton mass and twice its charge: $m_\alpha = 4m_p$, $q_\alpha = 2e$, while $q_p = e$. Therefore,

$$\frac{r_\alpha}{r_p} = \sqrt{\frac{m_\alpha}{m_p}\frac{q_p}{q_\alpha}} = \sqrt{4} \times \frac{1}{2} = 1.$$

The alpha particle's momentum is twice the proton's at equal energy; its doubled charge exactly compensates for this in the radius formula.

Step 3: Conclusion The required ratio is $\boxed{r_\alpha : r_p = 1:1}$; the two circular paths have equal radii.

Quick Tip to Solve: Check what is held equal: equal kinetic energy requires $r \propto \sqrt{m}/q$, whereas equal speed would give $r \propto m/q$.

9. State Bohr's quantization condition of angular momentum. Calculate the shortest wavelength of the Brackett series and state to which part of the electromagnetic spectrum does it belong. [2 marks]

Solution:

Concept:

- Bohr's quantization condition permits only those circular orbits for which the electron's orbital angular momentum is an integral multiple of $\hbar = h/(2\pi)$.
- For hydrogen, the Rydberg formula is $1/\lambda = R_H(1/n_f^2 - 1/n_i^2)$, where $n_i > n_f$ are the initial and final principal quantum numbers and $R_H = 1.097 \times 10^7\,\text{m}^{-1}$.

Step 1: State the allowed angular momenta If m_e, v_n and r_n denote the electron mass, orbital speed and orbital radius, respectively, then

$$L = m_e v_n r_n = \frac{nh}{2\pi}, \qquad n = 1, 2, 3, \ldots.$$

Thus the permitted angular momenta are discrete; n labels an allowed stationary orbit.

Step 2: Evaluate the Brackett series limit Brackett transitions terminate at $n_f = 4$. The shortest wavelength corresponds to the largest photon energy in this series, obtained at the limit $n_i \to \infty$. Therefore,

$$\frac{1}{\lambda_{\min}} = R_H\left(\frac{1}{4^2} - 0\right) = \frac{R_H}{16},$$

$$\lambda_{\min} = \frac{16}{1.097 \times 10^7} = 1.46 \times 10^{-6}\,\text{m}.$$

This is about $1460\,\text{nm}$, beyond the red end of the visible spectrum.

Step 3: Conclusion Bohr's condition is $m_e v_n r_n = nh/(2\pi)$. The shortest Brackett wavelength is $\boxed{1.46 \times 10^{-6}\,\text{m}}$, in the **infrared** region.

Quick Tip to Solve: At any hydrogen series limit set $n_i \to \infty$, giving $\lambda_{\min} = n_f^2/R_H$; Brackett has $n_f = 4$.

OR

9. Calculate the orbital period of the electron in the first excited state of hydrogen atom. [2 marks]

Solution:

Concept:

- In the Bohr model of hydrogen, $r_n = a_0 n^2$ and $v_n = v_1/n$, where $a_0 = 5.29 \times 10^{-11}\,\text{m}$ and $v_1 = 2.19 \times 10^6\,\text{m s}^{-1}$.
- The orbital period is the time for one revolution: $T_n = 2\pi r_n/v_n$. Hence $T_n = n^3 T_1$; the first excited state has $n = 2$.

Step 1: Find the radius and speed in the first excited state The ground state is $n = 1$, so the first excited state is $n = 2$. Applying the hydrogen orbit formulae gives

$$r_2 = 4a_0 = 4(5.29 \times 10^{-11}) = 2.116 \times 10^{-10}\,\text{m},$$

$$v_2 = \frac{v_1}{2} = 1.095 \times 10^6\,\text{m s}^{-1}.$$

The orbit is larger than the ground-state orbit and the electron moves more slowly, so the revolution time increases.

Step 2: Divide the circumference by the orbital speed One complete revolution covers the circumference $2\pi r_2$. Therefore,

$$T_2 = \frac{2\pi r_2}{v_2} = \frac{2\pi(2.116 \times 10^{-10})}{1.095 \times 10^6} = 1.21 \times 10^{-15}\,\text{s}.$$

The units are metres divided by metres per second, giving seconds. Equivalently, the n^3 scaling makes this period eight times the ground-state period.

Step 3: Conclusion The orbital period of the electron in the first excited state of hydrogen is $\boxed{T_2 \approx 1.21 \times 10^{-15}\,\text{s}}$.

Quick Tip to Solve: Translate "first excited state" into $n = 2$ before substituting; orbital periods scale as n^3, so $T_2 = 8T_1$.

10. Why a signal transmitted from a TV tower cannot be received beyond a certain distance? Write the expression for the optimum separation between the receiving and the transmitting antenna. [2 marks]

Solution:

Concept:

- Television signals at high frequencies propagate mainly as space waves along approximately straight paths. Their reliable reception requires a line of sight between the transmitting and receiving antennas.
- For an antenna of height $h \ll R$, its horizon distance is $d \simeq \sqrt{2Rh}$, where R is the Earth's radius. Both antenna heights contribute to the total range.

Step 1: Explain the limited reception distance Because the Earth's surface is curved, beyond a certain distance the Earth obstructs the direct path from the TV tower to the receiving antenna. TV waves are not reflected back effectively by the ionosphere at these frequencies and do not bend sufficiently around the Earth to maintain normal reception. Consequently, a direct TV broadcast has a finite line-of-sight range.

Step 2: Add the two horizon distances Let h_T and h_R be the heights of the transmitting and receiving antennas. A ray just reaching the horizon is tangent to the Earth. For either antenna, right-triangle geometry gives

$$d^2 = (R + h)^2 - R^2 = 2Rh + h^2 \simeq 2Rh.$$

At the limiting separation, the transmitter and receiver horizons just meet. Thus their individual horizon distances add:

$$d_{\max} = d_T + d_R = \sqrt{2Rh_T} + \sqrt{2Rh_R}.$$

All heights and R must be expressed in the same length unit to obtain $d_{\max}$ in that unit.

Step 3: Conclusion Earth's curvature limits TV reception to the line-of-sight range. The optimum

(maximum line-of-sight) antenna separation is $\boxed{d_{\max} = \sqrt{2Rh_T} + \sqrt{2Rh_R}}$.

Quick Tip to Solve: Add the two separate square roots for transmitter and receiver; do not place the sum of their heights inside one square root.

11. Why is wave theory of electromagnetic radiation not able to explain photoelectric effect? How does photon picture resolve this problem? [2 marks]

Solution:

Concept:

- Classical wave theory treats electromagnetic energy as continuously distributed over a wavefront, with intensity determining the energy incident per unit area per unit time.
- In the photon picture, one photon of frequency ν transfers energy $h\nu$ to one electron. Einstein's equation is $K_{\max} = h\nu - \phi$, where ϕ is the metal's work function.

Step 1: Compare the wave predictions with observations Continuous absorption would allow an electron to accumulate enough energy to escape at any frequency if illumination lasted sufficiently long. At very low intensity, a measurable waiting time would be expected. It also suggests that more intense light should provide greater electron energy. Instead, photoemission has a definite threshold frequency, occurs practically instantaneously above threshold, and its maximum kinetic energy depends on frequency rather than intensity. Classical wave theory cannot account for these observations together.

Step 2: Apply the one-photon energy balance An electron absorbs a photon's energy in a single interaction. Emission requires $h\nu \geq \phi$, which gives the threshold frequency

$$\nu_0 = \frac{\phi}{h}, \qquad K_{\max} = h\nu - \phi = h(\nu - \nu_0).$$

This explains both the threshold and the frequency dependence of electron energy without any energy-accumulation delay. At fixed frequency above threshold, increasing intensity increases the incident photon rate and hence the photoelectric current, but leaves $K_{\max}$ unchanged. Below threshold, greater intensity cannot compensate for insufficient energy per photon.

Step 3: Conclusion Wave theory fails to explain the threshold frequency, prompt emission and frequency-dependent maximum kinetic energy. The photon model explains all three through discrete absorption of $h\nu$ and $\boxed{K_{\max} = h\nu - \phi}$.

Quick Tip to Solve: Separate energy per photon from number of photons: frequency controls $K_{\max}$, while intensity controls current when $\nu > \nu_0$.

12. Plot a graph showing variation of de Broglie wavelength (λ) associated with a charged particle of mass m, versus $\frac{1}{\sqrt{V}}$, where V is the potential difference through which the particle is accelerated. How does this graph give us the information regarding the magnitude of the charge of the particle?

[2 marks]

Solution:

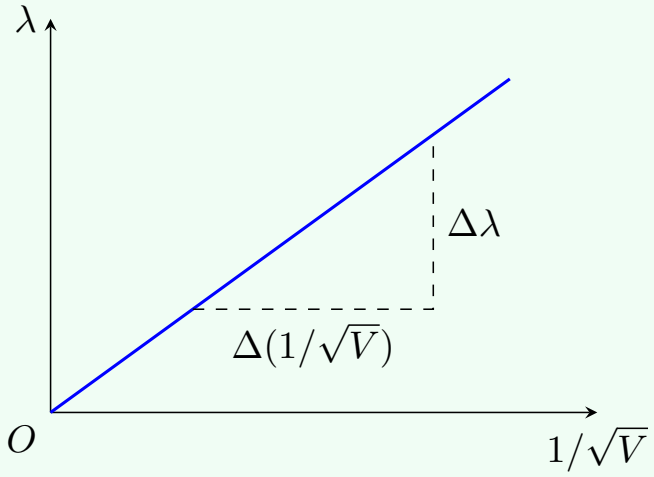
Concept:

- A particle of mass m and momentum p has de Broglie wavelength $\lambda = h/p$, where h is Planck's constant.
- For non-relativistic acceleration from rest through a potential difference of magnitude V, the kinetic energy gained is $K = |q|V = p^2/(2m)$, where $|q|$ is the charge magnitude.

Step 1: Express the wavelength in terms of the plotted variable Equating the electrical energy gained to kinetic energy gives

$$p = \sqrt{2m|q|V}, \qquad \lambda = \frac{h}{\sqrt{2m|q|}}\left(\frac{1}{\sqrt{V}}\right).$$

Thus, if $x = 1/\sqrt{V}$, the relation has the form $\lambda = Sx$, with constant positive slope $S = h/\sqrt{2m|q|}$. The graph is a straight line whose extension passes through the origin. The origin is a limiting extrapolation, since $x = 0$ would require an infinite accelerating potential in this classical model.



Straight-line variation; slope $S = \Delta\lambda/\Delta(1/\sqrt{V})$.

Step 2: Determine the charge from the slope Choose two well-separated points on the line and calculate

$$S = \frac{\Delta\lambda}{\Delta(1/\sqrt{V})}.$$

Squaring the slope relation gives $S^2 = h^2/(2m|q|)$, so the known mass and measured slope determine the charge magnitude. For a fixed mass, a steeper line corresponds to a smaller charge magnitude. The slope alone cannot determine the sign of the charge.

Step 3: Conclusion The λ-versus-$1/\sqrt{V}$ graph is a straight line through the origin, with slope $S = h/\sqrt{2m|q|}$; hence $\boxed{|q| = h^2/(2mS^2)}$.

Quick Tip to Solve: Write the wavelength as a constant times the horizontal-axis variable; square that constant before solving for the charge magnitude.

Section C (36 marks)

13. (a) Draw the equipotential surfaces corresponding to a uniform electric field in the z-direction.

(b) Derive an expression for the electric potential at any point along the axial line of an electric dipole. [3 marks]

Solution:

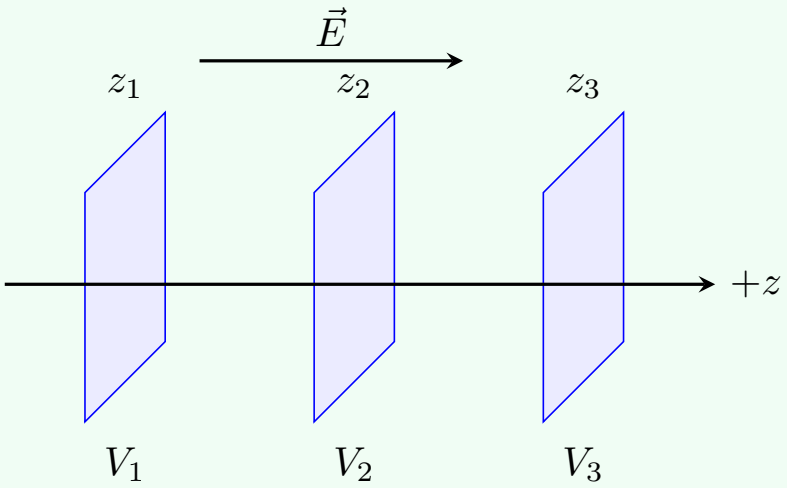
Concept:

- An equipotential surface has the same potential at every point. The electric field is normal to it, since $\vec{E} = -\boldsymbol{\nabla} V$.
- Electric potential is a scalar: the total potential is the algebraic sum $V = \sum_i q_i/(4\pi\varepsilon_0 r_i)$, taking zero potential at infinity.
- A dipole consists of charges $-q$ and $+q$, separated by $2a$; its dipole moment $\vec{p}$ has magnitude $p = 2aq$ and points from $-q$ towards $+q$.

Step 1: (a) Equipotential surfaces in a uniform field Let $\vec{E} = E\hat{z}$, with constant $E > 0$. Then

$$V(z) - V(0) = -\int_0^z E\, dz' = -Ez.$$

Therefore $V =$ constant requires $z =$ constant. These surfaces are parallel planes perpendicular to the z-axis, hence parallel to the xy-plane. The sketch shows three such planes. Potential decreases in the direction of the field, and equal potential differences correspond to equal plane separations, since $|\Delta V| = E|\Delta z|$.

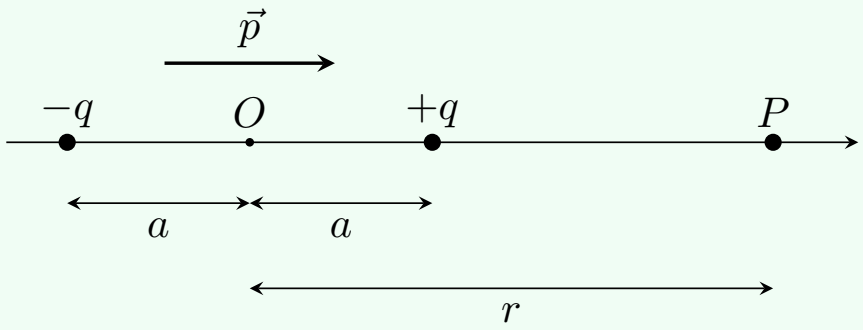


Parallel equipotential planes normal to $\vec{E} = E\hat{z}$, with $V_1 > V_2 > V_3$.

Step 2: (b) Add the potentials on the dipole axis Put the dipole centre at O, the negative charge at axial coordinate $-a$, and the positive charge at $+a$. Consider an external point P at distance $r > a$ from O, on the positive-charge side. Its distances from $+q$ and $-q$ are $r - a$ and $r + a$, respectively. Thus

$$V(P) = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r-a} - \frac{q}{r+a}\right)$$
$$= \frac{1}{4\pi\varepsilon_0}\frac{2aq}{r^2 - a^2} = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^2 - a^2}.$$

The minus sign is essential because the potential due to the negative charge is negative.



Dipole geometry for an external point on the positive axis.

Step 3: State the far-field form and the sign on the other side For $r \gg a$, neglecting a^2 against r^2 gives $V \simeq p/(4\pi\varepsilon_0 r^2)$ on the positive-charge side. At the same distance on the negative-charge side the potential has the opposite sign. More generally, at any signed axial coordinate x, including between the charges, use the positive distances:

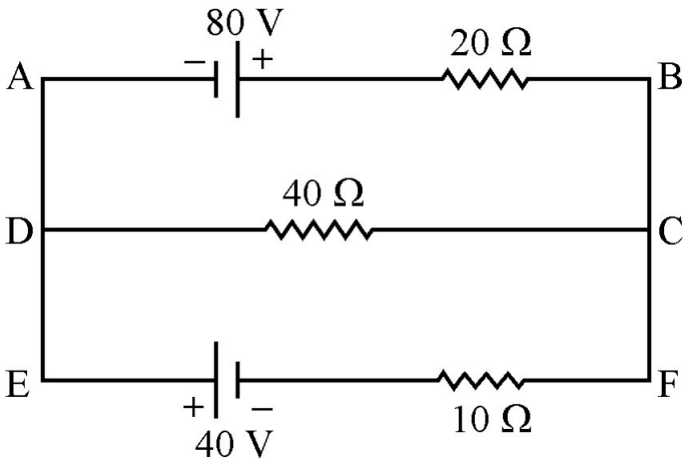
$$V(x) = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{|x-a|} - \frac{1}{|x+a|}\right), \qquad x \neq \pm a.$$

At the centre the two terms cancel; at the ideal point charges the potential is undefined.

Step 4: Conclusion (a) The equipotentials are xy-parallel planes $z =$ constant. (b) On the positive external axis, $\boxed{V = p/[4\pi\varepsilon_0(r^2 - a^2)]}$, reducing to $p/(4\pi\varepsilon_0 r^2)$ for a short dipole; the negative external side has the opposite sign.

Quick Tip to Solve: Use positive distances but signed charges when adding potentials; the dipole potential changes sign when you move to the opposite external axial side.

14. Using Kirchhoff's rules, calculate the current through the $40\,\Omega$ and $20\,\Omega$ resistors in the following circuit: [3 marks]



Solution:

Concept:

- Kirchhoff's junction rule expresses charge conservation: the total current entering a junction equals the total current leaving it.

- Kirchhoff's loop rule expresses energy conservation: the algebraic sum of potential changes around any closed loop is zero.
- Across a resistor, travelling with its assumed current gives a change $-IR$; crossing a cell from its negative to positive terminal gives $+\mathcal{E}$.

Step 1: Assign currents using the two common junctions The wires make A, D, E one common left-hand junction and B, C, F one common right-hand junction. Let I_1 flow through the upper $20\,\Omega$ resistor from A towards B, and let I_2 flow through the middle $40\,\Omega$ resistor from C towards D. The junction rule then makes the lower-branch current $I_1 - I_2$, directed from F towards E.

Step 2: Apply the loop rule to the upper and middle branches Traverse the loop $A \to B \to C \to D \to A$. The upper cell is crossed from negative to positive, and both resistors are traversed along their assumed currents. Therefore

$$80 - 20I_1 - 40I_2 = 0, \qquad I_1 + 2I_2 = 4. \tag{1}$$

Step 3: Apply the loop rule to the lower and middle branches Traverse $F \to E \to D \to C \to F$. The lower resistor is traversed along $I_1 - I_2$, the $40\,\text{V}$ cell from negative to positive, and the middle resistor opposite to I_2. Hence

$$-10(I_1 - I_2) + 40 + 40I_2 = 0, \qquad I_1 - 5I_2 = 4. \tag{2}$$

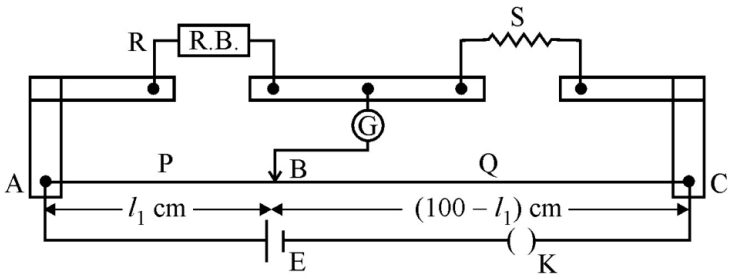
Subtracting (2) from (1) gives $7I_2 = 0$. Substitution in (1) then gives $I_1 = 4\,\text{A}$. As a check, the upper branch gives $V_B - V_A = 80 - (4)(20) = 0$, consistent with no potential difference across the middle resistor. The lower branch independently gives $V_E - V_F = 40 - (4)(10) = 0$.

Step 4: Conclusion The current through $40\,\Omega$ is $\boxed{0\,\text{A}}$. The current through $20\,\Omega$ is $\boxed{4\,\text{A}}$, from A towards B (left to right in the upper branch).

Quick Tip to Solve: Read each cell's polarity before writing loop equations; a zero middle current is valid when the two junction potentials are equal.

OR

14. What is end error in a metre bridge? How is it overcome? The resistances in the two arms of the metre bridge are $R = 5\,\Omega$ and S respectively. When the resistance S is shunted with an equal resistance, the new balance length is found to be $1.5\,l_1$, where l_1 is the initial balancing length. Calculate the value of S. [3 marks]



Solution:

Concept:

- A balanced metre bridge obeys $R/S = l/(100 - l)$, where l is the balance length in centimetres measured from the end next to R.

- Two equal resistances S connected in parallel have equivalent resistance $S/2$; hence shunting the right-hand resistance changes the balance position.

Step 1: Explain end error and its reduction End error is the systematic error caused by the finite resistances of the end connections, copper strips and contacts of a metre bridge. These add to the resistances of the two wire segments, so their resistance ratio is not exactly their measured length ratio. To reduce this error, interchange the resistances in the two gaps, repeat the measurement and take the mean of the two calculated values of the unknown resistance. Choose comparable gap resistances so that the null point is near the middle of the wire, where the relative effect of the end resistances is smaller.

Step 2: Write the two balance conditions For the numerical calculation use the ideal bridge relation. Initially the left gap contains $R = 5\,\Omega$, while the right gap contains S:

$$\frac{5}{S} = \frac{l_1}{100 - l_1}.$$

After shunting S with an equal resistance, the right-gap resistance is $S/2$, and the new balance length from the same end is $l_2 = 1.5l_1$. Thus

$$\frac{10}{S} = \frac{1.5l_1}{100 - 1.5l_1}.$$

Step 3: Eliminate the unknown resistance and substitute Divide the second balance equation by the first. Since the balance point lies inside the wire, $l_1 \neq 0$, so it cancels:

$$\begin{aligned} 2 &= \frac{1.5(100 - l_1)}{100 - 1.5l_1}, \\ 200 - 3l_1 &= 150 - 1.5l_1, \\ l_1 &= \frac{100}{3}\ \text{cm}. \end{aligned}$$

Substitute in the initial relation:

$$S = 5\frac{100 - l_1}{l_1} = 5\frac{200/3}{100/3} = 10\,\Omega.$$

The new length is 50 cm, consistent with equal gap resistances $R = S/2 = 5\,\Omega$.

Step 4: Conclusion The unknown resistance is $\boxed{S = 10\,\Omega}$. End error arises from finite end/contact resistances and is reduced by interchanging the gap resistances and averaging the results, with the balance point kept near the centre.

Quick Tip to Solve: Shunting a gap with an equal resistance halves that gap resistance. Divide the new balance equation by the old one to eliminate the unknown immediately.

15(a). Identify the part of the electromagnetic spectrum used in (i) radar and (ii) eye surgery. Write their frequency range. [3 marks total]

Solution:

Concept:

- The electromagnetic spectrum is classified by frequency ν or wavelength λ, related in vacuum by $c = \nu\lambda$, where c is the speed of light.
- Different spectral regions suit different applications: radar needs directional radio-frequency beams, whereas precision eye surgery uses ultraviolet laser radiation.

Step 1: Identify the radiation used in radar Radar uses **microwaves**. Their short wavelengths allow an antenna to produce a narrow beam, and the reflected signal helps locate an object. The approximate microwave frequency range is

$$10^9 \text{ Hz to } 10^{12} \text{ Hz}.$$

An individual radar system operates in a selected band within this region; it does not transmit across the whole range.

Step 2: Identify the radiation used in eye surgery **Ultraviolet radiation** is used in precision eye surgery, for example in an excimer laser for reshaping the cornea. The approximate ultraviolet frequency range is

$$10^{15} \text{ Hz to } 10^{17} \text{ Hz}.$$

These are conventional approximate spectrum limits; the surgical laser uses a particular wavelength within the ultraviolet region.

Step 3: Conclusion (i) Radar: microwaves, approximately 10^9 to 10^{12} Hz. (ii) Eye surgery: ultraviolet rays, approximately 10^{15} to 10^{17} Hz.

Quick Tip to Solve: Associate radar with a directed microwave beam and corneal surgery with ultraviolet lasers; always give frequency limits in hertz.

15(b). Prove that the average energy density of the oscillating electric field is equal to that of the oscillating magnetic field.

Solution:

Concept:

- The instantaneous electric and magnetic energy densities in vacuum are $u_E = \frac{1}{2}\varepsilon_0 E^2$ and $u_B = B^2/(2\mu_0)$, where ε_0 and μ_0 are the permittivity and permeability of free space.
- For a travelling electromagnetic wave, $E_0 = cB_0$, $c^2 = 1/(\varepsilon_0\mu_0)$, and the average of $\cos^2$ over one complete oscillation is $1/2$.

Step 1: Average the electric-field energy density At a fixed point in a sinusoidal electromagnetic wave, write the electric field as $E = E_0 \cos\omega t$, choosing the time origin conveniently. Here E_0 is its amplitude and ω its angular frequency. Average over one complete period:

$$\langle u_E \rangle = \frac{\varepsilon_0 E_0^2}{2}\langle \cos^2 \omega t \rangle = \frac{\varepsilon_0 E_0^2}{4}.$$

The field itself averages to zero, but its square and hence its stored energy do not.

Step 2: Average and compare the magnetic-field energy density The magnetic field oscillates in phase, so $B = B_0 \cos \omega t$. Its average energy density is

$$\langle u_B \rangle = \frac{B_0^2}{4\mu_0}.$$

Substitute $B_0 = E_0/c$ and then use $1/(\mu_0 c^2) = \varepsilon_0$:

$$\langle u_B \rangle = \frac{E_0^2}{4\mu_0 c^2} = \frac{\varepsilon_0 E_0^2}{4}.$$

Step 3: Conclusion Thus $\boxed{\langle u_E \rangle = \langle u_B \rangle}$. The oscillating electric and magnetic fields contribute equally to the average energy density of a travelling electromagnetic wave in vacuum.

Quick Tip to Solve: Average the squared fields, not the fields themselves: use $\langle \cos^2 \omega t \rangle = 1/2$, then substitute $E_0 = cB_0$.

16. Define the term wavefront. Using Huygens' wave theory, verify the law of reflection. [3 marks]

Solution:

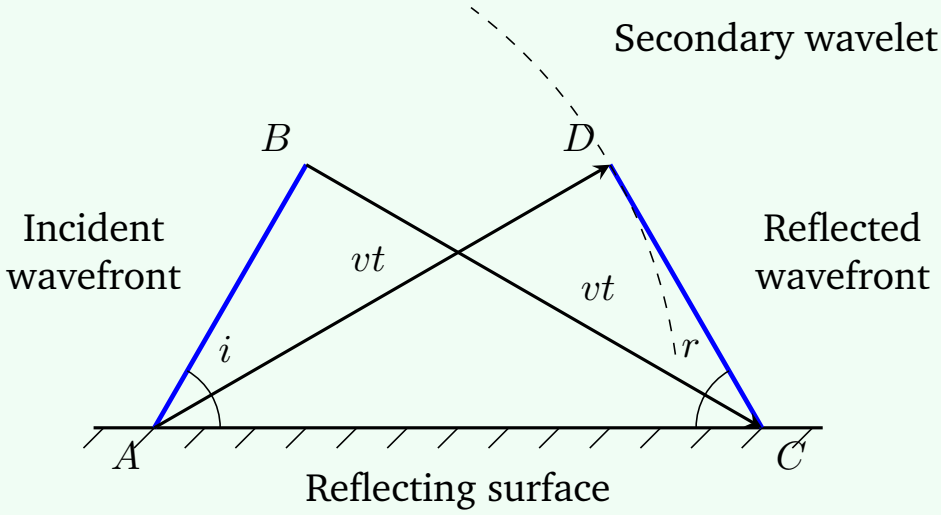
Concept:

- A wavefront is the locus of all points of a wave that oscillate in the same phase at a given instant. Rays in an isotropic medium are normal to the wavefront.
- Huygens' principle: every point on a wavefront acts as a source of secondary wavelets. The forward envelope of these wavelets gives the wavefront at a later instant.

Step 1: Construct the incident and reflected wavefronts Let AC be a plane reflecting surface. The plane incident wavefront AB first reaches the surface at A. While the disturbance at B travels along the incident ray BC to reach the surface at C, the secondary reflected wavelet from A travels a distance AD in the same medium. If the elapsed time is t and the wave speed is v, then

$$BC = vt, \qquad AD = vt.$$

Draw the tangent CD from C to the wavelet centred at A. This tangent is the reflected wavefront: wavelets from the intervening points on the surface have the same common envelope. The radius AD is perpendicular to CD and represents a reflected ray.



Huygens' construction: AB is incident, CD reflected, and $BC = AD = vt$.

Step 2: Compare the right triangles The incident ray BC is perpendicular to the incident wavefront AB, so $\angle ABC = 90^\circ$. The tangent construction gives $\angle ADC = 90^\circ$. In right triangles ABC and CDA, the hypotenuse AC is common and $BC = AD$. Hence the triangles are congruent by the right-angle–hypotenuse–side criterion, giving

$$\angle BAC = \angle DCA.$$

Equivalently, their sines are BC/AC and AD/AC, which are equal.

Step 3: Relate wavefront angles to ray angles The angle $\angle BAC$ between the incident wavefront and the reflecting surface equals the incidence angle i, since the ray and the normal are respectively perpendicular to these two lines. Similarly, $\angle DCA = r$, the reflection angle. Therefore $\sin i = \sin r$; both angles lie between 0° and 90°, so $i = r$. The construction lies in the plane containing the incident ray and the surface normal. The reflected ray also lies in this plane, verifying the coplanarity law.

Step 4: Conclusion A wavefront is a surface of constant phase. Huygens' construction verifies both laws of reflection: $\boxed{i = r}$, and the incident ray, reflected ray and normal at the point of incidence lie in the same plane.

Quick Tip to Solve: In Huygens' reflection construction, mark $BC = AD = vt$ first. The common hypotenuse AC then proves equality of the two wavefront angles and hence $i = r$.

OR

16. Define the term, "refractive index" of a medium. Verify Snell's law of refraction when a plane wavefront is propagating from a denser to a rarer medium. [3 marks]

Solution:

Concept:

- The absolute refractive index of a medium is $n = c/v$, where c is the speed of light in vacuum and v is its phase speed in that medium. It is dimensionless.
- According to Huygens' principle, every point on a wavefront emits secondary wavelets; their forward common tangent gives the new wavefront. Rays are normal to wavefronts.

Step 1: Set up the incident plane wavefront Let medium 1 be denser and medium 2 rarer, so $n_1 > n_2$ and $v_1 < v_2$. Let the plane incident wavefront AB first touch the plane interface at A. In time t, the point B reaches the interface at C, moving along the incident ray. Thus $BC = v_1 t$, and BC is perpendicular to AB. The line AC lies along the interface.

Step 2: Construct the refracted wavefront During the same time, the secondary wavelet from A travels into medium 2 a distance $AD = v_2 t$. Draw the tangent CD from C to this wavelet. Then CD is the refracted plane wavefront, and AD, being perpendicular to CD, is the refracted ray. If i and r are the angles made by the incident and refracted rays with the normal, the corresponding wavefronts make these same angles with the interface.

Step 3: Compare the two right triangles In right triangles ABC and ADC, the common hypotenuse is AC. Therefore,

$$\sin i = \frac{BC}{AC} = \frac{v_1 t}{AC}, \qquad \sin r = \frac{AD}{AC} = \frac{v_2 t}{AC}.$$

Dividing eliminates both t and AC:

$$\frac{\sin i}{\sin r} = \frac{v_1}{v_2} = \frac{c/n_1}{c/n_2} = \frac{n_2}{n_1}.$$

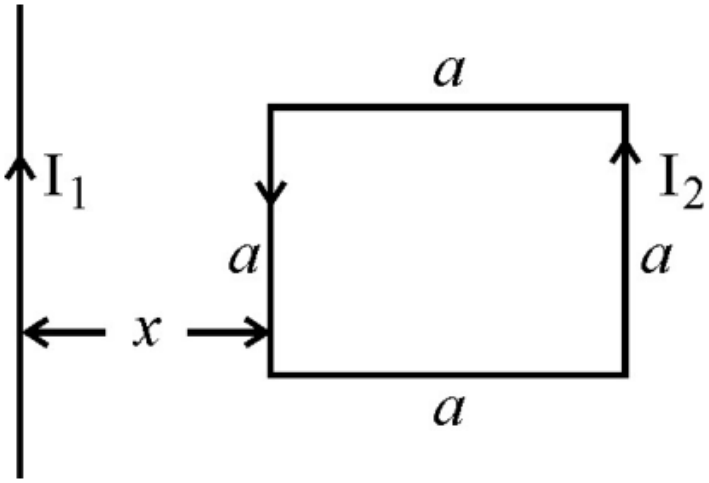
Since $v_2 > v_1$, the refracted ray bends away from the normal, so $r > i$ for oblique incidence. This transmitted-wave construction applies below the critical angle; at the critical angle $r = 90°$, and above it total internal reflection occurs.

Step 4: Conclusion Refractive index is $n = c/v$. Huygens' construction gives $n_1 \sin i = n_2 \sin r$, verifying Snell's law for transmission from a denser to a rarer medium.

Quick Tip to Solve: Use the same travel time in both media and divide the two sine ratios; for denser-to-rarer refraction, check that the ray bends away from the normal.

17. (a) Define mutual inductance and write its S.I. unit.

(b) A square loop of side 'a' carrying a current I_2 is kept at distance x from an infinitely long straight wire carrying a current I_1 as shown in the figure. Obtain the expression for the resultant force acting on the loop. [3 marks]



Solution:

Concept:

- Mutual inductance M relates the flux linkage of one coil to the current in the other: $N_2\Phi_{21} = MI_1$. For fixed geometry in a linear medium, $\varepsilon_2 = -M\, dI_1/dt$.
- A long straight wire produces $B(r) = \mu_0 I_1/(2\pi r)$. The force on a current element is $d\vec{F} = I_2\, d\vec{l} \times \vec{B}$.

Step 1: Define mutual inductance and its unit (a) Mutual inductance is the magnetic flux linkage with one coil per unit current in the other coil. Equivalently, it is the magnitude of induced emf in one coil when current in the other changes at unit rate. Its S.I. unit is the henry (H):

$$1\,\text{H} = 1\,\text{Wb A}^{-1} = 1\,\text{V s A}^{-1}.$$

Thus a mutual inductance of one henry produces an emf of one volt for a current-change rate of one ampere per second.

Step 2: Find the forces on the vertical sides (b) For upward current I_1, the magnetic field on the right of the wire points into the page. The nearer side of the loop, at distance x, carries current downward, opposite to I_1; it is therefore repelled to the right. The farther side, at distance $x + a$, carries current upward, parallel to I_1, and is attracted to the left. Each side has length a, so their force magnitudes are

$$F_{\text{near}} = \frac{\mu_0 I_1 I_2 a}{2\pi x}, \qquad F_{\text{far}} = \frac{\mu_0 I_1 I_2 a}{2\pi(x+a)}.$$

The magnetic field is stronger at the nearer side, making its repulsion larger than the farther side's attraction.

Step 3: Cancel the horizontal-side forces and combine At every distance r from the wire, matching elements on the upper and lower horizontal sides carry opposite currents in the same magnetic field. Their forces are therefore equal and opposite and cancel pairwise, even though the field varies along these sides. Taking rightward as positive, the remaining force is

$$\begin{aligned} F_x &= F_{\text{near}} - F_{\text{far}} \\ &= \frac{\mu_0 I_1 I_2 a}{2\pi}\left(\frac{1}{x} - \frac{1}{x+a}\right) \\ &= \frac{\mu_0 I_1 I_2 a^2}{2\pi x(x+a)}. \end{aligned}$$

Step 4: Conclusion Mutual inductance is flux linkage per unit current, with S.I. unit henry. The loop experiences a resultant force $F = \dfrac{\mu_0 I_1 I_2 a^2}{2\pi x(x+a)}$, directed away from the straight wire for the printed current directions.

Quick Tip to Solve: Check the nearer side's current first: opposite currents repel, and its stronger field determines the direction of the resultant force.

18. (a) Derive the expression for the torque acting on a current carrying loop placed in a magnetic field.

(b) Explain the significance of a radial magnetic field when a current carrying coil is kept in it.

[3 marks]

Solution:

Concept:

- A straight conductor of length l carrying current I in a magnetic field experiences $\vec{F} = I\vec{l} \times \vec{B}$. Two equal, opposite, non-collinear forces form a couple.
- The magnetic moment of an N-turn plane coil is $\vec{m} = NI\vec{A}$, where the area vector $\vec{A}$ is normal to the coil in the right-hand-rule direction.

Step 1: Identify the couple on a rectangular loop (a) Consider a rectangular coil of length l, breadth b, area $A = lb$, and N turns in a uniform magnetic field $\vec{B}$. Let its normal make an angle θ with $\vec{B}$. Choose the pair of sides of length l perpendicular to the field; they carry opposite currents and experience equal, opposite forces of magnitude $F = NIlB$. Their net force is zero, but their lines of action are separated, so they rotate the coil. The forces on the other two sides are collinear and make no contribution to this torque.

Step 2: Multiply force by the perpendicular separation The separation between the force lines is the projection $b\sin\theta$, not the full breadth unless $\theta = 90^\circ$. Hence the magnitude of the couple is

$$\tau = F(b\sin\theta) = NIlB\, b\sin\theta = NIAB\sin\theta.$$

Using $m = NIA$, the result can also be written as $\tau = mB\sin\theta$, or in vector form

$$\vec{\tau} = \vec{m} \times \vec{B}.$$

Its direction tends to align the magnetic moment with the field. For a single-turn loop, set $N = 1$. Torque vanishes when the normal is parallel to the field and is maximum when the plane of the coil is parallel to the field.

Step 3: Explain the purpose of a radial field (b) In a moving-coil galvanometer, a radial magnetic field keeps the field parallel to the plane of the rotating coil throughout its useful deflection. Thus the angle between the field and the coil's normal stays 90°. The torque is always $\tau = NIAB$, its maximum value for a given current, and contains no angle-dependent sine factor. If the suspension provides restoring torque $k\phi$, where k is the torsional constant and ϕ the angular deflection, equilibrium gives

$$NIAB = k\phi \quad \Rightarrow \quad \phi = \frac{NAB}{k}I.$$

Therefore deflection is directly proportional to current, giving a uniform scale and simplifying current measurement.

Step 4: Conclusion A current loop experiences $\vec{\tau} = \vec{m} \times \vec{B}$, with magnitude $NIAB\sin\theta$. A radial field maintains maximum torque $NIAB$ at every deflection and makes galvanometer deflection proportional to current.

Quick Tip to Solve: Measure θ from the coil's normal. In a radial field the plane is parallel to the field, so the normal is perpendicular and $\sin\theta = 1$.

19. Draw a labelled ray diagram of an astronomical telescope in the near point adjustment position.

A giant refracting telescope at an observatory has an objective lens of focal length $15\,\text{m}$ and an eyepiece of focal length $1.0\,\text{cm}$. If this telescope is used to view the Moon, find the diameter of the image of the Moon formed by the objective lens. The diameter of the Moon is $3.48 \times 10^6\,\text{m}$, and the radius of lunar orbit is $3.8 \times 10^8\,\text{m}$. [3 marks]

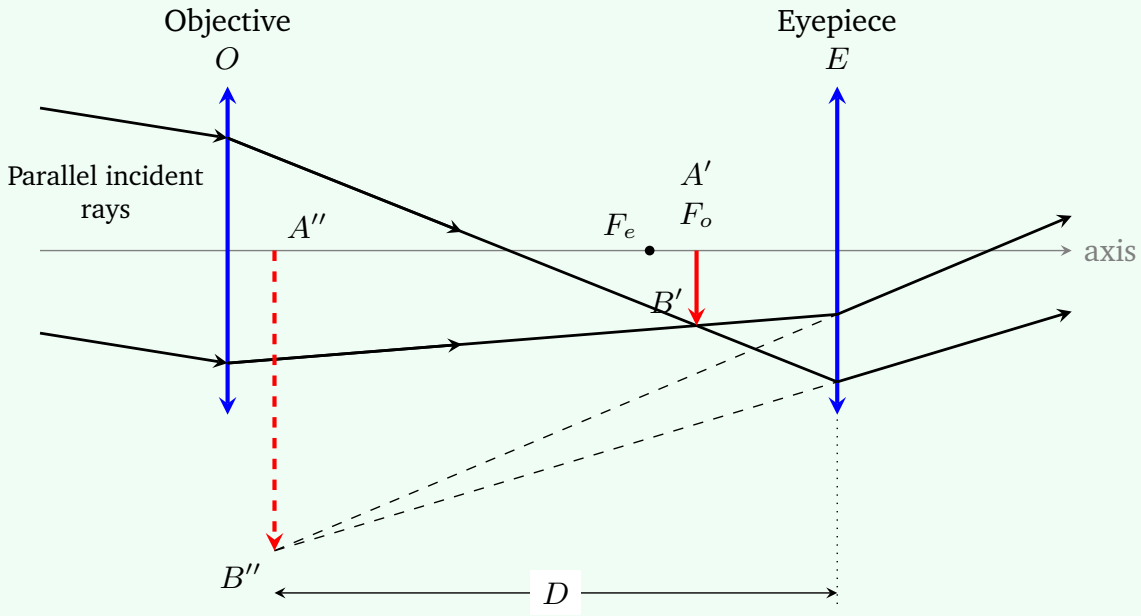
Solution:

Concept:

- An astronomical telescope uses a long-focal-length objective to form a real inverted intermediate image, which the short-focal-length eyepiece magnifies.
- For near-point adjustment, the intermediate image lies between the eyepiece and its first focus. The final image is virtual, inverted relative to the distant object, and at the least distance of distinct vision D from the eyepiece.
- A distant object of diameter d at distance R subtends a small angle $\theta \simeq d/R$. Its image diameter in the objective focal plane is $d' = f_o\theta$.

Step 1: Draw the near-point arrangement Parallel rays from one point of the distant object converge at B', forming the real image $A'B'$ near the objective focal plane F_o. Place the eyepiece so that $A'B'$ lies inside its focal length, between F_e and the eyepiece. The emergent rays diverge; their backward extensions meet at B''. Thus $A''B''$ is the enlarged virtual image at the near point. Solid arrows show actual light travelling left to right, while dashed lines are backward extensions. The sketch is schematic, not to scale. With the Cartesian convention, $v_e = -D$, so the eyepiece lens equation gives

$$\frac{1}{f_e} = \frac{1}{-D} - \frac{1}{u_e}, \qquad |u_e| = \frac{f_e D}{f_e + D} < f_e.$$



Near-point telescope adjustment: $A'B'$ is real and $A''B''$ is virtual; dashed rays are backward extensions.

Step 2: Calculate the Moon's angular diameter Here $R = 3.8 \times 10^8\,\text{m}$, $d = 3.48 \times 10^6\,\text{m}$, and $f_o = 15\,\text{m}$. Since R is enormously greater than the focal length, the objective image is essentially in its focal

plane. The small-angle approximation gives

$$\theta \simeq \frac{d}{R} = \frac{3.48 \times 10^6}{3.8 \times 10^8} = 9.16 \times 10^{-3}\,\text{rad}.$$

Step 3: Obtain the objective image diameter Apply the image-size relation in the objective focal plane:

$$d' = f_o\theta = 15 \times 9.16 \times 10^{-3}\,\text{m} = 0.1374\,\text{m}.$$

The given eyepiece focal length 1.0 cm affects the angular magnification and near-point setting, but it does not enter the size of the image formed by the objective alone.

Step 4: Conclusion The labelled diagram shows the near-point adjustment, with the final virtual image at D. The Moon's image formed by the objective has diameter $d' \simeq 0.137\,\text{m} = 13.7\,\text{cm}$ (about 14 cm to two significant figures).

Quick Tip to Solve: For the image made by the objective alone, use $d' = f_o d/R$; do not multiply by the telescope's angular magnification.

20(a). State Gauss's law for magnetism. Explain its significance. [3 marks total]

Solution:

Concept:

- Magnetic flux through a surface is $\Phi_B = \int \vec{B} \cdot d\vec{S}$, where $\vec{B}$ is the magnetic field and $d\vec{S}$ is the directed area element.
- For a closed surface, the area vector points outward. Flux leaving the surface is positive, whereas flux entering it is negative.

Step 1: State Gauss's law for magnetism The net magnetic flux through any closed surface is zero:

$$\boxed{\oint_S \vec{B} \cdot d\vec{S} = 0.}$$

Here S is any closed surface, irrespective of its shape, size or position. The law concerns the algebraic sum of the flux through the entire surface; it does not assert that the magnetic field is zero at its points.

Step 2: Explain the physical significance Magnetic field lines form continuous closed loops, so the total magnetic flux entering a closed surface equals the total flux leaving it. There is consequently no net magnetic source or sink enclosed by the surface. This expresses the absence of isolated magnetic poles in classical magnetism: north and south poles always occur together. Even when a bar magnet is cut into pieces, each piece becomes a magnet with both poles; cutting does not isolate a north or a south pole.

Step 3: Conclusion Gauss's law for magnetism is $\oint_S \vec{B} \cdot d\vec{S} = 0$. It signifies that magnetic field lines have no isolated starting or ending poles and that magnetic monopoles do not occur in the classical description.

Quick Tip to Solve: Remember that zero net flux means equal incoming and outgoing flux, not zero magnetic field everywhere on the closed surface.

20(b). Write the four important properties of the magnetic field lines due to a bar magnet.

Solution:

Concept:

- Magnetic field lines represent the direction and relative strength of $\vec{B}$, the magnetic field. The tangent at a point gives the field direction there.
- A bar magnet has north and south poles, but its magnetic field lines are continuous loops rather than lines with isolated endpoints.

Step 1: Property 1: continuous closed curves The field lines of a bar magnet form continuous closed loops. Outside the magnet they run from its north pole to its south pole; inside the magnet they continue from the south pole to the north pole. Thus they have no beginning or end at either pole.

Step 2: Property 2: tangent gives field direction The tangent drawn to a magnetic field line at any point gives the direction of the magnetic field at that point. It is the direction in which the north pole of a small freely suspended compass needle points when placed there.

Step 3: Property 3: closeness indicates strength The relative closeness of field lines indicates the strength of the magnetic field. Lines are crowded where the field is strong and farther apart where it is weak. For a bar magnet, the lines are especially crowded near its poles, indicating the stronger field there.

Step 4: Property 4: lines never intersect No two magnetic field lines intersect. If two lines crossed, their tangents would assign two different directions to the magnetic field at the same point. Since the field at a point has a unique direction, such an intersection is impossible.

Step 5: Conclusion The four properties are: closed loops with definite direction, tangents indicating the field direction, line density indicating relative field strength, and absence of intersections.

Quick Tip to Solve: Organise the four properties as closure, tangent, density and non-intersection; remember that the direction reverses inside the magnet.

OR

20. Write three points of differences between para-, dia- and ferro-magnetic materials, giving one example for each. [3 marks]

Solution:

Concept:

- Magnetic susceptibility χ_m describes the induced magnetisation: $M = \chi_m H$, where M is magnetic moment per unit volume and H is the magnetising field.
- For a linear isotropic material, relative permeability is $\mu_r = 1 + \chi_m$. Its value indicates whether the material weakens or strengthens the applied magnetic field inside it.

Step 1: Compare the response to an external field **Diamagnetic:** The induced magnetisation is weak and opposite to the applied field. The material is weakly repelled by a magnet and moves from

stronger to weaker regions of a non-uniform field.

Paramagnetic: Magnetisation is weak and along the applied field. The material is weakly attracted and moves from weaker to stronger regions of a non-uniform field.

Ferromagnetic: Magnetisation can be very large and along the applied field because magnetic domains align. The material is strongly attracted towards the stronger field region.

Step 2: Compare magnetic susceptibility **Diamagnetic:** χ_m is small and negative, so the induced moment opposes the field.

Paramagnetic: χ_m is small and positive, so the induced moment assists the field weakly.

Ferromagnetic: The effective susceptibility is large and positive in the usual magnetising region. It depends strongly on the applied field and magnetic history; ferromagnets are not described by one constant susceptibility over the entire magnetisation curve.

Step 3: Compare relative permeability and give examples **Diamagnetic:** μ_r is slightly less than unity. Example: bismuth.

Paramagnetic: μ_r is slightly greater than unity. Example: aluminium.

Ferromagnetic: The relative permeability can be much greater than unity in the ordinary unsaturated magnetising range. Example: iron.

Thus field response, susceptibility and relative permeability provide three corresponding points of comparison for all three classes. The qualifications for iron matter because its response becomes nonlinear and approaches saturation at sufficiently large applied fields.

Step 4: Conclusion Diamagnets such as bismuth are weakly repelled, with $\chi_m < 0$ and $\mu_r < 1$; paramagnets such as aluminium are weakly attracted, with small positive χ_m and $\mu_r > 1$; ferromagnets such as iron are strongly attracted and can have very large positive susceptibility and permeability.

Quick Tip to Solve: Use the same three headings for every class: field response, susceptibility and relative permeability; remember bismuth, aluminium and iron as examples.

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21. Define the term 'decay constant' of a radioactive sample. The rate of disintegration of a given radioactive nucleus is 10000 disintegrations/s and 5000 disintegrations/s after 20 hr and 30 hr respectively from start. Calculate the half life and initial number of nuclei at $t = 0$.

[3 marks]

Solution:

Concept:

- Radioactive decay obeys $-dN/dt = \lambda N$, where N is the number of undecayed nuclei and λ is the decay constant.
- Activity is $A = \lambda N = A_0 e^{-\lambda t}$. Both activity and number of nuclei halve in the half-life $T_{1/2} = \ln 2/\lambda$.

Step 1: Define the decay constant The decay constant is the probability of decay per nucleus per unit time, or equivalently the fractional rate of disintegration of a radioactive sample:

$$\lambda = -\frac{1}{N}\frac{dN}{dt}.$$

Its SI unit is s^{-1}. It is characteristic of the radionuclide and does not depend on the amount of the sample.

Step 2: Find the half-life from the two activities The activity falls from $10000\,\mathrm{s}^{-1}$ at $t_1 = 20\,\mathrm{h}$ to

$5000\,\mathrm{s}^{-1}$ at $t_2 = 30\,\mathrm{h}$. Thus

$$\frac{A_2}{A_1} = e^{-\lambda(t_2 - t_1)} = \frac{1}{2}.$$

A fall to half the original activity takes one half-life, so $T_{1/2} = 30 - 20 = 10\,\mathrm{h} = 36000\,\mathrm{s}$. Consequently,

$$\lambda = \frac{0.6931}{36000} = 1.925 \times 10^{-5}\,\mathrm{s}^{-1}.$$

Step 3: Work backwards to the initial sample At $20\,\mathrm{h}$, two half-lives have elapsed since the start. Hence $A(20\,\mathrm{h}) = A_0/4$, giving $A_0 = 40000\,\mathrm{s}^{-1}$. Using activity in disintegrations per second requires the decay constant in inverse seconds:

$$N_0 = \frac{A_0}{\lambda} = \frac{40000 \times 36000}{0.6931} = 2.08 \times 10^9.$$

As a check, after three half-lives the activity is $40000/8 = 5000\,\mathrm{s}^{-1}$, matching the observation at $30\,\mathrm{h}$.

Step 4: Conclusion The decay constant is the fractional decay rate, $\lambda = -(1/N)(dN/dt)$. The half-life is $\boxed{10\,\mathrm{h}}$, and the initial sample contains $\boxed{N_0 \simeq 2.08 \times 10^9 \text{ nuclei}}$.

Quick Tip to Solve: Use the time difference between the two activities to find the half-life; convert hours to seconds before dividing activity by the decay constant.

22. (a) Three photo diodes D_1, D_2 and D_3 are made of semiconductors having band gaps of 2.5 eV, 2 eV and 3 eV respectively. Which of them will not be able to detect light of wavelength 600 nm?
(b) Why photodiodes are required to operate in reverse bias? Explain. [3 marks]

Solution:

Concept:

- To produce an electron-hole pair by band-to-band absorption, a photon must supply energy $E_\gamma = hc/\lambda \geq E_g$, where E_g is the semiconductor band gap.
- In a reverse-biased photodiode, the junction electric field separates photogenerated carriers. The resulting reverse photocurrent increases with incident light intensity.

Step 1: Calculate the incident photon energy For the incident wavelength $\lambda = 600\,\mathrm{nm} = 600 \times 10^{-9}\,\mathrm{m}$,

$$\begin{aligned} E_\gamma &= \frac{(6.626 \times 10^{-34})(3.00 \times 10^8)}{600 \times 10^{-9}} \\ &= 3.313 \times 10^{-19}\,\mathrm{J} \\ &= \frac{3.313 \times 10^{-19}}{1.602 \times 10^{-19}}\,\mathrm{eV} \simeq 2.07\,\mathrm{eV}. \end{aligned}$$

Compare this energy with each band gap; wavelength alone does not decide detection without the material's threshold energy.

Step 2: Compare all three photodiodes For D_1, $E_g = 2.5\,\mathrm{eV} > 2.07\,\mathrm{eV}$, so the photons cannot excite electrons across the gap. For D_2, $E_g = 2.0\,\mathrm{eV} < 2.07\,\mathrm{eV}$, so electron-hole pairs can be generated and the light can be detected. For D_3, $E_g = 3.0\,\mathrm{eV} > 2.07\,\mathrm{eV}$, again the photon energy is insufficient. This comparison uses the band-gap detection model of the question.

Step 3: Explain reverse-bias operation Reverse bias widens the depletion region and establishes a

strong electric field across it. Electron-hole pairs generated in or near this region are rapidly separated: electrons move towards the n-side and holes towards the p-side, producing an additional reverse current in the external circuit. The reverse dark current is small, so even a small light-induced change is readily measured and follows the light intensity. Under forward bias a large majority-carrier current would mask this relatively small photocurrent. Reverse bias therefore gives sensitive and rapid light detection.

Step 4: Conclusion $\boxed{D_1 \text{ and } D_3}$ cannot detect $600\,\text{nm}$ light because their band gaps exceed the photon energy $2.07\,\text{eV}$; D_2 can. Reverse bias separates photocarriers efficiently and makes their current measurable against a small dark current.

Quick Tip to Solve: Use $E_\gamma(\text{eV}) \simeq 1240/\lambda(\text{nm})$; detection requires the band gap to be no greater than the photon energy.

---

23. (a) Describe briefly the functions of the three segments of n-p-n transistor.

(b) Draw the circuit arrangement for studying the output characteristics of n-p-n transistor in CE configuration. Explain how the output characteristics is obtained. [3 marks]

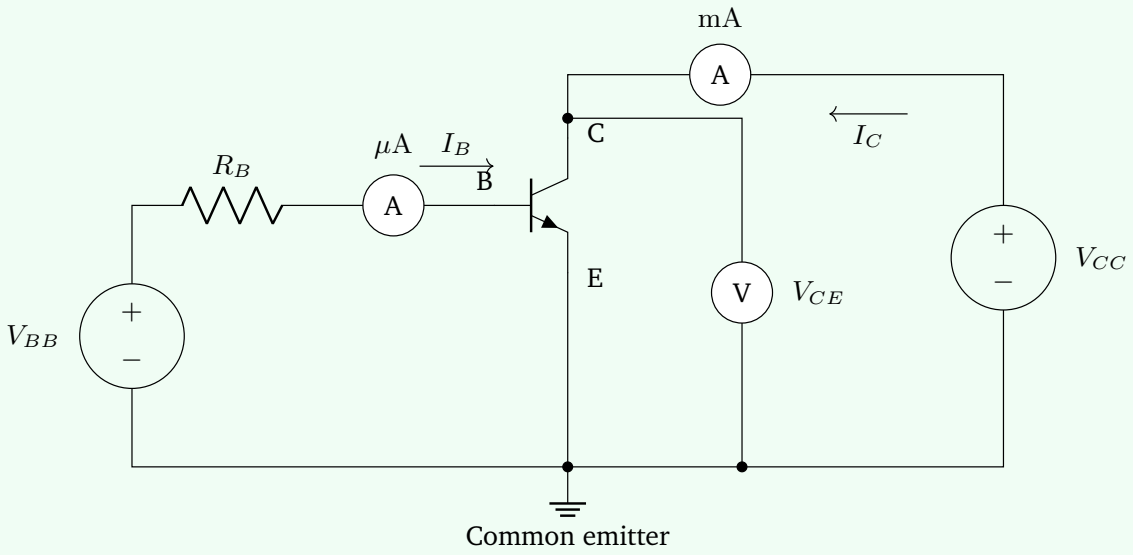
Solution:

Concept:

- An n-p-n transistor has a heavily doped emitter, a thin lightly doped base, and a moderately doped collector with a comparatively large area.
- In common-emitter (CE) operation the emitter is common to input and output. Output characteristics plot collector current I_C against collector-emitter voltage V_{CE} for fixed base current I_B.

Step 1: State the functions of the three segments The n-type emitter supplies a large number of majority carriers, electrons, by injection into the base when the emitter-base junction is forward biased. Heavy doping ensures efficient injection. The p-type base controls the flow of electrons from emitter to collector; its small thickness and light doping allow only a small fraction to recombine, producing the small base current. Most electrons reach the collector. The n-type collector collects these electrons through its reverse-biased junction with the base; its larger area also helps it dissipate the heat generated during operation.

Step 2: Arrange the common-emitter circuit Connect the emitter to the common negative terminal of both supplies. The adjustable input supply V_{BB}, base resistor R_B, and microammeter set and measure I_B. The positive output supply V_{CC} feeds the collector through a milliammeter to measure I_C. A voltmeter connected directly across collector and emitter measures V_{CE}. In the active region the base-emitter junction is forward biased and the collector-base junction is reverse biased. The emitter arrow points outwards for the n-p-n transistor.



Common-emitter circuit for n-p-n output characteristics; both DC supplies are adjustable.

Step 3: Obtain the output characteristic curves Adjust V_{BB} to establish a chosen base current and hold I_B constant. Increase V_{CC} in steps, noting the corresponding V_{CE} and I_C readings each time; readjust the input supply as needed to keep the measured I_B unchanged. Plot I_C on the vertical axis against V_{CE} on the horizontal axis. Repeat for several higher fixed base currents to obtain a family of curves. At very small V_{CE}, I_C rises rapidly in the saturation region; beyond the knee it is nearly independent of V_{CE} in the active region. Increasing I_B produces a higher collector-current curve. Keep the applied voltages below breakdown.

Step 4: Conclusion The emitter injects electrons, the thin base controls their flow, and the collector collects them. CE output characteristics are the family of I_C-versus-V_{CE} curves, each measured at a different constant I_B, using the circuit shown.

Quick Tip to Solve: For output characteristics, vary the collector supply while keeping the base-current meter reading fixed; both supplies share the emitter terminal.

OR

23. Draw the circuit diagram of a full wave rectifier and explain its working. Also, give the input and output waveforms. [3 marks]

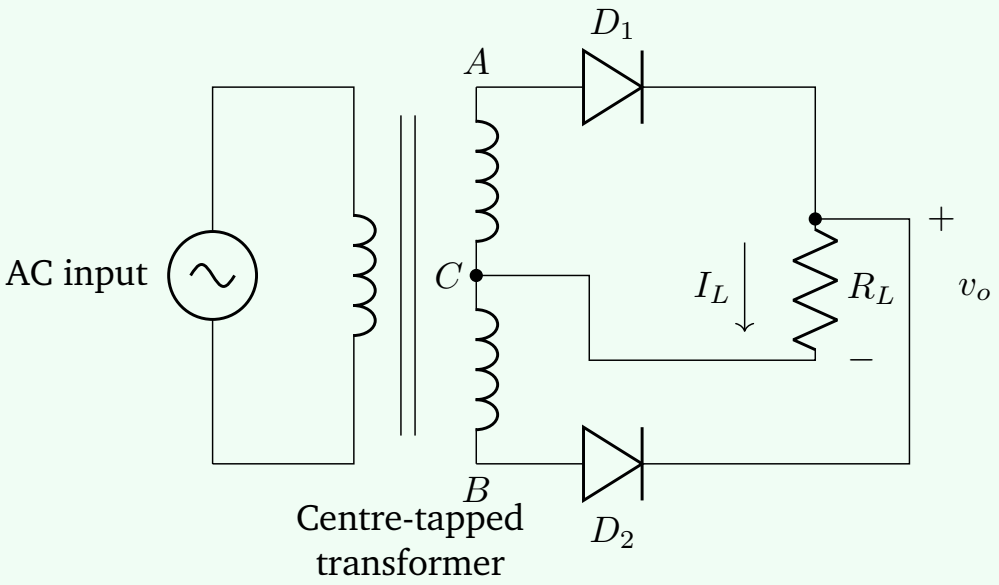
Solution:

Concept:

- A p-n junction diode conducts when forward biased and blocks current when reverse biased. An ideal diode is treated as a closed switch in forward bias and an open switch in reverse bias.
- A centre-tapped transformer supplies equal secondary voltages of opposite phase relative to the centre tap C. Two diodes therefore conduct on alternate half-cycles, sending current through the load in the same direction.

Step 1: Connect the full-wave rectifier Connect the secondary ends A and B to the anodes of D_1 and D_2, respectively. Join their cathodes to the upper end of the load resistance R_L, and connect its lower end to the centre tap C. Apply the alternating input to the primary winding. The output is taken

across R_L, with its upper terminal positive.



Centre-tapped full-wave rectifier; both cathodes join the positive output terminal.

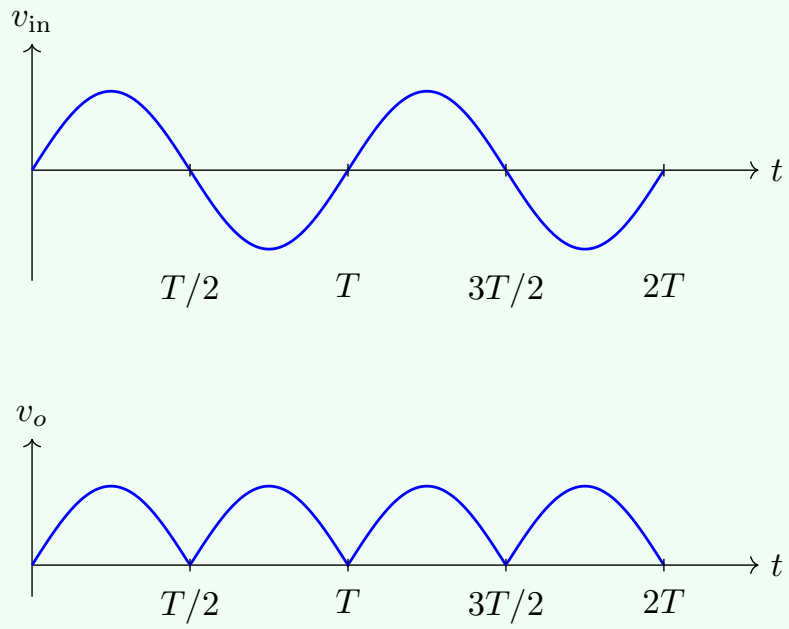
Step 2: Follow the current during both half-cycles When A is positive relative to C, B is negative. Diode D_1 is forward biased and D_2 is reverse biased. The current path is $A \to D_1 \to R_L \to C$, returning through the upper half of the secondary.

During the next half-cycle, B becomes positive relative to C. Diode D_2 conducts and D_1 blocks. Current follows $B \to D_2 \to R_L \to C$, returning through the lower half of the secondary. In both cases the load current flows downward from the joined cathodes towards C.

Step 3: Sketch and interpret the waveforms The input is sinusoidal and reverses polarity every half-period. Taking $v_{AC} = V_m \sin \omega t$ for one half of the secondary, the ideal output is

$$v_o = V_m |\sin \omega t|.$$

Thus the negative input half-cycle also produces a positive output pulse. The output is unidirectional but pulsating; without a smoothing filter it is not a constant voltage. Two output pulses occur in one input period T, so the ripple frequency is $2f$, where $f = 1/T$ is the input frequency.



Sinusoidal input and full-wave output, aligned on the same time scale; diode drops neglected.

Step 4: Conclusion The two diodes conduct alternately, rectifying both halves of the alternating input. The load receives a pulsating dc output of unchanged polarity, with ripple frequency twice the input frequency.

Quick Tip to Solve: Trace each current path back to the centre tap: both paths must cross the load in the same direction, and each input cycle must give two positive output pulses.

24. (a) If A and B represent the maximum and minimum amplitudes of an amplitude modulated wave, write the expression for the modulation index in terms of A & B.

(b) A message signal of frequency $20\,\text{kHz}$ and peak voltage $10\,\text{V}$ is used to modulate a carrier of frequency $2\,\text{MHz}$ and peak voltage of $15\,\text{V}$. Calculate the modulation index. Why the modulation index is generally kept less than one? [3 marks]

Solution:

Concept:

- In amplitude modulation, the carrier amplitude varies according to the instantaneous message signal. For a sinusoidal message, the modulation index is $\mu = A_m/A_c$, where A_m is the message peak amplitude and A_c is the unmodulated carrier peak amplitude.
- The envelope reaches $A = A_c + A_m$ at its maximum and $B = A_c - A_m$ at its minimum, provided $0 \le \mu \le 1$. An undistorted envelope must follow the message without crossing zero and folding over.

Step 1: (a) Express the index using the envelope amplitudes Adding and subtracting the maximum and minimum amplitudes isolates the carrier and message amplitudes:

$$A + B = (A_c + A_m) + (A_c - A_m) = 2A_c,$$
$$A - B = (A_c + A_m) - (A_c - A_m) = 2A_m.$$

Hence the ratio of the difference to the sum measures the fractional change in carrier amplitude:

$$\mu = \frac{A_m}{A_c} = \frac{A - B}{A + B}.$$

The numerator and denominator have identical amplitude units, so the index has no unit.

Step 2: (b) Substitute the peak voltages The message peak voltage is $A_m = 10\,\text{V}$ and the carrier peak voltage is $A_c = 15\,\text{V}$. Therefore,

$$\mu = \frac{10}{15} = \frac{2}{3} \approx 0.667.$$

As a percentage this is approximately 66.7%. The given frequencies $20\,\text{kHz}$ and $2\,\text{MHz}$ describe the rates of oscillation; their ratio is not the modulation index. As a check, the resulting envelope extremes are $A = 25\,\text{V}$ and $B = 5\,\text{V}$, whose difference-to-sum ratio is again $20/30 = 2/3$.

Step 3: Explain why the index is generally below unity When $\mu > 1$, the message amplitude exceeds the carrier amplitude. The envelope factor $1 + \mu \cos \omega_m t$ then changes sign during part of a message cycle. This produces overmodulation: the observed envelope folds over and no longer reproduces the message faithfully. An ordinary envelope detector consequently gives a distorted recovered signal. Keeping $\mu < 1$ prevents this and provides a margin against variations in the message amplitude. The limiting value $\mu = 1$ is ideal 100% modulation, at which the minimum envelope just reaches zero.

Step 4: Conclusion $\mu = \dfrac{A - B}{A + B}$, and for the given voltages $\mu = 2/3 \approx 0.667$, or 66.7%. It is generally kept below one to avoid overmodulation and distortion during envelope detection.

Quick Tip to Solve: Divide the message peak voltage by the carrier peak voltage, not their frequencies; for an envelope diagram use maximum minus minimum over maximum plus minimum.

Section D (15 marks)

25. (a) In a series LCR circuit connected across an ac source of variable frequency, obtain the expression for its impedance and draw a plot showing its variation with frequency of the source.

(b) What is the phase difference between the voltages across inductor and the capacitor at resonance in the LCR circuit?

(c) When an inductor is connected to a $200\,\text{V}$ dc voltage, a current of $1\,\text{A}$ flows through it. When the same inductor is connected to a $200\,\text{V}$, $50\,\text{Hz}$ ac source, only $0.5\,\text{A}$ current flows. Explain, why? Also, calculate the self inductance of the inductor. [5 marks]

Solution:

Concept:

- In a series LCR circuit, the same current flows through resistance R, inductance L and capacitance C. At angular frequency $\omega = 2\pi f$, the reactances are $X_L = \omega L$ and $X_C = 1/(\omega C)$.
- Relative to current, the resistor voltage is in phase, the inductor voltage leads by 90°, and the capacitor voltage lags by 90°. Applied voltage is their phasor sum.
- A real coil has both resistance and inductance. Its steady dc current is limited by resistance alone; its ac current is limited by impedance $Z = \sqrt{R^2 + (\omega L)^2}$. All ac voltage and current values used here are rms values.

Step 1: (a) Add the voltage phasors to obtain impedance Choose the current phasor as the reference. The rms voltage drops are $V_R = IR$, $V_L = IX_L$ and $V_C = IX_C$. The inductor and capacitor voltage

phasors point in opposite directions, so their resultant is perpendicular to V_R. By the right-triangle relation,

$$\begin{aligned} V^2 &= V_R^2 + (V_L - V_C)^2 \\ &= I^2\left[R^2 + (X_L - X_C)^2\right]. \end{aligned}$$

Defining the series impedance as $Z = V/I$ gives

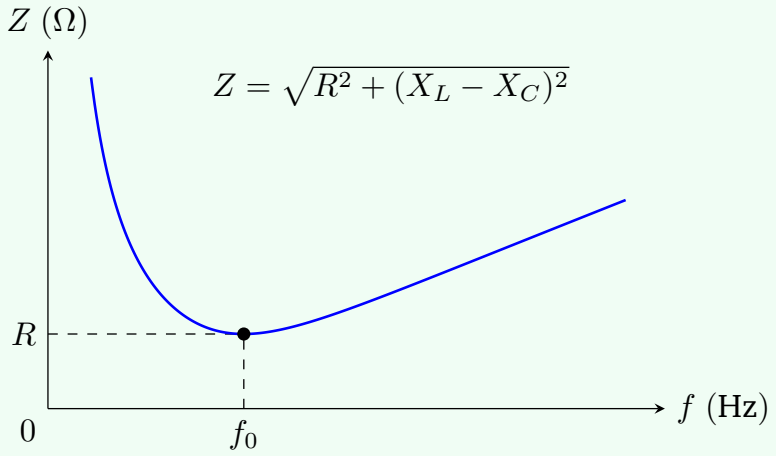
$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} = \sqrt{R^2 + \left(2\pi f L - \frac{1}{2\pi f C}\right)^2}.$$

Thus, reactances combine by their difference, whereas the resistance combines with that difference in quadrature.

Step 2: (a) Locate the minimum and sketch the frequency dependence The squared reactance term is nonnegative, so the minimum impedance occurs when $X_L = X_C$. Therefore,

$$\omega_0 L = \frac{1}{\omega_0 C}, \qquad f_0 = \frac{1}{2\pi\sqrt{LC}}, \qquad Z_{\min} = R.$$

As f approaches zero, the capacitive reactance becomes very large and Z tends to infinity. Increasing frequency reduces Z until resonance. Beyond f_0, inductive reactance dominates and impedance increases again, approaching $2\pi f L$ at high frequency. The sketch has a smooth minimum at (f_0, R); it does not touch zero for a circuit with nonzero resistance.



Qualitative variation of series LCR impedance with frequency; the minimum is R at resonance.

Step 3: (b) Compare the inductor and capacitor voltages at resonance The inductor voltage leads current by $\pi/2$, while the capacitor voltage lags current by $\pi/2$. Their phase separation is therefore

$$\Delta\phi = \frac{\pi}{2} - \left(-\frac{\pi}{2}\right) = \pi \text{ rad} = 180^\circ.$$

At resonance their magnitudes are equal, $V_L = V_C$, and the opposing voltage phasors cancel. This cancellation makes the source voltage and current in phase; it does not make the two reactive voltages mutually in phase.

Step 4: (c) Determine the coil resistance and self inductance For steady dc, $di/dt = 0$, so the self-induced emf vanishes after the switching transient. The wire resistance of the coil is consequently

$$R = \frac{V_{\text{dc}}}{I_{\text{dc}}} = \frac{200}{1} = 200\,\Omega.$$

With ac, the changing current produces a self-induced emf opposing its change. The additional inductive reactance makes the impedance exceed the resistance, explaining the smaller current. From the given ac readings,

$$Z = \frac{V_{\text{ac}}}{I_{\text{ac}}} = \frac{200}{0.5} = 400\,\Omega,$$
$$X_L = \sqrt{Z^2 - R^2} = \sqrt{400^2 - 200^2} = 200\sqrt{3}\,\Omega,$$
$$L = \frac{X_L}{2\pi f} = \frac{200\sqrt{3}}{2\pi \times 50} = \frac{2\sqrt{3}}{\pi}\,\text{H} \approx 1.10\,\text{H}.$$

The resistance inferred from dc must be retained in the ac calculation: treating the coil as a pure inductor would give an incorrect inductance.

Step 5: Conclusion The impedance is $Z = \sqrt{R^2 + (\omega L - 1/\omega C)^2}$, with minimum R at $f_0 = 1/(2\pi\sqrt{LC})$. At resonance, V_L and V_C differ in phase by $180°$. The coil draws less ac current because of its inductive reactance; its resistance is $200\,\Omega$ and self inductance is approximately $1.10\,\text{H}$.

Quick Tip to Solve: Use the dc reading to find the coil's resistance first. Then subtract squared impedances, $X_L^2 = Z^2 - R^2$, before dividing by $2\pi f$; never subtract R directly from Z.

OR

25. (a) Draw the diagram of a device which is used to decrease high ac voltage into a low ac voltage and state its working principle. Write four sources of energy loss in this device.

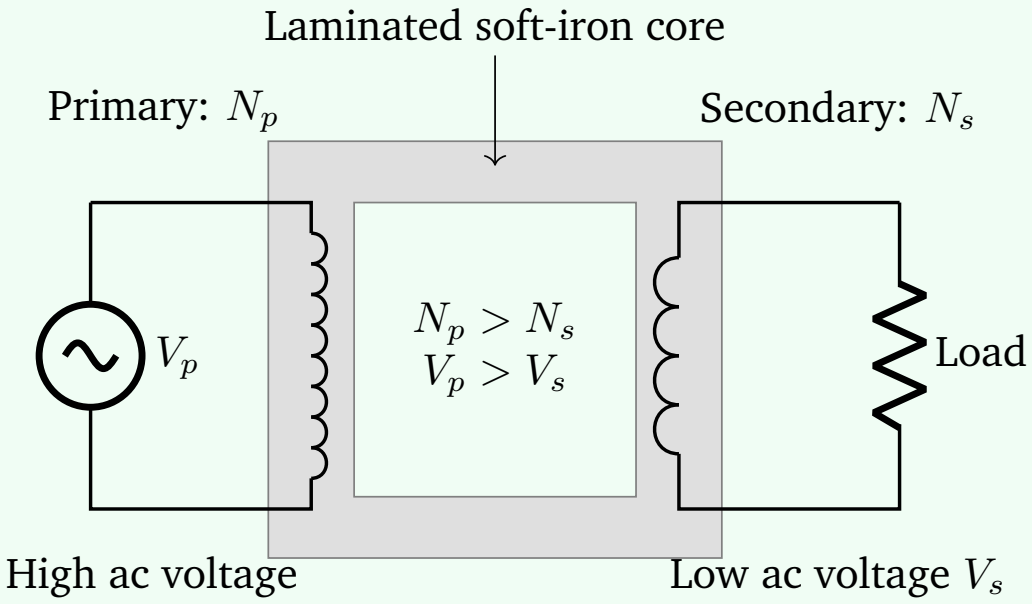
(b) A small town with a demand of $1200\,\text{kW}$ of electric power at $220\,\text{V}$ is situated $20\,\text{km}$ away from an electric plant generating power at $440\,\text{V}$. The resistance of the two wire line carrying power is $0.5\,\Omega$ per km. The town gets the power from the line through a 4000-$220\,\text{V}$ step-down transformer at a sub-station in the town. Estimate the line power loss in the form of heat. [5 marks]

Solution:

Concept:

- A transformer works by mutual induction: a changing magnetic flux in the primary coil induces an emf in the secondary coil linked by a common iron core.
- For an ideal transformer, $V_s/V_p = N_s/N_p$, where V is rms voltage and N is the number of turns; subscripts p and s denote primary and secondary.
- The power delivered at the receiving end is $P = VI$, and the heating loss in a transmission line of total resistance R is $P_{\text{loss}} = I^2R$.

Step 1: (a) Identify and draw the device The device is a **step-down transformer**. Two insulated copper windings are wound on a common laminated soft-iron core. The primary, connected to the high-voltage ac supply, has more turns than the secondary connected to the load: $N_p > N_s$. The closed iron core provides a path linking magnetic flux through both coils. The windings are electrically insulated from each other and from the core.



Step-down transformer: the primary has more turns than the secondary.

Step 2: Explain mutual induction and the voltage reduction Alternating current in the primary produces an alternating magnetic flux Φ in the core. This changing flux links the secondary and induces an alternating emf there. By Faraday's law,

$$e_p = -N_p \frac{d\Phi}{dt}, \qquad e_s = -N_s \frac{d\Phi}{dt}.$$

Neglecting winding resistance and flux leakage, the rms voltage ratio is therefore

$$\frac{V_s}{V_p} = \frac{N_s}{N_p} < 1.$$

Thus the secondary delivers a lower ac voltage. A steady dc supply cannot maintain changing core flux and hence cannot produce continuous transformer action.

Step 3: State four sources of energy loss **Copper loss:** current heats the primary and secondary winding resistances through I^2R dissipation.

Eddy-current loss: changing flux induces circulating currents in the iron core, which heat it; laminations reduce these currents.

Hysteresis loss: repeated magnetisation and demagnetisation dissipate energy in the core; soft iron with a narrow hysteresis loop reduces this loss.

Flux leakage: some primary flux does not link the secondary, so magnetic coupling and transfer of useful energy to the load are incomplete.

Step 4: (b) Find the transmission-line current Treat the sub-station transformer as ideal for the estimate. The town consumes $1200\,\text{kW} = 1.2 \times 10^6\,\text{W}$, so its transformer draws the same power from the line at its primary voltage of $4000\,\text{V}$. Thus

$$I_{\text{line}} = \frac{1.2 \times 10^6}{4000} = 300\,\text{A}.$$

The town-side voltage $220\,\text{V}$ determines the secondary current, not the current in the long transmission line. Similarly, the generating voltage $440\,\text{V}$ is not the receiving-end line voltage; a step-up

arrangement is needed at the plant.

Step 5: Calculate the line resistance and heating loss The quoted $0.5\,\Omega$ per kilometre is the resistance of the *two-wire line*. Hence for the stated route length,

$$R_{\text{line}} = 0.5 \times 20 = 10\,\Omega.$$

It must not be doubled again. The power converted into heat along the line is

$$P_{\text{loss}} = I_{\text{line}}^2 R_{\text{line}} = (300)^2 \times 10 = 9.0 \times 10^5\,\text{W}.$$

As a consistency check, the line voltage drop is $IR = 3000\,\text{V}$, so the sending-end line voltage would be $4000 + 3000 = 7000\,\text{V}$ in this resistive estimate.

Step 6: Conclusion The device is a step-down transformer with $N_s < N_p$, operating by mutual induction. Its principal losses are copper heating, eddy currents, hysteresis and flux leakage. The transmission-line heating loss is $\boxed{9.0 \times 10^5\,\text{W} = 900\,\text{kW}}$.

Quick Tip to Solve: Use the receiving transformer's high-side voltage, $4000\,\text{V}$, to find line current; the resistance given for the two-wire line already includes both wires.

---

26. (a) Describe any two characteristic features which distinguish between interference and diffraction phenomena. Derive the expression for the intensity at a point of the interference pattern in Young's double slit experiment.

(b) In the diffraction due to a single slit experiment, the aperture of the slit is 3 mm. If monochromatic light of wavelength 620 nm is incident normally on the slit, calculate the separation between the first order minima and the 3^{rd} order maxima on one side of the screen. The distance between the slit and the screen is 1.5 m. [5 marks]

Solution:

Concept:

- Coherent waves have a constant phase difference. Superposition adds their displacements, while the time average of the squared resultant displacement determines intensity.
- For slit separation d, screen distance D and transverse screen coordinate y, the Young's experiment path difference is $\Delta = d\sin\theta \simeq dy/D$, and the phase difference is $\phi = 2\pi\Delta/\lambda$.
- For a single slit of width a, minima occur at $a\sin\theta = m\lambda$. Secondary maxima lie approximately midway between successive minima; the third secondary maximum is near $a\sin\theta = 7\lambda/2$.

Step 1: (a) State two differences in the patterns First, in ideal two-slit interference the bright fringes have equal intensity when the two coherent sources have equal amplitudes. In single-slit diffraction, the central maximum is the brightest, and successive secondary maxima become weaker away from the centre. Second, the ideal Young's interference fringes have equal width $\lambda D/d$. In single-slit diffraction, the central maximum has width $2\lambda D/a$, twice the width of the secondary bright regions between successive minima. These comparisons refer to the small-angle pattern; finite slit widths can also produce a diffraction envelope over a real interference pattern.

Step 2: (a) Superpose the two coherent disturbances Let the disturbances at the observation point be

$x_1 = A_1 \cos \omega t$ and $x_2 = A_2 \cos(\omega t + \phi)$. On expanding the second cosine, their sum is

$$x = (A_1 + A_2 \cos \phi) \cos \omega t - A_2 \sin \phi \sin \omega t.$$

Hence the squared resultant amplitude is

$$A^2 = (A_1 + A_2 \cos \phi)^2 + (A_2 \sin \phi)^2 = A_1^2 + A_2^2 + 2A_1 A_2 \cos \phi.$$

Write $I_1 = KA_1^2$ and $I_2 = KA_2^2$, where the common constant K includes the time averaging factor. Multiplication by K gives

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \phi.$$

Step 3: (a) Express intensity in terms of screen position The path difference gives $\phi \simeq 2\pi dy/(\lambda D)$, for slits illuminated in phase. With equal single-slit intensities $I_1 = I_2 = I_0$,

$$I(y) = 2I_0 \left[1 + \cos\left(\frac{2\pi dy}{\lambda D}\right)\right] = 4I_0 \cos^2\left(\frac{\pi dy}{\lambda D}\right).$$

Constructive interference occurs for $\Delta = n\lambda$, yielding $I_{\max} = 4I_0$; destructive interference occurs for $\Delta = (n + 1/2)\lambda$, yielding zero intensity. For unequal amplitudes, the limiting intensities instead are $(\sqrt{I_1} \pm \sqrt{I_2})^2$.

Step 4: (b) Locate the minimum and third secondary maximum Convert the data to SI units: $a = 3 \times 10^{-3}$ m, $\lambda = 620 \times 10^{-9}$ m, and $D = 1.5$ m. Since the angles are small, $y \simeq D \sin \theta$. The first minimum on the chosen side is at

$$y_{\min,1} = \frac{\lambda D}{a} = \frac{620 \times 10^{-9} \times 1.5}{3 \times 10^{-3}} = 3.10 \times 10^{-4} \text{ m}.$$

Counting the secondary maxima outward from the central maximum, the third lies approximately halfway between the third and fourth minima:

$$y_{\max,3} \simeq \frac{7\lambda D}{2a} = 1.085 \times 10^{-3} \text{ m}.$$

As both points are on the same side, subtract their distances:

$$\Delta y \simeq y_{\max,3} - y_{\min,1} = \frac{5\lambda D}{2a} = 7.75 \times 10^{-4} \text{ m}.$$

The half-integer secondary-maximum positions are the standard approximate construction for this numerical calculation; the minima positions are exact within the small-angle approximation.

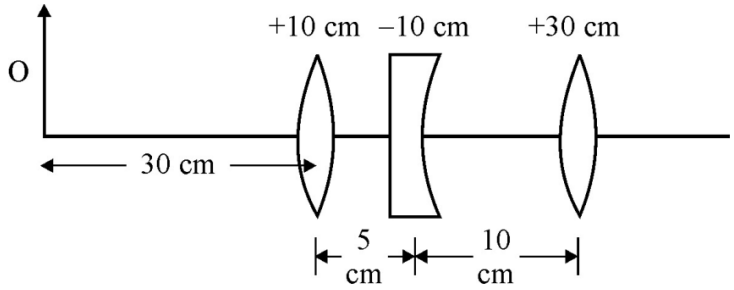
Step 5: Conclusion The interference intensity is $I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \phi$, or $I(y) = 4I_0 \cos^2[\pi dy/(\lambda D)]$ for equal coherent sources. The requested single-slit separation is approximately $\boxed{0.775 \text{ mm}}$.

Quick Tip to Solve: For a third secondary maximum use $7\lambda D/(2a)$, and subtract the first-minimum distance because both points lie on the same side.

OR

26. (a) Under what conditions is the phenomenon of total internal reflection of light observed? Obtain the relation between the critical angle of incidence and the refractive index of the medium.

(b) Three lenses of focal lengths $+10$ cm, -10 cm and $+30$ cm are arranged coaxially as in the figure given below. Find the position of the final image formed by the combination. [5 marks]



Solution:

Concept:

- Total internal reflection is possible when light travels from an optically denser medium of refractive index n_1 toward a rarer medium of index $n_2 < n_1$, with incidence angle greater than the critical angle.
- Snell's law is $n_1 \sin i = n_2 \sin r$; at the critical angle $i = C$, the refracted ray grazes the interface, so $r = 90°$.
- For each thin lens, use $1/v - 1/u = 1/f$. With light travelling left to right, distances measured to the right of that lens are positive and distances to the left are negative.

Step 1: (a) Give the conditions for total internal reflection Two conditions must hold together: the incident light must be travelling in the optically denser medium toward the optically rarer medium, and its angle of incidence, measured from the normal, must exceed the critical angle for the two media. At incidence equal to the critical angle, the refracted ray travels along the interface; total internal reflection occurs for $i > C$. It cannot occur for incidence from the rarer side.

Step 2: (a) Derive the critical-angle relation Increasing the incidence angle from the denser medium increases the refraction angle in the rarer medium. At the limiting value $i = C$, put $r = 90°$ in Snell's law:

$$n_1 \sin C = n_2 \sin 90° = n_2, \qquad \sin C = \frac{n_2}{n_1}.$$

Thus the refractive index of the denser medium relative to the rarer medium is

$$\mu_{12} = \frac{n_1}{n_2} = \frac{1}{\sin C}.$$

If the rarer medium is air, $n_2 \simeq 1$, so the usual relation for the denser medium is $\mu = 1/\sin C$. For incidence larger than C, Snell's law would require $\sin r > 1$, so there is no propagating refracted ray and the light is totally reflected.

Step 3: (b) Find the image that the first lens would form Label the lenses L_1, L_2, L_3 from left to right. The object is 30 cm to the left of L_1; hence $u_1 = -30$ cm and $f_1 = +10$ cm. Using centimetres throughout,

$$\frac{1}{v_1} = \frac{1}{f_1} + \frac{1}{u_1} = \frac{1}{10} - \frac{1}{30} = \frac{1}{15}, \qquad v_1 = +15 \text{ cm}.$$

Without the second lens, the emerging rays would therefore meet 15 cm to the right of L_1.

Step 4: (b) Use the virtual object for the second lens Lens L_2 is only $5\,\text{cm}$ to the right of L_1, so the converging rays reaching it are directed toward a point $15 - 5 = 10\,\text{cm}$ to its right. This point is a virtual object for L_2, giving $u_2 = +10\,\text{cm}$, not a negative object distance. Since $f_2 = -10\,\text{cm}$,

$$\frac{1}{v_2} = \frac{1}{f_2} + \frac{1}{u_2} = -\frac{1}{10} + \frac{1}{10} = 0.$$

The rays emerging from L_2 are parallel: its image is at infinity.

Step 5: (b) Pass the parallel rays through the third lens Parallel rays remain parallel during the $10\,\text{cm}$ propagation from L_2 to L_3. Thus the third lens has an object at infinity, and $1/u_3 = 0$. For $f_3 = +30\,\text{cm}$,

$$\frac{1}{v_3} = \frac{1}{f_3} + 0 = \frac{1}{30}, \qquad v_3 = +30\,\text{cm}.$$

The positive image distance identifies an image on the outgoing side of the third lens, where the refracted rays actually converge. Relative to L_1, its position is $5 + 10 + 30 = 45\,\text{cm}$ to the right.

Step 6: Conclusion Total internal reflection requires incidence from the denser medium with $i > C$, and $\sin C = n_2/n_1$, or $\mu = 1/\sin C$ for a medium relative to air. The combination forms a real final image $\boxed{30\,\text{cm}}$ to the right of the third lens, equivalently $45\,\text{cm}$ to the right of the first lens.

Quick Tip to Solve: Re-measure each object distance from the next lens: here the first image lies to the right of the diverging lens, so its object distance is positive.

27. (a) Describe briefly the process of transferring the charge between the two plates of a parallel plate capacitor when connected to a battery. Derive an expression for the energy stored in a capacitor.
(b) A parallel plate capacitor is charged by a battery to a potential difference V. It is disconnected from battery and then connected to another uncharged capacitor of the same capacitance. Calculate the ratio of the energy stored in the combination to the initial energy on the single capacitor. [5 marks]

Solution:

Concept:

- Capacitance is $C = Q/V$, where Q is the magnitude of charge on either plate and V is the potential difference. During charging, the instantaneous potential difference is $v = q/C$.
- The increase in electrostatic energy when a small positive charge dq is transferred against a potential difference v is $dU = v\,dq$.
- After the battery is disconnected, total charge is conserved. Capacitors connected in parallel reach the same potential difference, and their capacitances add.

Step 1: (a) Explain the transfer of charge Initially the two conducting plates are neutral. Connect one plate to the positive terminal and the other to the negative terminal of a battery. The battery removes electrons from the plate connected to its positive terminal and supplies the same number of electrons to the other plate through the external circuit. Thus the plates acquire equal and opposite charges, $+Q$ and $-Q$. Electrons do not cross the insulating gap between the plates. As the charges accumulate, the electric field and the potential difference between the plates increase. The resulting potential difference opposes further charging. The transient current falls to zero when the capacitor potential difference equals the battery voltage V, giving $Q = CV$. The battery does work to establish the separated charges, and energy is stored in the electric field between the plates.

Step 2: (a) Integrate the charging work Suppose that at an intermediate stage the plates carry charges

$+q$ and $-q$. Their potential difference is q/C, so the work needed to transfer a further infinitesimal charge dq from the negative plate to the positive plate is

$$dU = v\,dq = \frac{q}{C}\,dq.$$

Keeping the plate geometry and hence C fixed, integrate from the uncharged state to final charge Q:

$$U = \int_0^Q \frac{q}{C}\,dq = \frac{Q^2}{2C}.$$

Using $Q = CV$, this becomes $U = \frac{1}{2}QV = \frac{1}{2}CV^2$. The factor $1/2$ arises because the capacitor voltage rises from zero to V; it is not equal to V throughout charging. For ordinary charging through a resistive circuit, the battery supplies QV, while the remaining half is dissipated during the transient.

Step 3: (b) Find the common potential after connection Initially only the first capacitor is charged, so the available total charge on corresponding connected plates is $Q_{\text{i}} = CV$, and its stored energy is $U_{\text{i}} = \frac{1}{2}CV^2$. After disconnecting the battery, connect the initially uncharged capacitor of capacitance C across the first one. The equivalent capacitance is $2C$. If their common final potential difference is V_{f}, charge conservation gives

$$CV = (C + C)V_{\text{f}}, \qquad V_{\text{f}} = \frac{V}{2}.$$

Each capacitor now has charge magnitude $CV/2$. The voltage cannot remain V, since the disconnected battery can no longer supply extra charge.

Step 4: (b) Compare the final and initial energies Add the energy of both capacitors at their common final voltage:

$$U_{\text{f}} = 2\left[\frac{1}{2}C\left(\frac{V}{2}\right)^2\right] = \frac{CV^2}{4}.$$

Therefore

$$\frac{U_{\text{f}}}{U_{\text{i}}} = \frac{CV^2/4}{CV^2/2} = \frac{1}{2}.$$

Charge is conserved, but the energy stored in the capacitors decreases. The difference is dissipated in the connecting circuit, with possible electromagnetic radiation, during redistribution; it is not a violation of energy conservation.

Step 5: Conclusion The energy stored in a capacitor is $\boxed{U = Q^2/(2C) = QV/2 = CV^2/2}$. On sharing charge with an identical initially uncharged capacitor, the common voltage becomes $V/2$, and the final-to-initial stored-energy ratio is $\boxed{1:2}$.

Quick Tip to Solve: With the battery disconnected, conserve charge first. Doubling the total capacitance halves the voltage and hence halves $Q^2/(2C_{\text{total}})$.

OR

27. (a) Derive an expression for the electric field at any point on the equatorial line of an electric dipole.

(b) Two identical point charges, q each, are kept $2\,\text{m}$ apart in air. A third point charge Q of unknown magnitude and sign is placed on the line joining the charges such that the system remains in equilibrium. Find the position and nature of Q. [5 marks]

Solution:

Concept:

- An electric dipole consists of equal and opposite charges separated by a distance $2a$. Its dipole moment $\vec{p}$ has magnitude $p = 2aq_{\text{d}}$ and points from the negative to the positive charge.
- The electric field is the vector sum of the individual Coulomb fields. In air, use $k = 1/(4\pi\varepsilon_0)$, so a point charge q at distance R produces a field of magnitude $k|q|/R^2$.
- Equilibrium of a system of point charges requires zero net force on every charge, not merely on the newly introduced charge. Collinear forces must balance in both magnitude and direction.

Step 1: (a) Define the dipole geometry Place the dipole charges $-q_{\text{d}}$ and $+q_{\text{d}}$ at coordinates $(-a, 0)$ and $(a, 0)$, respectively. Let the observation point P on the equatorial line be $(0, r)$, at distance r from the dipole centre. The equatorial line is the perpendicular bisector of the charge separation, so the distance from either charge to P is

$$R = \sqrt{r^2 + a^2}.$$

The dipole moment is $\vec{p} = 2aq_{\text{d}}\,\hat{\mathbf{x}}$. The field of the positive charge points away from it, whereas that of the negative charge points towards it. Consequently their components along the equatorial line cancel and their components along the dipole axis reinforce towards the negative charge.

Step 2: (a) Add the field components Write the fields explicitly using the common distance R:

$$\vec{E}_{+} = \frac{kq_{\text{d}}}{R^3}(-a\hat{\mathbf{x}} + r\hat{\mathbf{y}}),$$
$$\vec{E}_{-} = \frac{kq_{\text{d}}}{R^3}(-a\hat{\mathbf{x}} - r\hat{\mathbf{y}}).$$

On adding, the y-components vanish, and therefore

$$\vec{E}_{\text{eq}} = -\frac{2kaq_{\text{d}}}{(r^2 + a^2)^{3/2}}\hat{\mathbf{x}} = -\frac{1}{4\pi\varepsilon_0}\frac{\vec{p}}{(r^2 + a^2)^{3/2}}.$$

This is the exact expression for a finite dipole. Its direction is opposite to $\vec{p}$. For a distant point, $r \gg a$, the denominator can be approximated by r^3, giving $\vec{E}_{\text{eq}} \simeq -\vec{p}/(4\pi\varepsilon_0 r^3)$. The distant-point approximation is not needed for the exact result.

Step 3: (b) Locate the third charge Denote the two identical given charges by q, as in the question, and place them $2\,\text{m}$ apart. For the third charge Q to experience zero net force, the fields of these two charges at its position must be equal and opposite. Outside the interval joining like charges their fields point in the same direction, so cancellation is impossible there. If Q lies a distance x from the left charge, between them, the distances are x and $2\,\text{m} - x$. Equating the field magnitudes gives

$$\frac{k|q|}{x^2} = \frac{k|q|}{(2\,\text{m} - x)^2}.$$

Both distances are positive, hence $x = 2\,\text{m} - x$, or $x = 1\,\text{m}$. Thus Q must be at the midpoint. Symmetry

makes its own net force zero there, whatever its nonzero magnitude.

Step 4: (b) Balance the force on either outer charge The outer charges repel one another. To cancel this outward force, the central charge must attract each outer charge, so Q has the opposite sign to q. The separation between the outer charges is $2\,\text{m}$, whereas each is $1\,\text{m}$ from Q. Therefore equilibrium of either outer charge requires

$$\frac{kq^2}{(2\,\text{m})^2} = \frac{k|qQ|}{(1\,\text{m})^2}, \qquad |Q| = \frac{|q|}{4}.$$

Combining this magnitude with the required opposite sign gives $Q = -q/4$. The other outer charge satisfies the same balance by symmetry. This checks all three charges, rather than checking only the central one.

Step 5: Conclusion At equatorial distance r, $\boxed{\vec{E} = -\vec{p}/[4\pi\varepsilon_0(r^2 + a^2)^{3/2}]}$, opposite to the dipole moment. The third charge must be $\boxed{Q = -q/4}$, located at the midpoint, $\boxed{1\,\text{m}}$ from each given charge; it is negative if q is positive and vice versa.

Quick Tip to Solve: For three-charge equilibrium, first cancel the field at the middle charge to find its position, then balance the force on an outer charge to determine the middle charge's sign and magnitude.