

CBSE Class 12 Physics Question Paper 2019 with Solution PDF-Set 55/1/2

Question Paper with Solutions

Conducted by the Central Board of Secondary Education (CBSE)



Time Allowed: 3 hours

Maximum Marks: 70

General Instructions

- (i) All questions are compulsory. There are 27 questions in all.
- (ii) This question paper has **four** sections : Section A, Section B, Section C and Section D.
- (iii) Section A contains **five** questions of **one** mark each, Section B contains **seven** questions of **two** marks each, Section C contains **twelve** questions of **three** marks each, and Section D contains **three** questions of **five** marks each.
- (iv) There is no overall choice. However, an internal choice(s) has been provided in **two** questions of **one** mark, **two** questions of **two** marks **four** questions of **three** marks and **three** questions of **five** marks weightage. You have to attempt only **one** of the choices in such questions.

You may use the following values of physical constants wherever necessary :

$$c = 3 \times 10^8 \text{ m/s}$$

$$h = 6.63 \times 10^{-34} \text{ Js}$$

$$e = 1.6 \times 10^{-19} \text{ C}$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ T m A}^{-1}$$

$$\epsilon_0 = 8.854 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$$

$$\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$$

$$\text{Mass of electron } (m_e) = 9.1 \times 10^{-31} \text{ kg}$$

$$\text{Mass of neutron} = 1.675 \times 10^{-27} \text{ kg}$$

$$\text{Mass of proton} = 1.673 \times 10^{-27} \text{ kg}$$

$$\text{Avogadro's number} = 6.023 \times 10^{23} \text{ per gram mole}$$

$$\text{Boltzmann constant} = 1.38 \times 10^{-23} \text{ JK}^{-1}$$

Section A (5 marks)

1. Define the term “threshold frequency”, in the context of photoelectric emission.

[1 mark]

Solution:

Concept:

- Each incident photon carries energy $h\nu$, where h is Planck's constant and ν is the light frequency.
- The work function ϕ is the minimum energy required to free an electron from the given metal surface.

Step 1: Apply the emission condition A photon can liberate an electron only when its energy is at least the work function: $h\nu \geq \phi$. Raising the intensity increases the number of photons but does not increase the energy of an individual photon.

Step 2: Identify the limiting frequency At the threshold, the emitted electron has zero maximum kinetic energy, so $h\nu_0 = \phi$. Frequencies below ν_0 cannot cause photoelectric emission from that surface, irrespective of the incident intensity.

Step 3: Conclusion Threshold frequency ν_0 is the minimum frequency of incident radiation required to produce photoelectric emission from a given metal surface; $\nu_0 = \phi/h$.

Quick Tip to Solve: Remember the threshold condition as photon energy equal to the metal work function.

OR

1. Define the term “Intensity” in photon picture of electromagnetic radiation.

[1 mark]

Solution:

Concept:

- In the photon picture, radiation consists of photons, each with energy $h\nu$.
- Intensity measures energy flow per unit area per unit time through a surface normal to propagation.

Step 1: Count photons crossing the surface Let N photons of frequency ν cross a perpendicular area A during time Δt . Their total transmitted energy is $E = N h \nu$. The number of photons alone is insufficient to specify energy intensity unless their frequency is also known.

Step 2: Convert photon flow to energy flow Dividing this energy by area and elapsed time gives $I = N h \nu / (A \Delta t)$. If $n = N / (A \Delta t)$ is the photon flux, then $I = n h \nu$. At fixed frequency, increasing photon flux increases the intensity in the same proportion.

Step 3: Conclusion Intensity is the photon energy passing per unit time through unit area normal to propagation: $I = n h \nu$, where n is photons per unit area per unit time. Its SI unit is W m^{-2} .

Quick Tip to Solve: Photon flux counts photons; multiply it by energy per photon to obtain intensity.

2. When unpolarised light is incident on the interface separating the rarer medium and the denser medium, Brewster angle is found to be 60° . Determine the refractive index of the denser medium.

[1 mark]

Solution:

Concept:

- Brewster's law is $\mu_{21} = \tan i_B$, where i_B is the polarising angle and $\mu_{21} = n_2/n_1$ is the denser medium's refractive index relative to the incident rarer medium.
- At this angle the reflected and refracted rays are mutually perpendicular.

Step 1: Identify the appropriate refractive index The light travels from the rarer medium into the denser medium and the given Brewster angle is $i_B = 60^\circ$. Thus the index determined directly by the angle is the relative refractive index n_2/n_1 . For incidence from air, this is effectively the absolute index of the denser medium.

Step 2: Substitute the given angle Using Brewster's law,

$$\mu_{21} = \tan 60^\circ = \sqrt{3} \approx 1.732.$$

The index is dimensionless. We use the tangent of the polarising angle, not its sine, since the refracted angle is complementary to it.

Step 3: Conclusion The refractive index of the denser medium relative to the rarer medium is $\sqrt{3} \approx 1.73$ (also its absolute index if the rarer medium is air).

Quick Tip to Solve: For a Brewster-angle question, use tangent of the given angle immediately.

3. In sky wave mode of propagation, why is the frequency range of transmitting signals restricted to less than 30 MHz?

[1 mark]

Solution:

Concept:

- Sky waves reach distant receivers after being returned towards Earth by the ionosphere.
- The ionosphere contains free electrons and has a limited ability to return radio waves; this ability depends on frequency and ionisation.

Step 1: State the requirement for sky waves The transmitted radio wave must be bent sufficiently by the ionised atmospheric layers to return to Earth. This effective return, often described as ionospheric reflection, permits reception beyond the direct line of sight.

Step 2: Explain the upper frequency restriction As the radio frequency increases, the ionosphere becomes less effective at bending the wave. Above roughly 30 MHz, the waves generally penetrate the ionosphere and escape into space instead of returning to the ground. Thus ordinary sky-wave communication cannot use these higher frequencies reliably.

Step 3: Conclusion The frequency is restricted to below about 30 MHz because higher-frequency signals generally pass through the ionosphere rather than being returned to Earth.

Quick Tip to Solve: Link the sky-wave upper limit to ionospheric penetration at high frequency.

3. On what factors does the range of coverage in ground wave propagation depend?

[1 mark]

Solution:

Concept:

- A ground wave travels along the Earth's surface and loses energy as it propagates.
- Its useful range ends when attenuation reduces the received signal below the level needed for reception.

Step 1: Identify the transmitted-signal factors The range depends on the transmitter power and on the frequency of the transmitted wave. Greater power permits a detectable signal at a greater distance. Higher frequencies generally suffer greater ground-wave attenuation, which is why lower-frequency signals are suited to this mode.

Step 2: Identify the ground-property factor The electrical conductivity of the ground along the propagation path also affects the losses. Better-conducting ground, such as seawater, generally supports a greater range than poorly conducting dry ground. Thus range is not determined by transmitter power alone.

Step 3: Conclusion The ground-wave coverage range depends mainly on the transmitted power, the signal frequency and the electrical conductivity of the ground.

Quick Tip to Solve: Recall three range factors: transmitter power, radio frequency and ground conductivity.

4. When a potential difference is applied across the ends of a conductor, how is the drift velocity of the electrons related to the relaxation time?

[1 mark]

Solution:

Concept:

- The relaxation time τ is the mean time between successive collisions of a conduction electron.
- An electron in electric field $\vec{E}$ experiences force $-e\vec{E}$, where e is the magnitude of its charge and m its mass.

Step 1: Find the acceleration between collisions The electron acceleration is $\vec{a} = -e\vec{E}/m$. Random thermal velocities have zero average, while the electric field produces a small nonzero average velocity directed opposite to the field.

Step 2: Relate mean velocity to relaxation time Over the average free-flight time τ , the field contributes the drift velocity $\vec{v}_d = \vec{a}\tau = -e\vec{E}\tau/m$. Hence its magnitude is $v_d = eE\tau/m$. For fixed potential difference V and conductor length L , $E = V/L$, so $v_d = eV\tau/(mL)$.

Step 3: Conclusion For a fixed applied electric field, the magnitude of electron drift velocity is directly proportional to relaxation time: $v_d \propto \tau$, with $v_d = eE\tau/m$.

Quick Tip to Solve: Keep the field fixed when stating proportionality; electron drift opposes the field.

5. Draw the equipotential surfaces due to an isolated point charge.

[1 mark]

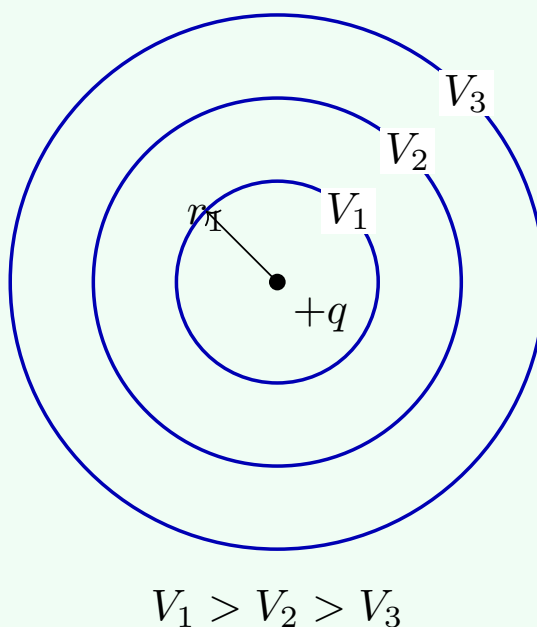
Solution:

Concept:

- The potential due to an isolated point charge q is $V = kq/r$, where r is the distance from the charge and $k = 1/(4\pi\epsilon_0)$.
- An equipotential surface contains points with the same potential; for a fixed nonzero q , this means the same distance r .

Step 1: Identify the shape For each selected value of V , the radius r is constant in every direction. The locus of all such points is a sphere with the point charge at its centre.

Step 2: Draw concentric spherical surfaces The sketch shows three concentric spherical equipotential surfaces as circular sections through their common centre. Their radii satisfy $r_1 < r_2 < r_3$. For the positive charge shown, their potentials satisfy $V_1 > V_2 > V_3$. For a negative charge, the shape remains spherical but the potentials are negative. The radial electric field is normal to every surface.



Circular sections of concentric spherical equipotential surfaces about a positive point charge.

Step 3: Conclusion The equipotential surfaces of an isolated point charge are concentric spheres centred on that charge.

Quick Tip to Solve: Set $V = kq/r$ constant: constant r immediately identifies a sphere centred on the charge.

Section B (14 marks)

6. Explain with the help of Einstein's photoelectric equation any two observed features in photoelectric effect which cannot be explained by wave theory. [2 marks]

Solution:

Concept:

- In Einstein's photon model, an electron absorbs energy $h\nu$ from one photon, where h is Planck's constant and ν is the frequency of incident light.
- Einstein's equation is $K_{\max} = h\nu - \phi = h(\nu - \nu_0)$, where ϕ is the metal's work function and $\nu_0 = \phi/h$ is its threshold frequency.

Step 1: Explain the existence of a threshold frequency An electron must receive at least the work function ϕ to escape the metal. Hence emission requires $h\nu \geq \phi$, or $\nu \geq \nu_0$. If $\nu < \nu_0$, an individual photon has insufficient energy and no photoelectrons are emitted, however intense the light is. Increasing intensity only increases the number of photons; it does not increase the energy of each photon. Classical wave theory instead allows continuous accumulation of energy and therefore cannot explain a sharp threshold frequency independent of intensity.

Step 2: Explain the dependence of maximum kinetic energy At a fixed frequency above threshold, $K_{\max} = h\nu - \phi$ has no intensity term. Thus increasing intensity does not change the maximum kinetic energy of the photoelectrons, although it can increase the photocurrent. Increasing frequency increases $K_{\max}$ linearly, with slope h . Classical wave theory associates greater intensity with greater energy delivered to electrons, so it incorrectly predicts an increase in their maximum kinetic energy with intensity.

Step 3: Conclusion Einstein's equation explains both the threshold frequency and the linear frequency dependence, together with intensity independence, of the maximum photoelectron kinetic energy.

Quick Tip to Solve: Use ν to determine photon energy and electron kinetic energy; use intensity to determine photon number and photocurrent.

7. A deuteron and an alpha particle having same momentum are in turn allowed to pass through a magnetic field $\vec{B}$, acting normal to the direction of motion of the particles. Calculate the ratio of the radii of the circular paths described by them. [2 marks]

Solution:

Concept:

- A particle of charge magnitude $|q|$, moving perpendicular to a uniform magnetic field B , experiences a force $F = |q|vB$, normal to its velocity.
- This magnetic force supplies the centripetal force. For momentum $p = mv$, the radius of the circular path is $r = p/(|q|B)$.

Step 1: Relate the orbit radius to momentum Since the field is normal to the motion, the magnetic force has its full magnitude and bends the path without doing work. Equating magnetic and centripetal forces gives

$$|q|vB = \frac{mv^2}{r} \Rightarrow r = \frac{mv}{|q|B} = \frac{p}{|q|B}.$$

Thus at fixed momentum and field, the radius is inversely proportional to the charge magnitude, rather than being determined by mass alone.

Step 2: Substitute the two nuclear charges A deuteron has one proton and one neutron, so its charge is $+e$. An alpha particle has two protons and two neutrons, so its charge is $+2e$. With the same momentum p in the same field B ,

$$r_d = \frac{p}{eB}, \quad r_\alpha = \frac{p}{2eB}, \quad \frac{r_d}{r_\alpha} = 2.$$

The different masses imply different speeds at equal momentum, but this is already accounted for by using p .

Step 3: Conclusion The required ratio, in the order deuteron to alpha particle, is $r_d : r_\alpha = 2 : 1$.

Quick Tip to Solve: When momenta are equal, use $r = p/(qB)$ directly: the radius ratio is the inverse of the charge ratio.

8. Two bulbs are rated (P_1, V) and (P_2, V) . If they are connected (i) in series and (ii) in parallel across a supply V , find the power dissipated in the two combinations in terms of P_1 and P_2 . [2 marks]

Solution:

Concept:

- A bulb rated (P, V) dissipates power $P = V^2/R$ at its rated voltage. Treating its resistance as constant in this standard circuit model gives $R = V^2/P$.
- For a combination connected across voltage V , total power is $P_{\text{total}} = V^2/R_{\text{eq}}$. Resistances add in series; conductances add in parallel.

Step 1: Obtain the bulb resistances and series power The bulb resistances inferred from their ratings are

$$R_1 = \frac{V^2}{P_1}, \quad R_2 = \frac{V^2}{P_2}.$$

In series, the same current passes through both bulbs and their resistances add:

$$R_s = R_1 + R_2 = V^2 \left(\frac{1}{P_1} + \frac{1}{P_2} \right), \quad P_s = \frac{V^2}{R_s} = \frac{P_1 P_2}{P_1 + P_2}.$$

The full supply voltage is shared by the bulbs; neither is separately connected across its rated voltage in this arrangement.

Step 2: Obtain the parallel power In parallel, each bulb is connected directly across the supply V , equal to its rated voltage. Thus their individual powers are P_1 and P_2 . Equivalently,

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{P_1 + P_2}{V^2}, \quad P_p = \frac{V^2}{R_p} = P_1 + P_2.$$

Total power adds because the supply delivers energy to both branches simultaneously.

Step 3: Conclusion The total powers dissipated in the two combinations are $P_s = \frac{P_1 P_2}{P_1 + P_2}$ in series and $P_p = P_1 + P_2$ in parallel.

Quick Tip to Solve: Convert each rating to $R = V^2/P$ before combining bulbs; rated powers add directly only when each bulb receives the rated voltage.

9. Calculate the radius of curvature of an equi-concave lens of refractive index 1.5, when it is kept in a medium of refractive index 1.4, to have a power of -5 D . [2 marks]

Solution:

Concept:

- For a thin lens immersed in a medium, the lens-maker formula is $P = \left(\frac{n_l}{n_m} - 1 \right) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$, where n_l and n_m are the refractive indices of the lens and surrounding medium.
- With light travelling from left to right, an equi-concave lens has $R_1 = -R$ and $R_2 = +R$, where R is the positive common radius. Power in dioptres requires radii in metres.

Step 1: Find the relative refractive index The lens is immersed in a medium, so its refractive index relative to that medium must be used:

$$\frac{n_l}{n_m} = \frac{1.5}{1.4} = \frac{15}{14}, \quad \frac{n_l}{n_m} - 1 = \frac{1}{14}.$$

Using $1.5 - 1$ would incorrectly give the power of the lens in air.

Step 2: Apply the signed radii in the lens-maker formula Substituting $P = -5\text{ D}$, $R_1 = -R$ and $R_2 = +R$,

$$-5 = \frac{1}{14} \left(-\frac{1}{R} - \frac{1}{R} \right) = -\frac{1}{7R}.$$

Hence $5R = 1/7$, giving $R = 1/35\text{ m}$. The negative power is consistent with a concave lens whose refractive index exceeds that of its surroundings.

Step 3: Conclusion The radius of curvature of each surface is $R = 0.0286\text{ m} = 2.86\text{ cm}$. With the stated sign convention, $R_1 = -2.86\text{ cm}$ and $R_2 = +2.86\text{ cm}$.

Quick Tip to Solve: For an equi-concave immersed lens, use $P = -2(n_l/n_m - 1)/R$, with R in metres.

OR

9. An equilateral glass prism has a refractive index 1.6 in air. Calculate the angle of minimum deviation of the prism, when kept in a medium of refractive index $\frac{4\sqrt{2}}{5}$. [2 marks]

Solution:

Concept:

- An equilateral prism has prism angle $A = 60^\circ$. Its refractive index relative to the surrounding medium is $n_{\text{rel}} = n_g/n_m$.
- At minimum deviation the ray path is symmetric: $i = e$ and $r_1 = r_2 = A/2$. The minimum deviation

δ_m obeys

$$n_{\text{rel}} = \frac{\sin[(A + \delta_m)/2]}{\sin(A/2)}.$$

Step 1: Calculate the refractive index relative to the medium The given refractive index of the glass in air is $n_g = 1.6 = 8/5$. For immersion in the given medium,

$$n_{\text{rel}} = \frac{8/5}{4\sqrt{2}/5} = \frac{2}{\sqrt{2}} = \sqrt{2}.$$

Thus the refractive index appearing in Snell's law at the prism faces is $\sqrt{2}$, rather than 1.6.

Step 2: Use the minimum-deviation relation With $A = 60^\circ$, each internal refraction angle at minimum deviation is 30° . Substitution gives

$$\sin\left(\frac{60^\circ + \delta_m}{2}\right) = \sqrt{2} \sin 30^\circ = \frac{1}{\sqrt{2}}.$$

The physical incidence angle is between 0° and 90° , so $(60^\circ + \delta_m)/2 = 45^\circ$. Solving gives $60^\circ + \delta_m = 90^\circ$. This also agrees with $\delta_m = i + e - A$ for $i = e = 45^\circ$.

Step 3: Conclusion The angle of minimum deviation of the immersed prism is $\boxed{\delta_m = 30^\circ}$.

Quick Tip to Solve: For an immersed prism, first divide glass index by medium index before using the minimum-deviation formula.

10. State Bohr's quantization condition of angular momentum. Calculate the shortest wavelength of the Brackett series and state to which part of the electromagnetic spectrum does it belong. [2 marks]

Solution:

Concept:

- Bohr's angular-momentum quantization condition permits stationary orbits for which $m_e v_n r_n = n\hbar = nh/(2\pi)$, where $n = 1, 2, 3, \dots$, m_e is the electron mass and h is Planck's constant.
- Hydrogen wavelengths satisfy $1/\lambda = R_H(1/n_f^2 - 1/n_i^2)$, where $R_H \simeq 1.097 \times 10^7 \text{ m}^{-1}$. Brackett transitions terminate at $n_f = 4$.

Step 1: State the allowed angular momentum For an electron revolving in a circular stationary orbit, Bohr requires

$$L = m_e v_n r_n = \frac{nh}{2\pi} = n\hbar, \quad n = 1, 2, 3, \dots$$

Thus the angular momentum can take integral multiples of $\hbar$; arbitrary intermediate values are not allowed in this model.

Step 2: Evaluate the Brackett series limit In the Brackett series, electrons fall from $n_i = 5, 6, \dots$ to $n_f = 4$. The shortest wavelength has the largest photon energy, obtained in the limiting transition $n_i \rightarrow \infty$:

$$\frac{1}{\lambda_{\min}} = R_H \left(\frac{1}{4^2} - 0 \right) = \frac{R_H}{16}, \quad \lambda_{\min} = \frac{16}{1.097 \times 10^7} \text{ m}.$$

This is approximately $1.46 \times 10^{-6} \text{ m}$, or 1460 nm. Its wavelength is longer than the visible range, placing it in the infrared region.

Step 3: Conclusion Bohr's condition and the Brackett series limit are

$$m_e v_n r_n = \frac{nh}{2\pi}, \quad \lambda_{\min} \simeq 1.46 \mu\text{m}.$$

This wavelength belongs to the **infrared** region.

Quick Tip to Solve: For the shortest wavelength of any hydrogen series, set the initial level to infinity and retain its fixed final level.

OR

10. Calculate the orbital period of the electron in the first excited state of hydrogen atom. [2 marks]

Solution:

Concept:

- For hydrogen in the Bohr model, the radius and speed in orbit n are $r_n = a_0 n^2$ and $v_n = v_1/n$, with $a_0 = 5.29 \times 10^{-11} \text{ m}$ and $v_1 = 2.19 \times 10^6 \text{ m s}^{-1}$.
- The orbital period is the circumference divided by speed: $T_n = 2\pi r_n / v_n$. It therefore scales as n^3 , since radius grows as n^2 while speed decreases as $1/n$.

Step 1: Identify the orbit and calculate its radius and speed The ground state is $n = 1$, so the first excited state is $n = 2$. The Bohr radius and electron speed for this state are

$$r_2 = 4a_0 = 4(5.29 \times 10^{-11}) = 2.116 \times 10^{-10} \text{ m},$$
$$v_2 = \frac{v_1}{2} = 1.095 \times 10^6 \text{ m s}^{-1}.$$

These are the radius and speed of the electron's circular orbit; the path length for a complete revolution is $2\pi r_2$.

Step 2: Calculate the time for one revolution Divide the circumference by the speed:

$$T_2 = \frac{2\pi(2.116 \times 10^{-10})}{1.095 \times 10^6} \text{ s} \simeq 1.21 \times 10^{-15} \text{ s}.$$

As a consistency check, $T_2 = 2^3 T_1 = 8T_1$, where $T_1 = 2\pi a_0 / v_1 \simeq 1.52 \times 10^{-16} \text{ s}$. Both methods give the same period to the accuracy of the constants used.

Step 3: Conclusion The orbital period of the electron in hydrogen's first excited state is $T_2 \simeq 1.21 \times 10^{-15} \text{ s}$.

Quick Tip to Solve: First excited means $n = 2$; use $T_n = n^3 T_1$, so its orbital period is eight times the ground-state period.

11. (a) Plot a graph showing variation of de Broglie wavelength (λ) associated with a charged particle of mass m , versus $\sqrt{V}$, where V is the accelerating potential.

(b) An electron, a proton and an alpha particle have the same kinetic energy. Which one has the shortest wavelength? [2 marks]

Solution:

Concept:

- The de Broglie wavelength is $\lambda = h/p$, where h is Planck's constant and p is momentum.
- For a nonrelativistic particle of mass m and charge magnitude q , acceleration from rest through potential difference V gives $K = qV$ and $p = \sqrt{2mK}$.

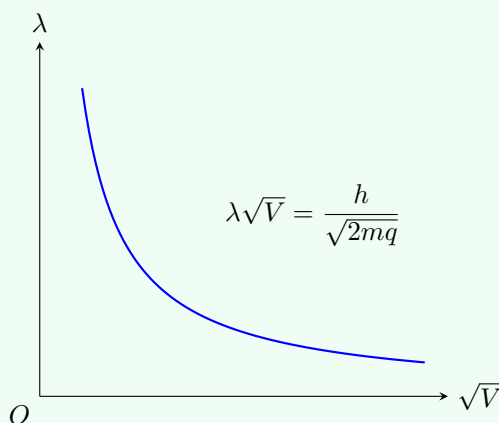
Step 1: (a) Express wavelength in terms of the plotted variable Combining the kinetic-energy and momentum relations gives

$$\lambda = \frac{h}{\sqrt{2mqV}}.$$

Put $x = \sqrt{V}$, as required by the printed question. Then

$$\lambda = \frac{h}{\sqrt{2mq}} \frac{1}{x}.$$

Thus λx is constant: the graph is a decreasing rectangular hyperbola in the positive quadrant. It approaches both axes asymptotically; it neither passes through the origin nor forms a straight line. The graph below represents the nonrelativistic relation.



Wavelength versus square root of accelerating potential (schematic).

Step 2: (b) Compare particles at equal kinetic energy If the common kinetic energy is K , then

$$\lambda = \frac{h}{\sqrt{2mK}} \Rightarrow \lambda \propto \frac{1}{\sqrt{m}}.$$

Since $m_\alpha > m_p > m_e$, their wavelengths satisfy $\lambda_\alpha < \lambda_p < \lambda_e$. Charge does not enter this comparison because kinetic energy, rather than accelerating potential, is fixed.

Step 3: Conclusion (a) The graph of λ against $\sqrt{V}$ is a decreasing rectangular hyperbola. (b) The alpha particle has the shortest de Broglie wavelength.

Quick Tip to Solve: Check the horizontal axis first: wavelength is inverse to square root V , but is linear in its reciprocal; at fixed kinetic energy compare masses only.

12. Why a signal transmitted from a TV tower cannot be received beyond a certain distance? Write the expression for the optimum separation between the receiving and the transmitting antenna. [2 marks]

Solution:

Concept:

- Television broadcasts use high-frequency electromagnetic waves whose direct reception is principally by line-of-sight or space-wave propagation.
- For an antenna of height h above a spherical Earth of radius R , its distance to the horizon is approximately $d = \sqrt{2Rh}$, when $h \ll R$.

Step 1: Explain the finite reception distance Space waves travel nearly in straight lines. The curved surface of the Earth therefore obstructs the direct path to a receiver beyond the radio horizon. The high frequencies used for TV are not usefully reflected by the ionosphere and cannot follow the Earth's curvature as ground waves do. Consequently direct reception is limited to the line-of-sight range of the transmitting and receiving antennas.

Step 2: Add the two horizon distances Let the transmitting and receiving antenna heights be h_T and h_R . For one antenna the tangent geometry gives

$$(R + h)^2 = R^2 + d^2, \quad d^2 = 2Rh + h^2 \simeq 2Rh.$$

The farthest direct link occurs when the two horizon distances meet. Hence the optimum (maximum line-of-sight) separation is

$$d_{\max} = \sqrt{2Rh_T} + \sqrt{2Rh_R}.$$

Here R is Earth's radius; if all lengths on the right are in metres, the separation is in metres. This approximation neglects atmospheric refraction and assumes unobstructed terrain.

Step 3: Conclusion Earth's curvature limits direct TV reception. The optimum separation is $d_{\max} = \sqrt{2Rh_T} + \sqrt{2Rh_R}$, where h_T and h_R are the antenna heights.

Quick Tip to Solve: Find each antenna's horizon range separately and add the two square roots; do not put the sum of the heights inside a single square root.

Section C (36 marks)

13. (a) State the underlying principle of a moving coil galvanometer.
- (b) Give two reasons to explain why a galvanometer cannot as such be used to measure the value of the current in a given circuit.
- (c) Define the terms: (i) voltage sensitivity and (ii) current sensitivity of a galvanometer. [3 marks]

Solution:

Concept:

- A current-carrying coil in a magnetic field experiences magnetic torque. In a radial field its magnitude is $\tau_m = NBIA$, with N turns, field B , current I , and coil area A .
- The suspension produces restoring torque $\tau_r = k\theta$, where k is the torsional constant and θ is the angular deflection. Equilibrium gives $\theta = NBIA/k$.

Step 1: (a) State the operating principle A moving coil galvanometer works on the torque exerted by a magnetic field on a current-carrying coil. A radial magnetic field keeps the plane of the coil parallel

to the field for every deflection, so the magnetic torque is $NBIA$. Balancing it against the restoring torque gives

$$NBIA = k\theta \Rightarrow \theta = \frac{NBA}{k}I.$$

Thus the deflection is directly proportional to the current, allowing small currents to be detected and compared on a uniform scale.

Step 2: (b) Explain why it is unsuitable as an ordinary ammeter First, the coil has appreciable resistance. Connecting it directly in series changes the total circuit resistance and hence changes the current that was to be measured; an ammeter should have very low resistance. Second, its fine coil and sensitive suspension can carry only a small current. An ordinary circuit current may drive it beyond full-scale deflection or damage it by heating. A low-resistance shunt in parallel diverts most of the current and converts it into an ammeter with a larger usable range.

Step 3: (c) Define voltage sensitivity and current sensitivity Voltage sensitivity is the angular deflection per unit potential difference applied across the galvanometer:

$$S_V = \frac{\theta}{V_g} = \frac{NBA}{kR_g}.$$

Here $V_g = IR_g$ and R_g is the galvanometer resistance; its unit is rad V^{-1} . Current sensitivity is the angular deflection per unit current through the galvanometer:

$$S_I = \frac{\theta}{I} = \frac{NBA}{k}.$$

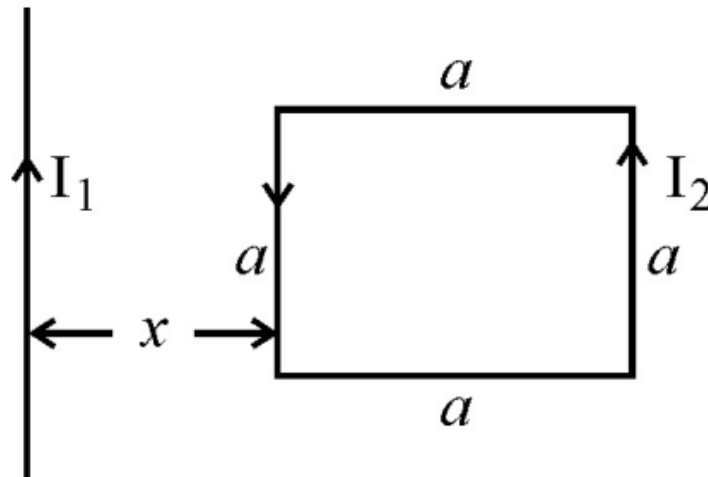
Its unit is rad A^{-1} , and $S_V = S_I/R_g$. A larger current sensitivity alone need not give larger voltage sensitivity if resistance also increases.

Step 4: Conclusion The galvanometer uses magnetic torque to produce $\theta \propto I$. Its appreciable resistance and small safe current range prevent direct use as an ordinary ammeter. Voltage sensitivity is θ/V_g , and current sensitivity is θ/I .

Quick Tip to Solve: Begin with the torque balance; divide deflection by current for current sensitivity, then use V equals I times coil resistance for voltage sensitivity.

14. (a) Define mutual inductance and write its S.I. unit.

(b) A square loop of side ' a ' carrying a current I_2 is kept at distance x from an infinitely long straight wire carrying a current I_1 as shown in the figure. Obtain the expression for the resultant force acting on the loop. [3 marks]



Solution:

Concept:

- Mutual inductance M is the magnetic flux linkage in one coil per unit current in the other: $N_2\Phi_{21} = MI_1$, for a fixed geometry and linear medium.
- The field of a long straight wire at perpendicular distance r is $B(r) = \mu_0 I_1 / (2\pi r)$. A current element experiences $d\vec{F} = I_2 d\vec{l} \times \vec{B}$.

Step 1: Define mutual inductance and its unit Mutual inductance is also numerically the induced emf in one coil when the current in the other changes at unit rate. Thus $\varepsilon_2 = -M dI_1/dt$, where the minus sign expresses Lenz's law. Its SI unit is the henry, symbol H, with $1 \text{ H} = 1 \text{ V s A}^{-1}$. Two coils have mutual inductance one henry if a change of current of 1 A s^{-1} in one induces an emf of magnitude 1 V in the other.

Step 2: Find the forces on the vertical sides In the printed figure the straight-wire current is upward, while the nearer side of the loop carries current downward. These antiparallel currents repel, so the nearer side is pushed away from the wire. The far side carries upward current and is attracted towards the wire. Each vertical side has length a and a constant distance from the wire, giving

$$F_{\text{near}} = \frac{\mu_0 I_1 I_2 a}{2\pi x}, \quad F_{\text{far}} = \frac{\mu_0 I_1 I_2 a}{2\pi(x+a)}.$$

The nearer-side force is greater because the magnetic field decreases with distance.

Step 3: Cancel the horizontal-side forces and combine The upper and lower sides have opposite current directions. At corresponding points they experience equal magnetic fields, since both span the same distances x to $x+a$. Their vertical forces are equal and opposite; indeed each has magnitude $\mu_0 I_1 I_2 \ln[(x+a)/x]/(2\pi)$. They therefore contribute no net force. Subtracting the vertical-side forces gives

$$F_{\text{net}} = \frac{\mu_0 I_1 I_2 a}{2\pi} \left(\frac{1}{x} - \frac{1}{x+a} \right) = \frac{\mu_0 I_1 I_2 a^2}{2\pi x(x+a)}.$$

Step 4: Conclusion Mutual inductance is flux linkage per unit current, measured in henry. The

resultant force is $\frac{\mu_0 I_1 I_2 a^2}{2\pi x(x+a)}$, directed away from the straight wire for the printed current directions.

Quick Tip to Solve: Read the current arrow on the nearer side first: it is antiparallel to the wire, so the stronger force is repulsive and fixes the net direction.

15. (a) Draw equipotential surfaces corresponding to the electric field that uniformly increases in magnitude along with the z -direction.

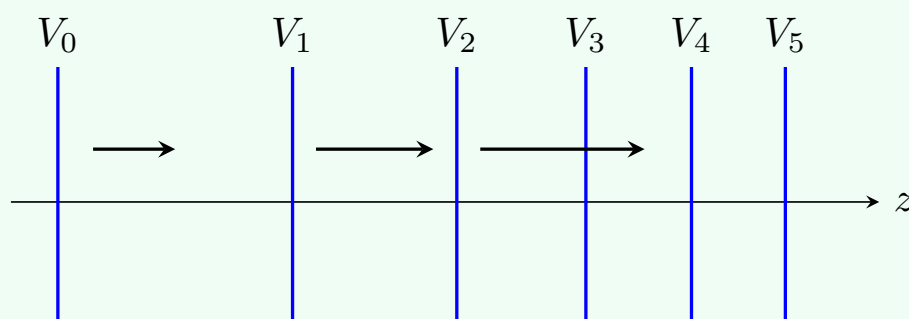
(b) Two charges $-q$ and $+q$ are located at points $(0, 0, -a)$ and $(0, 0, a)$. What is the electrostatic potential at the points $(0, 0, \pm z)$ and $(x, y, 0)$? [3 marks]

Solution:

Concept:

- Equipotential surfaces are perpendicular to $\vec{E}$, and $E_z = -dV/dz$. For a small separation, $|\Delta V| \simeq E\Delta z$.
- Potential is a scalar: $V = \frac{1}{4\pi\epsilon_0} \sum_i q_i/r_i$, taking zero potential at infinity. Here r_i is the distance from the observation point to charge q_i .

Step 1: Draw the equipotentials for an increasing field The electric field is along the z -direction and its magnitude increases as z increases. Hence the equipotentials are parallel planes perpendicular to the z -axis, each plane being parallel to the xy -plane. For equal successive potential differences, their separation decreases towards increasing z , because $\Delta z \simeq |\Delta V|/E$. If $E(z) = E_0 + kz$, with $k > 0$, integration gives $V(z) = V_0 - E_0 z - kz^2/2$; the potential decreases along the field. The drawing shows cross-sections of these planes, with closer spacing to the right.



$$\vec{E} \parallel +\hat{z}, \quad |\Delta V| \text{ equal}$$

$$V_0 > V_1 > V_2 > V_3 > V_4 > V_5$$

Sections of equipotential planes at equal potential intervals; the field grows towards increasing z .

Step 2: Add the two potentials on the positive axis At $(0, 0, +z)$, for $z > 0$, the distances from $+q$ at $+a$ and $-q$ at $-a$ are $|z - a|$ and $z + a$, respectively. Therefore, without making a far-field approximation,

$$V(0, 0, +z) = \frac{q}{4\pi\epsilon_0} \left(\frac{1}{|z - a|} - \frac{1}{z + a} \right).$$

For an external axial point $z > a$, this simplifies to

$$V(0, 0, +z) = \frac{1}{4\pi\epsilon_0} \frac{2qa}{z^2 - a^2}.$$

Step 3: Evaluate the negative axis and equatorial plane At $(0, 0, -z)$, the distances are interchanged, so

$$V(0, 0, -z) = \frac{q}{4\pi\epsilon_0} \left(\frac{1}{z+a} - \frac{1}{|z-a|} \right).$$

Thus the negative-axis potential is the negative of the positive-axis potential; for $z > a$ it is $-2qa/[4\pi\epsilon_0(z^2 - a^2)]$. At $(x, y, 0)$, both charges are equally distant, with $r = \sqrt{x^2 + y^2 + a^2}$, so their opposite contributions cancel:

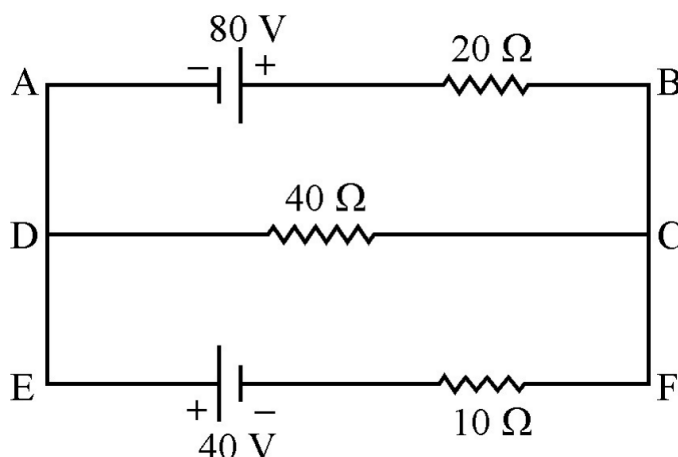
$$V(x, y, 0) = \frac{1}{4\pi\epsilon_0} \left(\frac{q}{r} - \frac{q}{r} \right) = 0.$$

The potential is undefined at either point charge itself, so the axial formulas exclude $z = a$.

Step 4: Conclusion Equipotentials are transverse parallel planes with decreasing separation towards increasing z for equal potential intervals. At external axial points $z > a$, $V(0, 0, \pm z) = \pm 2qa/[4\pi\epsilon_0(z^2 - a^2)]$; everywhere on the equatorial plane, $V(x, y, 0) = 0$.

Quick Tip to Solve: Write actual distances before adding signed potentials: equal distances on the equatorial plane give zero, while reversing an axial point reverses the sign.

16. Using Kirchhoff's rules, calculate the current through the $40\ \Omega$ and $20\ \Omega$ resistors in the following circuit: [3 marks]



Solution:

Concept:

- Kirchhoff's junction rule expresses charge conservation: the sum of currents entering a junction equals the sum leaving it.
- Kirchhoff's loop rule expresses energy conservation: the algebraic sum of potential changes round a closed loop is zero. A resistor gives $-IR$ along its current; a cell gives $+\epsilon$ on crossing from negative to positive terminal.

Step 1: Assign branch currents from the printed polarities The points A, D and E form the left node, while B, C and F form the right node. The upper 80 V cell has its positive terminal on the right, and the lower 40 V cell has its positive terminal on the left. Let i_1 flow from A to B through the $20\ \Omega$ resistor, and i_2 flow from F to E through the $10\ \Omega$ resistor. By the junction rule at the right node, the middle-branch current from C to D is $i_1 - i_2$. A negative result would indicate that a chosen current

actually flows in the reverse direction.

Step 2: Apply the loop rule to both independent loops Traverse A to B to C to D to A. The upper cell gives a rise of 80 V, and both resistor crossings follow their assigned currents:

$$80 - 20i_1 - 40(i_1 - i_2) = 0 \Rightarrow 60i_1 - 40i_2 = 80.$$

Now traverse D to C to F to E to D. Across the middle resistor we move opposite to its assigned current, giving a rise; the lower resistor gives a drop and its cell gives a rise:

$$40(i_1 - i_2) - 10i_2 + 40 = 0 \Rightarrow 40i_1 - 50i_2 = -40.$$

Step 3: Solve and check the branch currents Divide the first equation by 20 and the second by 10:

$$3i_1 - 2i_2 = 4, \quad 4i_1 - 5i_2 = -4.$$

Multiplying these equations by 5 and 2, respectively, and subtracting eliminates i_2 :

$$15i_1 - 10i_2 = 20, \quad 8i_1 - 10i_2 = -8, \quad 7i_1 = 28.$$

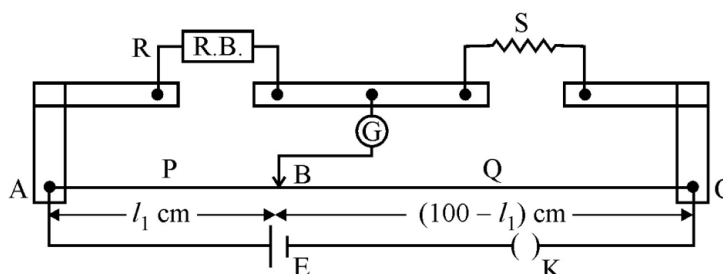
Thus $i_1 = 4$ A and $i_2 = (3i_1 - 4)/2 = 4$ A. The middle current $i_1 - i_2$ vanishes. As a check, the top branch's 80 V rise exactly equals its resistor drop $20 \times 4 = 80$ V, so the two nodes have equal potential. This agrees with zero current through the middle resistor.

Step 4: Conclusion The current through the 40Ω resistor is 0 A. The current through the 20Ω resistor is 4 A, flowing from A towards B (left to right).

Quick Tip to Solve: Preserve both cell polarities when writing loop equations: the outer loop gives $(80 + 40)/(20 + 10) = 4$ A, and equal node potentials then explain the zero middle current.

OR

16. What is end error in a metre bridge? How is it overcome? The resistances in the two arms of the metre bridge are $R = 5 \Omega$ and S respectively. When the resistance S is shunted with an equal resistance, the new balance length is found to be $1.5l_1$, where l_1 is the initial balancing length. Calculate the value of S . [3 marks]



Solution:

Concept:

- At balance, a metre bridge obeys $R/S = l/(100 - l)$, where l is the balance length in centimetres measured from the end adjacent to R .

- End error arises from the finite resistances of the end connections and copper strips. Two equal resistances S in parallel have equivalent resistance $S/2$.

Step 1: Explain end error and its reduction The copper strips, connecting leads and contacts at the ends of the bridge wire have small but nonzero resistances. These add to the effective resistances of the two wire portions, so their ratio is not exactly the ratio of the measured lengths. The resulting systematic error in the resistance obtained from the ideal balance formula is called end error. It is reduced by interchanging the known and unknown resistances in the two gaps, determining the unknown resistance in both arrangements and taking the mean. Choosing nearly equal gap resistances keeps the balance point near the middle of the wire and reduces the relative influence of the ends.

Step 2: Write the two balance conditions For the numerical calculation use the usual ideal bridge relation. Initially, with $R = 5\ \Omega$ in the left gap,

$$\frac{5}{S} = \frac{l_1}{100 - l_1}.$$

On shunting S with an equal resistance, the right-gap resistance becomes $S/2$, while the left gap remains unchanged. Since the new balance length is $1.5l_1$,

$$\frac{10}{S} = \frac{1.5l_1}{100 - 1.5l_1}.$$

The increased balance length is consistent with the increased ratio of left-gap to right-gap resistance.

Step 3: Eliminate the unknown resistance and substitute Divide the second balance equation by the first and cancel the nonzero l_1 :

$$\begin{aligned} 2 &= \frac{1.5(100 - l_1)}{100 - 1.5l_1}, \\ 200 - 3l_1 &= 150 - 1.5l_1, \\ l_1 &= \frac{100}{3} \text{ cm.} \end{aligned}$$

Hence $1.5l_1 = 50$ cm. Substitution in the initial balance equation gives

$$S = 5 \frac{100 - l_1}{l_1} = 5 \frac{200/3}{100/3} \Omega = 10\ \Omega.$$

Step 4: Conclusion End error is the systematic error caused by finite end-connection resistances; interchange the two gap resistances and average the results to reduce it. The unknown resistance is $S = 10\ \Omega$.

Quick Tip to Solve: Divide the two bridge equations first: the shunt doubles R/S , letting you find the balance length without S .

17. (a) Write the relation between half life and average life of a radioactive nucleus.

(b) In a given sample two isotopes A and B are initially present in the ratio of 1:2. Their half lives are 60 years and 30 years respectively. How long will it take so that the sample has these isotopes in the ratio of 2:1? [3 marks]

Solution:

Concept:

- Radioactive decay obeys $N(t) = N_0 e^{-\lambda t} = N_0 2^{-t/T_{1/2}}$, where N_0 is the initial number of nuclei and λ is the decay constant.
- The half-life is $T_{1/2} = \ln 2 / \lambda$ and the average life is $\tau = 1 / \lambda$.

Step 1: (a) Relate half-life and average life At the half-life the number of surviving nuclei is half its initial value, so $e^{-\lambda T_{1/2}} = 1/2$. Taking logarithms gives $\lambda T_{1/2} = \ln 2$. With average life $\tau = 1/\lambda$, the required relation is

$$T_{1/2} = (\ln 2)\tau \simeq 0.693\tau.$$

Equivalently, $\tau = T_{1/2}/0.693$. Both quantities have the same units of time; the average life exceeds the half-life.

Step 2: (b) Express the changing isotope ratio Let the initial populations be $N_{A0} = n$ and $N_{B0} = 2n$. For t measured in years, the surviving populations are

$$N_A = n 2^{-t/60}, \quad N_B = 2n 2^{-t/30}.$$

Therefore,

$$\frac{N_A}{N_B} = \frac{1}{2} 2^{-t/60+t/30} = \frac{1}{2} 2^{t/60}.$$

Isotope B decays faster because its half-life is shorter, so this ratio must increase from its initial value of $1/2$.

Step 3: Apply the required ratio and check The final ratio $2 : 1$ means $N_A/N_B = 2$, not $1/2$. Thus

$$\frac{1}{2} 2^{t/60} = 2 \Rightarrow 2^{t/60} = 4 = 2^2 \Rightarrow t = 120 \text{ years}.$$

As a check, A undergoes two half-lives and B undergoes four half-lives in this time. Their populations become $n/4$ and $2n/16 = n/8$, whose ratio is indeed $2 : 1$. The calculation refers to the surviving nuclei of the two isotopes.

Step 4: Conclusion $T_{1/2} = 0.693\tau$, where τ is the average life. The required isotope ratio is reached after 120 years.

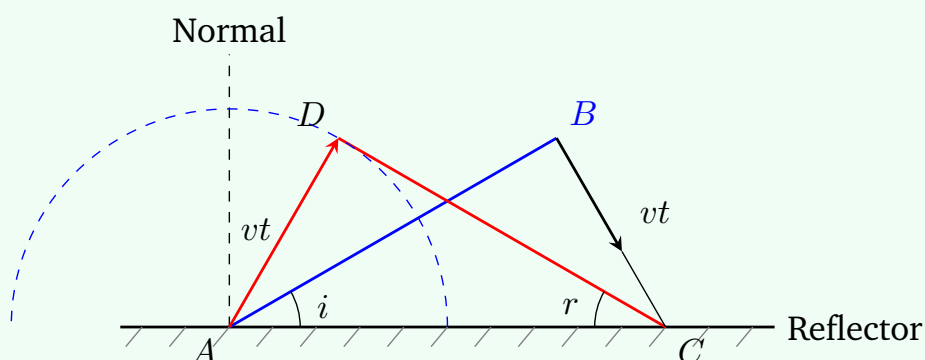
Quick Tip to Solve: Track the ratio of surviving populations: the common initial population cancels, and the shorter-lived isotope makes A/B increase.

Solution:**Concept:**

- A wavefront is the locus of points in a wave that are in the same phase at a given instant. Rays are normal to the wavefront in an isotropic medium.
- Huygens' principle states that every point on a wavefront acts as a source of secondary wavelets; the forward envelope of these wavelets forms the subsequent wavefront.
- During reflection the incident and reflected waves travel in the same medium, so both have the same speed v . Angles of incidence i and reflection r are measured from the surface normal.

Step 1: Define the wavefront and set up the construction A wavefront is a surface joining all points of a wave that have the same phase at a particular instant. For a plane wave it is a plane perpendicular to the direction of propagation. Consider the plane reflecting surface shown below and an incident plane wavefront AB . Point A touches the surface first, while point B is still approaching it. Let the incident ray through B , perpendicular to AB , meet the reflecting surface at C after time t .

Step 2: Construct the reflected wavefront In time t , point B travels the distance $BC = vt$. During this same time the secondary wavelet originating at A grows to radius $AD = vt$ in the incident medium. Draw the tangent CD from C to this wavelet; the envelope CD is the reflected wavefront. Since the radius to a tangent is perpendicular to it, $AD \perp CD$. Thus AD gives the direction of a reflected ray. Wavelets originating at the intervening points of the surface have this same tangent envelope.



Huygens' construction: AB is incident, CD is reflected, and $BC = AD = vt$.

Step 3: Compare the right triangles and verify reflection Triangles ABC and ADC are right-angled at B and D , respectively. They have the same hypotenuse AC and equal sides $BC = AD = vt$, so they are congruent. Equivalently,

$$\sin \angle BAC = \frac{BC}{AC} = \frac{AD}{AC} = \sin \angle DCA.$$

The angle between the incident wavefront AB and surface AC equals i , because their respective perpendiculars are the incident ray and the surface normal. Similarly, the angle between reflected wavefront CD and the surface equals r . For the physical angles between zero and a right angle, the above equality gives $i = r$. The construction takes place in the plane containing the incident ray and

surface normal; the reflected ray lies in that same plane.

Step 4: Conclusion A wavefront is a surface of equal phase. Huygens' construction verifies $i = r$; the incident ray, reflected ray and normal to the reflecting surface all lie in the same plane.

Quick Tip to Solve: Use the same travel time on both sides of the construction: $BC = AD = vt$, so the two right triangles share a hypotenuse and an equal leg.

OR

18. Define the term, “refractive index” of a medium. Verify Snell’s law of refraction when a plane wavefront is propagating from a denser to a rarer medium. [3 marks]

Solution:

Concept:

- The absolute refractive index of a medium is $n = c/v$, where c is the speed of light in vacuum and v is its speed in the medium.
- Huygens' principle states that every point of a wavefront acts as a source of secondary wavelets; their common tangent gives the new wavefront.
- A ray is perpendicular to its wavefront. The angles of incidence i and refraction r are measured from the normal to the interface.

Step 1: Define refractive index and set up the construction The refractive index compares the speed of light in vacuum with its speed in a medium: $n = c/v$, a dimensionless quantity. Let a plane wavefront AB travel in a denser medium of index n_1 and speed v_1 , towards a plane boundary with a rarer medium of index $n_2 < n_1$ and speed $v_2 > v_1$. Point A reaches the boundary first. In time t , the other end B reaches the boundary at C , so $BC = v_1 t$.

Step 2: Construct the refracted wavefront During the same time, the secondary wavelet from A in the rarer medium has radius $AD = v_2 t$. Draw the tangent CD from C to this wavelet. It is the refracted wavefront, and AD is perpendicular to CD . Since BC is perpendicular to the incident wavefront AB , triangles ABC and ADC are right triangles. The angles that AB and CD make with the boundary AC equal the corresponding ray angles i and r with the normal. Thus

$$\sin i = \frac{BC}{AC} = \frac{v_1 t}{AC}, \quad \sin r = \frac{AD}{AC} = \frac{v_2 t}{AC}.$$

Step 3: Obtain Snell's relation Taking the ratio cancels the common time and distance:

$$\frac{\sin i}{\sin r} = \frac{v_1}{v_2} = \frac{c/n_1}{c/n_2} = \frac{n_2}{n_1}.$$

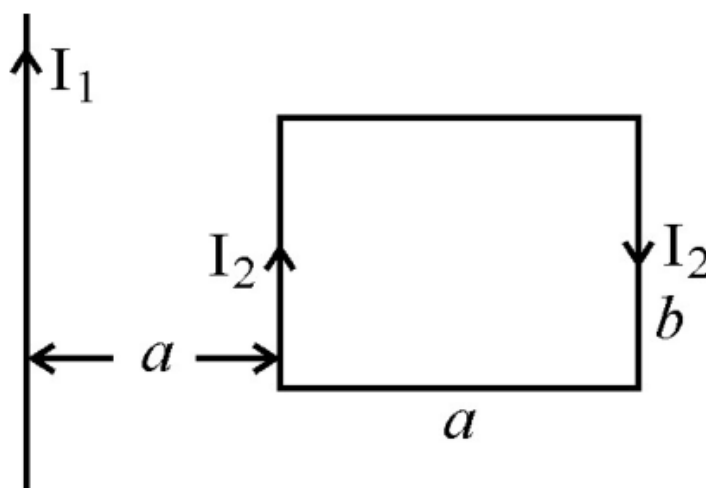
Since $v_2 > v_1$, the refracted angle exceeds the incident angle and the ray bends away from the normal. The tangent construction applies when i is below the critical angle; at the critical angle the refracted ray grazes the interface.

Step 4: Conclusion The refractive index is $n = c/v$. Huygens' construction gives $n_1 \sin i = n_2 \sin r$, verifying Snell's law for refraction from the denser to the rarer medium.

Quick Tip to Solve: Compare the two right triangles using the same hypotenuse AC; the two wavelet distances immediately give the sine ratio.

19. (a) Define the term 'self inductance' of a coil. Write its S.I. unit.

(b) A rectangular loop of sides a and b carrying current I_2 is kept at a distance ' a ' from an infinitely long straight wire carrying a current I_1 as shown in the figure. Obtain the expression for the resultant force acting on the loop. [3 marks]



Solution:

Concept:

- Self-inductance L relates a coil's flux linkage to its current: $N\Phi = LI$. The induced emf is $\varepsilon = -L dI/dt$.
- The magnetic field a distance r from a long straight wire is $B(r) = \mu_0 I_1 / (2\pi r)$.
- A straight conductor of length ℓ , carrying current perpendicular to a magnetic field, experiences force $F = I\ell B$. Parallel currents attract and antiparallel currents repel.

Step 1: (a) Define self-inductance and its unit Self-inductance is the flux linkage of a coil per unit current through it, $L = N\Phi/I$, for a coil with fixed geometry and a linear magnetic medium. Equivalently, it is the magnitude of the self-induced emf per unit rate of change of current: $L = |\varepsilon|/|dI/dt|$. Its S.I. unit is the henry, H. A coil has inductance 1 H if a current changing at 1 A s^{-1} induces an emf of magnitude 1 V, so $1 \text{ H} = 1 \text{ V s A}^{-1}$.

Step 2: (b) Find the forces on the two vertical sides In the printed figure, the loop has horizontal width a and vertical length b . Its nearer vertical side is at distance a from the wire, and its farther vertical side is at distance $2a$. The nearer side carries I_2 upwards, parallel to I_1 , and is attracted towards the wire:

$$F_{\text{near}} = I_2 b B(a) = \frac{\mu_0 I_1 I_2 b}{2\pi a}.$$

The farther side carries current downwards, opposite to I_1 , and is repelled away from the wire:

$$F_{\text{far}} = I_2 b B(2a) = \frac{\mu_0 I_1 I_2 b}{4\pi a}.$$

The nearer force is larger because the magnetic field decreases with distance.

Step 3: Cancel the horizontal-side forces and combine the remainder The top and bottom sides carry currents in opposite directions. For each pair of corresponding length elements at the same distance r from the straight wire, the field is identical, so the vertical forces have equal magnitudes and opposite directions. Their integrated forces therefore cancel, even though the field varies along each horizontal side. The two vertical-side forces oppose one another, giving

$$F_{\text{net}} = F_{\text{near}} - F_{\text{far}} = \frac{\mu_0 I_1 I_2 b}{2\pi} \left(\frac{1}{a} - \frac{1}{2a} \right) = \frac{\mu_0 I_1 I_2 b}{4\pi a}.$$

Step 4: Conclusion (a) $L = N\Phi/I$, with S.I. unit henry (H). (b) The resultant force on the loop has magnitude $\mu_0 I_1 I_2 b / (4\pi a)$ and acts towards the straight wire.

Quick Tip to Solve: Use b for the force-producing parallel sides and distances a and $2a$; subtract their forces because attraction and repulsion oppose each other.

20. (a) Describe briefly the functions of the three segments of n-p-n transistor.

(b) Draw the circuit arrangement for studying the output characteristics of n-p-n transistor in CE configuration. Explain how the output characteristics is obtained. [3 marks]

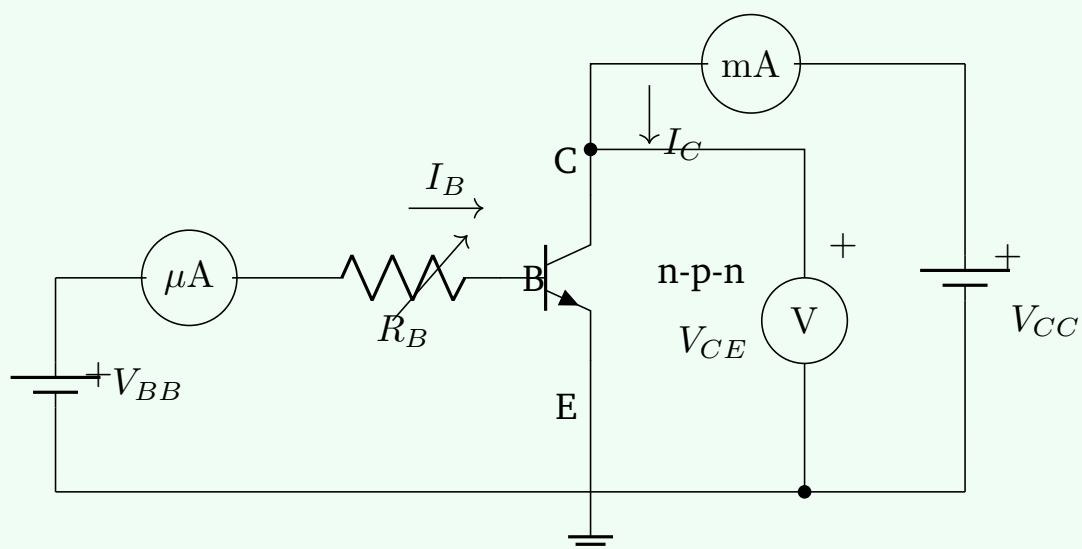
Solution:

Concept:

- An n-p-n transistor has a heavily doped n-type emitter, a thin lightly doped p-type base, and a larger moderately doped n-type collector.
- In active operation, the emitter-base junction is forward biased and the collector-base junction is reverse biased. The currents satisfy $I_E = I_B + I_C$.
- In common-emitter (CE) configuration, the emitter is common to the input and output circuits. Output characteristics plot collector current I_C against collector-emitter voltage V_{CE} at constant base current I_B .

Step 1: (a) Explain the three transistor segments The emitter supplies the majority charge carriers: its heavy n-type doping allows it to inject many electrons into the base when the emitter-base junction is forward biased. The base is very thin and lightly doped, so only a small fraction of these electrons recombines with holes, producing the small base current. Most injected electrons cross the base. The collector collects these electrons under the reverse bias of the collector-base junction, giving the collector current. It is made physically larger to dissipate the heat generated in the output circuit.

Step 2: (b) Set up the CE circuit Connect the emitter to the common negative terminals of the two d.c. supplies. Connect the positive terminal of the adjustable input supply V_{BB} to the base through a variable resistance R_B and a microammeter measuring I_B . Connect the collector to the positive terminal of an adjustable output supply V_{CC} through a milliammeter measuring I_C . A voltmeter across collector and emitter measures V_{CE} , with its positive terminal connected to the collector. The circuit below shows the n-p-n emitter arrow directed outwards.



CE circuit for measuring n-p-n transistor output characteristics.

Step 3: Obtain the family of output characteristics Adjust R_B to set a chosen base current I_B . Increase the collector supply gradually and record corresponding values of V_{CE} and I_C . During this sweep, readjust the input control when necessary so that I_B remains constant. Plot I_C on the vertical axis against V_{CE} on the horizontal axis. Repeat for several larger fixed base currents to obtain a family of curves. At very small V_{CE} , the current rises rapidly in the saturation region. In the active region, I_C is nearly independent of V_{CE} at a fixed I_B , and a larger I_B gives a larger I_C . Keep the output voltage below breakdown.

Step 4: Conclusion The emitter injects electrons, the thin base controls their flow with little recombination, and the collector collects them. CE output characteristics are the family of I_C -versus- V_{CE} curves measured at different constant values of I_B .

Quick Tip to Solve: For output characteristics, vary VCE while holding IB fixed; the emitter is the common return for both supplies.

OR

20. Draw the circuit diagram of a full wave rectifier and explain its working. Also, give the input and output waveforms. [3 marks]

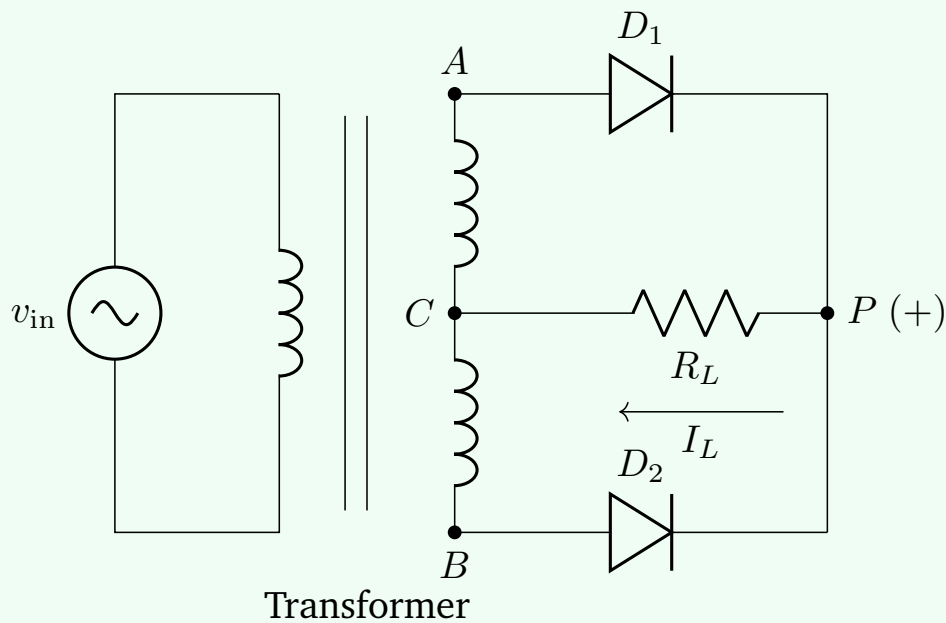
Solution:

Concept:

- A p-n junction diode conducts appreciably when forward biased and blocks current when reverse biased. A rectifier uses this one-way conduction to convert alternating voltage into unidirectional voltage.
- In a centre-tapped full-wave rectifier, the two ends of the secondary have opposite instantaneous polarities relative to the centre tap. The diodes conduct alternately, but load current has the same direction during both half-cycles.

Step 1: Circuit of the full-wave rectifier Connect the ends A and B of a centre-tapped transformer secondary to the anodes of diodes D_1 and D_2 . Join their cathodes at the positive output terminal P . Connect the load resistance R_L between P and the centre tap C . The transformer primary receives

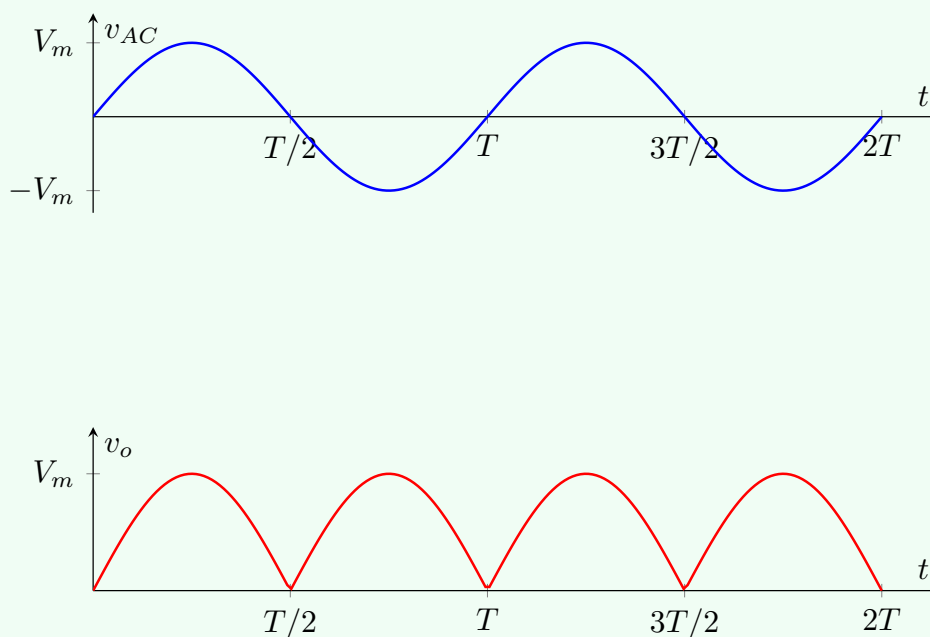
the alternating input. The secondary voltages v_{AC} and v_{BC} are equal in amplitude and opposite in phase.



Centre-tapped full-wave rectifier; both diode cathodes join at P.

Step 2: Working during the two half-cycles When A is positive relative to C, B is negative: D_1 is forward biased and D_2 is reverse biased. Current flows from A through D_1 , then from P through R_L to C. During the next half-cycle, B becomes positive relative to C. Now D_2 conducts and D_1 blocks; current flows from B through D_2 and again from P through R_L to C. Thus both half-cycles contribute to the output.

Step 3: Input and output waveforms For ideal diodes, write the input referred to one half of the secondary as $v_{AC} = V_m \sin \omega t$, where V_m is its peak voltage. The load voltage is then $v_o = V_m |\sin \omega t|$. The negative input half-cycles appear as positive output half-cycles. The output is pulsating dc; its ripple frequency is $2f$ for an input frequency f . Without a smoothing filter, the output is not a constant voltage. In practice, each conducting path also has one diode's forward voltage drop.



Sinusoidal input and full-wave rectified output for ideal diodes.

Step 4: Conclusion A centre-tapped full-wave rectifier uses alternate conduction of D_1 and D_2 to produce load current in the same direction throughout both half-cycles. Its output waveform consists of positive pulses at twice the input frequency.

Quick Tip to Solve: Trace both paths through the load: each must enter the same end of R_L , although the conducting diode changes.

21. (a) If A and B represent the maximum and minimum amplitudes of an amplitude modulated wave, write the expression for the modulation index in terms of A and B .

(b) A message signal of frequency 20 kHz and peak voltage 10 V is used to modulate a carrier of frequency 2 MHz and peak voltage of 15 V. Calculate the modulation index. Why the modulation index is generally kept less than one? [3 marks]

Solution:

Concept:

- The modulation index of an amplitude-modulated wave is $\mu = V_m/V_c$, where V_m is the peak message voltage and V_c is the peak unmodulated carrier voltage, for the stated AM convention.
- For undistorted AM, the envelope has maximum amplitude $A = V_c(1 + \mu)$ and minimum amplitude $B = V_c(1 - \mu)$. Ordinary envelope detection requires the envelope not to reverse sign.

Step 1: (a) Express the modulation index using the envelope amplitudes The message signal varies the carrier amplitude above and below its unmodulated value. Subtracting and adding the extreme amplitudes gives

$$A - B = 2\mu V_c, \quad A + B = 2V_c.$$

Dividing these two relations eliminates the unknown carrier amplitude:

$$\mu = \frac{A - B}{A + B}.$$

Here A and B are the positive maximum and minimum envelope amplitudes, rather than the instantaneous positive and negative peaks of the rapidly oscillating carrier.

Step 2: (b) Substitute the given message and carrier voltages The peak message voltage is $V_m = 10\text{ V}$, and the peak carrier voltage is $V_c = 15\text{ V}$. Therefore,

$$\mu = \frac{10\text{ V}}{15\text{ V}} = \frac{2}{3} \simeq 0.667.$$

The modulation percentage is $100\mu \simeq 66.7\%$. As a check, the envelope extremes are $A = 15 + 10 = 25\text{ V}$ and $B = 15 - 10 = 5\text{ V}$, which give the same ratio $(25 - 5)/(25 + 5) = 2/3$. The frequencies 20 kHz and 2 MHz describe the rates of variation of message and carrier; the modulation depth here depends on their amplitude ratio, not their frequency ratio.

Step 3: Explain why the modulation index is kept below unity When $\mu < 1$, the envelope factor $1 + \mu \cos \omega_m t$ remains positive, so its shape follows the message without an envelope crossing. If $\mu > 1$, the modulation is excessive: this factor changes sign near a message trough, producing envelope distortion and phase reversals. A simple envelope detector then fails to reproduce the original message faithfully. Unity corresponds to complete modulation, with the envelope just reaching zero; keeping the practical value below unity provides a margin against overmodulation.

Step 4: Conclusion The required expression is $\mu = (A - B)/(A + B)$. For the given voltages, $\mu = 2/3 \simeq 0.667$, or about 66.7% . It is generally kept below one to avoid overmodulation and distortion of the recovered message.

Quick Tip to Solve: Use peak message voltage divided by peak carrier voltage; the two frequencies do not enter this modulation-index calculation.

22. (a) State Gauss's law for magnetism. Explain its significance.

(b) Write the four important properties of the magnetic field lines due to a bar magnet.

[3 marks]

Solution:

Concept:

- Magnetic flux through a surface element is $d\Phi_B = \mathbf{B} \cdot d\mathbf{S}$, where $\mathbf{B}$ is the magnetic field and $d\mathbf{S}$ points along the outward normal for a closed surface.
- Magnetic field lines represent the direction and relative strength of a field. Their tangent gives its local direction, while greater line density indicates a stronger field.

Step 1: (a) State Gauss's law for magnetism The total magnetic flux through every closed surface is zero:

$$\oint_S \mathbf{B} \cdot d\mathbf{S} = 0.$$

The integral is over the entire closed surface S , using outward-directed surface normals. Flux leaving the surface contributes positively and flux entering it contributes negatively. Thus the outward and inward magnetic fluxes balance exactly; the result applies regardless of the shape of the surface.

Step 2: Explain the significance of the law The law expresses the absence of isolated magnetic poles in ordinary magnetism. Magnetic field lines have no isolated starting or ending points: those associated with a bar magnet form continuous loops. Even if a closed surface encloses only one end of a magnet, the field lines entering elsewhere balance those leaving near that end. Cutting a bar magnet

produces smaller magnets, each with both a north and a south pole, rather than a separated magnetic monopole. Zero net flux does not mean the magnetic field is zero at all points on the surface.

Step 3: (b) Four properties of the field lines of a bar magnet (1) The lines form continuous closed loops. Outside the bar magnet they run from its north pole to its south pole; inside, they return from south to north.

(2) The tangent to a field line at any point gives the direction of the magnetic field at that point, or the direction in which a small compass north pole would point.

(3) The lines are more closely spaced where the magnetic field is stronger. For a bar magnet they are crowded near the poles and more widely separated farther away.

(4) Two field lines never intersect. An intersection would assign two different directions to the magnetic field at the same point, whereas the field has a unique direction there.

Step 4: Conclusion Gauss's law for magnetism is $\oint_S \mathbf{B} \cdot d\mathbf{S} = 0$, signifying no isolated magnetic poles and continuous magnetic flux. Bar-magnet field lines form closed loops, give field direction through their tangents, indicate strength through their spacing, and never intersect.

Quick Tip to Solve: State the direction both outside and inside the magnet when explaining closed loops; zero net flux does not imply zero field.

OR

22. Write three points of differences between para-, dia- and ferro-magnetic materials, giving one example for each. [3 marks]

Solution:

Concept:

- Magnetic susceptibility χ_m measures the magnetisation M produced by an applied magnetising field H : $M = \chi_m H$.
- The relative permeability is $\mu_r = 1 + \chi_m$. It describes how the magnetic flux density in a material compares with that in vacuum for the same applied field.
- Diamagnetic induced moments oppose the applied field; paramagnetic permanent moments tend to align with it; ferromagnetic domains can align strongly with it.

Step 1: Compare their response to an external field Paramagnetic materials are weakly attracted by a magnet. Their net magnetisation is along the applied field, and a specimen tends to move from a weaker to a stronger field region. Diamagnetic materials are weakly repelled: their induced magnetisation opposes the applied field, and they move from a stronger to a weaker field region. Ferromagnetic materials are strongly attracted and acquire a large magnetisation along the field because many magnetic domains become aligned.

Step 2: Compare susceptibility For paramagnetic materials, χ_m is small and positive. For diamagnetic materials, it is small and negative. For ferromagnetic materials, it is large and positive, and its effective value depends on the field and the previous magnetisation history. Thus equal applied fields produce small parallel, small antiparallel and large parallel magnetisations, respectively.

Step 3: Compare relative permeability and give examples Paramagnetic materials have μ_r slightly greater than unity; aluminium is an example. Diamagnetic materials have μ_r slightly less than unity; bismuth is an example. Ferromagnetic materials have μ_r much greater than unity; iron is an example.

These are three comparisons: response to the field, susceptibility, and relative permeability. The susceptibility and permeability comparisons are related through $\mu_r = 1 + \chi_m$.

Step 4: Conclusion Paramagnetic aluminium is weakly attracted, with small positive χ_m and $\mu_r > 1$; diamagnetic bismuth is weakly repelled, with small negative χ_m and $\mu_r < 1$; ferromagnetic iron is strongly attracted, with large positive χ_m and $\mu_r \gg 1$.

Quick Tip to Solve: Remember the sign first: diamagnetism opposes the field, so susceptibility is negative; para and ferro align with the field, with ferro much stronger.

23. Identify the part of the electromagnetic spectrum used in (i) radar and (ii) eye surgery. Write their frequency range.

Prove that the average energy density of the oscillating electric field is equal to that of the oscillating magnetic field. [3 marks]

Solution:

Concept:

- Radar uses microwaves, while ultraviolet radiation is used in laser eye surgery. Electromagnetic spectral boundaries are approximate; frequency and wavelength satisfy $f = c/\lambda$.
- In vacuum, the instantaneous electric and magnetic energy densities are $u_E = \frac{1}{2}\epsilon_0 E^2$ and $u_B = B^2/(2\mu_0)$, respectively.
- The wave amplitudes satisfy $E_0 = cB_0$, where $c^2 = 1/(\mu_0\epsilon_0)$. The time average of $\cos^2$ over one cycle is $1/2$.

Step 1: Identify the two spectral regions (a) Radar transmits microwaves and detects their reflected signal from a target. A commonly quoted approximate microwave frequency range is 10^9 to 10^{12} Hz. Eye surgery uses ultraviolet radiation, for example an excimer laser, whose high-frequency photons can remove a controlled layer of corneal tissue. An approximate ultraviolet range is 10^{15} to 10^{17} Hz. These are broad spectral ranges rather than the operating frequency of one instrument; adjacent regions do not have sharp physical boundaries.

Step 2: Average the electric energy density (b) Consider a sinusoidal plane electromagnetic wave travelling in vacuum. At a fixed point, write $E = E_0 \cos(\omega t - kx)$ and $B = B_0 \cos(\omega t - kx)$; the two fields oscillate in phase. Therefore,

$$\begin{aligned}\langle u_E \rangle &= \frac{1}{2}\epsilon_0 E_0^2 \langle \cos^2(\omega t - kx) \rangle \\ &= \frac{1}{4}\epsilon_0 E_0^2.\end{aligned}$$

Here the brackets denote the time average over a complete oscillation.

Step 3: Average the magnetic energy density and compare Applying the same cycle average to the magnetic contribution gives

$$\langle u_B \rangle = \frac{B_0^2}{4\mu_0}.$$

Substitute $B_0 = E_0/c$ and then $1/(\mu_0 c^2) = \epsilon_0$:

$$\begin{aligned}\langle u_B \rangle &= \frac{E_0^2}{4\mu_0 c^2} \\ &= \frac{1}{4}\epsilon_0 E_0^2 = \langle u_E \rangle.\end{aligned}$$

Thus equal fractions of the wave's average energy density reside in the electric and magnetic fields. The average total energy density is their sum, $\epsilon_0 E_0^2/2$, with SI unit J m^{-3} .

Step 4: Conclusion Radar uses microwaves, approximately 10^9 – 10^{12} Hz; eye surgery uses ultraviolet radiation, approximately 10^{15} – 10^{17} Hz. The equal average energy densities are $\langle u_E \rangle = \langle u_B \rangle = \epsilon_0 E_0^2/4 = B_0^2/(4\mu_0)$.

Quick Tip to Solve: First average the squared field, introducing a factor of one-half, then use $E_0 = cB_0$; using peak fields without averaging doubles both energy densities.

24. Three photo diodes D_1 , D_2 and D_3 are made of semiconductors having band gaps of 2.5 eV, 2 eV and 3 eV respectively. Which of them will not be able to detect light of wavelength 600 nm?

Why photodiodes are required to operate in reverse bias? Explain.

[3 marks]

Solution:

Concept:

- A photodiode detects light through the generation of electron-hole pairs. A photon must have energy $E_\gamma = hc/\lambda$ at least equal to the semiconductor band gap E_g .
- Use $hc \simeq 1240 \text{ eV nm}$, so E_γ in electronvolts equals $1240/\lambda$ when wavelength is given in nanometres.
- Reverse bias widens the depletion region and strengthens its electric field, which separates and collects photogenerated carriers to produce a photocurrent.

Step 1: Calculate the incident photon energy (a) All three photodiodes receive light of wavelength 600 nm, so the available energy per photon is the same for them. Using the energy-wavelength relation,

$$E_\gamma = \frac{hc}{\lambda} = \frac{1240 \text{ eV nm}}{600 \text{ nm}} \simeq 2.07 \text{ eV}.$$

Detection requires a transition from the valence band into the conduction band, creating an electron-hole pair. A photon whose energy is smaller than the band gap cannot produce this transition in the ideal band-gap model used here.

Step 2: Compare the photon energy with each band gap For D_1 , the band gap is 2.5 eV, which is greater than 2.07 eV; the photon has insufficient energy to generate an electron-hole pair. For D_2 , the band gap is 2.0 eV, which is less than the incident photon energy, so the transition is energetically allowed. For D_3 , the band gap is 3.0 eV, also greater than the available photon energy. The comparison concerns energy per photon, not the total energy in a bright light beam.

Step 3: Explain the role of reverse bias (b) The reverse-biased junction has a wide depletion region with a strong electric field. When suitable photons are absorbed in or near this region, the field drives the generated electrons towards the n-side and holes towards the p-side, collecting them before substantial recombination. The resulting additional reverse current increases with light intensity. The dark reverse current is small, so the change caused by illumination is readily measured; reverse bias

also lowers junction capacitance and improves response speed. In forward bias, the much larger injected-carrier current can obscure the small photocurrent. These features make reverse bias the normal operating mode for the photodiode detector considered in this question.

Step 4: Conclusion D_1 and D_3 cannot detect the 600 nm light because their band gaps exceed 2.07 eV; D_2 can detect it. Reverse bias enables efficient separation and collection of photogenerated carriers and allows the light-dependent current to be measured against a small dark current.

Quick Tip to Solve: Compute $1240/\lambda$ in eV using wavelength in nm, then reject every diode with a larger band gap; greater light intensity does not increase energy per photon.

Section D (15 marks)

25. Describe any two characteristic features which distinguish between interference and diffraction phenomena. Derive the expression for the intensity at a point of the interference pattern in Young's double slit experiment.

In the diffraction due to a single slit experiment, the aperture of the slit is 3 mm. If monochromatic light of wavelength 620 nm is incident normally on the slit, calculate the separation between the first order minima and the 3rd order maxima on one side of the screen. The distance between the slit and the screen is 1.5 m. [5 marks]

Solution:

Concept:

- Interference is the superposition of waves from two coherent sources. Diffraction is the superposition of secondary wavelets from different portions of the same aperture.
- The intensity of a light wave is proportional to the square of its amplitude. A path difference Δ produces a phase difference $\phi = 2\pi\Delta/\lambda$, where λ is the wavelength.
- For a slit of width a , minima obey $a \sin \theta = n\lambda$. In the usual school-level approximation, the n -th secondary maximum obeys $a \sin \theta \simeq (n + \frac{1}{2})\lambda$, with $n = 1, 2, 3, \dots$

Step 1: (a) Distinguish the two patterns First, for two narrow, equally illuminated coherent slits, interference bright fringes have equal intensity; in single-slit diffraction the central maximum is strongest and successive secondary maxima become weaker. Second, ideal Young interference fringes have equal width and uniform spacing. In single-slit diffraction, the central maximum is twice as wide as a secondary maximum, so the central bright band is broader than the others. These comparisons refer to the ideal two-slit interference pattern without considering its diffraction envelope.

Step 2: (a) Derive the resultant intensity Let the two waves arriving at a point P have amplitudes A_1, A_2 , angular frequency ω , and phase difference ϕ :

$$y_1 = A_1 \cos \omega t, \quad y_2 = A_2 \cos(\omega t + \phi).$$

Adding their cosine and sine components gives the square of the resultant amplitude:

$$A^2 = (A_1 + A_2 \cos \phi)^2 + (A_2 \sin \phi)^2 = A_1^2 + A_2^2 + 2A_1A_2 \cos \phi.$$

If $I_1 = kA_1^2$ and $I_2 = kA_2^2$, where k is the same proportionality constant for both waves, then

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \phi.$$

For slit separation d , screen distance D , and transverse coordinate x , the small-angle path difference is $\Delta \simeq dx/D$. Thus $\phi = 2\pi dx/(\lambda D)$. For equal slit intensities $I_1 = I_2 = I_0$,

$$I(x) = 2I_0(1 + \cos \phi) = 4I_0 \cos^2\left(\frac{\pi dx}{\lambda D}\right).$$

Constructive interference has $\phi = 2m\pi$ and intensity $4I_0$; destructive interference has $\phi = (2m + 1)\pi$ and zero intensity.

Step 3: (b) Locate the required diffraction bands Use $a = 3 \times 10^{-3} \text{ m}$, $\lambda = 620 \times 10^{-9} \text{ m}$, and $D = 1.5 \text{ m}$. Since the angles are small, $y \simeq D \sin \theta$. The first minimum and the third secondary maximum on the same side therefore occur at

$$y_{\min,1} = \frac{\lambda D}{a}, \quad y_{\max,3} \simeq \frac{7\lambda D}{2a}.$$

The basic displacement is

$$\frac{\lambda D}{a} = \frac{620 \times 10^{-9} \times 1.5}{3 \times 10^{-3}} = 3.10 \times 10^{-4} \text{ m} = 0.310 \text{ mm}.$$

Hence the two positions are 0.310 mm and approximately 1.085 mm. Because they lie on the same side of the central maximum, subtract these distances:

$$\Delta y \simeq (3.5 - 1) \frac{\lambda D}{a} = 7.75 \times 10^{-4} \text{ m}.$$

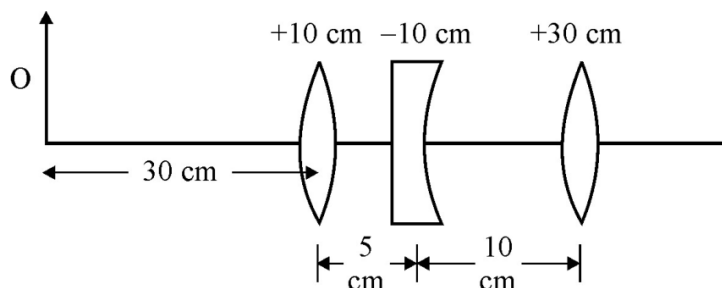
The secondary-maximum condition used here is approximate; unlike the minima, the maxima are not exactly midway between adjacent minima.

Step 4: Conclusion The interference intensity is $I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \phi$; for equal slits, $I(x) = 4I_0 \cos^2[\pi dx/(\lambda D)]$. Using the standard secondary-maximum approximation, the required separation is 0.775 mm.

Quick Tip to Solve: Count secondary maxima from the central band: the third is near $3.5\lambda D/a$, and same-side separations require subtraction.

25. Under what conditions is the phenomenon of total internal reflection of light observed? Obtain the relation between the critical angle of incidence and the refractive index of the medium.

Three lenses of focal lengths $+10\text{ cm}$, -10 cm and $+30\text{ cm}$ are arranged coaxially as in the figure given below. Find the position of the final image formed by the combination. [5 marks]



Solution:

Concept:

- Total internal reflection occurs when light travels from a medium of refractive index n_1 to one with $n_2 < n_1$, and its angle of incidence exceeds the critical angle C .
- Snell's law is $n_1 \sin i = n_2 \sin r$. The critical angle is the incidence angle for which the refracted ray just grazes the interface, so $r = 90^\circ$.
- For each thin lens, $1/v - 1/u = 1/f$. Distances are measured from that lens with the Cartesian sign convention; along the incident light direction they are positive.

Step 1: (a) State the conditions and obtain the critical-angle relation The two necessary conditions are: light must be incident from the optically denser medium toward the optically rarer medium, and the angle of incidence must be greater than the critical angle for that pair of media. At $i = C$, refraction still occurs with the refracted ray along the boundary. For $i > C$, no propagating refracted ray is possible in the rarer medium, and the light is reflected back into the denser medium. Applying Snell's law at the critical angle gives

$$n_1 \sin C = n_2 \sin 90^\circ = n_2, \quad \sin C = \frac{n_2}{n_1}.$$

Thus, if $\mu = n_1/n_2$ denotes the refractive index of the denser medium relative to the rarer one,

$$\mu = \frac{1}{\sin C}, \quad C = \sin^{-1}\left(\frac{1}{\mu}\right).$$

When the rarer medium is air, $n_2 \simeq 1$, and this becomes the usual relation between the medium's refractive index and its critical angle.

Step 2: (b) Find the image formed by the first lens Label the lenses L_1, L_2, L_3 from left to right. Light travels to the right. From the printed figure, the object is 30 cm to the left of L_1 , while L_2 is 5 cm to its right and L_3 is a further 10 cm to the right. For L_1 , $u_1 = -30\text{ cm}$ and $f_1 = +10\text{ cm}$. Hence

$$\frac{1}{v_1} = \frac{1}{f_1} + \frac{1}{u_1} = \frac{1}{10} - \frac{1}{30} = \frac{1}{15}, \quad v_1 = +15\text{ cm}.$$

In the absence of subsequent lenses, the rays would meet 15 cm to the right of L_1 .

Step 3: (b) Treat the first image as a virtual object for the second lens The converging rays reach L_2

before they meet. Their intended meeting point is $15 - 5 = 10$ cm to the right of L_2 ; it is therefore a virtual object for L_2 , with $u_2 = +10$ cm, not a negative object distance. Using $f_2 = -10$ cm,

$$\frac{1}{v_2} = \frac{1}{f_2} + \frac{1}{u_2} = -\frac{1}{10} + \frac{1}{10} = 0.$$

Thus the second lens makes the emerging rays parallel: its image is at infinity. The first lens's prospective focus coincides with the third lens's position, but the intervening concave lens changes the rays before they can reach that focus.

Step 4: (b) Find the final image produced by the third lens Since the rays reaching L_3 are parallel, its object is at infinity and $1/u_3 = 0$. For the convex lens with $f_3 = +30$ cm,

$$\frac{1}{v_3} = \frac{1}{f_3} + 0 = \frac{1}{30}, \quad v_3 = +30 \text{ cm}.$$

The image is real and lies on the right of the third lens. Relative to the first lens, its position is $5 + 10 + 30 = 45$ cm to the right. The 10 cm gap between the second and third lenses does not alter the parallel nature of the rays.

Step 5: Conclusion Total internal reflection requires denser-to-rarer incidence with $i > C$, and $\mu = 1/\sin C$. The lens combination forms its final real image 30 cm to the right of the third lens, or 45 cm to the right of the first lens.

Quick Tip to Solve: Translate each intermediate image distance to the next lens before assigning its sign; a convergence point beyond that lens is a virtual object with positive u .

- 26. (a) Describe briefly the process of transferring the charge between the two plates of a parallel plate capacitor when connected to a battery. Derive an expression for the energy stored in a capacitor. (b) A parallel plate capacitor is charged by a battery to a potential difference V . It is disconnected from battery and then connected to another uncharged capacitor of the same capacitance. Calculate the ratio of the energy stored in the combination to the initial energy on the single capacitor. [5 marks]**

Solution:

Concept:

- A capacitor stores equal and opposite plate charges. Its capacitance C relates the magnitude Q of either plate charge to the potential difference V : $Q = CV$.
- Charging requires work against the electric field already established between the plates. For an intermediate plate charge q , the instantaneous potential difference is $v = q/C$.
- After the battery is removed, total charge is conserved when capacitors are connected in parallel, and their final potential differences are equal. Electrostatic energy need not be conserved during charge redistribution.

Step 1: (a) Explain how the battery charges the plates Connect one plate to the positive terminal and the other to the negative terminal of the battery. The battery withdraws electrons from the plate connected to its positive terminal and supplies electrons to the other plate through the external circuit. Thus the plates acquire equal charges $+Q$ and $-Q$. No electrons cross the insulating gap between the plates. The battery's action therefore transfers charge between the plates through the connecting circuit. As charging proceeds, the electric field and the potential difference between the plates increase. Current ceases when the plate potential difference becomes equal to the battery emf;

then the final charge magnitude is $Q = CV$. The energy stored belongs to the electric field between the plates.

Step 2: (a) Derive the stored electrostatic energy Suppose the capacitor initially has no charge and has accumulated an intermediate plate charge of magnitude q . The potential difference at that instant is q/C . The small amount of work needed to transfer an additional positive charge dq from the negative plate to the positive plate, against this existing potential difference, is

$$dW = v dq = \frac{q}{C} dq.$$

The capacitor's capacitance is constant, so integrating from zero to the final charge Q gives

$$U = \int_0^Q \frac{q}{C} dq = \frac{Q^2}{2C}.$$

Using $Q = CV$, equivalent useful forms are

$$U = \frac{1}{2}QV = \frac{1}{2}CV^2.$$

The factor $1/2$ arises because the opposing potential difference increases continuously from zero to V ; it is not equal to V throughout charging.

Step 3: (b) Find the common potential after connecting the capacitors The initially charged capacitor has charge magnitude $Q_i = CV$ and energy $U_i = \frac{1}{2}CV^2$. The second capacitor is uncharged and also has capacitance C . On connecting the corresponding plates, the combination has capacitance $2C$. Since the battery has been disconnected, the conserved charge on the connected positive-side plates is CV . Thus

$$2CV_f = CV, \quad V_f = \frac{V}{2}.$$

Each capacitor consequently has charge magnitude $CV/2$.

Step 4: (b) Calculate the final energy and ratio The final energy is the sum of the energies of both capacitors at the common potential:

$$U_f = 2 \left[\frac{1}{2}C \left(\frac{V}{2} \right)^2 \right] = \frac{1}{4}CV^2.$$

Therefore

$$\frac{U_f}{U_i} = \frac{CV^2/4}{CV^2/2} = \frac{1}{2}.$$

During the transient redistribution, the missing electrostatic energy is dissipated in the connecting circuit, with possible electromagnetic radiation. Conservation of charge therefore does not imply conservation of the energy stored in the capacitors alone.

Step 5: Conclusion The stored energy of a capacitor is $U = Q^2/(2C) = QV/2 = CV^2/2$. For two equal capacitors after charge sharing, $V_f = V/2$ and the required final-to-initial energy ratio is $U_f : U_i = 1 : 2$.

Quick Tip to Solve: For identical capacitors, conserve charge first to obtain half the voltage; then sum both energies using the square of that voltage.

26. (a) Derive an expression for the electric field at any point on the equatorial line of an electric dipole.

(b) Two identical point charges, q each, are kept 2 m apart in air. A third point charge Q of unknown magnitude and sign is placed on the line joining the charges such that the system remains in equilibrium. Find the position and nature of Q . [5 marks]

Solution:

Concept:

- An electric dipole consists of charges $-q_d$ and $+q_d$ separated by $2a$. Its dipole moment $\vec{p}$ has magnitude $p = 2aq_d$ and points from the negative charge to the positive charge.
- Coulomb's law gives the field magnitude $k|q|/R^2$ and force magnitude $k|q_1q_2|/R^2$, where $k = 1/(4\pi\epsilon_0)$ in air to the usual approximation. Fields and forces are added as vectors.
- Equilibrium of a system of charges requires zero net force on every charge. Identical charges repel each other, so a third charge between them must have the opposite sign to provide balancing attractions.

Step 1: (a) Set up the equatorial-point geometry Place the dipole centre at the origin, with $-q_d$ at $x = -a$ and $+q_d$ at $x = +a$. Choose an equatorial point P at $(0, r)$, so its distance from either charge is

$$R = \sqrt{r^2 + a^2}.$$

Each field has magnitude $E_0 = kq_d/(r^2 + a^2)$. The field from the positive charge points away from that charge, while the field from the negative charge points towards the negative charge. Their components along the equatorial line have equal magnitudes and opposite directions and hence cancel. Their components parallel to the dipole axis both point along the negative x -direction, opposite to $\vec{p}$.

Step 2: (a) Add the axial components and obtain the field The magnitude of the axial projection factor is a/R , since the axial separation of either charge from the centre is a . The sum of the two axial components therefore has magnitude

$$E_{\text{eq}} = 2E_0 \frac{a}{R} = \frac{2kq_d a}{(r^2 + a^2)^{3/2}} = \frac{kp}{(r^2 + a^2)^{3/2}}.$$

Including its direction, the exact result for this finite dipole is

$$\vec{E}_{\text{eq}} = -\frac{1}{4\pi\epsilon_0} \frac{\vec{p}}{(r^2 + a^2)^{3/2}}.$$

For a short dipole viewed far away, $r \gg a$, one may neglect a^2 compared with r^2 , obtaining $\vec{E}_{\text{eq}} \simeq -k\vec{p}/r^3$. The exact expression applies at any point of the equatorial line; the inverse-cube expression requires this distance approximation.

Step 3: (b) Determine the third charge's position For the separate equilibrium problem, let the two printed identical charges q occupy $x = 0$ and $x = 2$ m. For a nonzero third charge Q to experience zero force, the fields due to the two equal charges must cancel at its location. Outside the interval, both fields point in the same direction, so cancellation is impossible. Between them the fields oppose. If Q is at a distance x from the left charge, equality of their magnitudes requires

$$\frac{k|q|}{x^2} = \frac{k|q|}{(2\text{ m} - x)^2}.$$

With $0 < x < 2 \text{ m}$, this gives $x = 1 \text{ m}$. Thus Q must lie at the midpoint, one metre from each charge.

Step 4: (b) Determine the sign and magnitude of the third charge Consider either end charge. Its repulsive force due to the other end charge is directed outwards and has magnitude

$$F_{qq} = \frac{kq^2}{(2 \text{ m})^2}.$$

To balance it, the midpoint charge must attract the end charge inwards, so its sign is opposite to that of q . Equating the two force magnitudes gives

$$\frac{k|qQ|}{(1 \text{ m})^2} = \frac{kq^2}{(2 \text{ m})^2}, \quad |Q| = \frac{|q|}{4}.$$

Hence $Q = -q/4$. The other end charge is balanced by symmetry, and the midpoint charge experiences equal and opposite forces from the end charges. All three net forces therefore vanish. If the given q is positive, Q is negative; if q is negative, Q is positive. This is an equilibrium configuration, without any claim that it is stable against displacement.

Step 5: Conclusion The equatorial field is $\vec{E} = -k\vec{p}/(r^2 + a^2)^{3/2}$, opposite to the dipole moment, reducing to $-k\vec{p}/r^3$ for $r \gg a$. The third charge must be $Q = -q/4$, placed at the midpoint, 1 m from each end charge.

Quick Tip to Solve: First cancel the forces on the middle charge to fix its position; then balance one end charge to determine the opposite sign and magnitude.

27. (a) In a series LCR circuit, connected across an ac source of variable frequency, obtain the expression for its impedance and draw a plot showing its variation with frequency of the ac source.

(b) What is the phase difference between the voltages across inductor and the capacitor at resonance in the LCR circuit?

(c) When an inductor is connected to a 200 V dc voltage, a current of 1 A flows through it. When the same inductor is connected to a 200 V, 50 Hz ac source, only 0.5 A current flows. Explain, why? Also, calculate the self inductance of the inductor.

[5 marks]

Solution:

Concept:

- In a series circuit, the same current flows through the resistor, inductor and capacitor. Their rms voltage magnitudes are $V_R = IR$, $V_L = IX_L$ and $V_C = IX_C$, where $X_L = \omega L$, $X_C = 1/(\omega C)$ and $\omega = 2\pi f$.
- The resistor voltage is in phase with current; the inductor voltage leads current by $\pi/2$, while the capacitor voltage lags current by $\pi/2$. Therefore voltage amplitudes must be added as phasors.
- At resonance, $X_L = X_C$. A real inductor has winding resistance r ; steady dc encounters this resistance, whereas ac encounters impedance $\sqrt{r^2 + (\omega L)^2}$.

Step 1: (a) Derive the impedance using voltage phasors Take the current phasor along the horizontal axis. The horizontal component of the applied voltage is IR , and its vertical component is $I(X_L - X_C)$, since the inductive and capacitive voltages point in opposite directions. Applying the phasor form of

Kirchhoff's voltage law and then Pythagoras gives

$$V^2 = (IR)^2 + \{I(X_L - X_C)\}^2,$$

$$Z = \frac{V}{I} = \sqrt{R^2 + (X_L - X_C)^2}$$

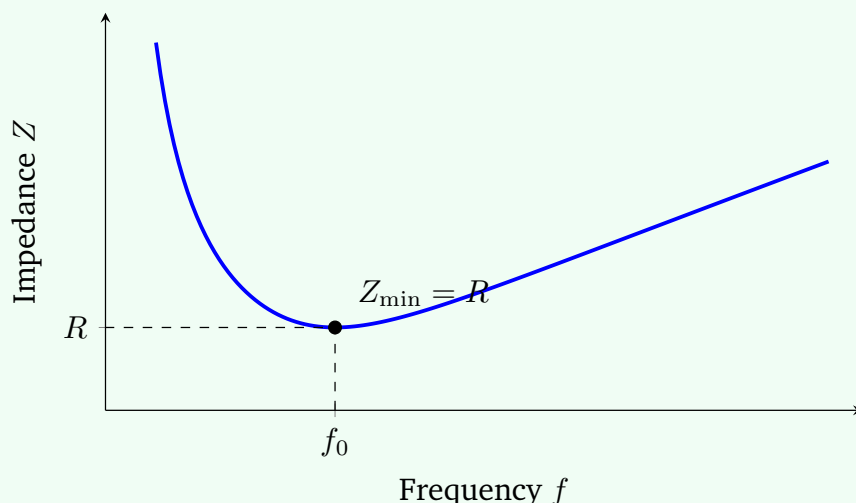
$$= \sqrt{R^2 + \left(2\pi fL - \frac{1}{2\pi fC}\right)^2}.$$

Also, the applied voltage makes an angle ϕ with current such that $\tan \phi = (X_L - X_C)/R$. Thus the circuit is capacitive below resonance and inductive above resonance.

Step 2: (a) Describe the impedance-frequency graph At very low positive frequency, X_C is large and the impedance is large. As frequency increases, X_C decreases while X_L increases. The difference of the reactances vanishes at

$$f_0 = \frac{1}{2\pi\sqrt{LC}}, \quad Z_{\min} = R.$$

Above f_0 , the increasing inductive reactance makes the impedance rise again. The minimum is nonzero for $R > 0$; the curve approaches large impedance as $f \rightarrow 0^+$ or as $f \rightarrow \infty$. The following schematic graph shows these features, with frequency normalized by f_0 .



Schematic variation of impedance with frequency; the minimum occurs at resonance.

Step 3: (b) Compare the inductor and capacitor voltages at resonance Relative to the common current, V_L has phase $+\pi/2$ and V_C has phase $-\pi/2$. Their phase difference is therefore π radians, or 180° . At resonance their magnitudes are equal, $V_L = V_C$, and their phasors cancel. This cancellation concerns their resultant voltage; each individual voltage can still be nonzero.

Step 4: (c) Find winding resistance and explain the reduced ac current For steady dc, the current is constant and the induced emf $-L di/dt$ vanishes. The given dc current therefore determines the winding resistance:

$$r = \frac{V_{\text{dc}}}{I_{\text{dc}}} = \frac{200}{1} = 200 \, \Omega.$$

With ac, the changing current produces a self-induced emf opposing its change. The inductor has an additional inductive reactance, so its impedance exceeds its winding resistance and the current is smaller. Treating the given ac voltage and current as rms values,

$$Z = \frac{V_{\text{ac}}}{I_{\text{ac}}} = \frac{200}{0.5} = 400 \, \Omega.$$

Step 5: (c) Calculate the self-inductance Resistance and inductive reactance combine quadratically, because their voltage phasors are perpendicular. Hence

$$\begin{aligned}X_L &= \sqrt{Z^2 - r^2} = \sqrt{400^2 - 200^2} \\&= 200\sqrt{3} \, \Omega, \\L &= \frac{X_L}{2\pi f} = \frac{200\sqrt{3}}{2\pi \times 50} \, \text{H} \\&= \frac{2\sqrt{3}}{\pi} \, \text{H} \approx 1.10 \, \text{H}.\end{aligned}$$

The winding resistance cannot be neglected: the finite steady dc current directly demonstrates its presence.

Step 6: Conclusion The series LCR impedance is $Z = \sqrt{R^2 + (2\pi fL - 1/(2\pi fC))^2}$, with its minimum R at $f_0 = 1/(2\pi\sqrt{LC})$. At resonance, the inductor and capacitor voltages differ in phase by 180° . The real inductor has resistance $200 \, \Omega$ and self-inductance $L \approx 1.10 \, \text{H}$; its inductive reactance reduces the ac current.

Quick Tip to Solve: First use steady dc to obtain the coil resistance, then use $X_L = \sqrt{Z^2 - r^2}$; do not subtract resistance directly from impedance.

OR

27. (a) Draw the diagram of a device which is used to decrease high ac voltage into a low ac voltage and state its working principle. Write four sources of energy loss in this device.

(b) A small town with a demand of 1200 kW of electric power at 220 V is situated 20 km away from an electric plant generating power at 440 V. The resistance of the two wire line carrying power is $0.5 \, \Omega$ per km. The town gets the power from the line through a 4000-220 V step-down transformer at a sub-station in the town. Estimate the line power loss in the form of heat. [5 marks]

Solution:

Concept:

- A transformer works by mutual induction: an alternating current in the primary produces a changing magnetic flux in the core, which induces an emf in the secondary.
- For an ideal transformer, $V_s/V_p = N_s/N_p$, where N_p, N_s are the numbers of primary and secondary turns and V_p, V_s are their rms voltages. A step-down transformer has $N_s < N_p$.
- The power delivered at rms voltage V is $P = VI$ for the resistive load assumed here. Heating in the complete transmission line is $P_{\text{loss}} = I^2 R_{\text{line}}$.

Step 1: (a) Construction and working of the step-down transformer The required device is a step-down transformer. Its two electrically insulated windings are wound on a common laminated soft-iron core. The primary, with more turns, is connected to the high-voltage ac supply; the secondary, with fewer turns, supplies the low-voltage load. The sketch below shows these essential features.

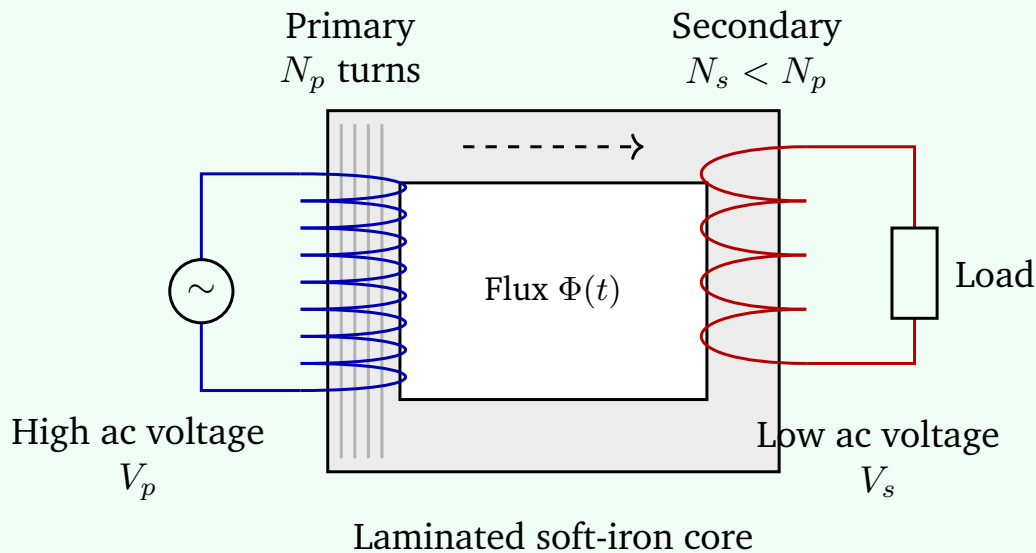
If the common magnetic flux is $\Phi(t)$, Faraday's law gives

$$e_p = -N_p \frac{d\Phi}{dt}, \quad e_s = -N_s \frac{d\Phi}{dt}.$$

Neglecting losses and flux leakage, the ratio of the rms voltages is therefore

$$\frac{V_s}{V_p} = \frac{N_s}{N_p} < 1.$$

Thus mutual induction produces the lower secondary ac voltage. The core guides the alternating flux through both windings. A steady dc supply cannot sustain this changing flux and hence cannot provide continuous transformer action.



Step-down transformer: a common laminated soft-iron core links a many-turn primary to a fewer-turn secondary.

Step 2: (a) Four sources of energy loss The four sources are: (i) copper loss, caused by I^2R heating in the primary and secondary winding resistances; (ii) eddy-current loss, caused by induced circulating currents heating the iron core; (iii) hysteresis loss, caused by repeated magnetisation and demagnetisation of the core during each ac cycle; and (iv) flux leakage, when some primary magnetic flux fails to link the secondary and so reduces effective energy transfer. Thick low-resistance copper wires reduce copper loss, insulated core laminations reduce eddy currents, a soft-iron core with a narrow hysteresis loop reduces hysteresis loss, and close magnetic coupling reduces leakage.

Step 3: (b) Determine the transmission-line current The town receives $1200 \text{ kW} = 1.2 \times 10^6 \text{ W}$ through an ideal 4000-220 V step-down transformer. Its secondary current is

$$I_s = \frac{1.2 \times 10^6}{220} = 5.45 \times 10^3 \text{ A}.$$

Conservation of transformer power gives $V_p I_p = V_s I_s$. The current in the transmission line, which feeds the 4000 V primary at the town, is consequently

$$I_{\text{line}} = I_p = \frac{1.2 \times 10^6}{4000} = 300 \text{ A}.$$

The 220 V town supply and 440 V generator rating are not the receiving-end line voltage. The generator output must be stepped up for transmission; the receiving transformer determines the line current used here.

Step 4: (b) Calculate the heat loss in the line The stated 0.5Ω per kilometre is the resistance of the

two-wire line together. Thus, over the 20 km separation, the total line resistance is

$$R_{\text{line}} = 0.5 \times 20 = 10 \, \Omega.$$

It must not be doubled again for the return conductor. Using the transmission current,

$$P_{\text{loss}} = I_{\text{line}}^2 R_{\text{line}} = (300)^2 \times 10 = 9.0 \times 10^5 \, \text{W} = 900 \, \text{kW}.$$

This estimate neglects transformer losses and treats the town demand as the power delivered. The line loses 900 kJ as heat every second while supplying the stated town demand.

Step 5: Conclusion (a) A step-down transformer, with $N_s < N_p$, lowers ac voltage by mutual induction. Its four energy-loss sources are copper heating, eddy currents, hysteresis and magnetic-flux leakage. (b) The transmission current is 300 A, the total two-wire resistance is $10 \, \Omega$, and the line heat loss is 900 kW.

Quick Tip to Solve: Use the receiving transformer's high-voltage side to find line current, and check whether the resistance per kilometre already includes both conductors.