

CBSE Class 12 Physics Question Paper 2019 with Solution PDF-Set 55/1/3



Question Paper with Solutions

Conducted by the Central Board of Secondary Education (CBSE)

Time Allowed: 3 hours

Maximum Marks: 70

General Instructions

- (i) All questions are compulsory. There are 27 questions in all.
- (ii) This question paper has four sections : Section A, Section B, Section C and Section D.
- (iii) Section A contains five questions of one mark each, Section B contains seven questions of two marks each, Section C contains twelve questions of three marks each, and Section D contains three questions of five marks each.
- (iv) There is no overall choice. However, an internal choice(s) has been provided in two questions of one mark, two questions of two marks, four questions of three marks and three questions of five marks weightage. You have to attempt only one of the choices in such questions.

You may use the following values of physical constants wherever necessary :

$$c = 3 \times 10^8 \text{ m/s}$$

$$h = 6.63 \times 10^{-34} \text{ Js}$$

$$e = 1.6 \times 10^{-19} \text{ C}$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ T m A}^{-1}$$

$$\epsilon_0 = 8.854 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$$

$$\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$$

$$\text{Mass of electron } (m_e) = 9.1 \times 10^{-31} \text{ kg}$$

$$\text{Mass of neutron} = 1.675 \times 10^{-27} \text{ kg}$$

$$\text{Mass of proton} = 1.673 \times 10^{-27} \text{ kg}$$

$$\text{Avogadro's number} = 6.023 \times 10^{23} \text{ per gram mole}$$

$$\text{Boltzmann constant} = 1.38 \times 10^{-23} \text{ JK}^{-1}$$

Section A (5 marks)

1. Distinguish between unpolarised and linearly polarised light.

[1 mark]

Solution:

Concept:

- Light is a transverse electromagnetic wave: its electric field oscillates perpendicular to its direction of propagation.
- Polarisation describes the directions in which the electric field oscillates in this transverse plane.

Step 1: Describe unpolarised light In unpolarised light, the direction of the electric-field vibrations varies randomly with time, with all directions perpendicular to the propagation direction equally represented. There is no fixed direction of vibration.

Step 2: Describe linearly polarised light In linearly polarised light, the electric field vibrates along one fixed direction perpendicular to propagation. Thus the electric-field vector and the direction of propagation lie in one fixed plane.

Step 3: Conclusion Unpolarised light has electric-field vibrations in all transverse directions, whereas linearly polarised light has them restricted to one fixed transverse direction.

Quick Tip to Solve: Compare the direction of the electric-field vibrations; both kinds of light remain transverse waves.

2. Define the term “threshold frequency”, in the context of photoelectric emission.

[1 mark]

Solution:

Concept:

- A photon of frequency ν carries energy $h\nu$, where h is Planck's constant.
- The work function ϕ is the minimum energy needed to liberate an electron from the surface of a given material.

Step 1: State the minimum energy condition Photoelectric emission requires the energy of an incident photon to be at least the work function. At the limiting frequency ν_0 , the most energetic emitted electrons have zero kinetic energy, so $h\nu_0 = \phi$.

Step 2: Identify the frequency cutoff Thus $\nu_0 = \phi/h$ is specific to the emitting material. Below this frequency, increasing the intensity cannot cause photoelectric emission, because it increases the number of photons without giving each photon the necessary energy.

Step 3: Conclusion Threshold frequency is the minimum frequency of incident radiation required to produce photoelectric emission from a given material: $\nu_0 = \phi/h$.

Quick Tip to Solve: Include both parts in the definition: a minimum incident frequency and a specified emitting material.

2. Define the term “Intensity” in photon picture of electromagnetic radiation.

[1 mark]

Solution:

Concept:

- Electromagnetic radiation consists of photons, each with energy $h\nu$, where h is Planck's constant and ν is frequency.
- Intensity is the radiant energy crossing unit area perpendicular to the direction of propagation per unit time.

Step 1: Count the incident photons Let N photons of frequency ν cross a perpendicular area A in time Δt . Their total energy is $Nh\nu$; therefore the photon flux per unit area is $N/(A\Delta t)$.

Step 2: Express the energy flux Dividing this energy by both area and time gives the intensity:

$$I = \frac{Nh\nu}{A\Delta t}.$$

At fixed frequency, greater intensity means more photons crossing unit area per second. Photon number alone is not intensity unless its energy, area and time are also accounted for.

Step 3: Conclusion In the photon picture, intensity is the energy carried by photons incident per unit area per unit time, $I = Nh\nu/(A\Delta t)$, measured in W m^{-2} .

Quick Tip to Solve: Multiply the photon flux per unit area by the energy of one photon to obtain intensity.

3. How is the drift velocity in a conductor affected with the rise in temperature?

[1 mark]

Solution:

Concept:

- For a metallic conductor, the magnitude of electron drift velocity is $v_d = eE\tau/m$, where E is the electric field, τ the relaxation time, e the magnitude of electron charge and m its mass.
- Relaxation time is the mean time between successive collisions of a conduction electron with the lattice.

Step 1: Examine the effect of heating A rise in temperature increases lattice vibrations in a metal, leading to more frequent collisions of the conduction electrons. Hence their relaxation time τ decreases.

Step 2: Keep the driving electric field fixed For the usual comparison at a fixed electric field, or fixed potential difference across a conductor of unchanged length, v_d is proportional to τ . Increased random thermal motion does not mean increased directed drift; more frequent collisions reduce the drift.

Step 3: Conclusion The drift velocity decreases with a rise in temperature of a metallic conductor, provided the applied electric field is held constant.

Quick Tip to Solve: Use $v_d = eE\tau/m$: heating a metal reduces τ , so at fixed E the drift speed falls.

4. In sky wave mode of propagation, why is the frequency range of transmitting signals restricted to less than 30 MHz? [1 mark]

Solution:

Concept:

- Sky wave propagation uses the ionosphere to return radio waves towards the Earth's surface, allowing reception beyond the horizon.
- Whether a wave returns depends on the response of the ionosphere's free electrons to its frequency.

Step 1: Identify the required ionospheric action A transmitted wave must be bent sufficiently by the ionosphere to return to Earth. This return is commonly described as reflection by the ionosphere; without it, a distant ground receiver cannot receive the signal by the sky wave route.

Step 2: Explain the upper frequency restriction At frequencies above about 30 MHz, the ionosphere generally cannot bend the waves enough to send them back to Earth. They penetrate the ionosphere and escape into space instead of providing the required return path.

Step 3: Conclusion The range is restricted to below about 30 MHz because higher-frequency waves generally pass through the ionosphere rather than being returned to Earth.

Quick Tip to Solve: Link the frequency limit to failure of ionospheric return, not to the horizon limit of space waves.

OR

4. On what factors does the range of coverage in ground wave propagation depend? [1 mark]

Solution:

Concept:

- A ground wave travels along the Earth's surface and loses energy through interaction with the ground.
- Its coverage is determined by whether its strength remains sufficient for useful reception after attenuation.

Step 1: Identify the signal factors Coverage depends on the transmitted power and the frequency of the signal. Greater transmitted power generally allows reception farther away. Higher frequencies suffer greater ground-wave attenuation, so lower-frequency signals generally have a larger range under comparable conditions.

Step 2: Identify the ground factor Coverage also depends on the nature and electrical conductivity of the ground along the path. A better-conducting surface, such as sea water, usually causes less attenuation than poorly conducting dry ground and therefore supports longer-range propagation.

Step 3: Conclusion The range of ground-wave coverage depends principally on transmitted power, signal frequency, and the nature or electrical conductivity of the ground.

Quick Tip to Solve: Recall the three factors together: transmitter power, wave frequency and ground conductivity.

5. Draw the pattern of electric field lines when a point charge $+q$ is kept near an uncharged conducting plate. [1 mark]

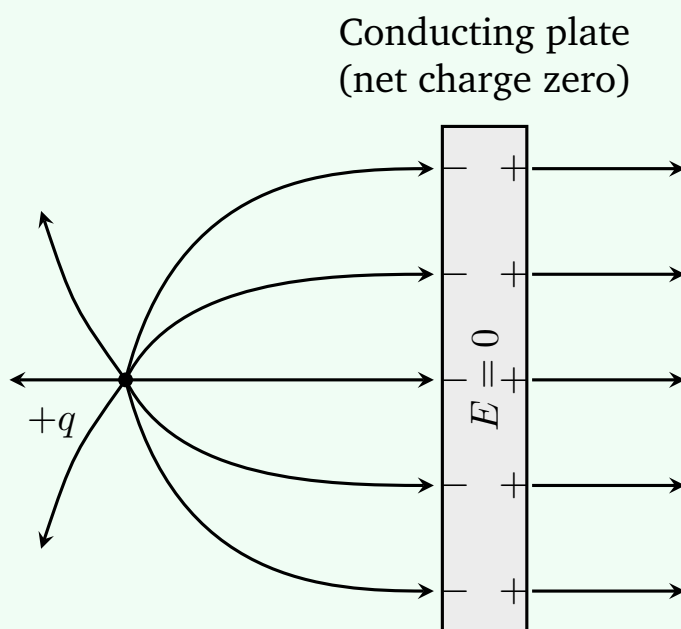
Solution:

Concept:

- Electric field lines start on positive charge and end on negative charge, or extend to infinity.
- A conductor in electrostatic equilibrium has zero field inside; external field lines meet its surface normally.

Step 1: Identify the induced charges The positive point charge $+q$ attracts free electrons towards the nearer face of the initially uncharged conducting plate. The nearer face becomes negative and the farther face positive. Equal and opposite induced charges leave the plate's net charge zero.

Step 2: Draw the field with its direction Lines leave $+q$, bend towards the nearer negative face and terminate perpendicular to it. Lines also emerge normally from the positive far face and extend outward. No field line passes through the conducting material. The diagram is schematic; the arrows show the direction of force on a positive test charge.



Field near a positive point charge and an isolated neutral conducting plate

Step 3: Conclusion The field lines point away from $+q$ towards the induced negative near face, while lines leave the induced positive far face; the electric field inside the plate is zero.

Quick Tip to Solve: First mark negative charge on the face nearest $+q$; then draw all surface intersections at right angles.

Section B (14 marks)

Define the terms in photoelectric effect:

6(a)(i). threshold frequency

[2 marks total]

Solution:

Concept:

- Einstein's photoelectric equation is $K_{\max} = h\nu - \phi$, where ϕ is the metal's work function and h is Planck's constant.
- A single absorbed photon supplies energy $h\nu$ to an electron; the work function ϕ is the minimum energy needed to liberate it from the surface.

Step 1: Define the threshold frequency The threshold frequency ν_0 is the minimum frequency of incident radiation required to eject photoelectrons from a given metal surface. It obeys $h\nu_0 = \phi$. Radiation below ν_0 produces no photoelectric emission, however high its intensity.

Step 2: Relate the threshold to the work function At the limiting frequency the emitted electron has zero maximum kinetic energy. Einstein's equation therefore becomes $0 = h\nu_0 - \phi$, giving $\nu_0 = \phi/h$. Increasing intensity below this frequency increases the number of photons but not their individual energies, so it cannot cause photoemission.

Step 3: Conclusion Threshold frequency is the minimum incident frequency required for photoelectric emission from a given metal: $\nu_0 = \phi/h$.

Quick Tip to Solve: For a frequency threshold, compare one photon's energy $h\nu$ with the work function, not the beam intensity.

6(a)(ii). stopping potential.

Solution:

Concept:

- Einstein's photoelectric equation is $K_{\max} = h\nu - \phi$, where ϕ is the metal's work function and h is Planck's constant.
- A retarding electric field removes kinetic energy eV_0 from an electron, where e is the magnitude of its charge.

Step 1: Define the stopping potential Stopping potential V_0 is the minimum magnitude of negative anode potential, relative to the emitting cathode, that reduces the photocurrent to zero. Even the fastest emitted electrons are then prevented from reaching the anode, so $eV_0 = h\nu - \phi$.

Step 2: Interpret the stopping condition The anode is made negative relative to the photocathode. At smaller reverse potentials some fast electrons still arrive; at the minimum reverse potential magnitude V_0 , even electrons with maximum kinetic energy cannot reach it. Thus the photocurrent just becomes zero and $V_0 = (h\nu - \phi)/e$.

Step 3: Conclusion Stopping potential is the minimum magnitude of negative anode potential, relative to the cathode, that reduces the photocurrent to zero; $eV_0 = K_{\max}$.

Quick Tip to Solve: Report V_0 as a positive magnitude; the actual anode-potential intercept is $-V_0$.

6(b). Plot a graph of photocurrent versus anode potential for a radiation of frequency ν and intensities I_1 and I_2 ($I_1 < I_2$).

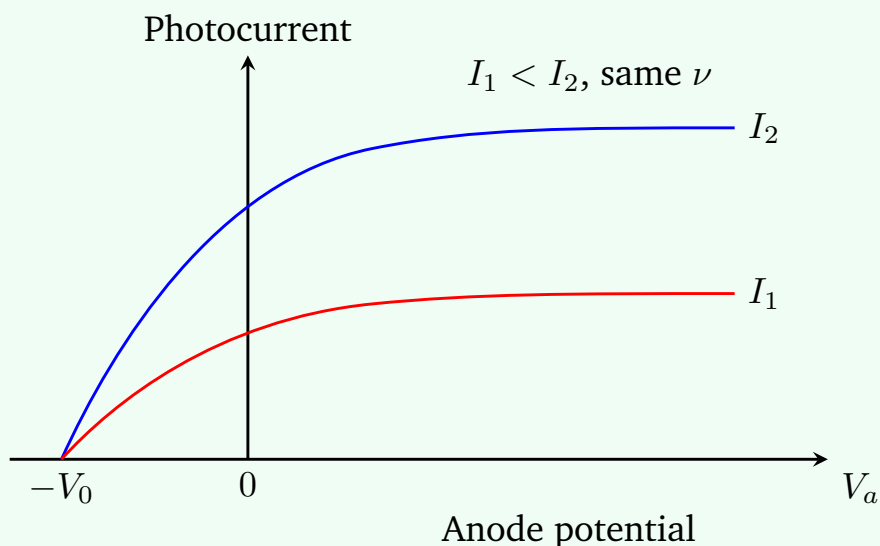
Solution:

Concept:

- Einstein's photoelectric equation is $K_{\max} = h\nu - \phi$, where ϕ is the metal's work function and h is Planck's constant.
- The stopping potential satisfies $eV_0 = K_{\max}$. At fixed frequency above threshold, saturation photocurrent is proportional to incident intensity.

Step 1: Fix the common stopping-potential intercept Both beams have the same frequency $\nu > \nu_0$, so Einstein's equation gives the same maximum electron kinetic energy. Consequently both current curves meet the potential axis at the same negative anode potential $-V_0$.

Step 2: Compare the two photocurrent curves For the same frequency $\nu > \nu_0$, increasing intensity from I_1 to I_2 increases the number of photoelectrons emitted per second. Thus the upper curve has greater saturation current. The maximum kinetic energy is unchanged, so both curves meet the potential axis at the same negative potential $-V_0$. At sufficiently positive anode potential all emitted electrons are collected and each current saturates.



Photocurrent versus anode potential at a fixed frequency above threshold

Step 3: Conclusion The curve for I_2 has a higher saturation photocurrent than the curve for I_1 , while their common stopping-potential intercept is $-V_0$.

Quick Tip to Solve: On an intensity comparison graph, keep the negative intercept fixed and raise only the saturation level for the stronger beam.

7. Why a signal transmitted from a TV tower cannot be received beyond a certain distance? Write the expression for the optimum separation between the receiving and the transmitting antenna. [2 marks]

Solution:

Concept:

- Television signals at high frequencies mainly use space-wave propagation: the transmitting and receiving antennas need a line-of-sight path.
- For antenna height $h \ll R$, the distance to the radio horizon is approximately $d = \sqrt{2Rh}$, where R is Earth's radius.

Step 1: Explain the limited range TV signals travel approximately along straight paths and do not follow Earth's curvature effectively. Beyond the radio horizon, Earth's curved surface obstructs the direct path from the transmitting tower to the receiver. Such high-frequency waves are also not normally returned by the ionosphere, so direct reception is limited to a certain distance.

Step 2: Add the two horizon distances For a tower of height h , the ray to the horizon is tangent to Earth. The right triangle formed by Earth's centre, the tangent point and the antenna gives

$$d^2 = (R + h)^2 - R^2 = 2Rh + h^2 \simeq 2Rh.$$

Let h_T and h_R be the transmitting and receiving antenna heights. At the limiting line-of-sight separation, the two horizon distances add:

$$d_{\max} = d_T + d_R = \sqrt{2Rh_T} + \sqrt{2Rh_R}.$$

Use the same length unit for R, h_T, h_R to obtain the separation in that unit; the approximation assumes both heights are small compared with Earth's radius.

Step 3: Conclusion Earth's curvature limits TV reception. The optimum maximum separation for direct space-wave reception is $d_{\max} = \sqrt{2Rh_T} + \sqrt{2Rh_R}$, neglecting atmospheric refraction.

Quick Tip to Solve: Find each antenna's horizon range separately and add them; do not put the sum of antenna heights inside one square root.

8. State Bohr's quantization condition of angular momentum. Calculate the shortest wavelength of the Brackett series and state to which part of the electromagnetic spectrum does it belong. [2 marks]

Solution:

Concept:

- Bohr's quantization condition allows electron orbital angular momentum only in integral multiples of $\hbar = h/(2\pi)$.
- For hydrogen, the Rydberg formula is $1/\lambda = R_H(1/n_f^2 - 1/n_i^2)$, where $n_i > n_f$ are the initial and final principal quantum numbers.

Step 1: State the angular momentum condition For an electron of mass m_e , speed v_n and orbital radius r_n , the allowed stationary circular orbits satisfy

$$L = m_e v_n r_n = \frac{nh}{2\pi} = n\hbar, \quad n = 1, 2, 3, \dots$$

Thus angular momentum cannot take arbitrary values; it is quantized, with n identifying the orbit.

Step 2: Identify the Brackett series limit All Brackett transitions terminate at $n_f = 4$. The shortest wavelength corresponds to the largest energy difference within this series, obtained at the limit $n_i \rightarrow \infty$, because the photon energy is hc/λ . Hence

$$\frac{1}{\lambda_{\min}} = R_H \left(\frac{1}{4^2} - 0 \right) = \frac{R_H}{16}.$$

Step 3: Substitute and identify the spectral region Using $R_H = 1.097 \times 10^7 \text{ m}^{-1}$,

$$\lambda_{\min} = \frac{16}{1.097 \times 10^7} \text{ m} = 1.459 \times 10^{-6} \text{ m} \simeq 1.46 \mu\text{m}.$$

This wavelength is longer than the visible red end of the spectrum, so the series limit lies in the infrared region.

Step 4: Conclusion Bohr's condition is $m_e v r = n h / (2\pi)$. The shortest Brackett wavelength is approximately $1.46 \times 10^{-6} \text{ m}$, in the infrared region.

Quick Tip to Solve: For the shortest wavelength of any hydrogen series, keep its final orbit fixed and set the initial quantum number to infinity.

OR

8. Calculate the orbital period of the electron in the first excited state of hydrogen atom. [2 marks]

Solution:

Concept:

- In Bohr's model of hydrogen, the radius of orbit n is $r_n = a_0 n^2$, where $a_0 = 5.29 \times 10^{-11} \text{ m}$ is the ground-state radius.
- The electron speed is $v_n = v_1/n$, with $v_1 = 2.19 \times 10^6 \text{ m s}^{-1}$. The orbital period is the circumference divided by speed: $T_n = 2\pi r_n / v_n$.

Step 1: Identify the excited orbit The ground state has principal quantum number $n = 1$, so the first excited state is $n = 2$. Its radius and speed are therefore

$$r_2 = 4a_0 = 2.116 \times 10^{-10} \text{ m},$$

$$v_2 = \frac{v_1}{2} = 1.095 \times 10^6 \text{ m s}^{-1}.$$

The electron travels once around a circular path of length $2\pi r_2$ in one orbital period.

Step 2: Substitute into the period formula Since the radius grows as n^2 while speed falls as $1/n$, the period varies as n^3 . Direct substitution for the second orbit gives

$$T_2 = \frac{2\pi(2.116 \times 10^{-10})}{1.095 \times 10^6} = 1.21 \times 10^{-15} \text{ s}.$$

The metre in the circumference cancels the metre in the speed, leaving seconds as required.

Step 3: Conclusion The orbital period of the electron in the first excited state is $T_2 \approx 1.21 \times 10^{-15} \text{ s}$.

Quick Tip to Solve: First excited means $n = 2$. Use $T_n \propto n^3$, so its period is eight times the ground-state period.

9. Why is wave theory of electromagnetic radiation not able to explain photoelectric effect? How does photon picture resolve this problem? [2 marks]

Solution:

Concept:

- Classical wave theory treats light energy as continuously distributed across the wavefront, with the energy incident per unit area per second determined by intensity.
- A photon of frequency ν carries energy $h\nu$. For photoemission, one electron absorbs one photon, and Einstein's equation is $K_{\max} = h\nu - \phi$, where ϕ is the metal's work function.

Step 1: Compare wave predictions with observations Wave theory suggests that an electron could gradually accumulate enough energy from light of any frequency; sufficiently intense illumination should therefore cause emission. It also predicts a measurable delay at low intensity and greater electron kinetic energy when intensity increases. Experiments instead show a threshold frequency below which emission never occurs, practically instantaneous emission above threshold, and a maximum kinetic energy controlled by frequency rather than intensity. Thus the classical energy-transfer picture cannot account for these observations.

Step 2: Apply the photon picture The electron receives $h\nu$ in a single absorption event. Emission requires $h\nu \geq \phi$, giving the threshold $\nu_0 = \phi/h$. Above threshold, the energy remaining after escape is

$$K_{\max} = h\nu - \phi = h(\nu - \nu_0).$$

No slow accumulation of wave energy is needed, explaining the negligible delay. At fixed frequency, increased intensity supplies more photons each second, increasing the photoelectron emission rate and hence the photocurrent under suitable collection conditions; it does not increase the energy of an individual photon or $K_{\max}$.

Step 3: Conclusion The photon picture explains the threshold frequency, prompt emission, frequency-dependent maximum kinetic energy and intensity-dependent photocurrent, which classical wave theory fails to explain.

Quick Tip to Solve: Separate photon energy from photon number: frequency sets $h\nu$ and $K_{\max}$, while intensity at fixed frequency sets the photon arrival rate.

10. Two bulbs are rated (P_1, V) and (P_2, V) . If they are connected (i) in series and (ii) in parallel across a supply V , find the power dissipated in the two combinations in terms of P_1 and P_2 . [2 marks]

Solution:

Concept:

- For a resistance R across a voltage V , electrical power is $P = V^2/R$. A bulb's rating specifies its power at the rated voltage.
- Resistances add in series, while conductances add in parallel. Treat each bulb's resistance as constant at the value inferred from its rating, as in the ideal circuit model.

Step 1: Express the bulb resistances using their ratings Both bulbs have the same rated voltage V . Applying the power relation separately gives

$$R_1 = \frac{V^2}{P_1}, \quad R_2 = \frac{V^2}{P_2}.$$

These resistances are then used to calculate the equivalent resistance of each combination; neither series bulb is individually connected across the full supply voltage.

Step 2: Calculate the total power in series The same current flows through the bulbs, and the supply V is applied across their combined resistance:

$$R_s = R_1 + R_2 = \frac{V^2(P_1 + P_2)}{P_1 P_2},$$

$$P_s = \frac{V^2}{R_s} = \frac{P_1 P_2}{P_1 + P_2}.$$

This is the total power dissipated by the two bulbs together in series.

Step 3: Calculate the total power in parallel Each parallel branch receives the full supply V , equal to its bulb's rated voltage. Therefore each bulb dissipates its rated power, and

$$P_p = \frac{V^2}{R_1} + \frac{V^2}{R_2} = P_1 + P_2.$$

Equivalently, this follows by using $1/R_p = 1/R_1 + 1/R_2$ in $P_p = V^2/R_p$.

Step 4: Conclusion The total power is $P_s = \frac{P_1 P_2}{P_1 + P_2}$ in series and $P_p = P_1 + P_2$ in parallel.

Quick Tip to Solve: Convert each rating to resistance first. Rated powers add directly only when both bulbs receive their rated voltage, as they do here in parallel.

11. Obtain the expression for the ratio of the de-Broglie wavelengths associated with the electron orbiting in the second and third excited states of hydrogen atom. [2 marks]

Solution:

Concept:

- The de Broglie wavelength of an electron is $\lambda = h/p = h/(m_e v)$, where p is its momentum, m_e its mass and v its speed.
- Bohr's angular-momentum condition $m_e v_n r_n = nh/(2\pi)$ is equivalent to the standing-wave condi-

tion $2\pi r_n = n\lambda_n$. For hydrogen, $r_n = a_0 n^2$, where a_0 is the Bohr radius.

Step 1: Relate the wavelength to the orbit number A stable orbit contains an integer number n of electron wavelengths around its circumference. Using the radius of the n th hydrogen orbit gives

$$\lambda_n = \frac{2\pi r_n}{n} = \frac{2\pi a_0 n^2}{n} = 2\pi a_0 n.$$

Hence the electron wavelength is proportional to n , even though the orbital radius is proportional to n^2 . This also follows from $v_n \propto 1/n$ and $\lambda_n = h/(m_e v_n)$.

Step 2: Identify the two excited states and form the ratio The ground state is $n = 1$, so the second excited state corresponds to $n = 3$, and the third excited state to $n = 4$. In the order requested,

$$\frac{\lambda_{\text{second excited}}}{\lambda_{\text{third excited}}} = \frac{\lambda_3}{\lambda_4} = \frac{2\pi a_0(3)}{2\pi a_0(4)} = \frac{3}{4}.$$

The common constants cancel, so no numerical value of the Bohr radius is required.

Step 3: Conclusion The required ratio of de Broglie wavelengths is $\lambda_3 : \lambda_4 = 3 : 4$.

Quick Tip to Solve: Add one to the excitation number to get n ; then use $\lambda_n \propto n$, not the radius law $r_n \propto n^2$.

12. A charged particle q is moving in the presence of a magnetic field $\vec{B}$ which is inclined to an angle 30° with the direction of the motion of the particle. Draw the trajectory followed by the particle in the presence of the field and explain how the particle describes this path. [2 marks]

Solution:

Concept:

- The magnetic force on a particle of charge q is $\vec{F} = q\vec{v} \times \vec{B}$. It is perpendicular to the velocity and does no work.
- Resolve the initial speed v into $v_{\parallel} = v \cos \theta$ along the uniform field and $v_{\perp} = v \sin \theta$ perpendicular to it.

Step 1: Resolve the velocity at the given angle Here $\theta = 30^\circ$, so

$$v_{\parallel} = \frac{\sqrt{3}}{2}v, \quad v_{\perp} = \frac{v}{2}.$$

There is no magnetic force along $\vec{B}$; hence $v_{\parallel}$ remains constant. The perpendicular component experiences a force of magnitude $|q|v_{\perp}B$, which acts as the centripetal force in the plane normal to the field.

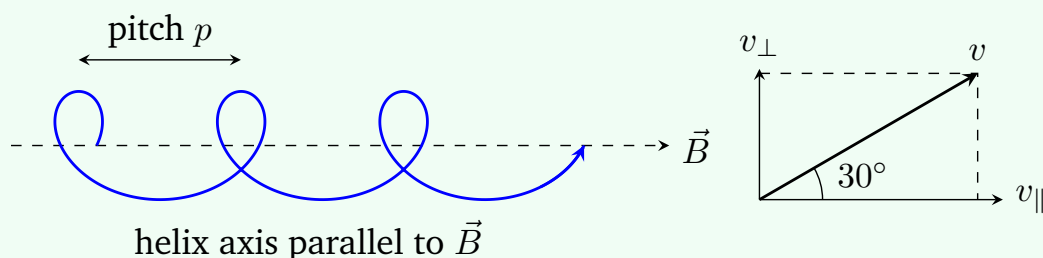
Step 2: Combine circular motion with uniform forward motion In the perpendicular plane, $mv_{\perp}^2/r = |q|v_{\perp}B$, giving

$$r = \frac{mv}{2|q|B}, \quad T = \frac{2\pi m}{|q|B}.$$

During every revolution the particle advances uniformly along the field through the pitch

$$p = v_{\parallel}T = \frac{\sqrt{3}\pi mv}{|q|B}.$$

Superposing these motions produces a helix with its axis parallel to $\vec{B}$. Both the radius and pitch remain constant. Reversing the sign of the charge reverses the sense of rotation, while the forward motion is unchanged.



Schematic helical path about the field axis, with initial velocity components shown separately.

Step 3: Conclusion The trajectory is a **helical path about the magnetic-field direction**: circular motion comes from $v \sin 30^\circ$, and uniform advance comes from $v \cos 30^\circ$.

Quick Tip to Solve: Resolve velocity along and across the field first: the parallel component sets the helix pitch, and the perpendicular component sets its radius.

Section C (36 marks)

13(a). Explain briefly how Rutherford scattering of α -particle by a target nucleus can provide information on the size of the nucleus.

[3 marks total]

Solution:

Concept:

- An alpha particle has charge $+2e$; a target nucleus of atomic number Z has charge $+Ze$. Their electrostatic repulsion converts the incident kinetic energy into potential energy during a head-on approach.
- In a head-on encounter, the distance of closest approach $r_{\min}$ follows from energy conservation. A larger incident kinetic energy probes a smaller separation.

Step 1: Interpret the large-angle scattering Most alpha particles pass through a thin foil with little deflection, whereas a few suffer large deflections or return backwards. Thus the positive charge and most of the atomic mass are concentrated in a very small central region, the nucleus.

Step 2: Use the distance of closest approach For an alpha particle incident head-on on a heavy target nucleus, its initial kinetic energy K is momentarily converted entirely into electrostatic potential energy at the distance of closest approach $r_{\min}$, neglecting target recoil:

$$K = \frac{1}{4\pi\epsilon_0} \frac{2Ze^2}{r_{\min}}, \quad r_{\min} = \frac{1}{4\pi\epsilon_0} \frac{2Ze^2}{K}.$$

The alpha particle turns back at this point. If scattering remains purely Rutherford scattering, the

nucleus lies within this closest-approach distance, so the result gives an upper limit to its radius. Raising the incident energy probes smaller distances; departures from the Coulomb-scattering law at very small separations give further information about the nuclear size. Closest approach is therefore not automatically equal to the nuclear radius.

Step 3: Conclusion For incident alpha-particle kinetic energy K , $r_{\min} = 2Ze^2/(4\pi\epsilon_0 K)$ gives an upper bound on the nuclear radius while purely Coulomb scattering applies.

Quick Tip to Solve: Use the alpha-particle charge $2e$ and distinguish a closest-approach upper bound from an exact nuclear radius.

13(b). Show that density of nucleus is independent of its mass number A .

Solution:

Concept:

- Nuclear radius follows $R = R_0 A^{1/3}$, where R_0 is a constant and A is the mass number. Nuclear mass is approximately Am_N , where m_N is the average nucleon mass.
- Average density is mass divided by volume: $\rho = M/V$. For a spherical nucleus $V = 4\pi R^3/3$; the cube of the empirical radius determines how volume depends on A .

Step 1: Write nuclear mass and volume The mass number counts the nucleons, so the mass is approximately $M = Am_N$, neglecting the small binding-energy contribution to mass. Treating the nucleus as approximately spherical, its volume is

$$V = \frac{4}{3}\pi R^3 = \frac{4}{3}\pi R_0^3 A.$$

Step 2: Cancel the mass-number dependence The average nuclear density is consequently

$$\rho = \frac{M}{V} = \frac{Am_N}{(4/3)\pi R_0^3 A} = \frac{3m_N}{4\pi R_0^3}.$$

The factor A cancels because both mass and volume are proportional to the number of nucleons. With m_N and R_0 approximately constant for different nuclei, the average density is approximately the same for all nuclei; small binding-energy and surface corrections are neglected.

Step 3: Conclusion $\rho = 3m_N/(4\pi R_0^3)$ contains no factor of A , so nuclear density is approximately independent of the mass number.

Quick Tip to Solve: Cube $R_0 A^{1/3}$ first: nuclear volume and mass both grow in direct proportion to A .

14. State the underlying principle of a cyclotron. Explain its working with the help of a schematic diagram. Obtain the expression for cyclotron frequency. [3 marks]

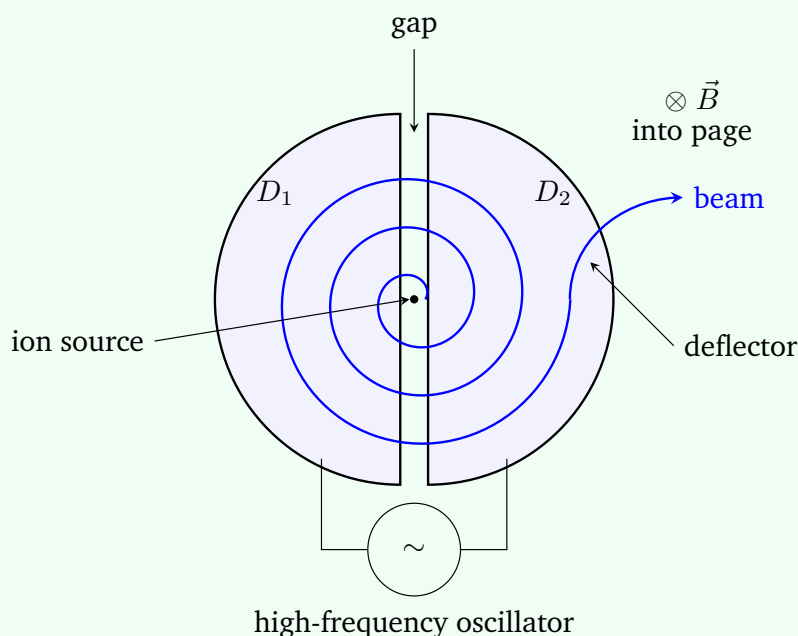
Solution:

Concept:

- A charged particle moving perpendicular to a uniform magnetic field follows a circular path, with $|q|vB = mv^2/r$. Here m is its mass and q its charge.
- For non-relativistic motion the revolution frequency is independent of speed and orbit radius.

An alternating electric field of this frequency accelerates the particle at each crossing of the gap between two hollow D-shaped electrodes (dees).

Step 1: State the principle and identify the apparatus A cyclotron uses resonance: the frequency of the alternating voltage across the dees equals the particle's natural revolution frequency in the magnetic field. Two hollow metal dees D_1 and D_2 are separated by a narrow gap in an evacuated chamber. A uniform magnetic field is perpendicular to their plane. A high-frequency oscillator is connected across the dees, and an ion source lies near the centre.



Cyclotron schematic: increasing orbits cross the accelerating gap; the field is into the page.

Step 2: Explain repeated acceleration A positive ion released near the centre accelerates across the gap toward the negatively charged dee. Inside a dee the electric field is zero, so the magnetic field bends the ion into a semicircle without changing its speed. By the time it reaches the gap again, the oscillator has reversed the dee polarities; the ion therefore gains energy at the next crossing as well. Each gain in speed increases the orbit radius $r = mv/(|q|B)$. Repeated gap crossings and semicircular paths of increasing radius carry the particle outward. A deflector near the outer edge directs the energetic beam toward a target. The electric field supplies energy; the magnetic field only steers the motion.

Step 3: Derive the cyclotron frequency Equating the magnetic force to the centripetal force gives

$$|q|vB = \frac{mv^2}{r}, \quad \frac{v}{r} = \frac{|q|B}{m}.$$

Hence the revolution period and frequency are

$$T = \frac{2\pi r}{v} = \frac{2\pi m}{|q|B}, \quad f_c = \frac{1}{T} = \frac{|q|B}{2\pi m}.$$

A semicircle takes $T/2 = \pi m/(|q|B)$, independent of speed. The alternating voltage reverses after this time, keeping every gap crossing accelerating. The result assumes constant mass in the non-relativistic regime; at relativistic speeds this simple resonance condition fails.

Step 4: Conclusion The cyclotron works by synchronous acceleration across the dee gap while a perpendicular magnetic field guides the particle in semicircles. The required oscillator frequency is

$$f_c = |q|B/(2\pi m).$$

Quick Tip to Solve: Derive the frequency from $v/r = |q|B/m$. The voltage reverses every half-revolution, but its full-cycle frequency is f_c , not $2f_c$.

15. Two infinitely long straight wires A_1 and A_2 carrying currents I and $2I$ flowing in the same directions are kept 'd' distance apart. Where should a third straight wire A_3 carrying current $1.5I$ be placed between A_1 and A_2 so that it experiences no net force due to A_1 and A_2 ? Does the net force acting on A_3 depend on the current flowing through it? [3 marks]

Solution:

Concept:

- The force per unit length between two long parallel wires separated by r , carrying currents I_1 and I_2 , has magnitude $F/L = \mu_0 I_1 I_2 / (2\pi r)$.
- Parallel currents attract and antiparallel currents repel. At a point between the two given wires, their magnetic fields are opposite; a third parallel wire is force-free where these fields cancel.

Step 1: Assign the distances and directions Place A_3 parallel to the given wires at distance x from A_1 , so its distance from A_2 is $d - x$, with $0 < x < d$. Take its current $I_3 = 1.5I$ in the same direction as the given currents. Its attraction towards A_1 opposes its attraction towards A_2 . Reversing I_3 reverses both forces and does not change their cancellation condition.

Step 2: Equate the two force magnitudes For any common length L , the forces balance when

$$\frac{\mu_0 I I_3}{2\pi x} = \frac{\mu_0 (2I) I_3}{2\pi (d - x)}.$$

Cancelling the common nonzero factors gives

$$\frac{1}{x} = \frac{2}{d - x}, \quad d - x = 2x.$$

Thus the weaker-current wire must be nearer: its smaller current is compensated by the smaller separation.

Step 3: Distinguish net force from the balance position Taking the direction towards A_2 as positive, for the chosen current direction,

$$\frac{F_{\text{net}}}{L} = \frac{\mu_0 I I_3}{2\pi} \left(\frac{2}{d - x} - \frac{1}{x} \right).$$

At a general position the net force is proportional to I_3 , so it does depend on that current. At the balance position the bracket is zero; the net force then stays zero for every value of I_3 . Therefore it is the zero-force position, rather than the general force, that is independent of the current in the third wire.

Step 4: Conclusion Place A_3 at $x = d/3$ from A_1 , or $2d/3$ from A_2 . The net force generally depends on I_3 , but at this position it is zero irrespective of I_3 .

Quick Tip to Solve: Between same-direction currents, set $I_1/r_1 = I_2/r_2$; the zero-field point lies closer to the wire carrying the smaller current.

16(a). Draw the equipotential surfaces due to an electric dipole.

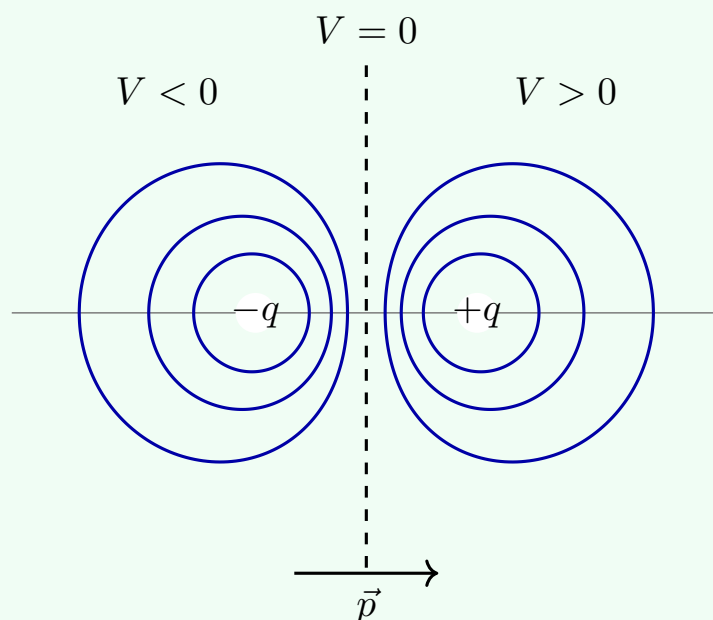
[3 marks total]

Solution:

Concept:

- For charges $+q$ and $-q$, the potential is $V = kq(1/r_+ - 1/r_-)$, where $k = 1/(4\pi\epsilon_0)$ and r_+, r_- are the distances from the charges. An equipotential surface has constant V .
- The electric field is normal to an equipotential surface. Moving a test charge along such a surface requires zero work because $W = q_0\Delta V = 0$.

Step 1: Sketch the constant-potential surfaces The figure shows cross-sections of equipotential surfaces in a plane containing the dipole axis. Rotate the closed curves about the horizontal dipole axis to obtain the corresponding three-dimensional surfaces. The right-hand surfaces have positive potential and the left-hand surfaces have negative potential. Their shapes are nearly spherical close to either charge and distorted farther away. The perpendicular bisector represents an entire plane of zero potential, because every point on it is equidistant from the charges. No two equipotential surfaces intersect; the electric field is perpendicular to them.



Cross-sections of dipole equipotential surfaces; the dashed line represents the zero-potential plane.

Step 2: Check the symmetry and zero-potential plane For any point on the perpendicular bisector, $r_+ = r_-$, so the two potentials cancel exactly. Reflection in this plane exchanges the distances from the positive and negative charges and therefore changes the sign of the potential. Thus corresponding surfaces on opposite sides have equal magnitudes but opposite signs. These curves are equipotential contours, so arrows are not placed along them as they would be on electric field lines.

Step 3: Conclusion The sketch represents positive equipotential surfaces around $+q$, negative surfaces around $-q$, and the perpendicular-bisector plane $V = 0$. Rotation of each closed contour about the dipole axis generates the corresponding surface.

Quick Tip to Solve: Draw paired nested contours around the two charges and mark the entire perpendicular-bisector plane $V = 0$; keep the contours free of field-line arrows.

16(b). Derive an expression for the electric field due to a dipole of dipole moment $\vec{p}$ at a point on its perpendicular bisector.

Solution:

Concept:

- The dipole consists of charges $-q$ and $+q$, separated by $2a$. Its dipole moment $\vec{p} = 2aq \hat{x}$ points from the negative to the positive charge.
- Coulomb's law gives the field magnitude of either charge as kq/R^2 , where $k = 1/(4\pi\epsilon_0)$. Add the electric fields vectorially, resolving along and perpendicular to the dipole axis.

Step 1: Specify the geometry and resolve the fields Place $-q$ at $(-a, 0)$, $+q$ at $(a, 0)$ and the equatorial point P at $(0, r)$, so each charge is at distance $R = \sqrt{r^2 + a^2}$ from P . The field of the positive charge points away from it, and that of the negative charge points towards it. Therefore

$$\begin{aligned}\vec{E}_+ &= \frac{kq}{R^3}(-a\hat{x} + r\hat{y}), \\ \vec{E}_- &= \frac{kq}{R^3}(-a\hat{x} - r\hat{y}).\end{aligned}$$

The components along the perpendicular bisector are equal and opposite. Both components parallel to the dipole axis point towards the negative charge.

Step 2: Add the components and obtain the distant-point limit On addition, the transverse components cancel while the axial components reinforce:

$$\vec{E} = \vec{E}_+ + \vec{E}_- = -\frac{2kqa}{(r^2 + a^2)^{3/2}}\hat{x} = -\frac{\vec{p}}{4\pi\epsilon_0(r^2 + a^2)^{3/2}}.$$

For a short dipole observed far away, $r \gg a$, the denominator is approximately r^3 . Although the scalar potential is zero on the equatorial plane, the electric field there is not zero: its axial components add.

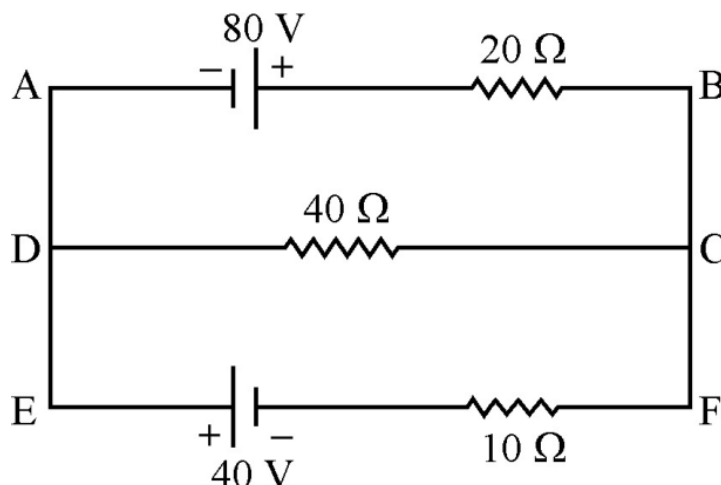
Step 3: Conclusion

$$\boxed{\vec{E}_{\text{eq}} = -\frac{\vec{p}}{4\pi\epsilon_0(r^2 + a^2)^{3/2}}}$$

For $r \gg a$, $\vec{E}_{\text{eq}} \simeq -\vec{p}/(4\pi\epsilon_0 r^3)$. Its direction is opposite to the dipole moment.

Quick Tip to Solve: Cancel the perpendicular-bisector components first; the two surviving components point from the positive charge towards the negative charge.

17. Using Kirchhoff's rules, calculate the current through the $40\ \Omega$ and $20\ \Omega$ resistors in the following circuit: [3 marks]



Solution:

Concept:

- Kirchhoff's junction rule expresses conservation of charge: the algebraic sum of currents entering a junction is zero.
- Kirchhoff's loop rule expresses conservation of energy: the algebraic sum of potential changes around a closed loop is zero. Crossing a cell from its negative to positive terminal gives $+\mathcal{E}$; moving along a resistor's current gives $-IR$.

Step 1: Choose branch-current directions The ideal connecting wires make A, D, E one equipotential node and B, C, F the other. Let I_1 flow through the top branch from A to B , and I_2 through the middle resistor from C to D . The junction rule then makes the bottom current $I_1 - I_2$, directed from F to E . A negative result would simply reverse an assumed direction.

Step 2: Apply the loop rule to the upper loop Traverse $A \rightarrow B \rightarrow C \rightarrow D \rightarrow A$. The 80 V cell is crossed from negative to positive, and both resistors are traversed along their assumed currents:

$$80 - 20I_1 - 40I_2 = 0, \quad I_1 + 2I_2 = 4.$$

The signs follow the actual cell polarity printed in the circuit.

Step 3: Apply the loop rule to the lower loop Traverse $D \rightarrow C \rightarrow F \rightarrow E \rightarrow D$. The middle resistor is traversed opposite to I_2 , the bottom resistor along $I_1 - I_2$, and the 40 V cell from negative to positive:

$$40I_2 - 10(I_1 - I_2) + 40 = 0, \quad I_1 - 5I_2 = 4.$$

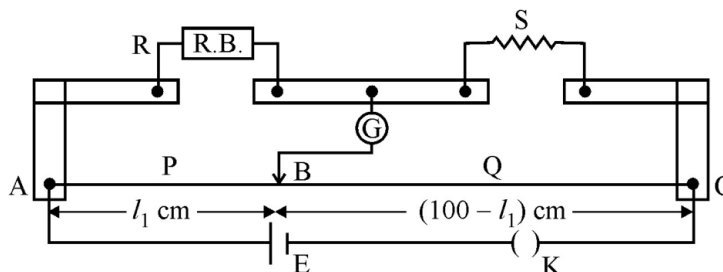
Subtracting this equation from the upper-loop equation gives $7I_2 = 0$. Substitution then determines I_1 . The bottom branch carries the same current magnitude as the top branch, so the two nodes have equal potential and no current is driven through the middle resistor.

Step 4: Conclusion The current through $40\ \Omega$ is $I_2 = 0\text{ A}$. The current through $20\ \Omega$ is $I_1 = 4\text{ A}$, directed from A to B (left to right). The bottom current is 4 A from F to E , consistent with the junction rule.

Quick Tip to Solve: Follow each printed cell polarity before writing the loop equation; here the outer-loop emfs aid one another and the middle nodes become equipotential.

17. What is end error in a metre bridge? How is it overcome? The resistances in the two arms of the metre bridge are $R = 5\ \Omega$ and S respectively. When the resistance S is shunted with an equal resistance, the new balance length is found to be $1.5l_1$, where l_1 is the initial balancing length. Calculate the value of S .

[3 marks]



Solution:

Concept:

- A metre bridge is a Wheatstone bridge. At balance, $R/S = l/(100 - l)$, where l is the balancing length in centimetres measured from the end adjacent to R , for a uniform wire of length 100 cm.
- Equal resistances connected in parallel have equivalent resistance $S_{eq} = S/2$; shunting S therefore changes the ratio of the bridge arms.

Step 1: Explain the end error and its reduction The copper strips, connecting leads and end contacts possess small, non-zero resistances. Consequently, the actual resistances of the two wire arms are not exactly proportional to their measured lengths. This produces an end error in the resistance calculated using the ideal metre-bridge relation. Interchange the resistances in the two gaps, repeat the balance measurement and take the mean of the resistance values obtained in the two arrangements to reduce the end correction. Choose comparable gap resistances so the balance point lies near the middle of the wire, and use clean, tight contacts to minimise contact resistance.

Step 2: Write the two balance conditions In the original arrangement, the unknown resistance S is in the right gap. Thus, neglecting the small end correction in the numerical calculation,

$$\frac{5}{S} = \frac{l_1}{100 - l_1}.$$

When an equal resistance is connected across S , the resistance in that gap becomes $S/2$. The new balance length is $l_2 = 1.5l_1$, measured from the same end. Hence,

$$\frac{5}{S/2} = \frac{1.5l_1}{100 - 1.5l_1}.$$

Step 3: Eliminate the unknown resistance and substitute Dividing the second balance equation by the first cancels both S and the non-zero length l_1 :

$$\begin{aligned} 2 &= \frac{1.5(100 - l_1)}{100 - 1.5l_1}, \\ 200 - 3l_1 &= 150 - 1.5l_1, \\ l_1 &= \frac{100}{3} \text{ cm.} \end{aligned}$$

The new balance is therefore at 50 cm. Using the first balance condition,

$$S = 5 \left(\frac{100 - 100/3}{100/3} \right) \Omega = 5 \times 2 \Omega.$$

As a consistency check, the new balance at the midpoint requires equal gap resistances, so $S/2 = R = 5 \Omega$.

Step 4: Conclusion The required resistance is $S = 10 \Omega$. End error arises from the finite resistance of the end connections; interchanging the gap resistances and averaging the measured results reduces it.

Quick Tip to Solve: Shunting a gap with an equal resistance halves that gap resistance. Keep the balance length measured from the same end in both bridge equations.

18(a). Describe briefly the functions of the three segments of n-p-n transistor.

[3 marks total]

Solution:

Concept:

- An n-p-n transistor has a thin p-type base between an n-type emitter and an n-type collector. Electrons are the principal carriers injected from the emitter.
- In normal active operation, the emitter-base junction is forward biased and the collector-base junction is reverse biased. Current conservation gives $I_E = I_C + I_B$, where the symbols denote emitter, collector and base currents.

Step 1: Describe the emitter and base The emitter is heavily doped so that it supplies a large number of electrons to the base through the forward-biased emitter-base junction. The base is very thin and lightly doped. Consequently, only a small fraction of the injected electrons recombines with holes in the base; this produces the small base current. Most electrons cross the base without recombination, so a small change in base current can control a much larger collector current.

Step 2: Describe the collector The collector receives most of the electrons injected by the emitter after they have crossed the base. The reverse-biased collector-base junction sweeps these electrons into the collector. It is moderately doped and generally made physically larger than the emitter so that it can dissipate the heat produced during operation. Thus the collector current is nearly equal to the emitter current, their small difference being the base current.

Step 3: Conclusion The emitter injects charge carriers, the thin base controls their passage with little recombination, and the collector collects most of them and dissipates heat.

Quick Tip to Solve: Link each region's function to its construction: heavily doped emitter supplies, thin lightly doped base controls, and larger collector collects and cools.

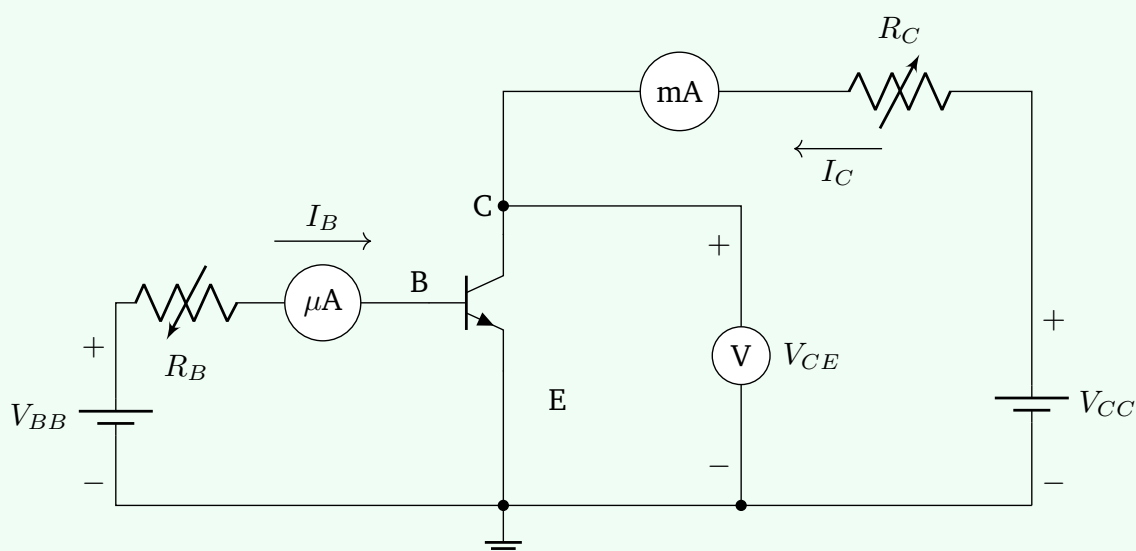
18(b). Draw the circuit arrangement for studying the output characteristics of n-p-n transistor in CE configuration. Explain how the output characteristics is obtained.

Solution:

Concept:

- In common-emitter (CE) configuration the emitter is common to the input base circuit and the output collector circuit. The output characteristic is a plot of collector current I_C against collector-emitter voltage V_{CE} at constant base current I_B .
- For an n-p-n transistor in the active region, the base is positive relative to the emitter and the collector is positive relative to the base. Approximately $I_C = \beta I_B$, where β is the common-emitter current gain.

Step 1: Connect the common-emitter circuit Connect the emitter to the negative terminals of both DC supplies. The base supply V_{BB} , variable resistor R_B and microammeter set and measure I_B . Connect the collector to the positive collector supply V_{CC} through variable resistor R_C and a milliammeter measuring I_C . Connect a voltmeter across collector and emitter, with its positive terminal at the collector, to measure V_{CE} . The n-p-n emitter arrow points outwards.



CE test circuit: the emitter is common to both supplies; the voltmeter measures collector-emitter voltage.

Step 2: Record readings at a constant base current Adjust R_B to choose a small fixed base current. Vary the collector supply or R_C to change V_{CE} , and record the corresponding I_C for each voltmeter reading. Throughout this sweep, monitor the microammeter and readjust R_B if necessary to keep I_B unchanged. Plot I_C on the vertical axis against V_{CE} on the horizontal axis.

Step 3: Obtain the family of output curves Repeat the procedure for successively larger fixed values of I_B , labelling each curve with its base current. At very low V_{CE} , the transistor is in saturation and I_C rises rapidly with voltage. In the active region, each curve is almost horizontal, showing that I_C depends mainly on I_B . A higher base current gives a higher collector-current curve. Keep the applied voltage below the breakdown region.

Step 4: Conclusion The CE output characteristics are the family of I_C -versus- V_{CE} curves, each obtained while holding I_B constant, using the circuit shown.

Quick Tip to Solve: During every output-voltage sweep, keep the microammeter reading fixed; changing the base current during a sweep mixes different characteristic curves.

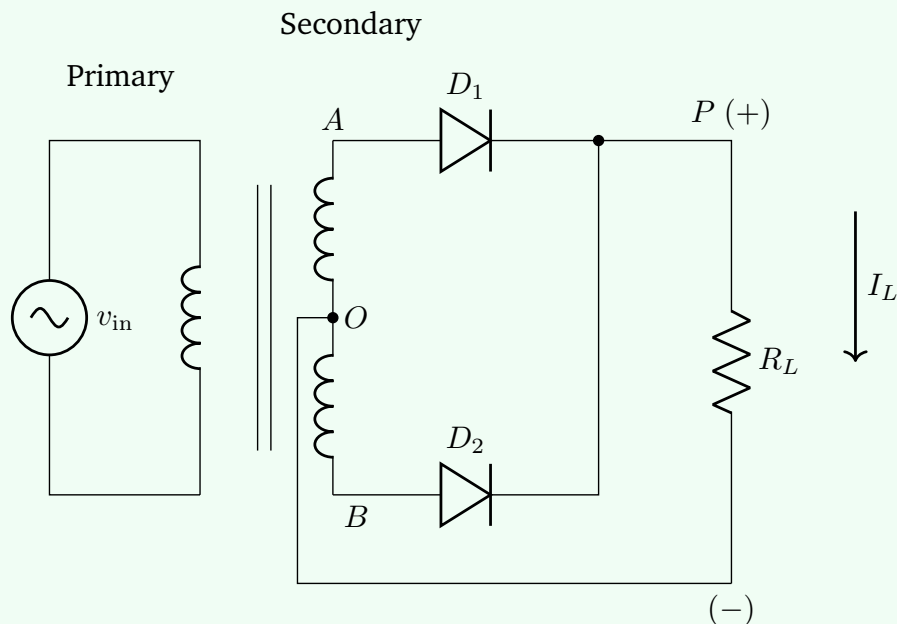
18. Draw the circuit diagram of a full wave rectifier and explain its working. Also, give the input and output waveforms. [3 marks]

Solution:

Concept:

- A junction diode conducts when forward biased and blocks current when reverse biased. Rectification converts alternating voltage into a unidirectional, pulsating output.
- In a centre-tapped full-wave rectifier, the two ends of the secondary have equal and opposite instantaneous voltages relative to the centre tap. The diodes conduct on alternate half-cycles.

Step 1: Connect the centre-tapped rectifier Connect the ends A and B of a centre-tapped transformer secondary to the anodes of D_1 and D_2 . Join their cathodes at P . Connect the load R_L between P and the centre tap O . Apply the sinusoidal input to the primary. The output is measured across R_L , with P positive relative to O .



Centre-tapped full-wave rectifier; the joined cathodes supply the positive load terminal.

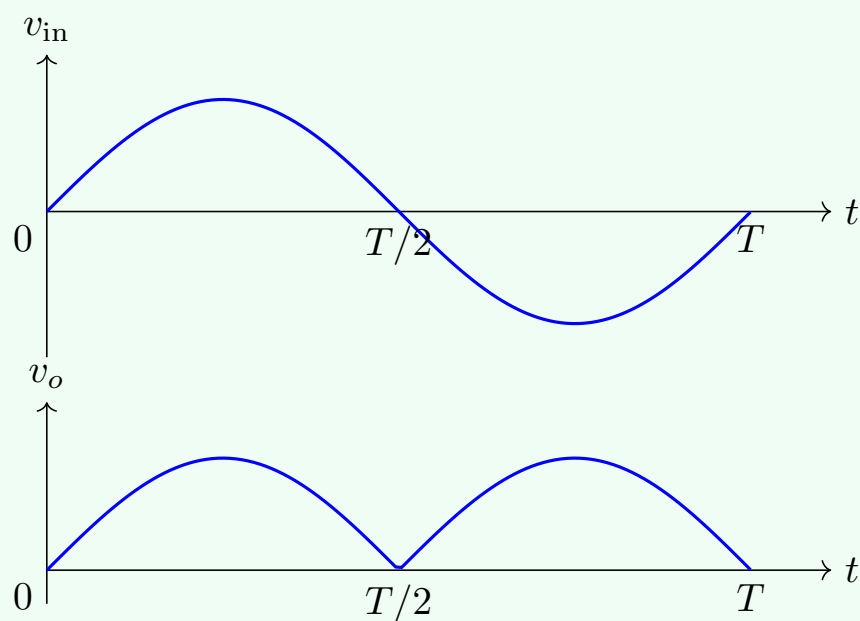
Step 2: Follow current during each half-cycle When A is positive relative to O , B is negative: D_1 is forward biased and D_2 is reverse biased. Conventional current follows $A \rightarrow D_1 \rightarrow P \rightarrow R_L \rightarrow O$, returning through the upper secondary winding.

During the next half-cycle, B is positive and A is negative relative to O . Now D_2 conducts and D_1 blocks; current follows $B \rightarrow D_2 \rightarrow P \rightarrow R_L \rightarrow O$, returning through the lower winding. Thus the current through the load has the same direction in both half-cycles.

Step 3: Obtain the input and output waveforms The input is sinusoidal and changes sign every half-cycle. For ideal diodes, if the voltage of one half of the secondary is $v_{AO} = V_m \sin \omega t$, then the load voltage is

$$v_o = V_m |\sin \omega t|.$$

Each negative half-cycle is turned into a positive output pulse. The output is pulsating direct voltage, not perfectly steady DC; its pulse frequency is $2f$ for input frequency f . A practical diode reduces the peak by its forward voltage drop.



Sinusoidal input and ideal full-wave output, with the same time scale and arbitrary voltage scales.

Step 4: Conclusion The two diodes rectify alternate half-cycles, giving current in the same direction through R_L . Both halves of the input contribute to the full-wave output, whose frequency is twice the input frequency.

Quick Tip to Solve: Trace the two paths separately: both must enter the same end of the load. Opposite load-current directions indicate a wrong diode connection.

19. Define the term wavefront. Using Huygens' wave theory, verify the law of reflection.

[3 marks]

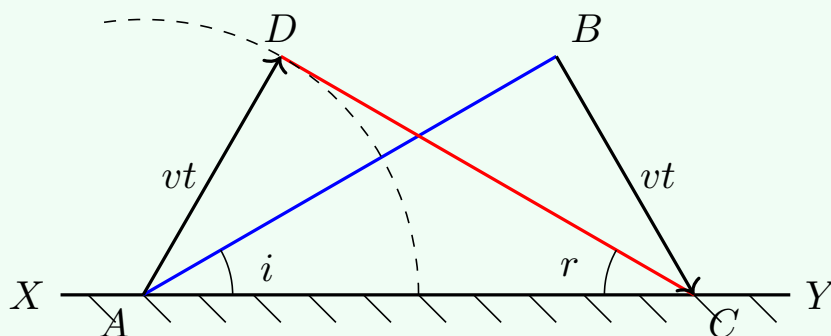
Solution:

Concept:

- A wavefront is the locus of points in a medium which oscillate in the same phase at a given instant. Rays are perpendicular to the wavefront in a homogeneous isotropic medium.
- Huygens' principle: every point on a wavefront acts as a source of secondary wavelets; their forward envelope gives the new wavefront. In reflection, incident and reflected waves travel in the same medium with the same speed v .

Step 1: Construct the incident and reflected wavefronts Let AB be a plane wavefront incident obliquely on the plane reflecting surface XY . Its end A reaches the surface first. During a time t , the disturbance at B reaches C on the surface, travelling along the incident ray BC . Hence $BC = vt$.

In the same time, the secondary reflected wavelet from A grows to radius $AD = vt$. Draw the tangent CD from C to this wavelet. This tangent is the reflected wavefront: $AD \perp CD$, while $BC \perp AB$. The construction below is a section in the plane of incidence.



Huygens' construction: incident wavefront AB , reflected wavefront CD , and equal paths $BC = AD$.

Step 2: Compare the two right triangles The angle between the incident ray and the normal is i . Since both ray and normal are perpendicular to the incident wavefront and surface respectively, $\angle BAC = i$. Similarly, the reflection angle r equals $\angle DCA$. From the right triangles ABC and ADC ,

$$\sin i = \frac{BC}{AC} = \frac{vt}{AC},$$

$$\sin r = \frac{AD}{AC} = \frac{vt}{AC}.$$

Therefore $\sin i = \sin r$. Both angles lie between 0° and 90° , so $i = r$. The incident ray, the normal and the reflected ray all lie in the construction plane, which is perpendicular to the reflecting surface.

Step 3: Conclusion A wavefront joins points in the same phase. Huygens' construction verifies that the angle of reflection equals the angle of incidence, $r = i$, and that the incident ray, reflected ray and normal are coplanar.

Quick Tip to Solve: In reflection, use the same speed on both paths: $BC = AD = vt$. Compare their ratios to the common side AC .

OR

19. Define the term, "refractive index" of a medium. Verify Snell's law of refraction when a plane wavefront is propagating from a denser to a rarer medium. [3 marks]

Solution:

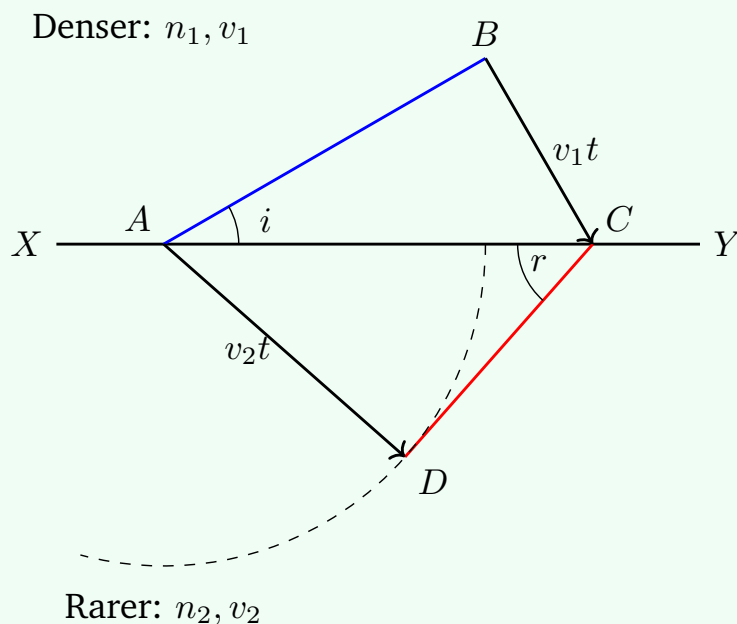
Concept:

- The absolute refractive index of a medium is $n = c/v$, where c is the speed of light in vacuum and v is its speed in that medium. It is dimensionless; greater optical density means a larger index and smaller light speed.
- Huygens' principle constructs the new wavefront as the forward envelope of secondary wavelets. The incident and refracted rays are normal to their respective wavefronts.

Step 1: Construct the refracted wavefront Let a plane wavefront AB approach the plane boundary XY from denser medium 1, of index n_1 and speed v_1 , towards rarer medium 2, of index $n_2 < n_1$ and speed $v_2 > v_1$. Point A reaches the boundary first. During time t , the point B travels along the incident ray to C , so $BC = v_1 t$.

During this same time, the secondary wavelet from A spreads into medium 2 with radius $AD = v_2 t$. Draw tangent CD to this wavelet from C ; it is the refracted wavefront. Thus $BC \perp AB$ and

$AD \perp CD$.



Huygens' refraction construction, with $AD = v_2 t > BC = v_1 t$; angles i, r are also the wavefront inclinations to the interface.

Step 2: Relate the angles to the two speeds Let i and r be the incidence and refraction angles measured from the normal. They equal the angles made by the corresponding wavefronts with the boundary: $i = \angle BAC$ and $r = \angle ACD$. The two right triangles share hypotenuse AC , giving

$$\begin{aligned}\sin i &= \frac{BC}{AC} = \frac{v_1 t}{AC}, \\ \sin r &= \frac{AD}{AC} = \frac{v_2 t}{AC}, \\ \frac{\sin i}{\sin r} &= \frac{v_1}{v_2} = \frac{c/n_1}{c/n_2} = \frac{n_2}{n_1}.\end{aligned}$$

For oblique incidence, $v_2 > v_1$ implies $r > i$: the ray bends away from the normal. The construction gives a propagating refracted ray when i is below the critical angle i_c , where $\sin i_c = n_2/n_1$. At i_c , $r = 90^\circ$; above it there is total internal reflection.

Step 3: Conclusion The refractive index is $n = c/v$. Huygens' construction verifies Snell's law, $n_1 \sin i = n_2 \sin r$; light passing obliquely from a denser to a rarer medium bends away from the normal when refraction is possible.

Quick Tip to Solve: Denser to rarer means $v_2 > v_1$, so draw the transmitted wavelet radius AD longer than BC ; this immediately gives $r > i$.

Identify the part of the electromagnetic spectrum used in (i) radar and (ii) eye surgery. Write their frequency range.

20(a)(i). radar

[3 marks total]

Solution:

Concept:

- Microwaves occupy the electromagnetic spectrum between radio waves of longer wavelength and infrared radiation of shorter wavelength.
- The wave relation is $c = f\lambda$, where f is frequency, λ is wavelength and c is the speed of light in vacuum.

Step 1: Identify the radiation used in radar Radar uses microwaves. Their relatively short wavelengths allow a narrow directional beam to be transmitted towards a distant object. The reflected signal returns to the receiver, enabling the object to be located.

Step 2: State the frequency range The approximate textbook microwave band extends from 10^9 Hz to 10^{12} Hz. A particular radar uses a selected frequency within this broad band; the quoted limits describe the spectral region rather than the bandwidth of one radar.

Step 3: Conclusion Radar uses **microwaves**, of approximate frequency range 10^9 to 10^{12} Hz.

Quick Tip to Solve: Associate radar with short-wavelength directional microwave beams; quote the frequency band in hertz.

20(a)(ii). eye surgery.

Solution:

Concept:

- Ultraviolet radiation has shorter wavelength and higher frequency than visible light.
- Photon energy is $E = hf$, where h is Planck's constant and f is the radiation frequency.

Step 1: Identify the radiation used in eye surgery The ultraviolet region is used in laser eye surgery. An ultraviolet excimer laser can remove very thin layers of corneal tissue with controlled precision, thereby reshaping the cornea.

Step 2: State the frequency range The approximate textbook ultraviolet frequency band is 10^{15} to 10^{17} Hz. These are broad approximate spectral limits; the surgical laser emits a selected wavelength within the ultraviolet region.

Step 3: Conclusion Eye surgery uses **ultraviolet radiation**, of approximate frequency range 10^{15} to 10^{17} Hz.

Quick Tip to Solve: For corneal reshaping, recall ultraviolet excimer radiation and its higher frequency than visible light.

20(b). Prove that the average energy density of the oscillating electric field is equal to that of the oscillating magnetic field.

Solution:

Concept:

- In vacuum the energy densities are $u_E = \frac{1}{2}\epsilon_0 E^2$ and $u_B = B^2/(2\mu_0)$, where E and B are instantaneous field magnitudes.
- For a plane electromagnetic wave, $E_0 = cB_0$, $c^2 = 1/(\mu_0\epsilon_0)$, and the average of $\cos^2$ over a cycle is $1/2$.

Step 1: Average the electric-field energy density At a fixed point, write $E = E_0 \cos \omega t$, choosing the time origin to suppress the phase. The energy density is quadratic in the field, so averaging the field itself would be incorrect:

$$\langle u_E \rangle = \frac{1}{2}\epsilon_0 E_0^2 \langle \cos^2 \omega t \rangle = \frac{1}{4}\epsilon_0 E_0^2.$$

Step 2: Average the magnetic-field energy density and compare The magnetic field oscillates in phase, $B = B_0 \cos \omega t$. Hence

$$\begin{aligned} \langle u_B \rangle &= \frac{B_0^2}{2\mu_0} \langle \cos^2 \omega t \rangle = \frac{B_0^2}{4\mu_0} \\ &= \frac{E_0^2}{4\mu_0 c^2} = \frac{1}{4}\epsilon_0 E_0^2. \end{aligned}$$

Thus each field supplies half of the average total electromagnetic energy density.

Step 3: Conclusion $\langle u_E \rangle = \langle u_B \rangle = \epsilon_0 E_0^2/4$. The electric and magnetic fields each carry half the average total wave energy density.

Quick Tip to Solve: Average the square of each field, then use $E_0 = cB_0$; averaging the oscillating field first would wrongly give zero energy.

21. Draw a labelled ray diagram of an astronomical telescope in the near point adjustment position. A giant refracting telescope at an observatory has an objective lens of focal length 15 m and an eyepiece of focal length 1.0 cm. If this telescope is used to view the Moon, find the diameter of the image of the Moon formed by the objective lens. The diameter of the Moon is 3.48×10^6 m, and the radius of lunar orbit is 3.8×10^8 m. [3 marks]

Solution:

Concept:

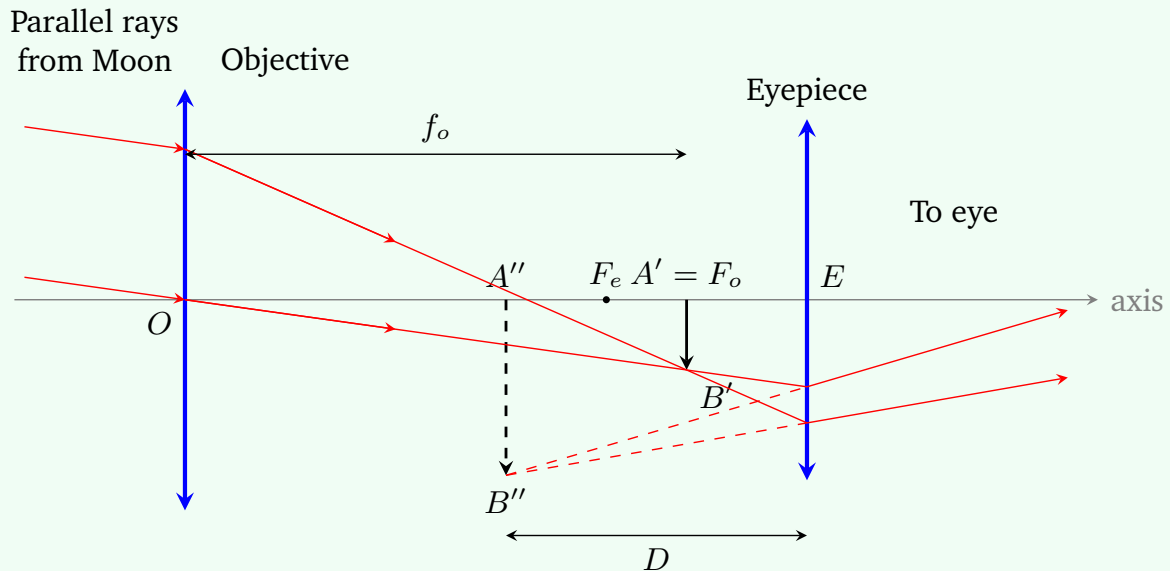
- The objective of a telescope forms a real inverted image of a distant object near its focal plane: image diameter $d = f_o \theta$, where θ is the object's small angular diameter in radians.
- For near-point adjustment, the intermediate image is inside the eyepiece's focal length, so the eyepiece forms an enlarged virtual image at the least distance of distinct vision D .
- For an object of diameter L at distance R , $\theta \simeq L/R$. The focal length f_e affects the eyepiece magnification, not the image diameter formed by the objective.

Step 1: Arrange the lenses for near-point adjustment Parallel rays from a distant point on the Moon meet at B' in the objective's focal plane, forming the real inverted image $A'B'$. Place the eyepiece so that $A'B'$ lies between its first focus F_e and its optical centre E . The rays emerging from the eyepiece diverge; their backward extensions meet at B'' . The final image $A''B''$ is virtual, enlarged

and inverted, at the near point on the objective side of the eyepiece. With the Cartesian convention, $v_e = -D$ gives

$$\frac{1}{f_e} = \frac{1}{-D} - \frac{1}{u_e}, \quad |u_e| = \frac{f_e D}{D + f_e} < f_e.$$

Thus the lens separation is $f_o + |u_e|$. In the schematic, the eye is taken close to the eyepiece and distances are not to scale.



Near-point adjustment: solid rays travel to the right; dashed lines are backward extensions to the virtual image. Schematic, not to scale.

Step 2: Determine the Moon's angular diameter The lunar orbit radius is the distance of the Moon from the telescope to the required approximation. Its angular diameter is small:

$$\theta \simeq \frac{L}{R} = \frac{3.48 \times 10^6}{3.8 \times 10^8} = 9.16 \times 10^{-3} \text{ rad.}$$

Step 3: Calculate the objective's image diameter Since the Moon is extremely far compared with $f_o = 15 \text{ m}$, its image distance is effectively f_o . Therefore

$$d = f_o \theta = 15 \times 9.16 \times 10^{-3} \text{ m} = 0.1374 \text{ m} = 13.74 \text{ cm.}$$

The stated 1.0 cm eyepiece focal length is unnecessary for this diameter: the question asks for the real image made by the objective before the eyepiece magnifies it.

Step 4: Conclusion The near-point ray diagram shows the real image inside the eyepiece focus and a final virtual image at D . The diameter of the Moon's image formed by the objective is $d \simeq 0.137 \text{ m} \simeq 13.7 \text{ cm}$.

Quick Tip to Solve: For a distant extended object, multiply its angular diameter L/R by the objective focal length; do not insert eyepiece magnification.

22(a). If A and B represent the maximum and minimum amplitudes of an amplitude modulated wave, write the expression for the modulation index in terms of A & B . [3 marks total]

Solution:

Concept:

- In sinusoidal amplitude modulation, $s(t) = [A_c + A_m \cos(2\pi f_m t)] \cos(2\pi f_c t)$, where A_c is the carrier peak amplitude and A_m is the amplitude of its variation due to the message.
- The modulation index is the dimensionless ratio $\mu = A_m/A_c$. For an undistorted envelope, $\mu \leq 1$.

Step 1: Express the maximum and minimum envelope amplitudes Let A_c be the unmodulated carrier amplitude and A_m the peak amplitude of the variation produced by the message. In ordinary amplitude modulation without envelope reversal, the positive and negative message peaks give

$$A = A_c + A_m, \quad B = A_c - A_m.$$

Here A and B are both envelope amplitudes, not the message and carrier amplitudes themselves.

Step 2: Eliminate the carrier and message amplitudes Adding the two relations gives $A + B = 2A_c$, while subtraction gives $A - B = 2A_m$. Consequently,

$$A_c = \frac{A + B}{2}, \quad A_m = \frac{A - B}{2}.$$

Dividing the message amplitude by the carrier amplitude cancels the common factor of two. The ratio is dimensionless because the numerator and denominator have identical units.

Step 3: Conclusion $\mu = \frac{A - B}{A + B}$, where A and B are the maximum and minimum envelope amplitudes respectively.

Quick Tip to Solve: When envelope extremes are given, use difference divided by sum; do not mistake their direct ratio for modulation index.

22(b). A message signal of frequency 20 kHz and peak voltage of 10 V is used to modulate a carrier of frequency 2 MHz and peak voltage of 15 V. Calculate the modulation index. Why the modulation index is generally kept less than one?

Solution:

Concept:

- In sinusoidal amplitude modulation, $s(t) = [A_c + A_m \cos(2\pi f_m t)] \cos(2\pi f_c t)$, where A_c is the carrier peak amplitude and A_m is the amplitude of its variation due to the message.
- The modulation index is the dimensionless ratio $\mu = A_m/A_c$. For an undistorted envelope, $\mu \leq 1$.

Step 1: Use the specified peak voltages The message peak voltage is $A_m = 10$ V and the carrier peak voltage is $A_c = 15$ V. Thus

$$\mu = \frac{10}{15} = \frac{2}{3} \simeq 0.667.$$

This corresponds to approximately 66.7% modulation. As a check, the maximum and minimum envelope amplitudes are 25 V and 5 V; their difference divided by their sum again gives $20/30 = 2/3$. The frequencies 20 kHz and 2 MHz do not enter this amplitude ratio.

Step 2: Explain the limit on modulation depth The envelope factor is $A_c[1 + \mu \cos(2\pi f_m t)]$. When

$\mu < 1$, it stays positive throughout the message cycle, so the envelope follows the message faithfully and a simple envelope detector can recover it. If $\mu > 1$, this factor changes sign near the negative message peaks. The envelope crosses zero and reverses phase, producing overmodulation; the detected envelope is distorted and no longer reproduces the original message. At $\mu = 1$, the envelope just touches zero. Keeping the index below one gives a margin against overmodulation due to signal fluctuations.

Step 3: Conclusion The modulation index is $\mu = 2/3 \simeq 0.667$, or 66.7%. It is generally kept below one to avoid overmodulation and distortion during envelope detection.

Quick Tip to Solve: Use a voltage ratio for modulation depth, not the frequency ratio; a positive minimum envelope amplitude checks that $\mu < 1$.

23(a). State Gauss's law for magnetism. Explain its significance.

[3 marks total]

Solution:

Concept:

- Magnetic flux through a surface is $\Phi_B = \int \vec{B} \cdot d\vec{A}$, where $\vec{B}$ is the magnetic field and $d\vec{A}$ is the directed area element.
- Magnetic field lines are continuous closed curves; their local direction and spacing represent the direction and strength of the field.

Step 1: State the closed-surface flux law Gauss's law for magnetism states that the net magnetic flux through any closed surface is zero:

$$\oint_S \vec{B} \cdot d\vec{A} = 0.$$

The vector $d\vec{A}$ points outward from each surface element. A field directed outward contributes positive flux, while one directed inward contributes negative flux.

Step 2: Interpret the physical significance The outgoing flux equals the magnitude of incoming flux. Magnetic field lines have no isolated starting or ending points; they form continuous closed curves. This expresses the absence of isolated magnetic poles in classical electromagnetism: north and south poles occur together. Cutting a bar magnet produces smaller magnets, each with both poles. Zero net flux does not mean that the field is zero everywhere on the surface; the positive and negative contributions cancel.

Step 3: Conclusion $\oint_S \vec{B} \cdot d\vec{A} = 0$. Magnetic poles occur in pairs, and the net outward flux through every closed surface vanishes.

Quick Tip to Solve: Use the closed integral sign and distinguish zero net flux from zero magnetic field at each point.

23(b). Write the four important properties of the magnetic field lines due to a bar magnet.

Solution:

Concept:

- Magnetic flux through a surface is $\Phi_B = \int \vec{B} \cdot d\vec{A}$, where $\vec{B}$ is the magnetic field and $d\vec{A}$ is the directed area element.
- Magnetic field lines are continuous closed curves; their local direction and spacing represent the direction and strength of the field.

Step 1: Describe the geometry and direction of field lines **1. Closed curves:** Magnetic field lines form continuous closed loops. Outside a bar magnet they run from its north pole to its south pole; inside the magnet they return from south to north. They do not terminate at either pole.

2. Local direction: The tangent to a field line at any point gives the direction of the magnetic field there, which is the direction in which a small north test pole would be urged.

Step 2: Describe uniqueness and relative strength **3. No intersections:** Two magnetic field lines cannot cross. If they did, their two tangents would assign two directions to the magnetic field at one point, whereas the field has a unique direction there.

4. Density represents strength: Closer field lines represent a stronger field. Lines are crowded near the poles of the bar magnet and farther apart in regions where the field is weaker. These lines are a representation of the field, not physical threads.

Step 3: Conclusion The four properties are closed-loop continuity, tangential field direction, non-intersection and line density proportional to relative field strength.

Quick Tip to Solve: Recall four distinct properties: loops, tangents, no crossings and spacing; include the return direction inside the magnet.

OR

23. Write three points of differences between para-, dia- and ferro-magnetic materials, giving one example for each. [3 marks]

Solution:

Concept:

- Magnetisation $\vec{M}$ is magnetic dipole moment per unit volume. Magnetic susceptibility χ_m describes the response to an applied magnetic field $\vec{H}$, with $\vec{M} = \chi_m \vec{H}$ for a linear isotropic material.
- Relative permeability is $\mu_r = \mu/\mu_0$. In the linear description, $\mu_r = 1 + \chi_m$; ferromagnets have a much stronger, generally nonlinear response because their magnetic domains can align.

Step 1: Compare attraction and induced magnetisation First difference: a paramagnetic material is weakly attracted towards a stronger applied field and develops a weak magnetisation along that field. A diamagnetic material is weakly repelled towards a weaker field and develops magnetisation opposite to the applied field. A ferromagnetic material is strongly attracted towards the stronger field and develops a large magnetisation along the field as its domains align.

Step 2: Compare magnetic susceptibility Second difference: paramagnetic susceptibility is small and positive, $\chi_m > 0$. Diamagnetic susceptibility is small and negative, $\chi_m < 0$. Ferromagnetic susceptibility is positive and very large over the usual magnetising range. Unlike the small linear

responses, its value depends on applied field and magnetic history; it should not be treated as a universal constant.

Step 3: Compare permeability and give examples Third difference: paramagnetic relative permeability is slightly greater than unity, diamagnetic relative permeability is slightly less than unity, and ferromagnetic relative permeability is generally much greater than unity. Thus, the internal field is weakly reinforced in paramagnets, weakly opposed in diamagnets and greatly enhanced in ferromagnets under ordinary magnetising conditions. An example of a paramagnet is aluminium; an example of a diamagnet is bismuth; an example of a ferromagnet is iron at room temperature.

Step 4: Conclusion Paramagnets such as aluminium have a weak positive response, diamagnets such as bismuth have a weak negative response, and ferromagnets such as iron have a strong positive response. The three comparisons above concern magnetic attraction, susceptibility and relative permeability.

Quick Tip to Solve: Recall the sign and size of susceptibility first: para is small positive, dia is small negative and ferro is large positive; then infer permeability using $\mu_r = 1 + \chi_m$.

24(a). Three photo diodes D_1 , D_2 and D_3 are made of semiconductors having band gaps of 2.5 eV, 2 eV and 3 eV respectively. Which of them will not be able to detect light of wavelength 600 nm?

[3 marks total]

Solution:

Concept:

- A photon of wavelength λ has energy $E = hc/\lambda$, where h is Planck's constant and c is the speed of light. Interband absorption produces electron-hole pairs when $E \geq E_g$, the semiconductor band gap.
- The detection threshold is $hc/\lambda \geq E_g$; a photon must supply at least the band-gap energy to excite an electron from the valence band into the conduction band.

Step 1: Calculate the incident photon energy Convert the wavelength: $600 \text{ nm} = 600 \times 10^{-9} \text{ m}$. Using $h = 6.63 \times 10^{-34} \text{ J s}$, $c = 3.0 \times 10^8 \text{ m s}^{-1}$ and $1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$,

$$\begin{aligned} E &= \frac{(6.63 \times 10^{-34})(3.0 \times 10^8)}{600 \times 10^{-9}} \\ &= 3.315 \times 10^{-19} \text{ J} \\ &= \frac{3.315 \times 10^{-19}}{1.6 \times 10^{-19}} \text{ eV} \simeq 2.07 \text{ eV}. \end{aligned}$$

Step 2: Compare each band gap with the photon energy For D_1 , the band gap is $2.5 \text{ eV} > 2.07 \text{ eV}$, so these photons cannot excite valence-band electrons into the conduction band. For D_2 , $2.0 \text{ eV} < 2.07 \text{ eV}$, so electron-hole pairs can be created. For D_3 , $3.0 \text{ eV} > 2.07 \text{ eV}$, so the photons again lack sufficient energy for interband excitation.

Step 3: Conclusion D_1 and D_3 cannot detect 600 nm light because their band gaps exceed the photon energy 2.07 eV. Diode D_2 can detect it.

Quick Tip to Solve: Use $E \text{ (eV)} \simeq 1240/\lambda \text{ (nm)}$, then reject every detector whose band gap exceeds that photon energy.

24(b). Why photodiodes are required to operate in reverse bias? Explain.

Solution:

Concept:

- A photodiode converts illumination into current when absorbed photons generate electron–hole pairs near the p–n junction.
- A reverse-biased photodiode collects light-generated carriers using the junction electric field. Its photocurrent adds to the small reverse dark current and varies with the incident light intensity.

Step 1: Explain separation and collection of photocarriers Reverse bias widens the depletion region and strengthens the junction electric field. When light creates electron–hole pairs in or near this region, the electric field separates them rapidly: electrons are driven towards the n-side and holes towards the p-side. This collection before recombination produces a reverse photocurrent whose magnitude increases with incident light intensity.

Step 2: Explain why the signal is readily detected In darkness the reverse current is very small, so the fractional change in current caused by illumination is large and measurable. Under forward bias the majority-carrier current is much larger and would mask the small light-induced change. Reverse bias therefore permits sensitive detection and rapid collection of the generated carriers. The applied voltage is maintained below the breakdown region for ordinary photodiode operation.

Step 3: Conclusion Photodiodes operate in reverse bias for efficient separation of light-generated carriers and sensitive measurement of photocurrent above the small dark current.

Quick Tip to Solve: Explain both benefits of reverse bias: rapid carrier separation and a large fractional signal above the small dark current.

Section D (15 marks)

25(a). Describe briefly the process of transferring the charge between the two plates of a parallel plate capacitor when connected to a battery. Derive an expression for the energy stored in a capacitor.

[5 marks total]

Solution:

Concept:

- A capacitor stores equal and opposite charges on its plates. Its capacitance is $C = q/v$, where q is the instantaneous positive plate charge and v is the instantaneous potential difference.
- The work needed to add charge dq against the existing potential difference is $dW = v dq$; this work increases the electrostatic field energy.

Step 1: Describe charging through the battery On connecting the initially uncharged plates to a battery, the battery removes electrons from the plate attached to its positive terminal and supplies electrons to the plate attached to its negative terminal through the external circuit. Thus the plates acquire equal and opposite charges; charge does not cross the insulating gap. The growing plate potential difference opposes further charge transfer. Charging stops when this potential difference equals the battery voltage V , giving final charge magnitude $Q = CV$.

Step 2: Integrate the incremental work During charging, when the charge magnitude is q , the capacitor voltage is $v = q/C$. Hence the incremental increase in stored energy is $dU = (q/C) dq$.

Starting with an uncharged capacitor and integrating to charge Q ,

$$U = \int_0^Q \frac{q}{C} dq = \frac{Q^2}{2C}.$$

Using $Q = CV$ gives the equivalent forms in terms of the final voltage. The factor $1/2$ arises because the capacitor voltage increases from zero to V ; its value is not V throughout the transfer. This integral is the stored field energy, distinct from the total work supplied by a fixed-voltage battery during ordinary charging.

Step 3: Conclusion The energy stored in the charged capacitor is $U = \frac{Q^2}{2C} = \frac{1}{2}QV = \frac{1}{2}CV^2$.

Quick Tip to Solve: Use the instantaneous capacitor voltage q/C inside the charging integral, then substitute the final relation $Q = CV$.

25(b). A parallel plate capacitor is charged by a battery to a potential difference V . It is disconnected from battery and then connected to another uncharged capacitor of the same capacitance. Calculate the ratio of the energy stored in the combination to the initial energy on the single capacitor.

Solution:

Concept:

- When the battery is disconnected, the isolated capacitor combination conserves its total charge; its voltage can change when another capacitor is connected.
- Connected in parallel, two capacitors share a common final voltage and have equivalent capacitance $C_{\text{eq}} = C_1 + C_2$. Their total stored energy is $U = \frac{1}{2}C_{\text{eq}}V_f^2$.

Step 1: Find the initial charge and energy Let each capacitor have capacitance C . Before connection, the charged capacitor has charge magnitude $Q = CV$, while the second capacitor is uncharged. The initial energy of the system is therefore just that of the first capacitor:

$$U_i = \frac{1}{2}CV^2.$$

Since the battery is removed, it cannot supply extra charge or maintain the original voltage during redistribution.

Step 2: Apply charge conservation and compute the final energy On joining the corresponding plates of the two capacitors, charge redistributes until both have a common voltage V_f . Conservation of charge on the connected plate pair gives

$$CV = (C + C)V_f, \quad V_f = \frac{V}{2}.$$

Each capacitor then carries charge magnitude $CV/2$. The energy stored in both capacitors together is

$$U_f = 2 \left[\frac{1}{2}C \left(\frac{V}{2} \right)^2 \right] = \frac{CV^2}{4}.$$

Dividing by the initial energy gives $U_f/U_i = (CV^2/4)/(CV^2/2) = 1/2$. The decrease in electrostatic energy is dissipated during the transient redistribution, for example as heat in the connecting wires

and electromagnetic radiation; total charge conservation does not imply conservation of stored electrostatic energy.

Step 3: Conclusion The required ratio of the total energy in the combination to the initial energy is $U_f : U_i = 1 : 2$.

Quick Tip to Solve: After disconnecting the battery, conserve charge first; two identical parallel capacitors halve the voltage and retain half the initial stored energy.

OR

25(a). Derive an expression for the electric field at any point on the equatorial line of an electric dipole. [5 marks total]

Solution:

Concept:

- An electric dipole consists of charges $-q$ and $+q$, separated by $2a$. Its dipole moment is $\vec{p} = 2qa \hat{x}$, directed from the negative to the positive charge.
- By superposition, the resultant field is the vector sum of the two Coulomb fields, with $k = 1/(4\pi\epsilon_0)$ in vacuum or approximately in air.

Step 1: Specify the equatorial point and individual fields Place the negative charge at $(-a, 0)$, the positive charge at $(a, 0)$, and the point P on the perpendicular bisector at $(0, r)$. Its distance from either charge is $R = \sqrt{r^2 + a^2}$. Each field has magnitude kq/R^2 . The positive-charge field points away from $+q$, whereas the negative-charge field points towards $-q$. Thus their components perpendicular to the dipole axis cancel and their components parallel to the axis add towards the negative charge.

Step 2: Add the components along the dipole axis The magnitude of the axial component of either field is its magnitude multiplied by a/R . Therefore

$$E = 2 \frac{kq}{R^2} \frac{a}{R} = \frac{k(2qa)}{(r^2 + a^2)^{3/2}}.$$

The resultant points opposite to $\vec{p}$. Consequently the exact vector expression is

$$\vec{E}_{\text{eq}} = -\frac{1}{4\pi\epsilon_0} \frac{\vec{p}}{(r^2 + a^2)^{3/2}}.$$

For a short dipole observed far away, $r \gg a$, replace $(r^2 + a^2)^{3/2}$ by r^3 . This approximation is unnecessary in the exact expression and must not be imposed on an arbitrary equatorial point.

Step 3: Conclusion $\vec{E}_{\text{eq}} = -\frac{\vec{p}}{4\pi\epsilon_0(r^2 + a^2)^{3/2}}$, directed opposite to the dipole moment. For $r \gg a$, $\vec{E}_{\text{eq}} \simeq -\vec{p}/(4\pi\epsilon_0 r^3)$.

Quick Tip to Solve: On the equatorial line, cancel the components perpendicular to the dipole axis and add the components pointing from the positive charge towards the negative charge.

25(b). Two identical point charges, q each, are kept 2 m apart in air. A third point charge Q of unknown magnitude and sign is placed on the line joining the charges such that the system remains in equilibrium. Find the position and nature of Q .

Solution:

Concept:

- For the system to remain in equilibrium, the net electric force on each of the three charges must vanish. Coulomb's force magnitude is $F = k|q_1q_2|/d^2$, where d is the separation.
- Identical charges repel. A third charge can balance their outward forces only if it lies between them and has the opposite sign.

Step 1: Locate the third charge using its equilibrium Let the identical charges lie at $x = 0$ and $x = L$, with $L = 2$ m. Outside this interval, the electric fields of the two identical charges point in the same direction, so their resultant cannot vanish. Between them, the forces on a nonzero charge Q are opposite. If Q is a distance x from the first charge, equal force magnitudes require

$$\frac{k|qQ|}{x^2} = \frac{k|qQ|}{(L-x)^2} \Rightarrow x = L - x = 1 \text{ m.}$$

Thus Q must be at the midpoint. This ensures its own equilibrium but the outer charges must also be checked.

Step 2: Balance the force on either outer charge The other outer charge repels an end charge with force magnitude kq^2/L^2 . The central charge must attract that end charge, so Q has the sign opposite to q . Equating the forces on either end charge gives

$$\frac{k|qQ|}{(L/2)^2} = \frac{kq^2}{L^2} \Rightarrow 4|qQ| = q^2 \Rightarrow |Q| = \frac{|q|}{4}.$$

Symmetry gives the same cancellation at the other end, and the two equal forces on the middle charge cancel. These conditions establish zero net force on all three charges; they do not assert stability against displacement.

Step 3: Conclusion Place $Q = -q/4$ at the midpoint, 1 m from each identical charge. It is negative if q is positive, and positive if q is negative.

Quick Tip to Solve: First balance the central charge to locate the midpoint; then balance an outer charge to determine the central charge's sign and magnitude.

26(a). In a series LCR circuit connected across an ac source of variable frequency, obtain the expression for its impedance and draw a plot showing its variation with frequency of the ac source. [5 marks total]

Solution:

Concept:

- In a series LCR circuit the same sinusoidal current flows through resistance R , inductance L and capacitance C . Their reactances are $X_L = \omega L$ and $X_C = 1/(\omega C)$, where $\omega = 2\pi f$.
- The resistor voltage is in phase with current, the inductor voltage leads current by $\pi/2$, and the capacitor voltage lags current by $\pi/2$. The applied voltage is their phasor sum.

Step 1: Add the component voltages as phasors Take the common current I as the horizontal

reference. The voltage $V_R = IR$ lies along it, while $V_L = IX_L$ points upwards and $V_C = IX_C$ downwards. Thus the net reactive voltage is $V_L - V_C$, perpendicular to V_R . By the right-triangle relation,

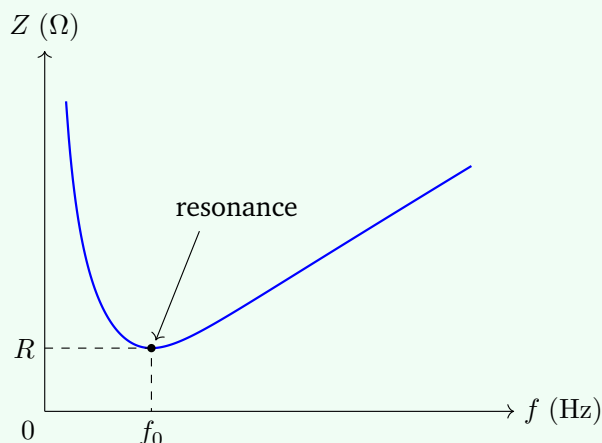
$$\begin{aligned} V^2 &= V_R^2 + (V_L - V_C)^2 \\ &= I^2 R^2 + I^2 (X_L - X_C)^2, \\ Z &= \frac{V}{I} = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C} \right)^2}. \end{aligned}$$

Opposite signs of the two reactive voltages are essential: their magnitudes cannot simply be added.

Step 2: Plot impedance against frequency At low frequency the capacitive reactance is large, so impedance decreases as frequency rises. Its minimum occurs when the net reactance vanishes:

$$2\pi f_0 L = \frac{1}{2\pi f_0 C}, \quad f_0 = \frac{1}{2\pi\sqrt{LC}}, \quad Z_{\min} = R.$$

Above f_0 , the growing inductive reactance makes impedance rise again. For ideal components, Z tends to infinity as $f \rightarrow 0^+$, and at very large f , $Z \approx 2\pi fL$. The following qualitative plot shows the minimum at resonance; the curve does not touch the frequency axis when $R > 0$.



Qualitative impedance-frequency curve for a series LCR circuit; resonance gives $Z = R$.

Step 3: Conclusion $Z = \sqrt{R^2 + (2\pi fL - 1/(2\pi fC))^2}$. It has its minimum $Z = R$ at resonance, $f_0 = 1/(2\pi\sqrt{LC})$, and rises on both sides as shown.

Quick Tip to Solve: Subtract the reactive voltages before squaring: inductor and capacitor voltages point oppositely in the phasor diagram.

26(b). What is the phase difference between the voltages across inductor and the capacitor at resonance in the LCR circuit?

Solution:

Concept:

- In a series LCR circuit the same sinusoidal current flows through resistance R , inductance L and capacitance C . Their reactances are $X_L = \omega L$ and $X_C = 1/(\omega C)$, where $\omega = 2\pi f$.
- The resistor voltage is in phase with current, the inductor voltage leads current by $\pi/2$, and the capacitor voltage lags current by $\pi/2$. The applied voltage is their phasor sum.

Step 1: Compare phases with the same reference current Take the current as $i = I_m \sin \omega t$. For an ideal inductor, $v_L = L di/dt = \omega L I_m \cos \omega t$, which leads the current by $\pi/2$. For a capacitor, integrating $i = C dv_C/dt$ gives $v_C = -I_m \cos \omega t / (\omega C)$, which lags the current by $\pi/2$. The two reactive voltages therefore have opposite signs at every instant.

Step 2: Compare the inductor and capacitor voltages The inductor voltage leads the common current by 90° , whereas the capacitor voltage lags it by 90° . Hence they differ in phase by 180° , or π rad. At resonance $X_L = X_C$, so these two voltages also have equal magnitudes and cancel in the phasor sum. This does not mean that their individual instantaneous voltages are zero.

Step 3: Conclusion The phase difference between the inductor and capacitor voltages at resonance is π rad $= 180^\circ$; they are equal in magnitude and opposite in phase.

Quick Tip to Solve: Use current as the common phase reference: add the inductor's 90° lead and capacitor's 90° lag.

26(c). When an inductor is connected to a 200 V dc voltage, a current of 1 A flows through it. When the same inductor is connected to a 200 V, 50 Hz ac source, only 0.5 A current flows. Explain, why? Also, calculate the self inductance of the inductor.

Solution:

Concept:

- A real inductor has winding resistance r . Its steady dc opposition is r , while its ac impedance is $Z_{\text{coil}} = \sqrt{r^2 + (\omega L)^2}$; ac voltage and current values below are rms values.
- Self-induction produces an emf $\varepsilon = -L dI/dt$, which opposes changes of current. Inductive reactance is $X_L = 2\pi f L$, measured in ohms.

Step 1: Determine the winding resistance and ac impedance After the dc current becomes steady, $dI/dt = 0$, so the induced emf in the coil is zero. Its winding resistance alone limits the current:

$$r = \frac{V_{\text{dc}}}{I_{\text{dc}}} = \frac{200}{1} = 200 \Omega.$$

With ac, the changing current produces a self-induced emf opposing the change. The coil therefore has inductive reactance in addition to its resistance, giving greater impedance and a smaller current at the same applied voltage:

$$Z_{\text{coil}} = \frac{V_{\text{ac}}}{I_{\text{ac}}} = \frac{200}{0.5} = 400 \Omega.$$

Step 2: Calculate the self-inductance The resistive and inductive voltage components are perpendicular, so subtract their squares to find the reactance, rather than subtracting resistance from impedance:

$$\begin{aligned} X_L &= \sqrt{Z_{\text{coil}}^2 - r^2} \\ &= \sqrt{400^2 - 200^2} = 200\sqrt{3} \Omega, \\ L &= \frac{X_L}{2\pi f} = \frac{200\sqrt{3}}{2\pi(50)} \text{ H} \approx 1.10 \text{ H}. \end{aligned}$$

This uses the usual assumption that the winding resistance remains the same in the two measurements and that the coil's capacitive effects are negligible at 50 Hz.

Step 3: Conclusion The ac current is smaller because the coil has inductive reactance as well as resistance. Its winding resistance is $200\ \Omega$ and self-inductance is $L \approx 1.10\text{ H}$.

Quick Tip to Solve: Use the dc reading to obtain the coil resistance first. Then use $X_L = \sqrt{Z_{\text{coil}}^2 - r^2}$, never $Z_{\text{coil}} - r$, before dividing by $2\pi f$.

OR

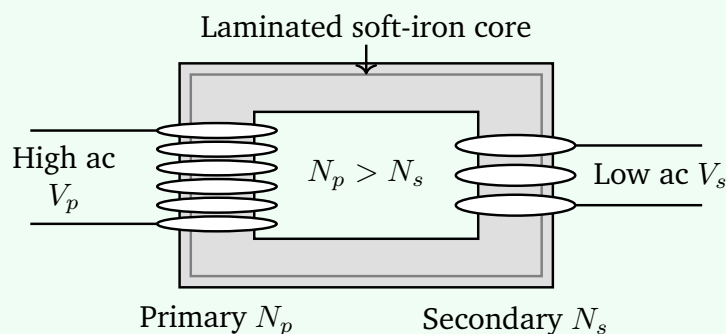
26(a). Draw the diagram of a device which is used to decrease high ac voltage into a low ac voltage and state its working principle. Write four sources of energy loss in this device. [5 marks total]

Solution:

Concept:

- A transformer operates on mutual induction: alternating current in its primary coil produces a changing magnetic flux linking its secondary coil.
- For an ideal transformer, $V_s/V_p = N_s/N_p$, where N_p, N_s are the primary and secondary turns and V_p, V_s are their rms voltages. A step-down transformer has $N_s < N_p$.

Step 1: Identify and draw the device The required device is a step-down transformer. Two insulated coils are wound on a common laminated soft-iron core. The primary has more turns and is connected to the high-voltage ac supply; the secondary has fewer turns and supplies the low-voltage ac output to the load. The core provides a path for the magnetic flux linking both coils.



Step-down transformer: more primary turns than secondary turns on a common laminated core.

Step 2: Explain mutual induction The alternating primary current establishes a time-varying core flux Φ . Faraday's law gives the induced emfs

$$e_p = -N_p \frac{d\Phi}{dt}, \quad e_s = -N_s \frac{d\Phi}{dt}.$$

Thus the secondary voltage is smaller because its number of turns is smaller. This operation requires changing flux; a steady dc supply cannot produce a sustained secondary emf.

Step 3: State four sources of energy loss (i) Copper loss: the resistance of the primary and secondary windings produces I^2R heating. (ii) Eddy-current loss: induced circulating currents heat the iron core; laminations reduce them. (iii) Hysteresis loss: repeated magnetisation and demagnetisation dissipate energy in the core. (iv) Flux leakage: some primary flux does not link the secondary, so energy transfer between the coils is incomplete. Close winding and a suitable closed core reduce leakage.

Step 4: Conclusion The device is a step-down transformer working on mutual induction, with

$N_s < N_p$. Its four principal sources of loss are copper heating, eddy currents, hysteresis and magnetic-flux leakage.

Quick Tip to Solve: Draw more primary turns than secondary turns, label the laminated core, and list copper, eddy-current, hysteresis and flux-leakage losses separately.

26(b). A small town with a demand of 1200 kW of electric power at 220 V is situated 20 km away from an electric plant generating power at 440 V. The resistance of the two wire line carrying power is 0.5Ω per km. The town gets the power from the line through a 4000-220 V step-down transformer at a sub-station in the town. Estimate the line power loss in the form of heat.

Solution:

Concept:

- For an ideal transformer supplying the town, input power equals output power. Taking the usual unity-power-factor estimate, power is $P = VI$, where V and I are rms values.
- Heating in a transmission line is $P_{\text{loss}} = I_{\text{line}}^2 R_{\text{line}}$. Use the high-voltage line current and the total resistance of the two-wire line.

Step 1: Find the current on the transmission side The town requires $P = 1200 \text{ kW} = 1.2 \times 10^6 \text{ W}$. Its substation transformer takes power at 4000 V and delivers it at 220 V. Neglecting transformer losses, the current entering the primary at the receiving end is

$$I_{\text{line}} = \frac{P}{V_{\text{primary}}} = \frac{1.2 \times 10^6}{4000} = 300 \text{ A}.$$

This same current flows along the transmission line. The 220 V town supply and the 440 V generator terminals are on different transformer sides, so neither voltage is used to calculate this line current.

Step 2: Determine the complete line resistance The printed resistance is 0.5Ω per kilometre for the two-wire line. For the stated distance of 20 km,

$$R_{\text{line}} = 0.5 \times 20 = 10 \Omega.$$

The two-wire resistance is already specified, so no extra factor of two is introduced for the return conductor.

Step 3: Calculate Joule heating in the line Substitute the line current and its resistance into the heating formula:

$$P_{\text{loss}} = I_{\text{line}}^2 R_{\text{line}} = (300)^2 (10) = 9.0 \times 10^5 \text{ W}.$$

The loss is substantial because even the stepped-up transmission voltage leaves a large current. As a consistency check, the line drop is $IR = 3000 \text{ V}$, so delivering 4000 V at the substation requires about 7000 V at the sending end; the plant therefore needs a step-up transformer.

Step 4: Conclusion The estimated line power loss as heat is $9.0 \times 10^5 \text{ W} = 900 \text{ kW}$, neglecting transformer losses.

Quick Tip to Solve: Use 4000 V to find the line current, and do not double a resistance already specified for the two-wire line.

27(a). Describe any two characteristic features which distinguish between interference and diffraction phenomena. Derive the expression for the intensity at a point of the interference pattern in Young's double slit experiment. [5 marks total]

Solution:

Concept:

- Interference follows the superposition of coherent waves: their phase difference is constant, so a stable distribution of intensity is formed.
- If the individual intensities are I_1 and I_2 and their phase difference is ϕ , the resultant intensity is obtained by squaring the resultant amplitude; intensity is proportional to amplitude squared.

Step 1: Distinguish the two phenomena **Origin:** In Young's interference experiment the two interfering waves come from two coherent slits. Single-slit diffraction is the interference of secondary wavelets coming from different portions of the same slit wavefront.

Pattern: With equal, narrow slits, ideal Young's bright fringes have equal peak intensity and equal width. In single-slit diffraction the central maximum is brightest and twice as wide as each adjacent secondary maximum; secondary peak intensities decrease away from the centre. The ideal Young's comparison neglects the individual slits' diffraction envelope.

Step 2: Add the two oscillations at a screen point Let the light disturbances be $e_1 = a_1 \sin \omega t$ and $e_2 = a_2 \sin(\omega t + \phi)$, where a_1, a_2 are their amplitudes and ω is the angular frequency. Superposition gives

$$e = (a_1 + a_2 \cos \phi) \sin \omega t + a_2 \sin \phi \cos \omega t.$$

Writing this as $e = A \sin(\omega t + \alpha)$ and comparing the sine and cosine coefficients,

$$A \cos \alpha = a_1 + a_2 \cos \phi,$$

$$A \sin \alpha = a_2 \sin \phi,$$

$$\begin{aligned} A^2 &= (a_1 + a_2 \cos \phi)^2 + a_2^2 \sin^2 \phi \\ &= a_1^2 + a_2^2 + 2a_1 a_2 \cos \phi. \end{aligned}$$

Step 3: Convert amplitude into intensity Let $I_1 = k a_1^2$, $I_2 = k a_2^2$, and $I = k A^2$, with the same proportionality constant k . Multiplication by k gives

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \phi.$$

For slit separation d , distance D , and transverse screen coordinate x , the path difference is $\Delta \simeq dx/D$, so $\phi = 2\pi dx/(\lambda D)$. For equal slit intensities $I_1 = I_2 = I_0$,

$$I(x) = 2I_0(1 + \cos \phi) = 4I_0 \cos^2 \left(\frac{\pi dx}{\lambda D} \right).$$

Thus phase differences $2n\pi$ produce intensity $4I_0$, while $(2n + 1)\pi$ produce zero intensity, as expected for constructive and destructive interference.

Step 4: Conclusion Interference combines waves from coherent slits; diffraction combines contribu-

tions across one aperture and has a wider, brighter central maximum. The interference intensity is $I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \phi$, reducing to $4I_0 \cos^2(\phi/2)$ for equal slit intensities.

Quick Tip to Solve: First add amplitudes, then square: adding the two intensities alone misses the interference term.

27(b). In the diffraction due to a single slit experiment, the aperture of the slit is 3 mm. If monochromatic light of wavelength 620 nm is incident normally on the slit, calculate the separation between the first order minima and the 3rd order maxima on one side of the screen. The distance between the slit and the screen is 1.5 m.

Solution:

Concept:

- For a single slit of width a , minima satisfy $a \sin \theta = n\lambda$. In the small-angle approximation, $y \simeq D\theta$, where D is the screen distance. Secondary maxima lie approximately midway between successive minima.
- For the n -th secondary maximum, the usual school approximation is $y_{\max, n} \simeq (n + 1/2)\lambda D/a$. This midpoint formula is approximate, whereas the minima positions follow the exact angular condition.

Step 1: Calculate the first-minimum position Use $a = 3 \times 10^{-3}$ m, $\lambda = 620 \times 10^{-9}$ m, and $D = 1.5$ m. The first minimum is at

$$y_{\min, 1} = \frac{\lambda D}{a} = \frac{620 \times 10^{-9} \times 1.5}{3 \times 10^{-3}} = 3.10 \times 10^{-4} \text{ m.}$$

Step 2: Locate the third secondary maximum and subtract Counting secondary maxima outward from the central maximum, the third lies between the third and fourth minima. Using the usual approximate midpoint position,

$$y_{\max, 3} \simeq \frac{7\lambda D}{2a} = 1.085 \times 10^{-3} \text{ m.}$$

Since both points are on the same side of the centre, subtract their distances:

$$\Delta y \simeq \left(\frac{7}{2} - 1\right) \frac{\lambda D}{a} = 7.75 \times 10^{-4} \text{ m.}$$

Step 3: Conclusion The required same-side separation is approximately $7.75 \times 10^{-4} \text{ m} = 0.775 \text{ mm}$, using the standard approximate position of a secondary diffraction maximum.

Quick Tip to Solve: For the third secondary diffraction maximum use $7\lambda D/(2a)$, then subtract $\lambda D/a$ because the specified features lie on the same side.

27(a). Under what conditions is the phenomenon of total internal reflection of light observed? Obtain the relation between the critical angle of incidence and the refractive index of the medium.

[5 marks total]

Solution:

Concept:

- Snell's law is $n_1 \sin i = n_2 \sin r$, where n_1, n_2 are the refractive indices of the incident and refracting media and angles are measured from the normal.
- The critical angle C is the angle of incidence in the optically denser medium for which the refracted ray just grazes the interface, $r = 90^\circ$.

Step 1: State the conditions for total internal reflection Light must travel from an optically denser medium to an optically rarer medium, so $n_1 > n_2$. Its angle of incidence in the denser medium must exceed the critical angle: $i > C$. At $i = C$ the refracted ray is along the interface; total internal reflection occurs only for larger incidence angles. For such angles Snell's law would demand $\sin r > 1$, which is impossible for a propagating refracted ray, so the incident light is reflected back into the denser medium.

Step 2: Derive the critical-angle relation Apply Snell's law at the limiting angle $i = C$, where $r = 90^\circ$:

$$n_1 \sin C = n_2 \sin 90^\circ = n_2.$$

Therefore

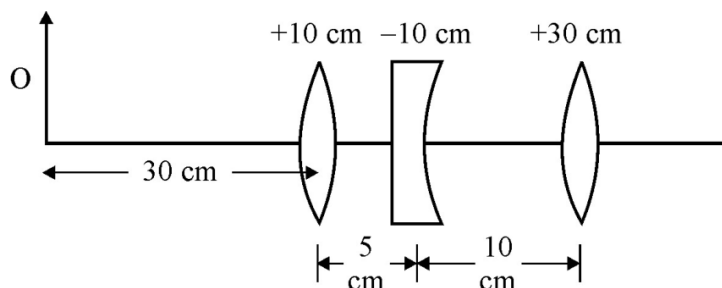
$$\sin C = \frac{n_2}{n_1}, \quad \frac{n_1}{n_2} = \frac{1}{\sin C}.$$

If the rarer medium is air, take $n_2 \simeq 1$. The refractive index μ of the denser medium relative to air is then $\mu = 1/\sin C$. This relation uses the refractive index relative to the rarer medium; it is not an absolute-index formula for an arbitrary pair of media.

Step 3: Conclusion Total internal reflection requires propagation from an optically denser to a rarer medium and $i > C$. The critical-angle relation is $\sin C = n_2/n_1$, or $\mu = 1/\sin C$ for refractive index relative to air.

Quick Tip to Solve: At $i = C$ the transmitted ray grazes the surface; use the strict inequality $i > C$ for total internal reflection.

27(b). Three lenses of focal lengths $+10\text{ cm}$, -10 cm and $+30\text{ cm}$ are arranged coaxially as in the figure given below. Find the position of the final image formed by the combination.



Solution:

Concept:

- For each thin lens, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$. With light travelling to the right, distances to the right of that lens are positive and distances to its left are negative.
- The image formed by one lens becomes the object for the next. If incident rays are converging toward a point on the outgoing side of a lens, that point is a virtual object and its object distance is positive.

Step 1: Find the image formed by the first lens Denote the lenses in order by L_1, L_2, L_3 . The object is 30 cm to the left of L_1 , whose focal length is $+10\text{ cm}$. Thus $u_1 = -30\text{ cm}$, and

$$\begin{aligned}\frac{1}{v_1} &= \frac{1}{f_1} + \frac{1}{u_1} \\ &= \frac{1}{10} - \frac{1}{30} = \frac{1}{15}, \\ v_1 &= +15\text{ cm}.\end{aligned}$$

Without the other lenses, the rays would meet 15 cm to the right of L_1 . This is the location toward which the rays are converging when they reach the second lens.

Step 2: Treat the converging rays at the second lens The second lens is 5 cm to the right of the first. The first lens's image therefore lies $15 - 5 = 10\text{ cm}$ to its right and acts as a virtual object for L_2 : $u_2 = +10\text{ cm}$. With $f_2 = -10\text{ cm}$,

$$\frac{1}{v_2} = \frac{1}{f_2} + \frac{1}{u_2} = -\frac{1}{10} + \frac{1}{10} = 0.$$

Hence the rays leaving L_2 are parallel; its image is at infinity. The positive object distance is essential because the incident rays are already converging toward a point on the outgoing side of this lens.

Step 3: Locate the image formed by the third lens The third lens receives the parallel beam, even though it is 10 cm beyond the second lens. Parallel incident rays are focused in its rear focal plane. With $f_3 = +30\text{ cm}$,

$$\frac{1}{v_3} = \frac{1}{f_3} + 0 = \frac{1}{30}, \quad v_3 = +30\text{ cm}.$$

The final image is real, since the rays actually converge after passing through L_3 . Its coordinate relative to L_1 is $5 + 10 + 30 = 45$ cm.

Step 4: Conclusion The final image is real and lies 30 cm to the right of the third lens, equivalently 45 cm to the right of the first lens.

Quick Tip to Solve: Track each intermediate image from the next lens: here $15 - 5 = +10$ cm is a virtual-object distance, so the concave lens sends out parallel rays.