

CBSE Class 12 Physics Question Paper 2019 with Solution PDF-Set 55/2/2



Question Paper with Solutions

Conducted by the Central Board of Secondary Education (CBSE)

Time Allowed: 3 hours

Maximum Marks: 70

General Instructions

- (i) All questions are compulsory. There are **27** questions in all.
- (ii) This question paper has **four** sections : Section A, Section B, Section C and Section D.
- (iii) Section A contains **five** questions of **one** mark each, Section B contains **seven** questions of **two** marks each, Section C contains **twelve** questions of **three** marks each, Section D contains **three** questions of **five** marks each.
- (iv) There is no overall choice. However, an internal choice(s) has been provided in **two** questions of **one** mark, **two** questions of **two** marks, **four** questions of **three** marks and **three** questions of **five** marks weightage. You have to attempt only **one** of the choices in such questions.

You may use the following values of physical constants wherever necessary :

$$c = 3 \times 10^8 \text{ m/s}$$

$$h = 6.63 \times 10^{-34} \text{ Js}$$

$$e = 1.6 \times 10^{-19} \text{ C}$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ T m A}^{-1}$$

$$\epsilon_0 = 8.854 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$$

$$\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$$

$$\text{Mass of electron } (m_e) = 9.1 \times 10^{-31} \text{ kg}$$

$$\text{Mass of neutron} = 1.675 \times 10^{-27} \text{ kg}$$

$$\text{Mass of proton} = 1.673 \times 10^{-27} \text{ kg}$$

$$\text{Avogadro's number} = 6.023 \times 10^{23} \text{ per gram mole}$$

$$\text{Boltzmann constant} = 1.38 \times 10^{-23} \text{ JK}^{-1}$$

Section A (5 marks)

1. Write the relation for the force acting on a charged particle q moving with velocity $\vec{v}$ in the presence of a magnetic field $\vec{B}$. [1 mark]

Solution:

Concept:

- The magnetic force on a moving charge is the magnetic part of the Lorentz force. Here q is the signed charge, $\vec{v}$ its velocity and $\vec{B}$ the magnetic field.
- A vector cross product determines both the magnitude and the direction of this force.

Step 1: Write the magnetic force law The required vector relation is

$$\vec{F} = q(\vec{v} \times \vec{B}).$$

If θ is the angle between the velocity and the magnetic field, its magnitude is $F = |q|vB \sin \theta$.

Step 2: Interpret the direction For a positive charge, the force points along $\vec{v} \times \vec{B}$, perpendicular to both vectors. For a negative charge the direction reverses. Motion parallel or antiparallel to the field gives zero magnetic force.

Step 3: Conclusion $\vec{F} = q(\vec{v} \times \vec{B})$.

Quick Tip to Solve: Use the signed charge in the vector law; use its absolute value when calculating the force magnitude.

2. The magnetic susceptibility of magnesium at 300 K is 1.2×10^5 . At what temperature will its magnetic susceptibility become 1.44×10^5 ? [1 mark]

Solution:

Concept:

- Curie's law for a paramagnetic material is $\chi = C/T$, where χ is magnetic susceptibility, T is absolute temperature and C is the Curie constant.
- For the same material, $\chi_1 T_1 = \chi_2 T_2$.

Step 1: Apply the inverse temperature dependence Using the Curie-law relation intended here, the printed data are $T_1 = 300$ K, $\chi_1 = 1.2 \times 10^5$ and $\chi_2 = 1.44 \times 10^5$. The factor 10^5 is common to the two susceptibilities and cancels in their ratio.

Step 2: Calculate the required temperature

$$T_2 = \frac{\chi_1 T_1}{\chi_2} = \frac{1.2 \times 10^5 \times 300}{1.44 \times 10^5} = 250 \text{ K}.$$

The susceptibility increases by a factor 1.2, so the temperature must decrease by the same factor.

Step 3: Conclusion The required temperature is 250 K, using Curie's law.

Quick Tip to Solve: Cancel the common power of ten before calculating; susceptibility and absolute temperature vary inversely.

2. The magnetic susceptibility χ of a given material is -0.5 . Identify the magnetic material. [1 mark]

Solution:

Concept:

- Magnetic susceptibility is defined by $M = \chi H$, where M is magnetisation and H is the applied magnetic field strength.
- A negative susceptibility characterises a diamagnetic material; positive susceptibility characterises paramagnetic or ferromagnetic behaviour.

Step 1: Read the sign of susceptibility The given value is $\chi = -0.5$. Its negative sign means that the induced magnetisation points opposite to the applied magnetic field.

Step 2: Identify the magnetic response A material whose induced magnetisation opposes the applied field is diamagnetic. Thus the classification follows directly from the sign, without requiring the chemical identity of the substance or a calculation of its magnitude.

Step 3: Conclusion The given material is diamagnetic.

Quick Tip to Solve: For classification from susceptibility, inspect the sign first: a negative value indicates diamagnetism.

3. Draw the pattern of electric field lines due to an electric dipole. [1 mark]

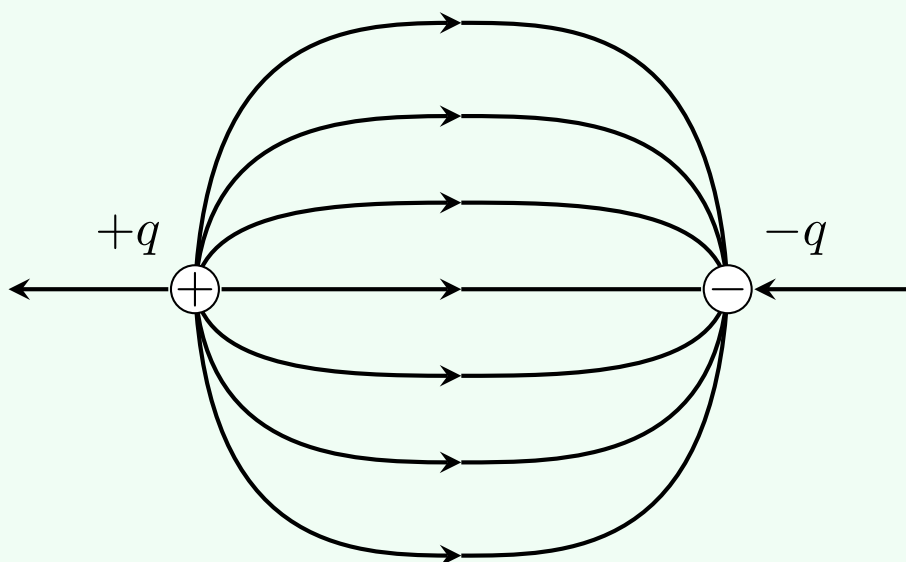
Solution:

Concept:

- An electric dipole consists of two equal and opposite charges, $+q$ and $-q$, separated by a small distance.
- Electric field lines originate on a positive charge and terminate on a negative charge. The tangent at a point gives the electric field direction.

Step 1: Draw the dipole field Place the two charges on a horizontal axis and draw a symmetric family of curved field lines connecting them. Add arrowheads pointing away from $+q$ and towards $-q$; the line between the charges also points from positive to negative.

Quick Tip to Solve: Put arrowheads on the sketch: each field line must leave the positive charge and enter the negative charge.



Electric field lines of a dipole, directed from the positive charge to the negative charge.

Step 2: Check the field-line pattern The lines do not intersect, because the field at a point has a unique direction. The pattern is symmetric about the dipole axis, and the lines are crowded near the charges where the field is stronger. On the outer axial portions, the arrows point away from the positive charge and towards the negative charge.

Step 3: Conclusion The diagram shows the required dipole pattern, with field-line direction from $+q$ to $-q$.

Quick Tip to Solve: Put arrowheads on the sketch: each field line must leave the positive charge and enter the negative charge.

4. Which part of the electromagnetic spectrum is used in RADAR? Give its frequency range. [1 mark]

Solution:

Concept:

- Microwaves are electromagnetic waves lying between radio waves and infrared radiation in the spectrum.
- RADAR transmits electromagnetic waves and detects their reflected signal to locate an object and determine its distance.

Step 1: Identify the spectral region RADAR uses microwaves. Their relatively short wavelengths allow a directed beam to be formed with a suitable antenna, so the reflected signal can reveal the direction of a target.

Step 2: State the frequency range The commonly quoted broad school-level microwave frequency range is approximately 10^9 to 10^{12} Hz. A particular RADAR operates within a selected band in this region; it does not need to transmit over the whole range.

Step 3: Conclusion RADAR uses microwaves, with an approximate frequency range 10^9 – 10^{12} Hz.

Quick Tip to Solve: Remember the pairing RADAR and microwaves, then quote frequency in hertz rather than a wavelength interval.

OR

4. How are electromagnetic waves produced by accelerating charges?

[1 mark]

Solution:

Concept:

- An accelerating charge produces time-dependent electric and magnetic fields.
- A changing electric field produces a magnetic field, and a changing magnetic field produces an electric field; these coupled disturbances can propagate through space.

Step 1: Explain the effect of acceleration When the velocity of a charge changes, its electric field changes with time. The disturbance in that field is accompanied by a changing magnetic field; it is not confined to the immediate location of the charge.

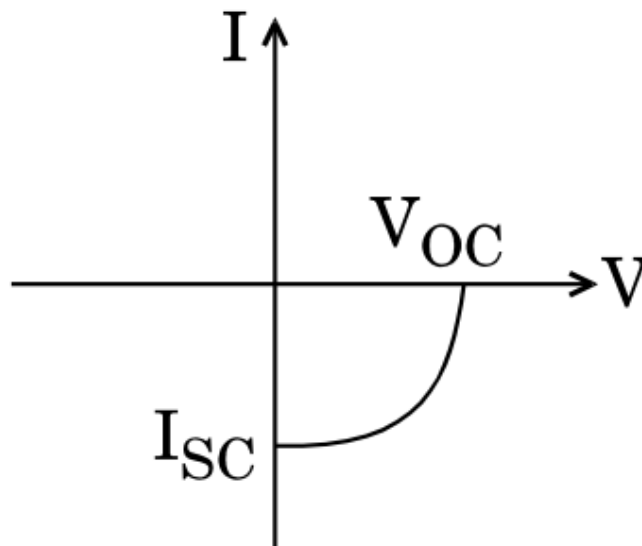
Step 2: Describe wave propagation The mutually coupled electric and magnetic field disturbances travel outwards from the accelerating charge as an electromagnetic wave. In vacuum their propagation speed is c , and the electric field, magnetic field and direction of propagation are mutually perpendicular.

Step 3: Conclusion Accelerating charges radiate electromagnetic waves through the coupled time-varying electric and magnetic fields they produce.

Quick Tip to Solve: Link acceleration to changing fields, then explain that the coupled electric and magnetic disturbance propagates.

5. Identify the semiconductor diode whose I-V characteristics are as shown.

[1 mark]



Solution:

Concept:

- A solar cell is an illuminated semiconductor p - n junction that converts light energy into electrical energy.
- Its output characteristic has a short-circuit current I_{SC} at zero voltage and an open-circuit voltage V_{OC} at zero current.

Step 1: Read the intercepts of the graph The curve meets the negative current axis at the labelled short-circuit current and the positive voltage axis at V_{OC} . Thus a current exists even when the terminal voltage is zero, and a voltage exists when the external current is zero.

Step 2: Identify the operating device This fourth-quadrant characteristic indicates a junction supplying electrical power under illumination. Light creates electron-hole pairs, and the junction field separates them to produce a photocurrent and a terminal voltage. These are the defining features of a solar cell.

Step 3: Conclusion The semiconductor diode is a solar cell.

Quick Tip to Solve: A fourth-quadrant curve labelled with short-circuit current and open-circuit voltage identifies a solar cell.

Section B (14 marks)

6. A photon and a proton have the same de-Broglie wavelength λ . Prove that the energy of the photon is $(2m\lambda c/h)$ times the kinetic energy of the proton. [2 marks]

Solution:

Concept:

- The de-Broglie relation is $p = h/\lambda$, where p is momentum and h is Planck's constant.
- A photon has energy $E_\gamma = pc$; a nonrelativistic proton of mass m has kinetic energy $K_p = p^2/(2m)$.

Step 1: Use the common wavelength Equal wavelengths imply equal momenta, not equal energies. For the photon,

$$E_\gamma = \frac{hc}{\lambda}.$$

For the proton the same momentum is $p = h/\lambda$. The symbol m in the requested factor denotes the proton mass, and c denotes the speed of light in vacuum.

Step 2: Compare the two energies Using the nonrelativistic kinetic-energy relation implicit in this question,

$$K_p = \frac{1}{2m} \left(\frac{h}{\lambda} \right)^2 = \frac{h^2}{2m\lambda^2}.$$

Division eliminates one power of h and one of λ :

$$\frac{E_\gamma}{K_p} = \frac{hc}{\lambda} \frac{2m\lambda^2}{h^2} = \frac{2m\lambda c}{h}.$$

This factor is dimensionless, as an energy ratio must be.

Quick Tip to Solve: First equate momenta from the common wavelength, then use the different energy-momentum relations.

Step 3: Conclusion $E_\gamma = (2m\lambda c/h)K_p$, as required for a nonrelativistic proton.

Quick Tip to Solve: First equate momenta from the common wavelength, then use the different energy-momentum relations.

7. How is the equation for Ampere's circuital law modified in the presence of displacement current? Explain. [2 marks]

Solution:

Concept:

- A steady conduction current obeys $\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I_c$.
- A changing electric flux $\Phi_E = \int \mathbf{E} \cdot d\mathbf{A}$ produces displacement current $I_d = \varepsilon_0 d\Phi_E/dt$ in vacuum.

Step 1: State the modified law Maxwell adds displacement current to the enclosed conduction current. The Ampere-Maxwell law is

$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0(I_c + I_d) = \mu_0 I_c + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}.$$

The surface used to calculate the flux is bounded by the same closed path as the magnetic-field integral. Here μ_0 and ε_0 are the permeability and permittivity of free space.

Step 2: Explain the need for displacement current Consider a charging capacitor. A surface intersecting the connecting wire encloses conduction current, whereas a surface passing between the capacitor plates encloses no conduction current. Nevertheless the electric field and flux between the plates change. Since $\Phi_E = Q/\varepsilon_0$,

$$I_d = \varepsilon_0 \frac{d}{dt} \left(\frac{Q}{\varepsilon_0} \right) = \frac{dQ}{dt} = I_c.$$

The extra term therefore gives the same magnetic circulation for either surface. Displacement current represents the magnetic effect of a changing electric field; it does not require charges to cross the dielectric gap.

Step 3: Conclusion The modified equation is $\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0[I_c + \varepsilon_0(d\Phi_E/dt)]$.

Quick Tip to Solve: For a charging capacitor, verify that the displacement current in the gap equals the current in the wire.

8. Under what conditions does the phenomenon of total internal reflection take place? Draw a ray diagram showing how a ray of light deviates by 90° after passing through a right-angled isosceles prism. [2 marks]

Solution:

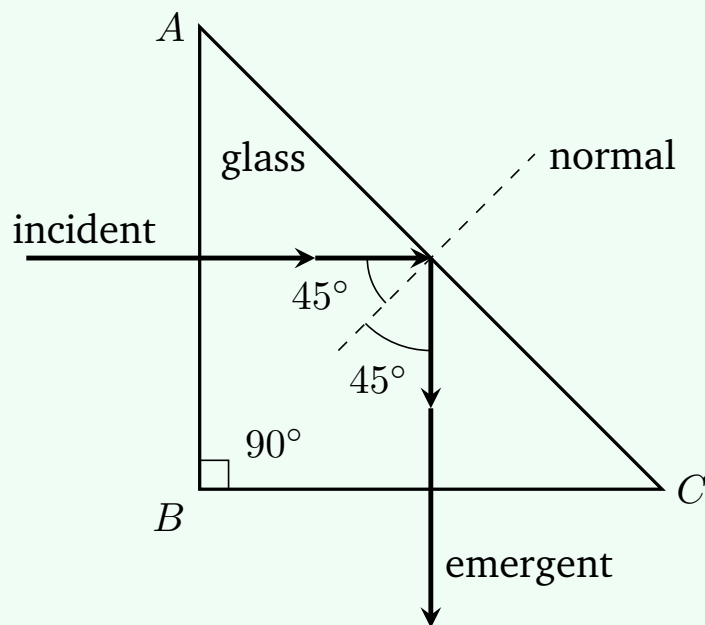
Concept:

- Total internal reflection requires light to travel from an optically denser medium to a rarer medium.
- The incidence angle must exceed the critical angle i_c , with $\sin i_c = n_2/n_1$ for $n_1 > n_2$.

Quick Tip to Solve: Enter a perpendicular face normally so that the only change of direction occurs at the hypotenuse.

Step 1: Specify both conditions Light must approach the boundary from the denser medium, and its angle of incidence, measured from the normal, must be greater than the critical angle. At $i = i_c$ the refracted ray grazes the surface; total internal reflection occurs only for $i > i_c$. A ray entering from air into glass cannot undergo total internal reflection at that entry surface.

Step 2: Trace the ray through the prism In a right-angled isosceles glass prism, send the incident ray normally through one of the perpendicular faces. It enters without bending and meets the hypotenuse at 45° . For a glass-air boundary with $i_c < 45^\circ$, the ray is totally reflected. The reflected ray then strikes the other perpendicular face normally and emerges without bending. Thus the incident horizontal ray becomes a vertically downward emergent ray. For ordinary glass, $n \simeq 1.5$ gives $i_c \simeq 41.8^\circ$, satisfying the required condition.



Normal entry and exit, with total internal reflection at the hypotenuse.

Step 3: Conclusion The conditions are denser-to-rarer travel and $i > i_c$; one reflection at the hypotenuse produces a deviation of 90° .

Quick Tip to Solve: Enter a perpendicular face normally so that the only change of direction occurs at the hypotenuse.

9. A photon emitted during the de-excitation of electron from a state n to the first excited state in a hydrogen atom, irradiates a metallic cathode of work function 2 eV, in a photo cell, with a stopping potential of 0.55 V. Obtain the value of the quantum number of the state n . [2 marks]

Solution:

Concept:

- Einstein's photoelectric equation is $E_\gamma = \phi + K_{\max}$, where ϕ is the metal's work function and $K_{\max} = eV_s$ is the maximum photoelectron kinetic energy at stopping potential V_s .

Quick Tip to Solve: First add the work function and stopping energy in eV; then use final quantum number 2 for the first excited state.

- The hydrogen energy levels are $E_n = -13.6/n^2$ eV. The first excited state has quantum number 2, so a photon emitted in a transition from $n > 2$ to this state has energy $13.6(1/4 - 1/n^2)$ eV.

Step 1: Obtain the incident photon energy The stopping potential is $V_s = 0.55$ V, hence the fastest emitted electrons have kinetic energy $eV_s = 0.55$ eV. Adding the energy needed to overcome the work function gives

$$E_\gamma = \phi + eV_s = 2 + 0.55 = 2.55 \text{ eV.}$$

The work function must be added because only the remaining photon energy appears as electron kinetic energy.

Step 2: Apply the hydrogen transition formula Equate the photon energy to the decrease in atomic energy for the transition to $n_f = 2$:

$$\begin{aligned} 13.6 \left(\frac{1}{4} - \frac{1}{n^2} \right) &= 2.55, \\ \frac{1}{4} - \frac{1}{n^2} &= \frac{2.55}{13.6} = \frac{3}{16}, \\ \frac{1}{n^2} &= \frac{1}{4} - \frac{3}{16} = \frac{1}{16}. \end{aligned}$$

Thus $n^2 = 16$. The principal quantum number is a positive integer. As a check, the energy released between the levels -0.85 eV and -3.40 eV is 2.55 eV, agreeing with the photoelectric data.

Step 3: Conclusion The initial state is $n = 4$; the photon is emitted in the transition $4 \rightarrow 2$.

Quick Tip to Solve: First add the work function and stopping energy in eV; then use final quantum number 2 for the first excited state.

OR

9. A hydrogen atom in the ground state is excited by an electron beam of 12.5 eV energy. Find out the maximum number of lines emitted by the atom from its excited state. [2 marks]

Solution:

Concept:

- Hydrogen energy levels are $E_n = -13.6/n^2$ eV, where n is the principal quantum number.
- An electron can excite the atom only if its available kinetic energy is at least the energy gap. From levels up to n , the maximum number of distinct spectral transitions is $N = n(n-1)/2$.

Step 1: Identify the highest accessible level The excitation energy from the ground state is

$$\Delta E_{1 \rightarrow n} = 13.6 \left(1 - \frac{1}{n^2} \right) \text{ eV.}$$

For $n = 3$, this is $13.6(1 - 1/9) = 12.09$ eV, which is less than 12.5 eV. For $n = 4$, it is $13.6(1 - 1/16) = 12.75$ eV, which exceeds the incident electron energy. Thus the largest accessible quantum number is 3. The electron may retain the excess kinetic energy; excitation does not require its entire energy to

Quick Tip to Solve: Compare the excitation energy with two neighbouring level gaps before using $n(n-1)/2$; never round the inferred quantum number upward.

be absorbed.

Step 2: Count all possible spectral transitions Among levels 3, 2, 1, the possible downward transitions are $3 \rightarrow 2$, $3 \rightarrow 1$ and $2 \rightarrow 1$. They have different energy gaps and hence different wavelengths. The maximum line count for the excited atoms is therefore

$$N = \frac{3(3-1)}{2} = 3.$$

This counts the possible lines across de-excitation events, including cascades through the intermediate level.

Step 3: Conclusion The maximum number of emitted spectral lines is $\boxed{3}$, corresponding to excitation up to $n = 3$.

Quick Tip to Solve: Compare the excitation energy with two neighbouring level gaps before using $n(n-1)/2$; never round the inferred quantum number upward.

10. Draw the ray diagram of an astronomical telescope showing image formation in the normal adjustment position. Write the expression for its magnifying power. [2 marks]

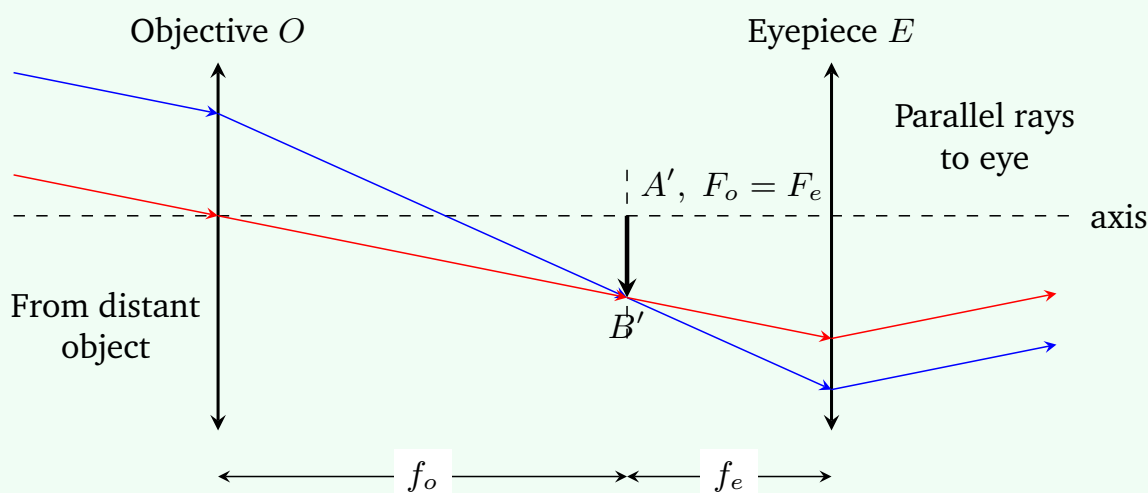
Solution:

Concept:

- In normal adjustment, the objective forms a real inverted image of a distant object in its focal plane. This plane coincides with the first focal plane of the eyepiece, so the final image is at infinity.
- Angular magnifying power is $M = \beta/\alpha$, where α is the angle subtended by the distant object at the objective and β is the angle subtended by the final image at the eye.

Step 1: Draw the image formation in normal adjustment Parallel rays from a distant off-axis point converge at B' after refraction through the objective O . The real inverted intermediate image $A'B'$ lies at the common focus $F_o = F_e$. Rays diverging from B' emerge parallel from the eyepiece E , giving a final virtual image at infinity for a relaxed eye. The lens separation is $f_o + f_e$.

Quick Tip to Solve: Put the intermediate image at the shared focal plane and show parallel emerging rays; use objective focal length divided by eyepiece focal length.



Astronomical telescope: common focal plane and final image at infinity (schematic).

Step 2: Obtain the angular magnification If the intermediate image has height h , then for small angles $|\alpha| \simeq h/f_o$ and $|\beta| \simeq h/f_e$. Taking their ratio gives $|M| = f_o/f_e$. The image is inverted relative to the distant object, so signed angular magnification is negative. Here f_o and f_e are the positive focal lengths of the objective and eyepiece respectively.

Step 3: Conclusion For an astronomical telescope in normal adjustment, $M = -f_o/f_e$; its magnifying power in magnitude is $|M| = f_o/f_e$.

Quick Tip to Solve: Put the intermediate image at the shared focal plane and show parallel emerging rays; use objective focal length divided by eyepiece focal length.

OR

10. Draw a labelled ray diagram to show image formation by a compound microscope and write the expression for its resolving power. [2 marks]

Solution:

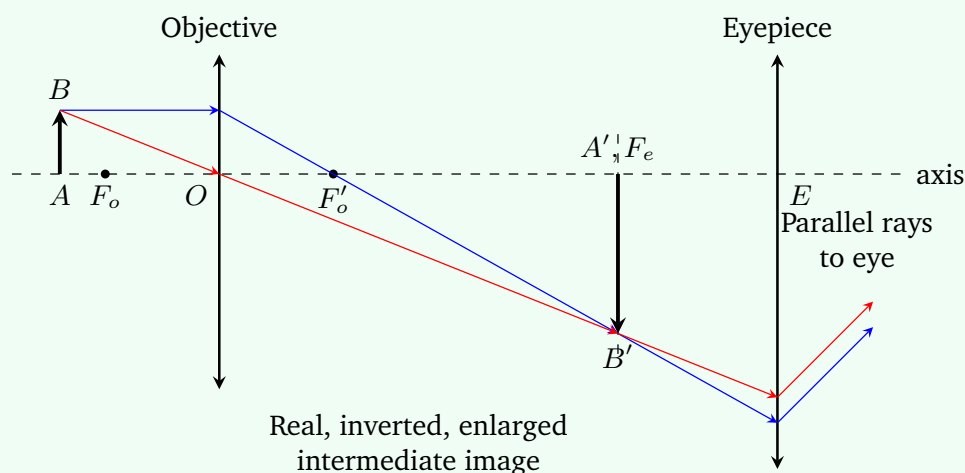
Concept:

- The objective forms a real, inverted, enlarged intermediate image of a small object placed just beyond its first focus. The eyepiece acts as a magnifier of this intermediate image.
- Resolving power is the reciprocal of the minimum resolvable separation $d_{\min}$. For a circular objective aperture, the Rayleigh criterion gives $d_{\min} = 1.22\lambda/(2\mu \sin \theta)$, where λ is the wavelength in vacuum, μ is the medium's refractive index, and θ is the half-angle of the accepted light cone.

Step 1: Construct the labelled ray diagram Place object AB just beyond the objective focus F_o . A ray from B parallel to the axis is refracted through F'_o ; a second ray passes undeviated through the objective's optical centre O . Their intersection gives the real inverted enlarged image $A'B'$. In the normal-adjustment diagram shown, $A'B'$ lies in the first focal plane F_e of the eyepiece E . Rays from B' emerge parallel, so the final magnified virtual image is at infinity. This is one standard operating

Quick Tip to Solve: Draw the objective's real inverted image first, then use it as the eyepiece's object; resolving power depends on numerical aperture and wavelength.

adjustment of a compound microscope.



Compound microscope in normal adjustment: enlarged intermediate image at the eyepiece's focal plane.

Step 2: Write and interpret the resolving power The numerical aperture is $NA = \mu \sin \theta$. Using the diffraction-limited separation above,

$$RP = \frac{1}{d_{\min}} = \frac{2\mu \sin \theta}{1.22\lambda} = \frac{NA}{0.61\lambda}.$$

Thus shorter wavelength or a larger numerical aperture makes closer object points distinguishable. This quantity measures the ability to distinguish detail, rather than the angular magnification produced by the eyepiece.

Step 3: Conclusion The labelled diagram shows a compound microscope with the final image at infinity. Its resolving power is $RP = 2\mu \sin \theta / (1.22\lambda)$.

Quick Tip to Solve: Draw the objective's real inverted image first, then use it as the eyepiece's object; resolving power depends on numerical aperture and wavelength.

11. A beam of light converges at a point P . Draw ray diagrams to show where the beam will converge if (i) a convex lens, and (ii) a concave lens is kept in the path of the beam. [2 marks]

Solution:

Concept:

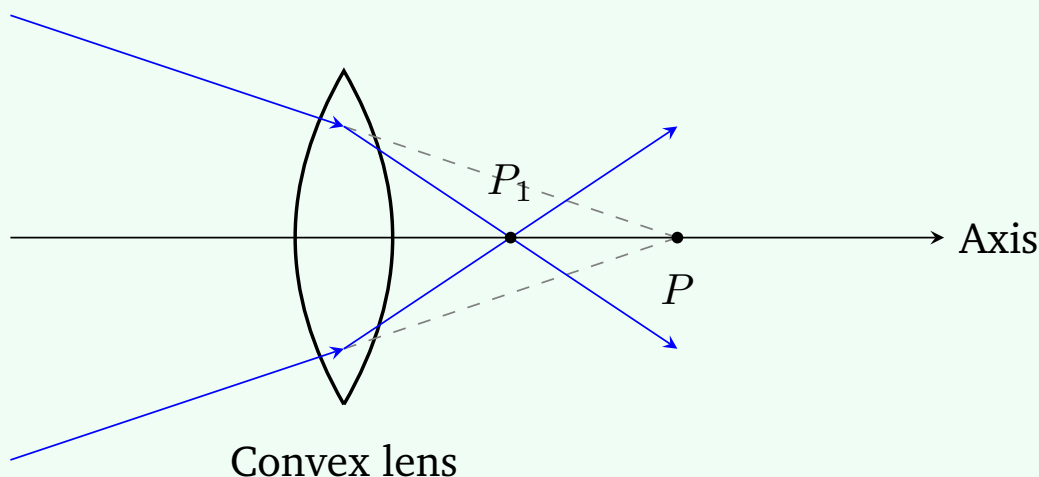
- A converging beam has a virtual object at its original convergence point P . If the lens is at a distance d before P , the Cartesian object distance is $u = +d$.
- The thin-lens formula is $1/v - 1/u = 1/f$, where v is the image distance and f is positive for a convex lens and negative for a concave lens.

Step 1: (i) Insert a convex lens The convex lens increases the convergence of the incoming rays. From $1/v = 1/d + 1/f$, we obtain

$$v = \frac{df}{d+f}.$$

Quick Tip to Solve: Treat the original meeting point as a virtual object: use positive u , then check the sign and size of v .

Since $d, f > 0$, $0 < v < d$. Thus the refracted rays meet at P_1 , between the lens and the original point P . The dashed lines in the diagram show where the rays would have travelled without the lens.

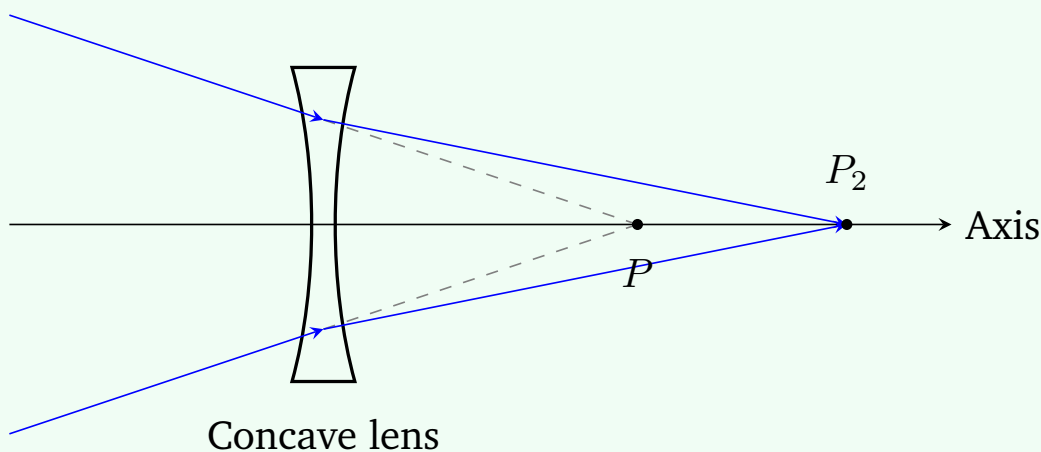


Convex lens: solid refracted rays meet before P ; dashed rays show the original beam.

Step 2: (ii) Insert a concave lens Let the magnitude of the concave focal length be F , so that $f = -F$. This lens reduces the convergence:

$$\frac{1}{v} = \frac{1}{d} - \frac{1}{F}, \quad v = \frac{dF}{F - d}.$$

For $d < F$, the beam still converges but $v > d$, so the meeting point P_2 lies beyond P , as drawn. The lens strength matters: if $d = F$, emergent rays are parallel and meet only at infinity; if $d > F$, they diverge and their backward extensions meet on the incident side. No finite real convergence occurs in those last two cases.



Concave lens with $d < F$: solid rays meet beyond P .

Step 3: Conclusion The convex lens brings the convergence point nearer, between the lens and P . A concave lens moves it beyond P when the incident convergence exceeds its diverging power ($d < F$); otherwise the beam becomes parallel or divergent.

Quick Tip to Solve: Treat the original meeting point as a virtual object: use positive u , then check the sign and size of v .

12. Write the relation between the height of a TV antenna and the maximum range up to which signals transmitted by the antenna can be received. How is this expression modified in the case of line of sight communication by space waves? In which range of frequencies, is this mode of communication used? [2 marks]

Solution:

Concept:

- Space waves travel approximately along straight lines, so the curvature of the Earth limits direct reception to the radio horizon.
- Let R be the radius of the Earth, h_T the transmitting antenna height and h_R the receiving antenna height, measured from the ground. The approximation assumes $h_T, h_R \ll R$.

Step 1: Range of a transmitting TV antenna At the limiting reception point, the ray from the antenna is tangent to the Earth. The Earth radius to the tangent point is perpendicular to this ray. Hence, by the right-triangle relation,

$$d_T^2 = (R + h_T)^2 - R^2 = 2Rh_T + h_T^2.$$

Neglecting h_T^2 compared with $2Rh_T$, the maximum horizon distance is $d_T \simeq \sqrt{2Rh_T}$. Use the same length unit for R and h_T . The surface distance has the same leading approximation.

Step 2: Include the receiving antenna and state the frequency range A receiving antenna raised to height h_R can receive beyond the transmitter's own ground horizon because it has an additional horizon distance $d_R \simeq \sqrt{2Rh_R}$. The maximum line-of-sight separation is therefore

$$d_{\max} = d_T + d_R = \sqrt{2Rh_T} + \sqrt{2Rh_R}.$$

Space-wave communication is used at frequencies above about 30 MHz, including VHF, UHF and microwave bands. At these frequencies sky-wave reflection by the ionosphere is generally ineffective, and direct line-of-sight links are used.

Step 3: Conclusion TV antenna range: $d_T \simeq \sqrt{2Rh_T}$. With both antenna heights included: $d_{\max} \simeq \sqrt{2Rh_T} + \sqrt{2Rh_R}$. The space-wave mode is used above approximately 30 MHz.

Quick Tip to Solve: Add the two horizon distances, not the two antenna heights inside one square root.

Section C (36 marks)

13. Prove that the magnetic moment of the electron revolving around a nucleus in an orbit of radius r with orbital speed v is equal to $evr/2$. Hence using Bohr's postulate of quantization of angular momentum, deduce the expression for the magnetic moment of hydrogen atom in the ground state. [3 marks]

Solution:

Concept:

- A charge in a circular orbit produces a current loop. Its magnetic moment magnitude is $\mu = IA$, where I is the current magnitude and A the enclosed area.

Quick Tip to Solve: First obtain current as charge divided by orbital period; then multiply by the circular area before applying quantization.

- For an electron, e denotes the positive magnitude of its charge and m_e its mass. Its orbital angular momentum magnitude is $L = m_e v r$.
- Bohr quantization gives $L = n\hbar = nh/(2\pi)$; the ground state corresponds to $n = 1$.

Step 1: Find the equivalent orbital current An electron travelling with orbital speed v in a circular orbit of radius r completes one revolution in the time

$$T = \frac{2\pi r}{v}.$$

The magnitude of charge passing a fixed point during one revolution is e . Therefore the equivalent conventional current has magnitude

$$I = \frac{e}{T} = \frac{ev}{2\pi r}.$$

Because the electron is negatively charged, the conventional current circulates opposite to its direction of motion.

Step 2: Calculate the magnetic moment of the orbit The circular orbit encloses area $A = \pi r^2$. Substituting the current into the current-loop expression gives

$$\mu = IA = \frac{ev}{2\pi r}(\pi r^2) = \frac{evr}{2}.$$

This establishes the required magnitude. The magnetic moment points normal to the orbital plane according to the right-hand rule for conventional current. Consequently it is antiparallel to the electron's orbital angular momentum; the vector relation is $\boldsymbol{\mu} = -(e/2m_e)\mathbf{L}$.

Step 3: Use Bohr quantization for the ground state Writing the magnitude as $\mu = (e/2m_e)(m_e v r)$ and inserting $m_e v r = n\hbar/(2\pi)$,

$$\mu_n = \frac{neh}{4\pi m_e} = \frac{ne\hbar}{2m_e}.$$

In the ground state $n = 1$, this becomes $\mu_1 = e\hbar/(2m_e)$. With $e = 1.602 \times 10^{-19}$ C, $\hbar = 1.055 \times 10^{-34}$ J s and $m_e = 9.109 \times 10^{-31}$ kg, its value is approximately 9.27×10^{-24} A m². This is the orbital magnetic moment in the Bohr model; electron spin is outside the requested derivation.

Step 4: Conclusion The orbital magnetic moment magnitude is $\mu = evr/2$. For ground-state hydrogen in the Bohr model, $\mu_1 = eh/(4\pi m_e) = e\hbar/(2m_e) = \mu_B$, the Bohr magneton, approximately 9.27×10^{-24} A m².

Quick Tip to Solve: First obtain current as charge divided by orbital period; then multiply by the circular area before applying quantization.

14. (a) How is the stability of hydrogen atom in Bohr model explained by de-Broglie's hypothesis?
(b) A hydrogen atom initially in the ground state absorbs a photon which excites it to $n = 4$ level. When it gets de-excited, find the maximum number of lines which are emitted by the atom. Identify the series to which these lines belong. Which of them has the shortest wavelength? [3 marks]

Solution:

Concept:

- The electron has de Broglie wavelength $\lambda = h/p = h/(m_e v)$. A stationary orbit is associated with a standing matter wave around the nucleus.
- Hydrogen energy levels obey $E_n = -13.6/n^2$ eV, and an emitted photon satisfies $hc/\lambda = E_i - E_f$, with $n_i > n_f$.
- Series are classified by the final level: Lyman has $n_f = 1$, Balmer has $n_f = 2$, and Paschen has $n_f = 3$.

Step 1: (a) Standing matter waves explain allowed stable orbits For the electron wave to join smoothly after one complete circuit, the circumference of the orbit must contain an integral number of wavelengths:

$$2\pi r = n\lambda = n \frac{h}{m_e v}.$$

Rearranging gives $m_e v r = nh/(2\pi)$, exactly Bohr's angular-momentum condition. The electron in a stationary state has a definite energy and does not continuously radiate; radiation occurs during transitions between such states.

Step 2: (b) Count and classify the possible emission lines Following excitation to $n = 4$, transitions can take place among levels 4, 3, 2, 1. The maximum number of distinct possible spectral lines, considering all possible decay paths, is

$$N = \frac{n(n-1)}{2} = \frac{4(4-1)}{2} = 6.$$

They are $4 \rightarrow 1, 3 \rightarrow 1, 2 \rightarrow 1$ in the Lyman series; $4 \rightarrow 2, 3 \rightarrow 2$ in the Balmer series; and $4 \rightarrow 3$ in the Paschen series. This usual spectral-line count refers to the possible spectrum over repeated excitations or a collection of atoms. A single atom during one individual decay cascade emits at most three photons, along $4 \rightarrow 3 \rightarrow 2 \rightarrow 1$, and does not emit all six lines in one cascade.

Step 3: Identify the shortest wavelength The shortest wavelength corresponds to the largest photon energy, hence to the largest energy-level difference. Among the available transitions this is $4 \rightarrow 1$, which belongs to the Lyman series. Its wavelength follows from

$$\frac{1}{\lambda} = R_H \left(1 - \frac{1}{4^2} \right) = \frac{15R_H}{16}, \quad \lambda = \frac{16}{15R_H} \simeq 97.3 \text{ nm}.$$

All other transitions have smaller energy differences and therefore longer wavelengths.

Step 4: Conclusion Allowed standing waves satisfy $2\pi r = n\lambda$, yielding Bohr quantization. The possible emission spectrum contains six lines: three Lyman, two Balmer and one Paschen. The $4 \rightarrow 1$ Lyman line has the shortest wavelength, about 97.3 nm.

Quick Tip to Solve: List pairs of initial and final levels; the final level names the series, and the largest energy drop gives the shortest wavelength.

15. A capacitor (C) and resistor (R) are connected in series with an ac source of voltage of frequency 50 Hz. The potential difference across C and R are respectively 120 V, 90 V, and the current in the circuit is 3 A. Calculate (i) the impedance of the circuit (ii) the value of the inductance, which when connected in series with C and R will make the power factor of the circuit unity. [3 marks]

Solution:

Concept:

- In a series RC circuit the same rms current I flows through both elements, so $V_R = IR$ and $V_C = IX_C$, where $X_C = 1/(\omega C)$.
- The resistor voltage is in phase with current, while the capacitor voltage lags current by 90° . Thus $Z = \sqrt{R^2 + X_C^2}$, and the source voltage is the phasor sum of the two voltages.
- For a series RLC circuit, unity power factor requires zero net reactance: $X_L = X_C$, with $X_L = \omega L$ and $\omega = 2\pi f$.

Step 1: (i) Determine the resistance and capacitive reactance Using the given current and voltage drops,

$$R = \frac{90}{3} = 30 \, \Omega, \quad X_C = \frac{120}{3} = 40 \, \Omega.$$

The two drops cannot be added arithmetically because their phases differ by a quarter cycle. Hence the impedance of the original circuit is

$$Z = \sqrt{30^2 + 40^2} = \sqrt{2500} = 50 \, \Omega.$$

As a check, the source rms voltage is $\sqrt{90^2 + 120^2} = 150 \, \text{V}$, giving $V/I = 150/3 = 50 \, \Omega$.

Step 2: (ii) Choose the inductance for unity power factor Adding an inductor in series makes the net reactance $X_L - X_C$. The power factor is $R/\sqrt{R^2 + (X_L - X_C)^2}$, so it becomes unity only when $X_L = X_C = 40 \, \Omega$. At the unchanged source frequency,

$$\omega = 2\pi(50) = 100\pi \, \text{rad s}^{-1}, \quad L = \frac{X_C}{\omega} = \frac{40}{100\pi} = 0.1273 \, \text{H}.$$

At this condition the inductive and capacitive voltage phasors cancel, leaving the source voltage in phase with the current. The original current was used to determine the component values; it need not remain 3 A after inserting the inductor.

Step 3: Conclusion (i) The impedance is $50 \, \Omega$. (ii) An inductance of $0.127 \, \text{H}$, approximately 127 mH, gives unity power factor at 50 Hz.

Quick Tip to Solve: Divide each voltage drop by the common current first; then use the 3-4-5 impedance triangle and set the added inductive reactance equal to the capacitive reactance.

15. The figure shows a series LCR circuit connected to a variable frequency 230 V source.

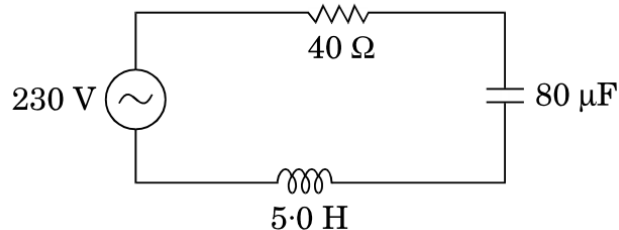
(a) Determine the source frequency which drives the circuit in resonance.

(b) Calculate the impedance of the circuit and amplitude of current at resonance.

(c) Show that potential drop across LC combination is zero at resonating frequency.

[3 marks]

230 V source.



Solution:

Concept:

- A series RLC circuit resonates when $X_L = X_C$, so $\omega_0 = 1/\sqrt{LC}$ and $f_0 = 1/(2\pi\sqrt{LC})$.
- Its impedance is $Z = \sqrt{R^2 + (X_L - X_C)^2}$. At resonance $Z = R$, and rms current is $I_{\text{rms}} = V_{\text{rms}}/R$.
- For a sinusoidal current the amplitude is $I_0 = \sqrt{2}I_{\text{rms}}$. Inductor and capacitor voltage phasors are opposite because they lead and lag the current respectively by 90° .

Step 1: (a) Calculate the resonant frequency From the printed figure, $L = 5.0 \text{ H}$, $C = 80 \mu\text{F} = 80 \times 10^{-6} \text{ F}$, and $R = 40 \Omega$. Thus

$$LC = 5.0(80 \times 10^{-6}) = 4.0 \times 10^{-4} \text{ s}^2,$$

$$\omega_0 = \frac{1}{0.020} = 50 \text{ rad s}^{-1}, \quad f_0 = \frac{50}{2\pi} = 7.96 \text{ Hz}.$$

This is the frequency at which the source drives the maximum steady-state current in this series circuit.

Step 2: (b) Find impedance and current amplitude At resonance the reactances cancel, so the source sees only the resistance:

$$Z = R = 40 \Omega.$$

Taking the specified 230 V AC source rating as its rms voltage,

$$I_{\text{rms}} = \frac{230}{40} = 5.75 \text{ A}, \quad I_0 = \sqrt{2}(5.75) = 8.13 \text{ A}.$$

The amplitude requested is the peak current, which is larger than the rms current by $\sqrt{2}$.

Step 3: (c) Show cancellation of the LC voltage At the calculated frequency,

$$X_L = \omega_0 L = 50(5.0) = 250 \Omega, \quad X_C = \frac{1}{50(80 \times 10^{-6})} = 250 \Omega.$$

The individual rms drops have equal magnitudes $I_{\text{rms}}X_L = I_{\text{rms}}X_C = 1437.5 \text{ V}$. They are opposite

Quick Tip to Solve: Keep rms and peak values separate: resonance gives $Z = R$, but multiply the rms current by the square root of two when the question asks for amplitude.

phasors, not zero individually. Their resultant across the series LC combination is therefore

$$\tilde{V}_{LC} = jI_{\text{rms}}(X_L - X_C) = 0.$$

The instantaneous voltages also cancel throughout each cycle in the ideal steady state.

Step 4: Conclusion (a) $f_0 = 7.96 \text{ Hz}$. (b) $Z = 40 \Omega$ and current amplitude $I_0 = 8.13 \text{ A}$. (c) The net potential drop across LC is 0 at resonance.

Quick Tip to Solve: Keep rms and peak values separate: resonance gives $Z = R$, but multiply the rms current by the square root of two when the question asks for amplitude.

16. What is the reason to operate photodiodes in reverse bias?

A p-n photodiode is fabricated from a semiconductor with a band gap of range of 2.5 to 2.8 eV. Calculate the range of wavelengths of the radiation which can be detected by the photodiode.

[3 marks]

Solution:

Concept:

- Incident photons can generate electron-hole pairs when their energy $E_\gamma = hc/\lambda$ is at least the semiconductor band gap E_g .
- Reverse bias establishes a strong electric field in the depletion layer, which separates photogenerated carriers and produces a reverse photocurrent.
- The long-wavelength threshold is $\lambda_c = hc/E_g$. Using $hc \simeq 1240 \text{ eV nm}$ makes the conversion from electron volts to nanometres direct.

Step 1: Explain why reverse bias is used The reverse current of an unilluminated p-n junction is very small. Light creates extra minority carriers, and the reverse electric field sweeps electrons and holes towards opposite sides of the junction before they readily recombine. The resulting change in reverse current is therefore easy to distinguish from the small dark current and varies with incident light intensity. In forward bias a large majority-carrier current would mask this comparatively small light-induced change. Reverse bias also widens the depletion layer and helps rapid collection of photogenerated carriers.

Step 2: Calculate the wavelength endpoints At the absorption threshold, the photon energy equals the band gap. Hence

$$\lambda_c = \frac{hc}{E_g} = \frac{1240}{E_g(\text{eV})} \text{ nm.}$$

Since wavelength is inversely proportional to energy, the larger band gap gives the shorter threshold wavelength:

$$\lambda_{c,\text{min}} = \frac{1240}{2.8} = 442.9 \text{ nm}, \quad \lambda_{c,\text{max}} = \frac{1240}{2.5} = 496.0 \text{ nm.}$$

Thus the wavelength interval corresponding to the stated band-gap range is approximately 443 to 496 nm, in the visible region. Physically these are long-wavelength cutoffs: for a particular band gap photons with $\lambda \leq \lambda_c$ have sufficient energy to create electron-hole pairs. The band gap alone does not

Quick Tip to Solve: Use wavelength in nm = 1240 divided by energy in eV; reverse the endpoint order because the larger energy corresponds to the shorter wavelength.

impose a lower wavelength bound on all detectable radiation.

Step 3: Conclusion Reverse bias gives a readily measurable photocurrent above the small dark current and efficiently separates the light-generated carriers. The wavelengths corresponding to 2.5–2.8 eV are $443\text{--}496\text{ nm}$, or $4.43 \times 10^{-7}\text{--}4.96 \times 10^{-7}\text{ m}$; these values specify the range of long-wavelength thresholds.

Quick Tip to Solve: Use wavelength in nm = 1240 divided by energy in eV; reverse the endpoint order because the larger energy corresponds to the shorter wavelength.

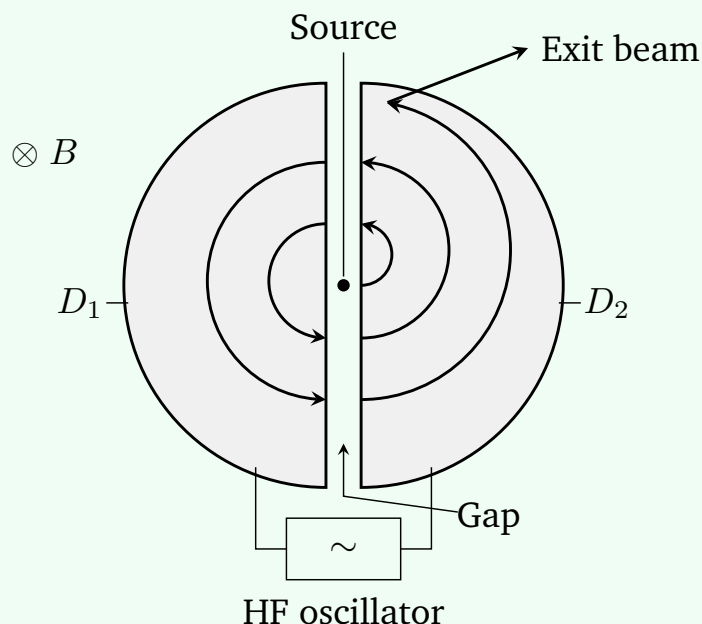
17. Draw a labelled diagram of cyclotron. Explain its working principle. Show that cyclotron frequency is independent of the speed and radius of the orbit. [3 marks]

Solution:

Concept:

- A cyclotron accelerates a charged particle by an alternating electric field across the gap between two hollow D-shaped electrodes (dees).
- A uniform magnetic field B , perpendicular to the dees, provides the centripetal force: $|q|vB = mv^2/r$. Here m , q , v and r are the particle's mass, charge, speed and orbit radius.
- The derivation assumes non-relativistic motion, so the particle's mass remains constant.

Step 1: Arrangement and working principle The two dees D_1 and D_2 are placed in an evacuated chamber between the poles of a magnet. An oscillator applies a high-frequency alternating potential difference between them. A charged particle is released near the centre. Inside a dee, the electric field is zero and the magnetic force bends its path into a semicircle without changing its speed. Across the gap, the electric field does work and increases its kinetic energy.



Cyclotron viewed perpendicular to the dees; the magnetic field points into the page.

Step 2: Repeated acceleration and extraction The oscillator reverses polarity during each semicircular

Quick Tip to Solve: Substitute $r/v = m/(|q|B)$ into $T = 2\pi r/v$; both speed and radius cancel at once.

journey. Thus the electric force accelerates the particle on every crossing of the gap. As its speed increases, $r = mv/(|q|B)$ increases, and successive semicircular paths form an outward spiral. Near the outer edge, a deflecting arrangement directs the energetic particle through an exit towards the target. The oscillator must remain synchronized with the orbital motion.

Step 3: Derive the cyclotron frequency Equating the magnetic force with the centripetal force gives

$$|q|vB = \frac{mv^2}{r}, \quad \frac{r}{v} = \frac{m}{|q|B}.$$

Hence the time for one full revolution is

$$T = \frac{2\pi r}{v} = \frac{2\pi m}{|q|B}, \quad f = \frac{1}{T} = \frac{|q|B}{2\pi m}.$$

The time in either dee is $T/2 = \pi m/(|q|B)$, which is constant even as the speed and radius increase. This allows a fixed-frequency oscillator to reverse the gap field at every crossing.

Step 4: Conclusion The cyclotron frequency is $f = |q|B/(2\pi m)$. It depends on the charge-to-mass ratio and magnetic field, and is independent of both particle speed and orbit radius in the non-relativistic regime.

Quick Tip to Solve: Substitute $r/v = m/(|q|B)$ into $T = 2\pi r/v$; both speed and radius cancel at once.

OR

17. (a) Derive, with the help of a diagram, the expression for the magnetic field inside a very long solenoid having n turns per unit length carrying a current I .

(b) How is a toroid different from a solenoid?

[3 marks]

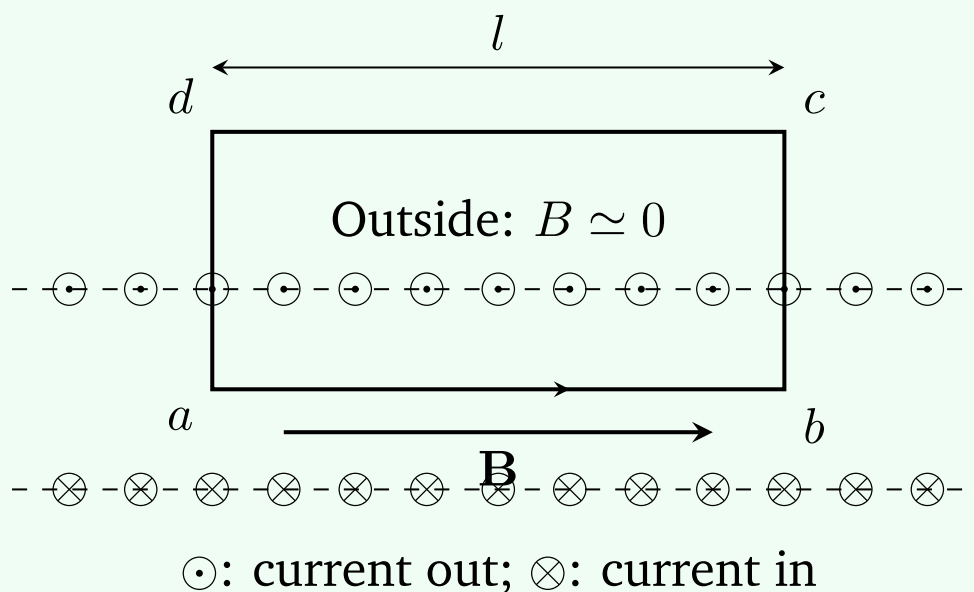
Solution:

Concept:

- Ampere's circuital law states $\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I_{\text{enc}}$, where I_{enc} is the net current enclosed by the loop.
- Deep inside a very long, closely wound solenoid, the magnetic field is nearly uniform and parallel to its axis. The external field is negligible in the ideal long-solenoid approximation.

Step 1: (a) Choose an Amperian rectangle Consider an air-core solenoid carrying current I , with n turns per unit length. Choose the rectangular path $abcd$, with ab of length l inside and parallel to the solenoid axis, and cd outside. The sides bc and da are perpendicular to the axial field. Each turn pierced by the rectangle carries current I in the same sense. Since the number of enclosed turns is nl , the enclosed current is $I_{\text{enc}} = nlI$.

Quick Tip to Solve: For the Amperian rectangle, only the internal axial side contributes; count nl enclosed turns before applying Ampere's law.



Long solenoid in longitudinal section, with a rectangular Amperian loop.

Step 2: Evaluate the circulation and obtain the field The internal side contributes Bl . The two transverse sides contribute zero because $\mathbf{B}$ is perpendicular to $d\mathbf{l}$, while the external side contributes negligibly for a very long solenoid. Therefore,

$$\oint \mathbf{B} \cdot d\mathbf{l} = Bl = \mu_0(nlI), \quad B = \mu_0 nI.$$

The direction is along the solenoid axis, fixed by the right-hand grip rule: curl the fingers in the direction of current in the turns, and the thumb gives the internal field direction. The result applies away from the ends; end effects spoil the uniform-field approximation near an open end.

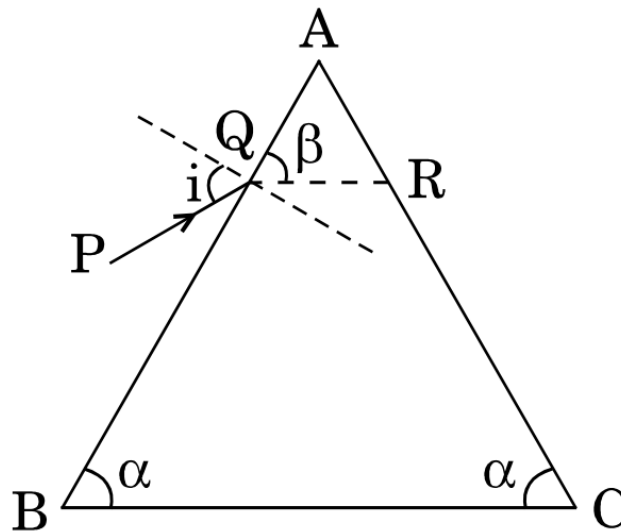
Step 3: (b) Compare a toroid with a solenoid A solenoid is a straight cylindrical coil with two open ends. A toroid is a solenoid bent into a closed circular ring, so it has no open ends. In a long straight solenoid the internal field is approximately uniform and axial, $B = \mu_0 nI$. In an ideal toroid the field follows concentric circles within the winding and has magnitude $B(r) = \mu_0 NI / (2\pi r)$, where N is the total number of turns and r the distance from the ring's centre. Thus it varies with r . A finite solenoid has leakage field near its ends, whereas the ideal toroid confines the field to its winding region, with zero field in the central hole and outside.

Step 4: Conclusion (a) The field inside a very long air-core solenoid is $B = \mu_0 nI$, along its axis. (b) A toroid is a closed ring-shaped coil with circular internal field lines, whereas a solenoid is an open-ended cylindrical coil with approximately straight axial field lines.

Quick Tip to Solve: For the Amperian rectangle, only the internal axial side contributes; count nl enclosed turns before applying Ampere's law.

18. A ray of light incident on the face AB of an isosceles triangular prism makes an angle of incidence i and deviates by angle β as shown in the figure. Show that in the position of minimum deviation $\angle\beta = \angle\alpha$. Also find out the condition when the refracted ray QR suffers total internal reflection.

[3 marks]



Solution:

Concept:

- For a prism of apex angle A , the internal angles of refraction obey $r_1 + r_2 = A$, with both angles measured from the normals to the faces.
- At minimum deviation, the ray path is symmetric: $i = e$ and $r_1 = r_2 = A/2$. Here i and e are the external incidence and emergence angles.
- Total internal reflection requires travel from a denser to a rarer medium and an incidence angle greater than the critical angle C . For a prism in air with refractive index μ , $\sin C = 1/\mu$.

Step 1: Relate the printed angle to the refraction angle The angles at B and C are both α , so the apex angle is $A = 180^\circ - 2\alpha$. In the printed figure, β is the angle AQR between the face QA and the internal ray QR . It is an angle to the face, whereas r_1 is measured from the normal. Thus

$$\beta + r_1 = 90^\circ.$$

Using the angle shown in the figure is essential: β here is not the usual overall angle between the incident and emergent rays of a prism.

Step 2: Apply the minimum-deviation condition At minimum deviation, the internal angles are equal. Therefore,

$$r_1 = r_2 = \frac{A}{2} = 90^\circ - \alpha.$$

Substituting in the complement relation at Q ,

$$\beta = 90^\circ - r_1 = 90^\circ - (90^\circ - \alpha) = \alpha.$$

Quick Tip to Solve: First identify which side of the normal an angle is measured from: the printed β is to the face, so $r_1 = 90^\circ - \beta$.

The internal ray is then parallel to the base BC , consistent with the symmetric path through the isosceles prism.

Step 3: Find the condition for total internal reflection at R For a general incident ray, let r_2 denote its angle of incidence on face AC , measured from the outward normal there. The prism relation gives

$$r_2 = A - r_1 = (180^\circ - 2\alpha) - (90^\circ - \beta) = 90^\circ - 2\alpha + \beta.$$

At R , the ray travels from the prism to air. Total internal reflection occurs when

$$r_2 > C, \quad 90^\circ - 2\alpha + \beta > \sin^{-1}(1/\mu).$$

Equivalently, using Snell's law at the first face, $r_1 = \sin^{-1}(\sin i/\mu)$, the condition is $A - \sin^{-1}(\sin i/\mu) > C$. At equality the refracted ray grazes face AC ; equality is not total internal reflection. For the parallel-to-base internal ray, $r_2 = 90^\circ - \alpha$, so the corresponding inequality is $90^\circ - \alpha > C$. If this holds, that ray cannot emerge to form the ordinary minimum-deviation transmitted path.

Step 4: Conclusion In the minimum-deviation position, $\beta = \alpha$. For total internal reflection of QR at AC , its incidence angle there must exceed the prism-air critical angle: $90^\circ - 2\alpha + \beta > C$, with $C = \sin^{-1}(1/\mu)$.

Quick Tip to Solve: First identify which side of the normal an angle is measured from: the printed β is to the face, so $r_1 = 90^\circ - \beta$.

19. Two large charged plane sheets of charge densities σ and -2σ C/m² are arranged vertically with a separation of d between them. Deduce expressions for the electric field at points (i) to the left of the first sheet, (ii) to the right of the second sheet, and (iii) between the two sheets. [3 marks]

Solution:

Concept:

- The field of a very large uniformly charged plane sheet is $E = |\sigma_s|/(2\epsilon_0)$ on either side, where σ_s is its surface charge density and ϵ_0 is the permittivity of free space. Edge effects are neglected.
- The electric field points away from a positive sheet and towards a negative sheet. By superposition, the resultant field is the vector sum of the fields of the two sheets.

Step 1: Set the direction and obtain the individual fields Take the first sheet, of density $+\sigma$, at $x = 0$ and the second, of density -2σ , at $x = d$. Let $\hat{x}$ point from the first sheet towards the second, and take $\sigma > 0$. A Gaussian pillbox enclosing a single sheet gives $2EA = |\sigma_s|A/\epsilon_0$. Hence the first sheet produces a field of magnitude $\sigma/(2\epsilon_0)$, while the second produces σ/ϵ_0 .

Step 2: Add the fields to the left of the first sheet For $x < 0$, the positive sheet's field is towards the left, whereas the negative sheet's field is towards the right. Therefore,

$$\mathbf{E}_{\text{left}} = -\frac{\sigma}{2\epsilon_0}\hat{x} + \frac{\sigma}{\epsilon_0}\hat{x} = \frac{\sigma}{2\epsilon_0}\hat{x}.$$

Step 3: Add the fields to the right of the second sheet For $x > d$, the positive sheet's field is towards

Quick Tip to Solve: First draw the direction of each sheet's field in each region; outside the sheets subtract their magnitudes, and between them add.

the right and the negative sheet's field towards the left. Thus,

$$\mathbf{E}_{\text{right}} = \frac{\sigma}{2\epsilon_0} \hat{\mathbf{x}} - \frac{\sigma}{\epsilon_0} \hat{\mathbf{x}} = -\frac{\sigma}{2\epsilon_0} \hat{\mathbf{x}}.$$

Step 4: Add the fields between the sheets For $0 < x < d$, both fields point towards the second sheet: away from the positive sheet and towards the negative sheet. Consequently,

$$\mathbf{E}_{\text{between}} = \frac{\sigma}{2\epsilon_0} \hat{\mathbf{x}} + \frac{\sigma}{\epsilon_0} \hat{\mathbf{x}} = \frac{3\sigma}{2\epsilon_0} \hat{\mathbf{x}}.$$

These fields are uniform within each region and independent of d in the large-sheet approximation.

Step 5: Conclusion (i) Left of the first sheet: $E = \sigma/(2\epsilon_0)$, towards the right. (ii) Right of the second sheet: $E = \sigma/(2\epsilon_0)$, towards the left. (iii) Between the sheets: $E = 3\sigma/(2\epsilon_0)$, from the $+\sigma$ sheet to the -2σ sheet.

Quick Tip to Solve: First draw the direction of each sheet's field in each region; outside the sheets subtract their magnitudes, and between them add.

OR

19. A spherical conducting shell of inner radius r_1 and outer radius r_2 has a charge Q .

(a) A charge q is placed at the centre of the shell. Find out the surface charge density on the inner and outer surfaces of the shell.

(b) Is the electric field inside a cavity (with no charge) zero, independent of the fact whether the shell is spherical or not? Explain. [3 marks]

Solution:

Concept:

- In electrostatic equilibrium, the electric field inside the conducting material is zero. Gauss's law states $\oint \mathbf{E} \cdot d\mathbf{A} = Q_{\text{enc}}/\epsilon_0$.
- Charge is conserved: the total charge on the conducting shell remains Q . For a spherical surface of radius r carrying uniform charge Q_s , the surface charge density is $\sigma_s = Q_s/(4\pi r^2)$.
- A connected conductor is equipotential. An empty cavity bounded by a surface at constant potential has constant potential throughout, and hence zero electric field, regardless of its shape.

Step 1: (a) Find the charge on the inner surface Place a spherical Gaussian surface within the metal, with radius r such that $r_1 < r < r_2$. Since the field in the metal vanishes, its electric flux is zero. If Q_{in} is the charge induced on the inner surface, Gauss's law gives

$$0 = \frac{q + Q_{\text{in}}}{\epsilon_0}, \quad Q_{\text{in}} = -q.$$

The point charge is at the centre of the spherical cavity, so all points of its inner surface are equivalent by symmetry. The induced inner charge is therefore uniformly distributed over area $4\pi r_1^2$.

Quick Tip to Solve: Use a Gaussian surface in the metal to find the inner charge first; then use the shell's own conserved charge to obtain the outer charge.

Step 2: (a) Find the outer-surface charge and the two densities The shell itself has total charge Q ; the inserted charge q is a separate charge. Thus the shell's inner and outer surface charges obey

$$Q_{\text{in}} + Q_{\text{out}} = Q, \quad Q_{\text{out}} = Q + q.$$

For the spherical shell in the stated arrangement, with no external charges specified, the outer charge is also uniform. Dividing each surface charge by its respective area gives

$$\sigma_{\text{in}} = -\frac{q}{4\pi r_1^2}, \quad \sigma_{\text{out}} = \frac{Q + q}{4\pi r_2^2}.$$

The sign of each expression describes the sign of the corresponding surface charge.

Step 3: (b) Explain the field in an empty cavity of any shape Yes, the electric field is zero inside a charge-free cavity in a conductor in electrostatic equilibrium. The entire conducting cavity wall is at one potential V_0 . Within the empty cavity there is no charge, so the potential satisfies Laplace's equation. The unique solution with the boundary held at V_0 is $V = V_0$ everywhere in the cavity. Therefore $\mathbf{E} = -\nabla V = 0$. This reasoning uses the equipotential boundary, not spherical symmetry, so it applies to a non-spherical cavity as well. Zero net enclosed charge alone would not suffice to prove zero field; the conducting boundary is essential. This empty-cavity result does not apply to part (a), where q is present inside the cavity.

Step 4: Conclusion (a) The inner and outer surface charge densities are $\boxed{-q/(4\pi r_1^2)}$ and $\boxed{(Q + q)/(4\pi r_2^2)}$, respectively. (b) An empty cavity has $\boxed{\mathbf{E} = 0}$ in electrostatic equilibrium, independently of whether the shell or cavity is spherical.

Quick Tip to Solve: Use a Gaussian surface in the metal to find the inner charge first; then use the shell's own conserved charge to obtain the outer charge.

20. A signal of low frequency f_m is to be transmitted using a carrier wave of frequency f_c . Derive the expression for the amplitude modulated wave and deduce expressions for the lower and upper sidebands produced. Hence, obtain the expression for modulation index. [3 marks]

Solution:

Concept:

- Amplitude modulation varies the amplitude of a high-frequency carrier in accordance with the instantaneous value of a low-frequency message, while keeping the carrier frequency and phase constant.
- For a sinusoidal message, the modulation index μ is the ratio of the amplitude variation to the unmodulated carrier amplitude. Normal undistorted AM uses $0 \leq \mu \leq 1$.
- The identity $\cos \alpha \cos \beta = \frac{1}{2}[\cos(\alpha + \beta) + \cos(\alpha - \beta)]$ resolves an AM wave into its carrier and sidebands.

Step 1: Write the message, carrier and modulated amplitude Let the message be $m(t) = A_m \cos \omega_m t$ and the unmodulated carrier be $c(t) = A_c \cos \omega_c t$, where $\omega_m = 2\pi f_m$, $\omega_c = 2\pi f_c$, and $f_c \gg f_m$. In the

Quick Tip to Solve: Expand the message-carrier product using the cosine identity; the sum and difference arguments immediately give the two sideband frequencies.

usual equal-amplitude-unit convention, the carrier amplitude becomes $A_c + A_m \cos \omega_m t$. Thus the amplitude-modulated wave is

$$s(t) = [A_c + A_m \cos \omega_m t] \cos \omega_c t = A_c [1 + \mu \cos \omega_m t] \cos \omega_c t,$$

where $\mu = A_m/A_c$. More generally, if a modulator converts message amplitude to carrier-amplitude change with proportionality constant k , replace A_m here by kA_m , so $\mu = kA_m/A_c$.

Step 2: Expand the wave and identify the sidebands Applying the product-to-sum identity yields

$$\begin{aligned} s(t) &= A_c \cos \omega_c t \\ &+ \frac{\mu A_c}{2} \cos[(\omega_c + \omega_m)t] \\ &+ \frac{\mu A_c}{2} \cos[(\omega_c - \omega_m)t]. \end{aligned}$$

The first term is the carrier at f_c . The upper sideband is $s_{\text{USB}}(t) = (\mu A_c/2) \cos[2\pi(f_c + f_m)t]$, with frequency $f_{\text{USB}} = f_c + f_m$. The lower sideband is $s_{\text{LSB}}(t) = (\mu A_c/2) \cos[2\pi(f_c - f_m)t]$, with frequency $f_{\text{LSB}} = f_c - f_m$. Both sideband amplitudes are $\mu A_c/2$, or $A_m/2$ in the stated convention. The frequencies bracket the carrier symmetrically.

Step 3: Obtain the modulation index from the envelope The envelope amplitude is $A(t) = A_c[1 + \mu \cos \omega_m t]$. For $\mu \leq 1$, its maximum and minimum values occur at message cosine values $+1$ and -1 , respectively:

$$A_{\text{max}} = A_c(1 + \mu), \quad A_{\text{min}} = A_c(1 - \mu).$$

Subtraction gives $A_{\text{max}} - A_{\text{min}} = 2\mu A_c$, while addition gives $A_{\text{max}} + A_{\text{min}} = 2A_c$. Dividing these expressions eliminates the unknown carrier amplitude and gives $\mu = (A_{\text{max}} - A_{\text{min}})/(A_{\text{max}} + A_{\text{min}})$. If $\mu > 1$, the signed amplitude crosses zero and envelope detection introduces distortion; the usual envelope-extrema formula is used for normal AM.

Step 4: Conclusion The AM wave is $s(t) = A_c[1 + \mu \cos(2\pi f_m t)] \cos(2\pi f_c t)$. Its lower and upper sideband frequencies are $f_c - f_m$ and $f_c + f_m$, each with amplitude $\mu A_c/2$. The modulation index is $\mu = A_m/A_c = (A_{\text{max}} - A_{\text{min}})/(A_{\text{max}} + A_{\text{min}})$ in the usual convention.

Quick Tip to Solve: Expand the message-carrier product using the cosine identity; the sum and difference arguments immediately give the two sideband frequencies.

21. Show, on a plot, variation of resistivity of (i) a conductor, and (ii) a typical semiconductor as a function of temperature.

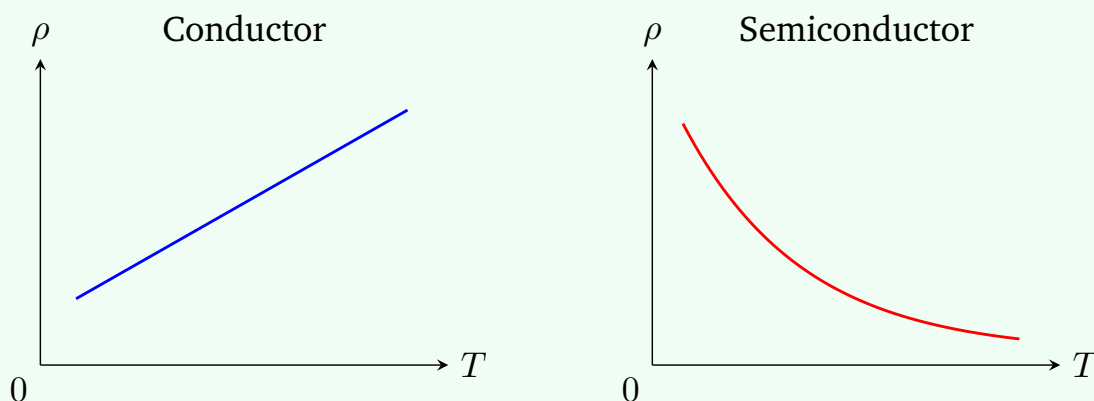
Using the expression for the resistivity in terms of number density and relaxation time between the collisions, explain how resistivity in the case of a conductor increases while it decreases in a semiconductor, with the rise of temperature. [3 marks]

Solution:

Concept:

- In the relaxation-time model, $\rho = m/(ne^2\tau)$, where m is the electron mass, n the number of mobile carriers per unit volume, e their charge magnitude, and τ the mean time between collisions.
- Temperature affects both carrier density and scattering. Resistivity increases if $n\tau$ decreases and decreases if $n\tau$ increases.

Step 1: Plot the two temperature dependences For a metallic conductor over an ordinary temperature range, resistivity rises approximately linearly: $\rho(T) = \rho(T_0)[1 + \alpha(T - T_0)]$, with $\alpha > 0$. For a typical semiconductor it falls strongly and nonlinearly as temperature rises. The plots below are qualitative; their vertical scales are independent, so the heights are not a comparison of absolute resistivities.



Qualitative resistivity curves with independent vertical scales.

Step 2: Explain the conductor curve A metal already has a large density of free conduction electrons. Raising the temperature changes n very little, but increases the thermal vibrations of the lattice ions. The electrons are scattered more frequently, reducing the relaxation time τ . With m , e , and approximately n fixed, $\rho \propto 1/\tau$; therefore the conductor's resistivity increases.

Step 3: Explain the semiconductor curve Heating a typical semiconductor excites additional electrons into the conduction band and creates holes. Its density of mobile charge carriers increases rapidly. Although stronger lattice vibrations also reduce carrier relaxation times, the increase in carrier density dominates this scattering effect. Thus the effective product $n\tau$ increases and resistivity decreases. More generally, $\sigma = e(n_e\mu_e + n_h\mu_h)$, where the subscripts denote electrons and holes; the growth of their populations outweighs the reduction of their mobilities.

Step 4: Conclusion A conductor has a rising ρ -versus- T curve because its relaxation time falls while carrier density is nearly fixed. A typical semiconductor has a falling curve because thermal generation of carriers dominates the increased scattering.

Quick Tip to Solve: In $\rho = m/(ne^2\tau)$, compare both n and τ ; keeping n fixed incorrectly predicts the semiconductor trend.

22. A $100\ \mu\text{F}$ parallel plate capacitor having plate separation of $4\ \text{mm}$ is charged by $200\ \text{V}$ dc. The source is now disconnected. When the distance between the plates is doubled and a dielectric slab of thickness $4\ \text{mm}$ and dielectric constant 5 is introduced between the plates, how will (i) its capacitance, (ii) the electric field between the plates, and (iii) energy density of the capacitor get affected? Justify your answer in each case. [3 marks]

Solution:

Concept:

- A disconnected capacitor retains its free charge Q . A dielectric layer of thickness t and relative permittivity K gives $C = \epsilon_0 A / (d - t + t/K)$, where d is the total plate separation.
- In each region, the energy density is $u = \frac{1}{2} \epsilon E^2$. At the air-dielectric interface, the normal electric displacement $D = Q/A$ is continuous in the absence of free interfacial charge.

Step 1: Find the conserved charge and new capacitance Initially, $C_0 = 100\ \mu\text{F}$, $d_0 = 4\ \text{mm}$, and $V_0 = 200\ \text{V}$. The retained charge is $Q = C_0 V_0 = 0.020\ \text{C}$. After the change, $d = 8\ \text{mm}$, with $4\ \text{mm}$ of air and $t = 4\ \text{mm}$ of dielectric. Treating the two layers as series capacitors gives

$$C' = \frac{\epsilon_0 A}{4\ \text{mm} + 4\ \text{mm}/5} = C_0 \frac{4}{4.8} = \frac{5}{6} C_0 = 83.3\ \mu\text{F}.$$

Thus capacitance decreases to five-sixths of its original value.

Step 2: Determine the field in each layer The initial field is $E_0 = V_0/d_0 = 5.0 \times 10^4\ \text{V m}^{-1}$. Since Q and A remain fixed, $D = Q/A = \epsilon_0 E_0$ remains fixed. Therefore

$$E_{\text{air}} = \frac{D}{\epsilon_0} = E_0, \quad E_{\text{slab}} = \frac{D}{5\epsilon_0} = \frac{E_0}{5} = 1.0 \times 10^4\ \text{V m}^{-1}.$$

The field is now piecewise uniform, rather than a single uniform value throughout the gap. As a check, $V' = E_{\text{air}}(0.004) + E_{\text{slab}}(0.004) = 240\ \text{V}$, which also equals Q/C' . The gap-averaged field is $V'/d = 3.0 \times 10^4\ \text{V m}^{-1} = 3E_0/5$.

Step 3: Compare local and average energy densities Initially $u_0 = \frac{1}{2} \epsilon_0 E_0^2 \simeq 0.0111\ \text{J m}^{-3}$. In air, $u_{\text{air}} = u_0$, while inside the dielectric,

$$u_{\text{slab}} = \frac{1}{2} (5\epsilon_0) (E_0/5)^2 = \frac{u_0}{5} \simeq 0.00221\ \text{J m}^{-3}.$$

The two layers have equal volumes, so the energy density averaged over the entire capacitor gap is $\bar{u} = (u_0 + u_0/5)/2 = 3u_0/5 \simeq 0.00664\ \text{J m}^{-3}$. Consistently, the total energy rises from $U_0 = \frac{1}{2} C_0 V_0^2 = 2\ \text{J}$ to $U' = Q^2/(2C') = 2.4\ \text{J}$, but the occupied gap volume doubles.

Step 4: Conclusion $C' = 83.3\ \mu\text{F}$. The air field is unchanged and the dielectric field becomes $E_0/5$; the gap-averaged field becomes $3E_0/5$. Local energy density is unchanged in air and becomes $u_0/5$ in the slab, giving average energy density $3u_0/5$.

Quick Tip to Solve: For a partly filled disconnected capacitor, keep Q fixed and calculate fields separately in air and dielectric before averaging.

23. Why is it difficult to detect the presence of an anti-neutrino during beta-decay? Define the term decay constant of a radioactive nucleus and derive the expression for its mean life in terms of the decay constant. [3 marks]

Solution:

Concept:

- An electron antineutrino emitted in beta-minus decay is electrically neutral and interacts with matter extremely weakly, so it usually escapes without a detectable interaction.
- Radioactive decay obeys $-dN/dt = \lambda N$, with $N(t) = N_0 e^{-\lambda t}$. Here λ is the decay constant and N_0 the initial number of undecayed nuclei.

Step 1: Explain why antineutrinos are difficult to detect The antineutrino has no electric charge, so it does not directly produce the ordinary electromagnetic ionisation track made by a charged beta particle. It interacts through the weak interaction with an extremely small probability. Consequently it can pass through large amounts of matter without depositing appreciable energy. Detection relies on rare interactions and therefore requires a large detector and many incident antineutrinos. Its weak interaction, rather than merely its small mass, is the essential reason for the difficulty.

Step 2: Define the decay constant The decay constant λ is the probability per unit time that a given undecayed nucleus decays; equivalently, it is the fractional decay rate of a radioactive sample. In a small interval dt , the number decaying is $dN_{\text{dec}} = \lambda N(t) dt$, so $\lambda = -(1/N)dN/dt$. Its SI unit is s^{-1} . Integrating the rate equation with $N(0) = N_0$ yields $N(t) = N_0 e^{-\lambda t}$.

Step 3: Derive the mean life from the decay-time distribution Mean life is the average lifetime of all nuclei in the original sample. Nuclei that decay between t and $t + dt$ have lifetime t , and their number is $\lambda N_0 e^{-\lambda t} dt$. Therefore

$$\begin{aligned}\tau &= \frac{1}{N_0} \int_0^{\infty} t \lambda N_0 e^{-\lambda t} dt \\ &= \lambda \int_0^{\infty} t e^{-\lambda t} dt \\ &= \left[-te^{-\lambda t} \right]_0^{\infty} + \int_0^{\infty} e^{-\lambda t} dt \\ &= 0 + \left[-\frac{e^{-\lambda t}}{\lambda} \right]_0^{\infty}.\end{aligned}$$

The boundary term vanishes because $te^{-\lambda t} \rightarrow 0$ as $t \rightarrow \infty$. The integral accounts for all nuclei, since eventually $N(t) \rightarrow 0$.

Step 4: Conclusion Antineutrinos are difficult to detect because they are neutral and interact extremely weakly with matter. The decay constant is the probability of decay per unit time, and the mean life is

$$\tau = 1/\lambda.$$

Quick Tip to Solve: Weight each lifetime t by the number decaying at that time, $\lambda N(t) dt$, rather than by the number still surviving.

23. (a) State two distinguishing features of nuclear force.

(b) Draw a plot showing the variation of potential energy of a pair of nucleons as a function of their separation. Mark the regions on the graph where the force is (i) attractive, and (ii) repulsive.

[3 marks]

Solution:

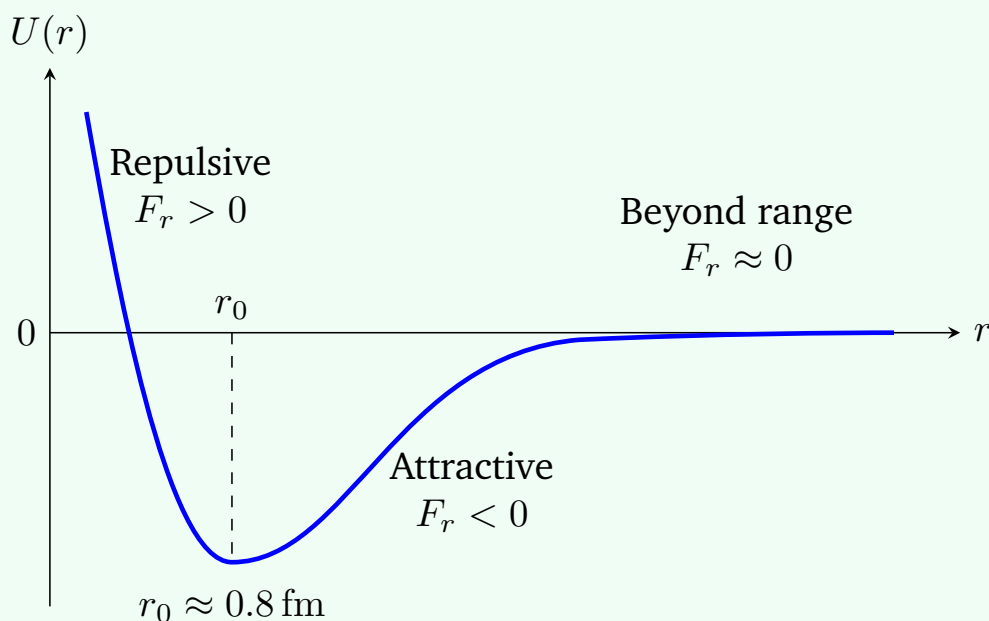
Concept:

- The nuclear force acts between nucleons (protons and neutrons); it is the strong interaction responsible for binding a nucleus.
- For a central interaction, the radial force is $F_r = -dU/dr$, where U is potential energy and r is the separation. Positive F_r means repulsion and negative F_r means attraction.
- Taking $U(\infty) = 0$, a negative potential energy indicates a bound configuration. The force follows the slope of the curve, rather than the sign of the potential energy itself.

Step 1: (a) State two distinguishing features First, the nuclear force is extremely strong but has a very short range: it is effective mainly over separations of the order of 1–2 fm and becomes negligible beyond a few femtometres. This allows closely spaced nucleons to remain bound despite electrostatic repulsion between protons. Second, it is approximately charge independent: after excluding the electrostatic interaction, the nuclear interaction between proton–proton, neutron–neutron and proton–neutron pairs is approximately the same for comparable spin states. Thus it acts on neutrons as well as protons and is not simply an electrical force.

Step 2: (b) Draw and interpret the potential-energy curve At very small separation, $U(r)$ rises steeply because of the repulsive core. As r increases, the curve falls to a negative minimum at $r = r_0$, approximately 0.8 fm, and then rises towards zero. The drawing is qualitative; its vertical scale is arbitrary. On the falling branch, $r < r_0$, $dU/dr < 0$, so $F_r > 0$: the force pushes the nucleons apart and is repulsive. On the rising branch, $r > r_0$ within the nuclear range, $dU/dr > 0$, so $F_r < 0$: the force pulls the nucleons together and is attractive. At the minimum, the slope and force are zero. Beyond the nuclear range, the curve is nearly flat at zero and the force is negligible.

Quick Tip to Solve: Read the slope, not the sign of the potential: a falling curve gives repulsion, whereas a rising curve gives attraction.



Qualitative nucleon potential: repulsion to the left of the minimum, attraction to its right within the interaction range.

Step 3: Conclusion Nuclear force is strong and short ranged, and approximately charge independent. The potential curve has a repulsive core, a negative minimum at r_0 , and an attractive outer branch tending to zero; the regions are identified from $F_r = -dU/dr$.

Quick Tip to Solve: Read the slope, not the sign of the potential: a falling curve gives repulsion, whereas a rising curve gives attraction.

24. Prove that in a common-emitter amplifier, the output and input differ in phase by 180° .

In a transistor, the change of base current by $30\ \mu\text{A}$ produces change of $0.02\ \text{V}$ in the base-emitter voltage and a change of $4\ \text{mA}$ in the collector current. Calculate the current amplification factor and the load resistance used, if the voltage gain of the amplifier is 400. [3 marks]

Solution:

Concept:

- In a common-emitter amplifier, the input is the base-emitter voltage and the output is the collector-emitter voltage. The emitter is common to both circuits.
- The small-signal current gain is $\beta = \Delta I_C / \Delta I_B$, and the input resistance is $r_i = \Delta V_{BE} / \Delta I_B$.
- With constant supply voltage V_{CC} , the collector voltage satisfies $V_{CE} = V_{CC} - I_C R_L$. Hence $\Delta V_{CE} = -R_L \Delta I_C$, and the voltage gain is $A_v = -\beta R_L / r_i$.

Step 1: Prove the phase reversal Consider a transistor biased in its active region. During the positive half-cycle of the input, the base-emitter voltage increases. The base current therefore increases, and the collector current increases by $\Delta I_C = \beta \Delta I_B$. The voltage drop $I_C R_L$ across the collector load

Quick Tip to Solve: Find the load directly from $R_L = |A_v| \Delta V_{BE} / \Delta I_C$; keep the minus sign only when discussing phase reversal.

increases. Since the supply voltage is constant, the collector-emitter voltage decreases:

$$\Delta V_{CE} = -R_L \Delta I_C = -\beta R_L \Delta I_B.$$

During the negative half-cycle, the base current and collector current decrease, the load voltage drop decreases, and the collector-emitter voltage increases. Thus every positive input change produces a negative output change, and conversely. More explicitly, if $v_i = V_m \sin \omega t$, then

$$v_o = -\frac{\beta R_L}{r_i} V_m \sin \omega t = \frac{\beta R_L}{r_i} V_m \sin(\omega t + \pi).$$

The input and collector output consequently differ in phase by π radians, or 180° , under the usual small-signal amplifier conditions.

Step 2: Calculate the current amplification factor Convert both current changes to amperes before taking their ratio:

$$\beta = \frac{4 \times 10^{-3}}{30 \times 10^{-6}} = \frac{400}{3} \approx 133.3.$$

This ratio is dimensionless. It is the common-emitter current amplification factor, because the input current is the base current and the output current is the collector current.

Step 3: Calculate the load resistance from the voltage gain The given voltage gain 400 is its magnitude; the signed common-emitter gain is negative. First calculate the input resistance:

$$r_i = \frac{0.02}{30 \times 10^{-6}} = \frac{2000}{3} \Omega \approx 666.7 \Omega.$$

Using $|A_v| = \beta R_L / r_i$,

$$R_L = \frac{|A_v| r_i}{\beta} = \frac{400(2000/3)}{400/3} \Omega = 2000 \Omega.$$

As an independent check, the output-voltage change has magnitude $400 \times 0.02 = 8 \text{ V}$. Dividing this by the collector-current change gives $R_L = 8 / (4 \times 10^{-3}) = 2000 \Omega$, confirming the result.

Step 4: Conclusion The common-emitter collector output is 180° out of phase with the input. The current amplification factor is $\beta \approx 133.3$, and the load resistance is $R_L = 2.0 \text{ k}\Omega$.

Quick Tip to Solve: Find the load directly from $R_L = |A_v| \Delta V_{BE} / \Delta I_C$; keep the minus sign only when discussing phase reversal.

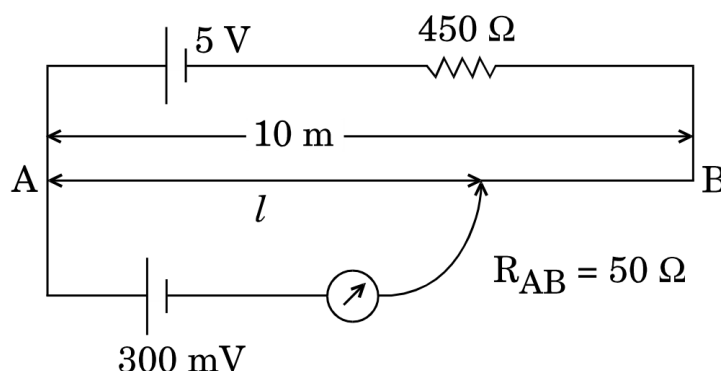
Section D (15 marks)

25. (a) Describe briefly, with the help of a circuit diagram, the method of measuring the internal resistance of a cell.

(b) Give reason why a potentiometer is preferred over a voltmeter for the measurement of emf of a cell.

(c) In the potentiometer circuit given below, calculate the balancing length l . Give reason, whether the circuit will work, if the driver cell of emf 5 V is replaced with a cell of 2 V, keeping all other factors constant.

[5 marks]



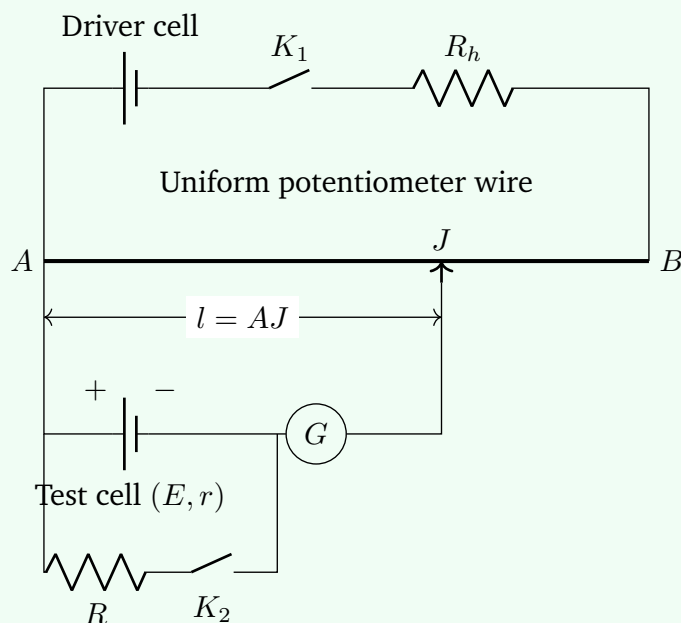
Solution:

Concept:

- A potentiometer compares potential differences by a null method. For a uniform wire carrying steady current, the potential drop over length l is kl , where k is the potential gradient.
- A cell of emf E and internal resistance r , delivering current i through an external resistance R , has terminal voltage $V = iR$, while $E = i(R + r)$.
- At balance, no current flows through the galvanometer branch. With K_2 open the test cell supplies no current; with K_2 closed it supplies current through R , and the null measures its loaded terminal voltage. The cell's positive terminal must connect to the high-potential end A , and the total wire drop must exceed the voltage being measured.

Step 1: (a) Set up the internal-resistance experiment Connect the uniform potentiometer wire AB in series with a driver cell, primary key K_1 , and rheostat R_h . Connect the positive terminal of the test cell to A , and its negative terminal through galvanometer G to jockey J . Connect a known resistance R and key K_2 in series across the test cell. The driver maintains the potential gradient; the shunt branch draws current from the test cell only when K_2 is closed. Choose a driver voltage sufficient to obtain both null points.

Quick Tip to Solve: First calculate the voltage across the potentiometer wire, not the driver's full emf. A proposed balance length longer than the wire immediately shows that no null point is possible.



Internal-resistance measurement: open K_2 for l_1 , close K_2 for l_2 .

Step 2: (a) Measure the two balance lengths Close K_1 , keep K_2 open, and move J until G shows zero deflection. Record $AJ = l_1$. At this null point the test cell supplies no current, so its emf is $E = kl_1$. Now close K_2 , allowing the cell to send current through R . Without changing the rheostat or primary current, locate the new null point and record $AJ = l_2$. This measures the loaded terminal voltage, $V = kl_2$, with $l_2 < l_1$.

Step 3: (a) Derive the internal resistance The load current is $i = V/R$, and the internal voltage drop is $E - V = ir$. Hence

$$r = \frac{E - V}{i} = R \left(\frac{E}{V} - 1 \right),$$

$$\frac{E}{V} = \frac{kl_1}{kl_2} = \frac{l_1}{l_2}.$$

Keeping k unchanged between the two readings is essential for cancelling it. Use a known R and the measured lengths to evaluate the cell's internal resistance.

Step 4: (b) Explain the preference over a voltmeter At potentiometer balance no current is drawn from the test cell, so there is no voltage drop across its internal resistance. The measured value is therefore its actual emf. A voltmeter of finite resistance draws a small current and reads the terminal voltage $V = E - ir$, which is below the emf when $r > 0$. The potentiometer avoids this loading error through its null measurement.

Step 5: (c) Find the current and potential gradient for the given circuit The printed diagram gives a 5 V driver, series resistance 450Ω , and a 10 m wire of resistance 50Ω . At balance the secondary branch

Quick Tip to Solve: First calculate the voltage across the potentiometer wire, not the driver's full emf. A proposed balance length longer than the wire immediately shows that no null point is possible.

draws no current. Thus

$$I = \frac{5}{450 + 50} = 0.010 \text{ A},$$
$$V_{AB} = IR_{AB} = 0.010 \times 50 = 0.50 \text{ V},$$
$$k = \frac{0.50}{10} = 0.050 \text{ V m}^{-1}.$$

The test cell has emf $300 \text{ mV} = 0.300 \text{ V}$. Apply the null condition $kl = E$, measuring l from A .

Step 6: (c) Calculate the length and test the replacement driver With the original driver,

$$l = \frac{0.300}{0.050} = 6.0 \text{ m}.$$

On replacing the driver by 2 V , with all resistances and wire length unchanged,

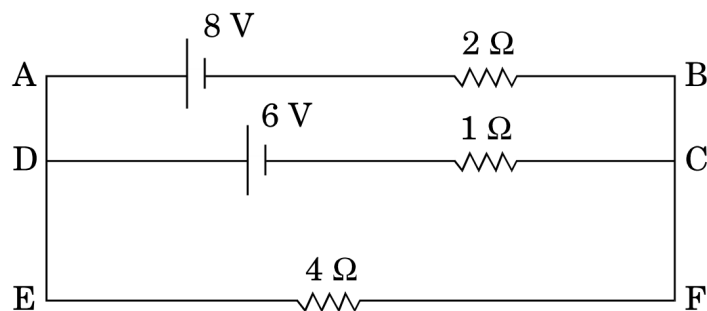
$$I' = \frac{2}{500} = 0.004 \text{ A},$$
$$V'_{AB} = 0.004 \times 50 = 0.200 \text{ V},$$
$$k' = 0.020 \text{ V m}^{-1},$$
$$l' = \frac{0.300}{0.020} = 15 \text{ m}.$$

This formal balance length lies beyond the available 10 m wire: even the full wire drop is only 0.200 V , below the test emf 0.300 V . Therefore a null point cannot be obtained with the replacement driver.

Step 7: Conclusion (a) The internal resistance is $r = R(l_1 - l_2)/l_2$, using open-circuit and loaded balance lengths at the same potential gradient. (b) A potentiometer measures emf without drawing current from the test cell at balance. (c) The balancing length is 6.0 m with the 5 V driver; the 2 V driver cannot produce balance because the wire drop is only 0.200 V .

Quick Tip to Solve: First calculate the voltage across the potentiometer wire, not the driver's full emf. A proposed balance length longer than the wire immediately shows that no null point is possible.

25. (a) State the working principle of a metre bridge used to measure an unknown resistance.
 (b) Give reason (i) why the connections between the resistors in a metre bridge are made of thick copper strips, (ii) why it is generally preferred to obtain the balance length near the mid-point of the bridge wire.
 (c) Calculate the potential difference across the 4Ω resistor in the given electrical circuit, using Kirchhoff's rules. [5 marks]



Solution:

Concept:

- A metre bridge works on the balanced Wheatstone bridge principle: at balance, no current flows through the galvanometer and the resistance ratios in the two arms are equal.
- For a uniform bridge wire, resistance is proportional to length. If an unknown resistance X occupies the left gap and a known resistance R the right gap, $X/R = l/(100 - l)$, with l measured in centimetres.
- Kirchhoff's junction rule expresses conservation of charge; the loop rule states that the algebraic sum of potential changes round a closed loop is zero.

Step 1: (a) Explain the bridge balance Connect the unknown and known resistances in the two gaps and move the jockey along the uniform metre wire until the galvanometer shows zero deflection. The potentials of the jockey and the junction of the two gap resistances are then equal. The balanced Wheatstone relation gives

$$\frac{X}{R} = \frac{r_{AJ}}{r_{JB}} = \frac{l}{100 - l}.$$

Hence the unknown resistance is found from $X = Rl/(100 - l)$. Uniform cross-section and material of the wire are essential for replacing the resistance ratio by its length ratio.

Step 2: (b)(i) Explain the use of thick copper strips Copper has low resistivity, and a large cross-sectional area further reduces resistance because $R_{\text{strip}} = \rho L/A$. Therefore the thick strips have negligible resistance compared with the gap resistances. They form nearly equipotential connections and prevent their own resistance from appreciably changing the bridge balance or the inferred unknown resistance.

Step 3: (b)(ii) Explain why balance is sought near the midpoint Near $l = 50$ cm, neither segment of the wire is very short, the two gap resistances are comparable, and the bridge has good sensitivity. For

Quick Tip to Solve: First combine A, D and E into one node and B, C and F into the other; check each cell's long plate before writing the loop equations.

a small uncertainty δl , differentiating the bridge relation gives

$$\frac{|\delta X|}{X} \simeq \left(\frac{1}{l} + \frac{1}{100-l} \right) |\delta l|.$$

The factor in brackets is smallest at the midpoint. Thus a fixed uncertainty in locating the balance point causes the least fractional error there, and small end or contact resistances have less influence than when balance lies close to an end.

Step 4: (c) Apply Kirchhoff's rules to the three branches The left-hand points A, D, E are one equipotential node, and the right-hand points B, C, F are another. Let $V = V_E - V_F$. Define the top and middle currents i_1, i_2 from right to left, and the bottom current i_3 from left to right. Both cells have their positive terminals on the left, as shown by the longer plates. The junction rule gives $i_3 = i_1 + i_2$. The loop rule, using the bottom branch and each cell branch, gives

$$8 - 2i_1 - 4i_3 = 0,$$

$$6 - i_2 - 4i_3 = 0.$$

Therefore $i_1 = (8 - 4i_3)/2$ and $i_2 = 6 - 4i_3$. Substitution into the junction rule yields

$$i_3 = 4 - 2i_3 + 6 - 4i_3,$$

$$7i_3 = 10, \quad i_3 = \frac{10}{7} \text{ A}.$$

The other currents are $i_1 = 8/7 \text{ A}$ and $i_2 = 2/7 \text{ A}$, both consistent with the chosen directions. The resistor voltage is $V = 4i_3 = 40/7 \text{ V}$.

Step 5: Conclusion The metre bridge uses the balanced Wheatstone relation $X/R = l/(100-l)$. Thick copper strips make connection resistance negligible; a midpoint balance improves sensitivity and reduces fractional error. The potential difference across the 4Ω resistor is $40/7 \text{ V} \simeq 5.71 \text{ V}$, with E at a higher potential than F .

Quick Tip to Solve: First combine A, D and E into one node and B, C and F into the other; check each cell's long plate before writing the loop equations.

26. (a) Derive an expression for the induced emf developed when a coil of N turns, and area of cross-section A , is rotated at a constant angular speed ω in a uniform magnetic field B .

(b) A wheel with 100 metallic spokes each 0.5 m long is rotated with a speed of 120 rev/min in a plane normal to the horizontal component of the Earth's magnetic field. If the resultant magnetic field at that place is $4 \times 10^{-4} \text{ T}$ and the angle of dip at the place is 30° , find the emf induced between the axle and the rim of the wheel. [5 marks]

Solution:

Concept:

- Faraday's law gives the induced emf as $\varepsilon = -d(N\Phi)/dt$, where $N\Phi$ is the total magnetic flux linkage of an N -turn coil.

Quick Tip to Solve: Use the field component normal to the wheel and integrate along one spoke; spokes sharing the axle and rim are parallel, so do not multiply by 100.

- Flux through a coil of area A in a uniform field B is $\Phi = BA \cos \theta$, where θ is the angle between the field and the coil's area normal.
- A conducting radial spoke rotating at angular speed ω in a perpendicular field $B_{\perp}$ develops a motional emf of magnitude $\varepsilon = \frac{1}{2} B_{\perp} \omega L^2$, where L is its length.

Step 1: (a) Express the flux linkage of the rotating coil Let the coil rotate about an axis in its plane perpendicular to the uniform magnetic field, so that the angle of its normal with the field changes at angular speed ω . Choose $t = 0$ when the normal is parallel to the field. Then $\theta = \omega t$. Each turn encloses the same area A , so the total linkage is

$$N\Phi = NBA \cos(\omega t).$$

This choice of initial orientation fixes the phase of the emf; another starting orientation would add a constant phase angle without changing the maximum emf.

Step 2: (a) Differentiate the flux linkage Applying Faraday's law to the coil gives

$$\begin{aligned}\varepsilon &= -\frac{d}{dt} [NBA \cos(\omega t)] \\ &= NBA\omega \sin(\omega t).\end{aligned}$$

Thus the instantaneous emf varies sinusoidally, with maximum magnitude $\varepsilon_0 = NBA\omega$ and frequency $f = \omega/(2\pi)$. At maximum flux the emf is zero because the instantaneous flux change is zero; when the coil normal is perpendicular to the field, the flux change and emf magnitude are greatest. The emf reverses after each half-turn, in accordance with Lenz's law.

Step 3: (b) Identify the effective magnetic field and angular speed The wheel rotates in a plane normal to the horizontal component of the Earth's magnetic field, so the component perpendicular to its plane is B_H . With resultant field $B = 4 \times 10^{-4} \text{ T}$ and dip $\delta = 30^\circ$,

$$\begin{aligned}B_H &= B \cos \delta = 4 \times 10^{-4} \cos 30^\circ = 2\sqrt{3} \times 10^{-4} \text{ T}, \\ f &= \frac{120}{60} = 2 \text{ s}^{-1}, \\ \omega &= 2\pi f = 4\pi \text{ rad s}^{-1}.\end{aligned}$$

The vertical component lies in the wheel's plane and does not contribute to the radial axle-to-rim emf.

Step 4: (b) Obtain and evaluate the axle-to-rim emf At radius r , an element dr of a spoke has tangential speed $v = \omega r$. Its radial motional emf is $d\varepsilon = B_H \omega r dr$. Integrating from the axle to the rim, with $L = 0.5 \text{ m}$, gives

$$\begin{aligned}|\varepsilon| &= \int_0^L B_H \omega r dr = \frac{1}{2} B_H \omega L^2 \\ &= \frac{1}{2} (2\sqrt{3} \times 10^{-4}) (4\pi) (0.5)^2 \\ &= \pi\sqrt{3} \times 10^{-4} \text{ V} \simeq 5.44 \times 10^{-4} \text{ V}.\end{aligned}$$

All 100 spokes connect the same axle and rim and are electrically in parallel; their emfs do not add in series. The number of spokes therefore does not multiply the potential difference. The sense of rotation is not specified, so only the magnitude, not which terminal is positive, can be fixed.

Quick Tip to Solve: Use the field component normal to the wheel and integrate along one spoke; spokes sharing the axle and rim are parallel, so do not multiply by 100.

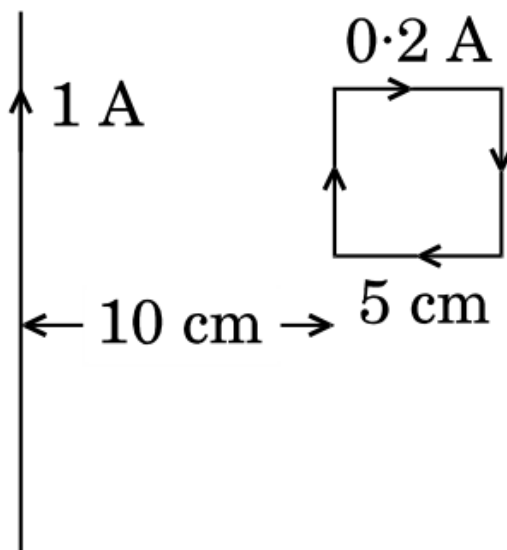
Step 5: Conclusion For the rotating coil, $\varepsilon(t) = NBA\omega \sin(\omega t)$, with peak emf $NBA\omega$. The magnitude of the wheel's axle-to-rim emf is $5.44 \times 10^{-4} \text{ V} = 0.544 \text{ mV}$.

Quick Tip to Solve: Use the field component normal to the wheel and integrate along one spoke; spokes sharing the axle and rim are parallel, so do not multiply by 100.

OR

26. (a) Derive the expression for the magnetic energy stored in an inductor when a current I develops in it. Hence, obtain the expression for the magnetic energy density.

(b) A square loop of sides 5 cm carrying a current of 0.2 A in the clockwise direction is placed at a distance of 10 cm from an infinitely long wire carrying a current of 1 A as shown. Calculate (i) the resultant magnetic force, and (ii) the torque, if any, acting on the loop. [5 marks]



Solution:

Concept:

- An ideal inductor of constant self-inductance L opposes a changing current with emf $\varepsilon = -L di/dt$. The external source supplies work against this back emf, storing energy in the magnetic field.
- For a long air-core solenoid, $L = \mu_0 n^2 A \ell$ and $B = \mu_0 n I$, where n is the number of turns per unit length, A the cross-sectional area and ℓ the length.
- A straight wire carrying current I_w produces $B(r) = \mu_0 I_w / (2\pi r)$. A loop element experiences $d\mathbf{F} = I_l d\mathbf{\ell} \times \mathbf{B}$, with direction given by the vector cross product.

Step 1: (a) Derive the energy stored in an inductor Let the instantaneous current rise from zero to i . The applied voltage needed to overcome the induced emf is $V = L di/dt$. Ignoring resistance, the

Quick Tip to Solve: Read the arrow showing the wire-to-loop distance carefully: use 10 cm for the nearer side and 15 cm for the farther side, then subtract the repulsive force from the attractive force.

power supplied goes entirely into the magnetic field:

$$\frac{dU}{dt} = Vi = Li \frac{di}{dt}, \quad dU = Li \, di.$$

Integrating from zero current to the final current I , for constant L , gives

$$U = \int_0^I Li \, di = \frac{1}{2} LI^2.$$

The energy is positive: the source does work against the induced emf during current growth. When the current decreases, the inductor can return this stored energy to the circuit.

Step 2: (a) Obtain magnetic energy per unit volume Consider a long air-core solenoid, neglecting its end effects. Its approximately uniform field occupies volume $A\ell$. Substituting its inductance into the energy expression,

$$U = \frac{1}{2} \mu_0 n^2 A \ell I^2, \quad u_B = \frac{U}{A\ell} = \frac{1}{2} \mu_0 n^2 I^2.$$

Since $nI = B/\mu_0$, this becomes

$$u_B = \frac{B^2}{2\mu_0}.$$

Thus the magnetic energy density in air or vacuum has units J m^{-3} . In a linear medium of constant permeability μ , the corresponding expression is $u_B = B^2/(2\mu)$.

Step 3: (b)(i) Find the forces on the two vertical sides The diagram gives the distance from the wire to the nearer vertical side, rather than to the loop centre. Write

$$a = 0.05 \text{ m}, \quad r_1 = 0.10 \text{ m}, \quad r_2 = r_1 + a = 0.15 \text{ m}, \quad I_w = 1 \text{ A}, \quad I_l = 0.2 \text{ A}.$$

To the right of the upward-current wire, its magnetic field points into the paper. The clockwise loop current flows upward along the nearer side and downward along the farther side. Hence the nearer side is attracted towards the wire, while the farther side is repelled. The respective magnitudes are

$$F_1 = \frac{\mu_0 I_w I_l a}{2\pi r_1} = 2.0 \times 10^{-8} \text{ N},$$

$$F_2 = \frac{\mu_0 I_w I_l a}{2\pi r_2} = 1.33 \times 10^{-8} \text{ N}.$$

The field is stronger on the nearer side, so the attractive force is larger.

Step 4: (b)(i) Include the horizontal sides and calculate the resultant The top side carries current to the right and experiences an upward force; the bottom side carries current to the left and experiences a downward force. At every corresponding distance r from the wire, these sides experience equal field strengths and equal, opposite force elements. Their resultant is therefore zero, even though the

Quick Tip to Solve: Read the arrow showing the wire-to-loop distance carefully: use 10 cm for the nearer side and 15 cm for the farther side, then subtract the repulsive force from the attractive force.

field varies along each side. The net force on the entire loop is

$$\begin{aligned}
 F_{\text{net}} &= F_1 - F_2 \\
 &= \frac{\mu_0 I_w I_l a}{2\pi} \left(\frac{1}{r_1} - \frac{1}{r_2} \right) \\
 &= 2 \times 10^{-7} (1)(0.2)(0.05) \left(\frac{1}{0.10} - \frac{1}{0.15} \right) \\
 &= 6.67 \times 10^{-9} \text{ N},
 \end{aligned}$$

directed horizontally towards the straight wire.

Step 5: (b)(ii) Determine whether the loop experiences a torque Take moments about the centre of the square. On each vertical side, the magnetic field is constant along the side, so its distributed horizontal force has its resultant at the side's midpoint. Its line of action passes through the loop centre and produces no moment about that centre. For corresponding elements of the top and bottom sides, the forces are equal and opposite; their moments about the centre cancel because their horizontal coordinates are the same. Thus the total torque about the loop centre is zero. This agrees with the orientation: the clockwise current gives the loop a magnetic moment into the paper, parallel to the wire's magnetic field. The loop is pulled towards the wire but has no tendency to rotate about its centre.

Step 6: Conclusion (a) The stored magnetic energy is $U = \frac{1}{2} LI^2$, and its density in air or vacuum is $u_B = B^2 / (2\mu_0)$. (b) The resultant force is $6.67 \times 10^{-9} \text{ N}$ towards the straight wire, and the torque about the loop centre is 0 .

Quick Tip to Solve: Read the arrow showing the wire-to-loop distance carefully: use 10 cm for the nearer side and 15 cm for the farther side, then subtract the repulsive force from the attractive force.

27. Explain, with the help of a diagram, how plane polarized light can be produced by scattering of light from the Sun.

Two polaroids P_1 and P_2 are placed with their pass axes perpendicular to each other. Unpolarised light of intensity I is incident on P_1 . A third polaroid P_3 is kept between P_1 and P_2 such that its pass axis makes an angle of 45° with that of P_1 . Calculate the intensity of light transmitted through P_1 , P_2 and P_3 . [5 marks]

Solution:

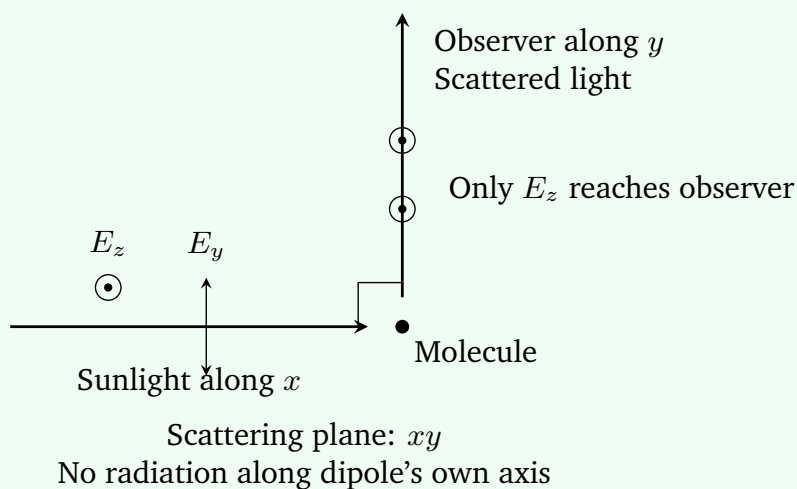
Concept:

- Unpolarised sunlight has electric-field vibrations in all directions perpendicular to its direction of propagation. Atmospheric molecules scatter this light by behaving as oscillating electric dipoles driven by the incident field.
- A dipole does not radiate along its oscillation axis. Therefore, light scattered at 90° to the incident beam is plane polarised: its electric field is perpendicular to the plane containing the incident and scattered rays.

Quick Tip to Solve: Write the physical order P_1, P_3, P_2 first; halve the unpolarised intensity only once, then use the angle to the immediately preceding pass axis.

- An ideal polaroid transmits half the intensity of unpolarised incident light. For plane polarised light, Malus's law gives $I_{\text{out}} = I_{\text{in}} \cos^2 \theta$, where θ is the angle between the incident electric-field direction and the pass axis.

Step 1: Explain polarisation by scattering Let sunlight travel along the x -direction and let the observer view the scattered light along the y -direction. The incident light can be resolved into electric-field components along y and z . These components drive atmospheric molecules to oscillate in those respective directions. The y -directed oscillations cannot radiate towards an observer on the y -axis, whereas the z -directed oscillations do radiate towards that observer. Consequently, only the z -component is present in the light observed at right angles to the sunlight. Its vibrations are perpendicular to the xy scattering plane, so it is plane polarised. Rotating an analyser in this scattered beam gives maximum transmission for its pass axis along z and extinction for its pass axis along x , in the ideal single-scattering picture.



Scattering at a right angle: dots denote electric-field vibrations along z , perpendicular to the page (the xy scattering plane).

Step 2: Determine the transmission through the first polaroid The actual order encountered by the light is P_1 , then P_3 , then P_2 . Since the beam incident on P_1 is unpolarised, the average of $\cos^2 \theta$ over all electric-field orientations is $1/2$. Thus the intensity immediately after P_1 is

$$I_1 = \frac{I}{2}.$$

This transmitted beam is plane polarised parallel to the pass axis of P_1 .

Step 3: Apply Malus's law at the middle polaroid The pass axis of P_3 makes 45° with that of P_1 , which is the polarisation direction of its incident light. Hence

$$I_3 = I_1 \cos^2 45^\circ = \frac{I}{2} \times \frac{1}{2} = \frac{I}{4}.$$

The beam emerging from P_3 is now polarised along the pass axis of P_3 ; that axis sets the reference direction for the next application of Malus's law.

Step 4: Apply Malus's law at the final polaroid The axes of P_1 and P_2 are perpendicular. The axis of

Quick Tip to Solve: Write the physical order P_1, P_3, P_2 first; halve the unpolarised intensity only once, then use the angle to the immediately preceding pass axis.

the intermediate polaroid is therefore at 45° to the axis of P_2 as well. Consequently,

$$I_2 = I_3 \cos^2 45^\circ = \frac{I}{4} \times \frac{1}{2} = \frac{I}{8}.$$

Although the outer polaroids are crossed, the inserted polaroid changes the polarisation direction of the light reaching P_2 . Applying the original 90° angle directly at P_2 would therefore be incorrect for this arrangement.

Step 5: Conclusion Sunlight scattered at 90° is plane polarised perpendicular to the scattering plane. With P_3 inserted, the transmitted intensities, in the question's requested order, are $P_1 : I/2$, $P_2 : I/8$, and $P_3 : I/4$. The final emerging intensity is $I/8$.

Quick Tip to Solve: Write the physical order P_1, P_3, P_2 first; halve the unpolarised intensity only once, then use the angle to the immediately preceding pass axis.

OR

27. (a) Why cannot the phenomenon of interference be observed by illuminating two pin holes with two sodium lamps?

(b) Two monochromatic waves having displacements $y_1 = a \cos \omega t$ and $y_2 = a \cos(\omega t + \phi)$ from two coherent sources interfere to produce an interference pattern. Derive the expression for the resultant intensity and obtain the conditions for constructive and destructive interference.

(c) Two wavelengths of sodium light of 590 nm and 596 nm are used in turn to study the diffraction taking place at a single slit of aperture 2×10^{-6} m. If the distance between the slit and the screen is 1.5 m, calculate the separation between the positions of the second maxima of diffraction pattern obtained in the two cases.

[5 marks]

Solution:

Concept:

- Coherent sources have the same frequency and maintain a constant phase difference. A steady interference pattern requires this fixed phase relationship.
- The superposition principle gives the resultant displacement as the sum of the individual displacements. For a fixed medium, intensity is proportional to the square of the resultant amplitude.
- For a slit of width b , diffraction minima satisfy $b \sin \theta = n\lambda$. Secondary maxima lie approximately midway between successive minima; the second secondary maximum has $b \sin \theta \simeq 5\lambda/2$. In the usual small-angle screen approximation, $x \simeq D \sin \theta$, where D is the screen distance.

Step 1: (a) Explain why two sodium lamps do not give stable fringes Two separate sodium lamps are independent sources. Although they emit light of nearly the same wavelength, their atoms emit separate wave trains with randomly changing phases. Thus the phase difference between the light reaching the two pinholes does not remain constant. Bright and dark positions change extremely rapidly, and the eye or detector records the time-averaged intensity. Consequently no steady interference fringes can be observed. For a stable pattern the two pinholes must instead be illuminated by the same source so that they act as coherent sources.

Quick Tip to Solve: For the second secondary maximum use the midpoint between the second and third minima, giving the factor $5/2$, and subtract wavelengths before substituting to find the separation directly.

Step 2: (b) Obtain the resultant displacement and amplitude Using superposition and the identity $\cos A + \cos B = 2 \cos[(A - B)/2] \cos[(A + B)/2]$,

$$\begin{aligned} y &= y_1 + y_2 \\ &= a \cos \omega t + a \cos(\omega t + \phi) \\ &= 2a \cos(\phi/2) \cos(\omega t + \phi/2). \end{aligned}$$

The resultant amplitude has magnitude $A = |2a \cos(\phi/2)|$; hence $A^2 = 4a^2 \cos^2(\phi/2)$. The phase shift of the resultant wave does not alter its time-averaged intensity.

Step 3: (b) Derive the intensity and the interference conditions Let the intensity of either wave alone be $I_0 = Ka^2$, where K is the common proportionality constant. The resultant intensity is

$$\begin{aligned} I &= KA^2 = 4I_0 \cos^2(\phi/2) \\ &= 2I_0(1 + \cos \phi). \end{aligned}$$

If the path difference is Δ , then $\phi = 2\pi\Delta/\lambda$. Constructive interference occurs when $\cos^2(\phi/2) = 1$, giving $\phi = 2n\pi$, $\Delta = n\lambda$, and $I_{\max} = 4I_0$. Destructive interference occurs when $\cos^2(\phi/2) = 0$, giving $\phi = (2n + 1)\pi$, $\Delta = (2n + 1)\lambda/2$, and $I_{\min} = 0$, for integer n . Complete cancellation is possible here because the two amplitudes are equal.

Step 4: (c) Find the separation of the second secondary maxima The second secondary maximum on either side of the central maximum lies approximately between the second and third minima. Using the conventional approximate position,

$$x_2 \simeq \frac{5\lambda D}{2b}.$$

Here $b = 2 \times 10^{-6}$ m, $D = 1.5$ m, and the wavelength difference is $(596 - 590)$ nm $= 6 \times 10^{-9}$ m. Comparing the maxima on the same side of the central maximum gives

$$\begin{aligned} \Delta x_2 &\simeq \frac{5D}{2b}(\lambda_2 - \lambda_1) \\ &= \frac{5 \times 1.5 \times 6 \times 10^{-9}}{2 \times 2 \times 10^{-6}} \text{ m} \\ &= 1.125 \times 10^{-2} \text{ m}. \end{aligned}$$

The larger wavelength places its corresponding maximum farther from the centre. This numerical result uses the standard approximate secondary-maximum formula and paraxial screen relation; secondary maxima are not exactly halfway between minima.

Step 5: Conclusion (a) Independent sodium lamps are incoherent and cannot produce steady fringes. (b) $I = 4I_0 \cos^2(\phi/2)$; constructive interference requires $\phi = 2n\pi$, and destructive interference requires $\phi = (2n + 1)\pi$. (c) The conventional approximate separation is 1.125×10^{-2} m $= 1.125$ cm.

Quick Tip to Solve: For the second secondary maximum use the midpoint between the second and third minima, giving the factor $5/2$, and subtract wavelengths before substituting to find the separation directly.