

CBSE Class 12 Physics Question Paper 2019 with Solution PDF-Set 55/3/1

Question Paper with Solutions

Conducted by the Central Board of Secondary Education (CBSE)



Time Allowed: 3 hours

Maximum Marks: 70

General Instructions

- (i) All questions are compulsory. There are 27 questions in all.
- (ii) This question paper has four sections : Section A, Section B, Section C and Section D.
- (iii) Section A contains five questions of one mark each, Section B contains seven questions of two marks each, Section C contains twelve questions of three marks each, Section D contains three questions of five marks each.
- (iv) There is no overall choice. However, an internal choice(s) has been provided in two questions of one mark, two questions of two marks, four questions of three marks and three questions of five marks weightage. You have to attempt only one of the choices in such questions.

You may use the following values of physical constants wherever necessary :

$$c = 3 \times 10^8 \text{ m/s}$$

$$h = 6.63 \times 10^{-34} \text{ Js}$$

$$e = 1.6 \times 10^{-19} \text{ C}$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ T m A}^{-1}$$

$$\epsilon_0 = 8.854 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$$

$$\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$$

$$\text{Mass of electron } (m_e) = 9.1 \times 10^{-31} \text{ kg}$$

$$\text{Mass of neutron} = 1.675 \times 10^{-27} \text{ kg}$$

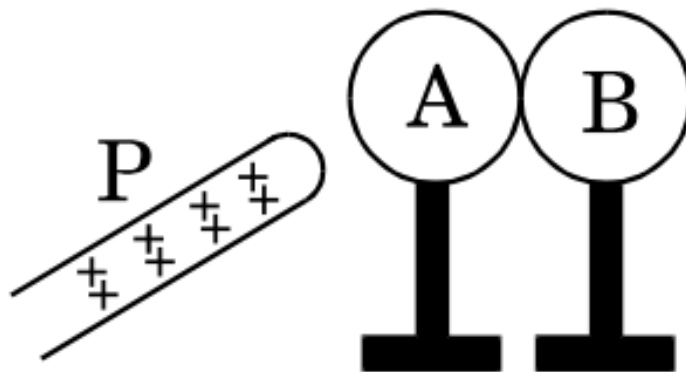
$$\text{Mass of proton} = 1.673 \times 10^{-27} \text{ kg}$$

$$\text{Avogadro's number} = 6.023 \times 10^{23} \text{ per gram mole}$$

$$\text{Boltzmann constant} = 1.38 \times 10^{-23} \text{ JK}^{-1}$$

Section A (5 marks)

1. Two metallic spheres A and B kept on insulating stands are in contact with each other. A positively charged rod P is brought near the sphere A as shown in the figure. The two spheres are separated from each other, and the rod P is removed. What will be the nature of charges on spheres A and B? [1 mark]



Solution:

Concept:

- Electrostatic induction redistributes free electrons in a conductor without contact with the charged rod.
- The insulating stands prevent exchange of charge with the Earth, so the total charge of the two initially neutral spheres remains zero.

Step 1: Follow the electrons while the spheres touch The positively charged rod attracts electrons towards the nearer sphere A. Electrons move from B to A through their contact, leaving an excess of electrons on A and a deficiency on B.

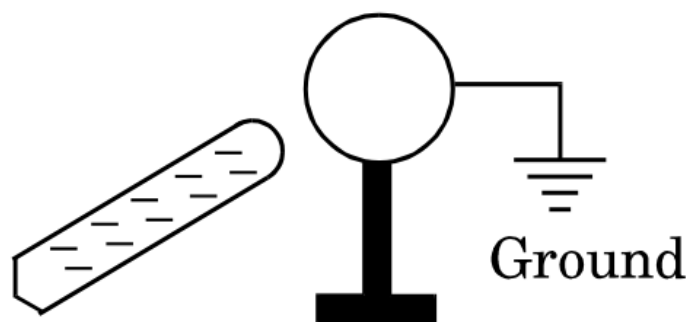
Step 2: Separate the spheres before removing the rod Separation breaks the conducting path while the induced charge separation is still present. On subsequently removing the rod, charge redistributes over each sphere, but electrons cannot flow back from A to B through the insulating stands.

Step 3: Conclusion Sphere A is negatively charged and sphere B is positively charged. Their net charges are equal in magnitude and opposite in sign.

Quick Tip to Solve: Track electron motion: a positive rod attracts electrons to the nearer sphere; separate the spheres before removing it to retain the charges.

OR

1. A metal sphere is kept on an insulating stand. A negatively charged rod is brought near it, then the sphere is earthed as shown. On removing the earthing, and taking the negatively charged rod away, what will be the nature of charge on the sphere? Give reason for your answer. [1 mark]



Solution:

Concept:

- A negatively charged rod repels the mobile electrons in a nearby metal conductor; the positively charged lattice remains fixed.
- Earthing provides a conducting path for electrons to leave or enter a conductor. Removing this path isolates its remaining net charge.

Step 1: Identify electron flow during earthing With the negative rod held near the sphere, electrons are repelled away from the rod. On connecting the sphere to the Earth, some electrons flow from the sphere into the Earth, leaving the sphere deficient in electrons.

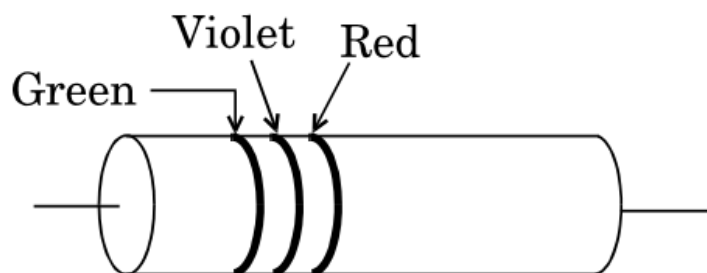
Step 2: Apply the stated order of disconnection The earth connection is removed first, so the electrons that escaped cannot return. When the rod is then taken away, the remaining charge spreads over the sphere's surface, but its net charge cannot change because the sphere is insulated.

Step 3: Conclusion The sphere becomes positively charged because electrons have flowed from it to the Earth and cannot return after the earth connection is removed.

Quick Tip to Solve: In charging by induction, disconnect the Earth before removing the rod; the retained charge is opposite to the rod's charge.

2. A carbon resistor is shown in the figure. Using colour code, write the value of the resistance.

[1 mark]



Solution:

Concept:

- In a resistor colour code, the first two bands give the two significant digits and the third band gives the power-of-ten multiplier.
- Green represents 5, violet represents 7 and red represents the multiplier 10^2 . The resulting resistance is in ohms.

Step 1: Read the significant digits The printed bands, read from the left, are green, violet and red. The green and violet bands therefore form the number 57. The third band is not an additional significant digit.

Step 2: Apply the multiplier The red band multiplies the two-digit number by one hundred:

$$R = 57 \times 10^2 \Omega = 5700 \Omega.$$

Since $1000 \Omega = 1 \text{ k}\Omega$, this can also be expressed in kilohms. No separate tolerance band is shown; its absence conventionally indicates a tolerance of $\pm 20\%$.

Step 3: Conclusion The resistance is $5.7 \text{ k}\Omega$ (with $\pm 20\%$ tolerance for the three-band code shown).

Quick Tip to Solve: Read green-violet as 57, then use red to append two zeros; the multiplier band does not make the number 572.

3. State the condition under which a large magnification can be achieved in an astronomical telescope.

[1 mark]

Solution:

Concept:

- An astronomical telescope first forms a real image with its objective; the eyepiece magnifies the angle subtended by this image.
- In normal adjustment, its angular magnifying power has magnitude $|M| = f_o/f_e$, where f_o and f_e are the focal lengths of the objective and eyepiece.

Step 1: Identify the controlling ratio Magnification depends on the ratio of the objective focal length to the eyepiece focal length. The negative sign sometimes written in $M = -f_o/f_e$ indicates image

Quick Tip to Solve: Remember the telescope ratio as objective over eyepiece: increase the numerator and decrease the denominator for larger magnification.

inversion and does not reduce the magnitude of magnification.

Step 2: Make the ratio large Choose an objective of long focal length and an eyepiece of short focal length. This makes the ratio f_o/f_e much greater than unity. Increasing the objective diameter mainly improves light collection and resolving power, rather than this focal-length ratio.

Step 3: Conclusion Large magnification requires $f_o \gg f_e$: a long-focal-length objective and a short-focal-length eyepiece.

Quick Tip to Solve: Remember the telescope ratio as objective over eyepiece: increase the numerator and decrease the denominator for larger magnification.

OR

3. How does the angle of minimum deviation of a glass prism vary if the incident violet light is replaced by red light? [1 mark]

Solution:

Concept:

- Under normal dispersion in glass, the refractive index is greater for violet light than for red light: $\mu_v > \mu_r$.
- For a prism of angle A , minimum deviation δ_m obeys $\mu = \sin[(A + \delta_m)/2] / \sin(A/2)$.

Step 1: Compare the refractive indices Replacing violet light by red light increases the wavelength. In the visible region, the refractive index of glass decreases as wavelength increases, so red light is refracted less strongly than violet light.

Step 2: Relate refractive index to minimum deviation The prism angle remains fixed. In the physical range of minimum-deviation angles, the prism formula shows that a smaller refractive index corresponds to a smaller angle of minimum deviation. Therefore changing the colour changes the minimum deviation even though the prism itself is unchanged.

Step 3: Conclusion The angle of minimum deviation decreases when violet light is replaced by red light: $\delta_{m,r} < \delta_{m,v}$.

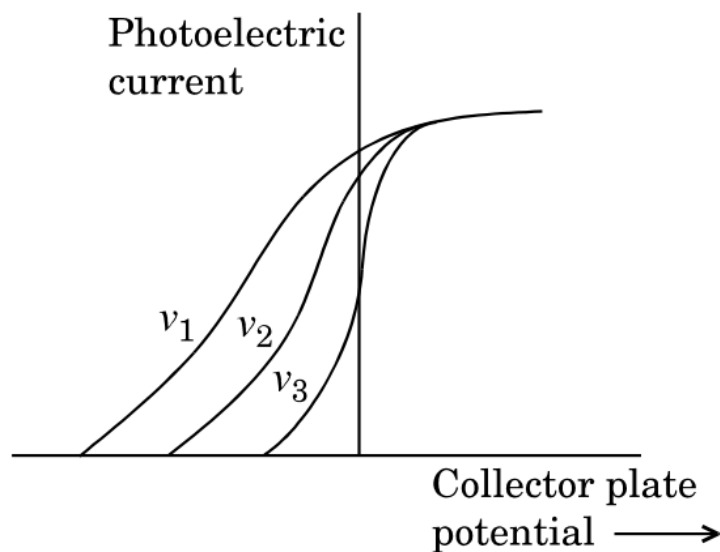
Quick Tip to Solve: For a glass prism, remember that violet deviates more and red deviates less; the same order holds for minimum deviation.

4. On the basis of the graphs shown in the figure, answer the following questions:

(a) Which physical parameter is kept constant for the three curves?

(b) Which is the highest frequency among ν_1 , ν_2 and ν_3 ?

[1 mark]



Solution:

Concept:

- In the standard photoelectric-current graph, saturation current indicates the intensity of the incident light for a given emitting surface.
- Einstein's photoelectric equation is $eV_0 = h\nu - \phi$, where V_0 is the magnitude of the stopping potential, ν the light frequency and ϕ the work function.

Step 1: Compare the saturation currents for part (a) All three curves approach the same saturation current at sufficiently positive collector potential. In the usual CBSE interpretation, this shows that the intensity of the incident radiation has been kept constant while its frequency is changed.

Step 2: Compare stopping potentials for part (b) The curve labelled ν_1 meets the horizontal axis farthest to the left. Thus it requires the largest magnitude of negative collector potential to stop the fastest photoelectrons. For the same metal, $eV_0 = h\nu - \phi$ then gives the greatest frequency for this curve. The graph implies $\nu_1 > \nu_2 > \nu_3$.

Step 3: Conclusion (a) The intensity of the incident light is kept constant. (b) The highest frequency is ν_1 .

Quick Tip to Solve: Use the plateau height to compare intensity and the magnitude of the negative-axis intercept to compare frequency.

5. Define amplitude modulation in communication system.

[1 mark]

Solution:

Concept:

- A carrier is a high-frequency wave used to carry a low-frequency information or message signal during transmission.
- Modulation varies a suitable parameter of the carrier in accordance with the instantaneous value of the message signal.

Step 1: Identify the parameter being varied In amplitude modulation, the amplitude of the carrier is varied in accordance with the instantaneous amplitude of the modulating signal. Thus the envelope of the modulated wave follows the message waveform.

Step 2: Specify what remains fixed The carrier frequency and phase remain unchanged in ideal amplitude modulation. For a sinusoidal message, a typical expression is

$$s(t) = A_c[1 + \mu \cos(\omega_m t)] \cos(\omega_c t),$$

where A_c is carrier amplitude, μ is modulation index, and ω_m and ω_c are message and carrier angular frequencies.

Step 3: Conclusion Amplitude modulation is the process of varying the amplitude of a high-frequency carrier wave in accordance with the instantaneous amplitude of the message signal, keeping the carrier frequency and phase unchanged.

Quick Tip to Solve: In the definition, name the varied carrier parameter explicitly: AM varies amplitude, while the message controls the envelope.

Section B (14 marks)

6. Five point charges, each of charge $+q$ are placed on five vertices of a regular hexagon of side l . Find the magnitude of the resultant force on a charge $-q$ placed at the centre of the hexagon. [2 marks]

Solution:

Concept:

- Superposition gives the net force as the vector sum of the individual electrostatic forces.
- In a regular hexagon, the distance from its centre to each vertex equals its side l . Coulomb's law gives the force magnitude $F_0 = kq^2/l^2$, where $k = 1/(4\pi\epsilon_0)$.

Step 1: Complete the symmetric arrangement Imagine placing an additional charge $+q$ at the empty sixth vertex. The force on the central negative charge due to each vertex is attractive and directed towards that vertex. Every force is cancelled by the equal force from the opposite vertex, so the vector sum of all six forces is zero.

Step 2: Remove the contribution of the sixth charge Let $\vec{F}_6$ denote the force due to the imagined

Quick Tip to Solve: Complete the regular polygon with an imaginary identical charge; the actual resultant is the negative of that missing charge's force.

charge and $\vec{F}_5$ the resultant due to the five actual charges. Then

$$\vec{F}_5 + \vec{F}_6 = 0, \quad \vec{F}_5 = -\vec{F}_6.$$

Thus the five-charge resultant has the same magnitude as one vertex force. Its direction is opposite to the empty vertex, along the radius towards the occupied vertex directly opposite it. The five magnitudes must not be added as scalars because their directions differ.

Step 3: Conclusion The required magnitude is $F = \frac{1}{4\pi\epsilon_0} \frac{q^2}{l^2}$, directed away from the empty vertex.

Quick Tip to Solve: Complete the regular polygon with an imaginary identical charge; the actual resultant is the negative of that missing charge's force.

OR

6. A simple pendulum consists of a small sphere of mass m suspended by a thread of length l . The sphere carries a positive charge q . The pendulum is placed in a uniform electric field of strength E directed vertically downwards. Find the period of oscillation of the pendulum due to the electrostatic force acting on the sphere, neglecting the effect of the gravitational force. [2 marks]

Solution:

Concept:

- A positive charge q in a uniform electric field E experiences a force qE in the field direction.
- Small angular oscillations satisfy $\ddot{\theta} + \omega^2\theta = 0$, with period $T = 2\pi/\omega$. Here θ is displacement from the downward vertical.

Step 1: Find the restoring torque Gravity is neglected as instructed, so the only constant downward force is the electric force qE . The thread tension acts along the thread and has zero torque about the support. At an angular displacement θ , the electric force produces a restoring torque

$$\tau = -qEl \sin \theta.$$

The negative sign shows that the torque brings the sphere back towards its vertical equilibrium position.

Step 2: Apply the small-angle approximation Treating the small sphere as a point mass gives its moment of inertia about the support as ml^2 . Therefore,

$$ml^2\ddot{\theta} = -qEl \sin \theta.$$

For small oscillations, $\sin \theta \simeq \theta$, so

$$\ddot{\theta} + \frac{qE}{ml}\theta = 0, \quad \omega = \sqrt{\frac{qE}{ml}}.$$

Equivalently, the electric force supplies an effective downward acceleration qE/m ; no gravitational

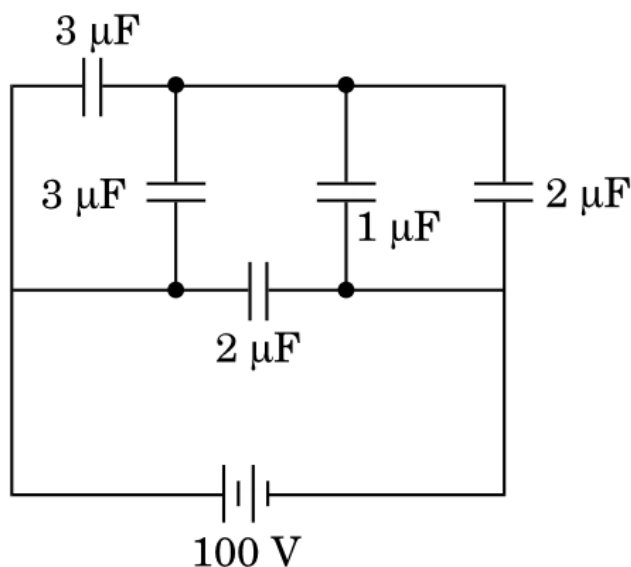
Quick Tip to Solve: Replace the usual pendulum acceleration g by qE/m when gravity is neglected and the electric field pulls the positive bob downwards.

acceleration is added.

Step 3: Conclusion The period of small oscillations is $T = 2\pi\sqrt{\frac{ml}{qE}}$.

Quick Tip to Solve: Replace the usual pendulum acceleration g by qE/m when gravity is neglected and the electric field pulls the positive bob downwards.

7. The figure shows a network of five capacitors connected to a 100 V supply. Calculate the total energy stored in the network. [2 marks]



Solution:

Concept:

- Parallel capacitors share both nodes, so their capacitances add. For two capacitors in series, $C_s = C_1 C_2 / (C_1 + C_2)$.
- The total electrostatic energy is $U = \frac{1}{2} C_{eq} V^2$, where C_{eq} is the equivalent capacitance across the supply voltage V .

Step 1: Identify the parallel pairs by their nodes Call the left supply node A , the upper central node B , and the right supply node C . The upper-left and left vertical capacitors, each $3\ \mu\text{F}$, both connect A to B . The $1\ \mu\text{F}$ and right vertical $2\ \mu\text{F}$ capacitors both connect B to C . Hence

$$C_{AB} = 3 + 3 = 6\ \mu\text{F}, \quad C_{BC} = 1 + 2 = 3\ \mu\text{F}.$$

Step 2: Reduce the network and calculate the energy The two equivalent groups form a series branch between A and C . This branch is in parallel with the middle horizontal $2\ \mu\text{F}$ capacitor, which is directly across the supply. Therefore,

$$C_s = \frac{6 \times 3}{6 + 3} = 2\ \mu\text{F}, \quad C_{eq} = 2 + 2 = 4\ \mu\text{F}.$$

Quick Tip to Solve: Follow continuous wires to identify shared nodes: the two $3\ \mu\text{F}$ capacitors are parallel despite being drawn perpendicular to each other.

With $V = 100\text{ V}$, convert microfarads to farads before substituting:

$$U = \frac{1}{2}(4 \times 10^{-6})(100)^2 \text{ J} = 2.0 \times 10^{-2} \text{ J}.$$

Step 3: Conclusion The total energy stored in the five-capacitor network is $\boxed{0.020 \text{ J}}$.

Quick Tip to Solve: Follow continuous wires to identify shared nodes: the two $3\text{ }\mu\text{F}$ capacitors are parallel despite being drawn perpendicular to each other.

8. A 0.5 m long solenoid of 10 turns/cm has area of cross-section 1 cm^2 . Calculate the voltage induced across its ends if the current in the solenoid is changed from 1 A to 2 A in 0.1 s . [2 marks]

Solution:

Concept:

- The inductance of a long air-core solenoid is $L = \mu_0 n^2 Al$, where n is turns per unit length, A its cross-sectional area, and l its length.
- Faraday's law for self-induction gives $\varepsilon = -L \Delta I / \Delta t$. The minus sign expresses opposition to the change in current.

Step 1: Convert the data and calculate the inductance The number of turns per metre is $n = 10 \times 100 = 1000\text{ m}^{-1}$, and the area is $A = 1\text{ cm}^2 = 10^{-4}\text{ m}^2$. With $l = 0.5\text{ m}$ and $\mu_0 = 4\pi \times 10^{-7}\text{ H m}^{-1}$,

$$\begin{aligned} L &= (4\pi \times 10^{-7})(1000)^2(10^{-4})(0.5)\text{ H} \\ &= 2\pi \times 10^{-5}\text{ H}. \end{aligned}$$

Here the given turn density is converted to SI units before it is squared.

Step 2: Apply the rate of change of current The current increases by $2 - 1 = 1\text{ A}$ in 0.1 s , so

$$\frac{\Delta I}{\Delta t} = 10\text{ A s}^{-1}.$$

The magnitude of the induced voltage is

$$|\varepsilon| = (2\pi \times 10^{-5})(10) = 2\pi \times 10^{-4}\text{ V}.$$

This is the average emf over the interval, and is constant if the current increases uniformly. Its polarity opposes the increase of current.

Step 3: Conclusion The induced voltage magnitude is $\boxed{6.28 \times 10^{-4}\text{ V}}$, or 0.628 mV .

Quick Tip to Solve: Convert turns per centimetre to turns per metre and square the converted value; also use $1\text{ cm}^2 = 10^{-4}\text{ m}^2$.

8. A small flat search coil of area 5 cm^2 with 140 closely wound turns is placed between the poles of a powerful magnet producing magnetic field 0.09 T and then quickly removed out of the field region. Calculate

(a) change of magnetic flux through the coil, and

(b) emf induced in the coil.

[2 marks]

Solution:

Concept:

- Magnetic flux through one turn is $\Phi = BA \cos \theta$, where θ is the angle between the field and the coil's normal. Flux linkage for N turns is $N\Phi$.
- Faraday's law gives the average induced emf $\bar{\epsilon} = -N\Delta\Phi/\Delta t$, where Δt is the time taken to remove the coil.

Step 1: Calculate the change of flux and flux linkage The printed question specifies neither orientation nor removal time. Under the usual intended arrangement with the plane of the coil perpendicular to the field, $\theta = 0$. Convert the area: $A = 5 \times 10^{-4} \text{ m}^2$. The final flux is zero outside the field. Taking the initial flux as positive,

$$\Delta\Phi = 0 - BA = -(0.09)(5 \times 10^{-4}) = -4.5 \times 10^{-5} \text{ Wb}.$$

Thus the flux decreases by $4.5 \times 10^{-5} \text{ Wb}$ per turn. For all 140 turns, the change in flux linkage is

$$N\Delta\Phi = 140(-4.5 \times 10^{-5}) = -6.3 \times 10^{-3} \text{ Wb-turn}.$$

Step 2: Express the induced emf using the removal time If the coil takes Δt seconds to leave the field, Faraday's law gives

$$|\bar{\epsilon}| = \frac{6.3 \times 10^{-3}}{\Delta t} \text{ V}.$$

The word "quickly" supplies no numerical time, so a numerical emf cannot be determined from the printed data. The induced current, if the coil forms a closed circuit, produces a field opposing the loss of the original magnetic flux. For an arbitrary initial orientation, both flux changes and the emf magnitude above acquire a factor $|\cos \theta|$.

Step 3: Conclusion For normal incidence of the field, (a) the flux decreases by $4.5 \times 10^{-5} \text{ Wb}$ per turn, or the flux linkage by $6.3 \times 10^{-3} \text{ Wb-turn}$; (b) $|\bar{\epsilon}| = 6.3 \times 10^{-3}/\Delta t \text{ V}$. A numerical emf requires the missing removal time.

Quick Tip to Solve: Keep per-turn flux separate from flux linkage; multiply by N once, then divide by the actual removal time to obtain an emf.

9. For paraxial rays, show that the focal length of a spherical mirror is one-half of its radius of curvature. [2 marks]

Solution:

Concept:

- The normal to a spherical mirror at any point M passes through its centre of curvature C ; the angle of incidence equals the angle of reflection.
- For paraxial rays the height h above the principal axis is small compared with the radius R , so $\sin \theta \simeq \tan \theta \simeq \theta$ when angles are in radians.

Step 1: Relate the ray angles to the mirror geometry Consider a concave mirror with pole P . A ray parallel to the principal axis strikes the mirror at M , at height h , and reflects through the principal focus F . Let the normal MC make angle θ with the principal axis. The incident ray is parallel to that axis, so the incidence angle is θ ; reflection makes the reflected ray inclined at 2θ to the axis. If H is the perpendicular projection of M on the axis, then

$$\sin \theta = \frac{h}{R}, \quad \tan(2\theta) = \frac{h}{FH}.$$

Step 2: Apply the paraxial approximation For a narrow aperture, H lies very close to P , so $FH \simeq FP = f$, where f and R here denote magnitudes. Using the small-angle approximations gives

$$\theta \simeq \frac{h}{R}, \quad 2\theta \simeq \frac{h}{f}.$$

Dividing the second relation by the first yields $2 \simeq R/f$. The same geometry applies to a convex mirror using backward extensions of reflected rays and its virtual focus.

Step 3: Conclusion For paraxial rays, $f = R/2$. With the Cartesian sign convention, f and R have the same sign for either kind of spherical mirror, so this relation also holds for signed distances.

Quick Tip to Solve: Use the normal through the centre of curvature: its angle is doubled in the reflected ray, giving the factor of two in $R = 2f$.

10. Obtain Bohr's quantisation condition for angular momentum of electron orbiting in n^{th} orbit in hydrogen atom on the basis of the wave picture of an electron using de Broglie hypothesis. [2 marks]

Solution:

Concept:

- De Broglie's hypothesis assigns wavelength $\lambda = h/p = h/(m_e v)$ to an electron of momentum $p = m_e v$, where h is Planck's constant.
- A stable circular orbit must accommodate a whole number of electron wavelengths so that the matter wave joins smoothly after each circuit and forms a standing wave.

Step 1: Impose the standing-wave condition Let the electron move with speed v_n in the n^{th} circular orbit of radius r_n . After one complete circuit, the matter wave must return with the same phase.

Quick Tip to Solve: Start with circumference $= n\lambda$, then replace λ by h/p ; the product pr is the orbital angular momentum.

Therefore the circumference must equal an integral number of wavelengths:

$$2\pi r_n = n\lambda_n, \quad n = 1, 2, 3, \dots$$

A nonintegral number would give a phase mismatch between successive circuits and would not sustain the standing-wave pattern required for an allowed orbit.

Step 2: Substitute the de Broglie wavelength In the nonrelativistic Bohr model, $\lambda_n = h/(m_e v_n)$. Hence the circumference condition becomes

$$2\pi r_n = \frac{nh}{m_e v_n}.$$

Multiplying by $m_e v_n$ and dividing by 2π gives

$$m_e v_n r_n = \frac{nh}{2\pi}.$$

Since the velocity is perpendicular to the radius in a circular orbit, the magnitude of the electron's orbital angular momentum is $L_n = m_e v_n r_n$.

Step 3: Conclusion Bohr's angular-momentum quantisation condition is

$$L_n = m_e v_n r_n = n\hbar = \frac{nh}{2\pi}.$$

Here $n = 1, 2, 3, \dots$ and $\hbar = h/(2\pi)$.

Quick Tip to Solve: Start with circumference $= n\lambda$, then replace λ by h/p ; the product pr is the orbital angular momentum.

11. Plot a graph showing the variation of undecayed nuclei N versus time t . From the graph, find out how one can determine the half-life and average life of the radioactive nuclei. [2 marks]

Solution:

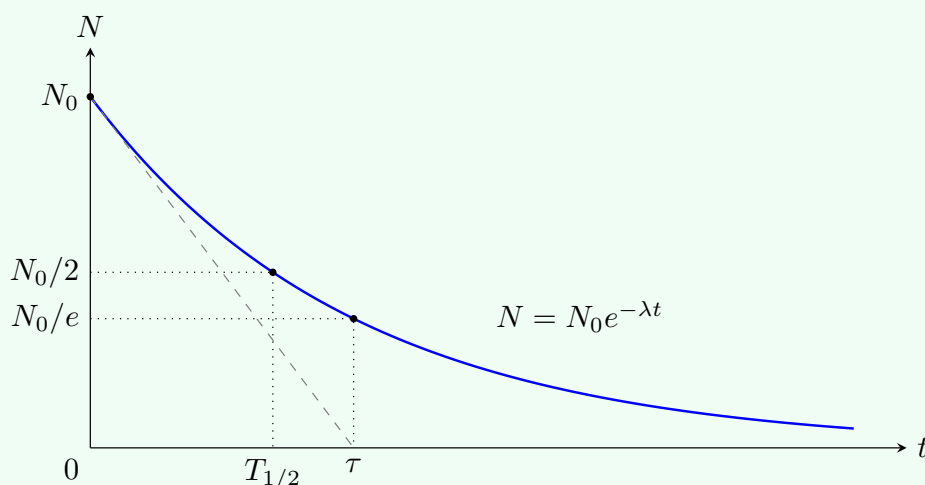
Concept:

- Radioactive decay follows $N(t) = N_0 e^{-\lambda t}$, where N_0 is the initial number of undecayed nuclei and λ is the decay constant.
- Half-life $T_{1/2}$ is the time for N to fall to $N_0/2$; mean or average life is $\tau = 1/\lambda$.

Step 1: Plot the decay curve and read the half-life Put time t on the horizontal axis and the number of undecayed nuclei N on the vertical axis. The curve begins at N_0 , falls exponentially, and approaches the time axis asymptotically. Draw a horizontal line from $N_0/2$ to the curve and drop a perpendicular to the time axis. The corresponding time is the half-life:

$$\frac{N_0}{2} = N_0 e^{-\lambda T_{1/2}} \Rightarrow T_{1/2} = \frac{\ln 2}{\lambda}.$$

Quick Tip to Solve: Read half-life at half the initial height; read mean life at N_0/e , or use the initial tangent's intercept on the time axis.



Exponential decay with half-life and mean life indicated; the dashed sloping line is the initial tangent.

Step 2: Determine the average life from the graph Draw the tangent to the curve at $t = 0$. Its slope is

$$\left. \frac{dN}{dt} \right|_{t=0} = -\lambda N_0,$$

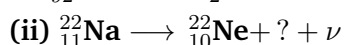
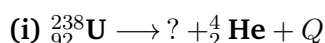
so its equation is $N = N_0 - \lambda N_0 t$. Setting $N = 0$ shows that this tangent cuts the time axis at $t = 1/\lambda = \tau$. Equivalently, read the time corresponding to the curve's height N_0/e , since $N(\tau) = N_0 e^{-1}$. This is a graphical way to obtain the mean lifetime, not the time at which all nuclei have decayed.

Step 3: Conclusion The graph gives $T_{1/2} = \ln 2 / \lambda$ at $N = N_0/2$, and $\tau = 1/\lambda = T_{1/2} / \ln 2$ from the initial tangent's time intercept or the point $N = N_0/e$.

Quick Tip to Solve: Read half-life at half the initial height; read mean life at N_0/e , or use the initial tangent's intercept on the time axis.

12. (a) Write two distinguishing features of nuclear forces.

(b) Complete the following nuclear reactions for α and β decay:



[2 marks]

Solution:

Concept:

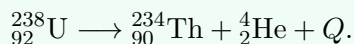
- Nuclear forces bind protons and neutrons inside the nucleus. Their short range and approximate charge independence distinguish them from the electrostatic force.
- In a nuclear reaction, the total mass number A and total atomic number Z are conserved. An alpha particle is ${}_2^4\text{He}$, while a positron is ${}_{+1}^0e$.

Step 1: State two features of nuclear forces (a) First, nuclear forces are very short-ranged: they are appreciable only over distances of the order of a few femtometres ($1 \text{ fm} = 10^{-15} \text{ m}$) and become

Quick Tip to Solve: Subtract the emitted particle's A and Z separately; unchanged A with the daughter's Z reduced by one identifies beta-plus decay.

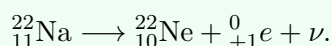
negligible at larger separations. Second, they are approximately charge independent: after excluding electrostatic effects, the nuclear interaction is nearly the same for proton–proton, neutron–neutron and proton–neutron pairs in comparable states.

Step 2: Balance the alpha-decay reaction (b)(i) The emitted alpha particle carries mass number 4 and atomic number 2. Thus the daughter has $A = 238 - 4 = 234$ and $Z = 92 - 2 = 90$, which identifies thorium:



Both totals are conserved: $234 + 4 = 238$ and $90 + 2 = 92$. The symbol Q denotes the energy released.

Step 3: Identify the particle in beta decay (b)(ii) Sodium changes into neon without changing mass number, while its atomic number decreases from 11 to 10. Therefore the missing particle has mass number 0 and charge number +1: it is a positron, so this is beta-plus decay.



Here ν is the emitted electron neutrino. Charge balance is $11 = 10 + 1$; an electron with charge number -1 would not balance the reaction.

Step 4: Conclusion Nuclear forces are short-ranged and approximately charge independent. The missing products are ${}_{90}^{234}\text{Th}$ in (i) and the positron ${}_{+1}^0e$ in (ii).

Quick Tip to Solve: Subtract the emitted particle's A and Z separately; unchanged A with the daughter's Z reduced by one identifies beta-plus decay.

Section C (36 marks)

13. (a) Explain briefly, using a proper diagram, the difference in behaviour of a conductor and a dielectric in the presence of external electric field.

(b) Define the term polarization of a dielectric and write the expression for a linear isotropic dielectric in terms of electric field. [3 marks]

Solution:

Concept:

- Free charges in a conductor redistribute until the net internal electric field is zero in electrostatic equilibrium.
- Bound charges in a dielectric form induced dipoles or align existing dipoles in an applied field.
- The polarization vector $\vec{P}$ is electric dipole moment per unit volume; its SI unit is C m^{-2} .

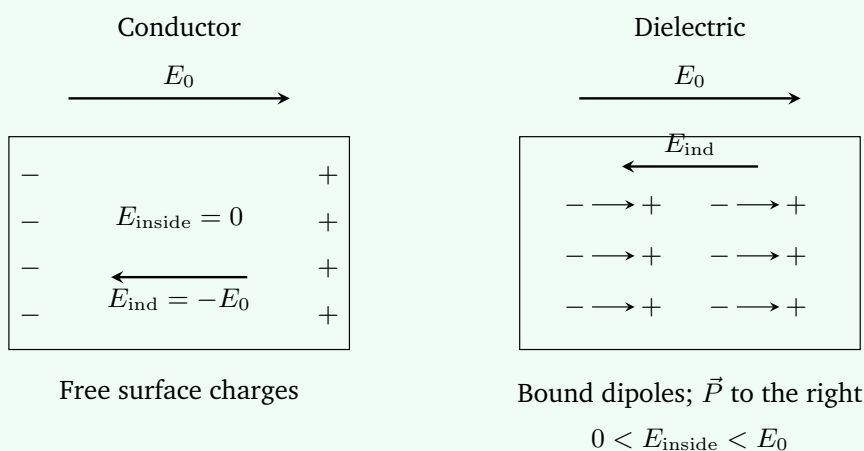
Step 1: (a) Describe the response of a conductor With $\vec{E}_0$ to the right, free electrons move to the left surface, leaving positive charge on the right surface. These charges produce a leftward induced field. Redistribution stops when

$$\vec{E}_{\text{inside}} = \vec{E}_0 + \vec{E}_{\text{ind}} = 0.$$

Thus the conductor becomes an equipotential; mobile charges on its surface produce the cancellation.

Quick Tip to Solve: Draw negative charge on the face where the applied field enters and positive charge where it leaves; use the resultant internal field in the polarization formula.

Step 2: (a) Describe the response of a dielectric In a dielectric, charges are bound: positive and negative charge centres separate slightly, or permanent dipoles partially align with the field. Polarization produces negative bound charge on the left face and positive bound charge on the right. The opposing induced field reduces, but does not cancel, the internal field.



Charge redistribution in a conductor and polarization of a dielectric in an applied field directed to the right.

Step 3: (b) Define polarization and give the constitutive relation Polarization is the net electric dipole moment per unit volume. For a linear isotropic dielectric, it is proportional to the resultant internal field $\vec{E}$:

$$\vec{P} = \frac{\sum_i \vec{p}_i}{\Delta V}, \quad \vec{P} = \epsilon_0 \chi_e \vec{E}.$$

Here $\vec{p}_i$ are the molecular dipole moments in volume ΔV , χ_e is dimensionless electric susceptibility, and ϵ_0 is vacuum permittivity. Thus $\vec{P}$ is parallel to $\vec{E}$.

Step 4: Conclusion A conductor has zero internal electrostatic field; a dielectric retains a reduced field, with polarization $\vec{P} = \epsilon_0 \chi_e \vec{E}$.

Quick Tip to Solve: Draw negative charge on the face where the applied field enters and positive charge where it leaves; use the resultant internal field in the polarization formula.

14. Twelve wires each having a resistance of $3\ \Omega$ are connected to form a cubical network. A battery of 10 V and negligible internal resistance is connected across the diagonally opposite corners of this network. Determine its equivalent resistance and the current along each edge of the cube. [3 marks]

Solution:

Concept:

- Ohm's law gives $I = V/R$. Resistances in series add, while n equal resistors of resistance r in parallel have equivalent resistance r/n .
- Symmetry about a cube's body diagonal makes the three vertices adjacent to either end of that diagonal equipotential. Identical branches between equal-potential groups carry equal currents.
- Kirchhoff's junction rule states that total current entering a vertex equals total current leaving it.

Quick Tip to Solve: For opposite corners of a cube, count edges in three groups: 3, 6, 3. Divide the edge resistance by these counts, then add the three section resistances.

Step 1: Group the vertices by symmetry Denote the positive-terminal corner by A and its diagonally opposite, negative-terminal corner by G . Let the three corners directly connected to A form group X , and the three directly connected to G form group Y . A rotation about the body diagonal AG interchanges the three members of each group without changing the battery connections. Therefore all vertices in X have one potential, and all vertices in Y have another. Joining vertices within each equipotential group would carry no additional current and leaves the network unchanged electrically.

Step 2: Reduce the network to three series sections There are three edges from A to group X , six edges connecting X to Y , and three edges from Y to G . All twelve resistors are thereby included. Within each section the edges are in parallel after the equipotential reduction. With each edge resistance $r = 3\ \Omega$,

$$R_{AX} = \frac{r}{3} = 1\ \Omega, \quad R_{XY} = \frac{r}{6} = 0.5\ \Omega, \quad R_{YG} = \frac{r}{3} = 1\ \Omega.$$

These three sections are in series, so

$$R_{\text{eq}} = \frac{r}{3} + \frac{r}{6} + \frac{r}{3} = \frac{5r}{6} = \frac{5(3)}{6} = 2.5\ \Omega.$$

Step 3: Determine the current in each set of edges The battery supplies a total current

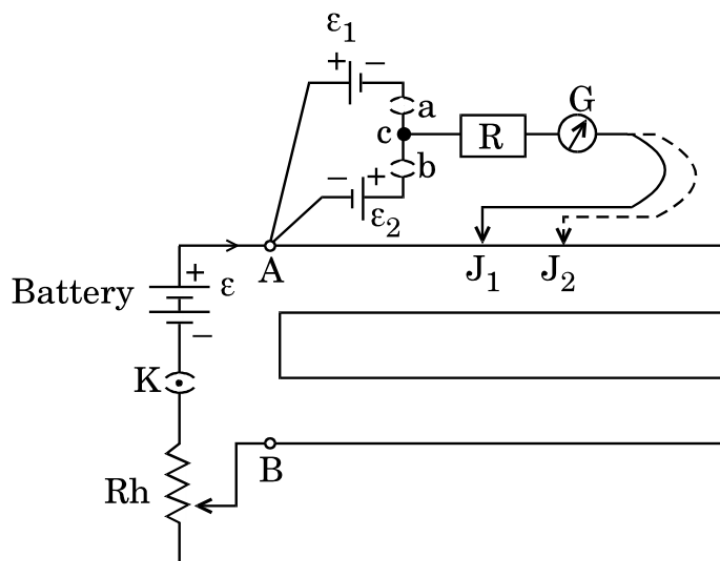
$$I = \frac{10\text{ V}}{2.5\ \Omega} = 4\text{ A}.$$

This divides equally among the three edges leaving A , giving $I/3 = 4/3\text{ A}$ in each. At each vertex of X , this current divides equally between the two outgoing edges to Y ; each of these six middle edges therefore carries $I/6 = 2/3\text{ A}$. At each vertex of Y , two middle-edge currents combine, giving $4/3\text{ A}$ in each of the three edges entering G . As a check, the potential drops in the successive sections are 4 V , 2 V , and 4 V , which add to the battery voltage.

Step 4: Conclusion The equivalent resistance is $2.5\ \Omega$. Each of the six edges meeting either battery corner carries $4/3\text{ A}$, while each of the other six edges carries $2/3\text{ A}$. Conventional current flows from the positive corner through X and Y to the negative corner.

Quick Tip to Solve: For opposite corners of a cube, count edges in three groups: 3, 6, 3. Divide the edge resistance by these counts, then add the three section resistances.

15. A student uses the circuit diagram of a potentiometer as shown in the figure. (a) For a steady current I passing through the potentiometer wire, he gets a null point for the cell ε_1 and not for ε_2 . Give reason for this observation and suggest how this difficulty can be resolved. (b) What is the function of resistance R used in the circuit? How will the change in its value affect the null point? (c) How can the sensitivity of the potentiometer be increased? [3 marks]



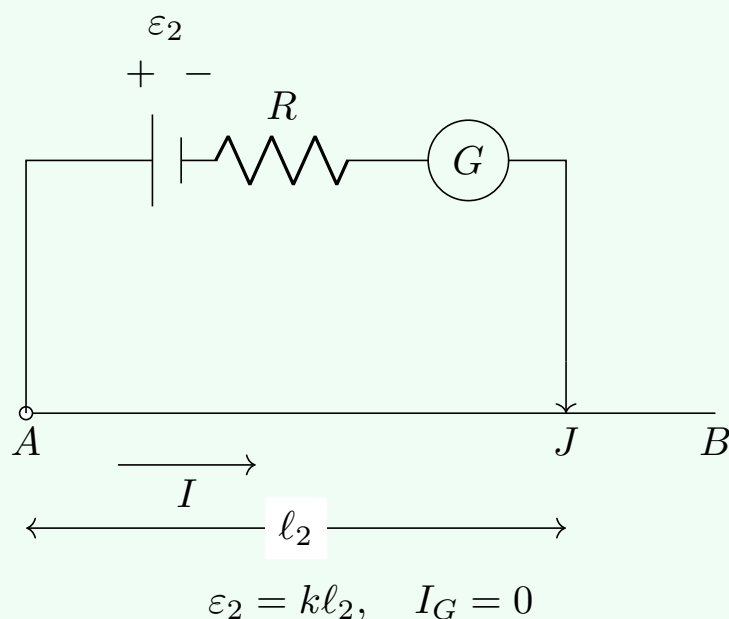
Solution:

Concept:

- A potentiometer balances a cell emf against the potential drop along a uniform wire: $\varepsilon = k\ell$, where k is potential gradient and ℓ is balancing length.
- The cell's positive terminal must connect to the high-potential end A . At balance, the galvanometer current is zero.
- Smaller k gives a larger change in balance length for the same small emf change: $\Delta\ell = \Delta\varepsilon/k$.

Step 1: (a) Inspect the cell polarities The driver sends current from A towards B , so $V_A > V_J$ for a jockey contact J farther along the wire. The positive terminal of ε_1 is connected to A , allowing its emf to oppose the wire's potential drop. In the printed circuit, however, the negative terminal of ε_2 is connected to A . Its positive terminal is therefore at $V_A + \varepsilon_2$, whereas the wire contact is at $V_A - k\ell$. The detector would have to balance a positive difference $\varepsilon_2 + k\ell$, which cannot vanish. Reverse the connections of ε_2 , joining its positive terminal to A . It can then balance at $\ell_2 = \varepsilon_2/k$, provided this length lies within the wire. If necessary, adjust the driving current so the total wire drop exceeds the cell emf.

Quick Tip to Solve: Trace the cell's positive terminal to end A before discussing emf magnitude; distinguish detector resistance R from the primary rheostat.



Correct balancing polarity for ε_2 ; the primary circuit maintains $V_A > V_B$.

Step 2: (b) Explain the series resistance Resistance R is in the galvanometer branch. It limits current away from balance and protects the galvanometer. It is not the rheostat R_h that sets the primary-wire current. At the null point $I_G = 0$, hence the voltage across R is $I_G R = 0$. Changing R therefore does not shift the true null position, although a large R reduces the visible deflection away from null.

Step 3: (c) Reduce the potential gradient Increase the sensitivity by reducing $k = V_{AB}/L$, while retaining an accessible balance point. For the same uniform wire, $k = I\rho/A_w$, where ρ is resistivity and A_w its cross-sectional area. Increasing the primary rheostat resistance reduces I , and hence k . Alternatively, use a longer potentiometer wire while keeping the same total potential difference across it. A small emf difference then corresponds to a more easily measured length difference.

Step 4: Conclusion Reverse ε_2 to obtain the correct balancing polarity. The protective resistance R does not change the null point. A smaller potential gradient increases potentiometer sensitivity.

Quick Tip to Solve: Trace the cell's positive terminal to end A before discussing emf magnitude; distinguish detector resistance R from the primary rheostat.

16. (a) Show that the time period (T) of oscillations of a freely suspended magnetic dipole of magnetic moment (m) in a uniform magnetic field (B) is given by $T = 2\pi\sqrt{\frac{I}{mB}}$, where I is a moment of inertia of the magnetic dipole.

(b) Identify the following magnetic materials:

(i) A material having susceptibility (χ_m) = -0.00015 .

(ii) A material having susceptibility (χ_m) = 10^{-5} .

[3 marks]

Solution:

Concept:

- A magnetic dipole of moment m at angle θ to a uniform magnetic field B experiences torque of magnitude $mB \sin \theta$.
- Rotational dynamics gives $\tau = I\ddot{\theta}$, where I is the moment of inertia about the suspension axis. A restoring torque proportional to angular displacement produces simple harmonic motion.
- Magnetic susceptibility is defined by $M = \chi_m H$. Diamagnetic materials have small negative χ_m ; paramagnetic materials have small positive χ_m .

Step 1: (a) Write the restoring-torque equation In stable equilibrium the dipole moment is parallel to the field. Turn the dipole through a small angle θ , measured in radians, and release it. The magnetic torque acts towards the equilibrium orientation, so its sign is opposite to the displacement:

$$I \frac{d^2\theta}{dt^2} = -mB \sin \theta.$$

Take the suspension to exert negligible torsional torque and neglect damping. The moment of inertia is about the axis of oscillation.

Step 2: Use the small-angle approximation For small angular oscillations, $\sin \theta \simeq \theta$. Thus

$$\frac{d^2\theta}{dt^2} + \frac{mB}{I}\theta = 0.$$

Compare this with the standard simple harmonic equation $\ddot{\theta} + \omega^2\theta = 0$, where ω is the angular frequency. This gives $\omega = \sqrt{mB/I}$. Since one oscillation corresponds to phase change 2π , the time period is

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{I}{mB}}.$$

The small-angle condition is essential for this amplitude-independent expression; at large angular amplitudes the exact sine torque is not linear.

Step 3: (b) Classify each susceptibility (i) For $\chi_m = -0.00015 = -1.5 \times 10^{-4}$, the susceptibility is small and negative. The induced magnetisation opposes the applied field, identifying a diamagnetic material.

(ii) For $\chi_m = 10^{-5}$, the susceptibility is small and positive. The magnetisation is weakly along the applied field, identifying a paramagnetic material. A small positive value here should not be confused with the very large positive susceptibility typical of a ferromagnetic material.

Step 4: Conclusion For small oscillations, $T = 2\pi\sqrt{I/(mB)}$. Material (i) is diamagnetic and material

Quick Tip to Solve: Put the minus sign in the torque first, then compare with the SHM equation; for susceptibility, inspect both its sign and its magnitude.

(ii) is paramagnetic.

Quick Tip to Solve: Put the minus sign in the torque first, then compare with the SHM equation; for susceptibility, inspect both its sign and its magnitude.

17. (a) How are eddy currents generated in a conductor which is subjected to a magnetic field?

(b) Write two examples of their useful applications.

(c) How can the disadvantages of eddy currents be minimized?

[3 marks]

Solution:

Concept:

- Faraday's law states that changing magnetic flux induces an emf: $\mathcal{E} = -d\Phi_B/dt$, where Φ_B is the magnetic flux through a closed path.
- A bulk conductor contains many closed conducting paths. Induced currents circulate in these paths and are called eddy currents; their direction follows Lenz's law.
- For an induced emf $\mathcal{E}$ and loop resistance R , the current is approximately $I = \mathcal{E}/R$, and resistive power loss is $P = \mathcal{E}^2/R$.

Step 1: (a) Describe how the circulating currents arise When the magnetic flux linked with regions inside a conductor changes, an emf is induced around closed paths within its body. The resulting currents circulate in loops, resembling eddies in water. Flux can change because the magnetic field varies with time or because the conductor moves through a non-uniform field, for example entering or leaving a field region. The induced current's magnetic effect opposes the flux change that produces it. A stationary conductor merely placed in a steady magnetic field does not sustain eddy currents, because its linked flux is then constant.

Step 2: (b) State two useful applications **Induction furnace:** An alternating magnetic field produces strong eddy currents in a metal charge. The currents generate Joule heat in the metal, raising its temperature and allowing it to melt without direct electrical contacts.

Electromagnetic damping: A conducting frame moving in a magnetic field develops eddy currents whose magnetic effect opposes its motion. In a moving-coil galvanometer this damping reduces oscillations, so the pointer settles rapidly at the required reading.

Step 3: (c) Reduce unwanted eddy-current losses Eddy currents in transformer and other AC machine cores cause unwanted heating and energy loss. Construct such cores from thin laminations insulated from one another instead of a solid metal block. The sheets are arranged parallel to the main magnetic flux so that the large transverse current loops are interrupted. Insulation increases the resistance of the unwanted loop paths and restricts their area, reducing the induced circulating currents and heating. High-resistivity core materials can further reduce these losses.

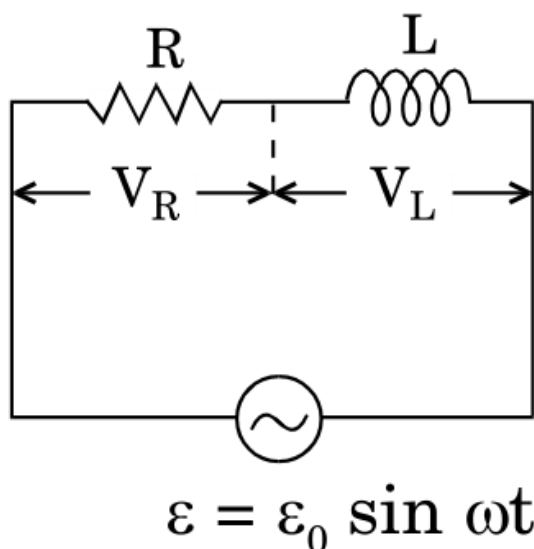
Step 4: Conclusion Changing linked magnetic flux produces eddy currents in a bulk conductor. They are useful for induction heating and electromagnetic damping; thin insulated core laminations reduce unwanted eddy-current heating.

Quick Tip to Solve: Begin with a change of magnetic flux, not just the presence of a magnetic field; laminations interrupt the large current loops responsible for heating.

18. (a) An ac circuit as shown in the figure has an inductor of inductance L and a resistor of resistance R connected in series. Using the phasor diagram, explain why the voltage in the circuit will lead the current in phase.

(b) The potential difference across the resistor is 160 V and that across the inductor is 120 V. Find the effective value of the applied voltage. If the effective current in the circuit be 1.0 A, calculate the total impedance of the circuit.

(c) What will be the potential difference in the circuit when direct current is passed through the circuit? [3 marks]



Solution:

Concept:

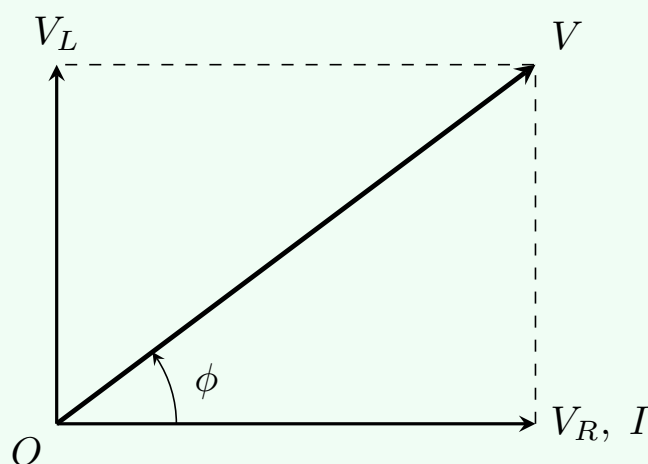
- In a series circuit the same current flows through both elements. The resistor voltage $V_R = IR$ is in phase with current, while the ideal-inductor voltage $V_L = IX_L$, where $X_L = \omega L$, leads current by 90° .
- Perpendicular voltage phasors give $V = \sqrt{V_R^2 + V_L^2}$; the impedance is $Z = V/I$. Here all voltage and current magnitudes are rms values.

Step 1: (a) Add the voltage phasors Take the current phasor along the positive horizontal direction. Draw V_R along it and V_L vertically upwards. The applied-voltage phasor is their vector sum, the diagonal shown below. It lies above the current reference, so the applied voltage leads current through an angle ϕ , with

$$\tan \phi = \frac{V_L}{V_R} = \frac{\omega L}{R} > 0.$$

The phase lead is between 0 and 90° ; the two component voltages must therefore not be added as ordinary scalar magnitudes.

Quick Tip to Solve: The voltage triangle is a scaled 3 : 4 : 5 triangle: 120 : 160 : 200. For steady dc, replace an ideal inductor by a connecting wire.



Series RL voltage phasors, with current as the reference.

Step 2: (b) Calculate the effective voltage and impedance Substitute the perpendicular rms voltage components:

$$V = \sqrt{160^2 + 120^2} = \sqrt{40000} = 200 \text{ V.}$$

With the effective current $I = 1.0 \text{ A}$,

$$Z = \frac{200}{1.0} = 200 \Omega.$$

Separately, $R = V_R/I = 160 \Omega$ and $X_L = V_L/I = 120 \Omega$, consistent with $Z = \sqrt{R^2 + X_L^2}$.

Step 3: (c) Use the steady direct-current limit For steady dc, $dI/dt = 0$, so the ideal inductor has no voltage drop: $V_L = L dI/dt = 0$. The entire circuit potential difference is then across the resistor,

$$V_{\text{dc}} = I_{\text{dc}}R = 160I_{\text{dc}} \text{ V,}$$

when I_{dc} is measured in amperes. For the same current of 1.0 A , this gives 160 V . The current must be specified to assign a numerical dc voltage; the quoted result uses the current given in part (b). Immediately after switching, a transient inductive voltage can exist.

Step 4: Conclusion The applied ac voltage leads the current; $V_{\text{rms}} = 200 \text{ V}$ and $Z = 200 \Omega$. For steady dc the inductor drop is zero and the circuit drop is IR , equal to 160 V for $I = 1.0 \text{ A}$.

Quick Tip to Solve: The voltage triangle is a scaled 3 : 4 : 5 triangle: 120 : 160 : 200. For steady dc, replace an ideal inductor by a connecting wire.

18. An ac circuit consists of a series combination of circuit elements X and Y . The current is ahead of the voltage in phase by $\frac{\pi}{4}$. If element X is a pure resistor of $100\ \Omega$,

(a) name the circuit element Y .

(b) calculate the rms value of current, if rms value of voltage is 141 V .

(c) what will happen if the ac source is replaced by a dc source?

[3 marks]

Solution:

Concept:

- Current leads the applied voltage in a series resistor-capacitor circuit. An inductor instead makes current lag, while a pure resistor produces no phase difference.
- For resistance R and capacitive reactance $X_C = 1/(\omega C)$, the current lead ϕ obeys $\tan \phi = X_C/R$. The impedance magnitude is $Z = \sqrt{R^2 + X_C^2}$, and $I_{\text{rms}} = V_{\text{rms}}/Z$.

Step 1: (a) Identify the second element from the phase lead The current is ahead of the total voltage by $\pi/4$. Since X is already a pure resistor, Y must provide a capacitive phase effect. Therefore Y is a capacitor. The resistor voltage is in phase with current and the capacitor voltage lags current by $\pi/2$; their resultant consequently lags the current.

Step 2: (b) Determine the reactance, impedance and rms current Use the magnitude of the current lead, $\phi = \pi/4$, together with $R = 100\ \Omega$:

$$X_C = R \tan \phi = 100 \tan(\pi/4) = 100\ \Omega.$$

Thus the resistance and reactance have equal magnitudes, but their voltage contributions are perpendicular:

$$Z = \sqrt{100^2 + 100^2} = 100\sqrt{2}\ \Omega.$$

For the given rms applied voltage,

$$I_{\text{rms}} = \frac{141}{100\sqrt{2}}\text{ A} = 0.997\text{ A} \simeq 1.0\text{ A}.$$

There is no additional factor of $\sqrt{2}$ to convert the supplied voltage, because it is already an rms value.

Step 3: (c) Replace the source by dc A charging current flows briefly on connecting the dc source. As the capacitor charges, the current decreases; once fully charged, its voltage equals the source voltage and the current becomes zero. Thus the capacitor behaves as an open circuit in the steady state, so no steady current flows through either element. This also agrees with the limiting expression $X_C = 1/(\omega C) \rightarrow \infty$ as the frequency approaches zero.

Step 4: Conclusion Y is a capacitor, $I_{\text{rms}} \simeq 1.0\text{ A}$, and replacing ac by dc gives only a transient charging current followed by zero steady current.

Quick Tip to Solve: A 45° current lead means $X_C = R$, so use $Z = R\sqrt{2}$. A series capacitor blocks steady dc after charging.

19. Name the radiation of the electromagnetic spectrum which is used for the following:

(a) Radar

(b) To photograph internal parts of human body

(c) For taking photographs of the sky during night and foggy conditions

Give the frequency range in each case.

[3 marks]

Solution:

Concept:

- Electromagnetic waves are classified by frequency ν or wavelength λ , related in vacuum by $c = \nu\lambda$, where $c = 3 \times 10^8 \text{ m s}^{-1}$.
- Different bands are useful because they interact differently with matter. Spectrum boundaries are approximate and can overlap in conventional classifications; higher frequency corresponds to shorter wavelength.

Step 1: (a) **Select the radiation used in radar** Radar uses **microwaves**, with an approximate frequency range of

$$10^9 \text{ to } 10^{12} \text{ Hz.}$$

Their short wavelengths allow a directional beam from a practical antenna. The waves reflected by a distant target return as echoes; the time delay can be used to determine distance. The stated interval describes the microwave band, rather than the operating frequency of every individual radar.

Step 2: (b) **Select the radiation used to image internal body parts** **X-rays** are used for radiographs of internal body structures. Their approximate frequency range is

$$10^{16} \text{ to } 10^{20} \text{ Hz.}$$

X-rays penetrate soft tissue relatively well, while bone absorbs them more strongly. This difference in absorption produces contrast on the detector, allowing internal structures to be photographed without exposing them directly.

Step 3: (c) **Select the radiation used for night and fog photography** **Infrared radiation** is used. A commonly quoted approximate frequency range is

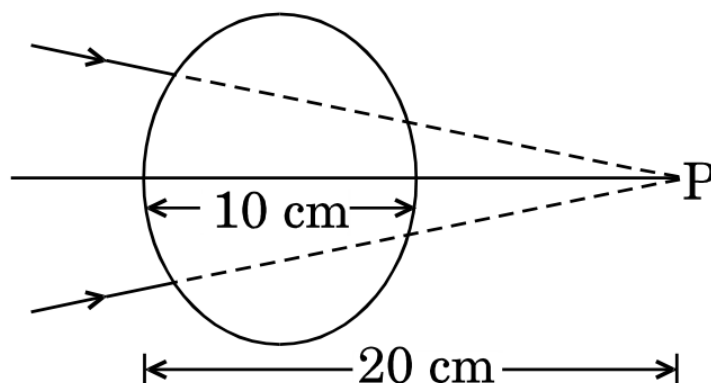
$$10^{11} \text{ to } 4 \times 10^{14} \text{ Hz.}$$

Infrared has wavelengths longer than visible light and is less strongly scattered in suitable atmospheric conditions, which makes it useful for photography through mist or haze. Infrared detectors can also register radiation from objects in darkness. The upper end of the band lies just below the red end of visible light; the range is conventionally quoted to order of magnitude.

Step 4: Conclusion (a) Radar: microwaves, approximately 10^9 – 10^{12} Hz. (b) Internal body imaging: X-rays, approximately 10^{16} – 10^{20} Hz. (c) Night and fog photography: infrared, approximately 10^{11} – 4×10^{14} Hz.

Quick Tip to Solve: Pair radar with microwaves, radiographs with X-rays, and night or fog photography with infrared. Check that the frequency order is microwave, infrared, then X-ray.

20. A converging beam of light travelling in air converges at a point P as shown in the figure. When a glass sphere of refractive index 1.5 is introduced in between the path of the beam, calculate the new position of the image. Also draw the ray diagram for the image formed. [3 marks]



Solution:

Concept:

- For paraxial refraction at a spherical surface, $n_2/v - n_1/u = (n_2 - n_1)/R$, where u , v and R are measured from that surface's pole and n_1, n_2 are the incident and refracted media's indices.
- Take the incident light direction, from left to right, as positive. A converging incident beam has a virtual object at its intended meeting point, so its object distance is positive.
- A ray is continuous at a refracting boundary. If the rays meet at that boundary itself, refraction changes their outgoing directions but cannot move their common point on the boundary.

Step 1: Identify the first surface's object distance and radius Let A and B denote the left and right poles of the sphere. The printed figure gives $AB = 10$ cm and $AP = 20$ cm. Therefore the sphere's radius is 5 cm. Without glass, the converging rays would meet at P , which acts as a virtual object for the first surface. Its centre of curvature also lies to the right of A . Thus

$$u_1 = +20 \text{ cm}, \quad R_1 = +5 \text{ cm}, \quad n_1 = 1, \quad n_2 = 1.5.$$

Step 2: Apply refraction at the air–glass surface Substitute these signed distances into the spherical-surface formula:

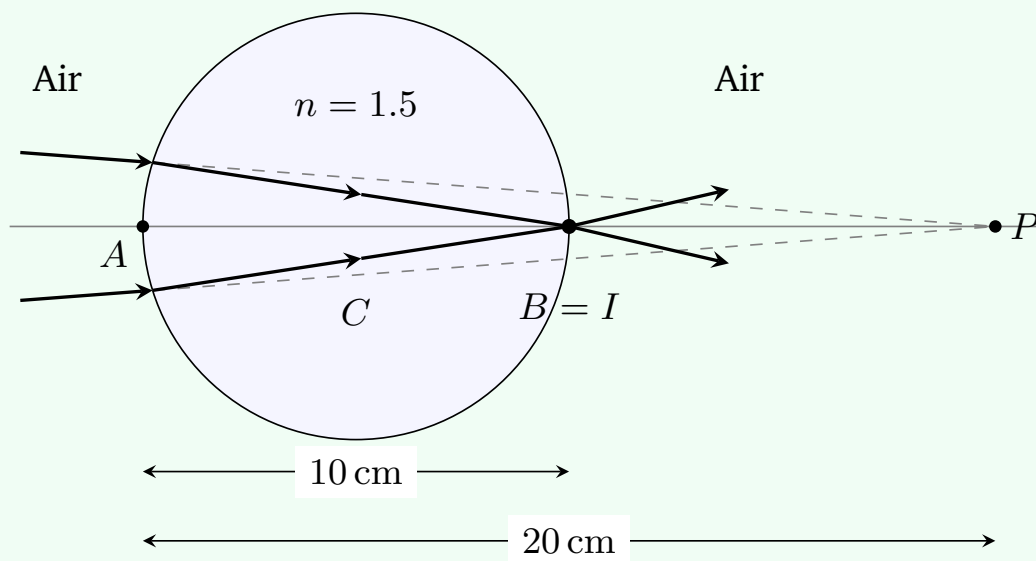
$$\begin{aligned} \frac{1.5}{v_1} - \frac{1}{20} &= \frac{1.5 - 1}{5}, \\ \frac{1.5}{v_1} &= 0.10 + 0.05 = 0.15 \text{ cm}^{-1}, \\ v_1 &= +10 \text{ cm}. \end{aligned}$$

This distance is measured from A , so the first refracted rays converge at the rear pole B , exactly one sphere diameter from A .

Step 3: Account for the second surface and construct the rays The convergence point lies on the glass–air boundary itself. The outgoing rays therefore share that same point and diverge after crossing it. There is no finite propagation distance left inside the sphere before the focus. In particular, do not substitute zero object distance directly into the second-surface formula, which contains its reciprocal.

Quick Tip to Solve: Read the two arrows from the same first-surface pole: the diameter is 10 cm, while the virtual-object distance is +20 cm. Compare the first image distance with the diameter before treating the second surface.

The ray diagram shows solid refracted paths meeting at B ; dashed continuations of the original incident rays show where P would have been without the sphere. The usual paraxial approximation is understood.



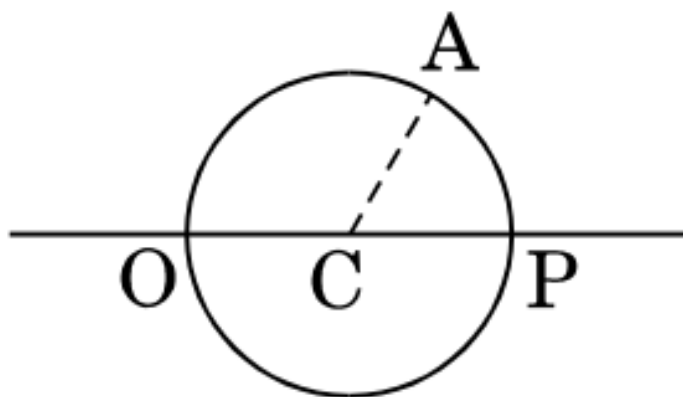
Paraxial rays focus at the rear pole B ; dashed paths show the original focus P .

Step 4: Conclusion The new real image is at the rear surface pole B : 10 cm from the first surface, or 10 cm before the original convergence point P .

Quick Tip to Solve: Read the two arrows from the same first-surface pole: the diameter is 10 cm, while the virtual-object distance is +20 cm. Compare the first image distance with the diameter before treating the second surface.

OR

20. A point ' O ' marked on the surface of a glass sphere of diameter 20 cm is viewed through glass from the position directly opposite to the point O . If the refractive index of the glass is 1.5, find the position of the image formed. Also, draw the ray diagram for the formation of the image. [3 marks]



Solution:

Concept:

- Refraction at a spherical surface obeys $n_2/v - n_1/u = (n_2 - n_1)/R$. Here n_1, n_2 are the refractive indices before and after refraction, and u, v, R are signed distances from the refracting surface's pole.
- Distances in the direction of incident light are positive. For a point viewed through the opposite surface, the point and the centre of curvature both lie on the incident side, giving negative u and R .
- A negative image distance indicates a virtual image on the incident side: backward extensions of the emergent rays meet there.

Step 1: Choose the surface through which the point is viewed Light from the marked point O travels inside the glass and leaves through the opposite surface, whose pole is P . Choose rightwards, from O towards P and the observer, as positive. Since $OP = 20$ cm, the radius is 10 cm. All distances in the refraction formula must now be measured from P , not from the sphere's centre C . Thus

$$u = -20 \text{ cm}, \quad R = -10 \text{ cm}, \quad n_1 = 1.5, \quad n_2 = 1.$$

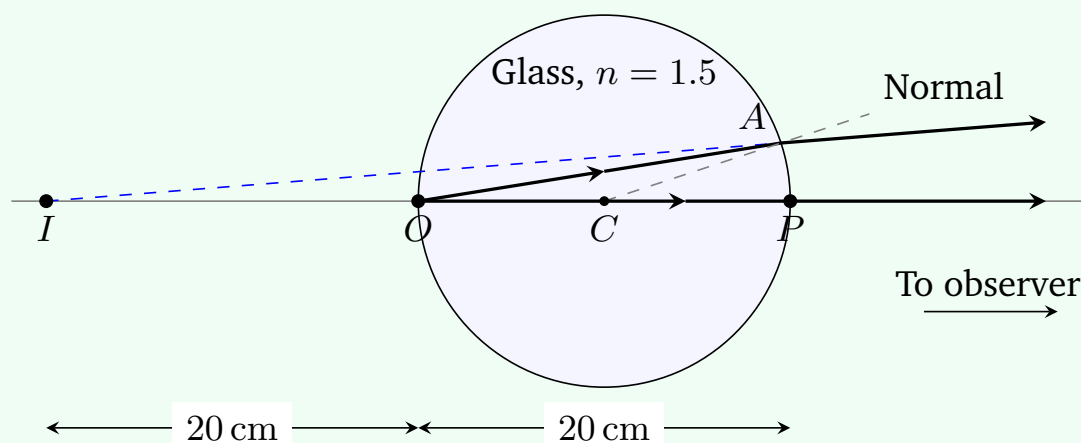
Step 2: Substitute the signed distances Apply the formula at the glass–air boundary:

$$\begin{aligned} \frac{1}{v} - \frac{1.5}{-20} &= \frac{1 - 1.5}{-10}, \\ \frac{1}{v} + 0.075 &= 0.050, \\ \frac{1}{v} &= -0.025 \text{ cm}^{-1}, \\ v &= -40 \text{ cm}. \end{aligned}$$

The negative sign places the image on the left of the viewing pole P . Since the actual point O is only 20 cm left of P , the apparent image lies another 20 cm beyond O .

Step 3: Trace the rays and their backward extensions An axial ray from O strikes the opposite pole normally and emerges undeviated. A second paraxial ray reaches the opposite curved surface above the axis and bends away from its radial normal on passing from glass to air. The emergent rays diverge, but their backward extensions meet at I , on the far side of O . Solid arrows show actual light travelling towards the observer; the dashed sloping line is only a backward extension. The construction is schematic and uses the paraxial approximation of the formula.

Quick Tip to Solve: Use the opposite surface as the origin: both $u = -20$ cm and $R = -10$ cm are negative. Convert the final distance from the viewing pole into a distance from O only at the end.



Backward extensions of rays emerging towards the observer locate the virtual image I .

Step 4: Conclusion The image is virtual, at $v = -40$ cm from the viewing surface P , i.e. 20 cm beyond the marked point O , away from the observer.

Quick Tip to Solve: Use the opposite surface as the origin: both $u = -20$ cm and $R = -10$ cm are negative. Convert the final distance from the viewing pole into a distance from O only at the end.

21. (a) Explain how an unpolarised light gets polarised when incident on the interface separating the two transparent media.

(b) Green light is incident at the polarising angle on a certain transparent medium. The angle of refraction is 30° . Find

(i) polarising angle, and

(ii) refractive index of the medium.

[3 marks]

Solution:

Concept:

- Unpolarised light has electric-field vibrations in all transverse directions. Polarisation restricts these vibrations to one direction.
- Brewster's law is $n_2/n_1 = \tan i_p$, where i_p is the polarising angle and n_1, n_2 are the refractive indices of the incident and transmitting media.

Step 1: (a) Explain polarisation at an interface Resolve the incident electric field into components parallel and perpendicular to the plane of incidence (the plane containing the incident ray and the normal). At a transparent dielectric interface these components are reflected by different amounts, so the reflected light is generally partially polarised. At the special incidence angle i_p , the parallel component is not reflected: the reflected beam has electric vibrations only perpendicular to the plane of incidence and is completely plane polarised. The transmitted beam is generally partially polarised. At this angle the reflected and refracted rays are mutually perpendicular.

Step 2: (b)(i) Use the perpendicular-ray condition Angles of incidence and refraction are measured

Quick Tip to Solve: When refraction at the polarising angle is given, first use $i_p = 90^\circ - r$, then use $n = \tan i_p$ for incidence from air.

from the normal. Since the reflection angle equals the incidence angle, the Brewster condition gives

$$i_p + r = 90^\circ.$$

Substituting the given refraction angle $r = 30^\circ$,

$$i_p = 90^\circ - 30^\circ = 60^\circ.$$

Step 3: (b)(ii) Apply Brewster's law Taking the incident medium as air, $n_1 \simeq 1$. The refractive index of the transparent medium is therefore

$$n = \tan i_p = \tan 60^\circ = \sqrt{3} \simeq 1.73.$$

This is also obtained from Snell's law as $n = \sin 60^\circ / \sin 30^\circ$, providing a check on the result for the given green light.

Step 4: Conclusion At the polarising angle the reflected light is completely polarised perpendicular to the plane of incidence. The required polarising angle is 60° , and the refractive index is $\sqrt{3} \simeq 1.73$.

Quick Tip to Solve: When refraction at the polarising angle is given, first use $i_p = 90^\circ - r$, then use $n = \tan i_p$ for incidence from air.

22. (a) Plot a graph to show the variation of stopping potential with frequency of incident radiation in relation to photoelectric effect.

(b) Use Einstein's photoelectric equation to show how from this graph, (i) Threshold frequency, and (ii) Planck's constant can be determined. [3 marks]

Solution:

Concept:

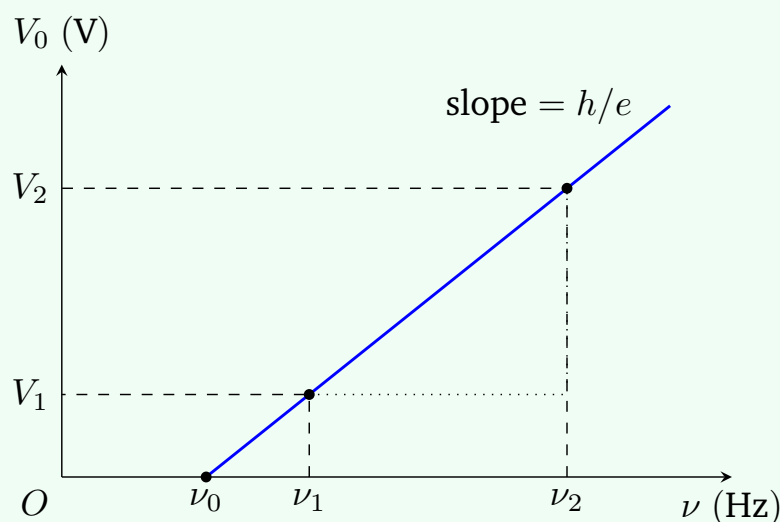
- Einstein's photoelectric equation is $h\nu = \phi + K_{\max}$, where h is Planck's constant, ν is incident frequency and ϕ is the work function.
- The positive magnitude of stopping potential V_0 satisfies $eV_0 = K_{\max}$, where e is the magnitude of electron charge. At threshold frequency ν_0 , $h\nu_0 = \phi$.

Step 1: (a) Obtain and plot the straight-line relation The retarding potential stops even the fastest photoelectrons when its electric potential-energy barrier equals their maximum kinetic energy. Substituting $K_{\max} = eV_0$ in Einstein's equation gives

$$eV_0 = h\nu - \phi, \quad V_0 = \frac{h}{e}\nu - \frac{\phi}{e}.$$

Thus, with frequency on the horizontal axis and stopping potential on the vertical axis, the graph is a straight line of positive slope. The measured photoemission branch begins at ν_0 . Below this frequency no photoelectrons are emitted, so a negative stopping potential is not a measured continuation of the line.

Quick Tip to Solve: For stopping potential against frequency, multiply the slope by e ; the slope itself is h/e , not h .



Stopping potential increases linearly above the threshold frequency.

Step 2: (b)(i) Read the threshold frequency At the point where the line meets the frequency axis, $V_0 = 0$ and the emitted electrons have zero maximum kinetic energy. Hence

$$0 = h\nu_0 - \phi, \quad \nu_0 = \frac{\phi}{h}.$$

The horizontal-axis intercept of the graph directly gives the threshold frequency of the photosensitive surface.

Step 3: (b)(ii) Determine Planck's constant from the slope Choose two well-separated points (ν_1, V_1) and (ν_2, V_2) on the straight line. Its slope is

$$m = \frac{V_2 - V_1}{\nu_2 - \nu_1} = \frac{h}{e}.$$

Multiply this measured slope by the known elementary charge:

$$h = em = e \frac{V_2 - V_1}{\nu_2 - \nu_1}.$$

With voltage in volts and frequency in hertz, the resulting value of h has units J s.

Step 4: Conclusion The V_0 -versus- ν graph is a straight line with frequency-axis intercept ν_0 and slope h/e . Thus the intercept gives the threshold frequency and $h = e \times \text{slope}$.

Quick Tip to Solve: For stopping potential against frequency, multiply the slope by e ; the slope itself is h/e , not h .

22. (a) How does one explain the emission of electrons from a photosensitive surface with the help of Einstein's photoelectric equation?

(b) Work function of aluminium is 4.2 eV. If two photons each of energy 2.5 eV are incident on its surface, will the emission of electrons take place? Justify your answer.

(c) The stopping potential in an experiment on photoelectric effect is 1.5 V. What is the maximum kinetic energy of the photoelectrons emitted? Calculate in Joules. [3 marks]

Solution:

Concept:

- In Einstein's model, one photon transfers its energy $h\nu$ to one electron. The work function ϕ is the minimum energy required to escape from the metal surface.
- Energy conservation gives $K_{\max} = h\nu - \phi$. Stopping potential V_0 measures this maximum kinetic energy through $K_{\max} = eV_0$, with $e = 1.6 \times 10^{-19}$ C.

Step 1: (a) Explain electron emission by photon absorption Incident radiation consists of photons, each carrying energy $h\nu$. When a photon is absorbed, its energy is delivered to an electron in a single interaction. Part of this energy is used to overcome the binding at the surface; the remainder appears as the kinetic energy of the emitted electron. For the most energetic electrons,

$$h\nu = \phi + K_{\max}.$$

Emission requires $h\nu \geq \phi$, or $\nu \geq \nu_0 = \phi/h$. Increasing intensity at fixed frequency increases the number of incident photons, but cannot overcome the threshold if each photon has insufficient energy.

Step 2: (b) Compare each photon energy with the work function For aluminium, $\phi = 4.2$ eV, whereas each incident photon carries only 2.5 eV. Thus

$$E_{\text{photon}} = 2.5 \text{ eV} < 4.2 \text{ eV} = \phi.$$

No photoelectric emission occurs in the ordinary single-photon photoelectric effect. The two photon energies are not added to liberate one electron in Einstein's model: each absorption must individually supply the work function. Although their combined energy is 5.0 eV, this is not the energy supplied by one photon to one electron in this model.

Step 3: (c) Convert stopping potential to maximum kinetic energy The stopping potential is the retarding potential just sufficient to stop the fastest photoelectrons. Using its positive magnitude $V_0 = 1.5$ V,

$$\begin{aligned} K_{\max} &= eV_0 \\ &= (1.6 \times 10^{-19} \text{ C})(1.5 \text{ V}) \\ &= 2.4 \times 10^{-19} \text{ J.} \end{aligned}$$

Equivalently the maximum kinetic energy is 1.5 eV; the conversion above expresses it in the requested SI unit.

Step 4: Conclusion Einstein's equation explains emission when one photon supplies at least the work function. Two photons of 2.5 eV each do not produce ordinary photoelectric emission from

Quick Tip to Solve: Compare the energy of each photon with the work function separately. For stopping potential in volts, $K_{\max}$ has the same numerical value in electron-volts.

aluminium with work function 4.2 eV. The maximum kinetic energy for a stopping potential of 1.5 V is 2.4×10^{-19} J.

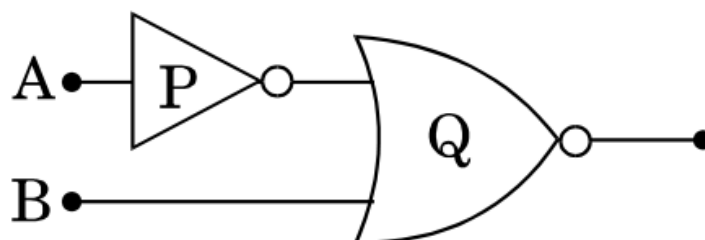
Quick Tip to Solve: Compare the energy of each photon with the work function separately. For stopping potential in volts, $K_{\max}$ has the same numerical value in electron-volts.

23. (a) (i) Write the truth tables of the logic gates marked P and Q in the given circuit.

(ii) Write the truth table for the circuit.

(b) Why are NOR gates considered as universal gates?

[3 marks]



Solution:

Concept:

- A NOT gate inverts its input: $X = \overline{A}$. An inversion bubble at the output of a gate denotes logical negation.
- A NOR gate negates the OR operation: $Y = \overline{X + B}$. Here $+$ denotes Boolean OR and juxtaposition denotes Boolean AND.
- A universal gate can implement NOT, OR and AND using only copies of that gate, so it can realise any Boolean logic function.

Step 1: (a)(i) Identify P and write its truth table Gate P is a triangle with a small circle at its output, so it is a NOT gate. Its input is A , and let its output be X . Therefore $X = \overline{A}$: a low input becomes high and a high input becomes low.

A	$X = \overline{A}$
0	1
1	0

Step 2: (a)(i) Identify Q and write its truth table Gate Q has the shape of an OR gate and an inversion circle at its output, making it a NOR gate. For its two inputs X and B , the output Y is high only when both inputs are low:

X	B	$Y = \overline{X + B}$
0	0	1
0	1	0
1	0	0
1	1	0

The circle is essential: an ordinary OR gate would give the opposite output in every row.

Step 3: (a)(ii) Form the truth table for the complete circuit The upper input to Q is the inverted value

Quick Tip to Solve: Write the intermediate output of P in a separate truth-table column before applying Q; the inversion circles must both be included.

of A , while B enters Q directly. Hence

$$Y = \overline{\overline{A} + B} = A\overline{B},$$

where the last equality follows from De Morgan's theorem. Compute the intermediate output before applying the NOR operation:

A	B	$X = \overline{A}$	Y
0	0	1	0
0	1	1	0
1	0	0	1
1	1	0	0

Thus a high circuit output requires A high and B low simultaneously.

Step 4: (b) Show why NOR is universal Write $U \downarrow V = \overline{U + V}$ for a NOR operation. Connecting the two inputs together gives NOT:

$$A \downarrow A = \overline{A}.$$

Inverting the output of a NOR gate with another NOR whose inputs are tied together gives OR:

$$(A \downarrow B) \downarrow (A \downarrow B) = A + B.$$

Finally, invert each input using NOR gates and then NOR the inverted inputs. De Morgan's theorem gives AND:

$$(A \downarrow A) \downarrow (B \downarrow B) = \overline{\overline{A} + \overline{B}} = AB.$$

Since NOT, OR and AND can all be obtained in this way, any logic circuit built from these basic operations can be realised using NOR gates alone.

Step 5: Conclusion P is NOT and Q is NOR. The full circuit gives $Y = A\overline{B}$, with outputs 0, 0, 1, 0 for inputs $(A, B) = (0, 0), (0, 1), (1, 0), (1, 1)$, respectively. NOR is universal because it can realise all three basic operations NOT, OR and AND.

Quick Tip to Solve: Write the intermediate output of P in a separate truth-table column before applying Q ; the inversion circles must both be included.

OR

23. (a) Explain how a potential barrier is developed in a p-n junction diode.

(b) Draw the circuit arrangement for studying the V-I characteristics of a p-n junction diode in reverse bias. Plot the V-I characteristics in this case. [3 marks]

Solution:

Concept:

- Diffusion carries majority electrons from the n region to the p region and majority holes from the p region to the n region because of the concentration difference.
- Uncovered, immobile ionised donors and acceptors form a depletion layer. Its electric field opposes

Quick Tip to Solve: For the reverse-bias circuit, check p-to-minus and n-to-plus first, then put the microammeter in series and the voltmeter across the diode.

further majority-carrier diffusion and produces the potential barrier.

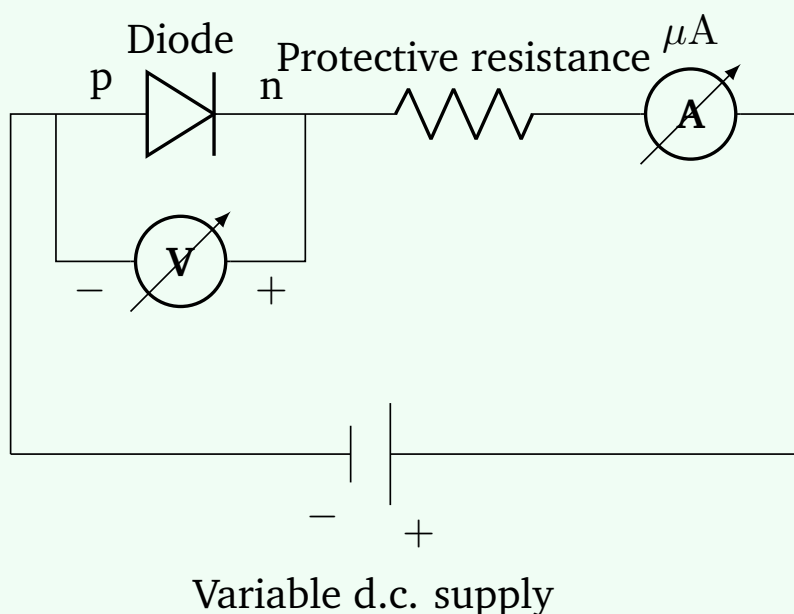
- In reverse bias the p side is connected to the negative terminal and the n side to the positive terminal. Reverse current is mainly due to minority carriers.

Step 1: (a) Formation of the potential barrier Immediately after a p-n junction is formed, electrons diffuse from n to p and holes diffuse from p to n. Electrons and holes recombine near the junction, leaving a region depleted of mobile charge carriers. The n side of this region contains fixed positive donor ions; the p side contains fixed negative acceptor ions.

These opposite space charges establish an electric field directed from n to p. The field drives a drift current opposite to the diffusion current and opposes further diffusion of majority carriers. The associated potential difference across the depletion layer is the potential barrier. At equilibrium, drift and diffusion currents balance, so the net current is zero; diffusion does not simply cease altogether.

Step 2: (b) Circuit for reverse-bias measurements Connect the p terminal of the diode to the negative terminal of a variable d.c. supply and the n terminal to the positive terminal. A protective resistance and a microammeter are in series, and a voltmeter is connected across the diode with its positive terminal on the n side. The microammeter is needed because the reverse current is very small before breakdown.

Increase the reverse voltage in small steps and record the diode voltage and current at each setting. Plot these readings. The series resistance limits current, especially near breakdown, and prevents damage to the junction.

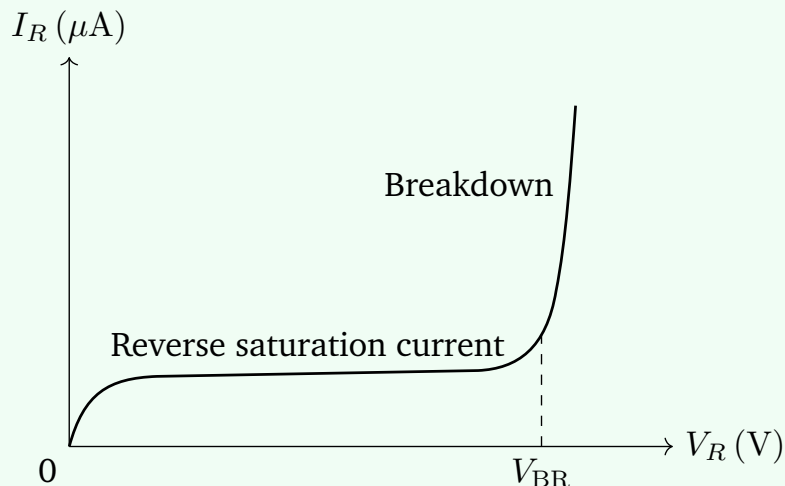


Reverse-bias circuit: the n terminal is positive relative to the p terminal.

Step 3: Reverse-bias V-I characteristic Reverse bias increases the barrier and widens the depletion layer. Majority-carrier conduction is suppressed, but thermally generated minority carriers give a small reverse saturation current. At a fixed temperature it is nearly independent of reverse voltage until breakdown. At the breakdown voltage V_{BR} , the reverse current increases very rapidly. On axes using signed diode voltage and current, this branch lies in the third quadrant; the graph below uses

Quick Tip to Solve: For the reverse-bias circuit, check p-to-minus and n-to-plus first, then put the microammeter in series and the voltmeter across the diode.

positive reverse magnitudes $V_R = -V$ and $I_R = -I$ for clarity.



Reverse characteristic plotted using reverse voltage and current magnitudes.

Step 4: Conclusion The space charge in the depletion layer creates the potential barrier. With p negative and n positive, the diode carries a small, nearly constant reverse current until breakdown, after which the current rises sharply.

Quick Tip to Solve: For the reverse-bias circuit, check p-to-minus and n-to-plus first, then put the microammeter in series and the voltmeter across the diode.

24. (a) What do you mean by bandwidth of a signal? Give its importance.

(b) Differentiate between Analog and Digital communication.

(c) Write the functions of transducer and repeaters.

[3 marks]

Solution:

Concept:

- A signal generally contains a range of frequency components. Its bandwidth is the difference between the highest and lowest frequencies that must be transmitted faithfully.
- Analog communication represents information by continuously varying quantities; digital communication represents it using discrete symbols, commonly binary 0 and 1.
- A transducer converts one form of energy into another. A repeater receives a signal and retransmits it to extend the communication distance.

Step 1: (a) Meaning and importance of bandwidth If the significant frequency components of a signal extend from $f_{\min}$ to $f_{\max}$, its bandwidth is

$$B = f_{\max} - f_{\min}.$$

The unit of bandwidth is hertz. For a baseband signal whose frequencies extend from zero to $f_{\max}$, its bandwidth is $f_{\max}$.

Quick Tip to Solve: Keep the three roles separate: bandwidth specifies the required frequency span, a transducer changes energy form, and a repeater extends transmission distance.

Bandwidth tells us the frequency range that a communication system must accommodate for faithful transmission of the information. The channel and associated circuits must pass the required components; otherwise the signal is distorted and information may be lost. It therefore helps determine the channel capacity and allocate frequency ranges to different communication services.

Step 2: (b) Analog and digital communication In analog communication, information is represented by a continuous electrical signal whose amplitude or another relevant parameter varies continuously with the message. For example, a microphone produces a voltage that follows variations in sound pressure. Noise added during transmission alters the signal, and ordinary amplification amplifies this unwanted contribution as well.

In digital communication, information is encoded as a sequence of discrete symbols, usually binary digits 0 and 1, represented by prescribed signal levels. Speech can be sampled, quantised and encoded into such a sequence. Within allowable noise limits, a receiver can distinguish the levels and regenerate the original symbols. Thus digital communication generally offers better resistance to accumulated noise than analog communication, as well as convenient digital processing and storage.

Step 3: (c) Functions of transducer and repeater A transducer converts energy from one form to another. An input transducer converts a physical message into an electrical signal suitable for communication: a microphone converts sound variations into electrical variations. At the receiving end, an output transducer such as a loudspeaker converts the recovered electrical signal back into sound.

A repeater, placed between the transmitter and receiver, receives a signal weakened during propagation, amplifies it and retransmits it towards the receiver. It combines receiver and transmitter functions and extends the communication range. In a digital system it may also regenerate the pulse sequence, reducing the accumulation of noise and distortion.

Step 4: Conclusion Bandwidth is the required frequency span of a signal. Analog communication uses continuous message variations, while digital communication uses discrete encoded symbols. Transducers convert the message between physical and electrical forms, and repeaters receive and retransmit signals to extend the range.

Quick Tip to Solve: Keep the three roles separate: bandwidth specifies the required frequency span, a transducer changes energy form, and a repeater extends transmission distance.

Section D (15 marks)

25. (a) State and explain the law used to determine magnetic field at a point due to a current element. Derive the expression for the magnetic field due to a circular current carrying loop of radius r at its centre.

(b) A long wire with a small current element of length 1 cm is placed at the origin and carries a current of 10 A along the X-axis. Find out the magnitude and direction of the magnetic field due to the element on the Y-axis at a distance 0.5 m from it. [5 marks]

Solution:

Concept:

- The Biot–Savart law gives the magnetic field of a steady current element: $d\vec{B} = \frac{\mu_0}{4\pi} \frac{I d\vec{\ell} \times \hat{\rho}}{\rho^2}$, where ρ points from the element to the observation point.
- The net field is the vector sum of the fields of all current elements. The cross product, or right-hand rule, determines the direction.

Step 1: (a) State and explain the Biot–Savart law For a short element of length $d\ell$ carrying current I , the magnetic field at a point a distance ρ away has magnitude

$$dB = \frac{\mu_0}{4\pi} \frac{I d\ell \sin \theta}{\rho^2}.$$

Here θ is the angle between the current element and the line joining it to the point, and μ_0 is the permeability of free space. Thus the field is proportional to I , $d\ell$, and $\sin \theta$, and inversely proportional to ρ^2 . Its direction is perpendicular to the plane of $d\vec{\ell}$ and ρ , along $d\vec{\ell} \times \hat{\rho}$.

Step 2: (a) Integrate the field around the circular loop At the centre of a circular loop of radius r , every element is at the same distance r . The element is tangential whereas the line towards the centre is radial, so $\theta = 90^\circ$. Every element therefore produces a field normal to the loop in the same direction. The magnitudes can be added directly:

$$\begin{aligned} B &= \oint dB = \frac{\mu_0 I}{4\pi r^2} \oint d\ell, \\ \oint d\ell &= 2\pi r, \\ B &= \frac{\mu_0 I}{4\pi r^2} (2\pi r) = \frac{\mu_0 I}{2r}. \end{aligned}$$

Curl the right-hand fingers in the current direction; the extended thumb gives the field direction along the loop's axis. As seen by an observer, anticlockwise current produces field towards the observer and clockwise current produces field away.

Step 3: (b) Substitute the current-element data We require the field of the specified small element, not of the whole long wire. With $\ell = 1 \text{ cm} = 10^{-2} \text{ m}$, $I = 10 \text{ A}$, and $\rho = 0.5 \text{ m}$, the element along X is

Quick Tip to Solve: Use the current-element formula for part (b); the infinite-wire formula would include the field of the entire wire and answer a different question.

perpendicular to the displacement along Y. Thus

$$B = 10^{-7} \frac{10 \times 10^{-2} \times \sin 90^\circ}{(0.5)^2} = 4.0 \times 10^{-8} \text{ T.}$$

Taking the current along positive X and the observation point on positive Y, $\hat{x} \times \hat{y} = \hat{z}$, so the field is along positive Z, perpendicular to the XY plane. At the corresponding point on negative Y its magnitude is unchanged and its direction is negative Z.

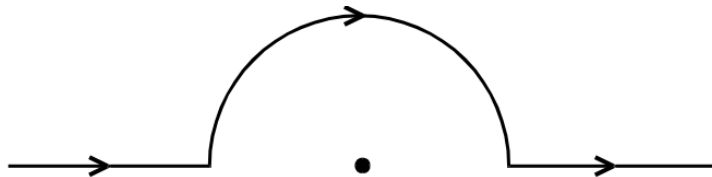
Step 4: Conclusion (a) Biot–Savart law gives $B = \frac{\mu_0 I}{2r}$ at the centre of a single circular loop, normal to its plane. (b) The element gives $B = 4.0 \times 10^{-8} \text{ T}$, along positive Z for a point on positive Y and current along positive X.

Quick Tip to Solve: Use the current-element formula for part (b); the infinite-wire formula would include the field of the entire wire and answer a different question.

OR

25. (a) Derive the expression for the magnetic field due to a current carrying coil of radius r at a distance x from the centre along the X-axis.

(b) A straight wire carrying a current of 5 A is bent into a semicircular arc of radius 2 cm as shown in the figure. Find the magnitude and direction of the magnetic field at the centre of the arc. [5 marks]



Solution:

Concept:

- Biot–Savart law: $d\vec{B} = \frac{\mu_0 I}{4\pi} \frac{d\vec{\ell} \times \hat{R}}{R^2}$, where R is the distance from the current element to the observation point.
- For a circular coil, symmetry cancels transverse field components on its axis; the axial components add. At the centre of a circular arc, every element contributes in the same normal direction.

Step 1: (a) Set up the axial-field geometry Place a circular coil of radius r , carrying current I , in the YZ plane with its centre at the origin. Let the observation point P be on the X-axis at distance x . Every element has the same separation $R = \sqrt{r^2 + x^2}$ from P . The tangent $d\vec{\ell}$ is perpendicular to the plane containing the radius and the axis, so it is perpendicular to the displacement from that element to P . Hence

$$dB = \frac{\mu_0 I d\ell}{4\pi R^2}.$$

Step 2: (a) Resolve and integrate the field If α is the angle between the element-to-point displacement and the X-axis, then $\sin \alpha = r/R$. Since $d\vec{B}$ is perpendicular to that displacement, its axial component

Quick Tip to Solve: At an arc's centre use its angle in radians; a semicircle gives half the full-loop field. Straight portions directed towards or away from that centre contribute zero.

is

$$dB_x = dB \sin \alpha = \frac{\mu_0 I r d\ell}{4\pi R^3}.$$

Diametrically opposite elements have equal and opposite transverse components, but their axial components have the same direction. Integrating around one complete turn gives

$$B_x = \frac{\mu_0 I r}{4\pi R^3} \oint d\ell = \frac{\mu_0 I r}{4\pi R^3} (2\pi r),$$
$$B = \frac{\mu_0 I r^2}{2(r^2 + x^2)^{3/2}}.$$

For N closely wound turns, multiply by N . The field points along the axis according to the right-hand grip rule. At $x = 0$, the expression becomes $\mu_0 N I / (2r)$, which agrees with the field at the centre of a coil.

Step 3: (b) Evaluate the semicircular-arc contribution At the arc's centre each tangent is perpendicular to its radius. For a circular arc subtending angle ϕ in radians, its length is $r\phi$; hence

$$B_{\text{arc}} = \frac{\mu_0 I}{4\pi r^2} (r\phi).$$

A semicircle has $\phi = \pi$, giving $B_{\text{arc}} = \mu_0 I / (4r)$. Substituting $I = 5 \text{ A}$ and $r = 2 \text{ cm} = 0.02 \text{ m}$,

$$B_{\text{arc}} = \frac{4\pi \times 10^{-7} \times 5}{4 \times 0.02} = 2.5\pi \times 10^{-5} \text{ T} \simeq 7.85 \times 10^{-5} \text{ T}.$$

Step 4: (b) Account for the straight portions and find direction The centre lies on the line of both straight portions. For every element on these portions, $d\vec{\ell}$ and the displacement to the centre are parallel or antiparallel; therefore their cross product is zero. They contribute no field at the centre. The arrows show current travelling clockwise along the upper semicircle, from its left end to its right end. The right-hand rule therefore gives a field into the plane of the paper.

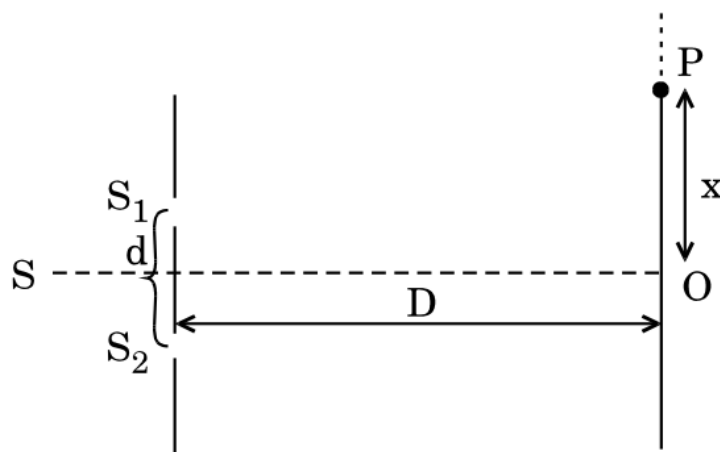
Step 5: Conclusion (a) The axial field of an N -turn coil is $B = \frac{\mu_0 N I r^2}{2(r^2 + x^2)^{3/2}}$; set $N = 1$ for a single turn. (b) The net field at the arc centre is $7.85 \times 10^{-5} \text{ T}$, directed into the paper.

Quick Tip to Solve: At an arc's centre use its angle in radians; a semicircle gives half the full-loop field. Straight portions directed towards or away from that centre contribute zero.

26. (a) Can the interference pattern be produced by two independent monochromatic sources of light? Explain.

(b) The intensity at the central maximum (O) in a Young's double slit experimental set-up shown in the figure is I_0 . If the distance OP equals one-third of the fringe width of the pattern, show that the intensity at point P , would equal $\frac{I_0}{4}$.

(c) In Young's double slit experiment, the slits are separated by 0.5 mm and screen is placed 1.0 m away from the slit. It is found that the 5^{th} bright fringe is at a distance of 4.13 mm from the 2^{nd} dark fringe. Find the wavelength of light used. [5 marks]



Solution:

Concept:

- A sustained interference pattern requires coherent sources: their frequency is the same and their relative phase remains constant during observation.
- For two equal slit intensities I_s , the resultant intensity is $I = 4I_s \cos^2(\phi/2)$, where ϕ is the phase difference. The central intensity is $I_0 = 4I_s$.
- In Young's experiment, fringe width $\beta = \lambda D/d$, phase difference $\phi = 2\pi y/\beta$, and the positions are $y_n^{\text{bright}} = n\beta$ and $y_n^{\text{dark}} = (n - \frac{1}{2})\beta$. Here D is screen distance, d slit separation and y distance from the central maximum.

Step 1: (a) Test the coherence of independent sources Two ordinary independent monochromatic light sources cannot produce a stable interference pattern. Although their light may have the same frequency, independent atomic emissions cause their relative phase to change randomly and rapidly. Thus the positions of maxima and minima change during the observation time. In the time-averaged intensity,

$$\langle I \rangle = I_1 + I_2 + 2\sqrt{I_1 I_2} \langle \cos \phi \rangle,$$

the interference term vanishes because $\langle \cos \phi \rangle = 0$. Monochromaticity alone therefore does not ensure coherence. Young's two slits are illuminated by one source to obtain a constant relative phase.

Step 2: (b) Convert the position of P into phase difference Under the usual small-angle conditions of

Quick Tip to Solve: Number the central bright fringe as zero and the first dark fringe at half a fringe width; subtract positions for fringes on the same side of the centre.

the experiment, the path difference at a transverse distance y is $\Delta = dy/D$. Since $OP = y = \beta/3$,

$$\Delta = \frac{d}{D} \frac{\beta}{3} = \frac{\lambda}{3}, \quad \phi = \frac{2\pi\Delta}{\lambda} = \frac{2\pi}{3}.$$

Both slits have equal intensity in the ideal arrangement. Therefore,

$$I_P = I_0 \cos^2 \left(\frac{\phi}{2} \right) = I_0 \cos^2 \left(\frac{\pi}{3} \right) = \frac{I_0}{4}.$$

Here I_0 is the central intensity from both slits together, not the intensity due to one slit.

Step 3: (c) Express the fringe separation in fringe widths Take the fifth bright and second dark fringes on the same side of the central maximum, as conventionally intended when their separation is specified. The central bright fringe is order zero. The second dark fringe is at $3\beta/2$, since the first dark fringe is at $\beta/2$. Hence,

$$y_5^{\text{bright}} = 5\beta, \quad y_2^{\text{dark}} = \frac{3\beta}{2},$$

$$4.13 \text{ mm} = 5\beta - \frac{3\beta}{2} = \frac{7\beta}{2}, \quad \beta = 1.18 \text{ mm}.$$

Step 4: (c) Calculate the wavelength in SI units Substituting $d = 0.5 \times 10^{-3} \text{ m}$, $D = 1.0 \text{ m}$ and $\beta = 1.18 \times 10^{-3} \text{ m}$,

$$\lambda = \frac{\beta d}{D} = \frac{(1.18 \times 10^{-3})(0.5 \times 10^{-3})}{1.0} \text{ m} = 5.90 \times 10^{-7} \text{ m}.$$

This is a visible-light wavelength. If the two specified fringes were instead on opposite sides, their distances would add; the same-side assumption is therefore explicit in this calculation.

Step 5: Conclusion (a) Independent monochromatic sources do not give a sustained pattern because they lack a fixed phase relation. (b) $I_P = I_0/4$. (c) The wavelength is $\lambda = 590 \text{ nm}$, taking the fringes on the same side of the centre.

Quick Tip to Solve: Number the central bright fringe as zero and the first dark fringe at half a fringe width; subtract positions for fringes on the same side of the centre.

26. (a) Derive the relation $a \sin \theta = \lambda$ for the first minimum of the diffraction pattern produced due to a single slit of width ' a ' using light of wavelength λ .

(b) State with reason, how the linear width of central maximum will be affected if (i) monochromatic yellow light is replaced with red light, and (ii) distance between the slit and the screen is increased.

(c) Using the monochromatic light of same wavelength in the experimental set-up of the diffraction pattern as well as in the interference pattern where the slit separation is 1 mm, 10 interference fringes are found to be within the central maximum of the diffraction pattern. Determine the width of the single slit, if the screen is kept at the same distance from the slit in the two cases. [5 marks]

Solution:

Concept:

- By the Huygens principle, every point across a uniformly illuminated slit acts as a coherent secondary source. The resultant amplitude in a direction depends on the path differences between these wavelets.
- For a single slit of width a , the first diffraction minima satisfy $a \sin \theta = \pm \lambda$, where λ is wavelength and θ is measured from the forward direction.
- For small diffraction angles and screen distance D , the central maximum width is $W = 2D\lambda/a$. In Young's experiment with slit separation d , the fringe width is $\beta = D\lambda/d$.

Step 1: (a) Pair wavelets from the two halves of the slit Consider a plane monochromatic wave incident normally on a slit of width a . Observe the far-field pattern in a direction making angle θ with the central axis. The path difference between rays from the two extreme edges is $a \sin \theta$. Divide the slit into two equal halves. Pair each point in the upper half with the point a distance $a/2$ below it in the lower half. Every such pair has the same path difference,

$$\Delta_{\text{pair}} = \frac{a}{2} \sin \theta.$$

Equal corresponding elements emit equal-amplitude wavelets. If their path difference is $\lambda/2$, their phase difference is π , and their amplitudes cancel.

Step 2: (a) Obtain the first minimum condition Set the pairwise path difference equal to half a wavelength:

$$\frac{a}{2} \sin \theta_1 = \frac{\lambda}{2} \Rightarrow a \sin \theta_1 = \lambda.$$

Every element in the upper half then cancels with its partner in the lower half, giving zero resultant amplitude. This is the first minimum on the positive side; the symmetric minimum is at $-\theta_1$. At $\theta = 0$, all wavelets are in phase, producing the central maximum. Thus the central maximum is the region between these two first minima.

Step 3: (b) Relate central width to wavelength and screen distance The first minima lie at $y = \pm D \tan \theta_1$. For the usual small angles, $\tan \theta_1 \simeq \sin \theta_1 \simeq \lambda/a$, so

$$W = 2D \tan \theta_1 \simeq \frac{2D\lambda}{a}.$$

Quick Tip to Solve: Compare the full diffraction width $2D\lambda/a$ with the interference fringe width $D\lambda/d$; their ratio is $2d/a$, independent of wavelength and screen distance.

- (i) Red light has a greater wavelength than yellow light. With D and a fixed, $W \propto \lambda$, so the central maximum becomes wider when yellow light is replaced by red light.
- (ii) With wavelength and slit width fixed, $W \propto D$. Increasing the slit-to-screen distance therefore increases the linear width of the central maximum in the same proportion. Its angular width remains unchanged in this case.

Step 4: (c) Compare the central width with ten interference fringes The wavelength and screen distance are the same in the two experiments. Ten interference fringe widths fit within the central diffraction maximum, so $W = 10\beta$. Using $d = 1 \text{ mm}$,

$$\frac{2D\lambda}{a} = 10 \frac{D\lambda}{d} \Rightarrow \frac{2}{a} = \frac{10}{d}.$$

The common factors D and λ cancel, giving

$$a = \frac{2d}{10} = \frac{2(1 \text{ mm})}{10} = 0.20 \text{ mm} = 2.0 \times 10^{-4} \text{ m}.$$

The factor of two is essential: the central diffraction maximum extends from the first minimum on one side to the first minimum on the other.

Step 5: Conclusion The first minimum obeys $a \sin \theta_1 = \lambda$. The central maximum widens both when yellow light is replaced by red light and when the screen is moved farther away. The required slit width is $a = 0.20 \text{ mm}$.

Quick Tip to Solve: Compare the full diffraction width $2D\lambda/a$ with the interference fringe width $D\lambda/d$; their ratio is $2d/a$, independent of wavelength and screen distance.

27(a). Draw a circuit diagram of an n-p-n transistor with its emitter-base junction forward biased and base-collector junction reverse biased. Briefly describe its working.

Explain how a transistor in its active state exhibits a low resistance at its emitter-base junction and high resistance at its base-collector junction. [5 marks total]

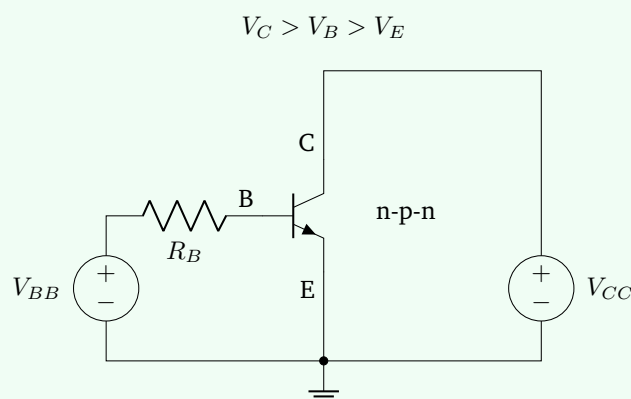
Solution:

Concept:

- In the active region of an n-p-n transistor, the emitter-base junction is forward biased and the collector-base junction is reverse biased: $V_C > V_B > V_E$.
- The emitter is heavily doped and the base is thin and lightly doped. Current continuity gives $I_E = I_B + I_C$, with I_C nearly equal to I_E .

Step 1: Connect the junctions with the required bias Connect the emitter to the common negative terminal of both supplies. The positive terminal of V_{BB} biases the base through R_B , while the positive terminal of V_{CC} connects to the collector. Choose $V_C > V_B > V_E$: the p-type base is positive relative to the emitter, and the n-type collector is positive relative to the base. The emitter arrow points outwards.

Quick Tip to Solve: For an active n-p-n transistor, check the potential order $V_C > V_B > V_E$ and draw the emitter arrow pointing out.



Active-region n-p-n bias circuit; both supplies share the emitter reference.

Step 2: Explain the transport of charge carriers Forward bias lowers the emitter-base potential barrier, so the heavily doped emitter injects a large number of electrons into the base. A small fraction recombines with holes in the thin, lightly doped base, producing the small base current. Most electrons cross the base and are swept into the collector by the electric field of the reverse-biased collector-base junction. This produces the collector current. Electron motion is from emitter to collector; conventional collector current enters the collector and leaves through the emitter.

Step 3: Compare the junction resistances At the forward-biased emitter-base junction, a small increase in forward voltage causes a large increase in injection current; hence its incremental resistance is low. At the reverse-biased collector-base junction, the depletion region is wider and a change in reverse voltage produces only a small change in collector current at fixed emitter injection. Its incremental resistance is therefore high. The collector current can still be large because it collects electrons supplied by the emitter; it is not merely the reverse leakage current of an isolated junction.

Step 4: Conclusion With $V_C > V_B > V_E$, the emitter injects electrons, the thin base passes most of them, and the collector collects them. Forward bias gives low emitter-base resistance, while reverse bias gives high collector-base resistance.

Quick Tip to Solve: For an active n-p-n transistor, check the potential order $V_C > V_B > V_E$ and draw the emitter arrow pointing out.

27(b). Derive the expression for the voltage gain of a transistor amplifier in CE configuration in terms of the load resistance R_L , current gain β_a and input resistance.

Explain why input and output voltages are in opposite phase.

Solution:

Concept:

- In a common-emitter (CE) amplifier, the emitter is common to the input and output. The small-signal current gain is $\beta_a = \Delta I_C / \Delta I_B$.
- The small-signal input resistance is $r_i = \Delta V_{BE} / \Delta I_B$. For a fixed collector supply and collector load R_L , the output change is $\Delta V_{CE} = -R_L \Delta I_C$.

Quick Tip to Solve: Start with $V_{CE} = V_{CC} - I_C R_L$: a rising collector current lowers the output voltage and fixes the minus sign in the gain.

Step 1: Express the input voltage through the input current Bias the transistor in its active region and apply a small alternating signal between base and emitter. Denote the resulting changes in input voltage and base current by $v_i = \Delta V_{BE}$ and ΔI_B . Using the input resistance at this operating point,

$$v_i = r_i \Delta I_B.$$

The signal must be small enough that the transistor remains in its active region and r_i and β_a can be regarded as constant.

Step 2: Relate the output voltage to the collector current The collector circuit obeys $V_{CE} = V_{CC} - I_C R_L$, with the emitter held at the common reference potential. The dc supply does not vary with the input signal. Therefore the alternating output voltage at the collector is

$$\begin{aligned} v_o &= \Delta V_{CE} = -R_L \Delta I_C, \\ \Delta I_C &= \beta_a \Delta I_B, \\ v_o &= -\beta_a R_L \Delta I_B. \end{aligned}$$

Dividing by the input signal gives the signed voltage gain:

$$A_v = \frac{v_o}{v_i} = \frac{-\beta_a R_L \Delta I_B}{r_i \Delta I_B} = -\frac{\beta_a R_L}{r_i}.$$

Step 3: Explain the phase reversal During a positive excursion of the input, the base-emitter forward bias increases, so the base current and collector current increase. The drop $I_C R_L$ increases, reducing the collector-emitter voltage. Thus a positive input excursion produces a negative output excursion. Similarly, when the input decreases, the collector current and load drop decrease and the collector voltage increases. For the usual small-signal operation, the output voltage is consequently 180° out of phase with the input; this is represented by the negative sign in A_v .

Step 4: Conclusion The CE voltage gain is $A_v = -\beta_a R_L / r_i$, with magnitude $|A_v| = \beta_a R_L / r_i$. The negative sign denotes the 180° phase difference between input and output voltages.

Quick Tip to Solve: Start with $V_{CE} = V_{CC} - I_C R_L$: a rising collector current lowers the output voltage and fixes the minus sign in the gain.

27. (a) Write the important considerations which are to be taken into account while fabricating a p-n junction diode to be used as a Light Emitting Diode (LED). What should be the order of band gap of an LED, if it is required to emit light in the visible range? Draw a circuit diagram and explain its action.

(b) Draw the V-I characteristics of an LED. State two advantages of LED lamps over conventional incandescent lamps. [5 marks]

Solution:

Concept:

- An LED is a heavily doped p-n junction that emits light by radiative recombination of injected electrons and holes when forward biased.
- The photon energy is approximately the semiconductor band gap: $E_g = h\nu = hc/\lambda$, where h is Planck's constant, ν is frequency and λ is wavelength.
- A series resistor limits the forward current. The LED's anode (p-side) connects towards the positive terminal and its cathode (n-side) towards the negative terminal of the supply.

Step 1: (a) Fabrication considerations Choose a direct-band-gap semiconductor so that electron-hole recombination releases energy efficiently as photons. Silicon and germanium are unsuitable for efficient LEDs because much of their recombination energy becomes heat. Suitable compound semiconductors, such as gallium arsenide phosphide, can provide a band gap corresponding to the required visible colour. The semiconductor must be heavily doped to provide sufficient carriers and a high rate of recombination under forward bias. Its band gap must match the desired wavelength. The junction is encapsulated in a transparent cover so that the emitted radiation can escape, with suitable contacts and provision to dissipate heat. The diode must operate within its rated current; excessive current heats and damages the junction.

Step 2: (a) Band gap for visible emission Visible wavelengths are approximately 400–700 nm. Using $hc \simeq 1240 \text{ eV nm}$, the corresponding photon energies are

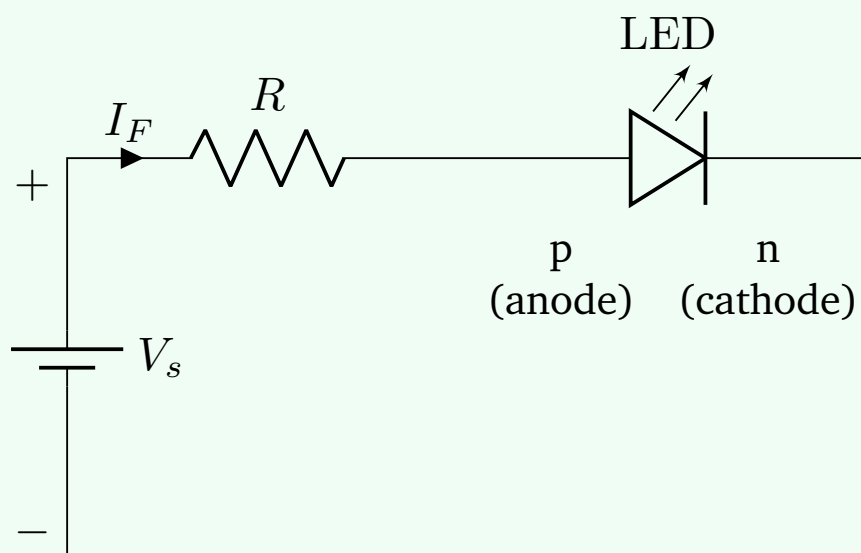
$$E_g(700 \text{ nm}) \simeq \frac{1240}{700} = 1.77 \text{ eV},$$

$$E_g(400 \text{ nm}) \simeq \frac{1240}{400} = 3.10 \text{ eV}.$$

Thus the required band gap is of the order of 2 eV, approximately 1.8–3.1 eV depending on the colour. A larger gap corresponds to a shorter emitted wavelength.

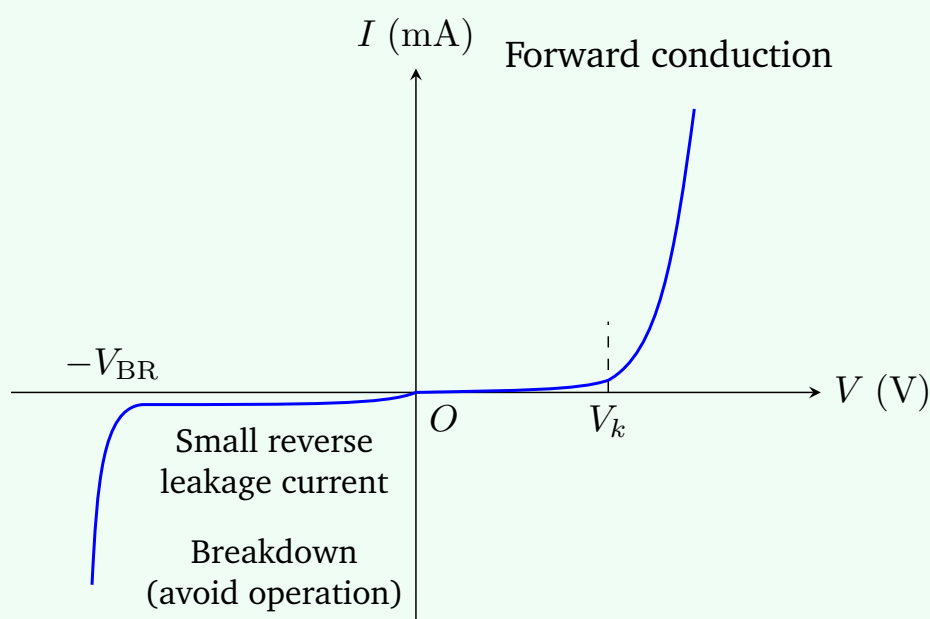
Step 3: (a) Circuit and action The circuit below forward biases the LED through a current-limiting resistor R . Electrons from the n-region and holes from the p-region are injected across the junction. Near the junction they recombine, releasing photons whose energies are approximately E_g . Increasing the safe forward current increases the recombination rate and hence the emitted light intensity. If the supply is V_s and the LED's operating forward voltage is V_F , choose $R \simeq (V_s - V_F)/I_F$ for the desired safe current I_F . Reverse bias does not give normal light emission and should be avoided beyond the LED's small reverse-voltage rating.

Quick Tip to Solve: Estimate the required colour using $E_g(\text{eV}) = 1240/\lambda(\text{nm})$, and always place a current-limiting resistor in series with the forward-biased LED.



Forward-biased LED with a series current-limiting resistor; arrows show emitted light.

Step 4: (b) V-I characteristics The characteristic resembles that of a p-n junction diode. The forward current is small below the knee voltage V_k , then rises steeply as forward voltage increases beyond the knee; appreciable light is produced in this forward-conduction region. The knee depends on the semiconductor and colour and is commonly about 1.8–3 V for visible LEDs. Under reverse bias, only a very small reverse leakage current flows until reverse breakdown. The schematic curve shows both branches; operation in reverse breakdown is avoided because it can damage the LED.



Schematic LED V-I characteristic (not to scale); the forward knee and reverse breakdown are indicated.

Step 5: (b) Two advantages over incandescent lamps First, LED lamps consume much less electrical power for the same useful light output, because they convert electrical energy into light more efficiently and waste less energy as heat. Second, they have a much longer operating life: there is no heated

Quick Tip to Solve: Estimate the required colour using $E_g(\text{eV}) = 1240/\lambda(\text{nm})$, and always place a current-limiting resistor in series with the forward-biased LED.

filament to evaporate or burn out, so replacement is needed less frequently.

Step 6: Conclusion An LED uses a heavily doped junction in a suitable direct-band-gap semiconductor, with a transparent light-extraction package. Visible emission requires a band gap of order 2 eV. Forward bias produces radiative electron-hole recombination; the forward V-I curve has a knee followed by a steep rise. LED lamps save electrical energy and have a longer life than incandescent lamps.

Quick Tip to Solve: Estimate the required colour using $E_g(\text{eV}) = 1240/\lambda(\text{nm})$, and always place a current-limiting resistor in series with the forward-biased LED.