

CBSE Class 12 Physics Question Paper 2019 with Solution PDF-Set 55/3/3

Question Paper with Solutions

Conducted by the Central Board of Secondary Education (CBSE)



Time Allowed: 3 hours

Maximum Marks: 70

General Instructions

- (i) All questions are compulsory. There are 27 questions in all.
- (ii) This question paper has four sections : Section A, Section B, Section C and Section D.
- (iii) Section A contains five questions of one mark each, Section B contains seven questions of two marks each, Section C contains twelve questions of three marks each, Section D contains three questions of five marks each.
- (iv) There is no overall choice. However, an internal choice(s) has been provided in two questions of one mark, two questions of two marks, four questions of three marks and three questions of five marks weightage. You have to attempt only one of the choices in such questions.

You may use the following values of physical constants wherever necessary :

$$c = 3 \times 10^8 \text{ m/s}$$

$$h = 6.63 \times 10^{-34} \text{ Js}$$

$$e = 1.6 \times 10^{-19} \text{ C}$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ T m A}^{-1}$$

$$\epsilon_0 = 8.854 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$$

$$\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$$

$$\text{Mass of electron } (m_e) = 9.1 \times 10^{-31} \text{ kg}$$

$$\text{Mass of neutron} = 1.675 \times 10^{-27} \text{ kg}$$

$$\text{Mass of proton} = 1.673 \times 10^{-27} \text{ kg}$$

$$\text{Avogadro's number} = 6.023 \times 10^{23} \text{ per gram mole}$$

$$\text{Boltzmann constant} = 1.38 \times 10^{-23} \text{ JK}^{-1}$$

Section A (5 marks)

1. Define intensity of radiation based on photon picture of light.

[1 mark]

Solution:

Concept:

- Each photon of frequency ν carries energy $E = h\nu$, where h is Planck's constant.
- Intensity is the radiant energy incident normally per unit area per unit time.

Step 1: Count the incident photons Let N photons strike a surface of area A , perpendicular to the beam, in time Δt . For monochromatic light, their total energy is $Nh\nu$.

Step 2: Express the energy flux Dividing this energy by area and time gives

$$I = \frac{Nh\nu}{A\Delta t}.$$

Thus, at a fixed frequency, intensity is proportional to the number of photons incident per unit area per unit time, rather than the energy of an individual photon. The SI unit is W m^{-2} .

Step 3: Conclusion Intensity is the energy carried by photons incident per unit area per unit time: $I = Nh\nu/(A\Delta t)$.

Quick Tip to Solve: Keep photon energy $h\nu$ separate from photon arrival rate when defining intensity.

2. State the condition under which a large magnification can be achieved in an astronomical telescope.

[1 mark]

Solution:

Concept:

- For an astronomical telescope in normal adjustment, the magnitude of angular magnification is $M = f_o/f_e$.
- Here f_o is the focal length of the objective and f_e is that of the eyepiece.

Step 1: Identify the controlling ratio The objective forms the real intermediate image of a distant object, and the eyepiece acts as a magnifier. The angular magnification therefore increases as the ratio f_o/f_e increases.

Step 2: Choose the focal lengths To obtain a large ratio, use an objective of large focal length and an eyepiece of small focal length, so that $f_o \gg f_e$. Increasing the objective aperture mainly improves light collection and resolving power; it does not replace the focal-length condition for large magnification.

Step 3: Conclusion A large magnification is achieved when the objective focal length is much greater than the eyepiece focal length: $f_o \gg f_e$.

Quick Tip to Solve: Remember telescope magnification as objective focal length divided by eyepiece focal length.

2. How does the angle of minimum deviation of a glass prism vary if the incident violet light is replaced by red light? [1 mark]

Solution:

Concept:

- For a prism of angle A , $\mu = \sin[(A + \delta_m)/2] / \sin(A/2)$, where δ_m is the minimum deviation.
- Under normal dispersion, glass has a smaller refractive index for red light than for violet light.

Step 1: Compare the refractive indices Red light has a longer wavelength than violet light. For ordinary glass in the visible region, the refractive index decreases with increasing wavelength, so $\mu_r < \mu_v$.

Step 2: Apply the minimum-deviation relation For the same prism angle A , the relation

$$\delta_m = 2 \sin^{-1} \left[\mu \sin \left(\frac{A}{2} \right) \right] - A$$

shows that a smaller refractive index gives a smaller minimum deviation. Consequently replacing violet light by red light reduces the angle of minimum deviation.

Step 3: Conclusion The angle of minimum deviation decreases: $\delta_{m,r} < \delta_{m,v}$.

Quick Tip to Solve: For glass, red is deviated less than violet; use that order before applying any formula.

3. Draw a plot of resistivity of copper as a function of temperature. [1 mark]

Solution:

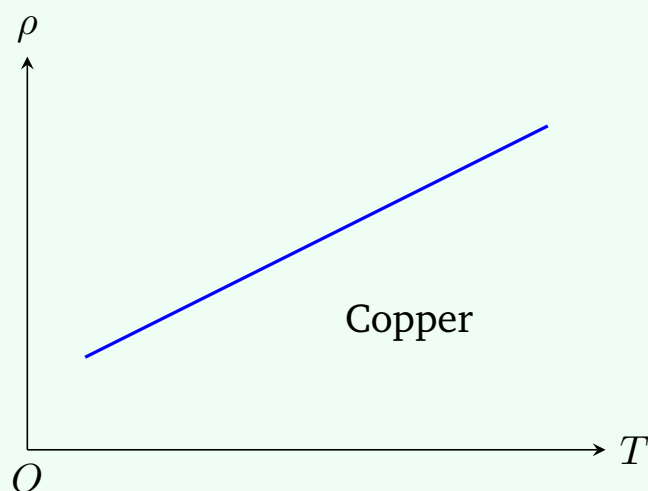
Concept:

- Copper is a metal with a positive temperature coefficient of resistivity.
- Over the usual moderate temperature range, $\rho(T) = \rho_0[1 + \alpha(T - T_0)]$, where $\alpha > 0$.

Step 1: Identify the trend As temperature increases, lattice vibrations become stronger and conduction electrons undergo more frequent scattering. Hence copper's resistivity increases with temperature.

Step 2: Plot resistivity against temperature Put temperature T on the horizontal axis and resistivity ρ on the vertical axis. In the ordinary temperature range, draw an approximately straight line with positive slope. The sketch below represents this standard metallic trend; the linear approximation is not intended to describe behaviour arbitrarily close to absolute zero.

Quick Tip to Solve: Label resistivity on the vertical axis and show a positive slope for copper.

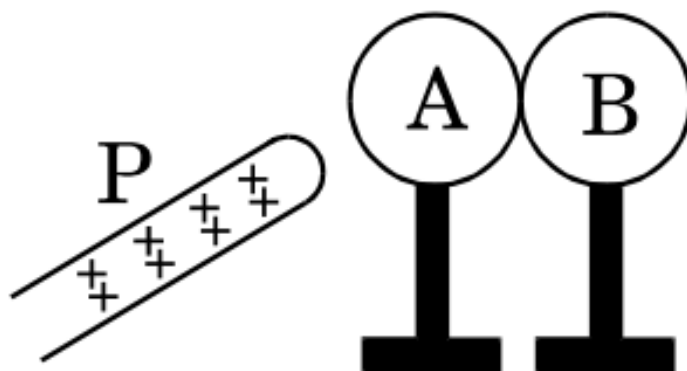


Resistivity of copper over the ordinary temperature range

Step 3: Conclusion The required plot is an approximately straight, rising ρ -versus- T graph in the ordinary temperature range.

Quick Tip to Solve: Label resistivity on the vertical axis and show a positive slope for copper.

4. Two metallic spheres A and B kept on insulating stands are in contact with each other. A positively charged rod P is brought near the sphere A as shown in the figure. The two spheres are separated from each other, and the rod P is removed. What will be the nature of charges on spheres A and B? [1 mark]



Solution:

Concept:

- Electrostatic induction redistributes free electrons in a conductor without contact with the charged inducing rod.
- The insulating stands prevent charge from flowing between the spheres and Earth.

Step 1: Redistribute charge while the spheres touch The positively charged rod near A attracts

Quick Tip to Solve: Follow electron motion towards the positive rod, then freeze the charges by separating the spheres.

electrons. Because A and B are in metallic contact, electrons move from B towards A. Sphere A acquires an excess of electrons, while sphere B is left with a deficiency of electrons. The combined spheres remain electrically neutral.

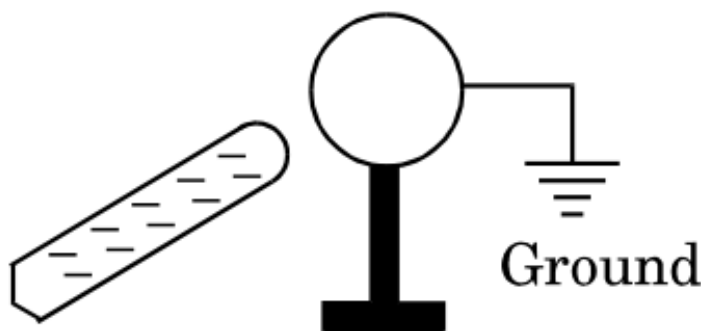
Step 2: Separate before removing the rod When the spheres are separated with the rod still present, the induced charges are trapped on the isolated spheres. Removing the rod afterwards changes the surface distribution but cannot transfer electrons between A and B. Thus the charges retain their opposite signs.

Step 3: Conclusion Sphere A becomes negatively charged and sphere B becomes positively charged.

Quick Tip to Solve: Follow electron motion towards the positive rod, then freeze the charges by separating the spheres.

OR

4. A metal sphere is kept on an insulating stand. A negatively charged rod is brought near it, then the sphere is earthed as shown. On removing the earthing, and taking the negatively charged rod away, what will be the nature of charge on the sphere? Give reason for your answer. [1 mark]



Solution:

Concept:

- A negatively charged rod repels the free electrons in a nearby metallic sphere.
- Earthing provides a path for electrons to flow between the sphere and Earth; removing it isolates the sphere.

Step 1: Allow electrons to escape through Earth On bringing the negative rod near the neutral sphere, its electrons are repelled away from the rod. When the sphere is earthed while the rod remains nearby, some electrons flow from the sphere to Earth. The sphere consequently develops a deficiency of electrons.

Step 2: Disconnect Earth before removing the rod Removing the earthing first prevents electrons from returning from Earth. Taking away the negative rod afterwards only allows the remaining charge to redistribute over the sphere's surface. The net deficiency of electrons persists, because the sphere is now insulated.

Step 3: Conclusion The sphere is positively charged, since electrons flow to Earth and cannot return

Quick Tip to Solve: In induction with earthing, disconnect Earth before removing the rod to retain the induced charge.

after earthing is removed.

Quick Tip to Solve: In induction with earthing, disconnect Earth before removing the rod to retain the induced charge.

5. Write the function of receiver in communication system.

[1 mark]

Solution:

Concept:

- A communication receiver accepts the signal arriving through the communication channel.
- Demodulation recovers the original message or information from a modulated carrier wave.

Step 1: Receive the transmitted signal The receiver picks up the transmitted signal at the receiving end of the channel. It selects the desired signal and amplifies it when necessary because the received signal is generally weak.

Step 2: Recover the message The receiver processes and demodulates the received signal to extract the original information. This recovered message is then made available in a usable form to the recipient, for example as sound from a loudspeaker.

Step 3: Conclusion The function of a receiver is to receive the transmitted signal and recover the original message or information from it.

Quick Tip to Solve: Link the receiver to the receiving end: it picks up the signal and extracts the message by demodulation.

Section B (14 marks)

6. Plot a graph showing the variation of undecayed nuclei N versus time t . From the graph, find out how one can determine the half-life and average life of the radioactive nuclei.

[2 marks]

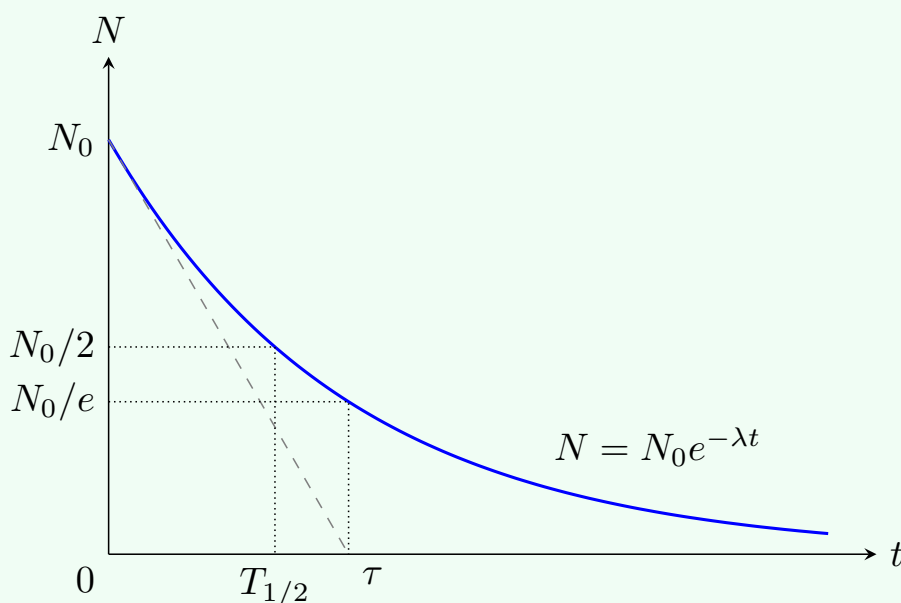
Solution:

Concept:

- Radioactive decay follows $N = N_0 e^{-\lambda t}$, where N_0 is the initial number of undecayed nuclei and λ is the decay constant.
- The half-life $T_{1/2}$ satisfies $N(T_{1/2}) = N_0/2$; the average life is $\tau = 1/\lambda$.

Step 1: Plot the exponential decay curve Take time t on the horizontal axis and the number N of undecayed nuclei on the vertical axis. The curve starts at N_0 at $t = 0$, decreases continuously, and approaches the time axis asymptotically. It never becomes exactly zero at a finite time in the exponential model.

Quick Tip to Solve: Use the curve for the half-value time; use the initial tangent's time-axis intercept for the average life.



Exponential decay, half-life and average life; the dashed straight line is the initial tangent.

Step 2: Read the half-life from the graph Locate $N_0/2$ on the vertical axis, draw a horizontal line to the decay curve, and project this intersection vertically onto the time axis. The corresponding time is $T_{1/2}$. Equivalently,

$$\frac{N_0}{2} = N_0 e^{-\lambda T_{1/2}} \Rightarrow T_{1/2} = \frac{\ln 2}{\lambda}.$$

Step 3: Determine the average life Draw the tangent to the decay curve at $t = 0$. Since the initial slope is $-\lambda N_0$, its equation is $N = N_0 - \lambda N_0 t$. Its intercept on the time axis is therefore $t = 1/\lambda = \tau$. One may also read τ as the time at which the curve falls to N_0/e , since $N(\tau) = N_0 e^{-1}$.

Step 4: Conclusion The graph gives $T_{1/2}$ at $N = N_0/2$, and gives the average life τ from the time-axis intercept of the initial tangent, or from $N = N_0/e$. Thus $\tau = T_{1/2} / \ln 2$.

Quick Tip to Solve: Use the curve for the half-value time; use the initial tangent's time-axis intercept for the average life.

7(a). Which property of nuclear force explains the constancy of binding energy per nucleon $\left(\frac{\text{BE}}{A}\right)$ for nuclei in the range $20 < A < 170$? [2 marks total]

Solution:

Concept:

- The nuclear force is short ranged: a nucleon interacts strongly only with its nearby neighbours.
- Saturation means that each nucleon is bound to approximately the same limited number of neighbours in a medium-sized nucleus.

Step 1: Identify the relevant nuclear-force property In nuclei with $20 < A < 170$, an individual

Quick Tip to Solve: Associate a nearly constant binding energy per nucleon with saturation: each nucleon binds mainly to its nearby neighbours.

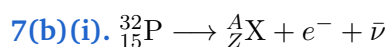
nucleon does not attract every other nucleon equally. The short range of the nuclear force limits its strong interactions to neighbouring nucleons, producing the saturation property.

Step 2: Relate saturation to binding energy per nucleon Since the number of close neighbours per nucleon is approximately constant, the contribution to binding energy per nucleon is also approximately constant. Thus the total binding energy grows roughly in proportion to the number of nucleons, $BE \propto A$, so BE/A remains nearly constant in the stated range.

Step 3: Conclusion The saturation property of the short-range nuclear force explains the nearly constant binding energy per nucleon for $20 < A < 170$.

Quick Tip to Solve: Associate a nearly constant binding energy per nucleon with saturation: each nucleon binds mainly to its nearby neighbours.

(b) Complete the following nuclear reactions:



Solution:

Concept:

- Nuclear reactions conserve mass number A , the number of nucleons, and charge number Z , the charge in units of the proton charge.
- A beta-minus electron has mass number zero and charge number -1 ; the antineutrino has zero mass number and zero charge.

Step 1: Conserve mass number In the given beta-minus decay the electron and antineutrino do not carry nucleons. Therefore the daughter nucleus has the same mass number as the phosphorus nucleus:

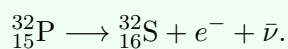
$$32 = A + 0 + 0, \quad A = 32.$$

Step 2: Conserve charge and identify the daughter Equating the charge numbers on the two sides gives

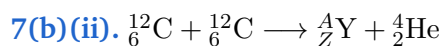
$$15 = Z + (-1) + 0, \quad Z = 16.$$

Atomic number 16 corresponds to sulphur. Physically, one neutron changes into a proton while emitting the electron and antineutrino, so the proton number increases by one while the total nucleon number stays unchanged.

Step 3: Conclusion $A = 32$, $Z = 16$, and X is sulphur:



Quick Tip to Solve: For beta-minus decay keep the mass number unchanged and increase the daughter atomic number by one.



Solution:

Concept:

- The total mass number and total charge number are separately conserved in a nuclear reaction.
- The emitted helium nucleus ${}^4_2\text{He}$ carries four nucleons and two units of positive charge.

Step 1: Balance the mass numbers The two carbon nuclei initially contain a total of $12 + 12 = 24$ nucleons. Four nucleons are carried away by the helium nucleus, so the mass number of the other product satisfies

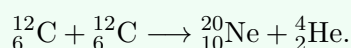
$$12 + 12 = A + 4, \quad A = 20.$$

Step 2: Balance the charge numbers Each carbon nucleus has charge number 6, so the initial total charge number is 12. After allowing for the helium nucleus, charge conservation gives

$$6 + 6 = Z + 2, \quad Z = 10.$$

Atomic number 10 identifies the remaining nucleus as neon. Both totals can be checked directly: the product mass numbers add to $20 + 4 = 24$, and their charge numbers add to $10 + 2 = 12$.

Step 3: Conclusion $A = 20$, $Z = 10$, and Y is neon:



Quick Tip to Solve: Add the two incoming carbon mass and charge numbers first, then subtract 4 and 2 for the emitted helium nucleus.

8. Obtain Bohr's quantisation condition for angular momentum of electron orbiting in n^{th} orbit in hydrogen atom on the basis of the wave picture of an electron using de Broglie hypothesis. [2 marks]

Solution:

Concept:

- According to de Broglie's hypothesis, an electron of momentum $p = mv$ has wavelength $\lambda = h/p = h/(mv)$, where h is Planck's constant.
- A stable circular orbit supports a standing electron wave: its circumference must contain an integral number of wavelengths. The orbital angular momentum is $L = mvr$.

Step 1: Apply the standing-wave condition Consider an electron in the n^{th} circular orbit of radius r_n . Its matter wave must return to the same phase after one complete revolution so that successive waves reinforce each other. Therefore the circumference equals an integer number n of wavelengths:

$$2\pi r_n = n\lambda, \quad n = 1, 2, 3, \dots$$

A non-integral number of wavelengths would fail this condition and would not form a stationary wave

Quick Tip to Solve: Begin with $2\pi r = n\lambda$, then insert $\lambda = h/(mv)$; the product mvr gives the required angular momentum.

around the orbit.

Step 2: Substitute the de Broglie wavelength Let the electron mass be m and its orbital speed be v_n . Substituting $\lambda = h/(mv_n)$ into the circumference condition and rearranging gives

$$\begin{aligned}2\pi r_n &= \frac{nh}{mv_n}, \\2\pi mv_n r_n &= nh, \\mv_n r_n &= \frac{nh}{2\pi}.\end{aligned}$$

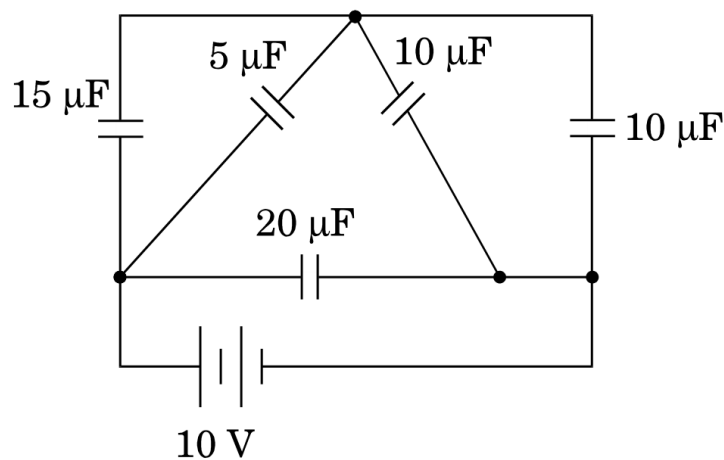
The quantity $mv_n r_n$ is the magnitude of the electron's orbital angular momentum.

Step 3: Conclusion Bohr's angular-momentum quantisation condition follows from the standing-wave requirement:

$$L_n = mv_n r_n = n \frac{h}{2\pi} = n\hbar, \quad n = 1, 2, 3, \dots$$

Quick Tip to Solve: Begin with $2\pi r = n\lambda$, then insert $\lambda = h/(mv)$; the product mvr gives the required angular momentum.

9. The figure shows a network of five capacitors connected to a 10 V battery. Calculate the charge acquired by the $5\ \mu\text{F}$ capacitor. [2 marks]



Solution:

Concept:

- Capacitors connected between the same two conducting nodes are in parallel: their capacitances add and their potential differences are equal.
- For a capacitor, the magnitude of charge on either plate is $Q = CV$, where C is capacitance and V is its potential difference. Series groups carry equal charge.

Step 1: Identify the parallel groups Call the top junction T , the lower left junction L , and the lower right junction R . The $15\ \mu\text{F}$ and $5\ \mu\text{F}$ capacitors both join L to T , so their equivalent capacitance is

Quick Tip to Solve: Trace the wires to identify nodes before combining capacitors: the sloping and outer capacitors here form two equal parallel groups.

$C_{LT} = 15 + 5 = 20 \mu\text{F}$. Both $10 \mu\text{F}$ capacitors join T to R , giving $C_{TR} = 10 + 10 = 20 \mu\text{F}$. These two groups form a series branch across the battery. The horizontal $20 \mu\text{F}$ capacitor is a separate parallel branch directly across L and R .

Step 2: Find the voltage across the required capacitor The two series groups have equal capacitances and equal charge, so the battery voltage divides equally between them:

$$V_{LT} = V_{TR} = \frac{10}{2} = 5 \text{ V}.$$

The $5 \mu\text{F}$ capacitor belongs to the L - T parallel group and therefore has this same 5 V across it. Its charge is

$$Q_5 = (5 \times 10^{-6})(5) = 25 \times 10^{-6} \text{ C}.$$

Step 3: Conclusion The charge acquired by the $5 \mu\text{F}$ capacitor is $25 \mu\text{C}$ in magnitude; its two plates carry equal and opposite charges.

Quick Tip to Solve: Trace the wires to identify nodes before combining capacitors: the sloping and outer capacitors here form two equal parallel groups.

10. A 0.5 m long solenoid of 10 turns/cm has area of cross-section 1 cm^2 . Calculate the voltage induced across its ends if the current in the solenoid is changed from 1 A to 2 A in 0.1 s . [2 marks]

Solution:

Concept:

- For a long air-core solenoid, the self-inductance is $L = \mu_0 n^2 A \ell$, where n is the number of turns per metre, A its cross-sectional area, and ℓ its length.
- A changing current produces a self-induced emf $\varepsilon = -L dI/dt$. The negative sign expresses Lenz's law: the emf opposes the current change.

Step 1: Convert the data and calculate the inductance The winding density must be expressed per metre:

$$n = 10 \text{ turns/cm} = 1000 \text{ m}^{-1}, \quad A = 1 \text{ cm}^2 = 10^{-4} \text{ m}^2.$$

With $\ell = 0.5 \text{ m}$ and $\mu_0 = 4\pi \times 10^{-7} \text{ H/m}$,

$$L = (4\pi \times 10^{-7})(1000)^2(10^{-4})(0.5) = 2\pi \times 10^{-5} \text{ H}.$$

This uses the usual long-solenoid approximation and assumes an air core since no magnetic core is specified.

Step 2: Apply the law of self-induction The current increases by $\Delta I = 2 - 1 = 1 \text{ A}$ in $\Delta t = 0.1 \text{ s}$. Therefore its average rate of change is 10 A/s , giving

$$|\varepsilon| = L \frac{\Delta I}{\Delta t} = (2\pi \times 10^{-5})(10) = 2\pi \times 10^{-4} \text{ V}.$$

For a uniform increase this is also the emf throughout the interval. The induced polarity tends to

Quick Tip to Solve: Convert turns per centimetre to turns per metre before squaring the winding density in the inductance formula.

oppose the increase of current; it does not reinforce it.

Step 3: Conclusion The magnitude of the voltage induced across the solenoid is $6.28 \times 10^{-4} \text{ V}$, or 0.628 mV.

Quick Tip to Solve: Convert turns per centimetre to turns per metre before squaring the winding density in the inductance formula.

OR

10. A small flat search coil of area 5 cm^2 with 140 closely wound turns is placed between the poles of a powerful magnet producing magnetic field 0.09 T and then quickly removed out of the field region. Calculate

(a) change of magnetic flux through the coil, and

(b) emf induced in the coil.

[2 marks]

Solution:

Concept:

- Magnetic flux through one turn is $\Phi = BA \cos \theta$, where θ is the angle between the magnetic field and the normal to the coil.
- Faraday's law gives the average induced emf as $\varepsilon_{\text{av}} = -N\Delta\Phi/\Delta t$, where $N\Delta\Phi$ is the change in flux linkage of an N -turn coil.

Step 1: Calculate the change in flux Using the standard search-coil arrangement with its plane perpendicular to the field, the initial flux per turn is

$$\Phi_i = BA = (0.09)(5 \times 10^{-4}) = 4.5 \times 10^{-5} \text{ Wb}.$$

Outside the magnet's field region, $\Phi_f = 0$. Hence

$$\Delta\Phi = \Phi_f - \Phi_i = -4.5 \times 10^{-5} \text{ Wb}$$

per turn. For all 140 turns the flux-linkage change is

$$\Delta(N\Phi) = 140\Delta\Phi = -6.3 \times 10^{-3} \text{ Wb-turn}.$$

Thus the flux decreases; the magnitude of the total linked-flux change is $6.3 \times 10^{-3} \text{ Wb-turn}$. If the coil has a different orientation, these values are multiplied by $\cos \theta$.

Step 2: Express the induced emf using the removal time If the removal takes Δt seconds, the average magnitude is

$$|\varepsilon_{\text{av}}| = \frac{N|\Delta\Phi|}{\Delta t} = \frac{6.3 \times 10^{-3}}{\Delta t} \text{ V},$$

with Δt expressed in seconds. The printed question gives no removal time, so "quickly removed" cannot yield a unique numerical emf. The emf exists during removal and its polarity opposes the loss of magnetic flux, as required by Lenz's law.

Quick Tip to Solve: Distinguish flux per turn from flux linkage, and check that a time interval is supplied before trying to calculate a numerical emf.

Step 3: Conclusion (a) For perpendicular incidence, the flux change is $-4.5 \times 10^{-5} \text{ Wb}$ per turn, or $-6.3 \times 10^{-3} \text{ Wb-turn}$ in flux linkage. (b) The average induced emf has magnitude $6.3 \times 10^{-3} / \Delta t \text{ V}$; a numerical voltage cannot be determined without the removal time.

Quick Tip to Solve: Distinguish flux per turn from flux linkage, and check that a time interval is supplied before trying to calculate a numerical emf.

11. In the case of a concave mirror of focal length f , when an object is kept between f and $2f$, show that its image is formed beyond $2f$. [2 marks]

Solution:

Concept:

- The mirror formula is $1/f_s = 1/u + 1/v$, where f_s , u , and v are signed focal length, object distance and image distance in the Cartesian convention.
- For a concave mirror, a real object and a real image lie in front of the mirror, so their signed distances are negative. Compare their positive distance magnitudes to establish the image position.

Step 1: Write the mirror equation using distance magnitudes Let $F = |f_s| > 0$ be the focal-length magnitude denoted by f in the question, and let $x > 0$ be the object's distance from the pole. The stated position means $F < x < 2F$. With $u = -x$ and $f_s = -F$, the mirror formula gives

$$-\frac{1}{F} = -\frac{1}{x} + \frac{1}{v}, \quad \frac{1}{v} = -\frac{x - F}{Fx}.$$

Since $x > F$, this yields $v < 0$: the image is real and lies in front of the mirror. Its positive distance magnitude is therefore

$$y = -v = \frac{Fx}{x - F}.$$

Step 2: Prove that the image distance exceeds twice the focal length Subtract $2F$ from the expression for y :

$$y - 2F = \frac{Fx - 2F(x - F)}{x - F} = \frac{F(2F - x)}{x - F}.$$

Both $2F - x$ and $x - F$ are strictly positive because the object lies between F and $2F$. Thus $y - 2F > 0$, establishing $y > 2F$. Equivalently, the image is beyond the centre of curvature. The limiting positions agree: as x approaches F , y tends to infinity; at $x = 2F$, $y = 2F$.

Step 3: Conclusion For an object strictly between f and $2f$, the image is real and forms at a distance $y > 2f$ in front of the concave mirror, hence beyond $2f$.

Quick Tip to Solve: To prove an image lies beyond a specified point, subtract that distance from the mirror-formula result and examine the signs.

12. Five point charges, each of charge $+q$ are placed on five vertices of a regular hexagon of side l . Find the magnitude of the resultant force on a charge $-q$ placed at the centre of the hexagon. [2 marks]

Solution:

Concept:

- Coulomb's law gives the force magnitude between two point charges: $F = k|q_1q_2|/r^2$, where $k = 1/(4\pi\epsilon_0)$ and r is their separation.
- Forces add as vectors. At the centre of a regular hexagon, forces from equal charges at opposite vertices cancel. The distance from its centre to each vertex equals the side length l .

Step 1: Use the symmetry of a complete hexagon Imagine placing one additional charge $+q$ at the unoccupied sixth vertex. Each positive vertex charge attracts the central charge $-q$ towards that vertex. With all six vertices occupied, there are three diametrically opposite pairs. The two forces in each pair have the same magnitude and opposite directions, so the total force at the centre is zero:

$$\vec{F}_{\text{five}} + \vec{F}_{\text{missing}} = \vec{0}.$$

Consequently, the force due to the actual five charges is $\vec{F}_{\text{five}} = -\vec{F}_{\text{missing}}$. Its magnitude equals that of the force which the hypothetical sixth charge would exert.

Step 2: Calculate the force magnitude and direction Joining the centre to two adjacent vertices makes an equilateral triangle, since the central angle is 60° . Thus the centre-to-vertex distance is l . By Coulomb's law, the hypothetical sixth charge exerts a force of magnitude

$$F_{\text{missing}} = \frac{1}{4\pi\epsilon_0} \frac{|(+q)(-q)|}{l^2} = \frac{q^2}{4\pi\epsilon_0 l^2}.$$

The actual resultant points opposite to the direction towards the vacant vertex: it points from the centre towards the occupied vertex diametrically opposite the vacancy. Simply adding the five magnitudes would ignore their different directions.

Step 3: Conclusion The magnitude of the resultant force is $F = \frac{q^2}{4\pi\epsilon_0 l^2}$, directed away from the vacant vertex towards the opposite occupied vertex.

Quick Tip to Solve: Complete the symmetric set with a hypothetical sixth charge, then reverse that charge's force instead of adding five vectors individually.

12. A simple pendulum consists of a small sphere of mass m suspended by a thread of length l . The sphere carries a positive charge q . The pendulum is placed in a uniform electric field of strength E directed vertically downwards. Find the period of oscillation of the pendulum due to the electrostatic force acting on the sphere, neglecting the effect of the gravitational force. [2 marks]

Solution:

Concept:

- A positive charge q in a uniform electric field E experiences a force $F_e = qE$ in the direction of the field.
- For small angular displacement θ , the tangential component of this downward force provides the restoring force. The thread has length l , and the sphere has mass m .

Step 1: Resolve the electric force along the arc Gravity is neglected as specified, so the only downward force is qE . Tension acts along the thread and has no tangential component. At displacement θ from the vertically downward equilibrium position, the electric force component along the arc is $-qE \sin \theta$. Its negative sign shows that it opposes the displacement. Newton's law along the arc gives

$$ml\ddot{\theta} = -qE \sin \theta.$$

Step 2: Use the small-oscillation approximation For small oscillations, $\sin \theta \simeq \theta$, so the equation becomes

$$\ddot{\theta} + \frac{qE}{ml}\theta = 0.$$

Comparing with the simple harmonic motion equation $\ddot{\theta} + \omega^2\theta = 0$, identify $\omega^2 = qE/(ml)$. The period is obtained from $T = 2\pi/\omega$. This approximation is the usual meaning of the period of a simple pendulum; large-amplitude oscillations have an amplitude-dependent period.

Step 3: Conclusion The period of small oscillations, with gravity neglected, is $T = 2\pi\sqrt{\frac{ml}{qE}}$.

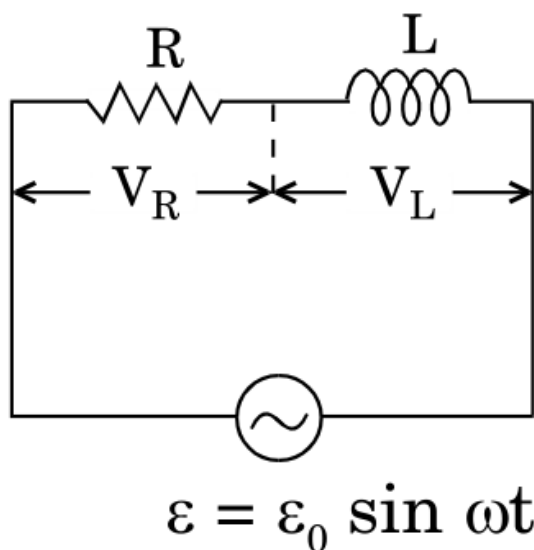
Quick Tip to Solve: Replace the usual downward acceleration g by qE/m in the small-angle pendulum formula; do not add gravity when the question excludes it.

Section C (36 marks)

13. (a) An ac circuit as shown in the figure has an inductor of inductance L and a resistor of resistance R connected in series. Using the phasor diagram, explain why the voltage in the circuit will lead the current in phase.

(b) The potential difference across the resistor is 160 V and that across the inductor is 120 V. Find the effective value of the applied voltage. If the effective current in the circuit be 1.0 A, calculate the total impedance of the circuit.

(c) What will be the potential difference in the circuit when direct current is passed through the circuit? [3 marks]



Solution:

Concept:

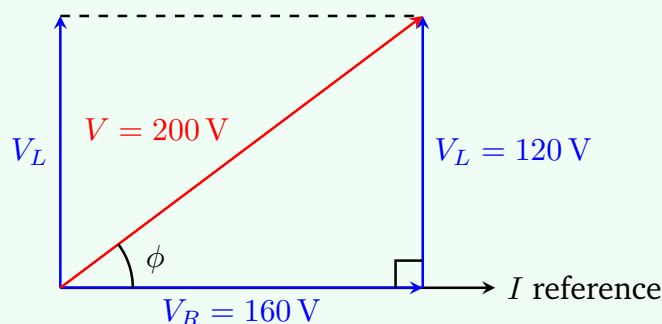
- In a series circuit the same current flows through R and the ideal inductor L . The resistor voltage $V_R = IR$ is in phase with current, whereas $V_L = IX_L$ leads it by 90° , with $X_L = \omega L$.
- Perpendicular voltage phasors add as $V = \sqrt{V_R^2 + V_L^2}$; the impedance magnitude is $Z = V/I$. All numerical voltages and currents here are effective (rms) values.

Step 1: Construct the voltage phasors for part (a) Take current I as the horizontal reference. Draw V_R along it and V_L vertically upward because the inductive voltage leads the current by a quarter cycle. Their vector sum V lies above the current axis and therefore leads current by an angle ϕ . For the ideal series RL circuit,

$$\tan \phi = \frac{V_L}{V_R} = \frac{X_L}{R}.$$

Since R and X_L are positive, $0 < \phi < 90^\circ$. Equivalently, the current lags behind the applied voltage. The diagram shows the head-to-tail sum as well as the two component directions.

Quick Tip to Solve: Use Pythagoras for the perpendicular RL voltage phasors; for steady dc, set the ideal inductor's voltage to zero before calculating the resistor drop.



Voltage phasors in a series RL circuit; current is the horizontal phase reference.

Step 2: Calculate the applied voltage and impedance for part (b) The component voltages must be added vectorially, rather than arithmetically, because their phases differ by 90° . Substituting the given values gives

$$V = \sqrt{(160)^2 + (120)^2} = \sqrt{40000} = 200 \text{ V},$$

$$Z = \frac{V}{I} = \frac{200}{1.0} = 200 \Omega.$$

Also $R = 160 \Omega$, $X_L = 120 \Omega$, and $\phi = \tan^{-1}(120/160) \simeq 36.9^\circ$, consistent with the phasor construction.

Step 3: Examine steady direct current for part (c) For steady dc, $dI/dt = 0$, so the voltage across the ideal inductor is $V_L = L dI/dt = 0$. The whole potential difference is therefore across the resistor: $V_{dc} = I_{dc}R$. If the stated current of 1.0 A is maintained in dc, the circuit potential difference is $1.0 \times 160 = 160 \text{ V}$. A different dc current would require $160I_{dc}$ volts; the steady dc formula specifies the assumption behind the numerical answer. During switching, current is changing and the inductor can have a transient voltage.

Step 4: Conclusion (a) The voltage leads the current, as shown by the phasor sum. (b) $V_{\text{rms}} = 200 \text{ V}$ and $Z = 200 \Omega$. (c) In steady dc, $V_L = 0$ and $V = I_{dc}R$, giving 160 V for the same 1.0 A current.

Quick Tip to Solve: Use Pythagoras for the perpendicular RL voltage phasors; for steady dc, set the ideal inductor's voltage to zero before calculating the resistor drop.

13. An ac circuit consists of a series combination of circuit elements X and Y . The current is ahead of the voltage in phase by $\frac{\pi}{4}$. If element X is a pure resistor of $100\ \Omega$,

(a) name the circuit element Y .

(b) calculate the rms value of current, if rms value of voltage is 141 V .

(c) what will happen if the ac source is replaced by a dc source?

[3 marks]

Solution:

Concept:

- In a series RC circuit, the voltage across the resistor is in phase with current, whereas the capacitor voltage lags current by $\pi/2$. The current therefore leads the applied voltage.
- If the current leads by angle ϕ , then $\tan \phi = X_C/R$, where $X_C = 1/(\omega C)$ is the capacitive reactance, R is resistance, and $Z = \sqrt{R^2 + X_C^2}$ is the impedance magnitude.

Step 1: Identify element Y in part (a) Since the current is ahead of the voltage, the non-resistive element must be capacitive. Thus Y is a capacitor. An inductor would make current lag the applied voltage, and another pure resistor would produce no phase difference. For the given lead angle,

$$X_C = R \tan \phi = 100 \tan \left(\frac{\pi}{4} \right) = 100\ \Omega.$$

This identifies the reactance at the operating ac frequency; a capacitance value cannot be obtained without that frequency.

Step 2: Calculate the rms current in part (b) Combine the resistance and reactance at right angles to obtain the impedance. Apply the rms form of Ohm's law:

$$Z = \sqrt{100^2 + 100^2} = 100\sqrt{2}\ \Omega,$$

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{Z} = \frac{141}{100\sqrt{2}}\text{ A} \simeq 0.997\text{ A} \simeq 1.0\text{ A}.$$

Using the full impedance is essential: dividing the source voltage by resistance alone would neglect the capacitor's phase-dependent voltage drop.

Step 3: Replace the ac source by dc in part (c) On connecting a dc source, the initially uncharged capacitor charges through the resistor, so a transient current may flow. As the capacitor voltage approaches the source voltage, the charging current decreases. In the steady state, an ideal capacitor blocks dc and behaves as an open circuit, so the series current becomes zero. There is then no voltage across the resistor and the dc source voltage appears across the capacitor. This steady-state result is distinct from the brief charging transient immediately after switching.

Step 4: Conclusion (a) Y is a capacitor with $X_C = 100\ \Omega$ at the given frequency. (b) $I_{\text{rms}} \simeq 1.0\text{ A}$. (c) After the charging transient, the capacitor blocks dc and $I_{\text{dc}} = 0$.

Quick Tip to Solve: A current lead identifies a capacitor; a 45° lead makes $X_C = R$, so immediately use $Z = \sqrt{2}R$.

14. Write the range of the wavelength of the following electromagnetic radiations:

(a) Infrared rays

(b) Ultraviolet rays

(c) γ -rays

Write one use of each of the above.

[3 marks]

Solution:

Concept:

- Electromagnetic radiation is classified by its wavelength λ or frequency ν , related in vacuum by $c = \lambda\nu$. Shorter wavelength therefore corresponds to higher frequency.
- The spectral boundaries are conventional approximate limits. Infrared lies on the long-wavelength side of visible light, ultraviolet on its short-wavelength side, and gamma radiation has the shortest wavelengths of these three bands.

Step 1: State the infrared range and a use for part (a) The usual school-level infrared range extends from about 700 nm to 1 mm, or equivalently

$$7 \times 10^{-7} \text{ m to } 10^{-3} \text{ m.}$$

One use is in television remote controls: an infrared emitter transmits coded pulses to a receiver in the television. This application uses infrared radiation beyond the red end of the visible spectrum.

Step 2: State the ultraviolet range and a use for part (b) The approximate ultraviolet range is 1 nm to 400 nm, or

$$10^{-9} \text{ m to } 4 \times 10^{-7} \text{ m.}$$

One use is sterilisation of water using ultraviolet lamps. Suitable ultraviolet wavelengths damage microorganisms and help disinfect the water. This band has shorter wavelengths and higher frequencies than visible light.

Step 3: State the gamma-ray range and a use for part (c) In the conventional school spectrum, gamma rays have wavelengths less than about 10^{-3} nm, i.e.

$$\lambda < 10^{-12} \text{ m.}$$

One use is radiotherapy for cancer, where controlled gamma radiation is directed at cancerous tissue to destroy tumour cells. Gamma rays have very high photon energies because photon energy increases with frequency. The quoted limit is a conventional spectral boundary; the X-ray and gamma-ray energy ranges can overlap.

Step 4: Conclusion Infrared: approximately 700 nm to 1 mm, used in television remote controls. Ultraviolet: approximately 1 nm to 400 nm, used in water sterilisation. Gamma rays: $\lambda < 10^{-12}$ m, used in cancer radiotherapy.

Quick Tip to Solve: First place infrared beyond red and ultraviolet beyond violet; then convert $1 \text{ nm} = 10^{-9} \text{ m}$ and $1 \text{ mm} = 10^{-3} \text{ m}$ carefully.

15. (a) Plot a graph to show the variation of stopping potential with frequency of incident radiation in relation to photoelectric effect.

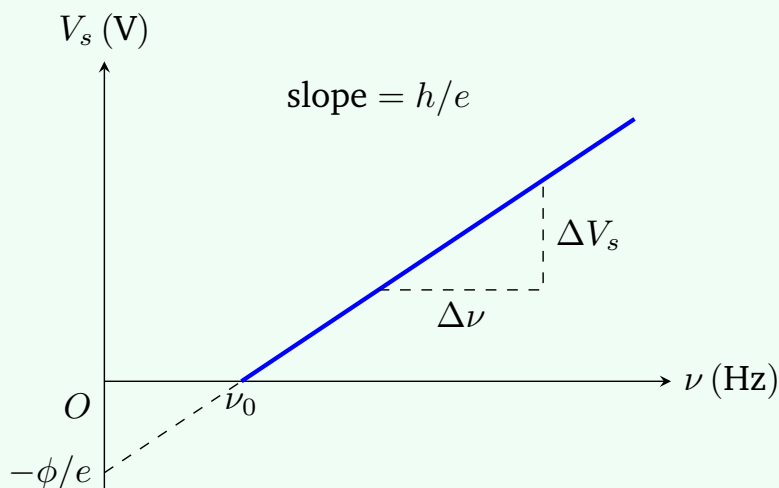
(b) Use Einstein's photoelectric equation to show how from this graph, (i) Threshold frequency, and (ii) Planck's constant can be determined. [3 marks]

Solution:

Concept:

- A photon of frequency ν has energy $h\nu$, where h is Planck's constant. The work function ϕ is the minimum energy needed to liberate an electron from the metal.
- Einstein's photoelectric equation is $K_{\max} = h\nu - \phi$. The stopping potential V_s satisfies $eV_s = K_{\max}$, where e is the magnitude of the electronic charge.
- The threshold frequency ν_0 obeys $h\nu_0 = \phi$. Photoelectric emission is possible only when $\nu \geq \nu_0$.

Step 1: (a) Plot stopping potential against frequency Take frequency ν on the horizontal axis and the positive magnitude of stopping potential V_s on the vertical axis. For a fixed photosensitive metal, the graph is a straight line starting at $(\nu_0, 0)$ and rising with increasing frequency. Below ν_0 , no photoelectrons are emitted, so there is no physical stopping-potential branch in that region. The broken extension in the figure is only an extrapolation of the straight line.



Stopping potential versus frequency; the dashed segment is an extrapolation.

Step 2: (b) Relate the straight line to Einstein's equation Substituting $K_{\max} = eV_s$ and $\phi = h\nu_0$ gives

$$eV_s = h\nu - \phi = h(\nu - \nu_0), \quad V_s = \frac{h}{e}\nu - \frac{\phi}{e}.$$

This is the straight-line form $y = mx + c$, with slope h/e and extrapolated vertical intercept $-\phi/e$. The slope is positive because more energetic photons leave the emitted electrons with greater maximum kinetic energy.

Step 3: (b)(i) Determine the threshold frequency At the horizontal-axis intercept, the stopping potential and maximum kinetic energy are zero. Hence $0 = h\nu_0 - \phi$. Read the frequency coordinate of this intercept directly from the graph; it gives $\nu_0 = \phi/h$. This is the minimum incident frequency needed for emission from this metal.

Quick Tip to Solve: First write $eV_s = h\nu - \phi$: the frequency-axis intercept gives ν_0 , while the slope must be multiplied by e to obtain h .

Step 4: (b)(ii) Determine Planck's constant from the slope Choose two well-separated points (ν_1, V_1) and (ν_2, V_2) on the straight line. Its slope is

$$m = \frac{V_2 - V_1}{\nu_2 - \nu_1} = \frac{h}{e}.$$

Multiply the measured slope by the electronic charge to obtain $h = em$. With potential measured in volts and frequency in hertz, this gives h in J s.

Step 5: Conclusion The graph is linear for $\nu \geq \nu_0$. Its horizontal-axis intercept gives the threshold frequency $\boxed{\nu_0}$, and its slope gives $\boxed{h = e \Delta V_s / \Delta \nu}$.

Quick Tip to Solve: First write $eV_s = h\nu - \phi$: the frequency-axis intercept gives ν_0 , while the slope must be multiplied by e to obtain h .

OR

15. (a) How does one explain the emission of electrons from a photosensitive surface with the help of Einstein's photoelectric equation?

(b) Work function of aluminium is 4.2 eV. If two photons each of energy 2.5 eV are incident on its surface, will the emission of electrons take place? Justify your answer.

(c) The stopping potential in an experiment on photoelectric effect is 1.5 V. What is the maximum kinetic energy of the photoelectrons emitted? Calculate in Joules. [3 marks]

Solution:

Concept:

- The energy of a photon is $h\nu$, where h is Planck's constant and ν is the incident frequency. The work function ϕ is the minimum energy required to remove an electron from the surface.
- Einstein's photoelectric equation is $h\nu = \phi + K_{\max}$. In the ordinary photoelectric effect, one electron absorbs the energy of one photon in a single interaction.
- The stopping potential V_s removes the maximum kinetic energy: $K_{\max} = eV_s$, with $e = 1.6 \times 10^{-19}$ C.

Step 1: (a) Explain emission using Einstein's equation When a photon falls on the photosensitive surface, it can transfer its whole energy $h\nu$ to an electron. The electron uses energy ϕ to overcome the attraction binding it to the metal. The remaining energy appears as kinetic energy. For an electron emitted with the greatest kinetic energy,

$$K_{\max} = h\nu - \phi.$$

Thus emission is possible only when $h\nu \geq \phi$, or $\nu \geq \nu_0 = \phi/h$. If the frequency is below this threshold, increasing the number of incident photons does not cause ordinary photoelectric emission. Electrons that lose energy within the metal emerge with kinetic energies smaller than $K_{\max}$.

Step 2: (b) Compare each photon's energy with aluminium's work function Here $\phi = 4.2$ eV, whereas

Quick Tip to Solve: Compare the energy of each photon with the work function; for stopping potential, multiply its numerical value by e to get kinetic energy in joules.

the energy of each photon is only 2.5 eV. Since

$$2.5 \text{ eV} < 4.2 \text{ eV},$$

neither photon supplies the energy required to liberate an electron. The energies of two incident photons are not added together for one electron in the single-photon photoelectric process described by Einstein's equation. Therefore the total incident energy 5.0 eV does not establish that emission occurs. There is no photoelectric emission under the stated conditions.

Step 3: (c) Calculate the maximum kinetic energy in joules A stopping potential of 1.5 V just prevents even the fastest emitted electrons from reaching the collector. The retarding electric field removes energy eV_s , so

$$\begin{aligned} K_{\max} &= eV_s \\ &= (1.6 \times 10^{-19} \text{ C})(1.5 \text{ V}) \\ &= 2.4 \times 10^{-19} \text{ J}. \end{aligned}$$

Equivalently, the energy is 1.5 eV; multiplying by $1.6 \times 10^{-19} \text{ J/eV}$ gives the same result. This stopping-potential experiment is a separate situation from part (b).

Step 4: Conclusion (a) Emission follows $h\nu = \phi + K_{\max}$ and requires $h\nu \geq \phi$. (b) No electrons are emitted because each 2.5 eV photon has insufficient energy. (c) $K_{\max} = 2.4 \times 10^{-19} \text{ J}$.

Quick Tip to Solve: Compare the energy of each photon with the work function; for stopping potential, multiply its numerical value by e to get kinetic energy in joules.

16. (a) Explain how an unpolarised light gets polarised when incident on the interface separating the two transparent media.

(b) Green light is incident at the polarising angle on a certain transparent medium. The angle of refraction is 30° . Find

(i) polarising angle, and

(ii) refractive index of the medium.

[3 marks]

Solution:

Concept:

- Unpolarised light has electric-field vibrations in all directions perpendicular to its propagation. Plane-polarised light has vibrations confined to one fixed direction perpendicular to propagation.
- The plane of incidence contains the incident ray and the normal to the interface. At Brewster's polarising angle i_p , the reflected light is completely polarised with its electric field perpendicular to this plane.
- At the polarising angle, the reflected and refracted rays are perpendicular: $i_p + r = 90^\circ$. Brewster's law is $n_2/n_1 = \tan i_p$, where n_1 and n_2 are the refractive indices of the incident and transmitting media.

Step 1: (a) Explain polarisation at a transparent interface Resolve the electric vibrations of incident unpolarised light into components parallel and perpendicular to the plane of incidence. At an oblique

Quick Tip to Solve: At polarising incidence, first use $i_p + r = 90^\circ$; then apply $\mu = \tan i_p$ for light entering the medium from air.

interface, these two components are reflected in different proportions, so the reflected beam is generally partially polarised. At the particular incidence angle i_p , called the polarising or Brewster angle, the parallel component has no reflected contribution. Consequently, the reflected light contains only electric vibrations perpendicular to the plane of incidence and is completely plane-polarised. The transmitted beam is generally partially polarised. At this angle the reflected and refracted rays make a right angle with one another.

Step 2: (b)(i) Find the polarising angle The angle of refraction is $r = 30^\circ$. Since incidence is at the polarising angle, use the perpendicularity relation

$$i_p + r = 90^\circ.$$

Therefore

$$i_p = 90^\circ - 30^\circ = 60^\circ.$$

Both incidence and refraction angles are measured from the normal to the interface, not from the surface.

Step 3: (b)(ii) Find the refractive index Taking the incident medium as air, with refractive index approximately unity, Snell's law gives the refractive index μ of the transparent medium:

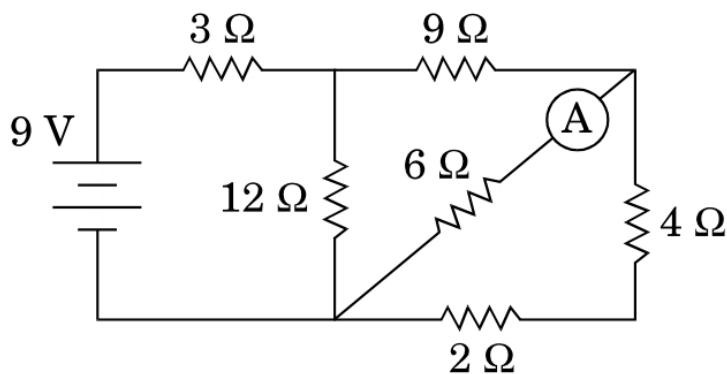
$$\mu = \frac{\sin i_p}{\sin r} = \frac{\sin 60^\circ}{\sin 30^\circ} = \frac{\sqrt{3}/2}{1/2} = \sqrt{3}.$$

The same result follows directly from Brewster's law, $\mu = \tan i_p = \tan 60^\circ$. Thus $\mu \approx 1.73$; refractive index is dimensionless. The specified green light fixes the wavelength at which this refractive index applies.

Step 4: Conclusion At Brewster's angle, the reflected beam is completely plane-polarised with electric vibrations perpendicular to the plane of incidence. The required values are $i_p = 60^\circ$ and, for incidence from air, $\mu = \sqrt{3} \approx 1.73$.

Quick Tip to Solve: At polarising incidence, first use $i_p + r = 90^\circ$; then apply $\mu = \tan i_p$ for light entering the medium from air.

17. In the circuit shown in the figure, find the value of the current shown in the ammeter A . [3 marks]



Solution:

Concept:

- For an ideal ammeter its resistance is negligible, so the diagonal branch has resistance 6Ω . Ohm's law gives $I = V/R$.
- Resistors in series carry the same current; resistors in parallel have the same potential difference. Reduce the circuit by identifying common junctions, not by the apparent orientation of the lines.

Step 1: Reduce the right-hand network Call the upper central junction P , the lower central junction Q , and the upper right junction R . The 4Ω and 2Ω resistors are in series between R and Q , since their common lower-right junction has no other branch. Their sum is 6Ω . This path is in parallel with the diagonal 6Ω resistor and ammeter, so

$$R_{RQ} = \frac{6 \times 6}{6 + 6} = 3\Omega.$$

The 9Ω resistor from P to R is in series with this equivalent resistance. Thus the right-hand path between P and Q has resistance $9 + 3 = 12\Omega$.

Step 2: Find the supply current and the branch current The direct 12Ω resistor between P and Q is in parallel with the right-hand 12Ω path. Therefore

$$R_{PQ} = \frac{12 \times 12}{12 + 12} = 6\Omega, \quad R_{\text{total}} = 3 + 6 = 9\Omega.$$

The battery supplies $I_{\text{total}} = 9/9 = 1\text{ A}$. Hence the potential difference across P, Q is $1 \times 6 = 6\text{ V}$. The current entering the right-hand path is $6/12 = 0.5\text{ A}$. At R , it divides equally between the two equal 6Ω paths. Equivalently, $V_R - V_Q = 0.5 \times 3 = 1.5\text{ V}$, which can be used directly in Ohm's law for the diagonal branch.

Step 3: Conclusion The ammeter reads $I_A = 1.5/6 = \boxed{0.25\text{ A}}$. With the printed battery polarity, the current in the diagonal flows from the upper-right junction towards the lower central junction.

Quick Tip to Solve: First combine the right-side 4-ohm and 2-ohm resistors; their 6-ohm sum matches the diagonal branch and makes two equal current divisions visible.

18. (a) Explain briefly, using a proper diagram, the difference in behaviour of a conductor and a dielectric in the presence of external electric field.

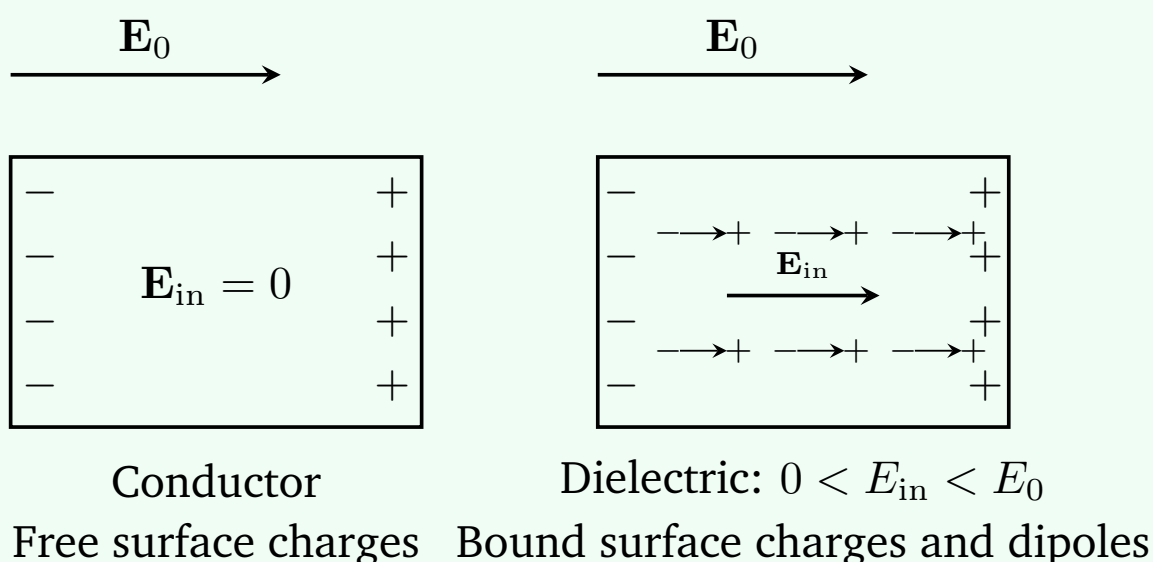
(b) Define the term polarization of a dielectric and write the expression for a linear isotropic dielectric in terms of electric field. [3 marks]

Solution:

Concept:

- A conductor contains mobile charges that redistribute under an applied electric field. In electrostatic equilibrium the net electric field inside its material is zero.
- A dielectric has bound charges. An electric field induces molecular dipoles, or partly aligns permanent dipoles, producing polarization and a field opposing the applied field.
- The polarization vector $\mathbf{P}$ is the electric dipole moment per unit volume. For a linear isotropic dielectric, $\mathbf{P} = \epsilon_0 \chi_e \mathbf{E}$, where $\mathbf{E}$ is the net internal field and χ_e is electric susceptibility.

Step 1: (a) Compare charge behaviour and internal fields Let the external field $\mathbf{E}_0$ point to the right. In a conductor, free electrons move to the left surface, leaving positive charge on the right surface. The induced surface-charge field points to the left and exactly cancels the applied field inside the conductor once electrostatic equilibrium is reached: $\mathbf{E}_{\text{in}} = 0$. The whole conductor is then an equipotential. Charge redistribution stops because no net field remains to cause a further drift of the mobile charges. In a dielectric the electrons cannot travel freely across the specimen. Their small displacement relative to positive nuclei produces induced dipoles; permanent dipoles, if present, tend to align with the field. Negative bound charge appears on the left face and positive bound charge on the right face. The polarization field opposes $\mathbf{E}_0$, reducing the net field, but it does not generally cancel it. Thus the internal field remains nonzero. For a uniform dielectric slab in the ideal uniform-field geometry, its magnitude is $E_{\text{in}} = E_0/K$, with relative permittivity $K > 1$.



Surface charges in a conductor and bound dipoles in a dielectric, with the same applied field.

Step 2: (b) Define polarization and state the constitutive relation If the vector sum of dipole moments

Quick Tip to Solve: Draw negative charge on the face opposite the external-field arrow; distinguish free surface charges in a conductor from bound charges and dipoles in a dielectric.

in a small volume ΔV is $\sum_i \mathbf{p}_i$, then the macroscopic polarization is

$$\mathbf{P} = \frac{\sum_i \mathbf{p}_i}{\Delta V}.$$

Its SI unit is C m^{-2} . A linear dielectric has polarization proportional to the field; isotropy means the response has the same value in every direction, so $\mathbf{P}$ is parallel to $\mathbf{E}$. Consequently

$$\mathbf{P} = \varepsilon_0 \chi_e \mathbf{E} = \varepsilon_0 (K - 1) \mathbf{E}.$$

Here ε_0 is the permittivity of free space, χ_e is dimensionless, and $K = 1 + \chi_e$. The field in this relation is the total field inside the dielectric, including the contribution of its bound charges.

Step 3: Conclusion A conductor cancels the external field in its interior at electrostatic equilibrium, whereas a dielectric polarizes and retains a reduced internal field. Polarization is dipole moment per unit volume, with $\mathbf{P} = \varepsilon_0 \chi_e \mathbf{E}$ for a linear isotropic dielectric.

Quick Tip to Solve: Draw negative charge on the face opposite the external-field arrow; distinguish free surface charges in a conductor from bound charges and dipoles in a dielectric.

19. (a) Obtain the expression for the current flowing through a conductor having number density of electrons n , area of cross-section A , in terms of the drift velocity v_d .

(b) How does the resistivity of a semiconductor change with rise of temperature? Explain. [3 marks]

Solution:

Concept:

- Electric current is the rate of charge transport through a cross-section: $I = \Delta Q / \Delta t$. The drift speed is the magnitude of the average directed velocity of charge carriers.
- In a conductor, n is the number of free electrons per unit volume, A the cross-sectional area, and e the magnitude of the electron charge. Electron motion is opposite to conventional current.
- Semiconductor conductivity is $\sigma = e(n\mu_e + p\mu_h)$, where n, p are the electron and hole densities and μ_e, μ_h are their mobilities. Resistivity is $\rho = 1/\sigma$.

Step 1: (a) Count the electrons crossing the section Consider a straight conductor with uniform free-electron number density and a steady drift speed v_d . During an interval Δt , electrons drift through a distance $v_d \Delta t$. Thus the electrons that cross a chosen cross-section are those contained in an adjoining cylinder of length $v_d \Delta t$ and area A . Its volume is $Av_d \Delta t$, so the number of electrons crossing is

$$N = nAv_d \Delta t.$$

Each electron carries charge of magnitude e ; the magnitude of the charge transported in this interval is therefore $\Delta Q = Ne = nAev_d \Delta t$. Random thermal motion contributes no net current because its directions average out; the directed drift alone produces the net transport.

Quick Tip to Solve: For drift-current derivations, count carriers in the swept cylinder of volume $Av_d \Delta t$; for semiconductors, remember that heating creates additional mobile carriers.

Step 2: Obtain the current and specify its direction Using the definition of current,

$$I = \frac{\Delta Q}{\Delta t} = nAev_d, \quad J = \frac{I}{A} = nev_d.$$

These equations use current magnitude and drift speed. If $\mathbf{v}_d$ denotes the actual electron drift velocity vector, then $\mathbf{J} = -nev_d$, since the electron charge is $-e$. Hence conventional current points opposite to the electron drift. The dimensions confirm the result: $\text{m}^{-3} \times \text{m}^2 \times \text{C} \times \text{m s}^{-1} = \text{A}$.

Step 3: (b) Explain the temperature dependence of semiconductor resistivity In the usual semiconductor temperature range, increasing temperature supplies thermal energy that excites more electrons from the valence band to the conduction band. Each excited electron also leaves a hole in the valence band. Thus the population of mobile electron-hole carriers increases. Although increased lattice vibrations can reduce carrier mobility, the growth in carrier concentration is the dominant effect in the standard semiconductor behaviour asked here. Since $\sigma = e(n\mu_e + p\mu_h)$ increases, its reciprocal ρ decreases. Semiconductors therefore have a negative temperature coefficient of resistivity in this usual regime, unlike the familiar positive coefficient of a metallic conductor.

Step 4: Conclusion (a) The magnitude of current is $I = nAev_d$, with conventional current opposite to electron drift. (b) Semiconductor resistivity decreases as temperature rises because thermal excitation increases the number of mobile electron-hole carriers.

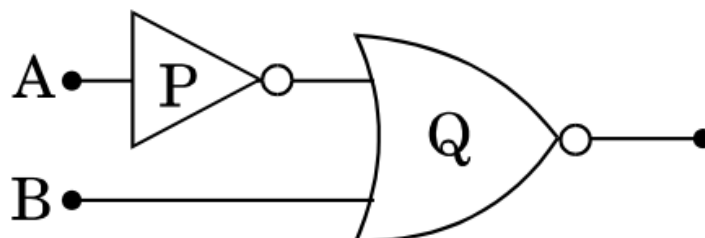
Quick Tip to Solve: For drift-current derivations, count carriers in the swept cylinder of volume $Av_d\Delta t$; for semiconductors, remember that heating creates additional mobile carriers.

20. (a)(i) Write the truth tables of the logic gates marked P and Q in the given circuit.

(ii) Write the truth table for the circuit.

(b) Why are NOR gates considered as universal gates?

[3 marks]



Solution:

Concept:

- A NOT gate inverts its input: $X = \overline{A}$. The bubble at the output of gate P indicates inversion.
- A NOR gate gives $Y = \overline{X + B}$, where $+$ means Boolean OR. Its output is 1 only when both inputs are 0.
- A universal gate can implement NOT, OR and AND operations, and hence any Boolean circuit, using only copies of that gate.

Step 1: (a)(i) Identify P and write its truth table P is a NOT gate. If its input is A, denote its output

Quick Tip to Solve: First insert a column for the output of P; then apply NOR to that column and B, not directly to A and B.

by X . Its truth table is

A	$X = \bar{A}$
0	1
1	0

Thus P reverses the logic level before it reaches Q.

Step 2: (a)(i) Write the truth table of Q Q has the shape of an OR gate with an inversion bubble at its output, so it is a NOR gate. For independent inputs X and B , its truth table is

X	B	$Y = \overline{X + B}$
0	0	1
0	1	0
1	0	0
1	1	0

Any high input to Q makes its output low.

Step 3: (a)(ii) Obtain the truth table of the complete circuit In the given connection, Q receives $X = \bar{A}$ and the uninverted input B . Therefore

$$Y = \overline{\bar{A} + B} = A \cdot \bar{B}.$$

Substitute the intermediate value X for every input combination:

A	B	$X = \bar{A}$	Y
0	0	1	0
0	1	1	0
1	0	0	1
1	1	0	0

Keeping the intermediate column prevents confusing a NOR gate's own truth table with that of the entire circuit.

Step 4: (b) Explain the universality of NOR Write $A \downarrow B = \overline{A + B}$ for NOR. A single NOR with its inputs joined gives NOT:

$$A \downarrow A = \bar{A}.$$

Inverting the result of a NOR gives OR:

$$(A \downarrow B) \downarrow (A \downarrow B) = A + B.$$

Invert each input with a NOR and feed these into another NOR to give AND:

$$(A \downarrow A) \downarrow (B \downarrow B) = \overline{\bar{A} + \bar{B}} = AB.$$

Since these basic operations can all be constructed from NOR gates alone, any logic function can be realised using them.

Step 5: Conclusion P is NOT and Q is NOR. The complete circuit has $Y = A\bar{B}$, so its output is 1 only

Quick Tip to Solve: First insert a column for the output of P; then apply NOR to that column and B, not directly to A and B.

for $A = 1, B = 0$. NOR is universal because it implements NOT, OR and AND.

Quick Tip to Solve: First insert a column for the output of P; then apply NOR to that column and B, not directly to A and B.

OR

20. (a) Explain how a potential barrier is developed in a p-n junction diode.

(b) Draw the circuit arrangement for studying the V-I characteristics of a p-n junction diode in reverse bias. Plot the V-I characteristics in this case. [3 marks]

Solution:

Concept:

- Diffusion carries majority carriers across a newly formed p-n junction because their concentrations differ on the two sides.
- Uncovered fixed ions form a depletion layer and an electric field. The associated potential difference opposes further majority-carrier diffusion.
- Reverse bias connects the p side to the negative terminal and the n side to the positive terminal; it increases the barrier and depletion width.

Step 1: (a) Formation of the depletion layer and potential barrier Initially the n region has many conduction electrons and the p region has many holes. Electrons diffuse from n to p and holes diffuse from p to n. Near the junction they recombine, leaving very few mobile carriers in a narrow region called the depletion layer.

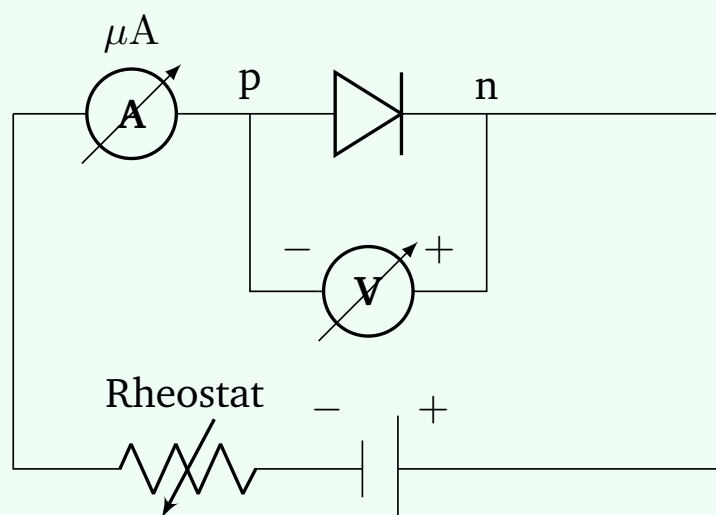
The n side of this layer contains immobile, positively charged ionised donor atoms; the p side contains immobile, negatively charged ionised acceptor atoms. Their space charge establishes an electric field directed from n to p. This field opposes the diffusion of majority carriers: it drives electrons towards n and holes towards p.

The corresponding potential barrier grows until the field-driven drift current balances the diffusion current. At thermal equilibrium the net current is zero. The barrier is therefore developed by carrier diffusion and recombination followed by the separation of fixed ionic charges, rather than by an externally applied battery.

Step 2: (b) Circuit arrangement for reverse bias Connect the p region (anode) to the negative terminal of a variable d.c. supply and the n region (cathode) to its positive terminal. Put a microammeter in series to measure the small reverse current, a rheostat in series to vary and limit the current, and a voltmeter across the diode. The voltmeter's positive terminal is on the n side.

Increase the reverse voltage gradually and record corresponding voltage and current readings. The series resistance protects the diode by limiting the current, particularly if breakdown is approached. The circuit below labels the battery terminals and diode regions so that the reverse connection is explicit.

Quick Tip to Solve: Check the battery connections first: reverse bias always places the positive terminal on n and the negative terminal on p.



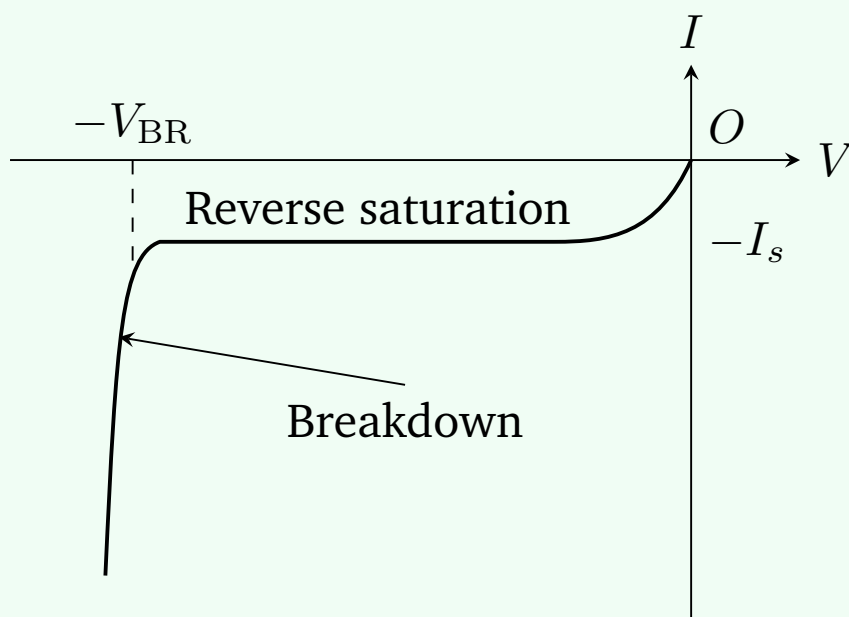
Variable d.c. supply

Reverse-bias measurement circuit: *p* to negative, *n* to positive; microammeter in series and voltmeter across the diode.

Step 3: (b) Reverse-bias V-I characteristic With diode voltage $V = V_p - V_n$, reverse voltage is negative, and the diode current measured positive from *p* to *n* is also negative. A very small minority-carrier current flows and soon becomes nearly independent of the applied reverse voltage; this is the reverse saturation current I_s .

At a sufficiently large reverse voltage, of magnitude V_{BR} , breakdown causes a sharp increase in reverse-current magnitude. Thus the characteristic lies in the third quadrant using the usual signed diode voltage and current convention. The sketch indicates this shape; its horizontal and vertical scales are schematic, since the saturation current is much smaller than a forward conduction current.

Quick Tip to Solve: Check the battery connections first: reverse bias always places the positive terminal on *n* and the negative terminal on *p*.



Signed reverse characteristic in the third quadrant; sharp current rise occurs at breakdown.

Step 4: Conclusion The potential barrier arises from the electric field of fixed ions left by diffusion and recombination. Reverse bias uses p-negative and n-positive connections, giving a small nearly constant reverse current until breakdown.

Quick Tip to Solve: Check the battery connections first: reverse bias always places the positive terminal on n and the negative terminal on p.

21. (a) How are eddy currents generated in a conductor which is subjected to a magnetic field?

(b) Write two examples of their useful applications.

(c) How can the disadvantages of eddy currents be minimized?

[3 marks]

Solution:

Concept:

- Faraday's law states that a changing magnetic flux induces an emf: $\mathcal{E} = -d\Phi_B/dt$, with Φ_B the magnetic flux through a loop.
- A bulk conductor contains closed conducting paths, so the induced emf drives circulating currents called eddy currents.
- Lenz's law gives their direction: the magnetic effect of an eddy current opposes the flux change that produces it. Resistive heating is given by $P = I^2 R$.

Step 1: (a) Explain how eddy currents arise When the magnetic flux through closed paths within a conductor changes, an emf is induced around those paths. Since the conductor provides low-resistance paths, currents circulate in loops within its bulk. These circulating currents, resembling eddies in a fluid, are called eddy currents.

The flux can change because the applied magnetic field varies with time, or because the conductor

Quick Tip to Solve: Always mention changing flux in the definition, and explain that insulated laminations break large electrical loops within a core.

moves through a nonuniform field or enters or leaves a field region. The induced currents create magnetic fields which oppose the change in flux according to Lenz's law. A conductor merely resting in a steady magnetic field does not develop sustained eddy currents, since its magnetic flux does not change.

Step 2: (b) State two useful applications **Induction furnace:** An alternating current in a surrounding coil produces a time-varying magnetic field. Strong eddy currents are induced in the metal charge. Their resistive heating raises its temperature and can melt it without direct electrical contact with the coil.

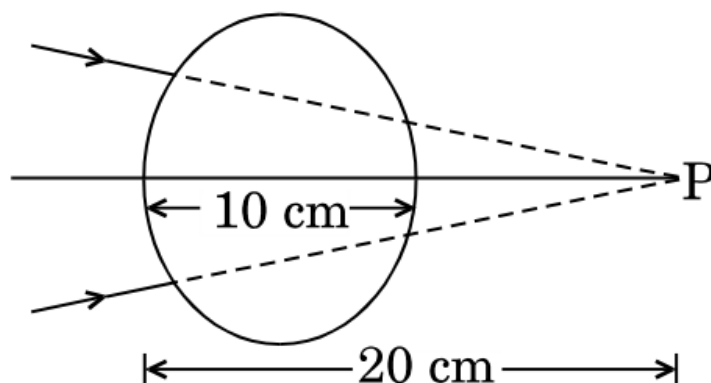
Electromagnetic braking: A conducting disc or wheel moving through a magnetic field experiences eddy currents. Their magnetic interaction produces a force or torque opposing the motion. The kinetic energy is dissipated as heat, thereby slowing the moving part without mechanical contact between braking surfaces.

Step 3: (c) Minimise unwanted eddy-current losses In transformer and other a.c. magnetic cores, replace a solid block by thin laminations separated by insulating coatings. The insulation interrupts large conducting loops across the core, and the thin sheets reduce the area of the remaining loops, lowering the induced emf. The restricted paths also offer higher resistance to eddy-current circulation. For a given induced emf, $I = \mathcal{E}/R$ and the dissipated power is $P = \mathcal{E}^2/R$. Thus reducing the induced emf and increasing path resistance suppress the currents and their unwanted heating. Using a suitable high-resistivity magnetic material further reduces these losses. Laminating the core preserves the desired magnetic path while breaking the unwanted electrical paths.

Step 4: Conclusion Eddy currents are induced loops of current caused by changing magnetic flux. They are useful in induction furnaces and electromagnetic brakes; their unwanted heating is reduced by using thin insulated core laminations and high-resistivity magnetic materials.

Quick Tip to Solve: Always mention changing flux in the definition, and explain that insulated laminations break large electrical loops within a core.

22. A converging beam of light travelling in air converges at a point P as shown in the figure. When a glass sphere of refractive index 1.5 is introduced in between the path of the beam, calculate the new position of the image. Also draw the ray diagram for the image formed. [3 marks]



Solution:

Concept:

- For paraxial refraction at a spherical surface, $\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$, where u , v and R are measured from that surface's pole.
- Take the incident-light direction, from left to right, as positive. A converging incident beam has a virtual object at its intended meeting point, so its object distance is positive.
- The diagram gives the sphere's diameter as 10 cm and the distance of P from its front pole as 20 cm. Thus its radius is 5 cm.

Step 1: Assign the distances at the first surface Let V_1 and V_2 be the front and rear poles, respectively, and C the centre. The rays would meet at P if the glass were absent. Therefore P is the virtual object for the first surface, with $u_1 = +20$ cm. The centre lies to the right of V_1 , giving $R_1 = +5$ cm. For entry into glass, $n_1 = 1$ and $n_2 = 1.5$.

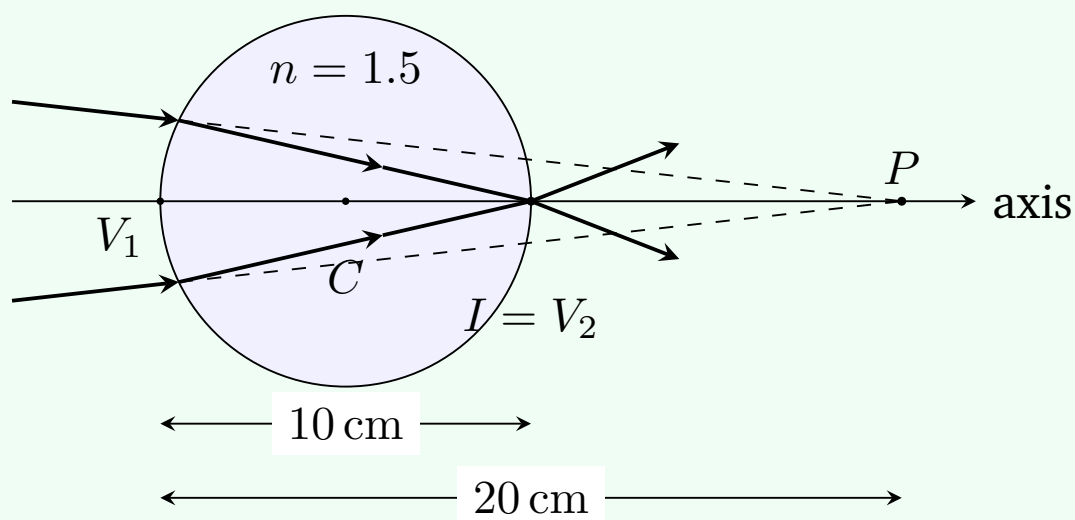
Step 2: Apply spherical refraction on entry Substituting with distances in centimetres,

$$\begin{aligned}\frac{1.5}{v_1} - \frac{1}{20} &= \frac{1.5 - 1}{5}, \\ \frac{1.5}{v_1} &= \frac{1}{10} + \frac{1}{20} = \frac{3}{20}, \\ v_1 &= +10 \text{ cm}.\end{aligned}$$

Thus the rays inside the sphere converge at a point one diameter from V_1 . This point coincides with the rear pole V_2 .

Step 3: Locate the final image and construct the rays Since the first-surface image lies at the second surface itself, the second refraction does not move the point at which the paraxial rays meet. The rays emerge diverging from that same point. Here the usual second-surface calculation with $u_2 = 0$ must be understood as a limiting case; direct division by zero is not valid. In the sketch, dashed incident-ray extensions show the original convergence at P, while the solid refracted rays meet at $I = V_2$. The construction is paraxial, as assumed by the spherical-refraction formula.

Quick Tip to Solve: Read both distances from the front pole: if the first-surface image distance equals the diameter, the image is at the rear pole and no finite-distance second-surface substitution is needed.



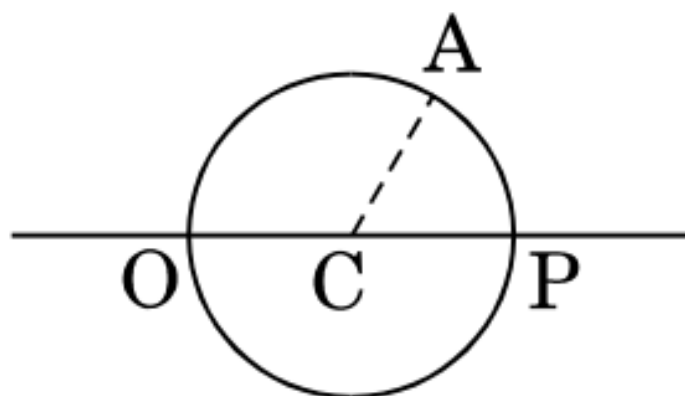
Paraxial ray construction; dashed lines show where the incident beam would meet without glass.

Step 4: Conclusion The new real image is at the rear surface of the sphere: 10 cm from the front pole, or 10 cm to the left of the original point P .

Quick Tip to Solve: Read both distances from the front pole: if the first-surface image distance equals the diameter, the image is at the rear pole and no finite-distance second-surface substitution is needed.

OR

22. A point 'O' marked on the surface of a glass sphere of diameter 20 cm is viewed through glass from the position directly opposite to the point O. If the refractive index of the glass is 1.5, find the position of the image formed. Also, draw the ray diagram for the formation of the image. [3 marks]



Solution:

Concept:

- Refraction at a spherical surface obeys $\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$, where distances are measured from the pole of the refracting surface.

Quick Tip to Solve: Measure both the object distance and radius from the observer-side pole; both are negative here, and a negative result for v identifies a virtual image.

- The point on the opposite surface is viewed through glass, so the relevant refraction is from glass ($n_1 = 1.5$) into air ($n_2 = 1$) at the surface nearest the observer.
- With light travelling to the right, distances to the right are positive and those to the left negative. A negative image distance denotes a virtual image on the object's side of the viewing surface.

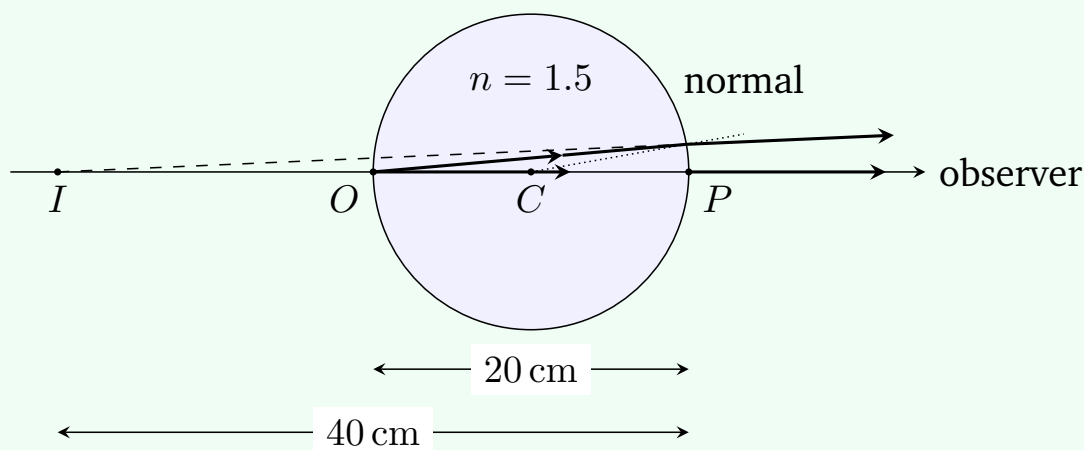
Step 1: Identify the object and viewing surface Place the marked point O at the left end of the sphere and the observer to the right of its opposite pole P , as in the paper. The diameter is 20 cm, so the radius is 10 cm. Rays from O travel through the glass before emerging at the surface near P . For this surface, the object lies 20 cm to the left, so $u = -20$ cm. The centre C also lies to the left of P , so $R = -10$ cm.

Step 2: Calculate the image distance Applying the refraction formula with distances in centimetres,

$$\begin{aligned}\frac{1}{v} - \frac{1.5}{-20} &= \frac{1 - 1.5}{-10}, \\ \frac{1}{v} + \frac{3}{40} &= \frac{1}{20}, \\ \frac{1}{v} &= -\frac{1}{40}, \\ v &= -40 \text{ cm}.\end{aligned}$$

The negative sign places the image on the left of the viewing pole P . The apparent position is farther from P than the actual object is; the curved viewing surface must be used rather than the plane-surface apparent-depth formula.

Step 3: Interpret the image and draw the rays The axial ray from O passes through C and P normally and emerges undeviated. A nearby oblique ray travels through glass and refracts away from the local normal as it enters air. Its backward extension meets the axial ray's extension at I . This is a virtual image because the emerging rays do not actually meet there. Since $PO = 20$ cm and $PI = 40$ cm, the image lies 20 cm beyond O , away from the observer. The ray diagram is a paraxial construction.



The backward extension of the refracted ray locates the virtual image I .

Step 4: Conclusion The image is virtual, 40 cm from the viewing pole P on its left, equivalently 20 cm beyond the marked point O , away from the observer.

Quick Tip to Solve: Measure both the object distance and radius from the observer-side pole; both are negative here, and a negative result for v identifies a virtual image.

23. (a) What do you mean by bandwidth of a signal? Give its importance.

(b) Differentiate between Analog and Digital communication.

(c) Write the functions of transducer and repeaters.

[3 marks]

Solution:

Concept:

- A message usually contains a range of frequency components. Its signal bandwidth is the difference between its highest and lowest significant frequencies.
- Analog communication conveys a continuously varying message, whereas digital communication conveys information using discrete symbols, commonly binary digits.
- A transducer converts one form of energy into another; a repeater restores a weakened signal and retransmits it to extend the communication range.

Step 1: (a) Define signal bandwidth and explain its importance If the significant frequency components of a signal extend from $f_{\min}$ to $f_{\max}$, its bandwidth is

$$\Delta f = f_{\max} - f_{\min}.$$

Bandwidth is measured in hertz. It specifies the frequency range required to carry the message. The communication channel must provide an adequate usable frequency range for the transmitted signal; otherwise some components are suppressed and the recovered message is distorted. Thus bandwidth is important in selecting the channel and allocating its available frequency range. A larger message bandwidth generally requires a larger transmission bandwidth, whose precise value also depends on the modulation used.

Step 2: (b) Distinguish analog and digital communication In analog communication, information is represented by a continuously varying electrical signal: its value can change smoothly over a range. For example, a microphone produces an analog voltage corresponding to speech. In digital communication, information is represented by discrete symbols, usually pulses encoding 0 and 1; an analog message can first be sampled and encoded into such symbols. Analog signals acquire noise and repeated amplification also amplifies that noise. Digital signals can be regenerated by identifying their discrete levels, making digital communication more resistant to accumulated noise, provided the noise does not prevent correct symbol detection.

Step 3: (c) State the functions of the two devices A transducer converts a physical input into a corresponding electrical signal for communication; for example, a microphone converts sound energy into an electrical signal. More generally, transducers convert energy between forms, so a loudspeaker performs the reverse conversion at the receiver. A repeater receives a signal weakened during propagation, restores its strength (and regenerates digital pulses where applicable), and retransmits it towards the next receiver. It therefore increases the distance over which a message can be communicated reliably.

Step 4: Conclusion Signal bandwidth is $f_{\max} - f_{\min}$ and determines the frequency range needed for faithful transmission. Analog communication uses continuous signals; digital communication uses discrete symbols. A transducer converts energy forms, while a repeater restores and retransmits a weakened signal.

Quick Tip to Solve: For each device, state its conversion or retransmission function and give a concrete example; do not confuse a repeater with the original message source.

24. Define the dipole moment of a magnetic dipole. Write its S.I. unit. Obtain the expression for the torque acting on a magnetic dipole placed in an external uniform magnetic field. [3 marks]

Solution:

Concept:

- A magnetic dipole consists of equal and opposite effective pole strengths separated by a small distance. Its magnetic dipole moment points from its south pole to its north pole.
- In a uniform magnetic field, the forces on the two poles are equal and opposite. Their resultant force is zero, but they form a couple when the dipole is inclined to the field.
- The moment of a couple equals the magnitude of either force multiplied by the perpendicular separation between the forces' lines of action.

Step 1: Define magnetic dipole moment and its unit Let the magnitude of each pole strength be p , and let the magnetic length, or separation of the effective poles, be 2ℓ . The magnetic dipole moment is defined as the product of pole strength and magnetic length:

$$m = p(2\ell).$$

Its vector $\vec{m}$ lies along the dipole axis, directed from the south pole to the north pole. The SI unit of magnetic dipole moment is A m^2 , equivalently J T^{-1} . For a plane current loop this same quantity is $m = IA$, where I is the current and A is the enclosed area; its direction follows the right-hand rule.

Step 2: Find the magnetic forces and their perpendicular separation Place the dipole in a uniform external field $\vec{B}$, making an angle θ between $\vec{m}$ and $\vec{B}$. The north pole experiences a force $p\vec{B}$ along the field, while the south pole experiences $-p\vec{B}$, opposite to the field. Both forces have magnitude $F = pB$, so their vector sum is zero. Since the pole separation makes angle θ with the force direction, the perpendicular distance between their parallel lines of action is $d = 2\ell \sin \theta$.

Step 3: Obtain the torque and its direction The two forces form a couple whose torque magnitude is

$$\begin{aligned}\tau &= Fd \\ &= (pB)(2\ell \sin \theta) \\ &= mB \sin \theta.\end{aligned}$$

The sense of this torque tends to align the dipole moment with the external field. By the right-hand rule, its direction is perpendicular to the plane containing $\vec{m}$ and $\vec{B}$, giving the vector expression $\vec{\tau} = \vec{m} \times \vec{B}$. The torque is zero for parallel or antiparallel orientations and is greatest when the dipole is perpendicular to the field. A uniform field therefore exerts an aligning couple without a net translational force on the ideal dipole.

Step 4: Conclusion Magnetic dipole moment is $m = p(2\ell)$, directed from south to north, with SI unit A m^2 . The torque in a uniform magnetic field is $\vec{\tau} = \vec{m} \times \vec{B}$, of magnitude $\tau = mB \sin \theta$.

Quick Tip to Solve: In the couple calculation, use the perpendicular separation $2\ell \sin \theta$, not the full pole separation; this immediately gives the correct angular dependence.

Section D (15 marks)

25. (a) Draw a circuit diagram of an n-p-n transistor with its emitter-base junction forward biased and base-collector junction reverse biased. Briefly describe its working. Explain how a transistor in its active state exhibits a low resistance at its emitter-base junction and high resistance at its base-collector junction.

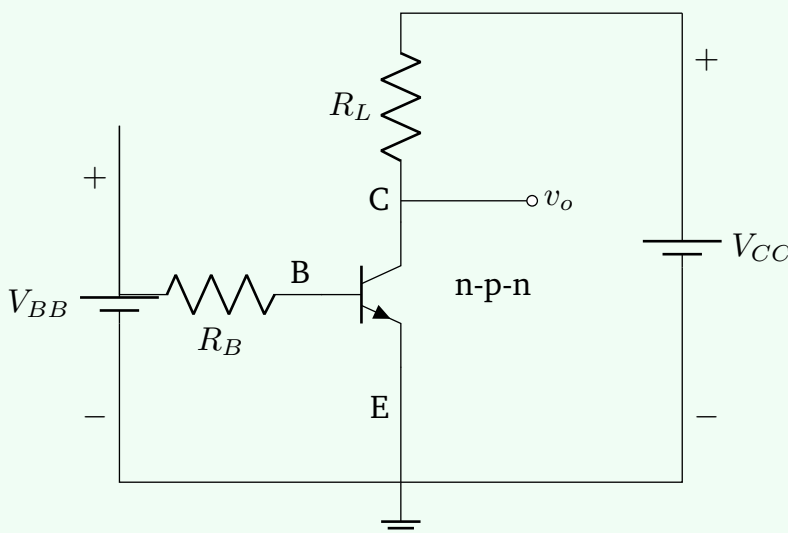
(b) Derive the expression for the voltage gain of a transistor amplifier in CE configuration in terms of the load resistance R_L , current gain β and input resistance. Explain why input and output voltages are in opposite phase. [5 marks]

Solution:

Concept:

- An n-p-n transistor operates in the active region when the emitter-base junction is forward biased and the collector-base junction is reverse biased: $V_C > V_B > V_E$.
- The emitter injects electrons, the thin base allows most to reach the collector, and the currents satisfy $I_E = I_B + I_C$.
- For small signal changes about the operating point, the CE current gain is $\beta_{ac} = \Delta I_C / \Delta I_B$ and the input resistance is $r_i = \Delta V_{BE} / \Delta I_B$.

Step 1: (a) Establish the active-region bias Connect both supply negative terminals to the emitter. Feed the base from the positive base supply through R_B , and the collector from the positive collector supply through R_L . Choose $V_C > V_B > V_E$. The p-type base is positive relative to the n-type emitter, so the emitter-base junction is forward biased. The n-type collector is positive relative to the p-type base, so the collector-base junction is reverse biased. The n-p-n emitter arrow points outwards.



Active-region n-p-n bias circuit; choose $V_C > V_B > V_E$. The emitter is common to the input and output.

Step 2: (a) Describe electron transport and junction resistance The heavily doped emitter injects electrons into the thin, lightly doped base. Few electrons recombine with base holes, giving a small base current. Most cross the base and are swept into the collector by the reverse-biased collector-base

Quick Tip to Solve: Start from $V_{CE} = V_{CC} - I_C R_L$: a rising collector current lowers the output voltage, giving the negative sign in CE voltage gain immediately.

field. Thus the collector current is controlled by emitter injection, and

$$I_E = I_C + I_B, \quad I_C \simeq \beta I_B.$$

Forward bias lowers the emitter-base barrier: a small voltage increase produces a large injection-current increase, so the junction has low incremental resistance. Reverse bias widens the collector-base depletion layer and suppresses majority-carrier flow. The collector current, supplied by emitter injection, changes little with collector-base voltage in the active region, giving high incremental resistance. Thus reverse bias permits collection of injected electrons while presenting high output resistance.

Step 3: (b) Relate the input signal to the collector-current change Superpose a small alternating input voltage on the forward base bias, keeping the transistor in the active region throughout the cycle. For a common emitter that is at signal ground, let the input voltage change be $v_i = \Delta V_{BE}$. With input resistance r_i ,

$$\Delta I_B = \frac{v_i}{r_i}.$$

Using the small signal CE current gain gives

$$\Delta I_C = \beta_{ac} \Delta I_B = \frac{\beta_{ac} v_i}{r_i}.$$

Treat β_{ac} and r_i as constant for these small signal changes. The collector supply is fixed and R_L is the effective collector load.

Step 4: (b) Derive voltage gain and explain phase reversal Applying the collector-loop voltage equation, with negligible signal voltage at the emitter,

$$V_{CE} = V_{CC} - I_C R_L.$$

Since the supply voltage does not change with the signal, the output-voltage change at the collector is

$$v_o = \Delta V_{CE} = -R_L \Delta I_C.$$

Substitute the collector-current change from the previous step:

$$v_o = -\frac{\beta_{ac} R_L}{r_i} v_i,$$

$$A_v = \frac{v_o}{v_i} = -\frac{\beta_{ac} R_L}{r_i}.$$

When the input voltage rises, the base current and hence collector current increase. The voltage drop across R_L increases, so the collector voltage falls. When the input falls, the collector voltage rises. The negative gain therefore represents a phase difference of 180° between the input and output voltages. The gain magnitude is $\beta_{ac} R_L / r_i$; the resistance ratio makes the voltage gain dimensionless.

Step 5: Conclusion The correctly biased n-p-n transistor has low emitter-base and high collector-base incremental resistance. Its CE voltage gain is $A_v = -\beta_{ac} R_L / r_i$, with magnitude $\beta_{ac} R_L / r_i$; the output is 180° out of phase with the input.

Quick Tip to Solve: Start from $V_{CE} = V_{CC} - I_C R_L$: a rising collector current lowers the output voltage, giving the negative sign in CE voltage gain immediately.

25. (a) Write the important considerations which are to be taken into account while fabricating a p-n junction diode to be used as a Light Emitting Diode (LED). What should be the order of band gap of an LED, if it is required to emit light in the visible range? Draw a circuit diagram and explain its action.

(b) Draw the V-I characteristics of an LED. State two advantages of LED lamps over conventional incandescent lamps. [5 marks]

Solution:

Concept:

- An LED is a heavily doped p-n junction made from a suitable semiconductor. Under forward bias, injected electrons and holes recombine near the junction and release energy as photons.
- Efficient light emission requires radiative recombination in a direct-band-gap material, such as a suitable gallium compound. Silicon and germanium are poor choices for visible LEDs because much of the recombination energy is transferred to the lattice instead of being emitted as light.
- The photon energy is approximately the band gap: $E_g = h\nu = hc/\lambda$, where h is Planck's constant, ν the frequency, c the speed of light and λ the wavelength.

Step 1: (a) Fabrication considerations Choose a semiconductor with a band gap corresponding to the required colour and with efficient radiative electron-hole recombination. The junction is heavily doped so that forward injection supplies a large number of carriers for recombination. The active junction should be arranged close enough to the emitting surface, and the package should have a transparent window or cover, so that the generated light can escape instead of being absorbed inside the device. Suitable contacts must supply current without obstructing the light; the junction and package must also dissipate heat during operation. A suitable compound semiconductor, for example an appropriate gallium arsenide phosphide composition for a visible LED, is therefore used rather than an ordinary silicon rectifier diode.

Step 2: (a) Band gap for visible light Visible photons have wavelengths approximately from 400 nm to 700 nm. Using $hc \simeq 1240 \text{ eV nm}$,

$$E_g(700 \text{ nm}) \simeq \frac{1240}{700} \text{ eV} = 1.77 \text{ eV},$$

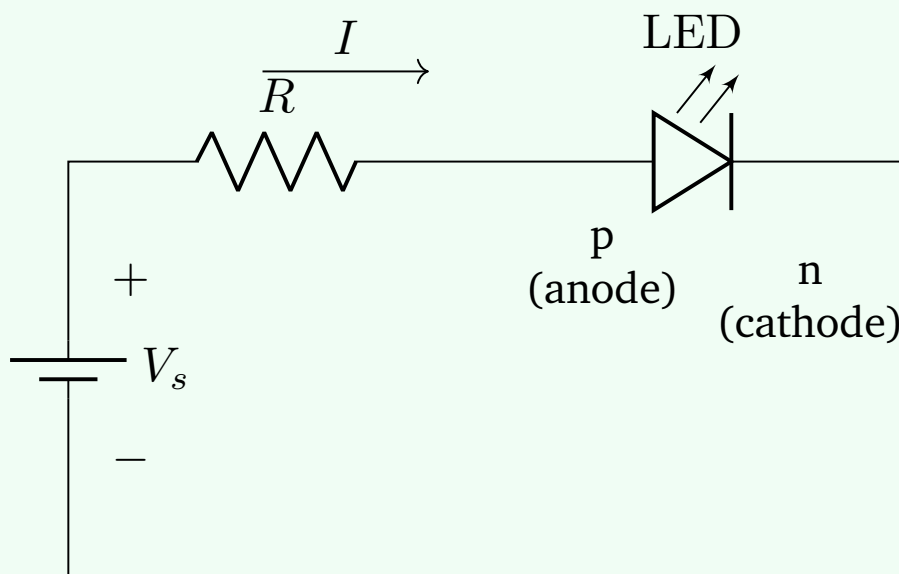
$$E_g(400 \text{ nm}) \simeq \frac{1240}{400} \text{ eV} = 3.10 \text{ eV}.$$

Thus the required band gap is of the order of a few electron volts, approximately 1.8–3.1 eV for visible emission. The precise value fixes the colour; it is not necessary for every visible LED to have exactly the same gap.

Step 3: (a) Circuit and action Connect the p-side (anode) of the LED to the positive terminal of a dc supply through a series current-limiting resistor R , and connect the n-side (cathode) to the negative terminal. This forward biases the junction and reduces its barrier. Electrons from the n-region and holes from the p-region are injected across the junction and recombine in the active region. In the chosen material, their energy is released principally as photons, so the LED glows. Above the turn-on voltage, current rises sharply; the resistor protects the junction from excessive current. For a desired

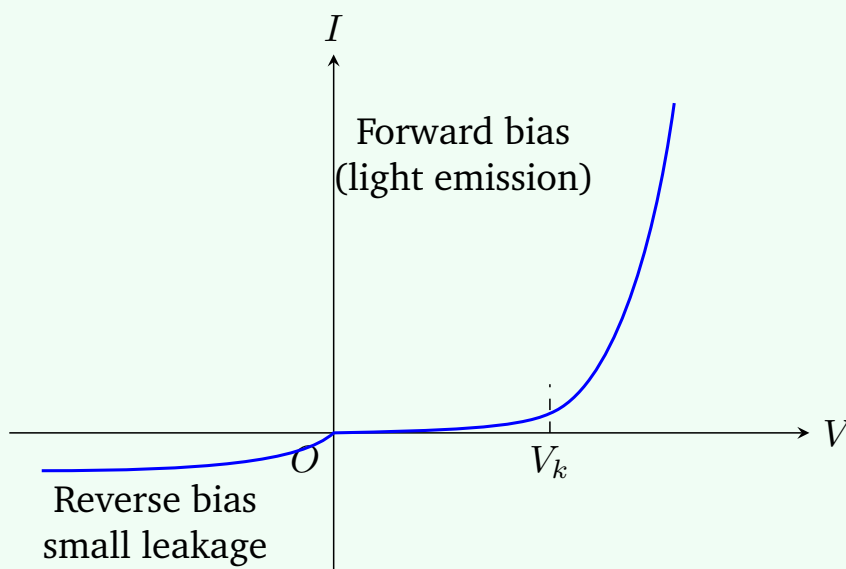
Quick Tip to Solve: Use $E_g(\text{eV}) \simeq 1240/\lambda(\text{nm})$ to check the colour, and always put a current-limiting resistor in series with the LED.

operating current I , choose approximately $R = (V_s - V_{\text{LED}})/I$, where V_s is the supply voltage and V_{LED} is the operating forward voltage.



Forward-biased LED with a series resistor; the two outward arrows denote light emission.

Step 4: (b) V-I characteristic In forward bias, current is small below the knee voltage V_k , then increases rapidly as the voltage increases. The knee depends on the LED material and colour, and is generally higher than that of an ordinary silicon diode. In reverse bias, only a very small leakage current flows before breakdown; this is not the light-emitting operating region. The sketch shows the forward operating branch and a schematic leakage branch, with leakage enlarged for visibility. It is qualitative and does not assert a numerical turn-on or reverse-breakdown voltage.



Qualitative LED characteristic; reverse leakage is exaggerated, and the device operates on the forward branch.

Step 5: (b) Two advantages over incandescent lamps First, LED lamps have higher luminous efficiency: they use less electrical energy for a comparable light output and waste less energy as heat. Second,

Quick Tip to Solve: Use $E_g(\text{eV}) \simeq 1240/\lambda(\text{nm})$ to check the colour, and always put a current-limiting resistor in series with the LED.

they have a much longer operating life because they do not depend on a hot filament that gradually deteriorates and burns out. These advantages reduce power consumption and the frequency of lamp replacement.

Step 6: Conclusion A visible LED needs an efficiently radiating junction with a band gap of the order of a few electron volts (about 1.8–3.1 eV). It emits light through carrier recombination under forward bias, using a series resistor to limit current. Its forward current rises rapidly beyond the knee, and LED lamps offer higher efficiency and longer life than incandescent lamps.

Quick Tip to Solve: Use $E_g(\text{eV}) \simeq 1240/\lambda(\text{nm})$ to check the colour, and always put a current-limiting resistor in series with the LED.

26. (a) State and explain the law used to determine magnetic field at a point due to a current element. Derive the expression for the magnetic field due to a circular current carrying loop of radius r at its centre.

(b) A long wire with a small current element of length 1 cm is placed at the origin and carries a current of 10 A along the X-axis. Find the magnitude and direction of the magnetic field due to the element on the Y-axis at a distance 0.5 m from it. [5 marks]

Solution:

Concept:

- Biot-Savart law gives the field due to a steady current element: $d\vec{B} = \frac{\mu_0}{4\pi} \frac{I d\vec{\ell} \times \vec{R}}{R^2}$, where $d\vec{\ell}$ points along the current and $\vec{R}$ points from the element to the observation point.
- The field magnitude is $dB = \frac{\mu_0}{4\pi} \frac{I d\ell \sin \theta}{R^2}$. Here θ is the angle between the current element and the line joining it to the point; the direction follows the right-hand rule.
- Fields of all elements add vectorially. At the centre of a circular loop all contributions are perpendicular to its plane and point in the same direction.

Step 1: (a) State and explain Biot-Savart law For a small element of a wire carrying steady current I , the magnetic field at a point is directly proportional to I , the element length $d\ell$ and $\sin \theta$, and inversely proportional to the square of its distance R . In vacuum the proportionality constant is $\mu_0/(4\pi)$:

$$dB = \frac{\mu_0}{4\pi} \frac{I d\ell \sin \theta}{R^2}.$$

Its direction is normal to the plane containing $d\vec{\ell}$ and $\vec{R}$, in the direction of $d\vec{\ell} \times \vec{R}$. Consequently a point on the extension of the element has zero field, whereas a point in a perpendicular direction has the largest field for fixed distance.

Step 2: Derive the field at the centre of the loop Let the loop have radius r and carry current I . For every element, its distance from the centre is r , and the tangent carrying current is perpendicular to the radius, so $\theta = 90^\circ$. Each element therefore produces

$$dB = \frac{\mu_0 I}{4\pi r^2} d\ell.$$

Quick Tip to Solve: At a loop's centre every element has the same distance and direction of field. For the numerical part, use only the stated 1 cm element and check the direction with $\hat{x} \times \hat{y} = \hat{z}$.

By the right-hand rule these contributions all point along the same normal to the loop. Their magnitudes can therefore be integrated directly around its circumference:

$$\begin{aligned} B &= \oint dB = \frac{\mu_0 I}{4\pi r^2} \oint d\ell \\ &= \frac{\mu_0 I}{4\pi r^2} (2\pi r) = \frac{\mu_0 I}{2r}. \end{aligned}$$

Curl the fingers of the right hand in the direction of current: the thumb gives the direction of this field. For N identical closely wound turns the result is multiplied by N .

Step 3: (b) Substitute the current-element data The field requested is due to the small element, so use its length rather than the formula for an entire long wire. Convert 1 cm to $\ell = 0.01$ m. With $I = 10$ A, $R = 0.5$ m and $\theta = 90^\circ$, the current-element approximation gives

$$\begin{aligned} B &= \frac{\mu_0}{4\pi} \frac{I\ell}{R^2} \\ &= 10^{-7} \frac{10 \times 0.01}{(0.5)^2} \text{ T} \\ &= 4.0 \times 10^{-8} \text{ T}. \end{aligned}$$

The element is small compared with its distance from the point, since $\ell/R = 0.02$, consistent with treating it as a short current element.

Step 4: Determine the direction from the vector product Take the observation point on the positive Y-axis. The current element points along $+\hat{x}$, while $\hat{R} = +\hat{y}$. Hence $\hat{x} \times \hat{y} = +\hat{z}$: the field is along the positive Z-axis, perpendicular to the XY-plane. If the specified point is instead on the negative Y-axis, the magnitude is unchanged and the direction reverses to $-\hat{z}$.

Step 5: Conclusion (a) Biot-Savart law gives $B = \mu_0 I / (2r)$ at the centre of a single circular loop, normal to its plane according to the right-hand rule. (b) The small element produces 4.0×10^{-8} T along $+\hat{z}$ at $y = +0.5$ m (along $-\hat{z}$ at $y = -0.5$ m).

Quick Tip to Solve: At a loop's centre every element has the same distance and direction of field. For the numerical part, use only the stated 1 cm element and check the direction with $\hat{x} \times \hat{y} = \hat{z}$.

26. (a) Derive the expression for the magnetic field due to a current carrying coil of radius r at a distance x from the centre along the X-axis.

(b) A straight wire carrying a current of 5 A is bent into a semicircular arc of radius 2 cm as shown in the figure. Find the magnitude and direction of the magnetic field at the centre of the arc. [5 marks]



Solution:

Concept:

- Biot-Savart law gives $dB = \frac{\mu_0}{4\pi} \frac{I d\ell \sin \theta}{R^2}$ for a current element at distance R , where θ is the angle between the element and the displacement to the point.
- On the axis of a circular coil the transverse components cancel by symmetry. Only the components along the axis add; for N identical closely wound turns the field is N times that of a single loop.
- At the centre of an arc of radius r subtending angle ϕ in radians, $B = \mu_0 I \phi / (4\pi r)$. Straight radial portions produce zero field there because their current elements are collinear with the displacement vector.

Step 1: (a) Define the axial geometry Place a circular loop of radius r in the YZ-plane with its centre at the origin. Let the observation point P be on the X-axis at distance x . Every element of the loop is at the same distance

$$R = \sqrt{r^2 + x^2}$$

from P . The element is tangent to the circle, and is perpendicular both to its radial direction and to the X-axis. Thus it is perpendicular to the displacement from the element to P , giving $\sin \theta = 1$. Its field magnitude is

$$dB = \frac{\mu_0 I d\ell}{4\pi(r^2 + x^2)}.$$

Step 2: Add the components along the axis The field of an element is perpendicular to the displacement vector. Its axial projection factor is r/R , as follows from the right triangle formed by the radius r , the axial distance x and the separation R . Therefore

$$dB_x = dB \frac{r}{R} = \frac{\mu_0 I r d\ell}{4\pi(r^2 + x^2)^{3/2}}.$$

For diametrically opposite elements the transverse components have equal magnitudes and opposite directions, and cancel. Their axial components have the same direction and add. Integrating around

Quick Tip to Solve: For a semicircle, halve the full-loop centre field and then check the current's clockwise sense. Before adding straight-wire fields, test whether each wire lies on the line through the observation point.

the entire loop gives

$$\begin{aligned} B_x &= \frac{\mu_0 I r}{4\pi(r^2 + x^2)^{3/2}} \oint d\ell \\ &= \frac{\mu_0 I r}{4\pi(r^2 + x^2)^{3/2}} (2\pi r) \\ &= \frac{\mu_0 I r^2}{2(r^2 + x^2)^{3/2}}. \end{aligned}$$

For a coil of N closely wound turns, $B = \mu_0 N I r^2 / [2(r^2 + x^2)^{3/2}]$. The field is along the axis, its sense being given by the right-hand rule. Setting $x = 0$ recovers the centre field $\mu_0 N I / (2r)$.

Step 3: (b) Separate the arc and straight portions In the printed figure the current flows from left to right and follows the upper semicircle clockwise. The two straight portions lie on the diameter line. For every element of either straight portion, the displacement towards the centre is parallel or antiparallel to the element. Since $d\vec{\ell} \times \hat{R} = 0$, each straight portion gives zero field at the centre. Only the semicircular arc contributes.

Step 4: Integrate over the semicircle and substitute Each arc element is perpendicular to the radius towards the centre and gives a field normal to the page in the same direction. For the semicircle the length is πr , so

$$\begin{aligned} B &= \frac{\mu_0 I}{4\pi r^2} \int_{\text{arc}} d\ell = \frac{\mu_0 I}{4\pi r^2} (\pi r) = \frac{\mu_0 I}{4r} \\ &= \frac{(4\pi \times 10^{-7}) \times 5}{4 \times 0.02} \text{ T} \\ &= 7.85 \times 10^{-5} \text{ T}. \end{aligned}$$

Here $r = 2 \text{ cm} = 0.02 \text{ m}$. Curling the right-hand fingers clockwise along the printed current makes the thumb point into the plane of the paper, which is the direction of the magnetic field.

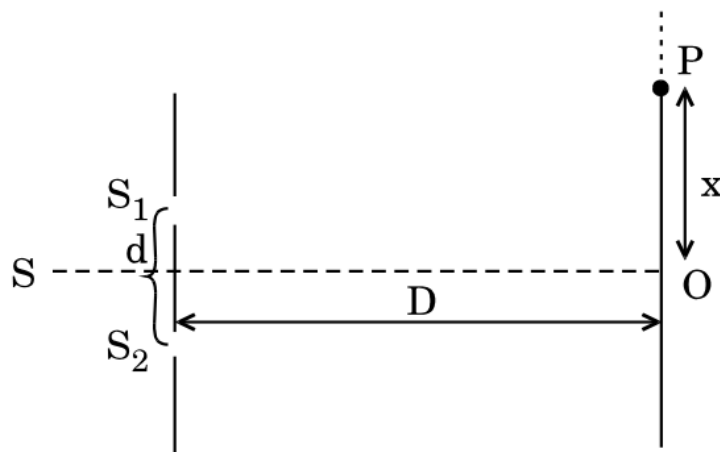
Step 5: Conclusion (a) On the coil axis, $B = \frac{\mu_0 N I r^2}{2(r^2 + x^2)^{3/2}}$, with $N = 1$ for a single loop. (b) The field at the semicircle's centre is $7.85 \times 10^{-5} \text{ T}$, directed into the plane of the paper; the straight portions contribute zero.

Quick Tip to Solve: For a semicircle, halve the full-loop centre field and then check the current's clockwise sense. Before adding straight-wire fields, test whether each wire lies on the line through the observation point.

27. (a) Can the interference pattern be produced by two independent monochromatic sources of light? Explain.

(b) The intensity at the central maximum (O) in a Young's double slit experimental set-up shown in the figure is I_0 . If the distance OP equals one-third of the fringe width of the pattern, show that the intensity at point P , would equal $\frac{I_0}{4}$.

(c) In Young's double slit experiment, the slits are separated by 0.5 mm and screen is placed 1.0 m away from the slit. It is found that the 5^{th} bright fringe is at a distance of 4.13 mm from the 2^{nd} dark fringe. Find the wavelength of light used. [5 marks]



Solution:

Concept:

- Sustained interference requires coherent sources: they have the same frequency and maintain a constant phase difference.
- For equal slit intensities I_s , the resultant intensity is $I = 4I_s \cos^2(\phi/2)$, where ϕ is the phase difference. The central intensity is $I_0 = 4I_s$.
- In Young's experiment, fringe width $\beta = \lambda D/d$, path difference $\Delta = dx/D$, and phase difference $\phi = 2\pi\Delta/\lambda$. Here d is slit separation and D is screen distance.

Step 1: (a) Test the coherence of independent sources Two independent ordinary monochromatic sources do not produce a sustained interference pattern. Monochromaticity establishes a definite frequency, but it does not establish a fixed relative phase. Light emission occurs in independent wave trains, so their relative phase changes randomly with time. Consequently, the positions of constructive and destructive interference fluctuate rapidly and the time-averaged interference term vanishes. The screen shows the sum of the individual intensities rather than stationary fringes. In Young's experiment both slits are illuminated from one source, so the two secondary sources have a stable phase relationship.

Step 2: (b) Relate the displacement to the phase difference At the central maximum O , the paths are equal and the waves arrive in phase. For P , let $x = OP = \beta/3$. Using the small-angle geometry of Young's experiment,

$$\Delta = \frac{dx}{D} = \frac{d}{D} \frac{\beta}{3} = \frac{d}{D} \frac{\lambda D}{3d} = \frac{\lambda}{3}.$$

Quick Tip to Solve: Write bright positions as $n\beta$ and dark positions as $(n - \frac{1}{2})\beta$ before taking their separation; the second dark fringe is at 1.5β , not 2β .

Hence the phase difference is

$$\phi = \frac{2\pi\Delta}{\lambda} = \frac{2\pi}{3}.$$

Step 3: (b) Calculate the intensity relative to the central value The slits in the ideal Young's arrangement contribute equal intensities. Their interference intensity is therefore

$$I(P) = 4I_s \cos^2 \frac{\phi}{2} = I_0 \cos^2 \frac{\pi}{3} = I_0 \left(\frac{1}{2}\right)^2.$$

It is essential to square the cosine: the amplitude ratio is $1/2$, whereas intensity is proportional to the square of amplitude.

Step 4: (c) Locate the fifth bright and second dark fringes Count the central bright fringe as order zero. On the same side of the centre, the fifth bright fringe is at $x_{B5} = 5\beta$. Dark fringes start at $\beta/2$, so the second dark fringe is at $x_{D2} = 3\beta/2$. The given separation is thus

$$\begin{aligned} x_{B5} - x_{D2} &= \left(5 - \frac{3}{2}\right)\beta = \frac{7}{2}\beta, \\ \beta &= \frac{2(4.13)}{7} \text{ mm} = 1.18 \text{ mm}. \end{aligned}$$

The positions are measured from the central maximum, not from the first bright or first dark fringe.

Step 5: (c) Substitute in the fringe-width formula Convert the lengths to SI units: $\beta = 1.18 \times 10^{-3} \text{ m}$, $d = 0.5 \times 10^{-3} \text{ m}$, and $D = 1.0 \text{ m}$. Therefore,

$$\lambda = \frac{\beta d}{D} = \frac{(1.18 \times 10^{-3})(0.5 \times 10^{-3})}{1.0} \text{ m} = 5.9 \times 10^{-7} \text{ m}.$$

Step 6: Conclusion (a) Independent ordinary monochromatic sources cannot yield sustained fringes because their phase difference is not constant. (b) $I(P) = I_0/4$. (c) The wavelength is $5.9 \times 10^{-7} \text{ m} = 590 \text{ nm}$, using the usual same-side fringe separation.

Quick Tip to Solve: Write bright positions as $n\beta$ and dark positions as $(n - \frac{1}{2})\beta$ before taking their separation; the second dark fringe is at 1.5β , not 2β .

27. (a) Derive the relation $a \sin \theta = \lambda$ for the first minimum of the diffraction pattern produced due to a single slit of width ' a ' using light of wavelength λ .

(b) State with reason, how the linear width of central maximum will be affected if (i) monochromatic yellow light is replaced with red light, and (ii) distance between the slit and the screen is increased.

(c) Using the monochromatic light of same wavelength in the experimental set-up of the diffraction pattern as well as in the interference pattern where the slit separation is 1 mm, 10 interference fringes are found to be within the central maximum of the diffraction pattern. Determine the width of the single slit, if the screen is kept at the same distance from the slit in the two cases. [5 marks]

Solution:

Concept:

- Huygens' principle treats every point of an illuminated slit as a source of secondary wavelets. Their superposition produces diffraction.
- For slit width a , the path difference between the extreme rays at angle θ is $a \sin \theta$. Diffraction minima satisfy $a \sin \theta = m\lambda$, with nonzero integer m .
- For a distant screen at distance D , the central diffraction maximum has linear width $W = 2\lambda D/a$ in the small-angle approximation. Young's interference fringe width is $\beta = \lambda D/d$, where d is the separation of the two slits.

Step 1: (a) Divide the slit into two equal halves Consider a plane monochromatic wave incident normally on the slit. At an observation point in direction θ , contributions from different parts of the slit travel different distances. Divide the slit of width a into two equal halves. Pair every element of the upper half with an element of the lower half displaced by $a/2$ across the slit. Each pair has the same separation, and hence the same path difference,

$$\Delta_{\text{pair}} = \frac{a}{2} \sin \theta.$$

Uniform illumination gives the members of a pair equal amplitudes. In the far-field arrangement their rays towards the observation point are effectively parallel, so the projected separation gives this path difference.

Step 2: (a) Apply pairwise destructive interference The members of each pair cancel when their path difference is $\lambda/2$, giving a phase difference of π . Thus,

$$\frac{a}{2} \sin \theta_1 = \frac{\lambda}{2} \implies a \sin \theta_1 = \lambda.$$

Every element in one half has a cancelling partner in the other, so the total resultant amplitude is zero. This is the first minimum on either side of the central maximum. At $\theta = 0$, all contributions arrive in phase instead of cancelling; that direction is the central maximum.

Step 3: (b) Determine the central maximum's width The first minima bound the central maximum at angles $+\theta_1$ and $-\theta_1$. With θ_1 small, $\sin \theta_1 \simeq \tan \theta_1 \simeq \theta_1$, so their screen positions are

$$y_{\pm} = \pm D \tan \theta_1 \simeq \pm \frac{D\lambda}{a}.$$

Quick Tip to Solve: Use the full central diffraction width $2\lambda D/a$, not its half-width. Dividing by the interference spacing immediately gives $W/\beta = 2d/a$.

Their separation is therefore

$$W = y_+ - y_- \simeq \frac{2D\lambda}{a}.$$

(i) Replacing yellow light with red light increases λ , so the central maximum becomes wider when D and a are unchanged. (ii) Increasing the slit-to-screen distance D also increases the linear width in direct proportion. The angular width is unchanged in the second case, but its projection on a more distant screen is larger.

Step 4: (c) Compare diffraction width with interference spacing Both arrangements use the same wavelength and screen distance. Ten interference fringe widths fit within the central diffraction maximum, so $W = 10\beta$. Substitute the two expressions before inserting numbers:

$$\begin{aligned}\frac{2\lambda D}{a} &= 10 \frac{\lambda D}{d}, \\ \frac{2d}{a} &= 10, \\ a &= \frac{2d}{10}.\end{aligned}$$

The common factors λ and D cancel because they are the same in both experiments. With $d = 1 \text{ mm}$,

$$a = \frac{2(1 \text{ mm})}{10} = 0.20 \text{ mm} = 2.0 \times 10^{-4} \text{ m}.$$

Step 5: Conclusion (a) Pairwise cancellation gives the first-minimum condition $a \sin \theta_1 = \lambda$. (b) The central maximum becomes wider with red light and also with increased screen distance. (c) The single-slit width is 0.20 mm.

Quick Tip to Solve: Use the full central diffraction width $2\lambda D/a$, not its half-width. Dividing by the interference spacing immediately gives $W/\beta = 2d/a$.