

CBSE Class 12 Physics Question Paper 2019 with Solution PDF-Set 55/4/1

Question Paper with Solutions

Conducted by the Central Board of Secondary Education (CBSE)



Time Allowed: 3 hours

Maximum Marks: 70

General Instructions

- (i) All questions are compulsory. There are 27 questions in all.
- (ii) This question paper has four sections : Section A, Section B, Section C and Section D.
- (iii) Section A contains five questions of one mark each, Section B contains seven questions of two marks each, Section C contains twelve questions of three marks each, Section D contains three questions of five marks each.
- (iv) There is no overall choice. However, an internal choice(s) has been provided in two questions of one mark, two questions of two marks, four questions of three marks and three questions of five marks weightage. You have to attempt only one of the choices in such questions.

You may use the following values of physical constants wherever necessary :

$$c = 3 \times 10^8 \text{ m/s}$$

$$h = 6.63 \times 10^{-34} \text{ Js}$$

$$e = 1.6 \times 10^{-19} \text{ C}$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ T m A}^{-1}$$

$$\epsilon_0 = 8.854 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$$

$$\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$$

$$\text{Mass of electron } (m_e) = 9.1 \times 10^{-31} \text{ kg}$$

$$\text{Mass of neutron} = 1.675 \times 10^{-27} \text{ kg}$$

$$\text{Mass of proton} = 1.673 \times 10^{-27} \text{ kg}$$

$$\text{Avogadro's number} = 6.023 \times 10^{23} \text{ per gram mole}$$

$$\text{Boltzmann constant} = 1.38 \times 10^{-23} \text{ JK}^{-1}$$

Section A (5 marks)

1. Two identical conducting balls A and B have charges $-Q$ and $+3Q$ respectively. They are brought in contact with each other and then separated by a distance d apart. Find the nature of the Coulomb force between them. [1 mark]

Solution:

Concept:

- Total charge is conserved when isolated conducting balls are brought into contact.
- Identical balls share the total charge equally; Coulomb force is repulsive for charges of the same sign.

Step 1: Find the charge on each ball The initial total charge is $(-Q) + 3Q = 2Q$. As the balls are identical, they have equal capacitances and acquire equal charges at a common potential. Thus, after contact, each ball has $q_A = q_B = 2Q/2 = Q$.

Step 2: Determine the nature of the interaction After separation, both balls have positive charge. Their product is $q_A q_B = Q^2 > 0$, so each ball experiences a force directed away from the other. In the usual point-charge approximation, its magnitude is $F = Q^2/(4\pi\epsilon_0 d^2)$.

Step 3: Conclusion The Coulomb force between the separated balls is **repulsive**, since each ball carries charge $+Q$.

Quick Tip to Solve: Add the initial charges before deciding attraction or repulsion: identical balls share the net charge equally.

OR

1. A metallic spherical shell has an inner radius R_1 and outer radius R_2 . A charge Q is placed at the centre of the shell. What will be the surface charge density on the (i) inner surface, and (ii) outer surface of the shell? [1 mark]

Solution:

Concept:

- The electric field inside the material of a conductor in electrostatic equilibrium is zero.
- For a spherical surface, the uniform surface charge density is $\sigma = q/(4\pi R^2)$.

Step 1: Find the induced charge on the inner surface Take a Gaussian surface lying within the metal. Its flux is zero, so the enclosed charge must vanish: $Q + q_{\text{in}} = 0$. Hence $q_{\text{in}} = -Q$. Because the charge is at the centre, spherical symmetry makes this induced charge uniform over the inner surface of radius R_1 .

Step 2: Use conservation of the shell charge The shell is taken to be initially neutral, as no net shell charge is specified. Therefore its inner and outer surface charges add to zero. The outer surface carries $q_{\text{out}} = +Q$, uniformly distributed over the sphere of radius R_2 . Divide each surface charge by its own surface area.

Quick Tip to Solve: Use zero field in the metal to fix the inner charge first; neutrality then fixes the outer charge, with a different area.

Step 3: Conclusion For the initially neutral shell, (i) $\sigma_{\text{in}} = -\frac{Q}{4\pi R_1^2}$, and (ii) $\sigma_{\text{out}} = +\frac{Q}{4\pi R_2^2}$.

Quick Tip to Solve: Use zero field in the metal to fix the inner charge first; neutrality then fixes the outer charge, with a different area.

2. Under what condition will the current in a wire be the same when connected in series and in parallel of n identical cells each having internal resistance r and external resistance R ? [1 mark]

Solution:

Concept:

- Let the emf of each identical cell be E . In series, emfs and internal resistances both add.
- For identical cells connected in parallel with like terminals together, the equivalent emf is E and internal resistance is r/n .

Step 1: Write the currents for the two combinations For the same external resistance R , the series combination supplies current $I_s = nE/(R + nr)$. The parallel combination supplies current $I_p = E/(R + r/n) = nE/(nR + r)$. Both expressions include the cells' internal resistance.

Step 2: Equate the currents and simplify For equal nonzero currents, the equal numerators imply equal denominators:

$$\begin{aligned} R + nr &= nR + r, \\ (n - 1)r &= (n - 1)R. \end{aligned}$$

For $n > 1$, division by $n - 1$ gives the required relation between external and internal resistances. If $n = 1$, the two arrangements are identical for any R .

Step 3: Conclusion For more than one cell, the currents are equal when $R = r$: the external resistance equals the internal resistance of one cell.

Quick Tip to Solve: For identical cells, compare the denominators after writing both currents with the same numerator nE .

3. The small ozone layer on top of the stratosphere is crucial for human survival. Why? [1 mark]

Solution:

Concept:

- The Sun emits ultraviolet radiation along with visible light and other electromagnetic waves.
- Ozone in the stratosphere absorbs much of the biologically harmful ultraviolet radiation before it reaches the Earth's surface.

Step 1: Identify the protective action of ozone Ozone molecules absorb strongly in the ultraviolet region. The layer therefore acts as a protective screen, reducing the amount of harmful solar ultraviolet radiation reaching living organisms on the ground.

Step 2: Relate ultraviolet absorption to human survival Excessive ultraviolet exposure damages

Quick Tip to Solve: For an ozone-layer question, name ultraviolet radiation and link its absorption to protection of skin, eyes and living cells.

living tissues and genetic material. It can cause skin cancer and eye damage such as cataracts. By limiting this exposure, the ozone layer protects human health and supports life; its role is absorption of harmful ultraviolet radiation, rather than blocking all sunlight.

Step 3: Conclusion The ozone layer is crucial because it absorbs harmful solar ultraviolet radiation and protects humans from its damaging effects.

Quick Tip to Solve: For an ozone-layer question, name ultraviolet radiation and link its absorption to protection of skin, eyes and living cells.

OR

3. Illustrate by giving suitable examples, how you can show that electromagnetic waves carry both energy and momentum. [1 mark]

Solution:

Concept:

- Electromagnetic waves transport energy; absorption can convert their energy into heat or electrical energy.
- Electromagnetic waves also carry momentum. A normally incident pulse of energy U transfers momentum U/c when fully absorbed, where c is the speed of light.

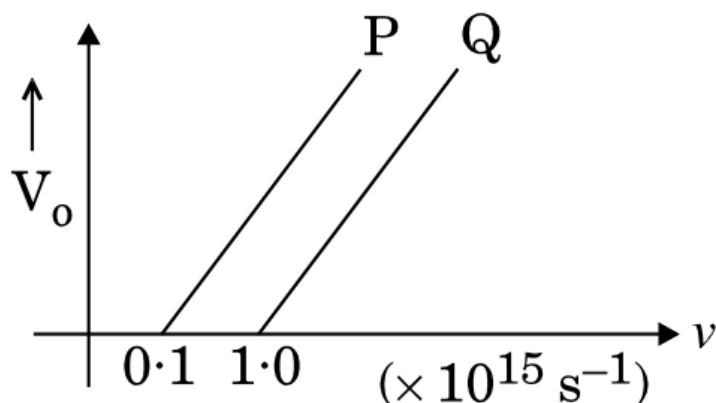
Step 1: Give an example showing energy transfer Sunlight warms the Earth and heats objects exposed to it. This temperature rise occurs because absorbed electromagnetic radiation transfers energy to matter. A solar cell converting sunlight into electrical energy is another direct example of energy transport by electromagnetic waves.

Step 2: Give an example showing momentum transfer Light falling on an absorbing or reflecting surface exerts radiation pressure, giving the surface a force. For example, sunlight pushes a solar sail. This mechanical push demonstrates momentum transfer: an absorbing surface receives momentum U/c , while normal reflection reverses the radiation momentum and transfers $2U/c$.

Step 3: Conclusion Solar heating demonstrates that electromagnetic waves carry energy; radiation pressure, such as the push on a solar sail, demonstrates that they carry momentum.

Quick Tip to Solve: Choose one heating or electrical example for energy and one force or pressure example for momentum.

4. The figure shows the variation of stopping potential V_o with the frequency ν of the incident radiations for two photosensitive metals P and Q. Which metal has smaller threshold wavelength? Justify your answer. [1 mark]



Solution:

Concept:

- Einstein's photoelectric equation gives $eV_o = h\nu - \phi = h(\nu - \nu_0)$, where ν_0 is the threshold frequency.
- The threshold wavelength is $\lambda_0 = c/\nu_0$; thus a larger threshold frequency means a smaller threshold wavelength.

Step 1: Read the threshold frequencies from the graph Threshold frequency is the horizontal-axis intercept where the stopping potential becomes zero. The line for P intersects the frequency axis to the left of the line for Q. Hence $\nu_{0Q} > \nu_{0P}$. The vertical positions at a common frequency are not the threshold wavelengths.

Step 2: Compare the corresponding wavelengths For both metals, light speed has the same value c . Since wavelength varies inversely with frequency, $\lambda_{0Q} = c/\nu_{0Q} < c/\nu_{0P} = \lambda_{0P}$. The larger intercept for Q also indicates a larger work function, consistent with needing higher-frequency incident light to start photoemission.

Step 3: Conclusion Metal Q has the smaller threshold wavelength, because its threshold frequency is larger and $\lambda_0 = c/\nu_0$.

Quick Tip to Solve: Read the frequency-axis intercept first; the line with the larger intercept has the smaller threshold wavelength.

5. Why is ground wave transmission of signals restricted to a frequency of 1500 kHz? [1 mark]

Solution:

Concept:

- Ground waves propagate along the Earth's surface and induce currents in the conducting ground.
- The induced currents dissipate electromagnetic energy as heat; attenuation of the ground wave

Quick Tip to Solve: For ground-wave frequency limits, connect induced ground currents to energy absorption, then state that attenuation rises with frequency.

increases with frequency.

Step 1: Explain why a ground wave loses energy As a ground wave travels along the surface, its electric field produces currents in the ground. The finite resistance of the ground causes energy loss, reducing the wave amplitude with distance. This absorption limits the usable transmission range.

Step 2: Relate the loss to the frequency restriction At higher frequencies, ground-wave attenuation becomes large, so the signal is absorbed strongly and cannot travel useful distances along the surface. Hence ground-wave transmission is practically restricted to frequencies up to about 1500 kHz, or 1.5 MHz; higher frequencies require another suitable propagation mode.

Step 3: Conclusion Ground-wave signals are restricted to about 1500 kHz because higher-frequency waves suffer excessive absorption and attenuation by the ground.

Quick Tip to Solve: For ground-wave frequency limits, connect induced ground currents to energy absorption, then state that attenuation rises with frequency.

Section B (14 marks)

6. Apply Gauss's law to show that for a charged spherical shell, the electric field outside the shell is, as if the entire charge were concentrated at the centre. [2 marks]

Solution:

Concept:

- Gauss's law states that $\oint \vec{E} \cdot d\vec{A} = Q_{\text{enc}}/\epsilon_0$, where Q_{enc} is the enclosed charge and ϵ_0 is the permittivity of free space.
- Spherical symmetry makes the field radial, with the same magnitude at every point at a fixed distance from the centre of a uniformly charged spherical shell.

Step 1: Choose the Gaussian surface Let the shell have radius R and total charge Q . To find the field outside it, choose a concentric imaginary sphere of radius $r > R$. This surface encloses the whole charge Q . By spherical symmetry, $\vec{E}$ is normal to the Gaussian surface and its magnitude $E(r)$ is constant on that surface. Thus the flux integral simplifies to $E(r)$ multiplied by the surface area; no charge is excluded from this Gaussian sphere.

Step 2: Apply the flux law For a positive charge, the field and the outward area element are parallel. Gauss's law therefore gives

$$E(r) 4\pi r^2 = \frac{Q}{\epsilon_0}, \quad \vec{E}(r) = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2} \hat{r}.$$

Here $\hat{r}$ points radially outward. A negative Q reverses the direction. The magnitude depends on the distance from the centre, not on the shell radius, provided the observation point is outside the shell.

Step 3: Conclusion Outside the charged spherical shell, $\vec{E} = \frac{Q}{4\pi\epsilon_0 r^2} \hat{r}$. This is exactly the electric field of a point charge Q placed at the centre, proving the required result.

Quick Tip to Solve: Choose a Gaussian sphere concentric with the shell so that the constant field comes outside the flux integral.

6. Two large parallel plane sheets have uniform charge densities $+\sigma$ and $-\sigma$. Determine the electric field (i) between the sheets, and (ii) outside the sheets. [2 marks]

Solution:

Concept:

- An effectively infinite plane sheet with surface charge density of magnitude σ produces a field $E_0 = \sigma/(2\epsilon_0)$ on each side, normal to the sheet.
- The field points away from a positively charged sheet and towards a negatively charged sheet. The resultant is the vector sum of the two fields.

Step 1: Add fields between the sheets Take the positively charged sheet on the left and the negatively charged sheet on the right. In the space between them, the positive sheet produces a field to the right, away from itself. The negative sheet also produces a field to the right, towards itself. Since both fields have the same direction, their magnitudes add:

$$E_{\text{between}} = \frac{\sigma}{2\epsilon_0} + \frac{\sigma}{2\epsilon_0} = \frac{\sigma}{\epsilon_0}.$$

The result is directed from the positive sheet to the negative sheet.

Step 2: Add fields outside the sheets To the left of both sheets, the positive sheet's field points left while the negative sheet's field points right. Each has magnitude $\sigma/(2\epsilon_0)$, so they cancel. To the right of both sheets their directions are again opposite and their magnitudes equal. Hence, on either exterior side,

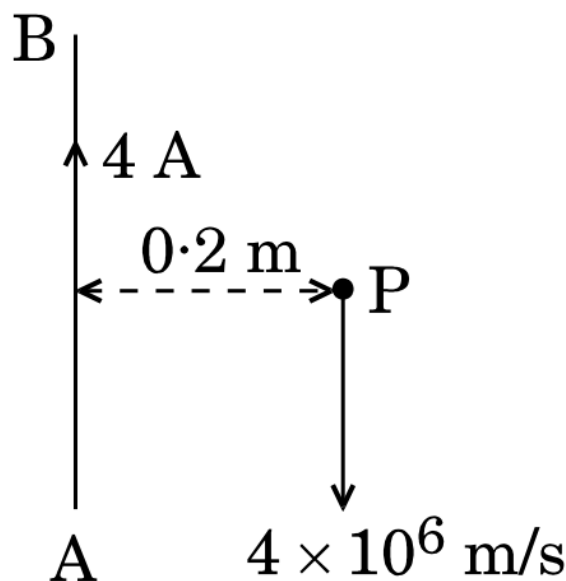
$$E_{\text{outside}} = \frac{\sigma}{2\epsilon_0} - \frac{\sigma}{2\epsilon_0} = 0.$$

For large finite sheets this result applies away from the edges, where fringing can be neglected.

Step 3: Conclusion (i) Between the sheets, $E = \sigma/\epsilon_0$, directed from $+\sigma$ to $-\sigma$. (ii) Outside the sheets, the net electric field is zero, neglecting edge effects.

Quick Tip to Solve: Draw the direction of each sheet's field first: the two fields add inside and cancel outside.

7. A long straight wire AB carries a current of 4 A. A proton P travels at $4 \times 10^6 \text{ m s}^{-1}$ parallel to the wire 0.2 m from it and in a direction opposite to the current as shown in the figure. Calculate the force which the magnetic field due to the current carrying wire exerts on the proton. Also specify its direction. [2 marks]



Solution:

Concept:

- The magnetic field at perpendicular distance r from a long straight wire carrying current I is $B = \mu_0 I / (2\pi r)$, where $\mu_0 = 4\pi \times 10^{-7} \text{ T m A}^{-1}$.
- A proton of charge $e = 1.6 \times 10^{-19} \text{ C}$ experiences $\vec{F} = e\vec{v} \times \vec{B}$. The right-hand rule determines both the field direction and the force direction.

Step 1: Calculate the field at the proton The wire carries $I = 4 \text{ A}$ from A to B, upwards in the figure, and the proton is $r = 0.2 \text{ m}$ to its right. Substitution gives

$$B = \frac{(4\pi \times 10^{-7})(4)}{2\pi(0.2)} = 4.0 \times 10^{-6} \text{ T}.$$

Curling the fingers of the right hand around the wire with the thumb pointing upwards shows that the magnetic field at P is into the plane of the paper.

Step 2: Calculate the force and its direction The proton moves downwards with speed $v = 4 \times 10^6 \text{ m s}^{-1}$. Its velocity lies in the plane of the paper and is perpendicular to the field, so $\sin \theta = 1$. Therefore

$$F = evB = (1.6 \times 10^{-19})(4 \times 10^6)(4 \times 10^{-6}) = 2.56 \times 10^{-18} \text{ N}.$$

For the positive proton, downward velocity crossed with an inward magnetic field points horizontally right. Thus the magnetic force is perpendicular to the wire and directed away from it; using the electron's force direction would reverse this result incorrectly.

Step 3: Conclusion The force is $2.56 \times 10^{-18} \text{ N}$, horizontally to the right in the figure, away from the current-carrying wire AB.

Quick Tip to Solve: For the positive proton use the right-hand rule directly; opposite motion to the wire current gives a force away from the wire.

8. A parallel plate capacitor of plate area A each and separation d , is being charged by an ac source. Show that the displacement current inside the capacitor is the same as the current charging the capacitor. [2 marks]

Solution:

Concept:

- Maxwell's displacement current is $I_d = \epsilon_0 d\Phi_E/dt$, where Φ_E is the electric flux through the area between the capacitor plates.
- The charging current in a connecting wire is $I = dQ/dt$, where Q is the instantaneous charge on the positive plate. For a parallel plate capacitor in air or vacuum, $C = \epsilon_0 A/d$.

Step 1: Express the electric flux between the plates Neglecting edge effects, the electric field between the plates is uniform and normal to their area A . At an instant when the plate charge is $Q(t)$, the field is $E(t) = Q(t)/(\epsilon_0 A)$. Equivalently, $E = V/d$ and $Q = CV$. The flux through an area spanning the gap and equal to the plate area is consequently

$$\Phi_E(t) = E(t)A = \frac{Q(t)}{\epsilon_0}.$$

The ac source makes this flux vary with time, even though charge does not cross the insulating gap as a conduction current.

Step 2: Differentiate the flux Substitute the flux into Maxwell's definition:

$$I_d = \epsilon_0 \frac{d}{dt} \left(\frac{Q}{\epsilon_0} \right) = \frac{dQ}{dt} = I.$$

The same result follows from $I_d = \epsilon_0 (A/d) dV/dt = C dV/dt$, which is the charging current. Since this equality holds instantaneously, it also holds as the ac voltage and current reverse. Consistent orientation of the flux surface gives the same current sign as in the charging lead.

Step 3: Conclusion The displacement current inside the capacitor is $I_d = dQ/dt = I$, equal at every instant to the current charging the plates.

Quick Tip to Solve: Write the gap flux as $\Phi_E = EA = Q/\epsilon_0$ before differentiating; the plate area then cancels immediately.

9. A beam of light converges at a point P. Now a convex lens is placed in the path of the convergent beam at 15 cm from P. At what point does a beam converge if the convex lens has a focal length 10 cm? [2 marks]

Solution:

Concept:

- The thin-lens formula in the Cartesian sign convention is $1/v - 1/u = 1/f$, where u and v are object and image distances measured from the lens.
- Distances along the incident light direction are positive. A convex lens has $f > 0$; a point towards which the incident rays already converge is a virtual object on the outgoing side of the lens.

Quick Tip to Solve: Treat the original convergence point P as a virtual object: its distance is positive in the lens formula.

Step 1: Identify the virtual object and signs Take the direction of propagation as left to right. Without the lens, the incident converging beam would meet at P. Since the lens intercepts the beam 15 cm before P, the point P is on the right of the lens and serves as a virtual object. Thus $u = +15$ cm, not -15 cm. The focal length of the convex lens is $f = +10$ cm. Using the sign for an ordinary real object here would describe a different arrangement.

Step 2: Find the new convergence point Insert the signed distances into the lens formula:

$$\frac{1}{v} - \frac{1}{15} = \frac{1}{10},$$

$$\frac{1}{v} = \frac{1}{10} + \frac{1}{15} = \frac{5}{30} = \frac{1}{6}.$$

Hence the image distance is positive. The convex lens increases the convergence of the incoming rays, so they meet on the outgoing side before reaching their original convergence point P. This physical check is consistent with adding the positive lens power to the beam's existing convergence.

Step 3: Conclusion The beam converges 6 cm beyond the lens, on its outgoing side. This new point is $15 - 6 = 9$ cm before P.

Quick Tip to Solve: Treat the original convergence point P as a virtual object: its distance is positive in the lens formula.

OR

9. An object is kept in front of a concave mirror of focal length 15 cm. The image formed is real and three times the size of the object. Calculate the distance of the object from the mirror. [2 marks]

Solution:

Concept:

- With the Cartesian sign convention, a concave mirror has negative focal length, and an object placed in front of it has $u < 0$.
- Mirror magnification is $m = -v/u$, where u and v are signed object and image distances. The mirror equation is $1/f = 1/v + 1/u$.

Step 1: Translate the real-image magnification into signed distances A real image formed by a concave mirror is inverted. Its size is three times the object size, so its signed magnification is $m = -3$, rather than $+3$. Consequently,

$$-\frac{v}{u} = -3 \quad \Rightarrow \quad v = 3u.$$

Both distances are negative because the real object and its real image lie in front of the mirror. The stated focal length gives $f = -15$ cm.

Step 2: Substitute in the mirror equation Substitute $v = 3u$ into the mirror equation, keeping all

Quick Tip to Solve: For a real image, make the magnification negative first. Here $m = -3$ immediately gives $v = 3u$, reducing the mirror equation to one unknown.

lengths in centimetres:

$$\begin{aligned}-\frac{1}{15} &= \frac{1}{3u} + \frac{1}{u} = \frac{4}{3u}, \\ 3u &= -60, \\ u &= -20 \text{ cm}.\end{aligned}$$

The corresponding image distance is $v = -60$ cm. This is consistent with an object between the focus at 15 cm and the centre of curvature at 30 cm, producing a magnified real image beyond the centre of curvature.

Step 3: Conclusion The object must be placed 20 cm in front of the concave mirror; its signed object distance is $u = -20$ cm.

Quick Tip to Solve: For a real image, make the magnification negative first. Here $m = -3$ immediately gives $v = 3u$, reducing the mirror equation to one unknown.

10. Explain giving reason, how the resolving power of a compound microscope depends on the

(a) frequency of the incident light.

(b) focal length of the objective lens.

[2 marks]

Solution:

Concept:

- Resolving power is the reciprocal of the smallest separation $d_{\min}$ between two object points that can be distinguished. For a microscope, $d_{\min} = 1.22\lambda/(2n \sin \theta)$, where n is the refractive index of the medium and θ is the objective's angular semi-aperture.
- Thus $\text{RP} = 2n \sin \theta / (1.22\lambda)$; since $\lambda = c/\nu$ for the vacuum wavelength, resolving power increases with frequency ν . The quantity $n \sin \theta$ is the numerical aperture.

Step 1: Explain the frequency dependence (a) Keep the objective and the surrounding medium unchanged. Increasing the frequency reduces the wavelength, so the diffraction images become smaller and closer points can be distinguished. Algebraically,

$$\text{RP} = \frac{2n \sin \theta}{1.22c} \nu \quad \Rightarrow \quad \text{RP} \propto \nu.$$

Therefore the resolving power increases in direct proportion to the frequency of the incident light.

Step 2: Explain the objective focal-length dependence (b) Compare objectives of the same clear aperture in the same medium. For an object near the objective's focus, an objective of radius a and focal length f_o subtends an angular semi-aperture satisfying approximately $\tan \theta = a/f_o$. A shorter focal length gives a larger θ , hence larger numerical aperture and greater resolving power. In the small-angle approximation,

$$\sin \theta \simeq \frac{a}{f_o}, \quad \text{RP} \simeq \frac{2na}{1.22\lambda f_o}.$$

Thus, for fixed aperture, resolving power increases when the objective focal length decreases; approximately $\text{RP} \propto 1/f_o$. The exact controlling quantity is numerical aperture, so focal length alone does

Quick Tip to Solve: Start from $\text{RP} \propto \sin \theta / \lambda$: higher frequency reduces λ , while a shorter objective of the same aperture increases θ .

not determine resolution if aperture also changes.

Step 3: Conclusion (a) Resolving power increases directly with light frequency. (b) For a fixed objective aperture and medium, it increases as the objective focal length decreases, approximately inversely with f_o .

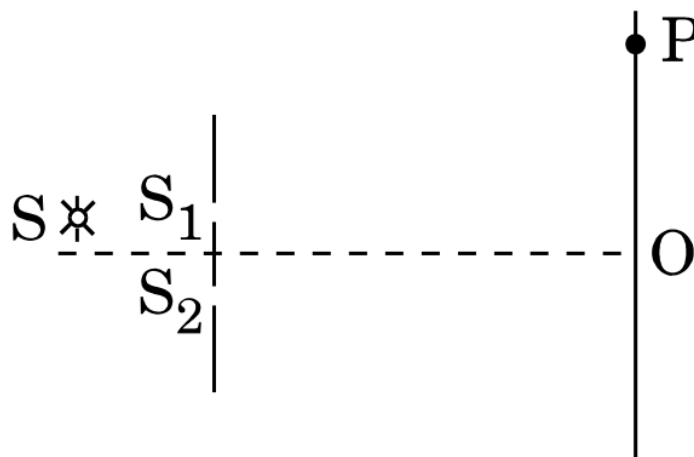
Quick Tip to Solve: Start from $RP \propto \sin \theta / \lambda$: higher frequency reduces λ , while a shorter objective of the same aperture increases θ .

11. The figure shows a modified Young's double slit experimental set-up. Here $SS_2 - SS_1 = \lambda/4$.

(a) Write the condition for constructive interference.

(b) Obtain an expression for the fringe width.

[2 marks]



Solution:

Concept:

- Constructive interference requires a total optical path difference of $m\lambda$, where m is an integer and λ is the wavelength.
- For slit separation d , screen distance $D \gg d$, and small upward screen displacement y , the outgoing path difference is $S_2P - S_1P \simeq dy/D$.

Step 1: Include the initial path difference (a) The two complete paths from the common source to P are $SS_1 + S_1P$ and $SS_2 + S_2P$. Thus constructive interference requires

$$(SS_2 - SS_1) + (S_2P - S_1P) = m\lambda.$$

Substituting the given initial difference gives

$$\frac{\lambda}{4} + S_2P - S_1P = m\lambda, \quad m = 0, \pm 1, \pm 2, \dots$$

The source-to-slit difference must be included because the light reaches the slits with a phase difference of $\pi/2$.

Step 2: Subtract consecutive bright-fringe positions (b) Measure y upward from O , towards P . With

Quick Tip to Solve: Include the source-to-slit difference when locating bright fringes; subtract adjacent positions to cancel this constant offset.

S_1 above S_2 , the bright-fringe positions satisfy

$$\frac{\lambda}{4} + \frac{dy_m}{D} = m\lambda, \quad y_m = \left(m - \frac{1}{4}\right) \frac{\lambda D}{d}.$$

Consequently, the spacing of adjacent bright fringes is

$$\beta = y_{m+1} - y_m = \frac{\lambda D}{d}.$$

The constant initial difference shifts the whole pattern, but cancels when adjacent fringe positions are subtracted.

Step 3: Conclusion Constructive interference requires $\lambda/4 + S_2P - S_1P = m\lambda$. The fringe width remains $\boxed{\beta = \lambda D/d}$.

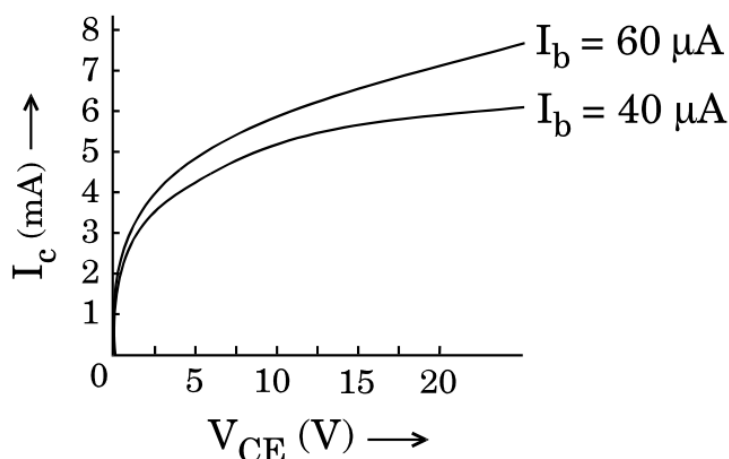
Quick Tip to Solve: Include the source-to-slit difference when locating bright fringes; subtract adjacent positions to cancel this constant offset.

12. A certain n-p-n transistor has the common emitter output characteristics as shown in the figure.

(a) Find the emitter current at $V_{CE} = 12.5 \text{ V}$ and $I_B = 60 \mu\text{A}$, and

(b) Current gain ' β ' at this point.

[2 marks]



Solution:

Concept:

- For an n-p-n transistor the emitter current is the sum of the collector and base currents: $I_E = I_C + I_B$. All currents must first be expressed in the same unit.
- The common-emitter small-signal current gain is $\beta = \Delta I_C / \Delta I_B$ at constant V_{CE} . The separate DC current gain is $\beta_{dc} = I_C / I_B$. Read collector current I_C from the curve labelled with the required base current I_B .

Step 1: Read the collector current and calculate the emitter current (a) Locate $V_{CE} = 12.5 \text{ V}$ halfway between the 10 V and 15 V positions on the horizontal axis. Move vertically to the upper curve,

Quick Tip to Solve: Read both curves at the same collector voltage for small-signal gain; use current differences for this gain and the single operating-point currents for the DC ratio.

labelled $I_B = 60 \mu\text{A}$, and read across to the current axis. The graph gives approximately $I_C = 6.0 \text{ mA}$. The lower curve corresponds to a different base current and is not the operating curve for this part. Convert the base current before adding it:

$$I_B = 60 \mu\text{A} = 0.060 \text{ mA}.$$

Thus the current leaving through the emitter is

$$I_E = 6.0 + 0.060 = 6.06 \text{ mA}.$$

This value is approximate because the collector current is read from a plotted curve.

Step 2: Calculate the common-emitter current gain (b) At the same $V_{CE} = 12.5 \text{ V}$, the lower curve for $I_B = 40 \mu\text{A}$ gives $I_C \simeq 5.0 \text{ mA}$. Use the two curves to find the common-emitter small-signal current gain:

$$\beta = \left. \frac{\Delta I_C}{\Delta I_B} \right|_{V_{CE}} = \frac{(6.0 - 5.0) \times 10^{-3}}{(60 - 40) \times 10^{-6}} = 50.$$

This dimensionless gain measures the change in collector current per change in base current. For completeness, if current gain is interpreted as the DC ratio at the specified operating point, it is

$$\beta_{\text{dc}} = \frac{I_C}{I_B} = \frac{6000 \mu\text{A}}{60 \mu\text{A}} = 100.$$

The two-characteristic construction gives the small-signal value; it must not be confused with the DC ratio.

Step 3: Conclusion The emitter current is $I_E \simeq 6.06 \text{ mA}$. From the two curves, the small-signal gain is $\beta \simeq 50$; the DC current ratio at the specified point is $\beta_{\text{dc}} \simeq 100$.

Quick Tip to Solve: Read both curves at the same collector voltage for small-signal gain; use current differences for this gain and the single operating-point currents for the DC ratio.

Section C (36 marks)

13. (a) Draw a graph showing the variation of current versus voltage in an electrolyte when an external resistance is also connected.

(b) (i) The graph between resistance (R) and temperature (T) for Hg is shown in figure (a). Explain the behaviour of Hg near 4 K.

(ii) In which region of the graph shown in figure (b) is the resistance negative and why? [3 marks]

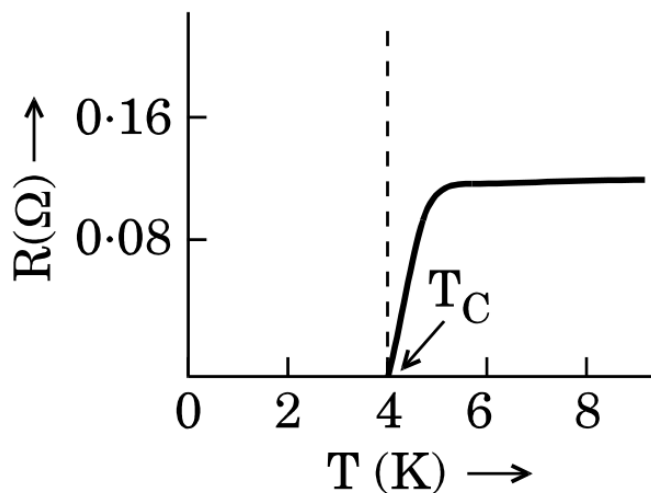


Figure (a)

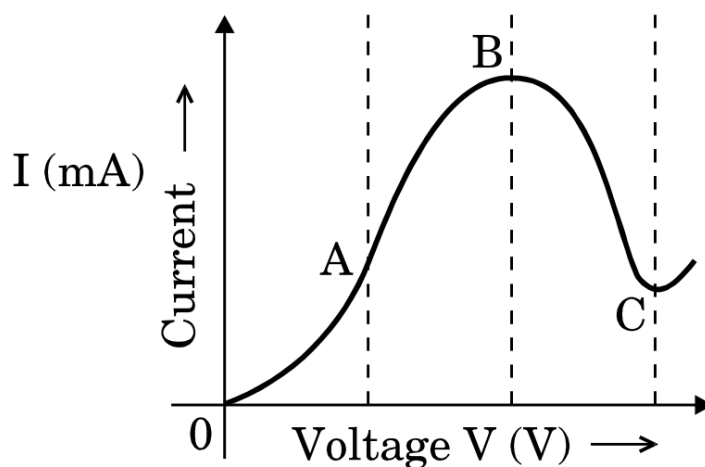


Figure (b)

Solution:

Concept:

- An electrolytic cell develops a back emf E_b . With external resistance R and electrolyte resistance r , its conducting characteristic is $I = (V - E_b)/(R + r)$.
- A superconductor has zero electrical resistance below its critical temperature T_c .

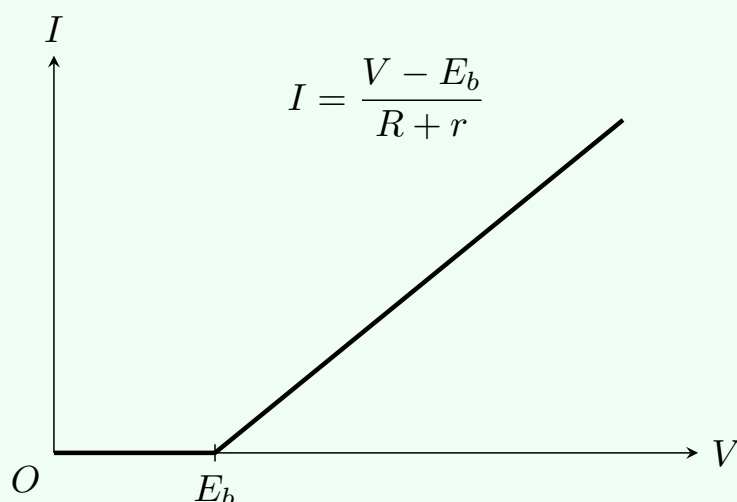
Quick Tip to Solve: On an I-V graph, find the falling-current segment to identify negative differential resistance; do not confuse its slope with the positive ratio V/I .

- Negative differential resistance means $dV/dI < 0$: current decreases as applied voltage increases.

Step 1: (a) Draw the electrolyte characteristic Take applied voltage V on the horizontal axis and current I on the vertical axis. Current is negligible until the voltage reaches the back emf E_b , the decomposition threshold in this simplified characteristic. Beyond this intercept, voltage drives conduction through the electrolyte and the external series resistance:

$$V = E_b + I(R + r), \quad I = \frac{V - E_b}{R + r}.$$

Thus the conducting portion is a straight line meeting the voltage axis at a positive voltage, rather than at the origin. Its slope is $1/(R + r)$; adding external resistance reduces this slope.



Current versus applied voltage for an electrolyte with external series resistance.

Step 2: (b)(i) Explain the mercury transition near 4 K In figure (a), mercury has a finite resistance above its critical temperature. When cooled through T_c , approximately 4 K in the given graph, its resistance falls abruptly to zero. Below T_c , mercury is superconducting, allowing current to flow without resistive energy dissipation. This is a sudden superconducting transition, rather than a gradual extrapolation of ordinary metallic resistance to zero.

Step 3: (b)(ii) Identify the negative-resistance region In figure (b), current falls from the maximum at B to the minimum at C while voltage increases. Therefore, along BC,

$$\frac{dI}{dV} < 0 \implies r_d = \frac{dV}{dI} < 0.$$

Hence BC is the negative differential resistance region. Along AB and beyond C the current rises with voltage, giving positive slope. The ratio V/I remains positive for positive voltage and current; the negative quantity is the incremental resistance, not that ratio.

Step 4: Conclusion (a) The graph has a positive voltage intercept E_b , followed by a rising straight line. (b)(i) Hg becomes superconducting with zero resistance below approximately 4 K. (b)(ii) BC has negative differential resistance because current decreases as voltage increases.

Quick Tip to Solve: On an I-V graph, find the falling-current segment to identify negative differential resistance; do not confuse its slope with the positive ratio V/I .

14. (a) For the circuit shown in the figure, how would the balancing length be affected, if

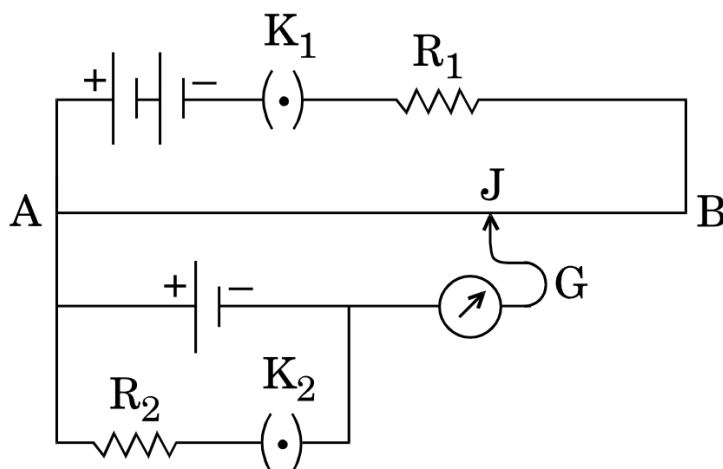
(i) R_1 is decreased,

(ii) R_2 is increased,

the other factors remaining the same in the circuit? Justify your answer in each case.

(b) Why is a potentiometer preferred over a voltmeter? Give reason.

[3 marks]



Solution:

Concept:

- At potentiometer balance, the measured voltage V equals kl , where k is the potential gradient of the uniform wire and l is the balancing length.
- The primary circuit determines k . For wire resistance R_w and length L , $k = I_p R_w / L$, where I_p is the primary current.
- A cell of emf E and internal resistance r , supplying an external load R_2 , has terminal voltage $V = ER_2 / (R_2 + r)$.

Step 1: (a)(i) Decrease the primary resistance R_1 In the given circuit, R_1 is in series with the driver cell and potentiometer wire AB. Decreasing R_1 increases the primary current I_p , hence increases the voltage drop per unit length k of AB. The secondary cell and its load are unchanged, so the voltage V being measured stays fixed. Using the null condition,

$$V = kl, \quad l = \frac{V}{k},$$

a larger potential gradient requires a shorter balancing length. The balance point J therefore moves towards A.

Step 2: (a)(ii) Increase the secondary load resistance R_2 R_2 is connected across the secondary cell through key K_2 . With that load connected, increasing R_2 reduces the current drawn from this cell and reduces its internal voltage loss. For internal resistance r ,

$$I_s = \frac{E}{R_2 + r}, \quad V = E - I_s r = \frac{ER_2}{R_2 + r}.$$

Thus the terminal voltage increases as R_2 increases. The primary circuit is unchanged, so k is

Quick Tip to Solve: Use $l = V/k$: first decide whether a changed resistor alters the wire's potential gradient or the test cell's terminal voltage.

unchanged and $l = V/k$ increases. J moves towards B, provided the new balance lies on AB. This dependence refers to the loaded-cell measurement with K_2 closed; if K_2 were open, varying the disconnected resistance would have no effect.

Step 3: (b) Explain the advantage of a potentiometer A potentiometer uses a null method. At balance, the galvanometer current is zero, so the measuring branch draws no current from the cell being tested. With its external load disconnected, the measured voltage is therefore the true emf. An ordinary voltmeter has finite resistance and draws current, causing an internal drop Ir in the cell; it generally reads the terminal voltage $E - Ir$, which is less than the emf. This absence of loading by the measuring branch makes the potentiometer preferable for accurate emf measurement.

Step 4: Conclusion Decreasing R_1 decreases the balancing length. Increasing R_2 , with K_2 closed, increases it. A potentiometer measures emf accurately by drawing no current from the test cell at balance.

Quick Tip to Solve: Use $l = V/k$: first decide whether a changed resistor alters the wire's potential gradient or the test cell's terminal voltage.

OR

14. State the underlying principle of meter bridge. Draw the circuit diagram and explain how the unknown resistance of a conductor can be determined by this method. [3 marks]

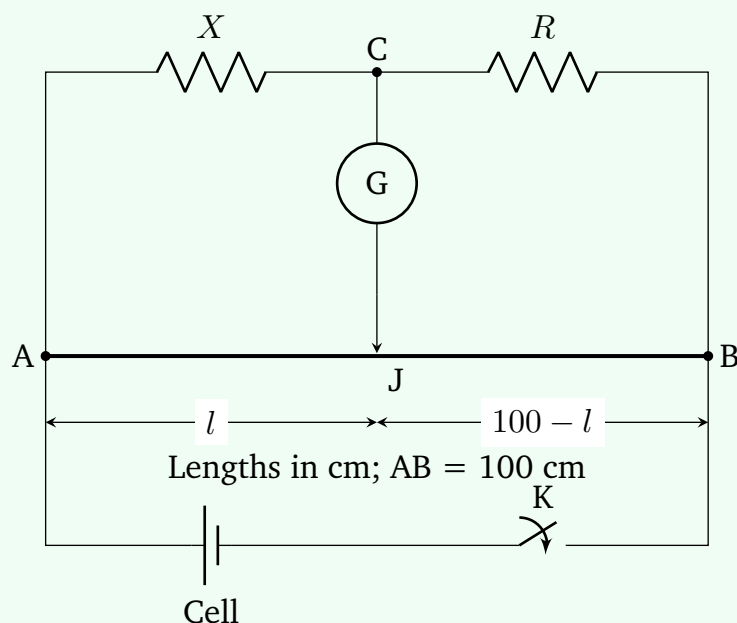
Solution:

Concept:

- A metre bridge works on the balanced Wheatstone bridge principle: no current flows through the galvanometer when the ratios of resistances in the two arms are equal.
- For a uniform wire, resistance is proportional to length: $R = \rho l/A$, where ρ is resistivity and A is cross-sectional area.

Step 1: Arrange the metre-bridge circuit Connect the unknown resistance X in the left gap and a known resistance R from a resistance box in the right gap. The uniform bridge wire AB is 100 cm long. Connect a cell and key K across A and B. Connect the galvanometer G between the junction C of the two gap resistances and a movable jockey J touching the wire. The diagram shows these connections; AJ is measured from the end next to the unknown resistance.

Quick Tip to Solve: Put each gap resistance above its adjacent wire length: X/R equals AJ/JB . Reversing the gap resistances reverses the length ratio.



Metre bridge with unknown resistance X in the left gap; G reads zero at J .

Step 2: Obtain the null point Close K and gently move the jockey along AB until the galvanometer shows zero deflection. At this balance point, C and J are at the same potential, so no current passes through G . Record $AJ = l$ cm; then $JB = (100 - l)$ cm. Select a suitable known resistance so that the balance point is near the middle of the wire, which improves the precision of the length ratio.

Step 3: Apply the balance condition and calculate X Let the resistances of wire segments AJ and JB be P and Q . At null, the upper branch current I_1 flows through both X and R , while the wire current I_2 flows through both wire segments. Equality of potentials at C and J gives

$$I_1 X = I_2 P, \quad I_1 R = I_2 Q.$$

Dividing the equations and using the same resistivity and cross-sectional area for both segments gives

$$\frac{X}{R} = \frac{P}{Q} = \frac{l}{100 - l}, \quad X = R \frac{l}{100 - l}.$$

Thus the measured balance length and known resistance determine the unknown resistance in ohms. Repeat with another suitable value of R , and interchange the gap resistances to reduce the effect of end resistances before taking the mean.

Step 4: Conclusion The metre bridge uses the null condition of a Wheatstone bridge. With X in the left gap, known R in the right gap, and balancing length l measured in centimetres from A , the unknown resistance is $X = Rl/(100 - l)$.

Quick Tip to Solve: Put each gap resistance above its adjacent wire length: X/R equals AJ/JB . Reversing the gap resistances reverses the length ratio.

15. A proton, a deuteron and an alpha particle, are accelerated through the same potential difference and then subjected to a uniform magnetic field $\vec{B}$, perpendicular to the direction of their motions. Compare (i) their kinetic energies, and (ii) if the radius of the circular path described by proton is 5 cm, determine the radii of the paths described by deuteron and alpha particle. [3 marks]

Solution:

Concept:

- A particle initially at rest gains kinetic energy $K = qV$ when accelerated through a potential difference V , where q is its positive charge.
- A perpendicular uniform magnetic field provides the centripetal force: $qvB = mv^2/r$. Thus $r = mv/(qB) = \sqrt{2mV/q}/B$.
- Taking the proton mass as m , a deuteron has mass approximately $2m$ and charge e , while an alpha particle has mass approximately $4m$ and charge $2e$.

Step 1: (i) Compare the kinetic energies Let the common accelerating potential difference be V . The electric field does work equal to charge times potential difference. Hence

$$K_p = eV, \quad K_d = eV, \quad K_\alpha = 2eV.$$

The proton and deuteron have equal charges and therefore gain equal energies despite their different masses. The alpha particle gains twice this energy because its charge is twice as large. Thus $K_p : K_d : K_\alpha = 1 : 1 : 2$. The subsequent magnetic field changes only the direction of velocity and does no work, so these kinetic energies remain unchanged.

Step 2: (ii) Derive the radius dependence Combining $\frac{1}{2}mv^2 = qV$ with $r = mv/(qB)$ gives

$$r = \frac{1}{B} \sqrt{\frac{2mV}{q}}.$$

Since V and B are common, compare $\sqrt{m/q}$, rather than mass alone. Therefore

$$\frac{r_d}{r_p} = \sqrt{\frac{2m/e}{m/e}} = \sqrt{2}, \quad \frac{r_\alpha}{r_p} = \sqrt{\frac{4m/(2e)}{m/e}} = \sqrt{2}.$$

For $r_p = 5$ cm, each of the other two radii is $5\sqrt{2}$ cm ≈ 7.07 cm. The equal radii follow from the equal mass-to-charge ratios of the deuteron and alpha particle.

Step 3: Conclusion $K_p : K_d : K_\alpha = 1 : 1 : 2$, and $r_d = r_\alpha = 5\sqrt{2}$ cm ≈ 7.07 cm.

Quick Tip to Solve: For equal accelerating potentials use $K \propto q$ and $r \propto \sqrt{m/q}$; equal energies alone do not imply equal radii.

16. (a) Briefly explain how a galvanometer is converted into an ammeter.

(b) A galvanometer coil has a resistance of $15\ \Omega$ and it shows full scale deflection for a current of 4 mA. Convert it into an ammeter of range 0 to 6 A. [3 marks]

Solution:

Concept:

- A galvanometer responds to a small current; its full-scale current is I_g and its coil resistance is G .
- An ammeter is obtained by connecting a small shunt resistance S in parallel with the galvanometer, so that most of the measured current bypasses the coil.
- Parallel branches have equal potential differences: $I_g G = (I - I_g)S$, where I is the required full-scale ammeter current.

Step 1: (a) Explain the conversion Connect a low resistance, called a shunt, across the galvanometer terminals. At the required maximum current I , only the safe full-scale current I_g flows through the coil and the remaining current $I - I_g$ flows through the shunt. Recalibrate the scale to read the total current. The parallel combination has a small effective resistance, so the resulting ammeter can be connected in series with the circuit without appreciably changing its current. Connecting a high resistance in series would instead produce a voltmeter and would not achieve this conversion.

Step 2: (b) Calculate the shunt resistance The given values are $G = 15\ \Omega$, $I_g = 4\ \text{mA} = 0.004\ \text{A}$, and $I = 6\ \text{A}$. At full scale, the current through the shunt is

$$I_s = I - I_g = 6 - 0.004 = 5.996\ \text{A}.$$

The voltage across both branches is $I_g G = 0.004 \times 15 = 0.060\ \text{V}$. Therefore

$$S = \frac{I_g G}{I - I_g} = \frac{0.004 \times 15}{5.996} = 0.0100067\ \Omega \approx 0.0100\ \Omega.$$

As a check, $I_s S$ is $0.060\ \text{V}$, the same as the coil voltage. The shunt is much smaller than G , as required to carry almost all the total current. The coil reaches full-scale deflection precisely when the combined current reaches $6\ \text{A}$.

Step 3: Conclusion Connect a shunt of $S \approx 0.0100\ \Omega$ in parallel with the $15\ \Omega$ galvanometer and calibrate the scale from 0 to 6 A.

Quick Tip to Solve: Convert milliamperes to amperes first, then use $S = I_g G / (I - I_g)$; the shunt carries the total current minus the coil current.

16. (a) Briefly explain how a galvanometer is converted into a voltmeter.

(b) A voltmeter of a certain range is constructed by connecting a resistance of $980\ \Omega$ in series with a galvanometer. When the resistance of $470\ \Omega$ is connected in series, the range gets halved. Find the resistance of the galvanometer. [3 marks]

Solution:

Concept:

- A voltmeter is connected across two circuit points and should have high resistance to draw negligible current from the circuit.
- Connect a large multiplier resistance R in series with a galvanometer of resistance G . Its voltage range is $V = I_g(G + R)$, where I_g is the coil's full-scale current.

Step 1: Explain how to obtain a high-resistance voltmeter (a) Connect a suitably high resistance in series with the galvanometer. This series resistance limits the current through the sensitive coil, so the coil reaches full-scale deflection only when the chosen maximum voltage is applied. Calibrate the scale in volts and connect the whole combination in parallel across the points whose potential difference is required. For a selected range V , the required series resistance is $R = V/I_g - G$.

Step 2: Write the full-scale voltage for each series resistance (b) The same galvanometer is used in both arrangements, so G and the full-scale current I_g remain unchanged. Let the initial voltage range be V , and let G be measured in ohms. With the first series resistance,

$$V = I_g(G + 980).$$

When the series resistance becomes $470\ \Omega$, the full-scale voltage is halved:

$$\frac{V}{2} = I_g(G + 470).$$

The total resistance, including the coil, is halved because the permitted full-scale current is unchanged. The added series resistance by itself need not halve.

Step 3: Eliminate the unknown full-scale current Divide the second equation by the first and rearrange:

$$\begin{aligned}\frac{1}{2} &= \frac{G + 470}{G + 980}, \\ G + 980 &= 2G + 940, \\ G &= 980 - 940 = 40\ \Omega.\end{aligned}$$

To check the result, the two complete instrument resistances are $40 + 980 = 1020\ \Omega$ and $40 + 470 = 510\ \Omega$. Their ratio is exactly $2 : 1$, giving the required ratio of voltage ranges at the same I_g .

Step 4: Conclusion A galvanometer becomes a voltmeter by adding a high resistance in series and calibrating in volts. The galvanometer coil resistance is $G = 40\ \Omega$.

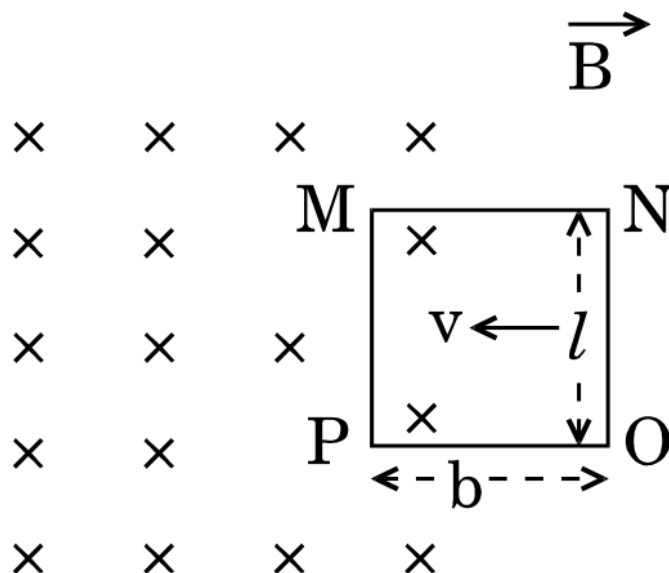
Quick Tip to Solve: For the same galvanometer, voltage range is proportional to total series resistance, including the coil; equate $G + 980 = 2(G + 470)$.

17. The figure shows a rectangular conducting frame MNOP of resistance R placed partly in a perpendicular magnetic field $\vec{B}$ and moved with velocity $\vec{v}$ as shown in the figure. Obtain the expressions for the

(a) force acting on the arm 'ON' and its direction, and

(b) power required to move the frame to get a steady emf induced between the arms MN and PO.

[3 marks]



Solution:

Concept:

- Faraday's law gives the magnitude of induced emf as $\mathcal{E} = |d\Phi_B/dt|$. For a rectangular loop entering a uniform field at speed v , $\mathcal{E} = Blv$, where l is its vertical side length.
- A current-carrying straight arm in the magnetic field experiences $\vec{F} = I\vec{l} \times \vec{B}$. An arm outside the field has zero magnetic force. Lenz's law makes the net magnetic force oppose motion.
- At steady speed, the external mechanical power is $P = F_{\text{ext}}v = I^2R$, where R is the resistance of the complete frame.

Step 1: Read the field region in the printed figure The crosses show a magnetic field into the page on the left. The frame moves left with speed v , so more of it enters the field. In the printed figure, the left arm MP lies in the field, whereas the right arm ON lies outside it. Thus there is a mismatch between the named arm ON in part (a) and the arm actually subject to magnetic drag. We keep the printed labels and distinguish the force on ON from the net force on the frame.

Step 2: Obtain the emf and current Let x be the horizontal length of the frame that is inside the field. While the frame is only partly immersed, the area in the field is lx and $dx/dt = v$. Hence

$$\mathcal{E} = \left| \frac{d(Blx)}{dt} \right| = Blv, \quad I = \frac{Blv}{R}.$$

The inward flux increases. The induced current therefore produces an outward field and flows anticlockwise: M to P to O to N to M. The emf between the upper and lower arms is generated across MP, with P at higher potential than M.

Quick Tip to Solve: First identify which vertical side lies among the field crosses. Use $\mathcal{E} = Blv$, then $I = \mathcal{E}/R$, and check the mechanical power using I^2R .

Step 3: Evaluate the force on the named arm and the frame (a) Literally, for arm ON in the printed figure, the external field is zero, so its magnetic force is $F_{ON} = 0$, with no direction. On the immersed arm MP, the current is downward and the field is into the page, so the force is rightward, opposing the leftward motion:

$$F_{MP} = BIl = \frac{B^2 l^2 v}{R}.$$

The vertical forces on the immersed portions of MN and PO cancel. Thus the net magnetic force on the frame is $B^2 l^2 v/R$ rightward. If part (a) intends the arm in the field, this is the usual nonzero answer, but it belongs to MP with the labels as printed.

Step 4: Determine the mechanical power for steady motion (b) An external leftward force equal to the rightward magnetic drag maintains constant speed. Its required power is

$$P = F_{\text{ext}} v = \frac{B^2 l^2 v^2}{R}.$$

This agrees with the rate of Joule heating, $I^2 R = (Blv/R)^2 R$, so the supplied mechanical energy becomes heat in the frame. The result holds while only part of the frame is in the uniform field; once the whole frame enters, the flux stops changing and the induced current vanishes.

Step 5: Conclusion As drawn, ON is outside the field and its magnetic force is zero. The immersed arm MP and hence the frame experience $F = B^2 l^2 v/R$ rightward, opposite to the motion. The required mechanical power is $P = B^2 l^2 v^2/R$.

Quick Tip to Solve: First identify which vertical side lies among the field crosses. Use $\mathcal{E} = Blv$, then $I = \mathcal{E}/R$, and check the mechanical power using $I^2 R$.

18. Draw a ray diagram to show the image formation of a distant object by a refracting telescope. Write the expression for its angular magnification in terms of the focal lengths of the lenses used. State the important considerations required to achieve large resolution and their consequent limitations.

[3 marks]

Solution:

Concept:

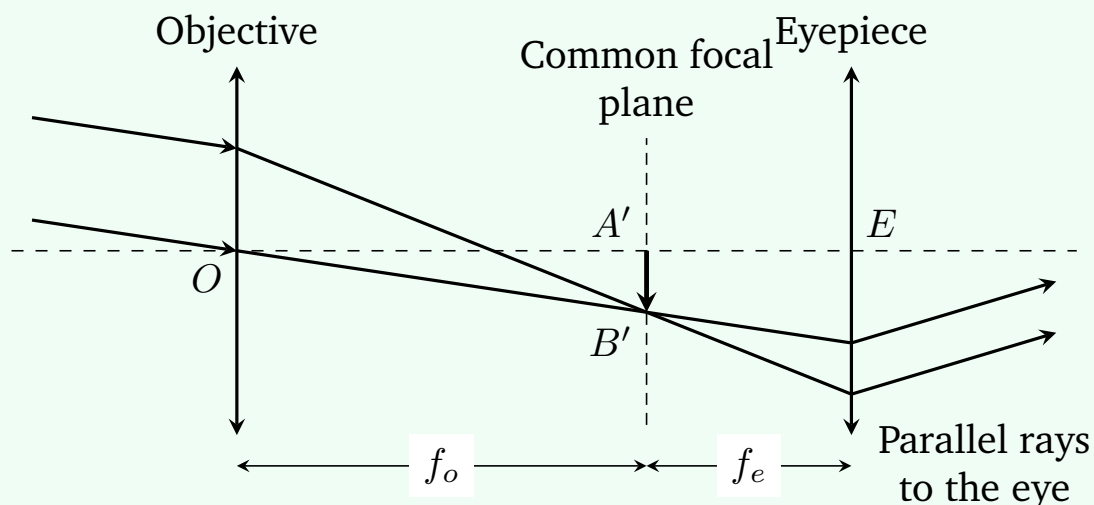
- An astronomical refracting telescope uses a converging objective of long focal length f_o and a converging eyepiece of short focal length f_e .
- In normal adjustment, the objective's focal plane coincides with the eyepiece's first focal plane. The final image is at infinity, so the eye views it without accommodation.
- Angular magnification is $M = \beta/\alpha$. For a circular aperture of diameter D , the diffraction limit is $\theta_{\min} = 1.22\lambda/D$, where λ is the wavelength.

Step 1: Image formation in normal adjustment Nearly parallel rays from a distant object enter the objective. It produces a real, inverted and diminished intermediate image $A'B'$ in its focal plane. The eyepiece is placed so that this image is also in its first focal plane; it therefore sends out parallel rays and acts as a magnifier. The final image is virtual, inverted relative to the object and at infinity. The lens separation is $f_o + f_e$. The diagram shows one off-axis bundle, with the intermediate image below

Quick Tip to Solve: Keep both focal planes coincident for normal adjustment; distinguish increasing magnification from reducing the diffraction-limited resolving angle.

the principal axis.

From distant
object



Refracting telescope in normal adjustment: the real inverted intermediate image lies at the common focal plane; emerging rays are parallel.

Step 2: Obtain the angular magnification If the intermediate image has height of magnitude h , small-angle geometry gives $|\alpha| \simeq h/f_o$ and $|\beta| \simeq h/f_e$. Hence, in normal adjustment,

$$M = -\frac{f_o}{f_e}, \quad |M| = \frac{f_o}{f_e}.$$

The negative sign represents inversion. A long focal-length objective and a short focal-length eyepiece give large angular magnification, but magnification alone does not determine whether two nearby objects can be distinguished.

Step 3: Resolution and practical limitations Large resolution requires a small minimum resolvable angle. Since $\theta_{\min} = 1.22\lambda/D$, the objective should have a large aperture; shorter wavelength also improves the diffraction limit. A large objective gathers more light and separates closer angular details. However, a large lens is heavy, difficult to manufacture accurately and can deform when supported at its edges. Thick glass absorbs light, and refracting objectives suffer from chromatic and spherical aberrations, requiring expensive correction. Atmospheric turbulence further limits useful resolution for observations from the ground. Thus simply increasing magnification cannot recover details blurred by diffraction, aberrations or the atmosphere.

Step 4: Conclusion In normal adjustment, $M = -f_o/f_e$ and the lens separation is $f_o + f_e$. High resolving power needs a large objective aperture, subject to lens fabrication, weight, aberration and atmospheric limitations.

Quick Tip to Solve: Keep both focal planes coincident for normal adjustment; distinguish increasing magnification from reducing the diffraction-limited resolving angle.

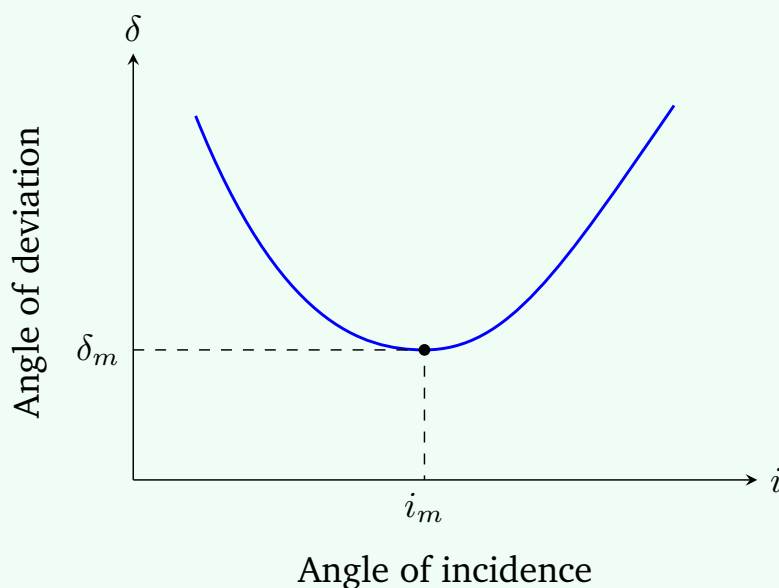
18. (a) Plot a graph for angle of deviation as a function of angle of incidence for a triangular prism.
 (b) Derive the relation for the refractive index of the prism in terms of the angle of minimum deviation and angle of prism.

[3 marks]

Solution:**Concept:**

- For a prism of apex angle A , the internal refraction angles satisfy $r_1 + r_2 = A$. If i is incidence and e emergence, the total deviation is $\delta = i + e - A$.
- At minimum deviation δ_m , the ray path is symmetric: $i = e$ and $r_1 = r_2$. Snell's law at the air-prism boundary gives $\mu = \sin i / \sin r_1$.

Step 1: (a) Deviation versus incidence For transmitted rays through the prism, the deviation initially decreases as the angle of incidence increases. It reaches a minimum value δ_m at an incidence angle i_m , and then increases. Therefore the graph has a smooth minimum, as shown schematically. The minimum occurs for symmetric passage through the prism. On either side of the minimum, the same deviation corresponds to two incidence angles related by reversibility of the ray path; the two incidence angles exchange the roles of incidence and emergence. The graph is not a straight line.



Schematic variation of deviation with incidence for a triangular prism.

Step 2: (b) Use the symmetric ray path At minimum deviation, $i = e = i_m$. From the geometry of the two refracting faces, $r_1 + r_2 = A$, and symmetry requires $r_1 = r_2$. Therefore

$$r_1 = r_2 = \frac{A}{2}.$$

Using the total deviation formula at this position gives

$$\delta_m = i_m + i_m - A = 2i_m - A, \quad i_m = \frac{A + \delta_m}{2}.$$

Quick Tip to Solve: At minimum deviation first write $i = e$ and $r_1 = r_2 = A/2$; then use $\delta_m = 2i - A$ before applying Snell's law.

Step 3: (b) Apply Snell's law The prism is surrounded by air, whose refractive index is taken as unity. At its first face, Snell's law gives

$$\mu = \frac{\sin i_m}{\sin r_1}.$$

Substituting $i_m = (A + \delta_m)/2$ and $r_1 = A/2$ expresses the prism's refractive index solely in terms of its apex angle and minimum deviation. All angles in the sine functions must be measured using the same angular convention.

Step 4: Conclusion The deviation-incidence graph has a minimum δ_m . For a prism in air,

$$\mu = \frac{\sin[(A + \delta_m)/2]}{\sin(A/2)}.$$

Quick Tip to Solve: At minimum deviation first write $i = e$ and $r_1 = r_2 = A/2$; then use $\delta_m = 2i - A$ before applying Snell's law.

19. (a) State briefly, with what purpose was Davisson and Germer experiment performed and what inference was drawn from this.

(b) Obtain an expression for the ratio of the accelerating potentials required to accelerate a proton and an α -particle to have the same de-Broglie wavelength associated with them. [3 marks]

Solution:

Concept:

- De Broglie proposed that a particle of momentum p has wavelength $\lambda = h/p$, where h is Planck's constant. Diffraction is evidence of wave behaviour.
- For a particle of mass m and charge magnitude q , accelerated from rest through potential difference V , the nonrelativistic kinetic energy is $qV = p^2/(2m)$. Hence $\lambda = h/\sqrt{2mqV}$.

Step 1: (a) Purpose and arrangement of the experiment Davisson and Germer investigated the scattering of an electron beam by a nickel crystal to test the wave nature of electrons predicted by de Broglie. Electrons were accelerated through a known potential and directed onto the crystal. A movable detector measured the intensity of scattered electrons at different angles. The crystal supplied regularly spaced atomic planes, allowing scattered electron waves to interfere.

Step 2: (a) Inference from the observations The scattered intensity showed a pronounced diffraction maximum at a particular angle, rather than only the smooth variation expected from a purely particle description. The wavelength inferred from the crystal diffraction condition agreed with the de Broglie wavelength calculated from the electrons' accelerating potential. Thus electrons exhibit wave behaviour, and the experiment provided direct confirmation of de Broglie's matter-wave hypothesis.

Step 3: (b) Equate the de Broglie wavelengths Let V_p and V_α be the accelerating potentials for the proton and alpha particle. Their charge magnitudes are e and $2e$, respectively. Equal wavelengths

Quick Tip to Solve: Equal wavelength means equal momentum, hence equal mqV , not equal kinetic energy; include the alpha particle's charge $2e$.

imply equal momentum magnitudes, so

$$\frac{h}{\sqrt{2m_p e V_p}} = \frac{h}{\sqrt{2m_\alpha (2e) V_\alpha}}.$$

Squaring and cancelling common factors gives

$$m_p V_p = 2m_\alpha V_\alpha, \quad \frac{V_p}{V_\alpha} = \frac{2m_\alpha}{m_p}.$$

Since $m_\alpha \simeq 4m_p$, this ratio is approximately 8. The larger mass and charge of the alpha particle mean that it needs a smaller potential for the same momentum.

Step 4: Conclusion Davisson-Germer electron diffraction confirmed the wave nature of electrons. For equal de Broglie wavelengths, $V_p/V_\alpha = 2m_\alpha/m_p \simeq 8$, so $V_p : V_\alpha = 8 : 1$.

Quick Tip to Solve: Equal wavelength means equal momentum, hence equal mqV , not equal kinetic energy; include the alpha particle's charge $2e$.

OR

19. (a) An electron and a proton are accelerated through the same potential. Which one of the two has

(i) greater value of de-Broglie wavelength associated with it, and

(ii) lesser momentum?

Justify your answer in each case.

(b) How is the momentum of a particle related with its de-Broglie wavelength? Show the variation on a graph. [3 marks]

Solution:

Concept:

- The de-Broglie wavelength is $\lambda = h/p$, where h is Planck's constant and p is the magnitude of the particle's momentum.
- A particle accelerated from rest through a potential difference of magnitude V gains kinetic energy $K = |q|V$. In the non-relativistic treatment, $K = p^2/(2m)$, with m the particle mass.
- Electron and proton charges have equal magnitudes e , while $m_p \simeq 1836m_e$. Their opposite charge signs determine the acceleration direction, but not the energy gained through the same accelerating potential magnitude.

Step 1: Compare the momenta for the same accelerating potential For each particle, equate the energy gained from the electric field to its kinetic energy:

$$eV = \frac{p^2}{2m} \Rightarrow p = \sqrt{2meV}.$$

Quick Tip to Solve: At the same accelerating voltage first write $p = \sqrt{2meV}$; invert this mass dependence for the wavelength. For the graph keep $p\lambda = h$ constant.

Thus the two particles gain the same kinetic energy, but their momenta are not equal. The ratio is

$$\frac{p_e}{p_p} = \sqrt{\frac{m_e}{m_p}} \simeq \frac{1}{\sqrt{1836}} < 1.$$

This establishes the mass dependence required in part (a)(ii): for a fixed accelerating potential, the heavier particle has the larger momentum. Equal kinetic energies must not be confused with equal velocities or equal momenta.

Step 2: Compare their de-Broglie wavelengths Substitute the momentum into the wavelength relation:

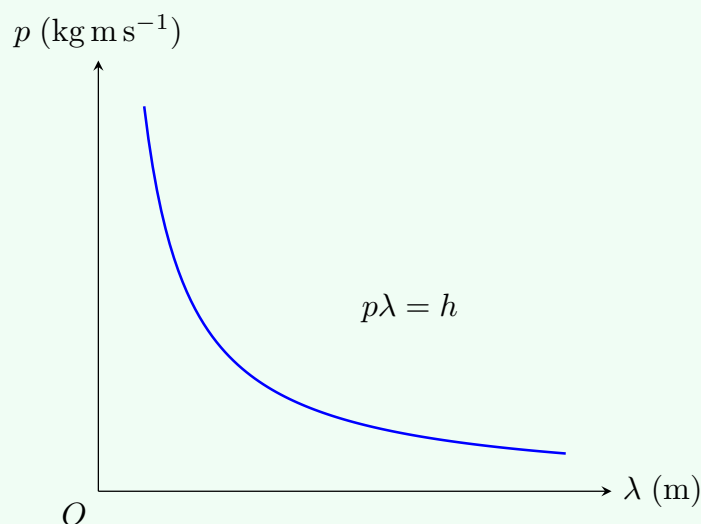
$$\lambda = \frac{h}{\sqrt{2meV}}, \quad \frac{\lambda_e}{\lambda_p} = \sqrt{\frac{m_p}{m_e}} \simeq \sqrt{1836} > 1.$$

The wavelength therefore varies inversely as the square root of mass at fixed V . This gives the justification for part (a)(i); the smaller electron mass produces a smaller momentum and hence a longer wavelength.

Step 3: Plot the momentum-wavelength relation For part (b), rearrange the de-Broglie relation as

$$p = \frac{h}{\lambda}, \quad p\lambda = h.$$

Plot momentum p vertically against wavelength λ horizontally. Both quantities are positive, so the graph lies in the first quadrant and is a decreasing rectangular hyperbola. As λ increases, p decreases; doubling λ halves p . The coordinate axes are asymptotes, and the curve does not intersect either axis for a finite positive wavelength or momentum. The sketch below shows this inverse variation; its scale is arbitrary.



Momentum versus de-Broglie wavelength (schematic)

Step 4: Conclusion (a)(i) The electron has the greater de-Broglie wavelength, because $\lambda \propto 1/\sqrt{m}$ at the same potential. (a)(ii) The electron has the lesser momentum, because $p \propto \sqrt{m}$. (b) $p = h/\lambda$; the graph of p against λ is a decreasing rectangular hyperbola.

Quick Tip to Solve: At the same accelerating voltage first write $p = \sqrt{2meV}$; invert this mass dependence for the wavelength. For the graph keep $p\lambda = h$ constant.

20. (a) Using Bohr's postulates, derive the expression for the orbital period of the electron moving in the n^{th} orbit of hydrogen atom.

(b) Write Rydberg's formula for wavelengths of the spectral lines of hydrogen atom spectrum. Mention to which series in the emission spectrum of hydrogen, H_α line belongs. [3 marks]

Solution:

Concept:

- In Bohr's hydrogen atom, electrostatic attraction provides the centripetal force: $m_e v_n^2 / r_n = e^2 / (4\pi\epsilon_0 r_n^2)$.
- Bohr's angular-momentum postulate is $m_e v_n r_n = nh / (2\pi)$, where $n = 1, 2, \dots$, m_e is electron mass and h is Planck's constant.
- The orbital period is the time for one revolution, $T_n = 2\pi r_n / v_n$. Emission occurs when an electron falls from a higher energy level to a lower one.

Step 1: (a) Obtain the radius and speed of the n^{th} orbit Using the force equation and the quantisation of angular momentum,

$$m_e v_n^2 r_n = \frac{e^2}{4\pi\epsilon_0}, \quad v_n = \frac{nh}{2\pi m_e r_n}.$$

Substitute the second expression into the first and solve for r_n :

$$\frac{n^2 h^2}{4\pi^2 m_e r_n} = \frac{e^2}{4\pi\epsilon_0} \Rightarrow r_n = \frac{\epsilon_0 n^2 h^2}{\pi m_e e^2}.$$

Substitution back into the angular-momentum equation then gives

$$v_n = \frac{e^2}{2\epsilon_0 n h}.$$

These results show that the orbit radius increases as n^2 , while the electron speed decreases as $1/n$.

Step 2: (a) Calculate the orbital period The distance covered in one circular revolution is $2\pi r_n$. Dividing it by the speed gives

$$\begin{aligned} T_n &= \frac{2\pi r_n}{v_n} \\ &= 2\pi \left(\frac{\epsilon_0 n^2 h^2}{\pi m_e e^2} \right) \left(\frac{2\epsilon_0 n h}{e^2} \right) \\ &= \frac{4\epsilon_0^2 h^3}{m_e e^4} n^3. \end{aligned}$$

Thus $T_n = T_1 n^3$, with $T_1 \simeq 1.52 \times 10^{-16}$ s. The factor n^3 follows from both the larger orbit circumference and the lower speed, rather than from the radius alone.

Step 3: (b) State the spectral formula and identify the line Rydberg's formula for an emitted hydrogen spectral line is

$$\frac{1}{\lambda} = R_H \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right), \quad n_2 > n_1,$$

where n_2 and n_1 are the initial and final principal quantum numbers, and $R_H \simeq 1.097 \times 10^7 \text{ m}^{-1}$. The printed H_α line belongs to the Balmer series, for which $n_1 = 2$. It is the first Balmer line, arising from

Quick Tip to Solve: Remember $r_n \propto n^2$ and $v_n \propto n^{-1}$, so the orbital period must scale as n^3 ; use this to check the derivation.

the transition $n_2 = 3$ to $n_1 = 2$, with wavelength approximately 656 nm.

Step 4: Conclusion The orbital period is $T_n = 4\pi^2 h^3 n^3 / (m_e e^4)$. Hydrogen wavelengths satisfy Rydberg's formula, and H_α belongs to the Balmer series ($3 \rightarrow 2$).

Quick Tip to Solve: Remember $r_n \propto n^2$ and $v_n \propto n^{-1}$, so the orbital period must scale as n^3 ; use this to check the derivation.

21. Draw a graph showing the variation of binding energy per nucleon with mass number of different nuclei. Write any two salient features of the curve. How does this curve explain the release of energy both in the processes of nuclear fission and fusion? [3 marks]

Solution:

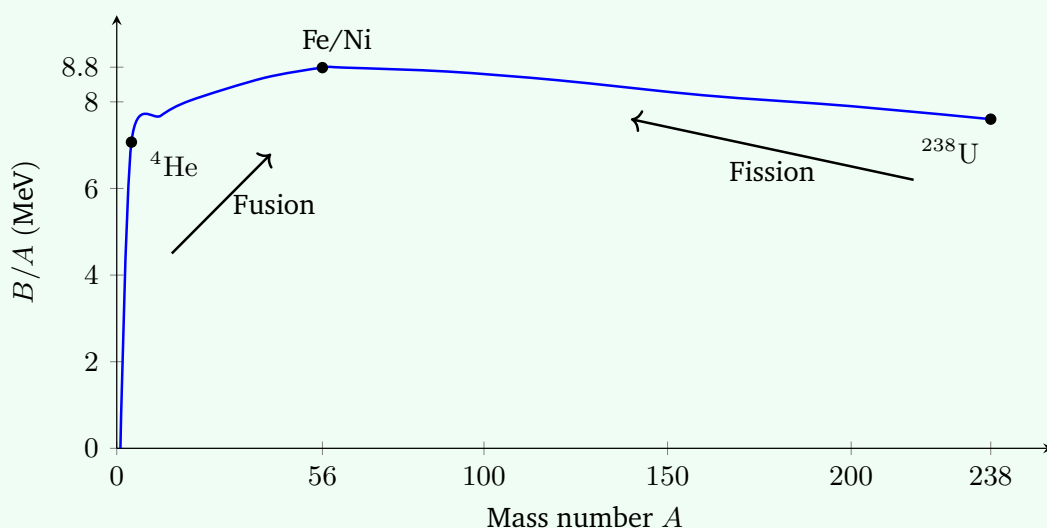
Concept:

- Nuclear binding energy B is the energy required to separate a nucleus completely into its constituent nucleons. Binding energy per nucleon B/A measures their average binding, where A is the mass number.
- Greater B/A generally means a more tightly bound nucleus. A reaction releases energy when the total binding energy of its products exceeds that of its reactants.
- The released energy is $Q = B_{\text{products}} - B_{\text{reactants}} = (m_{\text{initial}} - m_{\text{final}})c^2$; all participating particles must be included.

Step 1: Sketch the binding-energy curve and state two features Plot mass number A horizontally and binding energy per nucleon B/A , in MeV, vertically. The curve starts at zero for hydrogen, rises rapidly for light nuclei, reaches its broad maximum near iron and nickel, then falls gradually towards very heavy nuclei. The schematic curve below shows this overall trend; actual light nuclei show local variations.

Two salient features are: (i) B/A increases rapidly for light nuclei and reaches about 8.8 MeV near $A \simeq 56-62$; (ii) for medium-mass nuclei it is nearly constant around 8 MeV, and for heavier nuclei it slowly decreases, reaching about 7.6 MeV near uranium. The broad plateau reflects saturation of the short-range nuclear force, since a nucleon interacts strongly mainly with nearby nucleons.

Quick Tip to Solve: On either side of the broad iron-nickel peak, check whether the products move towards greater binding per nucleon; then confirm the total binding-energy balance.



Schematic binding energy per nucleon versus mass number, showing the broad iron-nickel maximum.

Step 2: Explain energy release in nuclear fission A very heavy nucleus, such as uranium, lies on the descending side of the curve. When it splits into two medium-mass nuclei, the fragments generally have greater binding energy per nucleon than the original heavy nucleus. Although emitted neutrons must also be included in the complete energy balance, the total binding energy of the final products increases. Their total rest mass is correspondingly smaller, so the difference is released as kinetic energy and radiation. The movement towards the high-binding region explains why fission of suitably heavy nuclei can release energy.

Step 3: Explain energy release in nuclear fusion Suitable light nuclei lie on the rising side of the curve. When they combine to form a more tightly bound nucleus, the product has greater binding energy per nucleon and the total binding energy increases. The corresponding loss of rest mass is released as energy, according to $Q = \Delta mc^2$. For example, fusion involving hydrogen isotopes leads towards the more strongly bound helium region. Thus fusion of suitable light nuclei and fission of suitable heavy nuclei both release energy by producing a more tightly bound collection of nucleons.

Step 4: Conclusion The curve peaks near iron and nickel. Fission of suitable heavy nuclei and fusion of suitable light nuclei release energy because their products have greater total binding energy and lower total rest mass.

Quick Tip to Solve: On either side of the broad iron-nickel peak, check whether the products move towards greater binding per nucleon; then confirm the total binding-energy balance.

22. Draw the energy band diagram of (i) n-type, and (ii) p-type semiconductors at temperature $T > 0\text{ K}$.

In the case of n-type Si-semiconductor, the donor energy level is slightly below the bottom of conduction band whereas in p-type semiconductor, the acceptor energy level is slightly above the top of valence band. Explain, giving examples, what role do these energy levels play in conduction and valence bands.

[3 marks]

Solution:

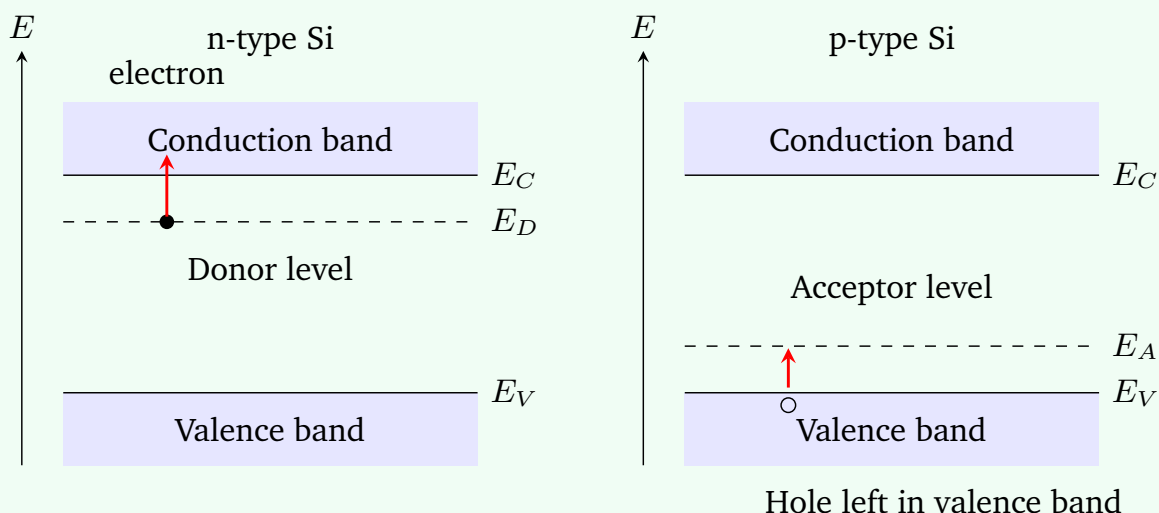
Concept:

- Doping silicon introduces impurity energy levels in its forbidden energy gap. The conduction-band edge is E_C , the valence-band edge is E_V , and $E_C - E_V$ is the band gap.
- Pentavalent donors produce a level E_D just below E_C ; trivalent acceptors produce a level E_A just above E_V . Small thermal energies can therefore create mobile majority carriers at ordinary temperatures.

Step 1: (i) Draw and explain the n-type energy bands In silicon doped with phosphorus, arsenic or antimony, four of the impurity's five valence electrons form covalent bonds with neighbouring Si atoms. The fifth electron is weakly bound and occupies a donor level E_D , slightly below the conduction band. At $T > 0\text{ K}$, thermal excitation can lift donor electrons through the small energy difference $E_C - E_D$ into the conduction band. These electrons can move under an applied electric field and carry current. After donating an electron, the impurity becomes a fixed positive ion; it does not move through the crystal. Electrons are the majority carriers, while holes are minority carriers.

Step 2: (ii) Draw and explain the p-type energy bands Silicon doped with boron, aluminium or gallium has trivalent impurities. Each impurity forms three complete covalent bonds and has one electron deficiency. Its acceptor level E_A lies slightly above the valence-band edge. A valence-band electron can acquire the small energy $E_A - E_V$ and enter this acceptor level. The electron fills the impurity deficiency but leaves a mobile hole in the valence band. Successive motion of valence electrons into empty bonds makes the hole move and contribute to conduction. The acceptor becomes a fixed negative ion. Holes are the majority carriers and electrons are minority carriers. At very low positive temperatures impurity ionisation may be incomplete; the diagrams show the allowed levels and thermally activated transitions.

Quick Tip to Solve: Remember the direction of the electron transition: donor to conduction band for n-type, and valence band to acceptor for p-type.



Impurity levels and electron excitation at $T > 0\text{ K}$; band separation is schematic.

Step 3: Conclusion Donor levels make it easy to supply mobile electrons to the conduction band of n-type silicon; acceptor levels make it easy to create mobile holes in the valence band of p-type silicon. The impurity ions remain fixed in both cases.

Quick Tip to Solve: Remember the direction of the electron transition: donor to conduction band for n-type, and valence band to acceptor for p-type.

23. With what considerations in view is a photodiode fabricated? Explain its working with the help of a suitable diagram. With the help of V-I characteristics, state how photodiode is used to detect optical signals. [3 marks]

Solution:

Concept:

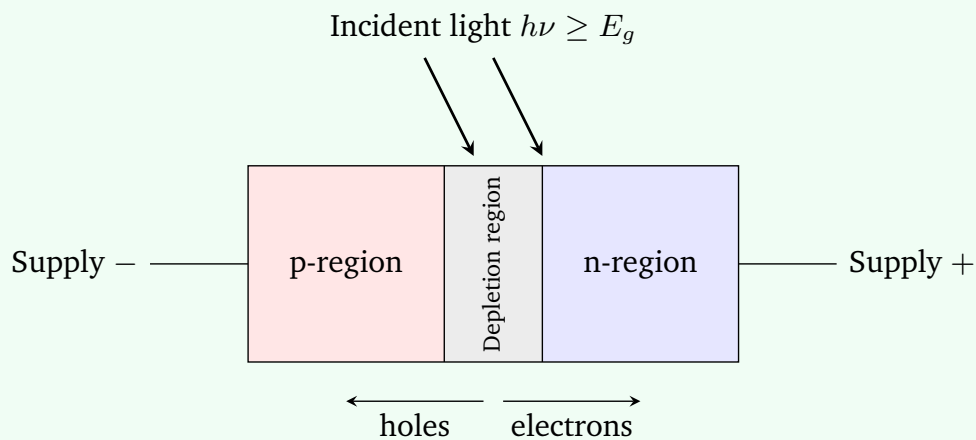
- A photodiode is a photosensitive p-n junction operated under reverse bias. Photons of energy $h\nu \geq E_g$, where E_g is the band gap, generate electron-hole pairs.
- The depletion-region field separates photogenerated carriers. At fixed reverse voltage, the reverse photocurrent increases with incident optical intensity.

Step 1: Fabrication considerations The junction is made accessible to incident light through a transparent window. Its photosensitive junction area is sufficiently large to collect the optical signal, and the depletion region is arranged so that absorbed light generates carriers where the electric field can separate them efficiently. The semiconductor is selected so that the photon energy of the light to be detected is at least its band gap. Reverse-bias operation widens the depletion region and enables rapid collection of carriers; a small dark current permits weak illumination to be detected clearly.

Step 2: Working under reverse bias Connect the n-region to the positive supply terminal and the p-region to the negative terminal. Light absorbed in or near the depletion region produces electron-hole pairs. The field drives electrons toward the n-side and holes toward the p-side, suppressing

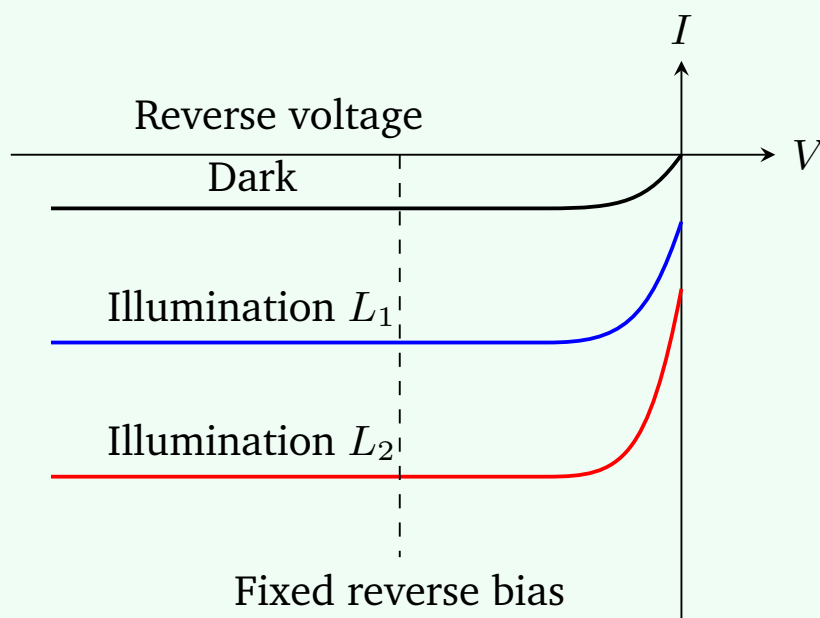
Quick Tip to Solve: In a signed V-I plot, stronger illumination moves the reverse-current curve downward; compare currents at the same reverse voltage.

recombination and producing a reverse photocurrent in the external circuit. Even without illumination a small reverse dark current exists. With illumination the reverse current contains this dark current plus the photogenerated contribution. A load resistor converts the change in current into a measurable voltage signal.



Reverse-biased illuminated junction; the supply connections indicate polarity.

Step 3: V-I characteristics and signal detection The graph uses signed diode current and voltage: the reverse region lies in the third quadrant. For a fixed reverse voltage below breakdown, increasing light intensity produces a larger magnitude of negative current. Away from very small reverse voltages the curves are nearly horizontal, so the current is chiefly controlled by illumination. Operate at a fixed reverse voltage and measure the current, or the load-resistor voltage $\Delta V = R_L \Delta I_{ph}$. Changes in an optical signal then produce corresponding changes in the electrical output. The increasing separation of the curves from the dark-current curve represents increasing incident intensity.



Reverse-bias characteristics below breakdown: $L_2 > L_1 > 0$.

Step 4: Conclusion A windowed, photosensitive reverse-biased junction detects light by converting

Quick Tip to Solve: In a signed V-I plot, stronger illumination moves the reverse-current curve downward; compare currents at the same reverse voltage.

photon-generated electron-hole pairs into reverse photocurrent. At fixed reverse bias, the photocurrent magnitude gives a measure of optical intensity.

Quick Tip to Solve: In a signed V-I plot, stronger illumination moves the reverse-current curve downward; compare currents at the same reverse voltage.

24. (a) What is amplitude modulation? Draw a diagram showing an amplitude modulated wave obtained by modulation of a carrier sinusoidal wave on a modulating signal.

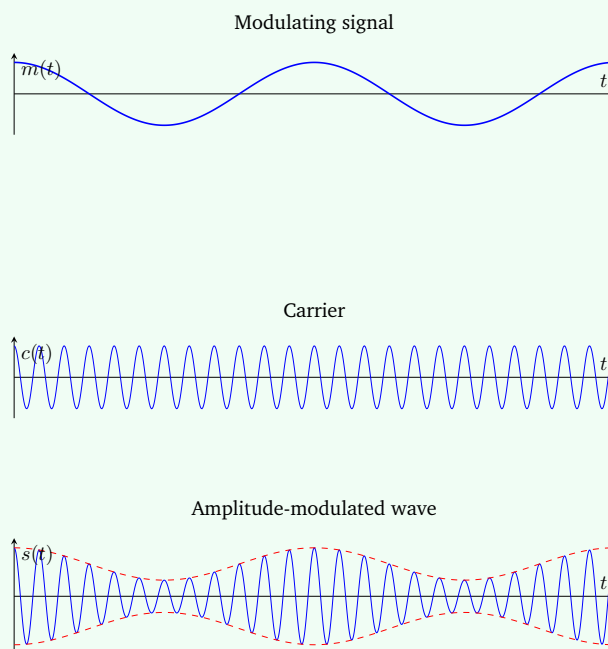
(b) Define the terms (i) modulation index, and (ii) side bands. Mention the significance of side bands. [3 marks]

Solution:

Concept:

- Amplitude modulation (AM) varies the amplitude of a high-frequency carrier in accordance with the instantaneous value of a lower-frequency message signal; carrier frequency and phase remain unchanged.
- For a sinusoidal message, $s(t) = A_c[1 + \mu \cos(2\pi f_m t)] \cos(2\pi f_c t)$, where $f_c \gg f_m$, A_c is carrier amplitude and μ is modulation index.

Step 1: (a) Amplitude-modulated waveform Let the message be $m(t) = A_m \cos(2\pi f_m t)$ and the unmodulated carrier be $c(t) = A_c \cos(2\pi f_c t)$. In conventional AM, the slowly varying amplitude factor is $A_c + A_m \cos(2\pi f_m t)$. The high-frequency oscillations therefore have upper and lower envelopes $\pm A_c[1 + \mu \cos(2\pi f_m t)]$. The upper envelope reproduces the shape of the message, shifted upward by A_c , while the oscillation period remains that of the carrier. The illustrated example uses $\mu = 0.5$, so the envelope never crosses zero.



Message, carrier and resulting AM waveform; dashed curves show the envelope.

Quick Tip to Solve: Read maximum and minimum envelope heights to find modulation index, then place sidebands one message frequency either side of the carrier.

Step 2: (b)(i) Modulation index Modulation index is the ratio of the modulating amplitude to the unmodulated carrier amplitude, for the conventional amplitude expression above:

$$\mu = \frac{A_m}{A_c} = \frac{A_{\max} - A_{\min}}{A_{\max} + A_{\min}}.$$

Here $A_{\max} = A_c(1 + \mu)$ and $A_{\min} = A_c(1 - \mu)$ are the maximum and minimum envelope amplitudes. Undistorted envelope detection requires $0 \leq \mu \leq 1$; $\mu > 1$ causes overmodulation and envelope distortion.

Step 3: (b)(ii) Side bands and their significance Expanding the AM expression using the product-to-sum identity gives

$$\begin{aligned} s(t) &= A_c \cos(2\pi f_c t) \\ &\quad + \frac{\mu A_c}{2} \cos[2\pi(f_c + f_m)t] \\ &\quad + \frac{\mu A_c}{2} \cos[2\pi(f_c - f_m)t]. \end{aligned}$$

The components above and below the carrier are the upper sideband at $f_c + f_m$ and lower sideband at $f_c - f_m$. For a message containing a range of frequencies they form corresponding frequency bands on either side of the carrier. These sidebands carry the message information; the unchanged carrier alone contains no message variation. For highest message frequency $f_{m,\max}$, conventional AM requires bandwidth $2f_{m,\max}$.

Step 4: Conclusion AM encodes the message in a carrier's amplitude envelope. Its modulation index specifies modulation depth, and its upper and lower sidebands carry the transmitted information.

Quick Tip to Solve: Read maximum and minimum envelope heights to find modulation index, then place sidebands one message frequency either side of the carrier.

Section D (15 marks)

25. (a) When a parallel plate capacitor is connected across a dc battery, explain briefly how the capacitor gets charged.

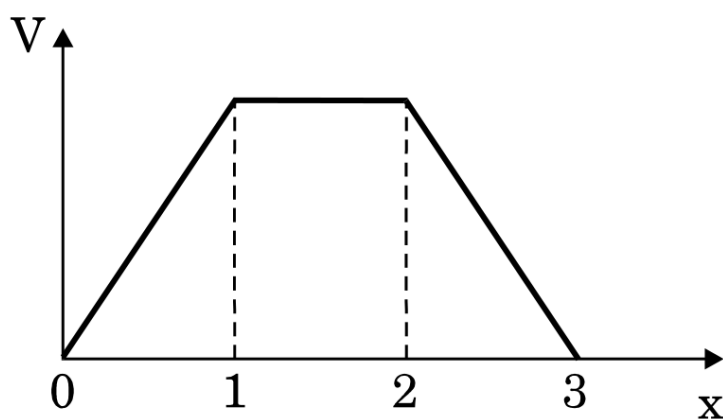
(b) A parallel plate capacitor of capacitance ' C ' is charged to ' V ' volt by a battery. After some time the battery is disconnected and the distance between the plates is doubled. Now a slab of dielectric constant $1 < k < 2$ is introduced to fill the space between the plates. How will the following be affected?

(i) The electric field between the plates of the capacitor.

(ii) The energy stored in the capacitor.

Justify your answer in each case.

(c) The electric potential as a function of distance ' x ' is shown in the figure. Draw a graph of the electric field E as a function of x . [5 marks]



Solution:

Concept:

- A capacitor stores charge on two conducting plates: $Q = CV$, where C is capacitance and V is the potential difference between the plates. The two plates acquire equal and opposite charges.
- For parallel plates of area A and separation d , neglecting edge effects, $C = \epsilon_0 A/d$ and $E = Q/(\epsilon_0 A)$. A dielectric completely filling the gap replaces ϵ_0 by $k\epsilon_0$.
- An isolated charged capacitor retains its free charge. Its energy is $U = Q^2/(2C)$, whereas the field along the x -direction is $E_x = -dV/dx$.

Step 1: (a) Explain charging by a dc battery On connecting the capacitor to the battery, the battery's negative terminal supplies electrons to the connected plate, making it negative. The positive terminal removes electrons from the other plate, making that plate positive. Electrons travel through the external conducting circuit; they do not cross the insulating gap between the plates. Thus the plates acquire charges $-Q$ and $+Q$. As charge accumulates, the capacitor's potential difference $V_C = Q/C$ increases and opposes further transfer of charge by the battery. The charging current consequently decreases. When V_C equals the battery voltage V , charging ceases in the ideal circuit and the final charge magnitude is $Q = CV$. The capacitor then stores energy in its electric field.

Step 2: (b) Identify the quantity conserved and the new capacitance Let the original plate separation

Quick Tip to Solve: First decide whether the battery remains connected: disconnected means use fixed Q . For a potential graph, reverse the sign of each segment's slope to obtain the electric field.

be d . Initially, the charge magnitude and energy are

$$Q = CV, \quad U_0 = \frac{1}{2}CV^2, \quad E_0 = \frac{V}{d}.$$

The battery is disconnected before the geometry is changed, so the free charge Q cannot flow away. Doubling the separation first reduces the capacitance to $C/2$. The slab then fills the entire new gap of width $2d$, giving

$$C_f = \frac{k\varepsilon_0 A}{2d} = \frac{kC}{2}, \quad V_f = \frac{Q}{C_f} = \frac{2V}{k}.$$

The voltage is therefore allowed to change; it is the charge, rather than the original battery voltage, that must be held fixed.

Step 3: (b)(i) Determine the electric field At fixed charge and plate area, doubling the gap alone does not change the ideal parallel-plate field: it remains $Q/(\varepsilon_0 A)$. When the dielectric fills the gap, its polarisation produces a field opposing the field of the free plate charges. The final field becomes

$$E_f = \frac{Q}{k\varepsilon_0 A} = \frac{E_0}{k}.$$

This also follows directly from $E_f = V_f/(2d) = (2V/k)/(2d) = V/(kd)$. Since $1 < k < 2$, the field decreases to a value between $E_0/2$ and E_0 . It is incorrect to halve the field merely because the plate separation has doubled: that argument would assume a fixed voltage, which no longer applies.

Step 4: (b)(ii) Determine the stored energy Using the conserved free charge gives

$$U_f = \frac{Q^2}{2C_f} = \frac{(CV)^2}{2(kC/2)} = \frac{CV^2}{k} = \frac{2}{k}U_0.$$

Because $1 < k < 2$, the multiplier $2/k$ lies between 1 and 2, so the final energy is greater than the original stored energy. Separating the charged plates requires external work and first raises the energy to $2U_0$. Inserting the dielectric subsequently reduces that energy by a factor k , leaving $2U_0/k$, still above its original value.

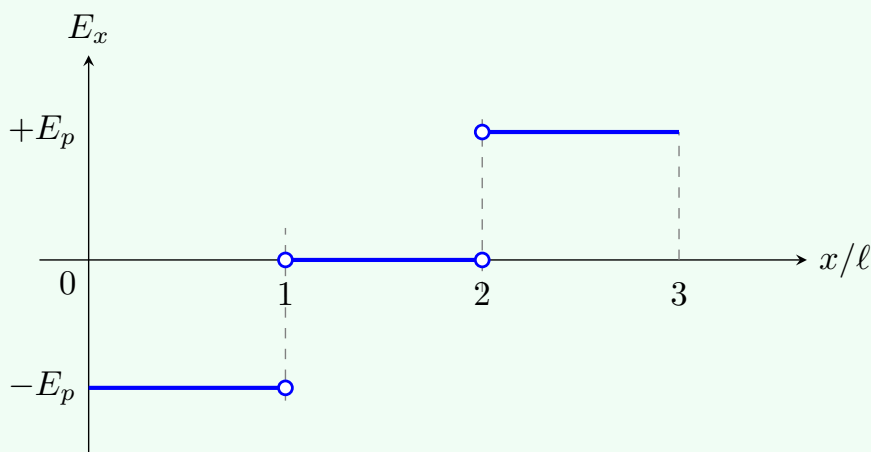
Step 5: (c) Obtain and sketch the electric field from the potential slope Let the height of the flat part of the given potential graph be V_p , and let ℓ be the distance represented by one unit on its horizontal axis. The potential rises linearly over $0 < x < \ell$, is constant over $\ell < x < 2\ell$, and falls linearly over $2\ell < x < 3\ell$. The slopes are respectively V_p/ℓ , zero, and $-V_p/\ell$. Therefore, putting $E_p = V_p/\ell > 0$,

$$E_x = \begin{cases} -E_p, & 0 < x < \ell, \\ 0, & \ell < x < 2\ell, \\ +E_p, & 2\ell < x < 3\ell. \end{cases}$$

The field graph has three horizontal segments: below the axis from 0 to 1, on the axis from 1 to 2, and above the axis from 2 to 3. The positive and negative levels have equal magnitudes because the two sloping parts of the potential graph have equal and opposite slopes. At 1 and 2 the idealised potential has sharp corners; the derivative is not uniquely defined there, so the graph shows open endpoints at the jumps. The given graph does not specify the potential outside the displayed interval,

Quick Tip to Solve: First decide whether the battery remains connected: disconnected means use fixed Q . For a potential graph, reverse the sign of each segment's slope to obtain the electric field.

so no field outside it is inferred.



Electric field versus position: $E_p = V_p/\ell$, with each horizontal unit representing ℓ .

Step 6: Conclusion (a) The battery transfers electrons through the external circuit until the capacitor voltage equals the battery voltage, leaving equal and opposite plate charges of magnitude CV . (b) With the battery disconnected, Q stays constant: the final field is E_0/k , which decreases, and the final energy is $(2/k)U_0$, which increases for $1 < k < 2$. (c) The field is a negative constant over 0–1, zero over 1–2, and an equal positive constant over 2–3, as shown.

Quick Tip to Solve: First decide whether the battery remains connected: disconnected means use fixed Q . For a potential graph, reverse the sign of each segment's slope to obtain the electric field.

OR

25. (a) Derive an expression for the potential energy of an electric dipole in a uniform electric field. Explain conditions for stable and unstable equilibrium.

(b) Is the electrostatic potential necessarily zero at a point where the electric field is zero? Give an example to support your answer. [5 marks]

Solution:

Concept:

- An electric dipole consists of charges $-q$ and $+q$ separated by distance $2a$. Its dipole moment $\mathbf{p}$, of magnitude $p = 2aq$, points from the negative to the positive charge.
- In a uniform field $\mathbf{E}$, the net force is zero, but the torque has magnitude $pE \sin \theta$, where θ is the angle between $\mathbf{p}$ and $\mathbf{E}$. The torque tends to align the dipole with the field.
- The change in potential energy equals the work done by an external agent in a slow rotation. Equilibrium is stable at a minimum of potential energy and unstable at a maximum.
- Electric field depends on the spatial variation of potential: $\mathbf{E} = -\nabla V$. Therefore a zero field specifies zero potential gradient, not necessarily zero potential.

Step 1: (a) Calculate work done in rotating the dipole Take the positive angular direction as increasing

Quick Tip to Solve: Use the sign of the second derivative of $-pE \cos \theta$ to test equilibrium; distinguish zero potential gradient from zero potential.

θ . The electric torque in this direction is $\tau_e = -pE \sin \theta$. To rotate the dipole slowly, without changing its kinetic energy, the external agent applies the opposite torque, $\tau_{\text{ext}} = pE \sin \theta$. Hence, for an infinitesimal rotation,

$$dU = dW_{\text{ext}} = pE \sin \theta d\theta.$$

Choose the reference $U = 0$ at $\theta = \pi/2$. Integrating from this position to an arbitrary orientation gives

$$U(\theta) - U(\pi/2) = \int_{\pi/2}^{\theta} pE \sin \phi d\phi = [-pE \cos \phi]_{\pi/2}^{\theta} = -pE \cos \theta.$$

Thus the familiar expression uses a stated energy reference; adding a constant would not alter the torque or equilibrium conditions.

Step 2: (a) Identify stable and unstable equilibrium Equilibrium requires zero torque, so $\sin \theta = 0$: the dipole is parallel or antiparallel to the field. Differentiate the energy to test each orientation:

$$\frac{dU}{d\theta} = pE \sin \theta, \quad \frac{d^2U}{d\theta^2} = pE \cos \theta.$$

At $\theta = 0$, the second derivative is positive and the energy is minimum, $U = -pE$. A small displacement produces a restoring torque towards alignment, so this is stable equilibrium. At $\theta = \pi$, the second derivative is negative and the energy is maximum, $U = +pE$. A small displacement produces a torque that increases the departure from the antiparallel position, so this is unstable equilibrium.

Step 3: (b) Give an example of zero field with nonzero potential Consider a conducting spherical shell of radius R carrying a nonzero charge Q . With potential taken to be zero at infinity, the field everywhere inside the shell is zero, whereas its potential is constant and equal to that on the surface:

$$E = 0 \quad (r < R), \quad V = \frac{1}{4\pi\epsilon_0} \frac{Q}{R} \neq 0.$$

There is no contradiction: the derivative of a nonzero constant is zero. Moving a test charge between two interior points requires no work because their potentials are equal, although bringing it from infinity to the interior involves a nonzero potential difference.

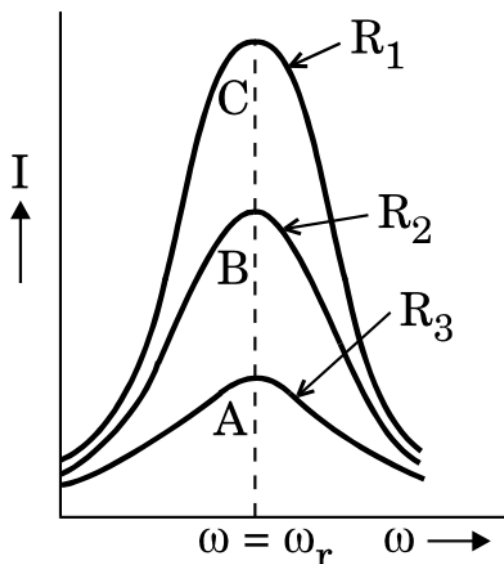
Step 4: Conclusion (a) With zero potential energy at $\theta = \pi/2$, $U = -\mathbf{p} \cdot \mathbf{E} = -pE \cos \theta$. Parallel alignment is stable and antiparallel alignment is unstable. (b) No: inside a charged conducting spherical shell, $E = 0$ while $V = Q/(4\pi\epsilon_0 R)$ is nonzero relative to infinity.

Quick Tip to Solve: Use the sign of the second derivative of $-pE \cos \theta$ to test equilibrium; distinguish zero potential gradient from zero potential.

26. (a) What do you understand by ‘sharpness of resonance’ for a series LCR resonant circuit? How is it related with the quality factor ‘Q’ of the circuit? Using the graphs given in the diagram, explain the factors which affect it. For which graph is the resistance (R) minimum?

(b) A $2\ \mu\text{F}$ capacitor, $100\ \Omega$ resistor and $8\ \text{H}$ inductor are connected in series with an ac source. Find the frequency of the ac source for which the current drawn in the circuit is maximum.

If the peak value of emf of the source is $200\ \text{V}$, calculate the (i) maximum current, and (ii) inductive and capacitive reactance of the circuit at resonance. [5 marks]



Solution:

Concept:

- For a series LCR circuit driven at angular frequency ω , impedance is $Z = \sqrt{R^2 + (\omega L - 1/(\omega C))^2}$ and current amplitude is $I_0 = V_0/Z$.
- Resonance occurs when inductive and capacitive reactances cancel: $\omega_0 = 1/\sqrt{LC}$. The quality factor is $Q = \omega_0 L/R = (1/R)\sqrt{L/C}$.
- The half-power bandwidth is $\Delta\omega = \omega_2 - \omega_1 = R/L$. Thus $Q = \omega_0/\Delta\omega$ measures how narrow the resonance is compared with its resonant frequency.

Step 1: (a) Explain sharpness and its relation to quality factor Sharpness of resonance describes how rapidly the current falls as the driving frequency moves away from the resonant frequency. A sharper resonance has a narrower peak and selects a smaller range of frequencies near resonance. The half-power frequencies ω_1 and ω_2 are the frequencies where the current amplitude falls to $1/\sqrt{2}$ times its resonant value. Since power is proportional to current squared, this corresponds to half the resonant power. For a series circuit,

$$\Delta\omega = \frac{R}{L}, \quad Q = \frac{\omega_0}{\Delta\omega} = \frac{\omega_0 L}{R} = \frac{1}{R} \sqrt{\frac{L}{C}}.$$

A higher Q means a narrower fractional bandwidth and hence a sharper resonance. For fixed L and C , reducing R decreases damping, increases Q , and narrows the peak. Increasing L or decreasing C , with other parameters fixed, also increases Q , although it changes the resonant frequency.

Quick Tip to Solve: At resonance set the total impedance equal to R , but keep each individual reactance nonzero; distinguish peak emf from rms emf before calculating current.

Step 2: (a) Compare the three curves in the printed graph The printed curves have a common resonant frequency and are compared for the same source amplitude and the same L, C . At resonance $Z = R$, so $I_{0,\text{res}} = V_0/R$. Curve C, labelled R_1 , has the tallest and narrowest peak; curve B, labelled R_2 , has an intermediate peak; curve A, labelled R_3 , is lowest and broadest. Therefore $R_1 < R_2 < R_3$ and $Q_1 > Q_2 > Q_3$. The tall narrow curve indicates both a larger resonance current and less damping; a broad low curve indicates more resistance and weaker frequency selectivity.

Step 3: (b) Find the frequency for maximum current Here $C = 2 \times 10^{-6} \text{ F}$, $L = 8 \text{ H}$, and $R = 100 \Omega$. Current is greatest when the impedance is smallest, requiring $X_L = X_C$. Hence

$$\begin{aligned}\sqrt{LC} &= \sqrt{8(2 \times 10^{-6})} = 4 \times 10^{-3} \text{ s}, \\ \omega_0 &= \frac{1}{4 \times 10^{-3}} = 250 \text{ rad s}^{-1}, \\ f_0 &= \frac{\omega_0}{2\pi} = \frac{250}{2\pi} = 39.8 \text{ Hz (approximately)}.\end{aligned}$$

This is the ordinary frequency of the ac source; angular frequency must be divided by 2π to express the answer in hertz.

Step 4: (b) Calculate maximum current and both reactances The given 200 V is the peak source emf, so use it with the peak current. At resonance the reactive contributions cancel in the total impedance:

$$I_{0,\text{res}} = \frac{V_0}{R} = \frac{200}{100} = 2 \text{ A}.$$

The individual reactances are nevertheless nonzero:

$$\begin{aligned}X_L &= \omega_0 L = 250 \times 8 = 2000 \Omega, \\ X_C &= \frac{1}{\omega_0 C} = \frac{1}{250(2 \times 10^{-6})} = 2000 \Omega.\end{aligned}$$

Their equality verifies resonance. The inductor and capacitor voltage phasors oppose each other, so their equal reactive voltage contributions cancel, leaving the source voltage balanced by the resistor voltage.

Step 5: Conclusion (a) Greater $Q = \omega_0/\Delta\omega = (1/R)\sqrt{L/C}$ gives sharper resonance. Curve C, labelled R_1 , has minimum resistance. (b) The maximum current occurs at $f_0 \approx 39.8 \text{ Hz}$; its peak value is 2 A , and $X_L = X_C = 2000 \Omega$ at resonance.

Quick Tip to Solve: At resonance set the total impedance equal to R , but keep each individual reactance nonzero; distinguish peak emf from rms emf before calculating current.

26. (a) Draw a schematic diagram of an ac generator. Explain its working and obtain the expression for the instantaneous value of the emf in terms of the magnetic field B , number of turns N of the coil of area A rotating with angular frequency ω . Show how an alternating emf is generated by a loop of wire rotating in a magnetic field.

(b) A circular coil of radius 10 cm and 20 turns is rotated about its vertical diameter with angular speed of 50 rad s^{-1} in a uniform horizontal magnetic field of $3.0 \times 10^{-2} \text{ T}$.

(i) Calculate the maximum and average emf induced in the coil.

(ii) If the coil forms a closed loop of resistance 10Ω , calculate the maximum current in the coil and the average power loss due to Joule heating. [5 marks]

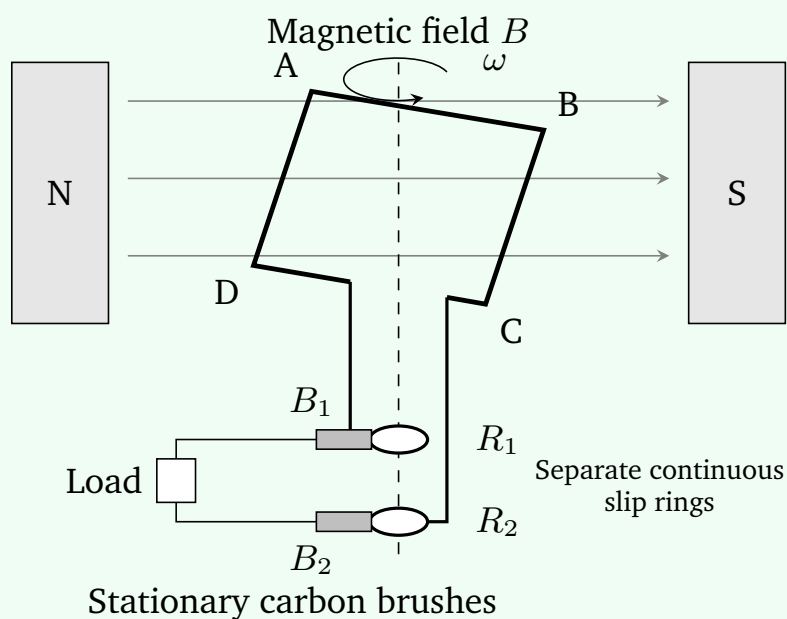
Solution:

Concept:

- An ac generator converts mechanical energy into electrical energy by electromagnetic induction. For a coil of N turns, Faraday's law gives $e = -d(N\Phi)/dt$, where Φ is the magnetic flux through one turn.
- In a uniform field B , a plane coil of area A has flux $\Phi = BA \cos \theta$, where θ is the angle between the field and the normal to the coil. At constant angular speed ω , choose $\theta = \omega t$.
- For a purely resistive closed coil of resistance R , $i = e/R$ and the average heating power is $P_{\text{av}} = I_{\text{rms}}^2 R$, with $I_{\text{rms}} = I_0/\sqrt{2}$ for a sinusoidal current.

Step 1: (a) Construction and operation of the generator A rectangular armature coil $ABCD$ rotates between the north and south pole pieces of a magnet. The two coil ends are connected to two separate, continuous slip rings R_1 and R_2 , which rotate with the coil. Stationary carbon brushes B_1 and B_2 touch these rings and connect the coil to an external load. A mechanical drive supplies the energy needed to rotate the coil. As it rotates, the magnetic flux linked with it changes and induces an emf; a current flows when the external circuit is closed. The separate slip rings retain the connection of each end to its brush throughout the rotation.

Quick Tip to Solve: Convert the radius to metres before finding $A = \pi r^2$; use peak emf for peak current but divide e_0^2/R by two for average heating power.



Schematic ac generator: rotating coil, separate slip rings, stationary brushes and external load.

Step 2: (a) Derive the instantaneous emf and its alternating nature Take $t = 0$ when the coil normal is parallel to the field. The total flux linkage and induced emf are

$$\begin{aligned}
 N\Phi(t) &= NBA \cos(\omega t), \\
 e(t) &= -\frac{d}{dt}[NBA \cos(\omega t)] \\
 &= NBA\omega \sin(\omega t) = e_0 \sin(\omega t).
 \end{aligned}$$

Thus the peak emf is $e_0 = NBA\omega$. The emf is zero at $\omega t = 0, \pi, 2\pi$, positive and maximum at $\pi/2$, and negative with maximum magnitude at $3\pi/2$. In the second half revolution, the sides of the coil move through the field in the opposite directions, so the induced polarity reverses. In particular, $e(t + \pi/\omega) = -e(t)$: successive half cycles generate opposite polarities at the brushes, producing an alternating output. The frequency is $f = \omega/(2\pi)$; choosing another initial coil orientation merely changes the phase of the sinusoid.

Step 3: (b)(i) Calculate the peak and average induced emf The radius is $r = 0.10$ m, so the circular coil area is $A = \pi r^2 = 0.01\pi$ m². Its vertical rotation axis is perpendicular to the horizontal field, so the flux varies sinusoidally with full amplitude BA . With $N = 20$, $B = 3.0 \times 10^{-2}$ T, and $\omega = 50$ rad s⁻¹,

$$e_0 = 20(3.0 \times 10^{-2})(0.01\pi)(50) = 0.30\pi \text{ V} \simeq 0.942 \text{ V}.$$

Over one complete revolution $T = 2\pi/\omega$, the positive and negative contributions cancel:

$$e_{\text{av}} = \frac{1}{T} \int_0^T e_0 \sin(\omega t) dt = 0.$$

Here the signed average emf is taken over a full cycle. If an average over a positive half cycle (or a mean rectified magnitude) is intended, it is $2e_0/\pi = 0.600$ V. This differs from the rms value and must

Quick Tip to Solve: Convert the radius to metres before finding $A = \pi r^2$; use peak emf for peak current but divide e_0^2/R by two for average heating power.

not be confused with it.

Step 4: (b)(ii) Calculate peak current and mean Joule heating power Treating the closed coil as the stated resistance $R = 10\ \Omega$, the instantaneous current is $i(t) = (e_0/R) \sin(\omega t)$. Therefore,

$$I_0 = \frac{e_0}{R} = \frac{0.30\pi}{10} = 0.030\pi\ \text{A} \simeq 0.0942\ \text{A}.$$

Heating depends on the square of the current, so it remains positive during both half cycles even though the signed mean current is zero. Since $\langle \sin^2(\omega t) \rangle = 1/2$,

$$\begin{aligned} P_{\text{av}} &= \langle i^2 R \rangle = \frac{I_0^2 R}{2} = \frac{e_0^2}{2R} \\ &= \frac{(0.30\pi)^2}{2(10)}\ \text{W} = 0.0045\pi^2\ \text{W} \simeq 0.0444\ \text{W}. \end{aligned}$$

Step 5: Conclusion The generator output is $e(t) = NBA\omega \sin(\omega t)$. For the given coil, $e_{\text{max}} = 0.942\ \text{V}$, the full-cycle signed average emf is zero (positive half-cycle average: $0.600\ \text{V}$), $I_{\text{max}} = 0.0942\ \text{A}$, and $P_{\text{av}} = 0.0444\ \text{W}$.

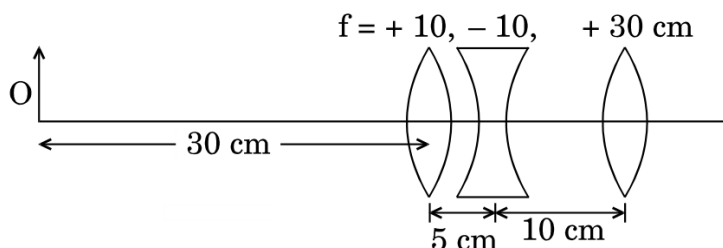
Quick Tip to Solve: Convert the radius to metres before finding $A = \pi r^2$; use peak emf for peak current but divide e_0^2/R by two for average heating power.

27. (a) Using the ray diagram for a system of two lenses of focal lengths f_1 and f_2 in contact with each other, show that the two lens system can be regarded as equivalent to a single lens of focal length f , where

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}.$$

Also write the relation for the equivalent power of the lens combination.

(b) Determine the position of the image formed by the lens combination given in the figure. [5 marks]



Solution:

Concept:

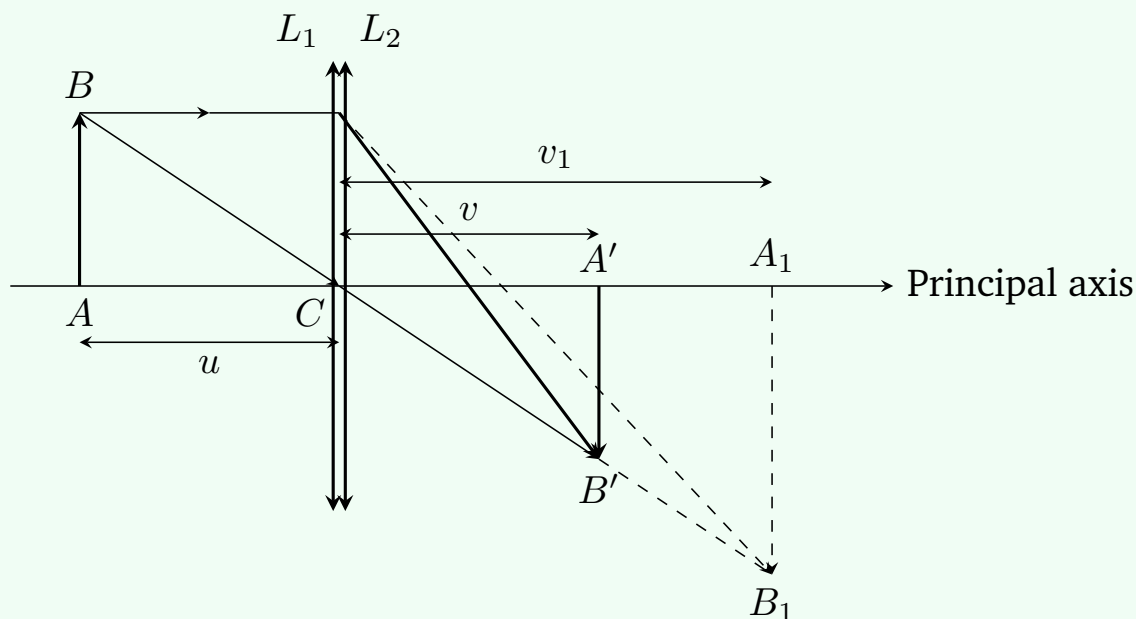
- The Cartesian lens formula is $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$. All distances are measured from the optical centre, with the direction of incident light taken positive.
- For thin lenses in contact, their separation is negligible. The image formed by the first lens acts as the object for the second lens, with the same signed distance from the common optical centre.
- Lens power is $P = 1/f$, with f in metres and P in dioptres. A converging lens has positive power

Quick Tip to Solve: After the first lens gives $v_1 = 15\ \text{cm}$, measure from the second lens: its virtual object is at $+10\ \text{cm}$. This cancels its $-10\ \text{cm}$ focal length and makes the outgoing rays parallel.

and a diverging lens has negative power.

- For separated lenses, first find the image from each lens and then measure the next object distance from the next lens. Parallel incident rays are focused at the focal plane of a converging lens.

Step 1: (a) Set up the contact-lens ray construction Place two thin lenses L_1, L_2 in contact at the common optical centre C . Let an object AB lie at signed distance u . The first lens alone would form the intermediate image A_1B_1 at distance v_1 ; the second lens changes the convergence of these rays to form the final image $A'B'$ at distance v . The ray through the common centre continues undeviated. In the diagram, dashed rays show the image that the first lens alone would produce, while the solid refracted ray reaches the final image.



Thin lenses in contact: dashed continuation gives the first-lens image; solid rays give the final image. The common plane is drawn schematically.

Step 2: (a) Add the two lens equations Applying the lens formula to L_1 ,

$$\frac{1}{v_1} - \frac{1}{u} = \frac{1}{f_1}.$$

Since the lenses are in contact, the object distance for L_2 is $u_2 = v_1$. For the illustrated converging rays this is a virtual object on the right of L_2 , so u_2 is positive. Therefore,

$$\frac{1}{v} - \frac{1}{v_1} = \frac{1}{f_2}.$$

Adding cancels the intermediate-image term:

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f_1} + \frac{1}{f_2}.$$

Quick Tip to Solve: After the first lens gives $v_1 = 15$ cm, measure from the second lens: its virtual object is at +10 cm. This cancels its -10 cm focal length and makes the outgoing rays parallel.

A single equivalent lens producing the same final image obeys $1/v - 1/u = 1/f$. Comparison gives

$$\boxed{\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}}, \quad \boxed{P = P_1 + P_2}.$$

The formula uses signed focal lengths and therefore also applies when one of the lenses is diverging. The contact condition is essential to the cancellation as written.

Step 3: (b) Find the image produced by the first lens Take the first lens at $x = 0$, the second at $x = 5$ cm, and the third at $x = 15$ cm. The object is at $x = -30$ cm. For the first lens, $u_1 = -30$ cm and $f_1 = +10$ cm. Thus,

$$\frac{1}{v_1} = \frac{1}{f_1} + \frac{1}{u_1} = \frac{1}{10} - \frac{1}{30} = \frac{1}{15}, \quad v_1 = +15 \text{ cm}.$$

The first lens alone would focus its rays at $x = 15$ cm. These converging rays encounter the second lens before reaching that point.

Step 4: (b) Determine the emergent rays from the second lens Relative to the second lens, the first image lies $15 - 5 = 10$ cm to its right, giving a virtual object at $u_2 = +10$ cm. With $f_2 = -10$ cm,

$$\frac{1}{v_2} = \frac{1}{f_2} + \frac{1}{u_2} = -\frac{1}{10} + \frac{1}{10} = 0.$$

Hence the second lens sends out a parallel bundle: its diverging action exactly cancels the convergence supplied by the first lens. The image after the second lens is at infinity, not at a finite location between the lenses.

Step 5: (b) Focus the parallel bundle with the third lens The third lens has $f_3 = +30$ cm. For a parallel incident bundle the object is at infinity, so $1/u_3 = 0$. Therefore,

$$\frac{1}{v_3} = \frac{1}{f_3} = \frac{1}{30}, \quad v_3 = +30 \text{ cm}.$$

Its separation from the second lens does not change the convergence of the parallel bundle. The final image is real and lies on the right of the third lens, at coordinate $x = 15 + 30 = 45$ cm measured from the first lens.

Step 6: Conclusion (a) The equivalent focal length and power are $\boxed{1/f = 1/f_1 + 1/f_2}$ and $\boxed{P = P_1 + P_2}$. (b) The final real image forms $\boxed{30 \text{ cm to the right of the third lens}}$, or 45 cm to the right of the first lens.

Quick Tip to Solve: After the first lens gives $v_1 = 15$ cm, measure from the second lens: its virtual object is at +10 cm. This cancels its -10 cm focal length and makes the outgoing rays parallel.

27. (a) Explain, using a suitable diagram, how unpolarized light gets linearly polarized by scattering.
 (b) Describe briefly the variation of the intensity of transmitted light when a polaroid sheet kept between two crossed polaroids is rotated. Draw the graph depicting the variation of intensity with the angle of rotation. How many maxima and minima would be observed when θ varies from 0 to π ?

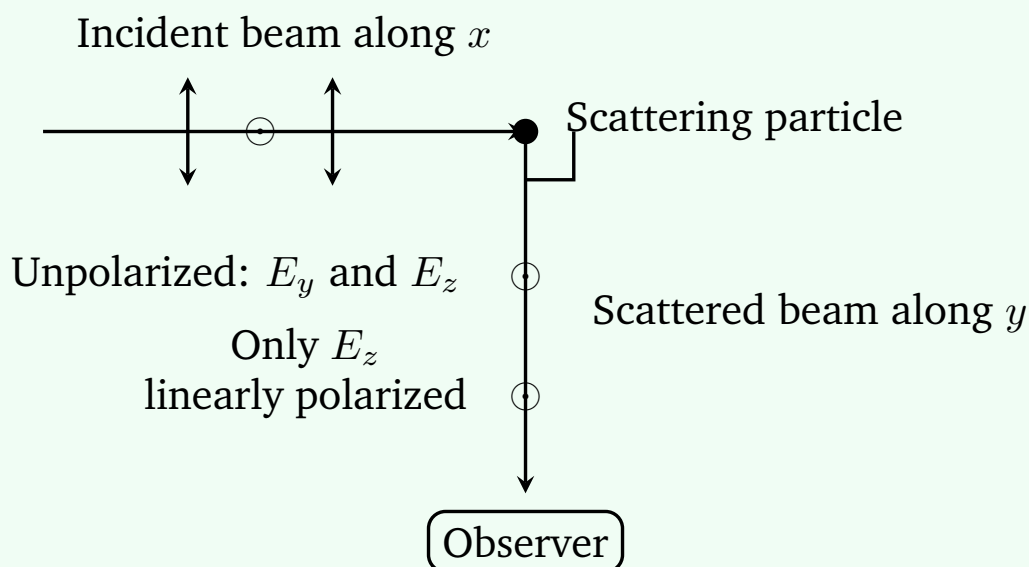
[5 marks]

Solution:**Concept:**

- Unpolarized light has electric-field vibrations in all directions perpendicular to its propagation direction. Scattering sets the electrons of a small particle into oscillation along the incident electric field.
- An oscillating electric dipole radiates most strongly perpendicular to its oscillation axis and does not radiate along that axis. Light scattered through 90° is therefore linearly polarized perpendicular to the plane of scattering.
- Malus's law is $I_{\text{out}} = I_{\text{in}} \cos^2 \phi$, where ϕ is the angle between the incident linear polarization and the transmission axis. An ideal first polaroid transmits half the intensity of unpolarized incident light.

Step 1: (a) Resolve the incident vibrations and identify the scattered polarization Let unpolarized light travel along the x -axis and let the observer receive scattered light along the y -axis, at right angles to the incident beam. The xy -plane is the plane of scattering. Resolve the incident electric field into its y - and z -components. These components drive the scattering electrons into oscillations along y and z , respectively. The electron oscillating along y cannot radiate towards the observer on its own oscillation axis. The electron oscillating along z can radiate in that direction, with its electric field along z . Thus the observer receives only one direction of electric-field vibration: perpendicular to the scattering plane. This is linear polarization. Complete polarization by this mechanism occurs for observation at 90° ; scattering at a general angle need not produce completely polarized light.

Quick Tip to Solve: Apply Malus's law twice: the middle sheet contributes $\cos^2 \theta$, and the final crossed sheet contributes $\sin^2 \theta$. Include both endpoints when counting minima.



Scattering at a right angle. Dots denote electric-field vibrations along z , perpendicular to the page; the page is the xy scattering plane.

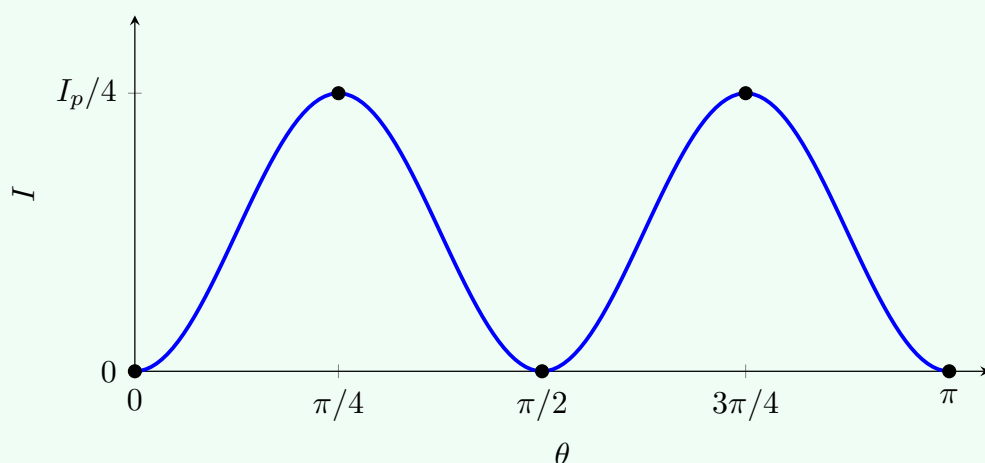
Step 2: (b) Apply Malus's law successively to the three polaroids Let the transmission axes of the fixed polaroids P_1 and P_3 be crossed, and let the middle sheet P_2 make an angle θ with P_1 . Write I_p for the intensity immediately after P_1 . The light emerging from P_2 has intensity $I_p \cos^2 \theta$ and is polarized along its axis. The angle between P_2 and P_3 is $\pi/2 - \theta$, giving

$$\begin{aligned} I(\theta) &= I_p \cos^2 \theta \cos^2(\pi/2 - \theta) \\ &= I_p \cos^2 \theta \sin^2 \theta \\ &= \frac{I_p}{4} \sin^2(2\theta). \end{aligned}$$

If the original incident beam is unpolarized and has intensity I_0 , then $I_p = I_0/2$, so $I(\theta) = I_0 \sin^2(2\theta)/8$. The middle polaroid changes the polarization direction, allowing some light through the final crossed sheet when its axis is oblique to both fixed axes.

Step 3: Locate the maxima and minima over the specified interval The intensity starts at zero for $\theta = 0$, increases to $I_p/4$ at $\pi/4$, falls to zero at $\pi/2$, rises again to $I_p/4$ at $3\pi/4$, and returns to zero at π . The maxima follow from $\sin^2(2\theta) = 1$; the minima follow from $\sin^2(2\theta) = 0$. The pattern repeats after a rotation of $\pi/2$. Both endpoints belong to the requested closed interval and are included in counting its zero-intensity minima.

Quick Tip to Solve: Apply Malus's law twice: the middle sheet contributes $\cos^2 \theta$, and the final crossed sheet contributes $\sin^2 \theta$. Include both endpoints when counting minima.



Transmitted intensity as the middle polaroid rotates; I_p is the intensity after the first polaroid.

Step 4: Conclusion Light scattered at 90° is linearly polarized perpendicular to the scattering plane. With a middle polaroid between two crossed fixed polaroids, $I = I_p \sin^2(2\theta)/4$. For $0 \leq \theta \leq \pi$, there are two maxima at $\pi/4, 3\pi/4$ and three minima at $0, \pi/2, \pi$.

Quick Tip to Solve: Apply Malus's law twice: the middle sheet contributes $\cos^2 \theta$, and the final crossed sheet contributes $\sin^2 \theta$. Include both endpoints when counting minima.