

CBSE Class 12 Physics Question Paper 2019 with Solution PDF-Set 55/4/2



Question Paper with Solutions

Conducted by the Central Board of Secondary Education (CBSE)

Time Allowed: 3 hours

Maximum Marks: 70

General Instructions

- (i) All questions are compulsory. There are **27** questions in all.
- (ii) This question paper has **four** sections : Section A, Section B, Section C and Section D.
- (iii) Section A contains **five** questions of **one** mark each, Section B contains **seven** questions of **two** marks each, Section C contains **twelve** questions of **three** marks each, Section D contains **three** questions of **five** marks each.
- (iv) There is no overall choice. However, an internal choice(s) has been provided in **two** questions of **one** mark, **two** questions of **two** marks, **four** questions of **three** marks and **three** questions of **five** marks weightage. You have to attempt only **one** of the choices in such questions.

You may use the following values of physical constants wherever necessary :

$$c = 3 \times 10^8 \text{ m/s}$$

$$h = 6.63 \times 10^{-34} \text{ Js}$$

$$e = 1.6 \times 10^{-19} \text{ C}$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ T m A}^{-1}$$

$$\epsilon_0 = 8.854 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$$

$$\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$$

$$\text{Mass of electron } (m_e) = 9.1 \times 10^{-31} \text{ kg}$$

$$\text{Mass of neutron} = 1.675 \times 10^{-27} \text{ kg}$$

$$\text{Mass of proton} = 1.673 \times 10^{-27} \text{ kg}$$

$$\text{Avogadro's number} = 6.023 \times 10^{23} \text{ per gram mole}$$

$$\text{Boltzmann constant} = 1.38 \times 10^{-23} \text{ JK}^{-1}$$

Section A (5 marks)

1. The small ozone layer on top of the stratosphere is crucial for human survival. Why? [1 mark]

Solution:

Concept:

- Solar radiation contains ultraviolet (UV) rays, whose photons can damage living tissues.
- Stratospheric ozone strongly absorbs much of the biologically harmful ultraviolet radiation.

Step 1: Identify the harmful radiation Ultraviolet radiation from the Sun can damage DNA and cause skin cancer and eye damage when exposure is excessive.

Step 2: Explain the protective role The ozone layer absorbs most of the harmful ultraviolet radiation before it reaches Earth's surface, greatly reducing human exposure to it.

Step 3: Conclusion The ozone layer is crucial because it shields humans from harmful solar ultraviolet rays.

Quick Tip to Solve: Link ozone protection specifically to absorption of ultraviolet rays, rather than all sunlight.

OR

1. Illustrate by giving suitable examples, how you can show that electromagnetic waves carry both energy and momentum. [1 mark]

Solution:

Concept:

- Electromagnetic radiation transports energy through its electric and magnetic fields.
- Radiation of energy U carries momentum $p = U/c$, where c is the speed of light in vacuum.

Step 1: Illustrate energy transfer Sunlight warms an absorbing surface such as a blackened metal plate. The temperature rise demonstrates that the incident electromagnetic waves transfer energy to the plate.

Step 2: Illustrate momentum transfer Light incident on a solar sail exerts radiation pressure and can accelerate it. This force demonstrates momentum transfer: complete absorption transfers U/c , while normal reflection transfers $2U/c$.

Step 3: Conclusion Heating by sunlight demonstrates the energy carried by electromagnetic waves; radiation pressure on a solar sail demonstrates their momentum.

Quick Tip to Solve: Pair a heating example with a radiation-pressure example to establish both energy and momentum.

2. Why is the sky wave transmission of signals restricted to a range of frequencies up to 30 MHz?

[1 mark]

Solution:

Concept:

- Sky-wave propagation uses the ionosphere to return radio signals towards Earth's surface.
- The return depends on sufficient refraction by the ionised atmospheric layers; their response becomes weaker as the wave frequency increases.

Step 1: State the condition for sky-wave transmission The transmitted wave must be bent back towards Earth by the ionosphere so that it reaches a distant ground receiver beyond the direct line of sight.

Step 2: Explain the upper frequency limit Above about 30 MHz, the ionosphere cannot normally return the waves to Earth. They penetrate the ionosphere and escape into space instead of forming the required sky-wave path.

Step 3: Conclusion Sky-wave transmission is normally restricted to frequencies up to about 30 MHz, because higher-frequency waves pass through the ionosphere.

Quick Tip to Solve: For the upper sky-wave frequency limit, explain penetration through the ionosphere, not ground absorption.

3. Two identical conducting balls A and B have charges $-Q$ and $+3Q$ respectively. They are brought in contact with each other and then separated by a distance d apart. Find the nature of the Coulomb force between them.

[1 mark]

Solution:

Concept:

- Total electric charge is conserved when isolated conducting balls are brought into contact.
- Identical conducting balls share their total charge equally on contact; like charges repel.

Step 1: Add the initial charges The charges can redistribute between the balls, but their combined charge remains

$$Q_{\text{total}} = -Q + 3Q = 2Q.$$

Step 2: Determine the charge after contact Since the balls are identical, equal potential after contact corresponds to equal charge. Each ball therefore receives

$$Q'_A = Q'_B = \frac{2Q}{2} = +Q.$$

After separation both balls have the same sign of charge, unlike their initial opposite signs.

Step 3: Conclusion The Coulomb force between the separated balls is **repulsive**, because each carries charge $+Q$.

Quick Tip to Solve: Determine the charges after contact before using the rule that like charges repel.

3. A metallic spherical shell has an inner radius R_1 and outer radius R_2 . A charge Q is placed at the centre of the shell. What will be the surface charge density on the (i) inner surface, and (ii) outer surface of the shell? [1 mark]

Solution:

Concept:

- In electrostatic equilibrium, the electric field inside the material of a conductor is zero.
- Surface charge density is $\sigma = q/A$; a sphere of radius R has area $4\pi R^2$.

Step 1: Find the induced inner charge A Gaussian surface lying within the metal has zero flux. Its enclosed charge must therefore vanish: $Q + q_{\text{inner}} = 0$. Hence the inner surface carries $-Q$, uniformly distributed because the charge is at the centre.

Step 2: Use neutrality of the shell Taking the initially uncharged, isolated shell implied in the question, its total charge remains zero. Thus the outer surface carries $+Q$, also uniformly distributed by spherical symmetry. Divide each surface charge by its own spherical area.

Step 3: Conclusion The surface charge densities are

$$(i) \sigma_{\text{inner}} = -\frac{Q}{4\pi R_1^2}, \quad (ii) \sigma_{\text{outer}} = +\frac{Q}{4\pi R_2^2}.$$

Quick Tip to Solve: Use opposite signs for the two induced surface charges and the correct radius for each area.

4. Why is the terminal voltage of a cell generally less than the emf of the cell? [1 mark]

Solution:

Concept:

- The emf $\mathcal{E}$ is the energy supplied by a cell per unit charge and equals its open-circuit voltage.
- A real cell has internal resistance r . When it supplies current I , an internal potential drop Ir occurs.

Step 1: Account for the internal voltage drop While the cell delivers current to an external circuit, some of the energy supplied per unit charge is dissipated inside the cell because of its internal resistance.

Step 2: Relate terminal voltage to emf Applying the loop rule to a discharging cell gives

$$\mathcal{E} = V + Ir, \quad V = \mathcal{E} - Ir.$$

Since $I > 0$ and $r > 0$, the internal drop reduces the voltage available at the terminals. With no current drawn, this drop vanishes.

Step 3: Conclusion The terminal voltage is generally less than the emf because of the potential drop Ir across the cell's internal resistance when it supplies current.

Quick Tip to Solve: Use $V = \mathcal{E} - Ir$ for a cell delivering current; at open circuit $V = \mathcal{E}$.

5. Plot a graph showing the variation of photoelectric current versus intensity of light.

[1 mark]

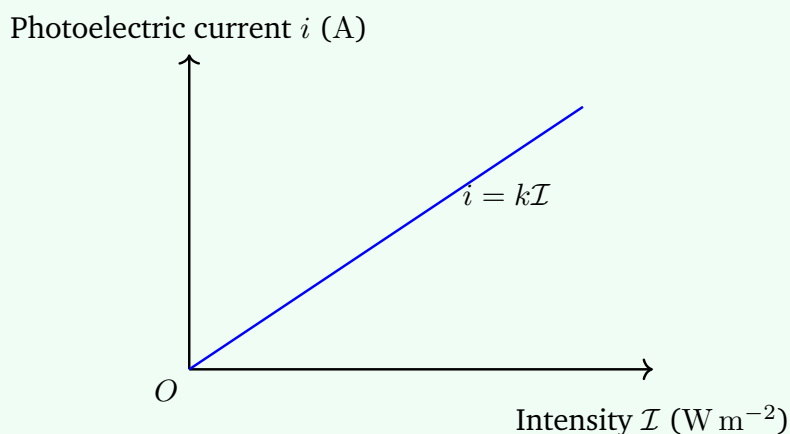
Solution:

Concept:

- At fixed light frequency above the threshold, the number of incident photons per second is proportional to the light intensity.
- For fixed collecting potential and illuminated area, photoelectric current i is proportional to light intensity $\mathcal{I}$.

Step 1: Connect intensity with electron emission Increasing intensity increases the number of photons incident per second. At the same frequency, this increases the number of photoelectrons emitted and collected per second in the same proportion.

Step 2: Plot the proportional relation Put intensity on the horizontal axis and photoelectric current on the vertical axis. The relation $i = k\mathcal{I}$, with positive constant k , gives a straight line through the origin. The frequency and collecting potential are kept fixed.



Photoelectric current at fixed frequency and collecting potential

Step 3: Conclusion The photoelectric-current versus light-intensity graph is a straight line with positive slope passing through the origin: $i \propto \mathcal{I}$.

Quick Tip to Solve: For current versus intensity, draw a straight line through the origin with both axes labelled.

Section B (14 marks)

6. A beam of light converges at a point P. Now a convex lens is placed in the path of the convergent beam at 15 cm from P. At what point does a beam converge if the convex lens has a focal length 10 cm? [2 marks]

Solution:

Concept:

- A converging incident beam has a virtual object at the point where its rays would meet in the absence of the lens.
- With distances measured along the incident light as positive, the lens formula is $1/v - 1/u = 1/f$, where u , v and f are the object distance, image distance and focal length.

Step 1: Identify the virtual object and signs Let the lens be at the origin and let the light travel towards the right. Before insertion of the lens, the incident rays would meet at P, 15 cm to the right of the lens. Thus P is a virtual object for this lens and $u = +15$ cm, not a negative object distance. Since the lens is convex, its focal length is $f = +10$ cm.

Step 2: Apply the lens formula Substituting the signed distances gives

$$\frac{1}{v} = \frac{1}{f} + \frac{1}{u} = \frac{1}{10} + \frac{1}{15} = \frac{1}{6} \text{ cm}^{-1}, \quad v = +6 \text{ cm}.$$

The positive image distance means that the emergent rays actually meet on the outgoing side of the lens. The convex lens adds convergence to the already converging beam, so the meeting point moves closer to the lens than P.

Step 3: Conclusion The beam converges 6 cm from the lens on the side of P, i.e. 9 cm before the original point P.

Quick Tip to Solve: A converging incident beam gives a positive virtual-object distance in the Cartesian lens formula.

OR

6. An object is kept in front of a concave mirror of focal length 15 cm. The image formed is real and three times the size of the object. Calculate the distance of the object from the mirror. [2 marks]

Solution:

Concept:

- For a spherical mirror, $1/f = 1/v + 1/u$, where f , u and v are signed focal, object and image distances.
- Mirror magnification is $m = -v/u$. A real image of a real object in a concave mirror is inverted, so a threefold real image has $m = -3$.

Step 1: Use the image size to relate the distances Taking the direction of incident light as positive, the object and a real image are both in front of the mirror and have negative distances. The concave

Quick Tip to Solve: First set magnification to negative for a real inverted mirror image, then substitute its distance relation.

mirror has $f = -15$ cm. Since the real image is inverted and three times the object size,

$$m = -\frac{v}{u} = -3 \Rightarrow v = 3u.$$

Using $+3$ would instead describe an erect image and would contradict the stated real image.

Step 2: Substitute into the mirror formula The mirror equation now becomes

$$-\frac{1}{15} = \frac{1}{3u} + \frac{1}{u} = \frac{4}{3u}, \quad u = -20 \text{ cm}.$$

It also gives $v = 3u = -60$ cm. Thus the object lies between the focus and the centre of curvature, while its image lies beyond the centre of curvature, consistent with a magnified real image.

Step 3: Conclusion The object is 20 cm in front of the concave mirror (signed distance $u = -20$ cm).

Quick Tip to Solve: First set magnification to negative for a real inverted mirror image, then substitute its distance relation.

7. Explain giving reason, how the magnifying power of a compound microscope depends on the (i) wavelength of incident light, and (ii) focal length of the objective lens. [2 marks]

Solution:

Concept:

- A compound microscope's magnifying power is the product of the objective's linear magnification and the eyepiece's angular magnification.
- Approximately, $M = (L/f_o)(D/f_e)$ for normal adjustment, where L is the effective tube length, f_o and f_e are the objective and eyepiece focal lengths, and D is the least distance of distinct vision.

Step 1: Dependence on wavelength In the standard geometrical-optics treatment, keeping the focal lengths, tube length and viewing condition fixed, magnifying power does not depend on the wavelength of the incident light. The expression $M = (L/f_o)(D/f_e)$ contains no wavelength. Magnification describes the increase in apparent angular size. Wavelength controls diffraction and hence resolving power, a different property. For real glass lenses, wavelength can also affect magnification indirectly through dispersion: under normal dispersion, shorter wavelength gives a larger refractive index n . By the lens-maker relation $1/f = (n - 1)(1/R_1 - 1/R_2)$, this reduces the focal lengths and can increase magnifying power when the microscope is refocused. This indirect effect is excluded only when the focal lengths are treated as fixed.

Step 2: Dependence on objective focal length The objective forms an enlarged intermediate image, which the eyepiece further magnifies. Keeping L , f_e and the final-image position fixed gives $M \propto 1/f_o$. Thus reducing the objective focal length increases magnifying power; for example, halving f_o approximately doubles it. For a final image at the near point, $M = (L/f_o)(1 + D/f_e)$, which gives the same inverse dependence on f_o and also contains no wavelength term.

Step 3: Conclusion (i) There is no direct wavelength dependence at fixed focal lengths; normal dispersion can increase magnification at shorter wavelengths by reducing focal lengths. (ii) It is inversely proportional to the focal length of the objective lens.

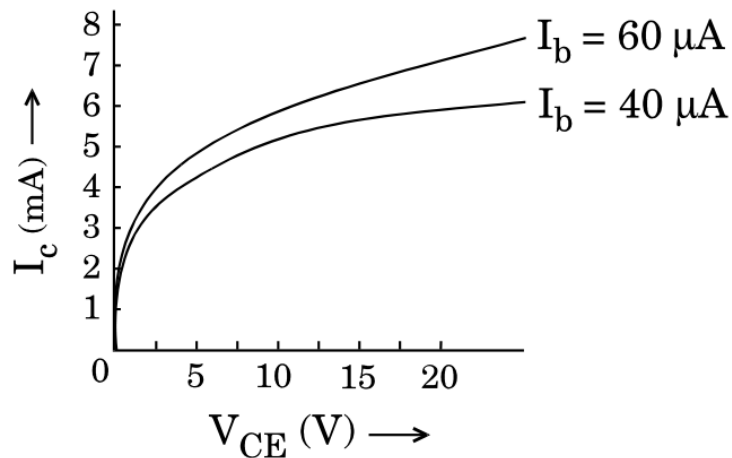
Quick Tip to Solve: Identify whether the question asks magnification or resolution before selecting a microscope formula.

8. The output characteristics of an n-p-n transistor in common emitter configuration is as shown in the figure.

(a) Find the emitter current at $V_{CE} = 10.0 \text{ V}$ and $I_b = 40 \mu\text{A}$, and

(b) Current gain β at this point.

[2 marks]



Solution:

Concept:

- For a transistor, the emitter current equals the sum of collector and base currents: $I_E = I_C + I_B$.
- The common-emitter dc current gain at an operating point is $\beta = I_C/I_B$, with both currents expressed in the same unit.

Step 1: Read the collector current at the given operating point Select the lower output-characteristic curve, labelled $I_B = 40 \mu\text{A}$. At $V_{CE} = 10.0 \text{ V}$, the corresponding collector current read from the vertical axis is approximately $I_C = 5.0 \text{ mA}$. The upper curve belongs to $I_B = 60 \mu\text{A}$ and must not be used. Since the graph gives collector current, it is necessary to add the base current to find the emitter current.

Step 2: Calculate emitter current and current gain Convert the base current to milliamperes: $40 \mu\text{A} = 0.040 \text{ mA}$. Hence

$$I_E = 5.0 + 0.040 = 5.04 \text{ mA}, \quad \beta = \frac{5.0 \text{ mA}}{0.040 \text{ mA}} = 125.$$

The gain is dimensionless because it is a ratio of currents. These results use the graph reading at the specified point; a difference quotient between the two curves would describe an incremental gain rather than the dc gain asked at this point.

Step 3: Conclusion (a) The emitter current is approximately 5.04 mA . (b) The common-emitter current gain is approximately $\beta = 125$.

Quick Tip to Solve: Use the curve labelled with the specified base current, and convert microamperes to milliamperes before dividing.

9. Apply Gauss's law to show that for a charged spherical shell, the electric field outside the shell is as if the entire charge were concentrated at the centre. [2 marks]

Solution:

Concept:

- Gauss's law states $\oint \vec{E} \cdot d\vec{A} = Q_{\text{enc}}/\epsilon_0$, where Q_{enc} is the enclosed charge and ϵ_0 is the permittivity of free space.
- For a uniformly charged spherical shell, spherical symmetry makes the electric field radial and its magnitude identical at every point at the same distance from the centre.

Step 1: Choose a concentric Gaussian sphere outside the shell Let the shell have radius R and total charge Q . Choose a spherical Gaussian surface of radius $r > R$, concentric with the shell. This surface encloses the entire charge Q . By spherical symmetry, the field is radial and its radial component E_r is constant over the Gaussian sphere. Its outward area element is also radial, so the flux reduces to E_r times the sphere's area.

Step 2: Apply Gauss's law and compare with a point charge Therefore,

$$\oint \vec{E} \cdot d\vec{A} = E_r(4\pi r^2) = \frac{Q}{\epsilon_0}, \quad \vec{E}(r) = \frac{Q}{4\pi\epsilon_0 r^2} \hat{r}.$$

Here $\hat{r}$ is the outward radial unit vector. For positive Q the field points outward, and for negative Q it points inward. The expression has precisely the inverse-square magnitude and radial direction of the Coulomb field of a point charge Q at the common centre. This comparison applies at every exterior point, not only on one chosen Gaussian sphere.

Step 3: Conclusion Outside a uniformly charged spherical shell, $\vec{E} = Q\hat{r}/(4\pi\epsilon_0 r^2)$; thus the entire charge acts as though concentrated at the centre.

Quick Tip to Solve: A concentric Gaussian sphere exploits spherical symmetry so the flux becomes field times total area.

OR

9. Two large parallel plane sheets have uniform charge densities $+\sigma$ and $-\sigma$. Determine the electric field (i) between the sheets, and (ii) outside the sheets. [2 marks]

Solution:

Concept:

- An infinite uniformly charged plane sheet produces a field of magnitude $\sigma/(2\epsilon_0)$ on each side, where σ is the magnitude of its surface charge density.
- The electric field points away from a positive sheet and towards a negative sheet. The net field is the vector sum of the two fields, by superposition.

Step 1: Add the fields between the sheets Neglect edge effects because the sheets are large. At a point between the sheets, the field of the positive sheet points towards the negative sheet. The field of

Quick Tip to Solve: Draw the direction of each sheet's field first: opposite charges give addition inside and cancellation outside.

the negative sheet also points towards that sheet, so these two equal fields reinforce:

$$E_{\text{between}} = \frac{\sigma}{2\epsilon_0} + \frac{\sigma}{2\epsilon_0} = \frac{\sigma}{\epsilon_0}.$$

The resultant is uniform in this approximation and is perpendicular to the sheets.

Step 2: Compare the fields outside the sheets On either exterior side, the positive sheet's field is away from the positive sheet whereas the negative sheet's field is towards the negative sheet. Their directions are opposite and their magnitudes are equal. Therefore,

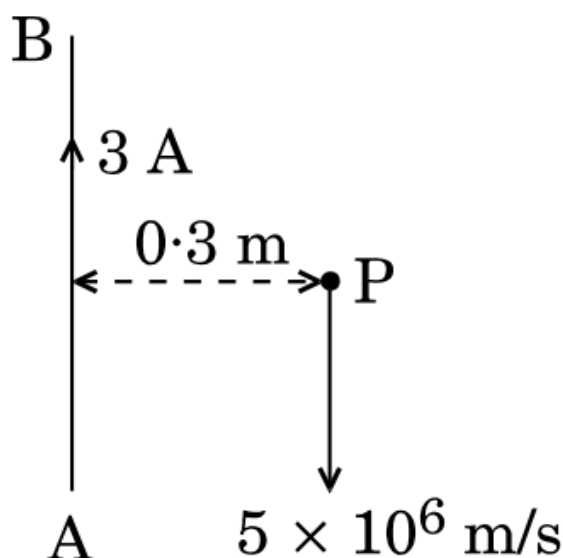
$$E_{\text{outside}} = \frac{\sigma}{2\epsilon_0} - \frac{\sigma}{2\epsilon_0} = 0.$$

The cancellation occurs on both exterior sides, rather than only on one side.

Step 3: Conclusion (i) Between the sheets, $E = \sigma/\epsilon_0$, directed normally from $+\sigma$ to $-\sigma$. (ii) Outside the sheets, $E = 0$, neglecting edge effects.

Quick Tip to Solve: Draw the direction of each sheet's field first: opposite charges give addition inside and cancellation outside.

10. A long straight wire AB carries a current of 3 A. A proton P travels at $5 \times 10^6 \text{ m s}^{-1}$ parallel to the wire 0.3 m from it and in a direction opposite to the current as shown in the figure. Calculate the force which the magnetic field due to the current carrying wire exerts on the proton. Also specify its direction. [2 marks]



Solution:

Concept:

- A long straight wire carrying current I creates a magnetic field $B = \mu_0 I / (2\pi r)$ at perpendicular distance r . Its direction follows the right-hand thumb rule.

Quick Tip to Solve: Determine the field with the wire's right-hand rule, then apply $\vec{v} \times \vec{B}$ for a positive proton; keep the two direction rules separate.

- A proton of charge $e = 1.6 \times 10^{-19} \text{ C}$ moving with velocity $\vec{v}$ experiences $\vec{F} = e(\vec{v} \times \vec{B})$. The magnitude is $evB \sin \theta$.

Step 1: Calculate the magnetic field at P Use the printed values $I = 3 \text{ A}$ and $r = 0.3 \text{ m}$. Thus,

$$B = \frac{(4\pi \times 10^{-7})(3)}{2\pi(0.3)} = 2.0 \times 10^{-6} \text{ T}.$$

The current flows upwards from A to B. At P, which lies to the right of the wire, the right-hand thumb rule gives a magnetic field into the plane of the paper.

Step 2: Evaluate the magnetic force and its direction The proton moves downwards at $v = 5 \times 10^6 \text{ m s}^{-1}$, perpendicular to the field. Hence $\sin \theta = 1$, and

$$F = evB = (1.6 \times 10^{-19})(5 \times 10^6)(2 \times 10^{-6}) = 1.6 \times 10^{-18} \text{ N}.$$

For a positive charge, the cross product of a downward velocity and an inward field points to the right. Thus the force is perpendicular to AB and directed away from the wire, rather than along the proton's velocity.

Step 3: Conclusion The force on the proton is $1.6 \times 10^{-18} \text{ N}$, directed to the right in the figure, away from the current-carrying wire AB.

Quick Tip to Solve: Determine the field with the wire's right-hand rule, then apply $\vec{v} \times \vec{B}$ for a positive proton; keep the two direction rules separate.

11. A parallel plate capacitor of plate area A each and separation d , is being charged by an ac source. Show that the displacement current inside the capacitor is the same as the current charging the capacitor. [2 marks]

Solution:

Concept:

- Displacement current is $I_d = \epsilon_0 d\Phi_E/dt$, where Φ_E is electric flux through a surface spanning the capacitor gap.
- For an ideal parallel-plate capacitor in air or vacuum, $C = \epsilon_0 A/d$, $Q = CV$ and $E = V/d = Q/(\epsilon_0 A)$. The charging current in its leads is $I_c = dQ/dt$.

Step 1: Express the electric flux in terms of plate charge Neglect fringing so that the electric field is uniform and perpendicular to a surface of area A inside the capacitor. For instantaneous plate charge $Q(t)$,

$$\Phi_E(t) = E(t)A = \frac{Q(t)}{\epsilon_0 A} A = \frac{Q(t)}{\epsilon_0}.$$

Although no conduction charge crosses the insulating gap, the ac source makes the plate charge and the electric flux vary with time.

Step 2: Differentiate the flux to find displacement current Substitute this expression in Maxwell's

Quick Tip to Solve: Use $EA = Q/\epsilon_0$ before differentiating; the factors of A and ϵ_0 then cancel immediately.

definition:

$$I_d = \varepsilon_0 \frac{d\Phi_E}{dt} = \varepsilon_0 \frac{d}{dt} \left(\frac{Q}{\varepsilon_0} \right) = \frac{dQ}{dt} = I_c.$$

Equivalently, $I_d = (\varepsilon_0 A/d) dV/dt = C dV/dt$, the same expression as the lead current. The equality holds instantaneously, including when the ac source reverses the direction of charging.

Step 3: Conclusion The displacement current inside the capacitor equals the charging current in the connecting wires: $I_d = I_c = dQ/dt = C dV/dt$.

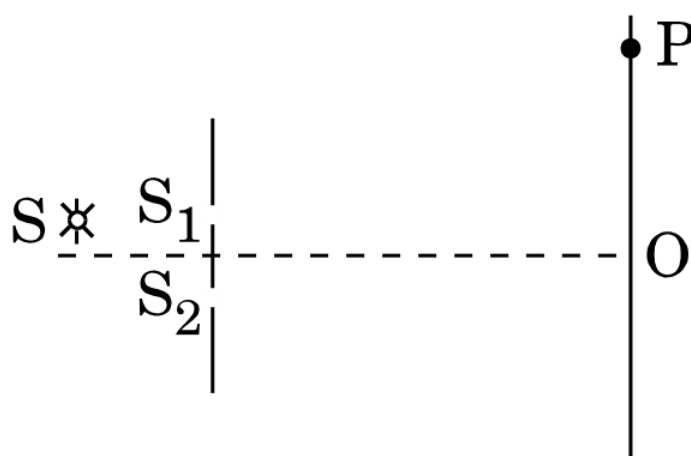
Quick Tip to Solve: Use $EA = Q/\varepsilon_0$ before differentiating; the factors of A and ε_0 then cancel immediately.

12. The figure shows a modified Young's double slit experimental set-up. Here $SS_2 - SS_1 = \lambda/4$.

(a) Write the condition for constructive interference.

(b) Obtain an expression for the fringe width.

[2 marks]



Solution:

Concept:

- Constructive interference requires the total path difference from the common source S to P to be $n\lambda$, with integer n .
- For slit separation $d = S_1S_2$, screen distance $D \gg d$, and small signed height $y = OP$ measured upwards, $S_2P - S_1P \simeq dy/D$.

Step 1: Include the source-path difference for part (a) Compare the complete paths through the two slits. Since $SS_2 - SS_1 = \lambda/4$, the bright-fringe condition is

$$(SS_2 + S_2P) - (SS_1 + S_1P) = n\lambda,$$

$$\frac{\lambda}{4} + S_2P - S_1P = n\lambda.$$

Thus the slit-to-screen path difference must be $(n - 1/4)\lambda$. At O it is zero, but the total difference is $\lambda/4$, so O is not a bright-fringe centre.

Step 2: Subtract adjacent fringe positions for part (b) Using the small-angle approximation in the

Quick Tip to Solve: Add the source-to-slit difference before finding maxima; this constant shifts the pattern but cancels from the fringe width.

preceding condition gives

$$y_n = \left(n - \frac{1}{4}\right) \frac{\lambda D}{d}, \quad \beta = y_{n+1} - y_n = \frac{\lambda D}{d}.$$

The fixed source-path difference shifts each fringe equally and cancels when successive positions are subtracted. The zero-total-path bright fringe is at $y_0 = -\beta/4$, below O.

Step 3: Conclusion (a) Constructive interference requires $\lambda/4 + S_2P - S_1P = n\lambda$. (b) The fringe width is $\beta = \lambda D/d$, unchanged by the source displacement.

Quick Tip to Solve: Add the source-to-slit difference before finding maxima; this constant shifts the pattern but cancels from the fringe width.

Section C (36 marks)

13. A particle of charge q and mass m is moving with velocity $\vec{v}$ in the positive x-direction.

(a) It is subjected to a uniform magnetic field $\vec{B}$ directed along negative z-direction. Explain briefly the trajectory it would describe.

(b) When the particle is subjected simultaneously to both the magnetic and electric fields directed along the z-axis and y-axis respectively, obtain the condition when the particle will go undeflected.

[3 marks]

Solution:

Concept:

- The magnetic force is $\vec{F}_B = q(\vec{v} \times \vec{B})$. It is perpendicular to velocity, does no work, and preserves speed.
- For motion perpendicular to a uniform magnetic field, magnetic force supplies centripetal force: $|q|vB = mv^2/r$, where m is particle mass and r is the radius.
- In simultaneous electric and magnetic fields, the Lorentz force is $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$. Undeflected motion requires this net force to vanish.

Step 1: Find the initial force and the trajectory in part (a) Here $\vec{v} = v\hat{x}$ and $\vec{B} = -B\hat{z}$, with $B > 0$. The initial force is

$$\vec{F}_B = qvB\hat{y},$$

since $\hat{x} \times (-\hat{z}) = \hat{y}$. For positive q the particle bends towards positive y ; for negative q it bends towards negative y . Throughout its motion the magnetic force stays perpendicular to velocity and has constant magnitude. It therefore supplies the centripetal force for uniform circular motion in the xy -plane:

$$|q|vB = \frac{mv^2}{r}, \quad r = \frac{mv}{|q|B}.$$

The speed and kinetic energy remain constant, and there is no helical motion because the velocity has no component parallel to the magnetic field.

Quick Tip to Solve: Check $\vec{v} \times \vec{B}$ before equating E and vB ; equal force magnitudes cause cancellation only when the force directions oppose.

Step 2: Balance the crossed-field forces in part (b) The exact condition, including directions, is

$$q(\vec{E} + \vec{v} \times \vec{B}) = 0, \quad \vec{E} = -\vec{v} \times \vec{B}.$$

Thus the electric and magnetic forces must oppose each other and have equal magnitudes. Since the velocity is perpendicular to $\vec{B}$,

$$|q|E = |q|vB, \quad v = \frac{E}{B}.$$

The stated axes alone do not assign signs to the fields in part (b). For $\vec{B} = +B\hat{z}$, the required field is $\vec{E} = +vB\hat{y}$. If $\vec{B}$ retains the negative- z direction of part (a), the required electric field is instead along negative y . These direction statements follow from the same vector equation for either sign of the charge.

Step 3: Conclusion (a) The particle executes uniform circular motion in the xy -plane, with radius $mv/(|q|B)$; the sense of bending depends on the sign of q . (b) It goes undeflected when $\vec{E} = -\vec{v} \times \vec{B}$, giving $v = E/B$ for oppositely directed electric and magnetic forces.

Quick Tip to Solve: Check $\vec{v} \times \vec{B}$ before equating E and vB ; equal force magnitudes cause cancellation only when the force directions oppose.

14. Draw a ray diagram to show the image formation of a distant object by a refracting telescope. Write the expression for its angular magnification in terms of the focal lengths of the lenses used. State the important considerations required to achieve large resolution and their consequent limitations.

[3 marks]

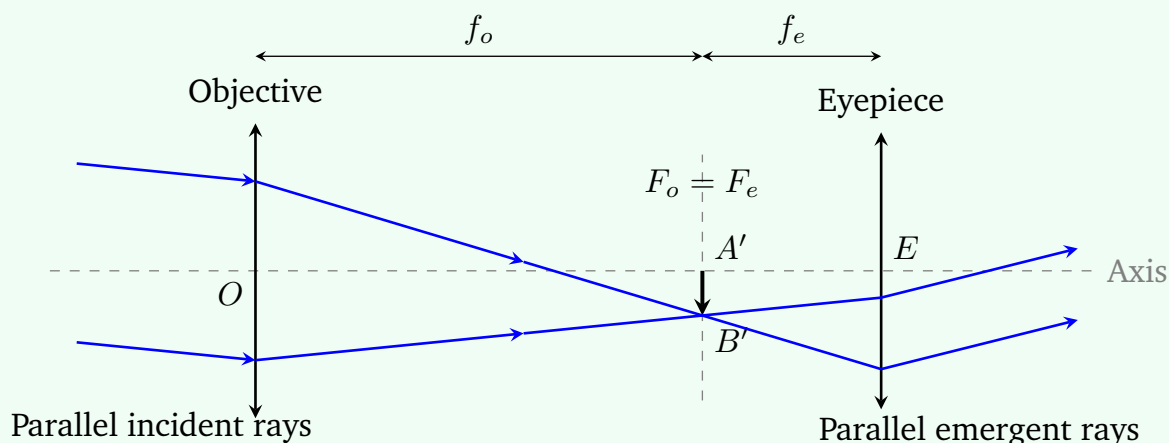
Solution:

Concept:

- A telescope's convex objective forms a real, inverted image of a distant object in its focal plane. A convex eyepiece magnifies the angular size of this intermediate image.
- In normal adjustment the final image is at infinity and the lens separation is $f_o + f_e$, where f_o and f_e are the objective and eyepiece focal lengths.
- Diffraction limits resolution: for an objective of diameter D , the smallest resolvable angle is approximately $\theta_{\min} = 1.22\lambda/D$, where λ is the wavelength.

Step 1: Trace rays in normal adjustment Parallel rays from a distant off-axis object converge at B' after refraction by the objective. The real, inverted image $A'B'$ lies at the common focal plane $F_o = F_e$. The eyepiece is placed so that this image is at its first focus. Rays emerging from the eyepiece are parallel, and the relaxed eye sees a magnified, inverted image at infinity. The diagram shows rays from one point of the distant object; A' is on the principal axis.

Quick Tip to Solve: Keep the two design goals separate: f_o/f_e sets magnification, while D/λ sets diffraction-limited resolving power.



Refracting telescope in normal adjustment; emergent rays are parallel.

Step 2: Express the angular magnification Let the intermediate image height have magnitude h . For small angles, the distant object subtends $|\alpha| \simeq h/f_o$ at the objective, whereas its image subtends $|\beta| \simeq h/f_e$ at the eye. Thus, using the sign convention that an inverted image has negative magnification,

$$M = \frac{\beta}{\alpha} = -\frac{f_o}{f_e}, \quad |M| = \frac{f_o}{f_e}.$$

A long-focal-length objective and a short-focal-length eyepiece give a large angular magnification.

Step 3: Relate high resolution to practical limitations High resolving power requires a large objective aperture, since increasing D decreases $\theta_{\min}$ and also collects more light from faint objects. Shorter wavelengths improve the diffraction limit as well. Increasing magnification alone cannot separate details already blurred by diffraction. A very large refracting objective is difficult to manufacture from homogeneous, defect-free glass and to polish accurately; it is heavy and can be supported only at its rim, so sagging can distort the image. Thick glass absorbs light, and chromatic and spherical aberrations must be controlled. Atmospheric turbulence further limits the useful resolution of a ground-based telescope.

Step 4: Conclusion In normal adjustment, $M = -f_o/f_e$ and the final image is inverted and at infinity. A large objective diameter improves resolution, but lens manufacture, weight, aberrations and atmospheric turbulence impose practical limits.

Quick Tip to Solve: Keep the two design goals separate: f_o/f_e sets magnification, while D/λ sets diffraction-limited resolving power.

14(a). Plot a graph for angle of deviation as a function of angle of incidence for a triangular prism.

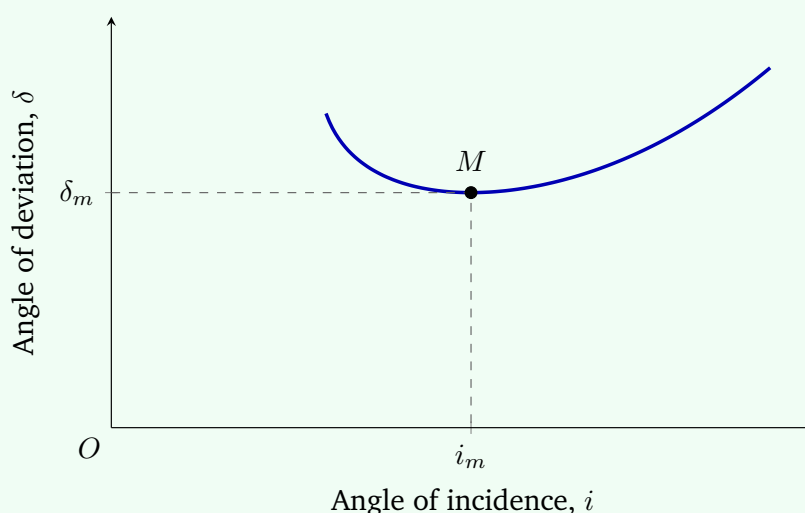
[3 marks total]

Solution:

Concept:

- For a prism of angle A , the deviation is $\delta = i + e - A$, where i and e are the angles of incidence and emergence, measured from the corresponding normals.
- For a fixed prism and wavelength, the deviation has a minimum value δ_m . At minimum deviation the ray passes symmetrically through the prism, so $i = e = i_m$.

Step 1: Plot the variation of deviation Take the angle of incidence i along the horizontal axis and the angle of deviation δ along the vertical axis. Over the range in which the ray emerges from the second face, increasing i initially decreases δ . The curve reaches a smooth minimum and then rises as i increases further. The following qualitative graph shows this variation; the two sides need not be mirror images.



Qualitative deviation curve for a triangular prism in air.

Step 2: Identify the minimum point Mark the lowest point M of the curve. Its horizontal coordinate is i_m , and its vertical coordinate is δ_m . Draw dotted projections onto the axes to show these values clearly. Since $i = e$ at this point, the deviation relation becomes

$$\delta_m = 2i_m - A.$$

Thus the minimum does not correspond to zero incidence or zero deviation for an ordinary refracting prism in air.

Step 3: Conclusion The δ -versus- i graph first falls, reaches $M(i_m, \delta_m)$, and then rises. At M , $i = e$ and the deviation is minimum.

Quick Tip to Solve: Label both axes, mark the lowest point, and project it onto the axes as i_m and δ_m ; do not draw the curve through the origin.

14(b). Derive the relation for the refractive index of the prism in terms of the angle of minimum deviation and angle of prism.

Solution:

Concept:

- In a prism of angle A , the internal refraction angles satisfy $r_1 + r_2 = A$, while the total deviation is $\delta = i + e - A$. Here i and e are the incidence and emergence angles.
- At minimum deviation δ_m , the path through the prism is symmetric: $i = e$ and $r_1 = r_2$. For a prism in air, Snell's law at its first face gives $\mu = \sin i / \sin r_1$, where μ is the refractive index relative to air.

Step 1: Apply the minimum-deviation condition Let r denote each internal angle at minimum deviation. The prism geometry gives

$$r_1 + r_2 = A, \quad 2r = A, \quad r = \frac{A}{2}.$$

The equality of the two internal angles makes the incidence and emergence angles equal by Snell's law, because the medium outside both faces is air.

Step 2: Express the incidence angle in terms of deviation Adding the deviations at the two refracting faces gives

$$\delta = (i - r_1) + (e - r_2) = i + e - A.$$

At minimum deviation, put $e = i$ and $\delta = \delta_m$. Hence

$$\delta_m = 2i - A, \quad i = \frac{A + \delta_m}{2}.$$

Step 3: Substitute into Snell's law At the air-to-prism interface, Snell's law is $n_{\text{air}} \sin i = n_{\text{prism}} \sin r$. Taking the ratio $\mu = n_{\text{prism}}/n_{\text{air}}$, and substituting the angles found above, gives

$$\mu = \frac{\sin i}{\sin r} = \frac{\sin((A + \delta_m)/2)}{\sin(A/2)}.$$

This relation refers to the wavelength for which the minimum deviation is measured; a prism's refractive index varies with wavelength.

Step 4: Conclusion The prism's refractive index relative to air is

$$\mu = \frac{\sin\left(\frac{A + \delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}.$$

Quick Tip to Solve: At minimum deviation, use symmetry first: $r = A/2$ and $i = (A + \delta_m)/2$. Substitute these into Snell's law to avoid reversing the sine ratio.

15. Using Bohr's postulates, derive the expression for the radius of the n^{th} orbit in which the electron is revolving in hydrogen atom. How does de-Broglie's hypothesis explain the stability of hydrogen atom? Explain. [3 marks]

Solution:

Concept:

- In a hydrogen atom, Coulomb attraction between the proton and electron provides the centripetal force for a circular orbit: $m_e v_n^2 / r_n = e^2 / (4\pi\epsilon_0 r_n^2)$.
- Bohr's angular-momentum condition is $m_e v_n r_n = n\hbar = nh/(2\pi)$, where $n = 1, 2, \dots$. The electron does not radiate while occupying an allowed stationary orbit.
- The de Broglie wavelength of an electron of momentum $p = m_e v_n$ is $\lambda = h/p$. A stationary matter wave must fit continuously around the orbit.

Step 1: Apply the force balance Treat the proton as fixed and let the electron have mass m_e , charge magnitude e , speed v_n and orbital radius r_n . Equating electrostatic attraction to the required centripetal force gives

$$\frac{m_e v_n^2}{r_n} = \frac{e^2}{4\pi\epsilon_0 r_n^2} \Rightarrow m_e v_n^2 r_n = \frac{e^2}{4\pi\epsilon_0}.$$

Here ϵ_0 is the permittivity of free space; only the magnitude of the attractive force is used.

Step 2: Use angular-momentum quantisation to obtain the radius Bohr's postulate gives $v_n = nh/(2\pi m_e r_n)$. Substitute this speed in the force-balance equation and simplify:

$$\begin{aligned} m_e \left(\frac{nh}{2\pi m_e r_n} \right)^2 r_n &= \frac{e^2}{4\pi\epsilon_0}, \\ \frac{n^2 h^2}{4\pi^2 m_e r_n} &= \frac{e^2}{4\pi\epsilon_0}, \\ r_n &= \frac{\epsilon_0 h^2}{\pi m_e e^2} n^2 = a_0 n^2. \end{aligned}$$

The Bohr radius is $a_0 \simeq 5.29 \times 10^{-11}$ m. Thus the radii grow as the square of the positive integer n , rather than varying continuously.

Step 3: Explain the stationary orbits using matter waves The electron's associated wave must return to the same phase after a complete revolution. A sustained standing wave is possible only when the circumference contains an integral number of wavelengths:

$$2\pi r_n = n\lambda = n \frac{h}{m_e v_n} \Rightarrow m_e v_n r_n = \frac{nh}{2\pi}.$$

This reproduces Bohr's quantisation condition. If the circumference is not an integral multiple of λ , successive wave circuits do not join in phase and cannot form an allowed stationary wave. The allowed stationary states therefore provide a wave interpretation of stable orbits. Within Bohr's model, these states do not radiate; radiation is emitted or absorbed only in a transition between two allowed energy levels. The lowest allowed state has $n = 1$, so there is no smaller allowed orbit into which the electron can continuously spiral.

Step 4: Conclusion The radius of hydrogen's n th Bohr orbit is $r_n = \epsilon_0 h^2 n^2 / (\pi m_e e^2) = a_0 n^2$. De

Quick Tip to Solve: First obtain v_n from $m_e v_n r_n = n\hbar$, then substitute into Coulomb force balance; the result must have $r_n \propto n^2$.

Broglie's standing-wave condition $2\pi r_n = n\lambda$ explains the quantisation of the stationary orbits and their wave-based stability.

Quick Tip to Solve: First obtain v_n from $m_e v_n r_n = n\hbar$, then substitute into Coulomb force balance; the result must have $r_n \propto n^2$.

16. How is a light emitting diode fabricated? Briefly explain the basic processes involved in the emission of spontaneous radiation from it. Write two advantages of LED lamps over the incandescent low power lamps. [3 marks]

Solution:

Concept:

- An LED is a heavily doped p - n junction made from a semiconductor in which electron-hole recombination can release energy as visible or infrared photons.
- In forward bias, electrons and holes are injected across the junction. Radiative recombination gives photon energy $h\nu \simeq E_g$, where E_g is the band gap and h is Planck's constant.

Step 1: Fabrication of the LED A heavily doped p - n junction is fabricated from a suitable compound semiconductor, such as gallium arsenide phosphide. Materials with efficient radiative recombination are selected; an ordinary silicon junction is unsuitable as an efficient light source. The junction is enclosed in a transparent cover with a window so that the light produced near the junction can emerge. Electrical contacts permit forward bias to be applied. The band gap of the material determines the photon energy and hence the colour of the emitted light.

Step 2: Injection and spontaneous emission Applying forward bias lowers the junction barrier. Electrons move from the n side into the p side and holes from the p side into the n side. These injected minority carriers recombine with majority carriers near the junction. An electron falling from a conduction-band state into an available valence-band state fills a hole and releases energy. In a radiative transition this energy appears as a photon, approximately satisfying

$$h\nu = \frac{hc}{\lambda} \simeq E_g.$$

This is spontaneous emission: recombination occurs without an incident photon stimulating it. The emitted photons have no fixed common phase, and the LED light is therefore incoherent.

Step 3: Two advantages over incandescent low-power lamps First, LED lamps convert electrical energy into light more efficiently, so they consume less power for comparable illumination and waste less energy as heat. Second, they have a much longer operating life: there is no heated filament that can burn out. These advantages reduce both electricity consumption and the frequency of replacement.

Step 4: Conclusion An LED is a heavily doped, forward-biased compound-semiconductor junction in a transparent enclosure. Injected electrons and holes recombine radiatively to emit photons spontaneously; LED lamps offer lower power consumption and longer life than incandescent lamps.

Quick Tip to Solve: Link fabrication to a heavily doped junction, emission to forward-bias recombination, and colour to the band gap.

17. (a) For the circuit shown in the figure, how would the balancing length be affected, if

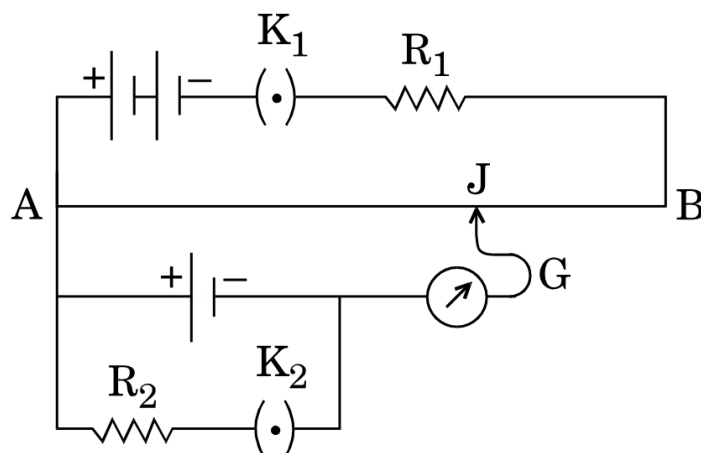
(i) R_1 is decreased,

(ii) R_2 is increased,

the other factors remaining the same in the circuit? Justify your answer in each case.

(b) Why is a potentiometer preferred over a voltmeter? Give reason.

[3 marks]



Solution:

Concept:

- For a uniform potentiometer wire of length L and resistance R_w , the potential gradient is $k = IR_w/L$, where I is the primary-circuit current. At balance, the measured potential difference is $V = k\ell$.
- A cell of emf E and internal resistance r , supplying an external resistance R_2 , has terminal voltage $V = ER_2/(R_2 + r)$. A potentiometer uses a null measurement, with zero galvanometer current at balance.

Step 1: (a)(i) Decrease the primary resistance R_1 The resistance R_1 is in the driving circuit of the potentiometer wire AB . Decreasing it increases the current through AB , since the driving source is unchanged. Hence the potential drop per unit length increases:

$$k = \frac{IR_w}{L}, \quad \ell = \frac{V}{k}.$$

The measured cell's terminal voltage V is unchanged because its load and other factors are fixed. A larger gradient therefore requires a shorter wire length to balance the same voltage. Thus the balancing length decreases. This argument concerns the gradient of the primary wire, rather than a change in the emf of the measured cell.

Step 2: (a)(ii) Increase the load resistance R_2 With key K_2 closed, R_2 is the load connected across the measured cell. Increasing R_2 reduces the current drawn from that cell and therefore reduces the internal drop across its internal resistance r . Its terminal voltage increases, as shown by

$$V = E - ir = \frac{ER_2}{R_2 + r}.$$

Quick Tip to Solve: Identify the circuit first: R_1 changes the wire gradient, while R_2 changes the measured cell's terminal voltage.

The driving circuit is unchanged in this case, so k remains constant. Since $\ell = V/k$, the balancing length increases. If K_2 were open, R_2 would be disconnected and changing it would have no effect; the stated increase is for the connected load shown in the measurement circuit.

Step 3: (b) Reason for preferring a potentiometer At the null point no current flows through the galvanometer branch. When the measured cell is unloaded, the potentiometer therefore measures its emf without drawing current from it and without introducing an internal-resistance voltage drop. A voltmeter has finite resistance and draws some current, so it measures the loaded terminal potential difference, which is smaller than the emf of a cell with nonzero internal resistance. The null method thus avoids loading error and enables a more accurate emf measurement.

Step 4: Conclusion Decreasing R_1 decreases the balancing length. Increasing the connected load resistance R_2 increases the balancing length. A potentiometer is preferred for accurate emf measurement because its null method draws no current from the unloaded cell at balance.

Quick Tip to Solve: Identify the circuit first: R_1 changes the wire gradient, while R_2 changes the measured cell's terminal voltage.

OR

17. State the underlying principle of meter bridge. Draw the circuit diagram and explain how the unknown resistance of a conductor can be determined by this method. [3 marks]

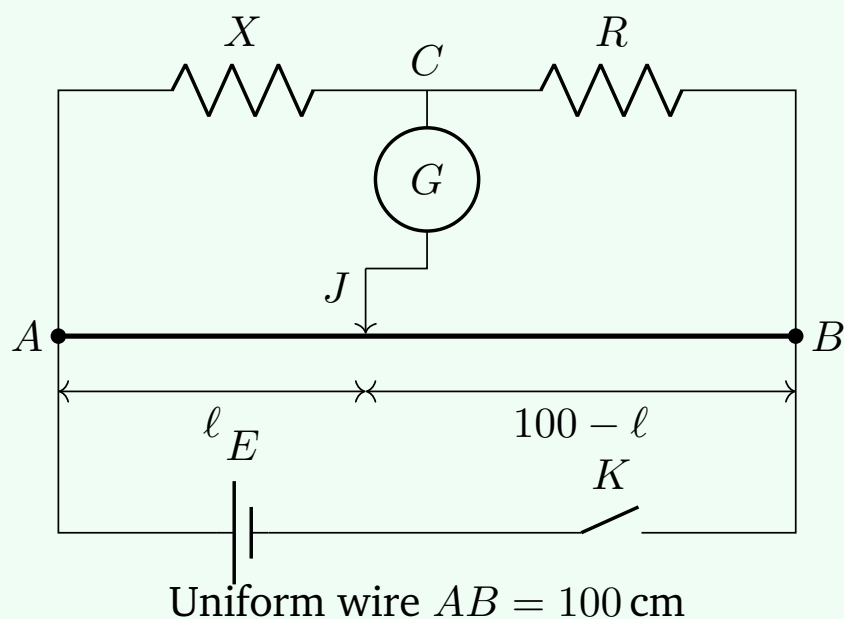
Solution:

Concept:

- A metre bridge is based on the balanced Wheatstone bridge principle. At balance, the ratios of resistances in the two arms are equal and the galvanometer carries no current.
- For a uniform wire, resistance is proportional to length: $R = \rho\ell/A$. Thus the resistances of its two portions have the ratio $\ell/(100 - \ell)$ when the lengths are measured in centimetres.

Step 1: Circuit arrangement and bridge principle Connect the unknown resistance X in the left gap and a known resistance R , obtained from a resistance box, in the right gap. The junction of X and R is C . Connect a cell and key across the ends A and B of a uniform wire of length 100 cm. Connect a galvanometer between C and a sliding jockey J on the wire. These connections form the four arms of a Wheatstone bridge: AC , CB , AJ and JB .

Quick Tip to Solve: Pair the left-gap resistance with the balance length measured from the left end before writing the bridge ratio.



Metre bridge with unknown X in the left gap and known R in the right gap; at balance, G has zero deflection.

Step 2: Obtain the balance point Close the key, choose a suitable value of R , and move the jockey gently along the wire until the galvanometer shows zero deflection. At this null point, C and J are at the same potential, so no current passes through the galvanometer. Record $AJ = \ell\text{ cm}$; then $JB = (100 - \ell)\text{ cm}$. The balanced bridge relation is

$$\frac{X}{R} = \frac{R_{AJ}}{R_{JB}}.$$

Since both wire portions have the same resistivity and cross-sectional area, their resistance ratio equals their length ratio.

Step 3: Calculate the unknown resistance Substituting the measured lengths gives

$$\frac{X}{R} = \frac{\ell}{100 - \ell}, \quad X = R \frac{\ell}{100 - \ell}.$$

Thus a known resistance and the observed balance length determine X . Choose R so that the balance point is near the middle of the wire, which improves sensitivity. For improved accuracy, interchange X and R , repeat the measurement using the corresponding relation, and average the two estimates; this reduces errors due to the resistance of the end connections. The jockey should make light contact rather than being dragged forcefully, to preserve the uniformity of the wire.

Step 4: Conclusion The metre bridge works on the null-balance principle of the Wheatstone bridge. With X in the left gap and R in the right gap, the unknown resistance is $X = R\ell/(100 - \ell)$, where ℓ is the balance length from A in centimetres.

Quick Tip to Solve: Pair the left-gap resistance with the balance length measured from the left end before writing the bridge ratio.

18. (a) Plot a graph showing the variation of potential energy of a pair of nucleons as a function of their separation. Also indicate the regions where the force is (i) attractive and (ii) repulsive.

(b) Write two characteristic properties of nuclear force.

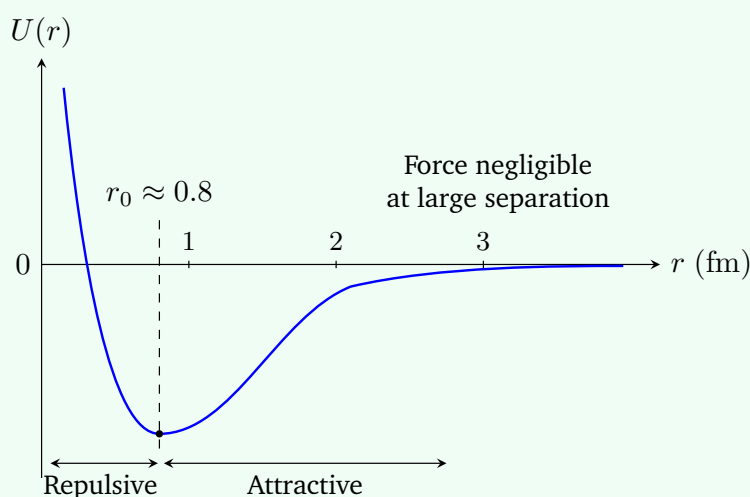
[3 marks]

Solution:

Concept:

- For nucleon separation r , the radial nuclear force is $F_r = -dU/dr$, where $U(r)$ is their potential energy and positive force increases separation.
- Set $U = 0$ at large separation. The strong nuclear interaction has a short range, with attraction outside a repulsive core.

Step 1: (a) Sketch the potential-energy curve At very small separation the potential energy is large and positive. As r increases it falls steeply, crosses zero and reaches a negative minimum at $r = r_0$, of order 0.8 fm. It then rises towards zero and becomes negligible beyond a few femtometres. The well represents a bound pair: energy must be supplied to separate the nucleons. The graph is schematic, since the precise interaction also depends on the nucleon state.



Schematic nuclear potential and the corresponding force regions.

Step 2: Identify the force from the slope To the left of the minimum, $dU/dr < 0$, hence $F_r > 0$: the force is repulsive and tends to increase separation. To the right of the minimum, within the interaction range, $dU/dr > 0$, hence $F_r < 0$: the force is attractive and tends to reduce separation. At the minimum $dU/dr = 0$; beyond the nuclear range the curve is almost flat and the force is negligible. The sign of the potential energy alone does not determine the force.

Step 3: (b) State two characteristic properties First, nuclear force is short ranged: it is appreciable only over distances of order 1–2 fm, and is negligible beyond a few femtometres. Second, it is approximately charge independent: after excluding electromagnetic effects, the nuclear interactions between proton–proton, neutron–neutron and proton–neutron pairs are approximately the same in comparable states.

Step 4: Conclusion The decreasing side of $U(r)$ is the repulsive region and the increasing side is the attractive region; the minimum separates them. Nuclear force is short ranged and approximately charge independent.

Quick Tip to Solve: Label attraction and repulsion using the slope through $F_r = -dU/dr$, not merely whether the curve lies above or below zero.

19. (a) Draw a graph showing the variation of current versus voltage in an electrolyte when an external resistance is also connected.

(b)(i) The graph between resistance (R) and temperature (T) for Hg is shown in the figure (a). Explain the behaviour of Hg near 4 K.

(ii) In which region of the graph shown in the figure (b) is resistance negative and why? [3 marks]

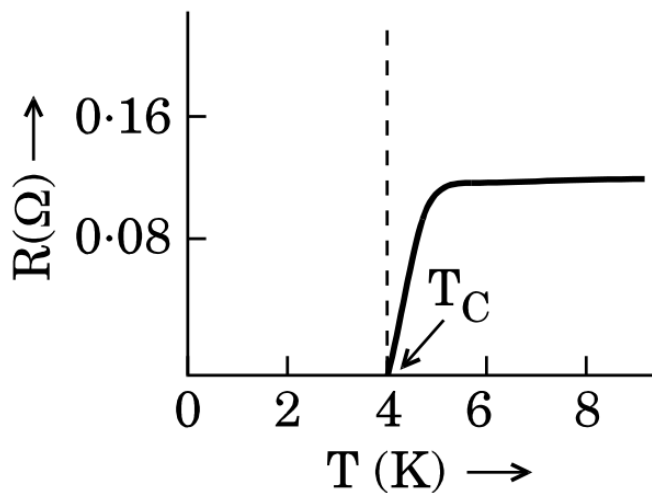


Figure (a)

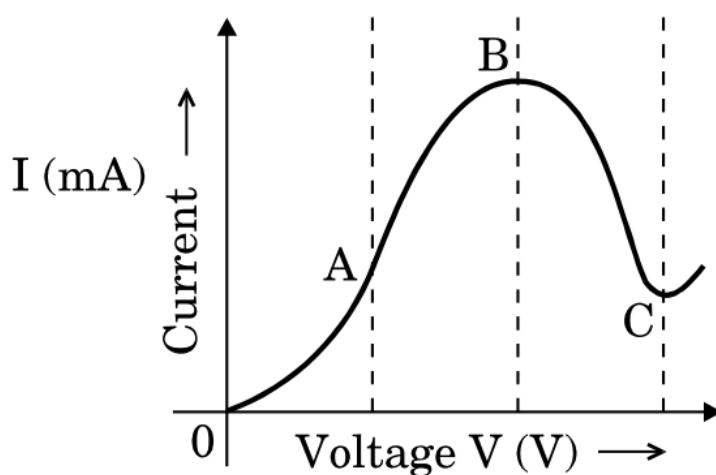


Figure (b)

Solution:

Concept:

- An electrolytic cell develops a back emf or decomposition threshold V_0 . Once conduction is established, $V = V_0 + I(R_{\text{ext}} + r)$, where r is the electrolyte resistance and R_{ext} the series resistance.
- A superconductor has zero electrical resistance below its critical temperature T_C . Differential

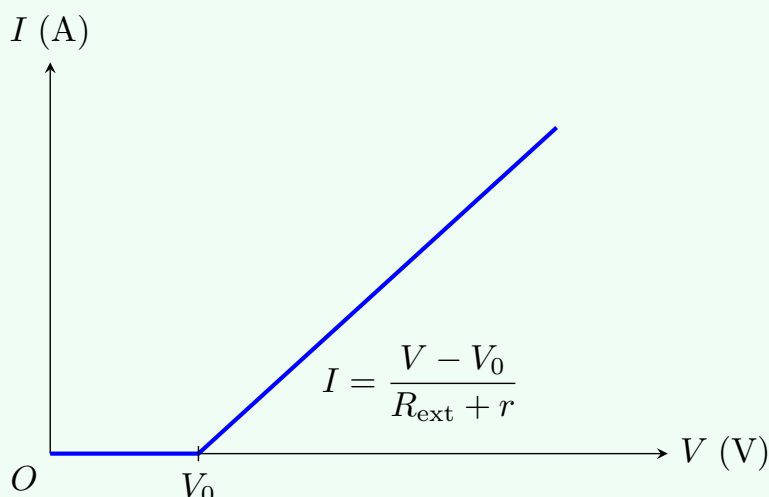
Quick Tip to Solve: For a curved current-voltage graph, identify negative resistance where current falls as voltage rises; use the slope, not the positive ratio V/I .

resistance is $r_d = dV/dI$, determined by the local slope of the characteristic.

Step 1: (a) Draw the electrolyte characteristic Put applied voltage V on the horizontal axis and current I on the vertical axis. In the idealized steady-current graph, appreciable electrolysis current starts only after the threshold V_0 . Beyond this threshold,

$$I = \frac{V - V_0}{R_{\text{ext}} + r}.$$

Thus the rising straight portion cuts the voltage axis at V_0 , not at the origin. Its slope is $1/(R_{\text{ext}} + r)$; increasing the external series resistance makes it less steep. The sketch shows the usual positive-voltage operating branch, neglecting the very small pre-threshold current.



Idealized electrolyte current versus applied voltage with a series resistance.

Step 2: (b)(i) Explain the transition of mercury On cooling mercury towards its critical temperature, approximately 4 K in the given graph, its resistance abruptly drops to zero. Below this temperature it is superconducting, so a current can flow without resistive energy loss. Above T_C , mercury is a normal conductor with finite resistance. This abrupt transition is different from the gradual resistance change of an ordinary metal.

Step 3: (b)(ii) Identify the negative-resistance region In figure (b), in the region from B to C , current decreases as the applied voltage increases. Therefore $dI/dV < 0$, and the differential resistance $r_d = dV/dI$ is negative. This is the negative-resistance region asked for. The static ratio V/I remains positive because both plotted coordinates are positive; it is the incremental response that is negative. The portions preceding B and following C have positive slope.

Step 4: Conclusion The electrolyte characteristic begins rising at a nonzero threshold voltage V_0 . Mercury becomes superconducting near 4 K, and the given characteristic has negative differential resistance in region BC .

Quick Tip to Solve: For a curved current-voltage graph, identify negative resistance where current falls as voltage rises; use the slope, not the positive ratio V/I .

20. (a) Briefly explain how a galvanometer is converted into an ammeter.

(b) A galvanometer coil has a resistance of $15\ \Omega$ and it shows full scale deflection for a current of 4 mA. Convert it into an ammeter of range 0 to 6 A. [3 marks]

Solution:

Concept:

- A galvanometer of resistance G reaches full-scale deflection at current I_g . A low shunt resistance S , connected in parallel, diverts the remaining current.
- Parallel branches have the same potential difference: $I_g G = (I - I_g)S$, where I is the desired full-scale ammeter current.

Step 1: (a) Explain the conversion Connect a suitable small resistance, called a shunt, in parallel with the galvanometer coil. When the total current is large, most of it passes through the shunt while only a small fraction flows through the delicate coil. Calibrate the deflection scale to indicate the total current rather than the coil current. The parallel combination has a very small effective resistance and is connected in series with the circuit whose current is to be measured, causing little change in that current.

Step 2: (b) Convert the full-scale current and find the shunt current The given values are $G = 15\ \Omega$, $I_g = 4\ \text{mA} = 0.004\ \text{A}$ and desired range $I = 6\ \text{A}$. At the top of this range, the galvanometer must still carry only its original full-scale current. The shunt therefore carries

$$I_s = I - I_g = 6 - 0.004 = 5.996\ \text{A}.$$

Step 3: Equate the branch voltages and calculate the resistance The potential difference across the coil at full scale is $I_g G = 0.004 \times 15 = 0.060\ \text{V}$. The same voltage appears across the shunt, so

$$\begin{aligned} S &= \frac{I_g G}{I - I_g} \\ &= \frac{0.004 \times 15}{6 - 0.004} \\ &= \frac{0.060}{5.996}\ \Omega = 0.0100067\ \Omega. \end{aligned}$$

As a check, the effective resistance is $R_A = 0.060/6 = 0.0100\ \Omega$, much smaller than the original coil resistance. Thus the instrument can measure the specified total current without sending that large current through the coil.

Step 4: Conclusion Connect a shunt of approximately $1.00 \times 10^{-2}\ \Omega$ in parallel with the galvanometer and calibrate the instrument for 0–6 A.

Quick Tip to Solve: Convert the galvanometer current from milliamperes to amperes first; the shunt carries $I - I_g$, not the total current I .

20. (a) Briefly explain how a galvanometer is converted into a voltmeter.

(b) A voltmeter of a certain range is constructed by connecting a resistance of $980\ \Omega$ in series with a galvanometer. When the resistance of $470\ \Omega$ is connected in series, the range gets halved. Find the resistance of the galvanometer. [3 marks]

Solution:

Concept:

- A galvanometer of resistance G reaches full-scale deflection at current I_g . A voltmeter measures the potential difference across two points and is connected in parallel with them.
- A large series resistance S limits the galvanometer current. Its full-scale voltage range is $V = I_g(G + S)$, by Ohm's law.

Step 1: (a) Add a high resistance in series Connect a suitable high resistance S in series with the galvanometer and calibrate its scale in volts. Connect this entire combination across the points whose potential difference is to be measured. For a required maximum reading V , the series resistance is

$$S = \frac{V}{I_g} - G.$$

The large total resistance makes the instrument draw a small current, reducing its effect on the circuit. At full scale, the same current I_g passes through both the galvanometer and its series resistance.

Step 2: (b) Write the two voltage ranges Let the original voltage range be V . Since the same galvanometer is used in both arrangements, its resistance G and full-scale current I_g are unchanged. The two ranges therefore satisfy

$$\begin{aligned} V &= I_g(G + 980\ \Omega), \\ \frac{V}{2} &= I_g(G + 470\ \Omega). \end{aligned}$$

The resistance of the galvanometer must be included in both total resistances; halving the external resistance alone would not halve the voltage range.

Step 3: Eliminate the full-scale current Divide the first equation by the second and rearrange:

$$\begin{aligned} 2 &= \frac{G + 980\ \Omega}{G + 470\ \Omega}, \\ 2G + 940\ \Omega &= G + 980\ \Omega, \\ G &= 40\ \Omega. \end{aligned}$$

As a check, the two total resistances become $1020\ \Omega$ and $510\ \Omega$, which have the required ratio $2 : 1$.

Step 4: Conclusion Convert the galvanometer into a voltmeter by adding a high resistance in series. The galvanometer resistance is $G = 40\ \Omega$.

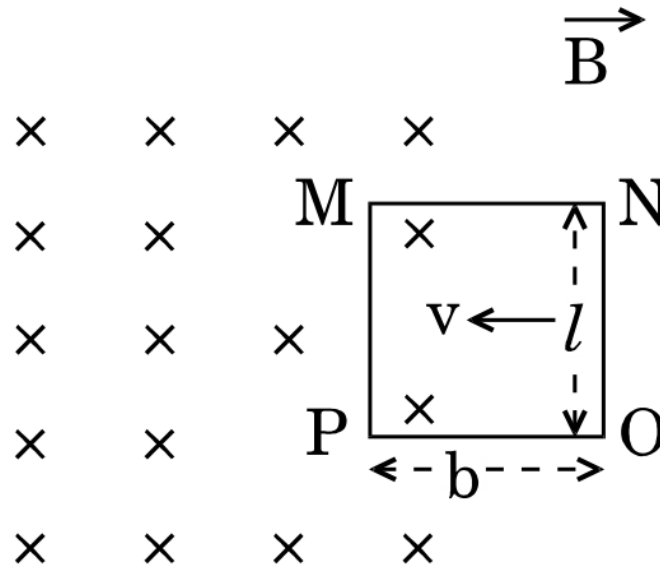
Quick Tip to Solve: For the same galvanometer, voltage range is proportional to total resistance: compare $G + S$, not just the added resistance S .

21. The figure shows a rectangular conducting frame MNOP of resistance R placed partly in a perpendicular magnetic field $\vec{B}$ and moved with velocity $\vec{v}$ as shown in the figure. Obtain the expressions for the

(a) force acting on the arm 'ON' and its direction, and

(b) power required to move the frame to get a steady emf induced between the arms MN and PO.

[3 marks]



Solution:

Concept:

- Faraday's law gives the induced emf magnitude $\mathcal{E} = |d\Phi_B/dt|$. For a straight active length l moving perpendicular to a uniform field, $\mathcal{E} = Blv$.
- A conductor carrying current I experiences $\vec{F} = I\vec{l} \times \vec{B}$. Lenz's law makes the net magnetic force oppose the motion that changes the flux.
- At steady speed, the applied mechanical power equals the Joule heating: $P = F_{\text{ext}}v = I^2R$, where R is the resistance of the complete frame.

Step 1: Identify the field region in the printed figure The crosses denote a magnetic field into the page. As drawn, the left arm MP lies in the field, while the right arm ON lies outside it. The frame moves left, so the area within the field increases. If x is the width currently immersed, the flux magnitude is Blx , and $dx/dt = v$. Hence

$$\mathcal{E} = Blv, \quad I = \frac{Blv}{R}.$$

The increasing inward flux induces an outward magnetic field, so the induced current is anticlockwise: $M \rightarrow P \rightarrow O \rightarrow N \rightarrow M$.

Step 2: (a) Distinguish the named arm from the force on the frame Taken literally with the printed figure, the named arm ON is outside the magnetic field. Its magnetic force is therefore $F_{ON} = 0$, with no direction. The nonzero horizontal retarding force acts on the immersed arm MP . Current there is downward; with the field into the page, $I\vec{l} \times \vec{B}$ points right, opposite the leftward velocity. Its

Quick Tip to Solve: First check which arm is inside the crosses. Use $\mathcal{E} = Blv$, then $I = \mathcal{E}/R$; apply BIl only to a conductor actually in the field.

magnitude is

$$F_{MP} = BIl = \frac{B^2 l^2 v}{R}.$$

Magnetic forces on the immersed portions of the two horizontal arms are equal and opposite, so their vertical contributions cancel. Thus $B^2 l^2 v / R$ rightward is the net drag on the frame. If the question's reference to ON was intended to name the immersed arm, this is the corresponding usual retarding-force result; the printed labels themselves have been retained here.

Step 3: (b) Calculate the power needed for steady motion An external force of the same magnitude must act leftward to balance the magnetic drag. For constant B , l , and v , the induced emf remains steady while the frame is partly inside the field. Thus

$$P = F_{\text{ext}} v = \frac{B^2 l^2 v^2}{R},$$
$$I^2 R = \left(\frac{Blv}{R} \right)^2 R = \frac{B^2 l^2 v^2}{R}.$$

The agreement expresses conservation of energy: mechanical work becomes heat in the resistance. Once the frame is wholly inside the uniform field, its flux stops changing and this induced current ceases.

Step 4: Conclusion For the printed geometry, $F_{ON} = 0$. The retarding force on the immersed arm MP , and hence on the frame, is $B^2 l^2 v / R$ rightward. The required mechanical power is $P = B^2 l^2 v^2 / R$.

Quick Tip to Solve: First check which arm is inside the crosses. Use $\mathcal{E} = Blv$, then $I = \mathcal{E}/R$; apply BIl only to a conductor actually in the field.

22. (a) State briefly, with what purpose was Davisson and Germer experiment performed and what inference was drawn from this.

(b) Obtain an expression for the ratio of the accelerating potentials required to accelerate a proton and an α -particle to have the same de-Broglie wavelength associated with them. [3 marks]

Solution:

Concept:

- The de Broglie relation $\lambda = h/p$ associates a wavelength λ with a particle of momentum p ; h is Planck's constant. Diffraction provides evidence of wave behaviour.
- A nonrelativistic particle of mass m and charge magnitude q , accelerated from rest through a potential difference V , gains kinetic energy $qV = p^2/(2m)$. Therefore $\lambda = h/\sqrt{2mqV}$.

Step 1: (a) State the purpose and inference of the experiment Davisson and Germer investigated the scattering of a beam of electrons by a nickel crystal. The experiment supplied an experimental test of the wave nature of moving electrons predicted by de Broglie. The intensity of scattered electrons showed pronounced maxima at particular angles, characteristic of diffraction by the regularly spaced crystal planes. The wavelength inferred from the diffraction agreed with the de Broglie wavelength

Quick Tip to Solve: At equal de Broglie wavelength, momentum is fixed and mqV is constant; include both the alpha particle's fourfold mass and twofold charge.

calculated from the accelerating potential. Thus electrons exhibit wave behaviour, and de Broglie's matter-wave hypothesis was experimentally verified.

Step 2: (b) Express the wavelengths in terms of the potentials Let the proton and alpha-particle accelerating potentials be V_p and V_α . Their charge magnitudes are $q_p = e$ and $q_\alpha = 2e$, and the alpha-particle mass is approximately $m_\alpha = 4m_p$. Using energy conservation for acceleration from rest,

$$\lambda_p = \frac{h}{\sqrt{2m_p e V_p}}, \quad \lambda_\alpha = \frac{h}{\sqrt{2m_\alpha (2e) V_\alpha}}.$$

The nonrelativistic expression is appropriate to the usual board-level accelerating-potential relation.

Step 3: Equate the wavelengths and find the ratio Equal wavelengths require equal momenta, so the squared denominators in the two expressions must be equal:

$$m_p e V_p = m_\alpha (2e) V_\alpha, \\ \frac{V_p}{V_\alpha} = \frac{m_\alpha (2e)}{m_p e} = \frac{(4m_p)(2e)}{m_p e} = 8.$$

For the same momentum, the heavier alpha particle needs one quarter of the proton's kinetic energy. Since its charge is twice as large, the required accelerating potential is one eighth of the proton's.

Step 4: Conclusion Davisson and Germer observed electron diffraction, confirming the wave nature of electrons and the de Broglie relation. Equal de Broglie wavelengths require $V_p : V_\alpha = 8 : 1$, taking $m_\alpha \simeq 4m_p$.

Quick Tip to Solve: At equal de Broglie wavelength, momentum is fixed and mqV is constant; include both the alpha particle's fourfold mass and twofold charge.

OR

22. (a) An electron and a proton are accelerated through the same potential. Which one of the two has

- (i) greater value of de-Broglie wavelength associated with it, and
- (ii) lesser momentum?

Justify your answer in each case.

(b) How is the momentum of a particle related with its de-Broglie wavelength? Show the variation on a graph. [3 marks]

Solution:

Concept:

- The de-Broglie wavelength of a particle is $\lambda = h/p$, where h is Planck's constant and p is the magnitude of its momentum.
- A particle of charge magnitude e , accelerated from rest through potential difference V , gains kinetic energy eV . In the non-relativistic treatment, $p = \sqrt{2meV}$.

Quick Tip to Solve: For equal accelerating potentials and equal charge magnitudes, use $p \propto \sqrt{m}$ and $\lambda \propto 1/\sqrt{m}$; do not assume equal momenta from equal energies.

Step 1: Compare the momenta at equal accelerating potential (a)(ii) The electron and proton have equal charge magnitudes, so the same accelerating potential gives them equal kinetic energies, although their speeds are different. Using $K = p^2/(2m) = eV$,

$$p_e = \sqrt{2m_e eV}, \quad p_p = \sqrt{2m_p eV}, \quad \frac{p_e}{p_p} = \sqrt{\frac{m_e}{m_p}}.$$

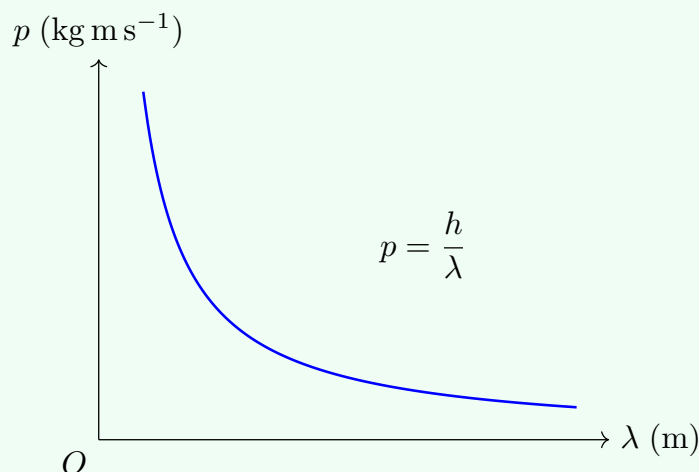
Since $m_e < m_p$, the electron has the lesser momentum. Equal kinetic energy does not mean equal momentum: at fixed kinetic energy, momentum increases with the square root of mass.

Step 2: Compare the associated wavelengths (a)(i) Substitute the momentum into the de-Broglie relation:

$$\lambda = \frac{h}{\sqrt{2m_e eV}}, \quad \frac{\lambda_e}{\lambda_p} = \sqrt{\frac{m_p}{m_e}} > 1.$$

Thus the electron has the greater wavelength. The smaller electron mass gives smaller momentum for the same energy gain, and wavelength is inversely proportional to that momentum.

Step 3: Plot momentum against wavelength (b) Rearranging the de-Broglie relation gives $p = h/\lambda$, or $p\lambda = h$. Therefore a graph of p on the vertical axis against λ on the horizontal axis is a decreasing rectangular hyperbola in the first quadrant. Doubling the wavelength halves the momentum. Both quantities are positive, and the curve approaches the axes asymptotically without meeting them at any finite positive value.



Momentum decreases inversely with de-Broglie wavelength (schematic).

Step 4: Conclusion At the same accelerating potential, the **electron has the greater de-Broglie wavelength and the lesser momentum**. The momentum–wavelength relation is $p = h/\lambda$, represented by a rectangular hyperbola.

Quick Tip to Solve: For equal accelerating potentials and equal charge magnitudes, use $p \propto \sqrt{m}$ and $\lambda \propto 1/\sqrt{m}$; do not assume equal momenta from equal energies.

23. (a) What is amplitude modulation? Draw a diagram showing an amplitude modulated wave obtained by modulation of a carrier sinusoidal wave on a modulating signal.

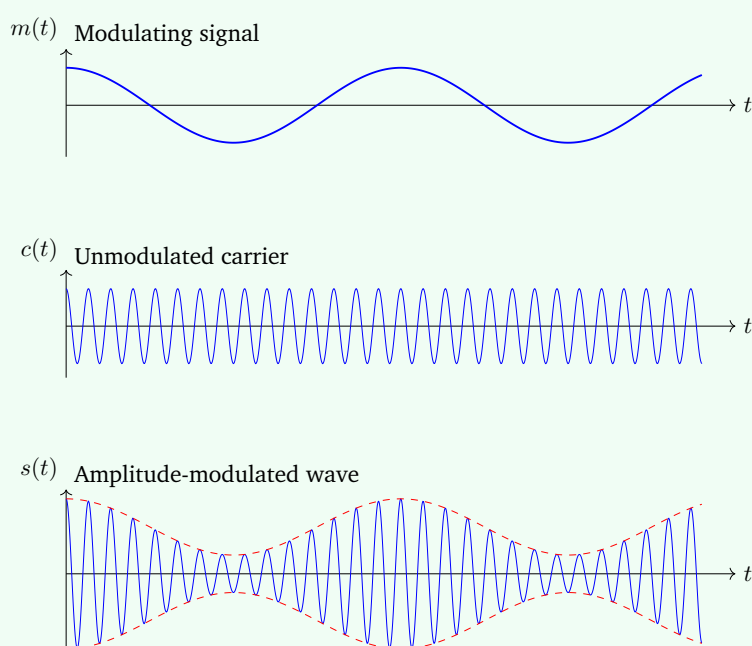
(b) Define the terms (i) modulation index, and (ii) side bands. Mention the significance of side bands. [3 marks]

Solution:

Concept:

- In amplitude modulation (AM), the amplitude of a high-frequency carrier varies according to the instantaneous value of a low-frequency information signal; the carrier frequency and phase remain unchanged.
- For a single sinusoidal signal, an AM wave is $s(t) = A_c[1 + \mu \cos(2\pi f_m t)] \cos(2\pi f_c t)$, where A_c is the unmodulated carrier amplitude, f_m is the signal frequency, $f_c \gg f_m$ is the carrier frequency and μ is the modulation index.

Step 1: Describe the modulated wave and its envelope (a) Amplitude modulation makes the carrier amplitude rise and fall with the modulating signal. The closely spaced oscillations in the diagram are the carrier oscillations, while their slowly varying envelope reproduces the shape of the information signal. For ordinary undistorted AM, $0 \leq \mu \leq 1$. The diagram illustrates $\mu = 0.6$; dashed curves indicate the upper and lower envelopes.



Sinusoidal message, carrier and resulting amplitude-modulated wave; amplitudes are schematic.

Step 2: Define and measure the modulation index (b)(i) The modulation index is the ratio of the amplitude of the modulating signal to that of the unmodulated carrier, when both amplitudes are expressed on the same modulation scale:

$$\mu = \frac{A_m}{A_c} = \frac{A_{\max} - A_{\min}}{A_{\max} + A_{\min}}.$$

Here $A_{\max} = A_c + A_m$ and $A_{\min} = A_c - A_m$ are the maximum and minimum amplitudes of the AM

Quick Tip to Solve: Read the message from the slow AM envelope. Locate the sidebands by subtracting and adding the message frequency to the carrier frequency.

envelope. Thus μ measures the depth of modulation; $\mu = 1$ corresponds to 100 percent modulation.

Step 3: Identify the sidebands and their significance (b)(ii) Using the product-to-sum identity, the modulated wave becomes

$$\begin{aligned}s(t) &= A_c \cos(2\pi f_c t) \\ &+ \frac{\mu A_c}{2} \cos[2\pi(f_c + f_m)t] \\ &+ \frac{\mu A_c}{2} \cos[2\pi(f_c - f_m)t].\end{aligned}$$

Sidebands are the frequency components generated on either side of the carrier by modulation. For a single sinusoidal signal, the upper sideband is at $f_c + f_m$ and the lower sideband is at $f_c - f_m$. For a band of signal frequencies, each becomes a frequency band. The sidebands carry the information about the message; the unmodulated carrier alone contains no message. Each sideband contains the message information, and the total AM bandwidth is $2f_m$ for a single tone, or twice the highest modulating frequency in general.

Step 4: Conclusion AM varies the carrier amplitude according to the message. Its modulation index is $\mu = A_m/A_c$, and its information-bearing sidebands occur at $f_c - f_m$ and $f_c + f_m$ for sinusoidal modulation.

Quick Tip to Solve: Read the message from the slow AM envelope. Locate the sidebands by subtracting and adding the message frequency to the carrier frequency.

24. Draw the energy band diagram of (i) n-type, and (ii) p-type semiconductors at temperature $T > 0$ K.

In the case of n-type Si-semiconductor, the donor energy level is slightly below the bottom of conduction band whereas in p-type semiconductor, the acceptor energy level is slightly above the top of valence band. Explain, giving examples, what role do these energy levels play in conduction and valence bands. [3 marks]

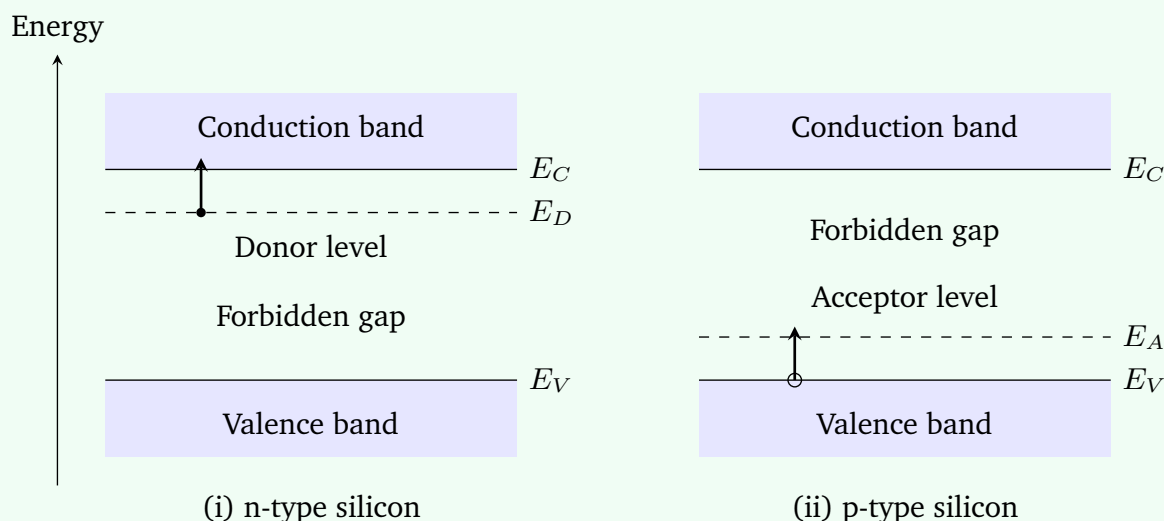
Solution:

Concept:

- The conduction-band edge E_C and valence-band edge E_V enclose the forbidden energy gap. Impurities introduce permitted energy levels inside this gap.
- A donor level E_D lies just below E_C , while an acceptor level E_A lies just above E_V . Thus the required excitation energy is much smaller than the full band gap.

Step 1: Show the impurity levels in the band diagrams Energy increases upwards in both diagrams. For n-type silicon, a donor electron can acquire thermal energy and enter the conduction band across the small separation $E_C - E_D$. For p-type silicon, a valence electron can acquire thermal energy and enter an acceptor level across $E_A - E_V$, leaving a mobile hole in the valence band. The upward arrows show electron excitation, not motion of the impurity atoms.

Quick Tip to Solve: Place the donor level near the conduction band and the acceptor level near the valence band; in both diagrams the electron excitation arrow points upwards.



Energy bands for doped silicon at $T > 0\text{ K}$; small upward arrows represent thermal excitation of electrons.

Step 2: Explain conduction in n-type silicon Doping silicon with a pentavalent impurity such as phosphorus or arsenic supplies five valence electrons per impurity atom. Four form covalent bonds with neighbouring silicon atoms; the fifth is weakly bound and corresponds to the donor level. At a suitable nonzero temperature, thermal energy promotes this electron into the conduction band, leaving a fixed positive donor ion. Electrons are the majority carriers and holes are the minority carriers. The shallow donor level therefore increases conduction-band electron concentration without requiring excitation across the whole forbidden gap.

Step 3: Explain conduction in p-type silicon A trivalent impurity such as boron, aluminium or gallium has only three valence electrons, leaving one silicon bond deficient in an electron. Its acceptor level can receive an electron from the valence band with little thermal energy. The impurity becomes a fixed negative ion, while the missing valence electron produces a hole. Successive valence electrons filling this vacancy make the hole mobile; holes are the majority carriers and electrons the minority carriers. The impurity ions themselves remain bound to the lattice. At very low temperatures ionisation can be incomplete, so $T > 0$ does not mean that every impurity is ionised.

Step 4: Conclusion Donor levels just below E_C supply conduction electrons in n-type silicon (for example Si doped with P); acceptor levels just above E_V create valence-band holes in p-type silicon (for example Si doped with B). Their shallow positions make thermal generation of the majority carriers easy.

Quick Tip to Solve: Place the donor level near the conduction band and the acceptor level near the valence band; in both diagrams the electron excitation arrow points upwards.

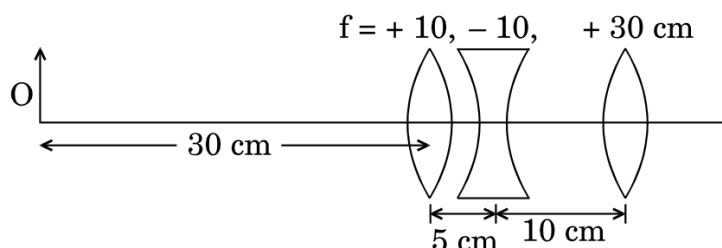
Section D (15 marks)

25. (a) Using the ray diagram for a system of two lenses of focal lengths f_1 and f_2 in contact with each other, show that the two lens system can be regarded as equivalent to a single lens of focal length f , where

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}.$$

Also write the relation for the equivalent power of the lens combination.

(b) Determine the position of the image formed by the lens combination given in the figure. [5 marks]



Solution:

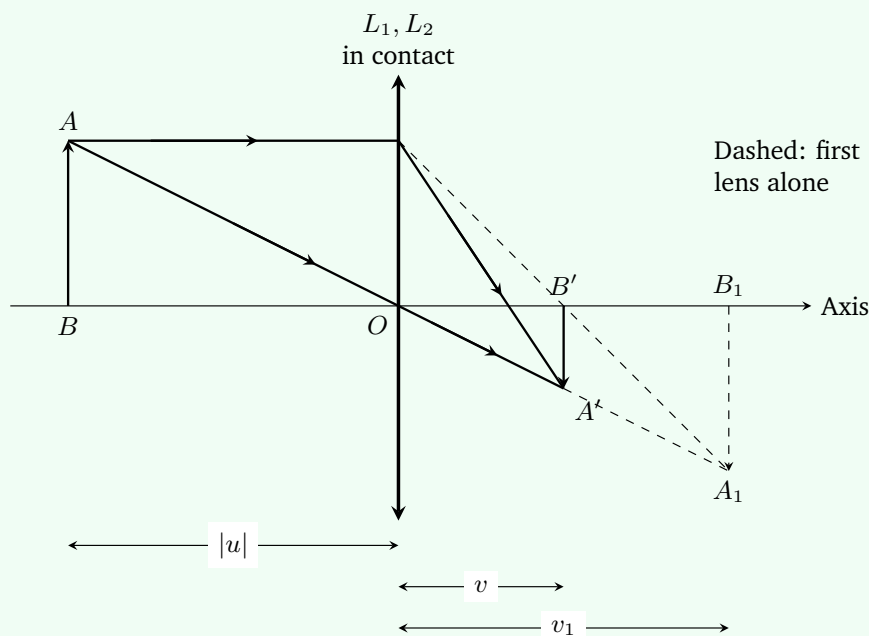
Concept:

- For a thin lens in air, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$, where all distances are measured from its optical centre using the Cartesian sign convention, positive in the direction of incident light.
- The image formed by the first lens acts as the object for the next lens. Thin lenses in contact have a common optical-centre plane; separated lenses require a new object distance at each lens.
- Lens power is $P = 1/f$, with f in metres and P in dioptries (D).

Step 1: (a) Trace the rays and apply the first lens formula Let two thin lenses L_1, L_2 of focal lengths f_1, f_2 be in contact at O . An object AB is at distance u . Without the second lens, the first lens would form A_1B_1 at v_1 ; the dotted ray in the diagram shows this intermediate image. The second lens changes the convergence to form the final image $A'B'$ at v . For the first lens,

$$\frac{1}{v_1} - \frac{1}{u} = \frac{1}{f_1}.$$

Quick Tip to Solve: After each separated lens, subtract the lens separation from its image distance to obtain the next object distance. Converging incident rays mean a positive virtual-object distance.



Two thin lenses in contact at O , shown at a common plane. The dashed path gives the first lens's image; solid paths give the final image.

Step 2: (a) Apply the second lens formula and add Since lens separation is negligible, the object distance for the second lens is $u_2 = v_1$, including its sign. In the drawn case, rays incident on L_2 are converging towards A_1B_1 , so this is a virtual object to its right. Hence

$$\frac{1}{v} - \frac{1}{v_1} = \frac{1}{f_2}.$$

Adding the two equations eliminates the intermediate-image distance:

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f_1} + \frac{1}{f_2}.$$

A single equivalent lens forming the same final image from the same object obeys $1/v - 1/u = 1/f$. Comparing gives

$$\boxed{\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}}, \quad \boxed{P = P_1 + P_2}.$$

Diverging lenses contribute negative focal lengths and negative powers; power addition applies algebraically.

Step 3: (b) Find the image produced by the first lens The figure gives $f_1 = +10$ cm, $f_2 = -10$ cm, $f_3 = +30$ cm, with separations 5 cm and 10 cm. These lenses are separated, so the contact formula from part (a) cannot be applied directly. For the first lens, the object is on the left, giving $u_1 = -30$ cm:

$$\frac{1}{v_1} = \frac{1}{10} + \frac{1}{-30} = \frac{1}{15} \text{ cm}^{-1}, \quad v_1 = +15 \text{ cm}.$$

Thus, without the remaining lenses, the first image would be 15 cm to the right of the first lens.

Step 4: (b) Use the virtual object for the concave lens The concave lens is 5 cm to the right of the first

Quick Tip to Solve: After each separated lens, subtract the lens separation from its image distance to obtain the next object distance. Converging incident rays mean a positive virtual-object distance.

lens, so the incoming rays converge towards a point $15 - 5 = 10$ cm to its right. Its object is virtual, and $u_2 = +10$ cm, not negative. Therefore

$$\frac{1}{v_2} = \frac{1}{f_2} + \frac{1}{u_2} = -\frac{1}{10} + \frac{1}{10} = 0.$$

The image is at infinity: rays emerging from the second lens are parallel.

Step 5: (b) Focus the parallel rays with the third lens The third lens receives the parallel beam even though it is 10 cm beyond the second lens. Its object is effectively at infinity, so $1/u_3 = 0$. Thus

$$\frac{1}{v_3} = \frac{1}{30} + 0, \quad v_3 = +30 \text{ cm}.$$

The positive image distance places the real final image on the right of the third lens. Its distance from the first lens is $5 + 10 + 30 = 45$ cm.

Step 6: Conclusion (a) For two thin lenses in contact, $\boxed{1/f = 1/f_1 + 1/f_2}$ and $\boxed{P = P_1 + P_2}$. (b) The final real image is $\boxed{30 \text{ cm}}$ to the right of the third lens, or 45 cm to the right of the first lens.

Quick Tip to Solve: After each separated lens, subtract the lens separation from its image distance to obtain the next object distance. Converging incident rays mean a positive virtual-object distance.

OR

- 25. (a) Explain, using a suitable diagram, how unpolarized light gets linearly polarized by scattering.**
(b) Describe briefly the variation of the intensity of transmitted light when a polaroid sheet kept between two crossed polaroids is rotated. Draw the graph depicting the variation of intensity with the angle of rotation. How many maxima and minima would be observed when θ varies from 0 to π ? [5 marks]

Solution:

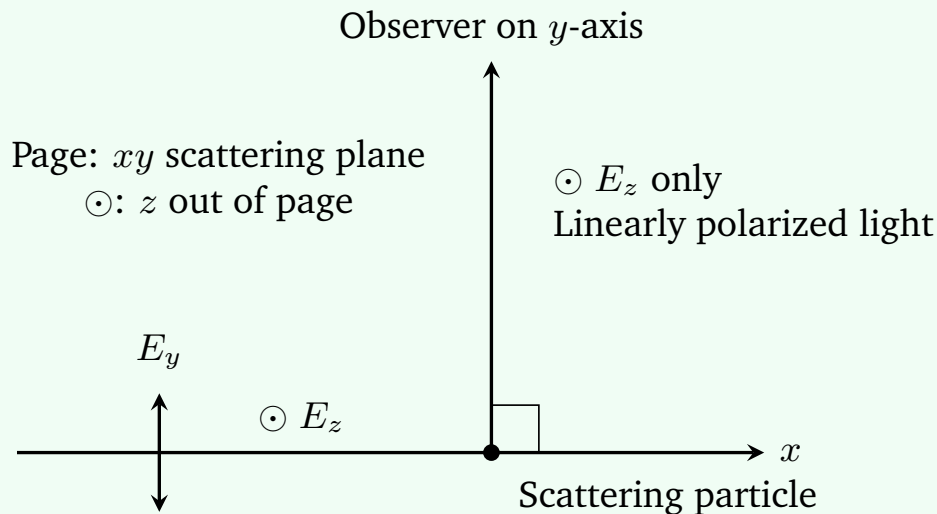
Concept:

- Unpolarized light has electric-field vibrations in all directions perpendicular to its propagation. Scattering by small particles makes their electrons oscillate and re-radiate light.
- An oscillating electric dipole emits no radiation along its oscillation axis. Light scattered through 90° is therefore linearly polarized perpendicular to the scattering plane.
- Malus's law is $I_{\text{out}} = I_{\text{in}} \cos^2 \phi$, where ϕ is the angle between the incident polarization and the transmission axis of an ideal polaroid.

Step 1: (a) Identify the surviving vibration in scattered light Let an unpolarized beam travel along the x -axis and let the observer receive light scattered along the y -axis, at right angles to the incident beam. The scattering plane is the xy -plane. Resolve the incident electric-field vibrations into components along y and z ; these drive the electrons of a small scattering particle in the corresponding directions. An electron oscillating along y cannot radiate towards the observer on the y -axis. The electron's z -oscillation does radiate towards this observer. Thus only the electric-field component along z ,

Quick Tip to Solve: Apply Malus's law twice: multiply $\cos^2 \theta$ by $\sin^2 \theta$. Define the reference intensity before counting the peaks, and include both endpoints when counting zero-intensity orientations.

perpendicular to the scattering plane, is present in the received beam. The light scattered through 90° is linearly polarized. In the diagram, a circled dot represents the z -direction out of the page; the page is the scattering plane. At other scattering angles the light is generally only partially polarized.



Incident unpolarized light

Scattering at right angles: only the out-of-page electric-field vibration reaches the observer.

Step 2: (b) Apply Malus's law at the rotating sheet and the analyser Let the crossed outer polaroids be P and A , with transmission axes along z and y , respectively. Define I_0 as the intensity of the plane-polarized light immediately after P . Insert a middle sheet M whose axis makes angle θ with that of P . The intensity after M is

$$I_M = I_0 \cos^2 \theta.$$

The transmitted light is now polarized along the axis of M . Since the axis of A is perpendicular to that of P , its squared projection factor is $\sin^2 \theta$. Consequently,

$$\begin{aligned} I(\theta) &= I_M \sin^2 \theta \\ &= I_0 \cos^2 \theta \sin^2 \theta = \frac{I_0}{4} \sin^2(2\theta). \end{aligned}$$

Insertion of the middle sheet permits transmission except when its axis coincides with either outer axis. If the original unpolarized intensity before P is I_u , then $I_0 = I_u/2$ and the same result is $I = (I_u/8) \sin^2(2\theta)$.

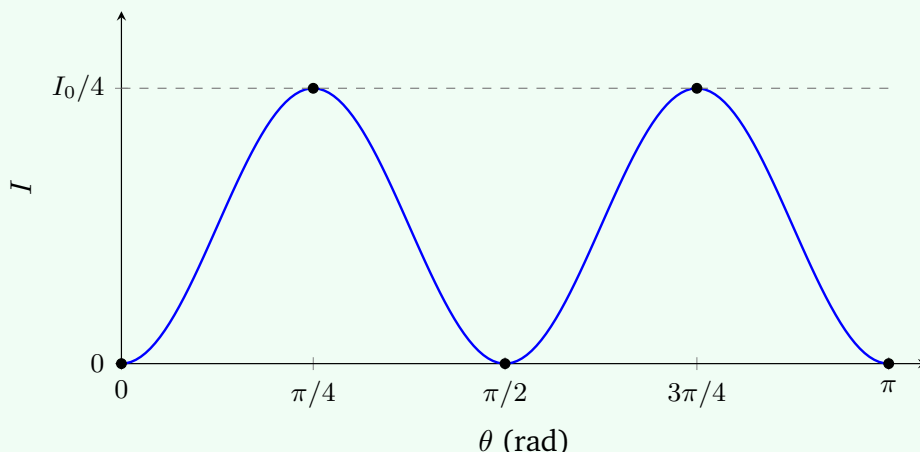
Step 3: Locate the maxima and minima on the intensity graph The intensity has period $\pi/2$ in θ . It rises from zero to a maximum, returns to zero, and repeats this variation over the second half of the stated interval. Maxima occur when $\sin^2(2\theta) = 1$, while minima occur when $\sin(2\theta) = 0$:

$$\begin{aligned} I_{\max} &= I_0/4, & \theta &= \pi/4, 3\pi/4; \\ I_{\min} &= 0, & \theta &= 0, \pi/2, \pi. \end{aligned}$$

Both endpoints are included in the requested sweep from 0 to π , so the zero intensity is observed at

Quick Tip to Solve: Apply Malus's law twice: multiply $\cos^2 \theta$ by $\sin^2 \theta$. Define the reference intensity before counting the peaks, and include both endpoints when counting zero-intensity orientations.

the starting orientation, at the middle orientation, and at the final orientation.



Transmitted intensity for a middle polaroid rotated through $0 \leq \theta \leq \pi$.

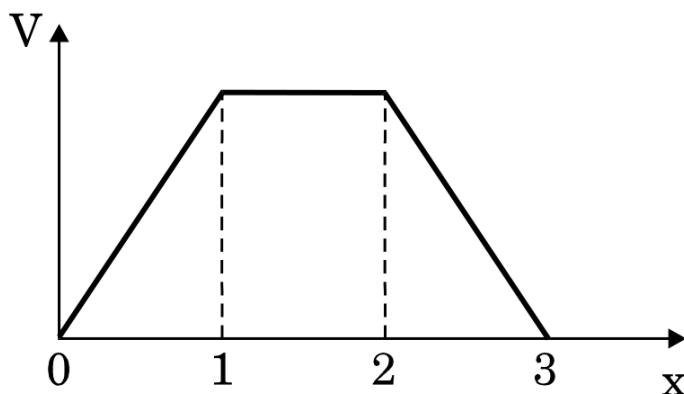
Step 4: Conclusion At 90° scattering, the light is linearly polarized perpendicular to the scattering plane. With the middle polaroid rotating between crossed outer polaroids, $I = (I_0/4) \sin^2(2\theta)$, where I_0 is the intensity after the first polaroid. From 0 to π , there are **two maxima and three minima**, including the endpoints.

Quick Tip to Solve: Apply Malus's law twice: multiply $\cos^2 \theta$ by $\sin^2 \theta$. Define the reference intensity before counting the peaks, and include both endpoints when counting zero-intensity orientations.

26. (a) When a parallel plate capacitor is connected across a dc battery, explain briefly how the capacitor gets charged.

(b) A parallel plate capacitor of capacitance C is charged to V volt by a battery. After some time the battery is disconnected and the distance between the plates is doubled. Now a slab of dielectric constant $1 < k < 2$ is introduced to fill the space between the plates. How will the following be affected: (i) the electric field between the plates, (ii) the energy stored in the capacitor? Justify your answer in each case.

(c) The electric potential as a function of distance x is shown in a graph. Draw a graph of the electric field E as a function of x . [5 marks]



Solution:

Concept:

- A battery transfers electrons between capacitor plates until the potential difference of the capacitor equals the battery voltage; the final charges are equal and opposite, with magnitude $Q = CV$.
- For a parallel plate capacitor of area A , separation d , and vacuum between the plates, $C = \epsilon_0 A/d$ and $E = Q/(\epsilon_0 A)$. A dielectric filling the gap replaces ϵ_0 by $k\epsilon_0$.
- An isolated capacitor retains its free charge. Its energy is $U = Q^2/(2C)$, while the electric field along x is the negative potential gradient, $E_x = -dV/dx$.

Step 1: (a) Explain charging by the battery Initially the uncharged plates have no potential difference. The battery removes electrons from the plate connected to its positive terminal and supplies electrons to the plate connected to its negative terminal. The plates therefore acquire charges $+Q$ and $-Q$, and an electric field builds up in the gap. Electrons travel through the connecting wires; they do not cross the insulating gap between the plates. As charge accumulates, the capacitor potential difference $V_C = Q/C$ increases and opposes further transfer by the battery. The charging current consequently decreases. Charging is complete when V_C equals the battery voltage V , giving $Q = CV$. In steady state there is no charging current in the ideal dc circuit.

Step 2: (b) Determine the final capacitance and conserved charge Let the original separation be d . Initially,

$$C = \frac{\epsilon_0 A}{d}, \quad Q_i = CV, \quad E_i = \frac{V}{d}, \quad U_i = \frac{1}{2} CV^2.$$

The battery is disconnected before either subsequent change. Thus the free charge on each plate remains $Q_f = Q_i = CV$. Doubling the separation first reduces the capacitance to $C/2$. Filling the enlarged gap completely with dielectric then gives

$$C_f = \frac{k\epsilon_0 A}{2d} = \frac{kC}{2}.$$

Since $1 < k < 2$, this final capacitance is smaller than the original capacitance. The isolated capacitor's voltage is therefore allowed to change; it is not held at the original battery voltage.

Step 3: (b)(i) Compare the electric fields For fixed free charge and fixed plate area, the field before dielectric insertion is independent of plate separation, ignoring edge effects. Thus merely doubling d leaves the field equal to E_i . Polarisation of the dielectric produces a field opposing the field of the free plate charges. After insertion,

$$E_f = \frac{Q_f}{k\epsilon_0 A} = \frac{E_i}{k} = \frac{V}{kd}.$$

Hence the field decreases by the factor k . As a consistency check, the final voltage is $V_f = Q_f/C_f = 2V/k$, and dividing by the final separation $2d$ again gives $E_f = V/(kd)$.

Step 4: (b)(ii) Compare the stored energies Because charge is conserved, use the energy formula containing Q :

$$U_f = \frac{Q_f^2}{2C_f} = \frac{(CV)^2}{2(kC/2)} = \frac{CV^2}{k},$$
$$\frac{U_f}{U_i} = \frac{CV^2/k}{CV^2/2} = \frac{2}{k} > 1.$$

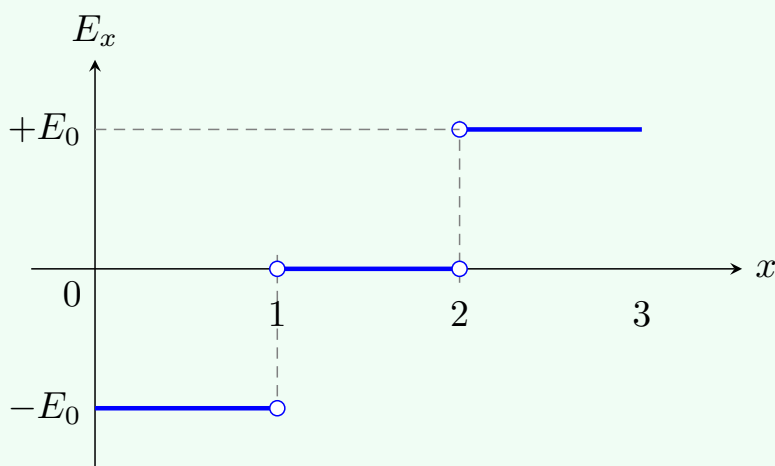
Quick Tip to Solve: Once the battery is disconnected, keep charge fixed and use $Q^2/(2C)$; for a potential graph, read each slope and reverse its sign to obtain the field.

Therefore the final stored energy increases relative to the initial energy. Separating the attracting plates requires external work and initially doubles the stored energy; dielectric insertion subsequently reduces that increased energy by a factor k . The overall increase is consistent with $k < 2$, even though the final electric field is smaller.

Step 5: (c) Convert each potential slope to electric field Denote the plateau potential by V_0 and the distance from $x = 0$ to $x = 1$ by ℓ . The three printed intervals have equal width. Set $E_0 = V_0/\ell > 0$. On the first interval potential rises linearly, so its positive slope gives a constant negative field. On the middle interval potential is constant, so the field is zero. On the last interval potential falls with the same slope magnitude, so the field is a constant positive value:

$$E_x(x) = \begin{cases} -E_0, & 0 < x < 1, \\ 0, & 1 < x < 2, \\ +E_0, & 2 < x < 3. \end{cases}$$

The field graph consequently consists of three horizontal segments. At $x = 1$ and $x = 2$, the idealised potential has sharp corners, so its derivative has no unique value exactly at those points. Open endpoints indicate these jumps; the graph does not assign a sloping field segment there. The printed potential does not specify behaviour outside the displayed range.



Electric field corresponding to the three slopes of the printed potential graph; $E_0 = V_0/\ell$.

Step 6: Conclusion The battery charges the capacitor until its voltage equals V , with $Q = CV$. After disconnection, doubling the gap and filling it with dielectric gives $E_f = E_i/k$, so the electric field decreases, and $U_f = (2/k)U_i = CV^2/k$, so the stored energy increases because $1 < k < 2$. The field graph is $-E_0$, zero and $+E_0$ on the successive intervals $(0, 1)$, $(1, 2)$ and $(2, 3)$.

Quick Tip to Solve: Once the battery is disconnected, keep charge fixed and use $Q^2/(2C)$; for a potential graph, read each slope and reverse its sign to obtain the field.

26. (a) Derive an expression for the potential energy of an electric dipole in a uniform electric field. Explain conditions for stable and unstable equilibrium.

(b) Is the electrostatic potential necessarily zero at a point where the electric field is zero? Give an example to support your answer. [5 marks]

Solution:

Concept:

- A dipole of charges $-q, +q$, separated by $2a$, has moment $\vec{p}$ directed from negative to positive charge, with magnitude $p = 2aq$.
- A uniform electric field exerts torque $\vec{\tau} = \vec{p} \times \vec{E}$. The potential-energy increase equals work done by an external agent during slow rotation against this torque.
- Equilibrium requires $dU/d\theta = 0$; a potential-energy minimum is stable and a maximum is unstable. Electric field is the negative potential gradient, $\vec{E} = -\nabla V$.

Step 1: (a) Calculate the work required to rotate the dipole Let θ be the angle between $\vec{p}$ and the uniform field $\vec{E}$. The electric torque tends to reduce this angle. To turn the dipole slowly in the direction of increasing θ , an external agent applies the opposing torque of magnitude $pE \sin \theta$. There is no change in kinetic energy, so the external work is stored as potential energy:

$$dU = dW_{\text{ext}} = pE \sin \theta d\theta.$$

Choose the conventional zero of potential energy at $\theta = \pi/2$. Integrating from this reference orientation gives

$$\begin{aligned} U(\theta) - U(\pi/2) &= pE \int_{\pi/2}^{\theta} \sin \theta' d\theta' \\ &= -pE \cos \theta. \end{aligned}$$

Thus the orientation-dependent potential energy is $U = -pE \cos \theta = -\vec{p} \cdot \vec{E}$. An arbitrary additive constant would represent a different energy reference without changing the torque.

Step 2: (a) Test the two equilibrium orientations Equilibrium occurs when $\tau = 0$, hence $\sin \theta = 0$. The two distinct orientations are $\theta = 0$ and $\theta = \pi$. Their stability follows from

$$\frac{dU}{d\theta} = pE \sin \theta, \quad \frac{d^2U}{d\theta^2} = pE \cos \theta.$$

At $\theta = 0$, the dipole is parallel to the field, $U = -pE$ is minimum, and the second derivative is positive. A small angular displacement produces a restoring torque, so this is stable equilibrium. At $\theta = \pi$, the dipole is antiparallel to the field, $U = +pE$ is maximum, and the second derivative is negative. A small displacement takes it farther from this orientation, so this is unstable equilibrium.

Step 3: (b) Distinguish zero field from zero potential The potential need not be zero where the electric field is zero. Zero field means zero spatial gradient of potential at that point; it does not set the value of potential. For example, consider two equal positive point charges $+q$ separated by $2a$. At their midpoint, their equal and opposite electric-field vectors cancel. With potential zero at infinity,

Quick Tip to Solve: For dipole stability, locate the minimum of $-pE \cos \theta$. For zero-field examples, cancel the field vectors first and then add the scalar potentials separately.

however, their scalar potentials add:

$$\vec{E}_{\text{mid}} = 0, \quad V_{\text{mid}} = \frac{1}{4\pi\epsilon_0} \left(\frac{q}{a} + \frac{q}{a} \right) = \frac{2q}{4\pi\epsilon_0 a} \neq 0.$$

This explicitly gives a point with zero electric field and nonzero electrostatic potential.

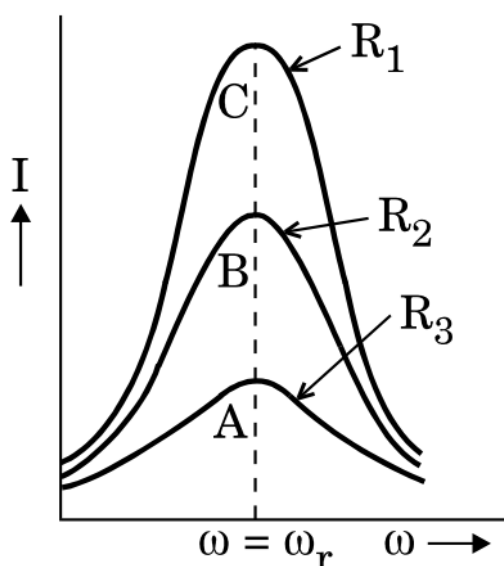
Step 4: Conclusion $U = -\vec{p} \cdot \vec{E}$, taking $U = 0$ for a perpendicular dipole. Parallel alignment is stable and antiparallel alignment unstable. Zero electric field does not require zero potential, as demonstrated at the midpoint of two equal positive charges.

Quick Tip to Solve: For dipole stability, locate the minimum of $-pE \cos \theta$. For zero-field examples, cancel the field vectors first and then add the scalar potentials separately.

27. (a) What do you understand by ‘sharpness of resonance’ for a series LCR resonant circuit? How is it related with the quality factor ‘Q’ of the circuit? Using the graphs given in the diagram, explain the factors which affect it. For which graph is the resistance (R) minimum?

(b) A $2 \mu\text{F}$ capacitor, 100Ω resistor and 8 H inductor are connected in series with an ac source. Find the frequency of the ac source for which the current drawn in the circuit is maximum.

If the peak value of emf of the source is 200 V , calculate the (i) maximum current, and (ii) inductive and capacitive reactance of the circuit at resonance. [5 marks]



Solution:

Concept:

- A series LCR circuit has impedance $Z = \sqrt{R^2 + (\omega L - 1/\omega C)^2}$. The current amplitude is $I_0 = E_0/Z$, where E_0 is the applied emf amplitude.
- At resonance, $\omega_r L = 1/(\omega_r C)$, so $\omega_r = 1/\sqrt{LC}$, $Z = R$, and the current is maximum.
- The quality factor is $Q = \omega_r L/R = (1/R)\sqrt{L/C}$. It equals resonance frequency divided by the half-power bandwidth: $Q = \omega_r/(\omega_2 - \omega_1)$.

Quick Tip to Solve: At series resonance use $Z = R$ for current, but calculate each reactance separately. Convert ω_r to hertz by dividing by 2π , and keep peak and rms values distinct.

Step 1: (a) Define sharpness and relate it to the quality factor Sharpness of resonance describes how rapidly the current falls when the source frequency is moved away from the resonant frequency. A narrow resonance peak selects a small range of frequencies and corresponds to greater sharpness. Let ω_1 and ω_2 be the two half-power frequencies, at which the current amplitude is $I_{0,\max}/\sqrt{2}$. For a series circuit,

$$\Delta\omega = \omega_2 - \omega_1 = \frac{R}{L}, \quad Q = \frac{\omega_r}{\Delta\omega} = \frac{1}{R} \sqrt{\frac{L}{C}}.$$

A larger Q therefore gives a smaller fractional bandwidth and a sharper resonance. With the other components fixed, decreasing R , increasing L , or decreasing C increases Q . Changing L or C also changes the resonant frequency; changing only R leaves ω_r unchanged.

Step 2: (a) Compare the three printed resonance curves For the same applied emf amplitude and the same L, C , the peak current is $I_{0,\max} = E_0/R$, while the bandwidth is R/L . The tallest, narrowest curve has peak C and is labelled R_1 ; the middle curve has peak B and is labelled R_2 ; the lowest, broadest curve has peak A and is labelled R_3 . Consequently,

$$R_1 < R_2 < R_3, \quad Q_1 > Q_2 > Q_3.$$

The resistance is minimum for the curve through C, labelled R_1 . All three peak at the common frequency shown because their inductance and capacitance are the same.

Step 3: (b) Calculate the frequency for maximum current Convert the capacitance to SI units: $C = 2 \times 10^{-6}$ F. Using $L = 8$ H, the resonant angular frequency and ordinary frequency are

$$\begin{aligned} \sqrt{LC} &= \sqrt{8(2 \times 10^{-6})} = 4 \times 10^{-3} \text{ s}, \\ \omega_r &= \frac{1}{4 \times 10^{-3}} = 250 \text{ rad s}^{-1}, \\ f_r &= \frac{\omega_r}{2\pi} = \frac{250}{2\pi} \simeq 39.8 \text{ Hz}. \end{aligned}$$

At this frequency the inductive and capacitive reactances cancel in the total impedance, leaving only the resistance.

Step 4: (b)(i) Find the maximum current The supplied 200 V is the peak emf. Therefore the maximum instantaneous current, which is the current amplitude at resonance, is

$$I_{0,\max} = \frac{E_0}{R} = \frac{200}{100} = 2.00 \text{ A}.$$

If an rms value is desired, it is $I_{\text{rms}} = I_{0,\max}/\sqrt{2} \simeq 1.41$ A; it must not be confused with the requested peak current.

Step 5: (b)(ii) Calculate the two reactances at resonance Evaluate each reactance using $\omega_r = 250 \text{ rad s}^{-1}$:

$$\begin{aligned} X_L &= \omega_r L = 250(8) = 2000 \Omega, \\ X_C &= \frac{1}{\omega_r C} = \frac{1}{250(2 \times 10^{-6})} = 2000 \Omega. \end{aligned}$$

Each reactance is nonzero. Their opposite contributions cancel in $X_L - X_C$, so the net reactance vanishes and the impedance is 100Ω .

Quick Tip to Solve: At series resonance use $Z = R$ for current, but calculate each reactance separately. Convert ω_r to hertz by dividing by 2π , and keep peak and rms values distinct.

Step 6: Conclusion Greater Q means sharper resonance. Curve C, labelled R_1 , has minimum resistance. Numerically, $f_r \simeq 39.8$ Hz, the maximum current is 2.00 A, and $X_L = X_C = 2000 \Omega$.

Quick Tip to Solve: At series resonance use $Z = R$ for current, but calculate each reactance separately. Convert ω_r to hertz by dividing by 2π , and keep peak and rms values distinct.

OR

27. (a) Draw a schematic diagram of an ac generator. Explain its working and obtain the expression for the instantaneous value of the emf in terms of the magnetic field B , number of turns N of the coil of area A rotating with angular frequency ω . Show how an alternating emf is generated by a loop of wire rotating in a magnetic field.

(b) A circular coil of radius 10 cm and 20 turns is rotated about its vertical diameter with angular speed of 50 rad s^{-1} in a uniform horizontal magnetic field of $3.0 \times 10^{-2} \text{ T}$.

(i) Calculate the maximum and average emf induced in the coil.

(ii) If the coil forms a closed loop of resistance 10Ω , calculate the maximum current in the coil and the average power loss due to Joule heating. [5 marks]

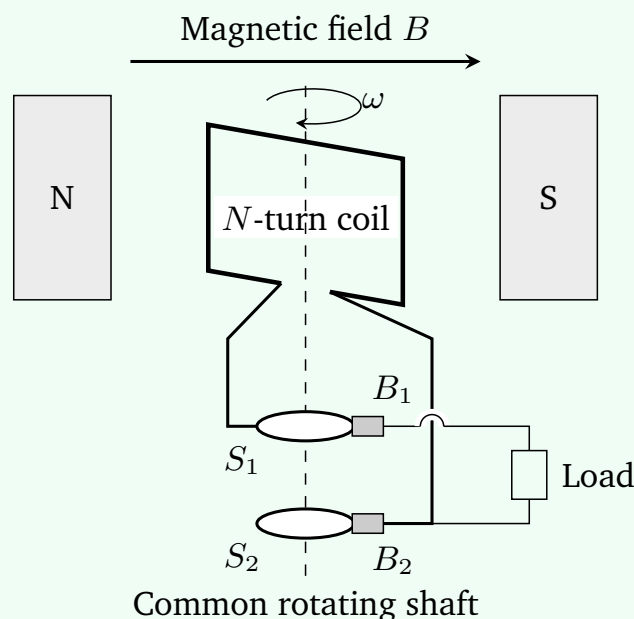
Solution:

Concept:

- Faraday's law gives the induced emf as $\varepsilon = -d(N\Phi)/dt$, where $N\Phi$ is the magnetic flux linkage of the coil.
- For a coil of area A , whose normal makes angle θ with a uniform field B , $\Phi = BA \cos \theta$. Uniform rotation gives $\theta = \omega t$ for a suitable choice of time origin.
- For a sinusoidal emf, $\varepsilon_{\text{rms}} = \varepsilon_0/\sqrt{2}$. A purely resistive closed coil dissipates average power $\overline{P} = \varepsilon_{\text{rms}}^2/R$.

Step 1: (a) Construction and working of the ac generator A rectangular armature coil of N turns rotates mechanically between the poles of a magnet. Its two ends connect to two separate slip rings S_1, S_2 , which rotate with the coil on the same shaft. Stationary carbon brushes B_1, B_2 touch the rings and connect the coil to the external load. The schematic below shows the connections; the dashed line is the rotation axis. Mechanical work supplies the electrical energy delivered to the load. As the coil rotates, its magnetic flux linkage changes and an emf is induced. Slip rings maintain the connections while allowing the polarity across the brushes to reverse every half-turn.

Quick Tip to Solve: Use $A = \pi r^2$ with radius in metres, and use rms current for average heating; a zero signed average emf does not imply zero power.



Schematic ac generator: rotating coil and slip rings, stationary brushes and load.

Step 2: (a) Derive the instantaneous emf and its alternating nature Choose $t = 0$ when the coil's normal is parallel to the field. At time t , the normal has turned through $\theta = \omega t$, so the flux linkage is

$$N\Phi = NBA \cos \omega t.$$

Applying Faraday's law,

$$\varepsilon = -\frac{d}{dt}(NBA \cos \omega t) = NBA\omega \sin \omega t = \varepsilon_0 \sin \omega t.$$

Thus the amplitude is $\varepsilon_0 = NBA\omega$. The emf is zero when the normal is parallel or antiparallel to the field, and its magnitude is maximum when the plane of the coil is parallel to the field. During the first half-turn $\sin \omega t$ is positive; during the next half-turn it is negative. Consequently the output alternates, with period $T = 2\pi/\omega$. A different initial orientation changes only the phase of the sine function.

Step 3: (b)(i) Maximum emf and its average Convert the radius to metres before calculating the area:

$$r = 0.10 \text{ m}, \quad A = \pi r^2 = 0.01\pi \text{ m}^2.$$

The vertical rotation axis is perpendicular to the horizontal field, so the full sinusoidal flux variation applies. Using $N = 20$, $B = 3.0 \times 10^{-2} \text{ T}$ and $\omega = 50 \text{ rad s}^{-1}$,

$$\varepsilon_0 = 20(3.0 \times 10^{-2})(0.01\pi)(50) = 0.30\pi \text{ V} \simeq 0.942 \text{ V}.$$

The average of the signed emf over a complete revolution is

$$\bar{\varepsilon} = \frac{1}{T} \int_0^T \varepsilon_0 \sin \omega t \, dt = 0.$$

Quick Tip to Solve: Use $A = \pi r^2$ with radius in metres, and use rms current for average heating; a zero signed average emf does not imply zero power.

Positive and negative half-cycles cancel. If an average over a positive half-cycle (or the rectified average) is intended, its value is $2\varepsilon_0/\pi = 0.600\text{ V}$; this is distinct from the full-cycle average and from the rms emf.

Step 4: (b)(ii) Maximum current and average Joule power Treat the coil as the stated resistive loop of resistance $R = 10\ \Omega$, neglecting self-inductance. Ohm's law gives

$$I_0 = \frac{\varepsilon_0}{R} = \frac{0.30\pi}{10} = 0.030\pi\text{ A} \simeq 0.0942\text{ A}.$$

Joule heating depends on the square of the current, so both half-cycles produce positive heating. Since the average of $\sin^2 \omega t$ is $1/2$,

$$\overline{P} = I_{\text{rms}}^2 R = \frac{I_0^2 R}{2} = \frac{(0.30\pi)^2}{2(10)} = 0.0045\pi^2\text{ W} \simeq 0.0444\text{ W}.$$

Step 5: Conclusion The generator produces $\varepsilon = NBA\omega \sin \omega t$. For the circular coil, $\varepsilon_0 = 0.942\text{ V}$ and the full-cycle average emf is 0 (positive half-cycle average 0.600 V). The maximum current is 0.0942 A , and the average Joule power is 0.0444 W .

Quick Tip to Solve: Use $A = \pi r^2$ with radius in metres, and use rms current for average heating; a zero signed average emf does not imply zero power.