

CBSE Class 12 Physics Question Paper 2019 with Solution PDF-Set 55/4/3

Question Paper with Solutions

Conducted by the Central Board of Secondary Education (CBSE)



Time Allowed: 3 hours

Maximum Marks: 70

General Instructions

- (i) All questions are compulsory. There are 27 questions in all.
- (ii) This question paper has four sections : Section A, Section B, Section C and Section D.
- (iii) Section A contains five questions of one mark each, Section B contains seven questions of two marks each, Section C contains twelve questions of three marks each, Section D contains three questions of five marks each.
- (iv) There is no overall choice. However, an internal choice(s) has been provided in two questions of one mark, two questions of two marks, four questions of three marks and three questions of five marks weightage. You have to attempt only one of the choices in such questions.

You may use the following values of physical constants wherever necessary :

$$c = 3 \times 10^8 \text{ m/s}$$

$$h = 6.63 \times 10^{-34} \text{ Js}$$

$$e = 1.6 \times 10^{-19} \text{ C}$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ T m A}^{-1}$$

$$\epsilon_0 = 8.854 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$$

$$\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$$

$$\text{Mass of electron } (m_e) = 9.1 \times 10^{-31} \text{ kg}$$

$$\text{Mass of neutron} = 1.675 \times 10^{-27} \text{ kg}$$

$$\text{Mass of proton} = 1.673 \times 10^{-27} \text{ kg}$$

$$\text{Avogadro's number} = 6.023 \times 10^{23} \text{ per gram mole}$$

$$\text{Boltzmann constant} = 1.38 \times 10^{-23} \text{ JK}^{-1}$$

Section A (5 marks)

1. Plot a graph of stopping potential (V_0) versus the frequency (ν) of incident radiation in photoelectric emission. [1 mark]

Solution:

Concept:

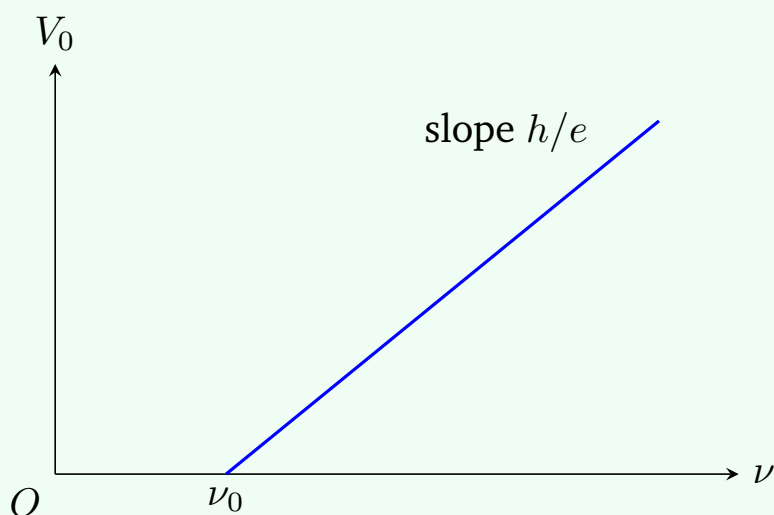
- Einstein's photoelectric equation is $h\nu = \phi + K_{\max}$, where h is Planck's constant and ϕ is the work function.
- The stopping potential obeys $eV_0 = K_{\max}$, with e the positive elementary charge and threshold frequency $\nu_0 = \phi/h$.

Step 1: Obtain the straight-line equation Combining the energy equation with the definition of stopping potential gives

$$V_0 = \frac{h}{e}\nu - \frac{\phi}{e} = \frac{h}{e}(\nu - \nu_0).$$

Thus the slope is positive and equal to h/e .

Step 2: Locate the threshold on the graph At $\nu = \nu_0$, the maximum kinetic energy and stopping potential are zero. Draw the physical line from $(\nu_0, 0)$ upwards to the right. Below threshold, no photoelectrons are emitted, so a negative continuation is not a physical stopping potential.



Stopping potential versus incident frequency

Step 3: Conclusion The required graph is a straight line of slope h/e , meeting the frequency axis at $\nu_0 = \phi/h$.

Quick Tip to Solve: Place the frequency intercept to the right of the origin; start the physical graph at that threshold.

2. Name the charge carriers for the flow of current in a (i) conductor and (ii) electrolyte. [1 mark]

Solution:

Concept:

- Electric current is the directed transport of electric charge.
- Charge carriers depend on the medium: metallic conduction involves mobile electrons, whereas electrolytic conduction involves mobile ions.

Step 1: Identify carriers in a conductor In an ordinary metallic conductor, free electrons move through the lattice under an applied electric field. The positive ions remain bound at lattice sites and do not carry the conduction current. Electron drift is opposite to the conventional current direction.

Step 2: Identify carriers in an electrolyte An electrolyte contains positive ions (cations) and negative ions (anions). Cations drift toward the negative electrode and anions toward the positive electrode. Both types of ions contribute to current; their opposite charges and opposite drift directions give the same conventional current direction.

Step 3: Conclusion (i) Conductor: free electrons. (ii) Electrolyte: positive and negative ions.

Quick Tip to Solve: For a metal name electrons; for an electrolyte always include both cations and anions.

3. Why are the antennas in space wave mode of propagation generally mounted at heights of many wavelengths above the ground? [1 mark]

Solution:

Concept:

- Space waves propagate mainly along a line-of-sight path between transmitting and receiving antennas.
- The Earth's surface and nearby obstacles limit the unobstructed path; ground-reflected waves can also interfere with the direct wave.

Step 1: Explain the need for elevation At low antenna heights, the ground and local obstacles obstruct the direct propagation path. Mounting antennas at heights of many wavelengths provides a clearer line of sight and reduces the adverse influence of the ground on the radiated and received wave.

Step 2: Relate height to communication range Raising an antenna extends its radio horizon. For a smooth Earth, the approximate horizon distance is $d \simeq \sqrt{2Rh}$, where R is the Earth's radius and h the antenna height. Hence higher antennas allow space-wave signals to reach farther before the Earth's curvature blocks the path.

Step 3: Conclusion Antennas are mounted high to provide an unobstructed line-of-sight path and increase the range of space-wave communication.

Quick Tip to Solve: Link a high antenna to a clear line of sight and a larger radio horizon.

4. Two identical conducting balls A and B have charges $-Q$ and $+3Q$ respectively. They are brought in contact with each other and then separated by a distance d apart. Find the nature of the Coulomb force between them. [1 mark]

Solution:

Concept:

- Charge is conserved when the two conducting balls are brought into contact.
- Identical conducting balls share the total charge equally; like charges repel according to Coulomb's law.

Step 1: Find the charges after contact The total initial charge is $(-Q) + 3Q = 2Q$. Since the balls are identical, charge redistributes equally while they touch. After separation, each ball has charge $q_A = q_B = (2Q)/2 = +Q$.

Step 2: Determine the interaction after separation Both balls now carry positive charge. Their Coulomb interaction is therefore repulsive, acting along the line joining their centres and pushing them apart. In the usual point-charge approximation at separation d , its magnitude is $F = Q^2/(4\pi\epsilon_0 d^2)$, where ϵ_0 is the vacuum permittivity.

Step 3: Conclusion The force between the separated balls is repulsive because both carry $+Q$.

Quick Tip to Solve: Add the initial charges first, divide equally for identical balls, then compare their final signs.

OR

4. A metallic spherical shell has an inner radius R_1 and outer radius R_2 . A charge Q is placed at the centre of the shell. What will be the surface charge density on the (i) inner surface, and (ii) outer surface of the shell? [1 mark]

Solution:

Concept:

- In electrostatic equilibrium the electric field inside the metal of a conductor is zero.
- A charge at the centre of a spherical cavity induces uniform charge on its spherical inner surface. The uncharged shell remains neutral overall.

Step 1: Find charge on the inner surface Take a spherical Gaussian surface lying within the metal. Since its electric flux is zero, the enclosed charge must vanish: $Q + q_{\text{inner}} = 0$. Hence $q_{\text{inner}} = -Q$. Central placement and spherical symmetry distribute this charge uniformly over area $4\pi R_1^2$.

Step 2: Find charge on the outer surface With the shell initially uncharged, conservation of its net charge requires $q_{\text{inner}} + q_{\text{outer}} = 0$, so $q_{\text{outer}} = +Q$. Spherical symmetry makes the outer distribution uniform, over area $4\pi R_2^2$. Surface density equals the charge divided by the corresponding surface area.

Step 3: Conclusion For the initially neutral shell, $\sigma_{\text{inner}} = -\frac{Q}{4\pi R_1^2}$ and $\sigma_{\text{outer}} = +\frac{Q}{4\pi R_2^2}$.

Quick Tip to Solve: Use zero field inside the metal for the inner charge, then neutrality for the outer charge.

5. The small ozone layer on top of the stratosphere is crucial for human survival. Why?

[1 mark]

Solution:

Concept:

- Ozone in the stratosphere absorbs much of the Sun's harmful ultraviolet radiation.
- Ultraviolet photons can damage biological molecules and living tissues when exposure is excessive.

Step 1: Identify the radiation screened by ozone Solar radiation includes ultraviolet radiation as well as visible and infrared radiation. The ozone layer strongly absorbs harmful ultraviolet wavelengths before they reach the Earth's surface, thereby reducing the ultraviolet exposure of living organisms.

Step 2: Explain its importance for survival Excessive ultraviolet exposure damages cellular DNA and can cause skin cancer and eye damage. By screening much of this radiation, the ozone layer protects humans and other life on Earth. Its small thickness does not prevent it from providing this essential absorption.

Step 3: Conclusion The ozone layer is crucial because it absorbs harmful solar ultraviolet radiation and protects humans from its damaging effects.

Quick Tip to Solve: Name ultraviolet radiation specifically and connect its absorption with protection of living tissues.

OR

5. Illustrate by giving suitable examples, how you can show that electromagnetic waves carry both energy and momentum.

[1 mark]

Solution:

Concept:

- Electromagnetic waves transport energy in their electric and magnetic fields.
- They also transport momentum: an absorbed wave of energy U transfers momentum U/c , where c is the speed of light.

Step 1: Give an example demonstrating energy transport Sunlight warms a surface exposed to it, and a solar cell converts incident light into electrical energy. The warming or electrical output shows that energy has arrived with the electromagnetic radiation and has been transferred to matter.

Step 2: Give an example demonstrating momentum transport Light exerts radiation pressure on an illuminated surface. A solar sail is pushed by sunlight because the photons transfer momentum to it. For normally incident radiation of energy U , complete absorption transfers U/c , while perfect reflection transfers $2U/c$. This force demonstrates that electromagnetic waves carry momentum.

Step 3: Conclusion Solar heating or solar-cell output demonstrates energy transport; radiation pressure, such as the push on a solar sail, demonstrates momentum transport.

Quick Tip to Solve: Use a heating example for energy and radiation pressure on a surface for momentum.

Section B (14 marks)

6. Two long straight wires carrying currents of 2 A and 5 A in the same direction are kept parallel, 10 cm apart from each other. Calculate the force acting between them and write its nature. [2 marks]

Solution:

Concept:

- A long straight wire carrying current I_1 produces the magnetic field $B_1 = \mu_0 I_1 / (2\pi r)$ at a perpendicular distance r .
- A parallel wire of length L , carrying current I_2 , experiences force $F = I_2 L B_1$. Here $\mu_0 = 4\pi \times 10^{-7} \text{ N A}^{-2}$.

Step 1: Identify the appropriate force quantity The separation is $r = 10 \text{ cm} = 0.10 \text{ m}$, and the currents are $I_1 = 2 \text{ A}$ and $I_2 = 5 \text{ A}$. Since the length of the wires is not specified, the computable quantity is the force per unit length. For parallel wires, the current direction is perpendicular to the magnetic field of the other wire, so the magnetic force formula gives $F/L = \mu_0 I_1 I_2 / (2\pi r)$.

Step 2: Substitute the values and determine the direction

$$\frac{F}{L} = \frac{(4\pi \times 10^{-7})(2)(5)}{2\pi(0.10)} = 2.0 \times 10^{-5} \text{ N m}^{-1}.$$

The right-hand rule for the field around either wire, followed by $\mathbf{F} = I\mathbf{L} \times \mathbf{B}$, places the force on each wire towards the other wire. The forces have equal magnitudes and opposite directions. For any common length L , the force magnitude is $F = (2.0 \times 10^{-5} \text{ N m}^{-1})L$.

Step 3: Conclusion The force per unit length is $2.0 \times 10^{-5} \text{ N m}^{-1}$, and it is attractive because the currents flow in the same direction.

Quick Tip to Solve: When no wire length is given, calculate F/L ; convert centimetres to metres before substituting.

7. A parallel plate capacitor of plate area A each and separation d , is being charged by an ac source. Show that the displacement current inside the capacitor is the same as the current charging the capacitor. [2 marks]

Solution:

Concept:

- Maxwell's displacement current is $I_d = \epsilon_0 d\Phi_E/dt$, where Φ_E is electric flux through the gap.
- For ideal parallel plates in air or vacuum, $E = Q/(\epsilon_0 A)$, and the charging current in a lead is $I = dQ/dt$. Fringing is neglected.

Step 1: Express the flux between the plates Let $Q(t)$ denote the instantaneous positive charge on a plate. The field inside the capacitor is uniform and normal to its plates. For an area A spanning the plate gap,

$$\Phi_E = EA = \frac{Q(t)}{\epsilon_0 A} A = \frac{Q(t)}{\epsilon_0}.$$

Quick Tip to Solve: First write $\Phi_E = EA = Q/\epsilon_0$; differentiating immediately gives $I_d = dQ/dt$.

The area cancels because the field is inversely proportional to the plate area for a fixed charge.

Step 2: Differentiate the flux and compare with the lead current Using Maxwell's definition,

$$I_d = \varepsilon_0 \frac{d\Phi_E}{dt} = \varepsilon_0 \frac{d}{dt} \left(\frac{Q}{\varepsilon_0} \right) = \frac{dQ}{dt} = I.$$

Equivalently, with instantaneous voltage $V(t)$, $E = V/d$ and $C = \varepsilon_0 A/d$, giving $I_d = (\varepsilon_0 A/d) dV/dt = C dV/dt = I$. This relation holds at every instant during alternating charging and discharging, including the sign change of the current. There is no conduction through the ideal insulating gap; the changing electric field supplies the displacement-current contribution.

Step 3: Conclusion $I_d = I$: the displacement current inside the capacitor equals the instantaneous current charging it.

Quick Tip to Solve: First write $\Phi_E = EA = Q/\varepsilon_0$; differentiating immediately gives $I_d = dQ/dt$.

8. Apply Gauss's law to show that for a charged spherical shell, the electric field outside the shell is as if the entire charge were concentrated at the centre. [2 marks]

Solution:

Concept:

- Gauss's law states $\oint \mathbf{E} \cdot d\mathbf{S} = Q_{\text{enc}}/\varepsilon_0$.
- For a uniformly charged spherical shell, spherical symmetry makes the exterior field radial and its magnitude constant on any concentric sphere.

Step 1: Choose the Gaussian surface and evaluate its flux Let the shell have radius R and total charge Q . To find the field at an exterior point a distance $r > R$ from the centre, choose a concentric spherical Gaussian surface of radius r . Its area is $4\pi r^2$. For positive charge, the field and outward area vector are parallel everywhere; for negative charge the signed radial component is negative. With E_r denoting the signed radial component,

$$\oint \mathbf{E} \cdot d\mathbf{S} = E_r(4\pi r^2).$$

Step 2: Apply Gauss's law and compare with a point charge The Gaussian sphere encloses the entire charge Q , irrespective of the shell radius. Therefore

$$E_r(4\pi r^2) = \frac{Q}{\varepsilon_0}, \quad \mathbf{E} = \frac{Q}{4\pi\varepsilon_0 r^2} \hat{\mathbf{r}} \quad (r > R).$$

Here $\hat{\mathbf{r}}$ points radially outwards. This expression has exactly the inverse-square dependence and direction of the field of a point charge Q at the centre. The field points outwards for $Q > 0$ and inwards for $Q < 0$; its magnitude is $|Q|/(4\pi\varepsilon_0 r^2)$.

Step 3: Conclusion Outside the shell, $\mathbf{E} = \frac{Q}{4\pi\varepsilon_0 r^2} \hat{\mathbf{r}}$. Thus its field is the same as if the total charge were concentrated at its centre.

Quick Tip to Solve: Choose a concentric Gaussian sphere outside the shell: symmetry turns the flux integral into $E_r 4\pi r^2$.

8. Two large parallel plane sheets have uniform charge densities $+\sigma$ and $-\sigma$. Determine the electric field (i) between the sheets, and (ii) outside the sheets. [2 marks]

Solution:

Concept:

- A large uniformly charged plane sheet produces field magnitude $E_0 = \sigma/(2\epsilon_0)$ on either side, neglecting edge effects.
- The field points away from a positive sheet and towards a negative sheet. Superposition requires adding field vectors, not merely their magnitudes.

Step 1: Add the fields in the region between the sheets Take the positive sheet to the left and the negative sheet to the right, and let $\hat{n}$ point from the positive sheet towards the negative sheet. Between the sheets the positive sheet produces a rightward field because its field points away from it. The negative sheet also produces a rightward field because its field points towards it. Hence

$$\mathbf{E}_{\text{between}} = \left(\frac{\sigma}{2\epsilon_0} + \frac{\sigma}{2\epsilon_0} \right) \hat{n} = \frac{\sigma}{\epsilon_0} \hat{n}.$$

Step 2: Add the fields in each exterior region On the left of both sheets, the positive sheet produces a leftward field while the negative sheet produces a rightward field. Their magnitudes are equal, so they cancel. On the right of both sheets, the positive sheet produces a rightward field and the negative sheet produces a leftward field, which again cancel:

$$\mathbf{E}_{\text{outside}} = \mathbf{0}.$$

These results use the large-plane-sheet approximation; near finite edges the fields need not be perfectly uniform and cancellation is not exact.

Step 3: Conclusion (i) Between the sheets, $E = \sigma/\epsilon_0$, directed from $+\sigma$ to $-\sigma$. (ii) Outside the sheets, $E = 0$.

Quick Tip to Solve: Draw the field direction of each sheet mentally: between opposite sheets the arrows agree, and outside they oppose.

9. Out of the two optical instruments, a microscope and a telescope, which one plays the role in magnifying the objects and in resolving the two objects kept close to each other? Explain, giving example. [2 marks]

Solution:

Concept:

- Magnification increases the apparent angular size of an object. Resolving power is the ability to distinguish two nearby objects as separate images.
- A microscope is used to magnify tiny nearby objects; a telescope is especially useful for resolving

Quick Tip to Solve: Use a cell as the microscope example and a binary star as the telescope example; distinguish size increase from seeing two separate images.

distant objects with a small angular separation. Both instruments have magnification and a finite resolution.

Step 1: Explain the magnifying role of a microscope A microscope produces a large image of a very small object placed close to its objective. Its objective first produces an enlarged real image, and the eyepiece further magnifies that image so that details can be examined comfortably. For example, a small biological cell or a microorganism that is too small to examine clearly with the unaided eye can be observed with a microscope. This is the magnifying role intended in the comparison.

Step 2: Explain the resolving role of a telescope A telescope helps distinguish distant objects that appear close together in the sky. For example, two nearby stars of a binary system may appear as one bright point to the unaided eye, but a telescope with sufficient resolving power shows two distinct stars. Its large objective aperture reduces the diffraction-limited angular separation: for a circular aperture of diameter D , $\theta_{\min} \simeq 1.22\lambda/D$. Increasing magnification alone cannot separate overlapping diffraction images; sufficient aperture is needed to resolve them. A telescope also magnifies angular size, and a microscope also resolves fine detail, so the distinction describes their usual practical roles.

Step 3: Conclusion A microscope plays the usual role of magnifying very small nearby objects, such as cells; a telescope resolves close distant objects, such as the two stars in a binary system.

Quick Tip to Solve: Use a cell as the microscope example and a binary star as the telescope example; distinguish size increase from seeing two separate images.

10. A beam of light converges at a point P. Now a convex lens is placed in the path of the convergent beam at 15 cm from P. At what point does a beam converge if the convex lens has a focal length 10 cm? [2 marks]

Solution:

Concept:

- The thin-lens formula is $1/v - 1/u = 1/f$, where distances are measured from the optical centre using the Cartesian sign convention.
- A converging incident beam has a virtual object at the point where it would meet without the lens. A convex lens has positive focal length.

Step 1: Identify the virtual object Take the incident light to travel from left to right, so rightward distances are positive. The point P lies on the outgoing side of the inserted lens, because the incident rays are already converging towards it. Hence this point acts as a virtual object, with $u = +15$ cm. The convex lens has $f = +10$ cm. Using a negative object distance here would incorrectly describe a diverging incident beam.

Step 2: Apply the lens formula Substitution gives

$$\frac{1}{v} = \frac{1}{f} + \frac{1}{u} = \frac{1}{10} + \frac{1}{15} = \frac{5}{30} = \frac{1}{6} \text{ cm}^{-1}.$$

Thus $v = +6$ cm. The positive image distance shows that the transmitted rays actually meet on the outgoing side. The lens adds convergence, so the focus moves closer to the lens than the original

Quick Tip to Solve: For an already converging beam, treat its intended focus as a virtual object: use positive u in the Cartesian lens formula.

point P .

Step 3: Conclusion The beam converges 6 cm beyond the lens, on the side of P , i.e. 9 cm before the original point P .

Quick Tip to Solve: For an already converging beam, treat its intended focus as a virtual object: use positive u in the Cartesian lens formula.

OR

10. An object is kept in front of a concave mirror of focal length 15 cm. The image formed is real and three times the size of the object. Calculate the distance of the object from the mirror. [2 marks]

Solution:

Concept:

- For a spherical mirror, $1/f = 1/u + 1/v$, with u , v , and f measured from the pole in the Cartesian convention.
- Transverse magnification is $m = -v/u$. A real image of a real object formed by a concave mirror is inverted, so its magnification is negative.

Step 1: Use the size and nature of the image Take the incident direction as positive. The concave mirror focus and the object are in front of the mirror, so $f = -15$ cm and $u < 0$. The real image is inverted and three times the object size; therefore $m = -3$, rather than $+3$. From $-v/u = -3$, we get $v = 3u$. Both image and object distances are negative, consistent with a real image in front of the mirror.

Step 2: Substitute into the mirror equation Using $v = 3u$ gives

$$\frac{1}{-15} = \frac{1}{u} + \frac{1}{3u} = \frac{4}{3u}, \quad u = \frac{4(-15)}{3} = -20 \text{ cm.}$$

The corresponding image distance is $v = -60$ cm, which indeed gives $-v/u = -3$. The object lies between the focus at 15 cm and the centre of curvature at 30 cm, as expected for an enlarged real image.

Step 3: Conclusion The object must be placed 20 cm in front of the concave mirror.

Quick Tip to Solve: Write the signed magnification first: a real image three times larger has $m = -3$, which immediately gives $v = 3u$.

11. For a CE transistor amplifier, the audio signal voltage across the collector resistance of $2\text{ k}\Omega$ is 2 V . If the current amplification factor of the transistor is 100, calculate the input signal voltage and the base current, given the base resistance as $1\text{ k}\Omega$. [2 marks]

Solution:

Concept:

- For a CE amplifier, the signal current gain is $\beta = \Delta I_C / \Delta I_B$, where ΔI_C and ΔI_B are collector and base signal-current amplitudes.
- Ohm's law gives $V_o = \Delta I_C R_C$ and $V_i = \Delta I_B R_B$ for the given collector and base resistances.

Step 1: Calculate the collector and base signal currents The output audio signal voltage is the voltage across the collector resistance. Convert $R_C = 2\text{ k}\Omega = 2000\ \Omega$. The collector-current signal is therefore

$$\Delta I_C = \frac{V_o}{R_C} = \frac{2}{2000}\text{ A} = 1.0 \times 10^{-3}\text{ A}.$$

With current gain $\beta = 100$, the base-current signal is

$$\Delta I_B = \frac{\Delta I_C}{\beta} = \frac{10^{-3}}{100}\text{ A} = 10^{-5}\text{ A} = 10\ \mu\text{A}.$$

Step 2: Calculate the input signal voltage The base resistance is $R_B = 1\text{ k}\Omega = 1000\ \Omega$. Hence

$$V_i = \Delta I_B R_B = (10^{-5})(1000)\text{ V} = 10^{-2}\text{ V}.$$

As a consistency check, the magnitude of voltage gain is $\beta R_C / R_B = 100 \times 2 = 200$, and $V_o / 200 = 2 / 200 = 0.01\text{ V}$. The CE voltage phase inversion changes the relative phase, but not the requested positive signal amplitudes.

Step 3: Conclusion The input signal voltage is $0.01\text{ V} = 10\text{ mV}$, and the base signal current is $10\ \mu\text{A}$.

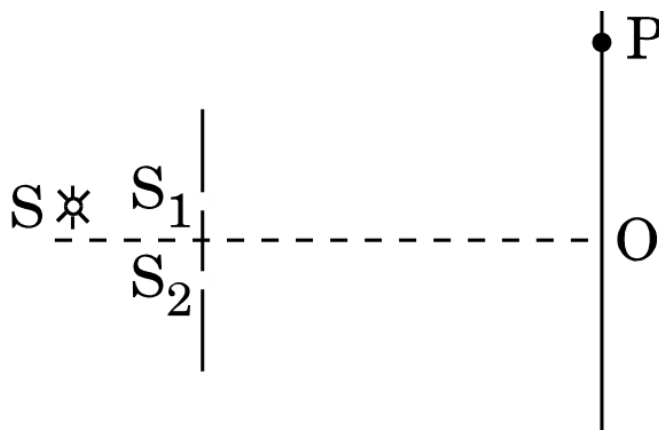
Quick Tip to Solve: First divide the output voltage by collector resistance, then divide that current by β ; only then multiply by base resistance.

12. The figure shows a modified Young's double slit experimental set-up. Here $SS_2 - SS_1 = \lambda/4$.

(a) Write the condition for constructive interference.

(b) Obtain an expression for the fringe width.

[2 marks]



Solution:

Concept:

- Compare complete source-to-screen paths through the two slits.
- A bright fringe requires total path difference $n\lambda$, with integer n . Let d be slit separation and D the screen distance.

Step 1: (a) Write the constructive interference condition Including the given initial path difference, the complete path difference at P is: $\Delta = (SS_2 + S_2P) - (SS_1 + S_1P) = \lambda/4 + (S_2P - S_1P)$. A bright fringe occurs when $\Delta = n\lambda$. Equivalently,

$$S_2P - S_1P = \left(n - \frac{1}{4}\right)\lambda, \quad n \in \mathbb{Z}.$$

Step 2: (b) Obtain the fringe width Measure y upwards from O , towards slit S_1 . In the usual paraxial limit $D \gg d$, $S_2P - S_1P \simeq dy/D$. Hence the bright-fringe positions satisfy

$$\frac{dy_n}{D} + \frac{\lambda}{4} = n\lambda, \quad y_n = \left(n - \frac{1}{4}\right) \frac{\lambda D}{d}.$$

Subtract positions for successive integer orders: $\beta = y_{n+1} - y_n = \lambda D/d$. The source-path offset cancels in this difference.

Step 3: Conclusion Constructive interference requires $(SS_2 + S_2P) - (SS_1 + S_1P) = n\lambda$. The fringe width is $\beta = \lambda D/d$.

Quick Tip to Solve: Include both source-to-slit and slit-to-screen paths; a fixed source-path offset shifts fringe positions but cancels when their spacing is calculated.

Section C (36 marks)

13. Use Bohr's postulates to derive the expressions for the potential and kinetic energy of the electron moving in the n^{th} orbit of the hydrogen atom. How is the total energy of the electron expressed in terms of its kinetic and potential energies? [3 marks]

Solution:

Concept:

- Bohr's angular-momentum postulate is $m_e v_n r_n = n\hbar$, with $\hbar = h/(2\pi)$, electron mass m_e , orbit radius r_n , and positive integer n .
- The Coulomb attraction supplies centripetal force: $m_e v_n^2 / r_n = k e^2 / r_n^2$, where $k = 1/(4\pi\epsilon_0)$ and e is the magnitude of electron charge.
- Take potential energy to be zero for electron and proton at infinite separation; their attractive Coulomb potential energy is $U(r) = -ke^2/r$.

Step 1: Find the allowed orbital radius From angular-momentum quantisation, $v_n = n\hbar/(m_e r_n)$. Substitution into the centripetal-force equation yields

$$\frac{n^2 \hbar^2}{m_e r_n^3} = \frac{k e^2}{r_n^2}, \quad r_n = \frac{n^2 \hbar^2}{m_e k e^2} = \frac{\epsilon_0 n^2 h^2}{\pi m_e e^2}.$$

The allowed radius grows as n^2 , taking the proton as stationary.

Step 2: Derive the kinetic energy Multiplying the force equation by r_n gives $m_e v_n^2 = k e^2 / r_n$. Therefore

$$K_n = \frac{1}{2} m_e v_n^2 = \frac{k e^2}{2 r_n} = \frac{m_e k^2 e^4}{2 n^2 \hbar^2} = \boxed{\frac{m_e e^4}{8 \epsilon_0^2 h^2 n^2}}.$$

Numerically, $K_n = 13.6/n^2$ eV.

Step 3: Derive the potential energy Using the zero at infinite separation, the electron-proton pair has

$$U_n = -\frac{k e^2}{r_n} = -\frac{m_e k^2 e^4}{n^2 \hbar^2} = \boxed{-\frac{m_e e^4}{4 \epsilon_0^2 h^2 n^2}}.$$

Comparison gives $U_n = -2K_n$, or numerically $U_n = -27.2/n^2$ eV.

Step 4: Express the total energy Total mechanical energy is the sum of kinetic and potential energy:

$$E_n = K_n + U_n = K_n - 2K_n = -K_n = \frac{U_n}{2}.$$

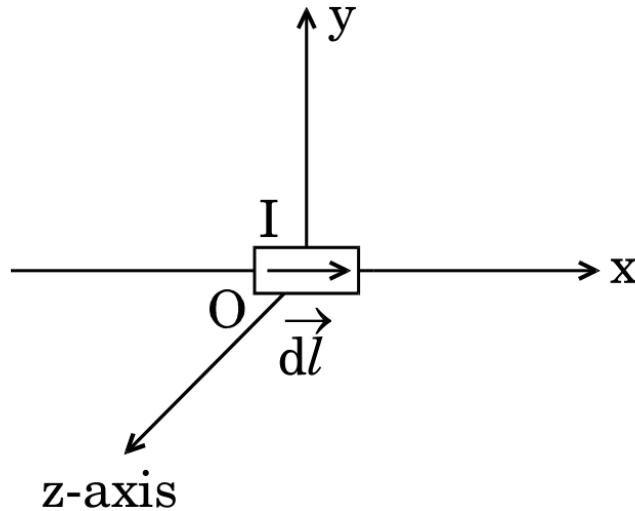
Consequently $E_n = -m_e e^4 / (8 \epsilon_0^2 h^2 n^2) = -13.6/n^2$ eV. The negative total energy signifies a bound electron.

Step 5: Conclusion $K_n = \frac{m_e e^4}{8 \epsilon_0^2 h^2 n^2}$, $U_n = -2K_n$, and $E_n = K_n + U_n = -K_n = U_n/2$.

Quick Tip to Solve: After obtaining $m_e v_n^2 = k e^2 / r_n$, compare it with $U_n = -k e^2 / r_n$: the relations $U_n = -2K_n$ and $E_n = -K_n$ follow immediately.

14. (a) Depict the magnetic field lines due to a circular current carrying loop showing the direction of field lines.

(b) A current I is flowing in a conductor placed along the x-axis as shown in the figure. Find the magnitude and direction of the magnetic field due to a small current element $d\vec{l}$ lying at the origin at points (i) $(0, d, 0)$ and (ii) $(0, 0, d)$. [3 marks]



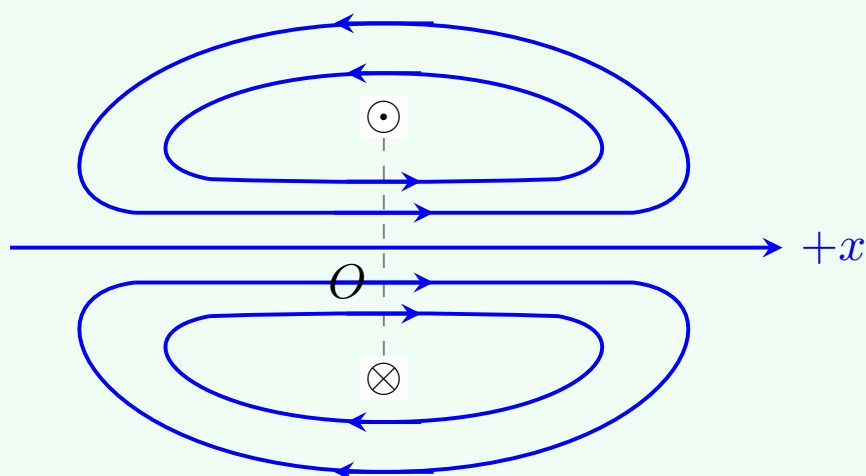
Solution:

Concept:

- Biot-Savart law for a small current element: $d\vec{B} = \frac{\mu_0}{4\pi} \frac{I d\vec{l} \times \hat{r}}{r^2}$, where $\hat{r}$ points from the element towards the observation point.
- Magnetic field lines form closed curves. The right-hand grip rule gives the field direction for a circular loop: curl the fingers with current and the thumb gives the axial field direction.

Step 1: Draw the field of the circular loop The sketch shows a side view of a loop in the yz -plane. The upper dot denotes current towards the viewer ($+z$); the lower cross denotes current away from the viewer ($-z$). Thus the field through the centre is along $+x$. Near the centre the lines are nearly straight and parallel, while outside the loop they bend round and return, making closed curves. Reversing the current reverses every field arrow.

Quick Tip to Solve: Use the cyclic products $\hat{i} \times \hat{j} = \hat{k}$ and $\hat{k} \times \hat{i} = \hat{j}$; reversing their order changes the sign.



$\odot$: I towards viewer ($+z$)
 $\otimes$: I away from viewer ($-z$)

Side view of a circular current loop and its magnetic field lines

Step 2: Calculate the field at $(0, d, 0)$ The printed arrow shows current along $+x$, so $d\vec{l} = dl \hat{i}$. At $P = (0, d, 0)$, the displacement is $\vec{r} = d \hat{j}$, perpendicular to the element. Therefore

$$d\vec{B}_P = \frac{\mu_0 I dl}{4\pi d^2} (\hat{i} \times \hat{j}) = \frac{\mu_0 I dl}{4\pi d^2} \hat{k}.$$

The magnitude is $\mu_0 I dl / (4\pi d^2)$, and the direction is the positive z -axis. This is the contribution of the small element only, rather than the field of an entire long wire.

Step 3: Calculate the field at $(0, 0, d)$ At $Q = (0, 0, d)$, $\vec{r} = d \hat{k}$, again perpendicular to $d\vec{l}$. Using $\hat{i} \times \hat{k} = -\hat{j}$,

$$d\vec{B}_Q = \frac{\mu_0 I dl}{4\pi d^2} (\hat{i} \times \hat{k}) = -\frac{\mu_0 I dl}{4\pi d^2} \hat{j}.$$

Hence the magnitude equals that at P , since the distance and the angle are the same, but the direction is the negative y -axis.

Step 4: Conclusion The loop has closed magnetic field lines directed along its axis through the centre. For the printed current element, $dB = \mu_0 I dl / (4\pi d^2)$ at both points: along $+z$ at $(0, d, 0)$, and along $-y$ at $(0, 0, d)$.

Quick Tip to Solve: Use the cyclic products $\hat{i} \times \hat{j} = \hat{k}$ and $\hat{k} \times \hat{i} = \hat{j}$; reversing their order changes the sign.

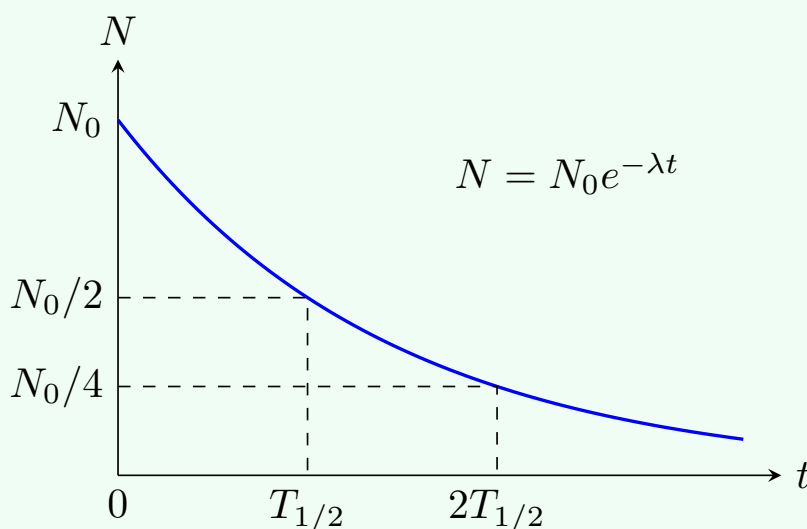
15. Plot a graph showing the variation of number of undecayed nuclei in a radioisotope as a function of time. Hence define half life of a nucleus and obtain its relation with average life. [3 marks]

Solution:

Concept:

- Radioactive decay obeys $dN/dt = -\lambda N$, where N is the number of undecayed nuclei and λ is the decay constant. Thus $N(t) = N_0 e^{-\lambda t}$.
- Half-life $T_{1/2}$ is the time for the undecayed population to fall to half its initial value. Average life τ is the mean time a nucleus survives before decay.

Step 1: Plot the decay curve The graph starts at $N = N_0$ when $t = 0$ and decreases exponentially towards zero. It does not reach zero at any finite time in the continuous decay model. At $t = T_{1/2}$, $N = N_0/2$; at $t = 2T_{1/2}$, $N = N_0/4$. Each equal half-life interval halves the remaining population, so the curve becomes progressively less steep.



Exponential decay of the number of undecayed nuclei

Step 2: Obtain the half-life By definition, substituting $N = N_0/2$ and $t = T_{1/2}$ into the decay law gives

$$\frac{N_0}{2} = N_0 e^{-\lambda T_{1/2}}, \quad e^{-\lambda T_{1/2}} = \frac{1}{2}.$$

Taking natural logarithms yields $\lambda T_{1/2} = \ln 2$, and therefore

$$T_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda}.$$

Half-life is a statistical property of a large collection of nuclei, rather than a fixed decay time for each individual nucleus.

Step 3: Obtain average life and relate it to half-life The number of nuclei decaying between t and

Quick Tip to Solve: Mark $N_0/2$ at one half-life and $N_0/4$ at two half-lives; this distinguishes exponential decay from a straight-line decrease.

$t + dt$ is $-dN = \lambda N_0 e^{-\lambda t} dt$. The mean survival time is therefore

$$\begin{aligned}\tau &= \frac{1}{N_0} \int_0^\infty t \lambda N_0 e^{-\lambda t} dt \\ &= \lambda \int_0^\infty t e^{-\lambda t} dt = \frac{1}{\lambda}.\end{aligned}$$

Here integration by parts gives $\int_0^\infty t e^{-\lambda t} dt = 1/\lambda^2$. Combining this result with the half-life expression establishes the required relation.

Step 4: Conclusion $T_{1/2} = (\ln 2)\tau \simeq 0.693\tau$, or equivalently $\tau \simeq 1.443T_{1/2}$. The decay graph is $N = N_0 e^{-\lambda t}$.

Quick Tip to Solve: Mark $N_0/2$ at one half-life and $N_0/4$ at two half-lives; this distinguishes exponential decay from a straight-line decrease.

16. (a) For the circuit shown in the figure, how would the balancing length be affected, if

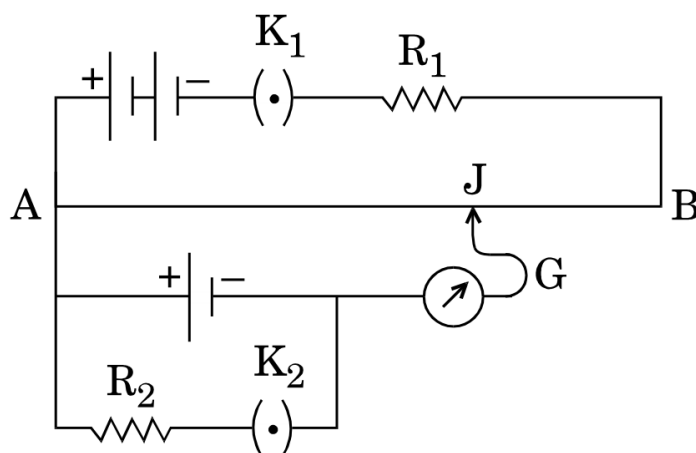
(i) R_1 is decreased,

(ii) R_2 is increased,

the other factors remaining the same in the circuit? Justify your answer in each case.

(b) Why is a potentiometer preferred over a voltmeter? Give reason.

[3 marks]



Solution:

Concept:

- At potentiometer balance, the potential difference being measured is $V = kl$, where k is the potential gradient along the uniform wire and $l = AJ$ is the balancing length.
- For wire length L , resistance R_{AB} , and primary current I_p , $k = I_p R_{AB}/L$. A cell of emf E and internal resistance r , supplying load R_2 , has terminal voltage $V = ER_2/(R_2 + r)$.

Step 1: Effect of decreasing the primary resistance R_1 With K_1 closed, R_1 controls the current in the primary circuit containing the driving cell and wire AB . Decreasing R_1 decreases the total primary resistance, so the wire current I_p increases. Consequently $k = I_p R_{AB}/L$ increases. The secondary

Quick Tip to Solve: Trace which circuit a resistor belongs to: R_1 changes the wire gradient, whereas the closed R_2 branch changes the test cell terminal voltage.

cell voltage remains unchanged because its circuit is unchanged. Since $l = V/k$, the balancing length decreases: the jockey balance point moves towards A .

Step 2: Effect of increasing the load resistance R_2 With K_2 closed, R_2 is the external load across the secondary cell. Increasing R_2 reduces the current drawn from that cell and therefore reduces the internal voltage drop Ir . Its terminal voltage consequently increases, as is also seen from

$$V = \frac{ER_2}{R_2 + r}.$$

The primary circuit and potential gradient k are unchanged. Thus $l = V/k$ increases and the balance point moves towards B , provided a balance lies within AB . If K_2 is open, R_2 is disconnected: changing it has no effect and the potentiometer measures emf. For an ideal cell with $r = 0$, changing the load likewise produces no change.

Step 3: Explain the preference over a voltmeter At the balance point, the galvanometer current is zero, so the potentiometer measuring branch draws no current from the test cell. With the load disconnected, it measures the true emf without causing any internal voltage drop. A voltmeter of finite resistance draws a small current from the cell; its reading is then the terminal potential difference $E - Ir$, which is less than the emf. The potentiometer is therefore preferred for accurate emf measurement by the null method.

Step 4: Conclusion Decreasing R_1 decreases the balancing length. Increasing R_2 increases the balancing length when K_2 is closed and the test cell has internal resistance. A potentiometer measures emf without drawing current from the cell at balance.

Quick Tip to Solve: Trace which circuit a resistor belongs to: R_1 changes the wire gradient, whereas the closed R_2 branch changes the test cell terminal voltage.

OR

16. State the underlying principle of meter bridge. Draw the circuit diagram and explain how the unknown resistance of a conductor can be determined by this method. [3 marks]

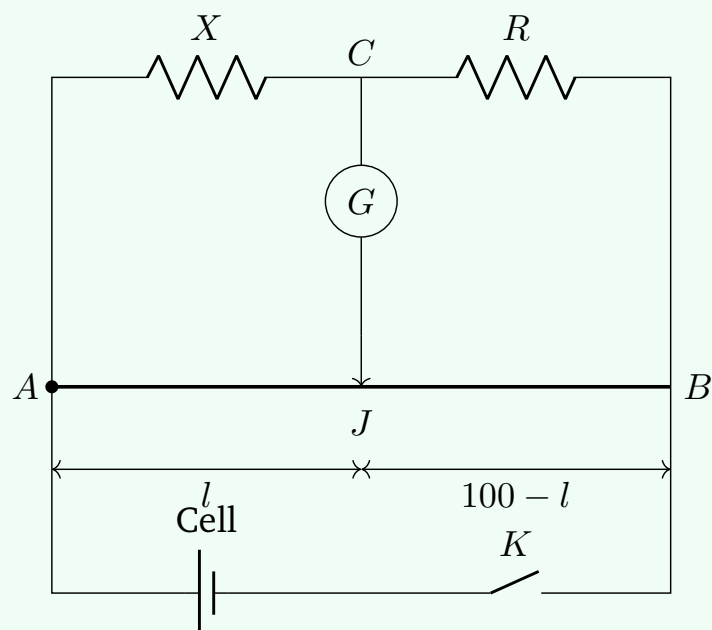
Solution:

Concept:

- A metre bridge works on the balanced Wheatstone bridge principle: when the galvanometer current is zero, the ratios of resistances in the two pairs of arms are equal.
- For a uniform wire, resistance is proportional to length: $R = \rho L/A$, where ρ is resistivity and A is its cross-sectional area.

Step 1: Connect the metre bridge Place the unknown resistance X in the left gap and a known resistance R , usually a resistance box, in the right gap. The uniform wire AB has length 100 cm. Connect a cell and key K across A and B , and a galvanometer G between the central junction C of the gap resistances and a sliding jockey J on the wire. The galvanometer detects the potential difference between C and J .

Quick Tip to Solve: Associate each wire length with the resistance in the gap on the same side; this prevents reversing the ratio.



Metre bridge: unknown X in the left gap, known R in the right gap, and jockey J at the null point.

Step 2: Obtain the null point and calculate the resistance Close the key and touch the jockey gently at different positions until the galvanometer shows zero deflection. If $AJ = l$ cm, then $JB = (100 - l)$ cm. At balance, C and J are at the same potential, so the Wheatstone bridge condition gives

$$\frac{X}{R} = \frac{R_{AJ}}{R_{JB}} = \frac{l}{100 - l}, \quad X = R \frac{l}{100 - l}.$$

The middle equality follows because both wire segments have the same material and cross-sectional area. Select R so that the balance point lies near the middle of the wire, where the measurement is more reliable. Repeat with other suitable known resistances and average the estimates. Interchanging X and R and repeating can reduce errors due to the end connections.

Step 3: Conclusion The unknown resistance is $X = Rl/(100 - l)$, with l measured from the end next to X . The measurement uses the null-deflection condition of a Wheatstone bridge.

Quick Tip to Solve: Associate each wire length with the resistance in the gap on the same side; this prevents reversing the ratio.

17. (a) Draw a graph showing the variation of current versus voltage in an electrolyte when an external resistance is also connected.

(b)(i) The graph between resistance (R) and temperature (T) for Hg is shown in the figure (a). Explain the behaviour of Hg near 4 K.

(ii) In which region of the graph shown in the figure (b) is the resistance negative, and why? [3 marks]

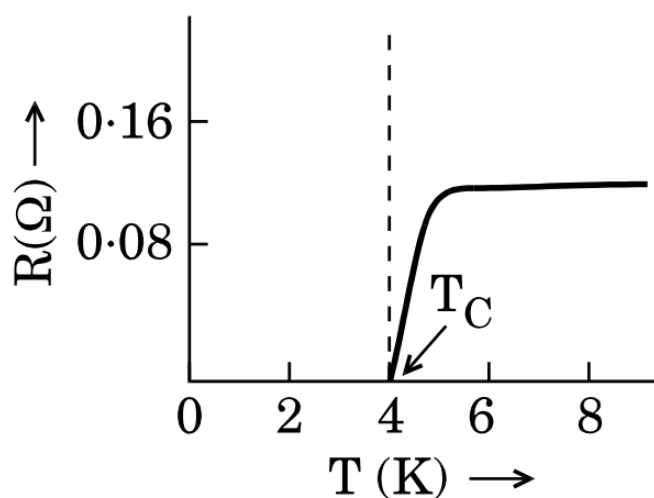


Figure (a)

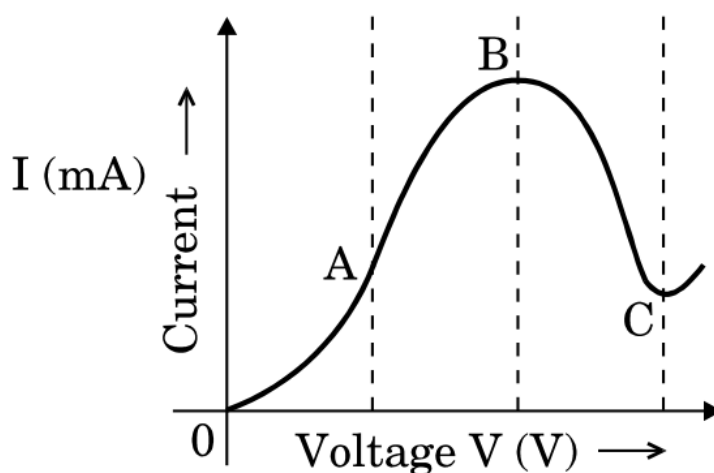


Figure (b)

Solution:

Concept:

- An electrolyte develops a back emf or decomposition threshold V_0 ; appreciable steady electrolysis current requires an applied voltage sufficient to overcome it.
- A superconductor has zero electrical resistance below its critical temperature T_c . Negative differential resistance means $r_d = dV/dI < 0$, not a negative value of the ratio V/I .

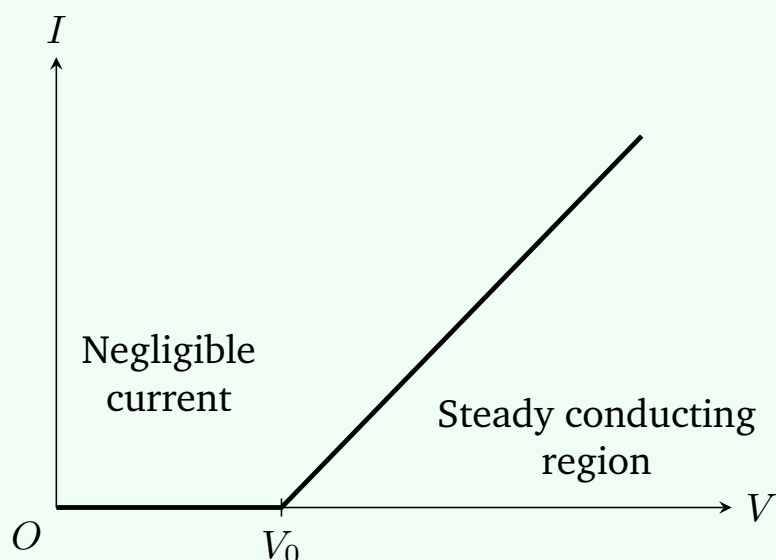
Step 1: (a) Draw the current-voltage characteristic Plot current I vertically and the applied voltage

Quick Tip to Solve: On an I-V graph, a descending segment indicates negative differential resistance; do not judge it using V/I .

V horizontally. With an external resistance in series, the steady current is negligible below the decomposition voltage V_0 . Above this threshold it rises approximately linearly. If R_{ext} is the external resistance and r the electrolyte's effective resistance, then the idealised relation in the conducting region is

$$I = \frac{V - V_0}{R_{\text{ext}} + r}, \quad V > V_0.$$

Thus the rising line extrapolates to V_0 on the voltage axis rather than passing through the origin. The added series resistance reduces its slope. The diagram shows the idealised threshold and conducting region; the actual onset can be rounded.



Idealised I-V characteristic of an electrolyte with an external series resistance.

Step 2: (b)(i) Explain the behaviour of mercury near 4 K In figure (a), the resistance of Hg drops abruptly to zero on cooling through its critical temperature, approximately $T_c = 4.2$ K, shown near 4 K. Below T_c , mercury becomes superconducting and a current can persist without resistive dissipation. On heating above T_c , it returns to its ordinary conducting state and has a finite resistance. The sudden transition, rather than a gradual extrapolation to zero, identifies superconductivity.

Step 3: (b)(ii) Identify the negative-resistance region In figure (b), the current decreases as the applied voltage increases from the peak B to the valley C . Therefore $dI/dV < 0$ on the descending segment BC , so its differential resistance dV/dI is negative. The increasing segments before B and after C have positive differential resistance. The plotted voltage and current remain positive, so the ordinary static resistance V/I is positive; the word negative here describes the incremental response.

Step 4: Conclusion (a) The electrolyte characteristic has a voltage threshold followed by an approximately linear rise. (b)(i) Hg becomes superconducting below its critical temperature near 4 K. (b)(ii) Region BC has negative differential resistance because current falls while voltage rises.

Quick Tip to Solve: On an I-V graph, a descending segment indicates negative differential resistance; do not judge it using V/I .

18. (a) Briefly explain how a galvanometer is converted into an ammeter.

(b) A galvanometer coil has a resistance of $15\ \Omega$ and it shows full scale deflection for a current of $4\ \text{mA}$. Convert it into an ammeter of range 0 to 6 A. [3 marks]

Solution:

Concept:

- A galvanometer is converted into an ammeter by connecting a low resistance, called a shunt S , in parallel with its coil.
- Parallel branches have the same potential difference. If the coil resistance is G , the full-scale coil current is I_g , and the required full-scale total current is I , then $I_g G = (I - I_g)S$.

Step 1: (a) Explain the conversion into an ammeter Connect a small shunt resistance across the galvanometer terminals. Most of the measured current then flows through the shunt, while only a small, safe fraction flows through the galvanometer coil. Choose the shunt so that, at the desired maximum current, the coil carries exactly its full-scale current. Calibrate the scale to read the total current through the parallel combination. The resulting low-resistance instrument is connected in series in the circuit whose current is measured; its low resistance minimises the change in that current.

Step 2: (b) Calculate the required shunt resistance The given coil resistance is $G = 15\ \Omega$, full-scale galvanometer current is $I_g = 4\ \text{mA} = 0.004\ \text{A}$, and required full-scale current is $I = 6\ \text{A}$. Thus the shunt carries

$$I_s = I - I_g = 6 - 0.004 = 5.996\ \text{A}.$$

Equating the potential differences across the two parallel branches gives

$$\begin{aligned} I_g G &= I_s S, \\ S &= \frac{I_g G}{I - I_g} = \frac{0.004 \times 15}{5.996}\ \Omega \\ &= 0.0100067\ \Omega \approx 0.0100\ \Omega. \end{aligned}$$

As a check, the coil voltage at full scale is $0.004 \times 15 = 0.060\ \text{V}$. The calculated shunt also has $I_s S = 0.060\ \text{V}$, as required. Its resistance is far smaller than $15\ \Omega$, so almost the whole current bypasses the delicate coil.

Step 3: Conclusion Connect a shunt of $S \approx 0.0100\ \Omega$ in parallel with the galvanometer to obtain an ammeter of range 0 to 6 A.

Quick Tip to Solve: Convert the coil current from mA to A first, then use the shunt current $I - I_g$ in the denominator.

18. (a) Briefly explain how a galvanometer is converted into a voltmeter.

(b) A voltmeter of a certain range is constructed by connecting a resistance of $980\ \Omega$ in series with a galvanometer. When the resistance of $470\ \Omega$ is connected in series, the range gets halved. Find the resistance of the galvanometer. [3 marks]

Solution:

Concept:

- A galvanometer measures current. A high resistance connected in series limits the current so that a specified potential difference produces full-scale deflection.
- If G is the galvanometer resistance, I_g its full-scale current and R_s the series resistance, the voltmeter range is $V = I_g(G + R_s)$.

Step 1: Explain conversion into a voltmeter Connect a suitable high resistance R_s in series with the galvanometer. The combination is connected in parallel across the two points whose potential difference is to be measured. For a desired range 0 to V , choose

$$R_s = \frac{V}{I_g} - G.$$

At the maximum indicated voltage the current is exactly I_g ; at smaller voltages it is proportionally smaller. The scale can therefore be calibrated in volts. The high total resistance reduces the current drawn from the measured circuit and helps avoid disturbing its potential difference.

Step 2: Write the two full-scale voltage equations The same galvanometer is used in both arrangements, so I_g is unchanged. With a series resistance of $980\ \Omega$, let the range be V . With $470\ \Omega$, the range is $V/2$. Hence

$$\begin{aligned} V &= I_g(G + 980\ \Omega), \\ \frac{V}{2} &= I_g(G + 470\ \Omega). \end{aligned}$$

Dividing the first equation by the second eliminates both the unknown full-scale current and the original range.

Step 3: Solve for the coil resistance The ratio gives

$$\begin{aligned} G + 980\ \Omega &= 2(G + 470\ \Omega), \\ 980\ \Omega - 940\ \Omega &= G. \end{aligned}$$

As a check, the two total resistances are $1020\ \Omega$ and $510\ \Omega$; their ratio is exactly two, as required for the ranges at the same full-scale current.

Step 4: Conclusion A galvanometer becomes a voltmeter by adding a high series resistance and calibrating its scale in volts. The galvanometer resistance is $G = 40\ \Omega$.

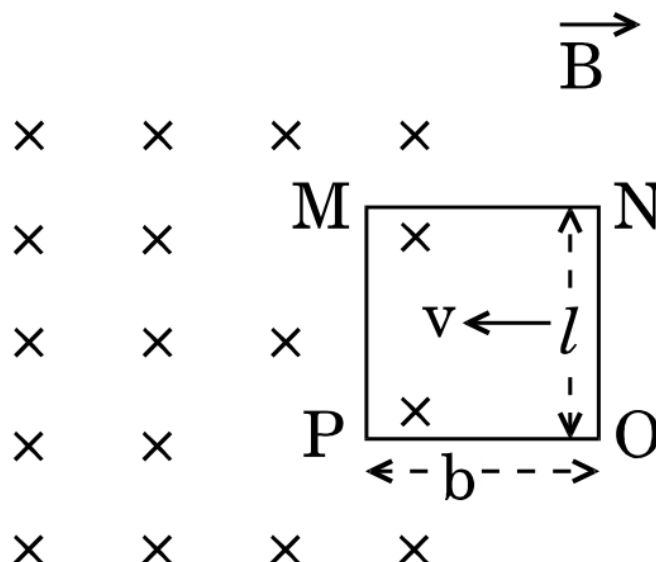
Quick Tip to Solve: Compare total resistances, including the galvanometer coil: halving the range halves $G + R_s$, not the external resistance alone.

19. The figure shows a rectangular conducting frame MNOP of resistance R placed partly in a perpendicular magnetic field $\vec{B}$ and moved with velocity $\vec{v}$ as shown in the figure. Obtain the expressions for the

(a) force acting on the arm 'ON' and its direction, and

(b) power required to move the frame to get a steady emf induced between the arms MN and PO.

[3 marks]



Solution:

Concept:

- A conducting arm of length l , moving with speed v perpendicular to a uniform magnetic field B , develops motional emf $\varepsilon = Blv$. The frame resistance is R , so $I = Blv/R$.
- Magnetic force on an immersed current-carrying arm is $\vec{F} = I\vec{l} \times \vec{B}$; the external power needed for steady translation is $P = Fv = I^2R$.

Step 1: Find the induced current and its sense The frame moves left into the region marked by crosses, where the magnetic field is into the page. If its immersed area increases at rate lv , then the inward flux increases at rate Blv . Lenz's law requires an outward induced field, so the induced current is anticlockwise: down MP , along PO , up ON , then along NM . The emf and current magnitudes are

$$\varepsilon = Blv, \quad I = \frac{Blv}{R}.$$

These expressions hold while only one vertical arm is inside the uniform field. Once the entire frame is immersed, its flux no longer changes during uniform translation.

Step 2: Identify the arm actually acted on by the field In the printed figure the crosses lie on the left side and extend across MP ; the right-hand arm ON is outside that field region. Taken literally, the magnetic force on the stated arm ON is therefore

$$F_{ON} = 0.$$

The nonzero horizontal magnetic force acts on the immersed arm MP . Its current is downward and

Quick Tip to Solve: First locate which vertical arm lies within the field. Then use $\varepsilon = Blv$, $I = \varepsilon/R$, $F = BIl$ and $P = Fv$; verify that $P = I^2R$.

the field is inward, giving a rightward force, opposite the leftward motion:

$$F_{MP} = BIl = \frac{B^2 l^2 v}{R}.$$

If the reference to ON is intended to name the active immersed arm, the usual requested drag-force magnitude is $B^2 l^2 v/R$, directed against the velocity. A force cannot be assigned to a segment outside the magnetic field merely because it carries induced current.

Step 3: Calculate the power supplied at constant speed To maintain the leftward velocity, the applied force must balance the rightward magnetic drag. The transverse forces on the horizontal arms cancel, so the required mechanical power is

$$P = F_{MP}v = \frac{B^2 l^2 v^2}{R}.$$

This equals the heat generated in the frame:

$$I^2 R = \left(\frac{Blv}{R} \right)^2 R = \frac{B^2 l^2 v^2}{R}.$$

The steady induced emf between the horizontal arms is maintained by constant B , l and v during partial immersion.

Step 4: Conclusion For the literal printed placement, $F_{ON} = 0$; the active immersed arm experiences drag $F = B^2 l^2 v/R$ opposite $\vec{v}$. The power required is $P = B^2 l^2 v^2/R$. The arm named in part (a) is inconsistent with the depicted field placement.

Quick Tip to Solve: First locate which vertical arm lies within the field. Then use $\varepsilon = Blv$, $I = \varepsilon/R$, $F = BIl$ and $P = Fv$; verify that $P = I^2 R$.

20. Draw a ray diagram to show the image formation of a distant object by a refracting telescope. Write the expression for its angular magnification in terms of the focal lengths of the lenses used. State the important considerations required to achieve large resolution and their consequent limitations.

[3 marks]

Solution:

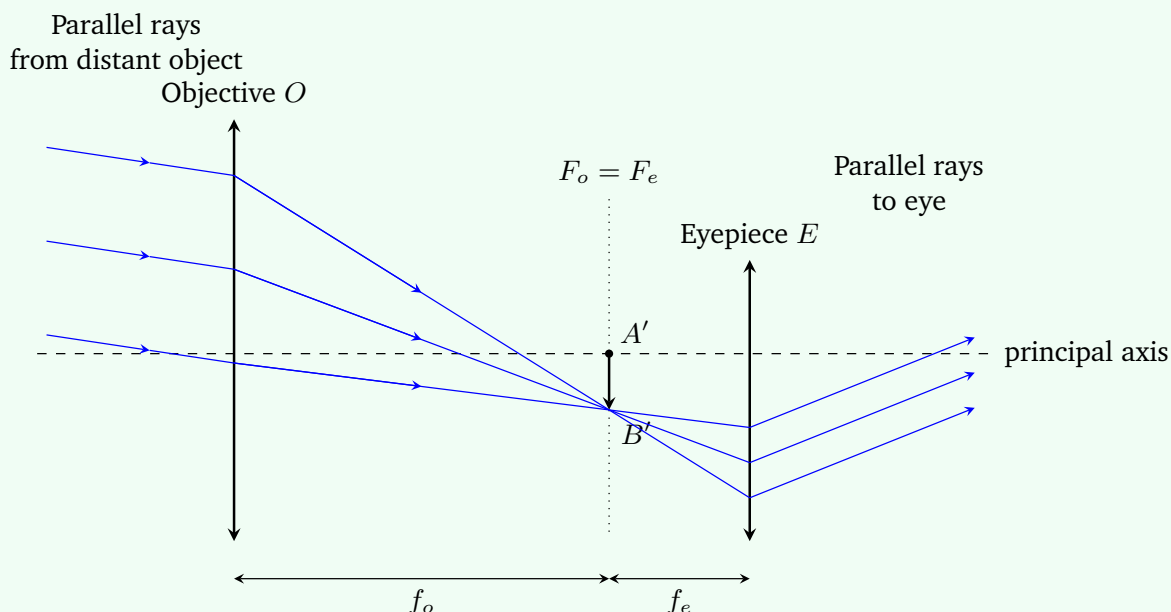
Concept:

- A refracting astronomical telescope uses a convex objective of long focal length f_o and a convex eyepiece of short focal length f_e . The objective forms a real inverted image of a distant object.
- In normal adjustment, the objective's image lies at the eyepiece's first focal plane. The final image is at infinity, and angular magnification is $M = -f_o/f_e$.
- Diffraction limits angular resolution: for a circular objective aperture of diameter D , the minimum resolvable angular separation is approximately $\theta_{\min} = 1.22\lambda/D$, where λ is wavelength.

Step 1: Trace rays for normal adjustment Parallel rays from a distant off-axis point enter the objective O and converge to a real, inverted intermediate image $A'B'$ in its focal plane. Place the eyepiece E so

Quick Tip to Solve: Keep the two design requirements distinct: f_o/f_e controls magnification, whereas objective diameter D controls diffraction-limited resolution.

that this image is also in the eyepiece's first focal plane, marked $F_o = F_e$. The eyepiece then sends the rays out parallel; the eye sees the magnified virtual final image at infinity. The lens separation is $f_o + f_e$. The schematic below shows an off-axis bundle; arrows indicate propagation from left to right.



Refracting telescope in normal adjustment: the objective image is in the common focal plane, and emergent rays are parallel.

Step 2: Obtain angular magnification If the intermediate image has height magnitude h , the small incident angle has magnitude $\alpha \simeq h/f_o$, while the emergent angle has magnitude $\beta \simeq h/f_e$. Therefore

$$|M| = \frac{\beta}{\alpha} = \frac{f_o}{f_e}, \quad M = -\frac{f_o}{f_e}.$$

The negative sign states that the final image is inverted. Thus a long focal length objective and short focal length eyepiece increase angular magnification, but magnification alone does not improve resolution.

Step 3: Explain how to increase resolution and its limitations A large diameter objective reduces the diffraction angle $1.22\lambda/D$, letting the instrument distinguish two nearby distant points. It also collects more light, making faint objects visible. For a specified wavelength the resolving power is proportional to D ; shorter wavelengths also reduce the diffraction limit. However, making a large refracting objective requires a heavy lens whose surfaces must be accurately ground and polished. A lens is supported mainly at its rim, so its weight can deform it and spoil the image. Large lenses are costly and difficult to fabricate from homogeneous glass; absorption and chromatic aberration impose further limitations. Atmospheric turbulence also limits practical ground-based resolution, even when the objective's diffraction limit is small.

Step 4: Conclusion In normal adjustment the lens separation is $f_o + f_e$ and $M = -f_o/f_e$. High resolution requires a large objective aperture, but lens fabrication, weight and deformation, optical aberrations and the atmosphere restrict practical performance.

Quick Tip to Solve: Keep the two design requirements distinct: f_o/f_e controls magnification, whereas objective diameter D controls diffraction-limited resolution.

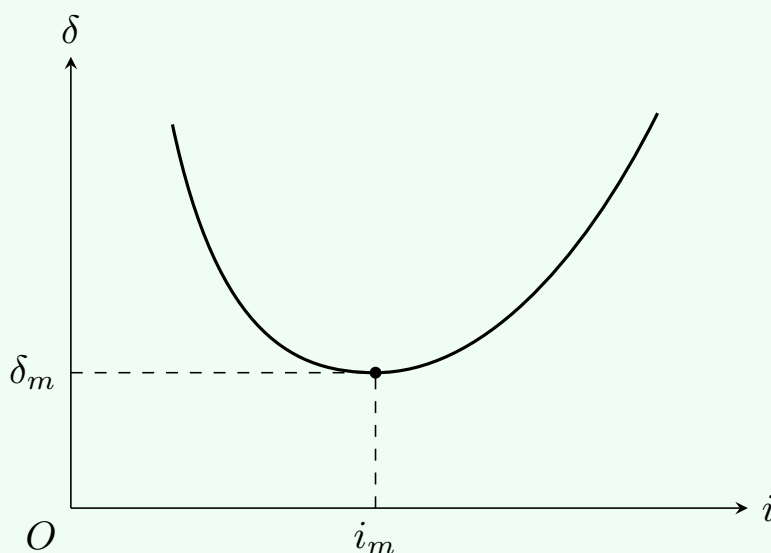
20. (a) Plot a graph for angle of deviation as a function of angle of incidence for a triangular prism.
 (b) Derive the relation for the refractive index of the prism in terms of the angle of minimum deviation and angle of prism.

[3 marks]

Solution:**Concept:**

- For a prism of angle A , the internal refraction angles obey $r_1 + r_2 = A$, and the deviation is $\delta = i + e - A$, where i and e are the incidence and emergence angles.
- At minimum deviation δ_m , the light path is symmetric: $i = e = i_m$ and $r_1 = r_2 = A/2$. Snell's law for a prism in air is $\mu = \sin i / \sin r_1$.

Step 1: (a) Plot deviation against incidence Take the angle of incidence i on the horizontal axis and deviation δ on the vertical axis. For the rays which emerge from the prism, deviation first decreases as incidence increases, reaches the minimum δ_m at $i = i_m$, and then increases. Thus the curve has a rounded minimum. Its two branches represent the two reciprocal light paths through the prism; the minimum corresponds to equal incidence and emergence angles. The diagram is schematic, so the angles are not assigned numerical values.



Schematic deviation-incidence curve with its minimum labelled.

Step 2: (b) Use the symmetric path at minimum deviation The prism geometry gives $r_1 + r_2 = A$. Since the minimum-deviation path is symmetric, $r_1 = r_2$, hence each internal angle is $A/2$. Also, using $\delta = i + e - A$ with $i = e = i_m$,

$$\delta_m = 2i_m - A, \quad i_m = \frac{A + \delta_m}{2}.$$

Applying Snell's law at the air-prism entrance gives

$$\mu = \frac{\sin i_m}{\sin(A/2)} = \frac{\sin[(A + \delta_m)/2]}{\sin(A/2)}.$$

Quick Tip to Solve: At minimum deviation write $i = e$ and $r_1 = r_2$ first; these two equalities immediately supply both angles needed in Snell's law.

Here μ is the refractive index of the prism relative to air, for the wavelength used in measuring the minimum deviation.

Step 3: Conclusion The deviation-incidence graph has its minimum at the symmetric path, and the required relation is $\mu = \frac{\sin[(A + \delta_m)/2]}{\sin(A/2)}$.

Quick Tip to Solve: At minimum deviation write $i = e$ and $r_1 = r_2$ first; these two equalities immediately supply both angles needed in Snell's law.

21. Draw the energy band diagram of (i) n-type, and (ii) p-type semiconductors at temperature $T > 0$ K.

In the case of n-type Si-semiconductor, the donor energy level is slightly below the bottom of conduction band whereas in p-type semiconductor, the acceptor energy level is slightly above the top of valence band. Explain, giving examples, what role do these energy levels play in conduction and valence bands. [3 marks]

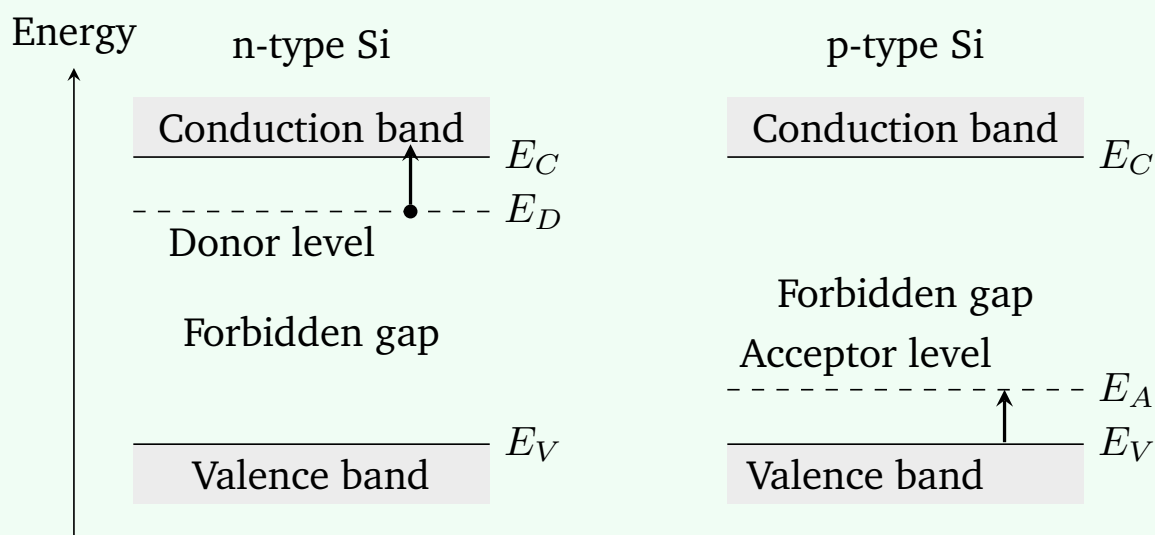
Solution:

Concept:

- Conduction requires mobile charge carriers: conduction-band electrons or valence-band holes. Doping silicon creates impurity levels within the forbidden gap between the conduction-band edge E_C and valence-band edge E_V .
- The donor level E_D lies just below E_C , whereas the acceptor level E_A lies just above E_V . These small energy separations permit thermal ionisation at ordinary temperatures without exciting carriers across the full band gap.

Step 1: Draw the impurity levels in the two band diagrams The diagrams show increasing electron energy upwards. In n-type silicon, the donor level is inside the forbidden gap close to the bottom of the conduction band. In p-type silicon, the acceptor level is inside the gap close to the top of the valence band. The arrows identify the electron transitions that create mobile carriers at $T > 0$ K. The band diagrams are schematic and their energy separations are not to scale.

Quick Tip to Solve: Track the electron arrow: donor to conduction band creates an electron, whereas valence band to acceptor leaves a hole.



Impurity energy levels and thermally activated electron transitions in doped silicon.

Step 2: Explain donor action in n-type silicon A pentavalent impurity such as phosphorus or arsenic supplies five valence electrons. Four form covalent bonds with neighbouring silicon atoms; the fifth is weakly bound and occupies a donor state. Only the small energy $E_C - E_D$ is required to transfer it to the conduction band. Thermal energy therefore releases donor electrons, leaving fixed positive donor ions. These free conduction-band electrons carry current and are the majority carriers; holes are minority carriers. At very low nonzero temperatures the donors need not all be ionised.

Step 3: Explain acceptor action in p-type silicon A trivalent impurity such as boron or aluminium has one electron fewer than needed for four complete covalent bonds. Its acceptor state can receive an electron from the valence band using the small energy $E_A - E_V$. This electron occupies the acceptor level and leaves a hole in the valence band. The acceptor becomes a fixed negative ion, while the hole moves as neighbouring valence electrons fill the vacancy. Holes are the majority mobile carriers in p-type material; electrons are minority carriers.

Step 4: Conclusion Donors provide mobile electrons to the conduction band; acceptors remove electrons from the valence band and thereby create mobile holes. Their proximity to the respective band edges makes thermal carrier generation much easier than excitation across the full forbidden gap.

Quick Tip to Solve: Track the electron arrow: donor to conduction band creates an electron, whereas valence band to acceptor leaves a hole.

22. (a) State briefly, with what purpose was Davisson and Germer experiment performed and what inference was drawn from this.

(b) Obtain an expression for the ratio of the accelerating potentials required to accelerate a proton and an α -particle to have the same de-Broglie wavelength associated with them. [3 marks]

Solution:

Concept:

- De Broglie's relation $\lambda = h/p$ associates wavelength λ with momentum p , where h is Planck's constant. Diffraction is evidence of wave behaviour.
- For a nonrelativistic particle of mass m and positive charge q , initially at rest and accelerated through a potential difference V , $K = qV = p^2/(2m)$, giving $\lambda = h/\sqrt{2mqV}$.

Step 1: (a) State the purpose and inference of the experiment Davisson and Germer studied the scattering of electrons by a nickel crystal to test the wave nature of electrons and verify de Broglie's hypothesis. They directed an accelerated electron beam onto the crystal and measured the scattered-electron intensity at different angles. The crystal's regularly spaced atomic planes act as a diffraction grating for electron waves. Sharp intensity maxima, corresponding to electron diffraction, were observed. The wavelength obtained from the diffraction condition agreed with the wavelength h/p calculated from the accelerating potential. Thus the experiment established that moving electrons exhibit wave behaviour and experimentally confirmed the de Broglie relation.

Step 2: (b) Set the wavelengths of the proton and alpha particle equal Let V_p and V_α be the accelerating potentials for the proton and alpha particle. Equality of their wavelengths implies equality of their momenta, since the same constant h appears in both de Broglie relations. Equivalently,

$$\frac{h}{\sqrt{2m_p e V_p}} = \frac{h}{\sqrt{2m_\alpha (2e) V_\alpha}}.$$

Squaring and cancelling the common factors gives

$$m_p e V_p = m_\alpha (2e) V_\alpha, \quad \frac{V_p}{V_\alpha} = \frac{2m_\alpha}{m_p}.$$

Using the standard approximation $m_\alpha = 4m_p$, we obtain $V_p/V_\alpha = 8$. The alpha particle's larger mass and double charge mean that a much smaller accelerating potential is needed for it to reach the same momentum.

Step 3: Conclusion Electron diffraction in the Davisson-Germer experiment verified the wave nature of electrons. For equal de Broglie wavelengths, $V_p/V_\alpha = 2m_\alpha/m_p \simeq 8$, so the required potential ratio is $V_p : V_\alpha = 8 : 1$.

Quick Tip to Solve: Equal wavelength means equal momentum, so set the products $m q V$ equal; remember that an alpha particle has charge $2e$ as well as mass about $4m_p$.

22. (a) An electron and a proton are accelerated through the same potential. Which one of the two has

(i) greater value of de-Broglie wavelength associated with it, and

(ii) lesser momentum?

Justify your answer in each case.

(b) How is the momentum of a particle related with its de-Broglie wavelength? Show the variation on a graph. [3 marks]

Solution:

Concept:

- The de-Broglie relation is $\lambda = h/p$, where h is Planck's constant and p is the magnitude of the particle's momentum.
- On acceleration from rest through a potential difference of magnitude V , a particle of charge magnitude e gains kinetic energy eV . In the non-relativistic regime, $p^2/(2m) = eV$.

Step 1: (a)(i) Compare the wavelengths Both the electron and proton have the same charge magnitude e , so their kinetic energies after acceleration through the same potential are equal. Using $p = \sqrt{2meV}$,

$$\lambda = \frac{h}{\sqrt{2meV}}, \quad \frac{\lambda_e}{\lambda_p} = \sqrt{\frac{m_p}{m_e}} > 1.$$

Since the electron's mass is much smaller, the electron has the greater de-Broglie wavelength. Numerically the wavelength ratio is about $\sqrt{1836} \simeq 43$. Equal kinetic energy does not imply equal momentum when the masses differ.

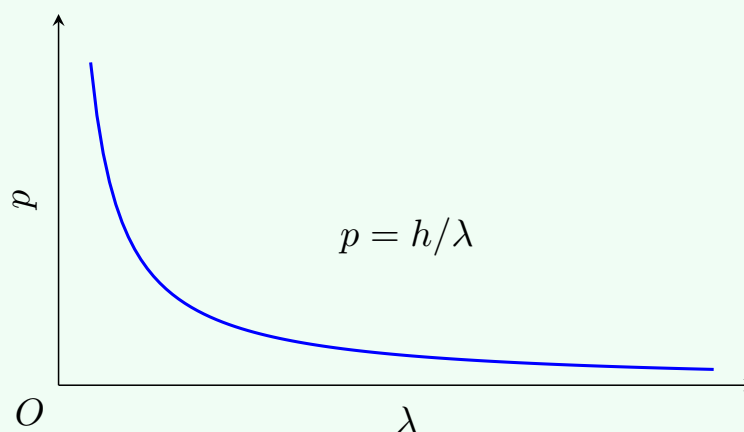
Step 2: (a)(ii) Compare the momenta At fixed accelerating potential the momentum varies as the square root of mass:

$$\frac{p_e}{p_p} = \sqrt{\frac{m_e}{m_p}} < 1.$$

Thus the electron has the lesser momentum. This also agrees with the inverse wavelength-momentum relation: the particle having the longer wavelength must have the smaller momentum.

Step 3: (b) Obtain the momentum-wavelength graph Rearranging the de-Broglie equation gives $p = h/\lambda$, or $p\lambda = h$. Therefore momentum is inversely proportional to wavelength. The plot of p against positive λ is a decreasing rectangular hyperbola in the first quadrant. Its two coordinate axes are asymptotes: for very small wavelength the momentum is large, while increasing wavelength reduces the momentum towards zero. Doubling the wavelength halves the momentum; the graph is not a straight line.

Quick Tip to Solve: Equal accelerating potentials give equal kinetic energies for these two particles; compare masses using $p \propto \sqrt{m}$, then invert to compare wavelengths.



Momentum varies inversely with de-Broglie wavelength.

Step 4: Conclusion The electron has both the greater de-Broglie wavelength and the lesser momentum. For any particle, $p = h/\lambda$; its momentum-wavelength graph is a rectangular hyperbola.

Quick Tip to Solve: Equal accelerating potentials give equal kinetic energies for these two particles; compare masses using $p \propto \sqrt{m}$, then invert to compare wavelengths.

23. (a) What is amplitude modulation? Draw a diagram showing an amplitude modulated wave obtained by modulation of a carrier sinusoidal wave on a modulating signal.

(b) Define the terms (i) modulation index, and (ii) side bands. Mention the significance of side bands. [3 marks]

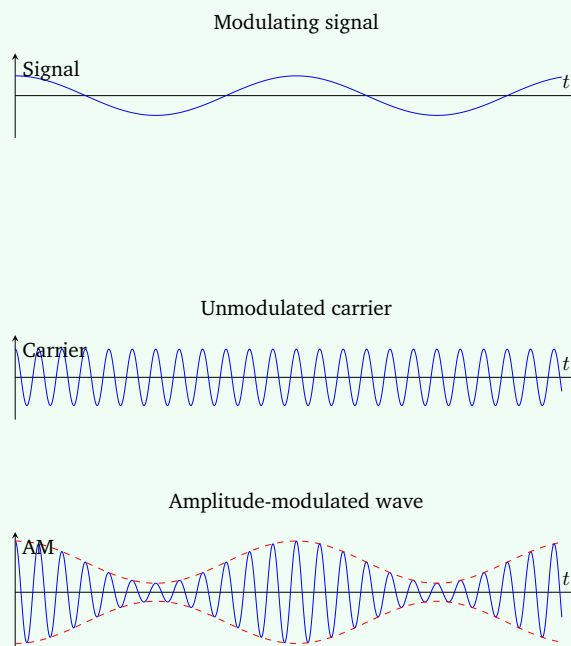
Solution:

Concept:

- A high-frequency carrier transports a low-frequency information signal. In amplitude modulation (AM), its amplitude varies according to the instantaneous value of the signal.
- For a sinusoidal signal, the AM voltage is $v(t) = A_c[1 + \mu \cos(\omega_m t)] \cos(\omega_c t)$, where A_c is the carrier amplitude and μ is the modulation index.

Step 1: (a) Define amplitude modulation and show the waveform Amplitude modulation is the process of varying the amplitude of a high-frequency sinusoidal carrier in accordance with the instantaneous amplitude of the modulating signal, keeping the carrier frequency and phase constant. In the diagram, the signal is slowly varying, the unmodulated carrier has constant amplitude, and the AM wave has an envelope that reproduces the signal's shape. The carrier oscillations remain rapid even where their amplitude becomes smaller. The dashed curves are the upper and lower envelopes, not additional carrier waves.

Quick Tip to Solve: Trace the outer envelope to identify the message in an AM wave; use its largest and smallest heights to find the modulation index.



A sinusoidal signal, constant-amplitude carrier and resulting AM wave (schematic).

Step 2: (b)(i) Define modulation index The modulation index is the ratio of the modulating-signal amplitude A_m to the unmodulated carrier amplitude A_c :

$$\mu = \frac{A_m}{A_c} = \frac{A_{\max} - A_{\min}}{A_{\max} + A_{\min}}.$$

Here $A_{\max}$ and $A_{\min}$ are the maximum and minimum envelope amplitudes. For faithful ordinary envelope detection, $0 \leq \mu \leq 1$; $\mu > 1$ causes overmodulation and distortion.

Step 3: (b)(ii) Identify side bands and their significance Expanding the AM expression using the product-to-sum identity gives

$$\begin{aligned} v(t) &= A_c \cos(\omega_c t) \\ &+ \frac{\mu A_c}{2} \cos[(\omega_c + \omega_m)t] \\ &+ \frac{\mu A_c}{2} \cos[(\omega_c - \omega_m)t]. \end{aligned}$$

Side bands are the frequency components generated on either side of the carrier frequency by modulation: the upper side band is at $f_c + f_m$ and the lower side band at $f_c - f_m$. For a general signal they form bands rather than single frequencies. They carry the information contained in the modulating signal; an unmodulated carrier alone carries no message. For highest signal frequency f_m , the required AM bandwidth is $2f_m$.

Step 4: Conclusion AM varies the carrier amplitude with the message signal. Its modulation index is $\mu = A_m/A_c$; the information-bearing side bands lie above and below f_c , at $f_c \pm f_m$ for a single sinusoidal signal.

Quick Tip to Solve: Trace the outer envelope to identify the message in an AM wave; use its largest and smallest heights to find the modulation index.

24. How is Zener diode fabricated? Draw its I-V characteristic and use this to explain the working of a Zener diode as a voltage regulator. [3 marks]

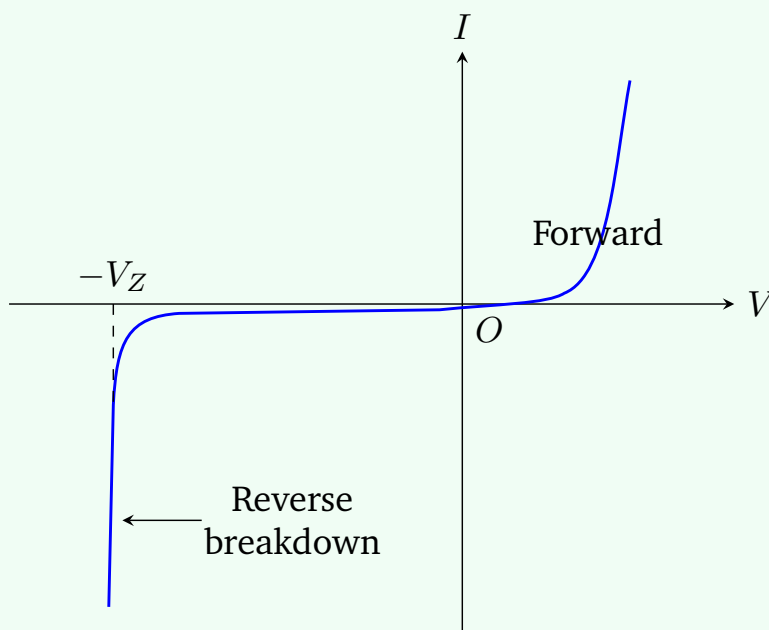
Solution:

Concept:

- A Zener diode is designed to operate in reverse breakdown. Its reverse voltage remains nearly constant at the Zener voltage V_Z over an allowed range of reverse currents.
- A series resistor limits current; the reverse-biased Zener is connected across the load to keep the load voltage approximately equal to V_Z .

Step 1: Describe fabrication A Zener diode is fabricated by heavily doping both the p and n sides of a p-n junction. Heavy doping produces a very thin depletion layer. Consequently a strong electric field develops across the junction even at a relatively small reverse voltage. The doping is selected to obtain the desired breakdown voltage. It is designed to withstand reverse breakdown within its current and power ratings, provided the external circuit limits the current.

Step 2: Explain the I-V characteristic In forward bias the diode behaves like an ordinary p-n junction diode: current rises rapidly after the forward knee. In reverse bias, only a small leakage current flows until the reverse voltage reaches V_Z . Beyond this knee, a large change in reverse current produces only a very small change in voltage. The steep reverse-breakdown branch shown near $V = -V_Z$ is the basis of voltage regulation. On the graph, signed reverse voltage and current are both negative; V_Z denotes the positive magnitude of the breakdown voltage.



Zener I-V characteristic with its steep reverse-breakdown branch.

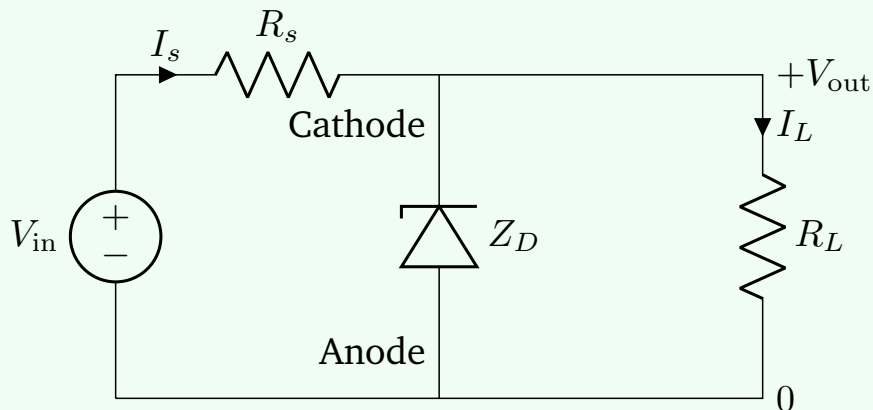
Step 3: Explain the regulator circuit Connect the Zener in reverse bias across the load R_L , with its cathode at the positive output and anode at the return. Connect a resistor R_s between the supply and

Quick Tip to Solve: In a Zener regulator, put the cathode at the positive output and remember that changes in supply or load are accommodated mainly by changes in Zener current.

this parallel combination. In breakdown,

$$V_{\text{out}} \simeq V_Z, \quad I_s = \frac{V_{\text{in}} - V_Z}{R_s} = I_Z + I_L.$$

If the input voltage increases, the extra voltage falls across R_s , and the Zener current increases while the output remains approximately V_Z . If the input falls, the Zener current decreases. Likewise, when the load current increases, the Zener takes less current; when the load current decreases, it takes more. Thus the diode absorbs the current variation while maintaining nearly constant load voltage. Regulation requires I_Z to remain above the breakdown knee and below its maximum rated current; R_s provides essential current limiting.



Zener voltage regulator; the diode cathode connects to the positive output.

Step 4: Conclusion Heavy doping gives the Zener a thin depletion region and a controlled reverse-breakdown voltage. Its nearly constant voltage in breakdown, with a series current-limiting resistor and the diode across the load, provides $V_{\text{out}} \simeq V_Z$.

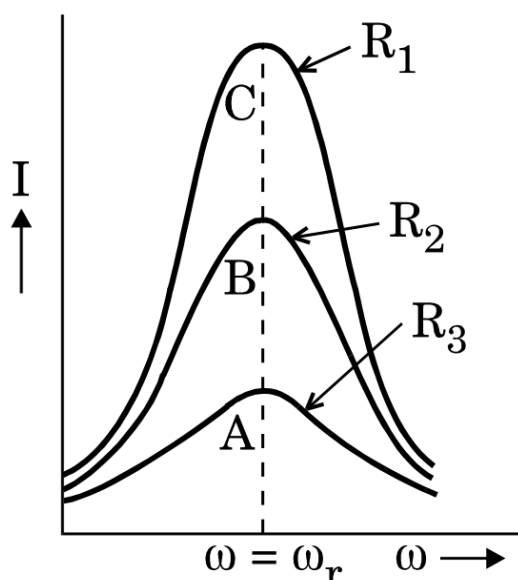
Quick Tip to Solve: In a Zener regulator, put the cathode at the positive output and remember that changes in supply or load are accommodated mainly by changes in Zener current.

Section D (15 marks)

25. (a) What do you understand by ‘sharpness of resonance’ for a series LCR resonant circuit? How is it related with the quality factor ‘Q’ of the circuit? Using the graphs given in the diagram, explain the factors which affect it. For which graph is the resistance (R) minimum?

(b) A $2\ \mu\text{F}$ capacitor, $100\ \Omega$ resistor and $8\ \text{H}$ inductor are connected in series with an ac source. Find the frequency of the ac source for which the current drawn in the circuit is maximum.

If the peak value of emf of the source is $200\ \text{V}$, calculate the (i) maximum current, and (ii) inductive and capacitive reactance of the circuit at resonance. [5 marks]



Solution:

Concept:

- In a series LCR circuit, the impedance is $Z = \sqrt{R^2 + (\omega L - 1/\omega C)^2}$, where R is resistance, L inductance, C capacitance and ω the source angular frequency. The current amplitude is $I_0 = E_0/Z$.
- Resonance occurs when $X_L = X_C$, so $\omega_r = 1/\sqrt{LC}$, $f_r = 1/(2\pi\sqrt{LC})$, and the impedance takes its minimum value R .
- The quality factor is $Q = \omega_r L/R = (1/R)\sqrt{L/C}$. Equivalently, $Q = \omega_r/\Delta\omega$, where $\Delta\omega$ is the bandwidth between the two half-power frequencies.

Step 1: (a) Meaning of sharpness and its relation to quality factor Sharpness of resonance describes how narrowly the large current response is confined around the resonant frequency. A sharp resonance has a tall, narrow current-frequency peak: the current falls rapidly when the source frequency moves away from ω_r . A broad peak corresponds to less sharp resonance and poorer frequency selectivity. At the half-power frequencies, the current is $I_{0,\text{max}}/\sqrt{2}$; their separation is $\Delta\omega$. For a series circuit,

$$\Delta\omega = \frac{R}{L}, \quad Q = \frac{\omega_r}{\Delta\omega} = \frac{\omega_r L}{R}.$$

Thus a larger Q means a smaller bandwidth relative to the resonant frequency and a sharper resonance. Peak height alone is not the definition of sharpness; the width of the response must also be considered.

Quick Tip to Solve: At series resonance first set $X_L = X_C$ and $Z = R$; keep the given peak emf paired with peak current, and divide angular frequency by 2π to obtain hertz.

Step 2: (a) Interpret the given three resonance curves For the same source amplitude and fixed L and C , all three curves peak at the common angular frequency ω_r . At resonance,

$$I_{0,\max} = \frac{E_0}{R}.$$

Reducing R raises the peak current, reduces $\Delta\omega = R/L$, and increases Q ; increasing R lowers and broadens the peak. The upper, narrowest curve C , labelled R_1 , therefore has the smallest resistance. The intermediate curve B is labelled R_2 , and the lowest, broadest curve A is labelled R_3 , giving $R_1 < R_2 < R_3$. More generally, $Q = (1/R)\sqrt{L/C}$ shows that, with the other quantities fixed, increasing L or decreasing C increases sharpness as measured by Q . Changing either L or C also shifts the resonant frequency; the supplied graphs specifically illustrate the effect of changing resistance with a common resonant frequency.

Step 3: (b) Find the source frequency for maximum current The current is maximum when the inductive and capacitive reactances cancel. Convert the capacitance to SI units: $C = 2 \times 10^{-6}$ F, while $L = 8$ H and $R = 100 \Omega$. Hence

$$\omega_r = \frac{1}{\sqrt{8(2 \times 10^{-6})}} = \frac{1}{4 \times 10^{-3}} = 250 \text{ rad s}^{-1},$$

$$f_r = \frac{250}{2\pi} = 39.79 \text{ Hz}.$$

The angular frequency and ordinary frequency differ by 2π ; the requested source frequency is expressed in hertz.

Step 4: (b)(i) Calculate the maximum current At resonance the circuit impedance is purely resistive, $Z_r = R = 100 \Omega$. Since the specified emf of 200 V is a peak value, it must be used with the peak current:

$$I_{0,\max} = \frac{E_0}{R} = \frac{200}{100} = 2.0 \text{ A}.$$

If an rms current is required, its value is $I_{\text{rms}} = I_{0,\max}/\sqrt{2} = 1.414$ A. The maximum instantaneous current asked for here is the peak current.

Step 5: (b)(ii) Evaluate the two reactances at resonance Using $\omega_r = 250 \text{ rad s}^{-1}$,

$$X_L = \omega_r L = 250 \times 8 = 2000 \Omega,$$

$$X_C = \frac{1}{\omega_r C} = \frac{1}{250(2 \times 10^{-6})} = 2000 \Omega.$$

These equal reactance magnitudes give opposite reactive voltage contributions, so their net contribution to impedance is zero. The individual reactances are large even though the circuit impedance is only 100Ω at resonance.

Step 6: Conclusion (a) Larger $Q = \omega_r/\Delta\omega = (1/R)\sqrt{L/C}$ means sharper resonance. Curve C , labelled R_1 , has minimum resistance, with $R_1 < R_2 < R_3$. (b) The required frequency is $f_r \approx 39.8$ Hz, the maximum peak current is 2.0 A, and $X_L = X_C = 2000 \Omega$.

Quick Tip to Solve: At series resonance first set $X_L = X_C$ and $Z = R$; keep the given peak emf paired with peak current, and divide angular frequency by 2π to obtain hertz.

25. (a) Draw a schematic diagram of an ac generator. Explain its working and obtain the expression for the instantaneous value of the emf in terms of the magnetic field B , number of turns N of the coil of area A rotating with angular frequency ω . Show how an alternating emf is generated by a loop of wire rotating in a magnetic field.

(b) A circular coil of radius 10 cm and 20 turns is rotated about its vertical diameter with angular speed of 50 rad s^{-1} in a uniform horizontal magnetic field of $3.0 \times 10^{-2} \text{ T}$.

(i) Calculate the maximum and average emf induced in the coil.

(ii) If the coil forms a closed loop of resistance 10Ω , calculate the maximum current in the coil and the average power loss due to Joule heating. [5 marks]

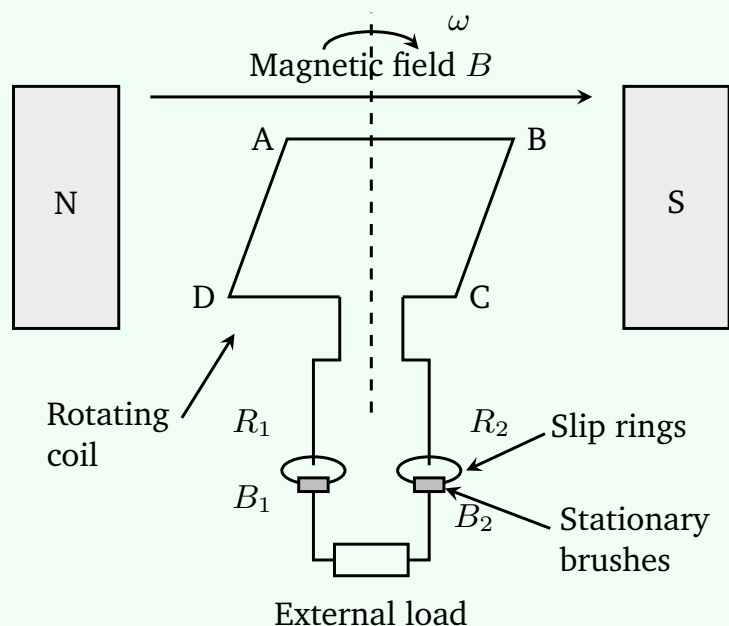
Solution:

Concept:

- Faraday's law gives the induced emf: $e = -d\Phi_N/dt$, where Φ_N is the magnetic flux linked with all N turns.
- For a plane coil of area A in uniform field B , $\Phi_N = NBA \cos \theta$, where θ is the angle between the field and the coil's normal.
- For sinusoidal emf across a resistance R , $I_0 = e_0/R$ and $P_{\text{av}} = I_0^2 R/2$. The signed average emf over a complete cycle is zero.

Step 1: (a) Describe the generator and its working An ac generator converts mechanical energy into electrical energy by electromagnetic induction. A rectangular coil $ABCD$, with N turns, rotates between the north and south poles of a magnet. Its two ends connect separately to two continuous slip rings R_1 and R_2 , which rotate with the coil. Stationary brushes B_1 and B_2 touch the rings and connect the coil to the external load. As the coil rotates, its orientation relative to the magnetic field changes and hence its linked flux changes. This produces an emf. The rotation is maintained by an external mechanical source. Continuous slip rings retain the connections to the two coil ends as their polarity reverses; consequently, the external current reverses direction every half revolution.

Quick Tip to Solve: Convert the radius to metres before using $A = \pi r^2$. For ac, specify the averaging interval: use $2e_0/\pi$ for the positive half cycle and $e_0^2/(2R)$ for mean heating power.



Schematic ac generator: the coil and slip rings rotate; brushes and the load remain stationary.

Step 2: Derive the instantaneous emf and show its alternation Choose $t = 0$ when the coil's normal is parallel to the magnetic field. For uniform rotation, $\theta = \omega t$. The linked flux and emf are therefore

$$\begin{aligned}\Phi_N(t) &= NBA \cos(\omega t), \\ e(t) &= -\frac{d}{dt}[NBA \cos(\omega t)] \\ &= NBA\omega \sin(\omega t) = e_0 \sin(\omega t).\end{aligned}$$

The peak magnitude is $e_0 = NBA\omega$. The emf is zero when the normal is parallel or antiparallel to the field, and has maximum magnitude when the normal is perpendicular to the field. During the first half revolution $\sin(\omega t)$ is positive; during the next half it is negative. Thus the induced emf alternates in sign. Reversing the chosen terminal polarity reverses the overall sign but does not change the amplitude.

Step 3: (b)(i) Calculate the maximum and average emf Convert the radius to SI units: $r = 10 \text{ cm} = 0.10 \text{ m}$. The field is horizontal and the rotation axis is the vertical diameter, so the coil's normal sweeps through all angles to the field.

$$\begin{aligned}A &= \pi r^2 = \pi(0.10)^2 = 0.01\pi \text{ m}^2, \\ e_0 &= NBA\omega \\ &= 20(3.0 \times 10^{-2})(0.01\pi)(50) \\ &= 0.30\pi \text{ V} \simeq 0.94 \text{ V}.\end{aligned}$$

The positive and negative contributions cancel over one complete rotation, of period $T = 2\pi/\omega$:

$$e_{\text{av,cycle}} = \frac{1}{T} \int_0^T e_0 \sin(\omega t) dt = 0.$$

Quick Tip to Solve: Convert the radius to metres before using $A = \pi r^2$. For ac, specify the averaging interval: use $2e_0/\pi$ for the positive half cycle and $e_0^2/(2R)$ for mean heating power.

If average emf is intended in the usual ac half-cycle sense, its value is

$$e_{\text{av,half}} = \frac{1}{\pi} \int_0^{\pi} e_0 \sin \theta \, d\theta = \frac{2e_0}{\pi} = 0.60 \text{ V}.$$

This is also the average magnitude over a complete cycle; it must be distinguished from the zero signed cycle average.

Step 4: (b)(ii) Calculate the peak current and mean heating loss Taking the closed loop to be purely resistive, with the specified resistance $R = 10 \, \Omega$, Ohm's law gives

$$I_0 = \frac{e_0}{R} = \frac{0.30\pi}{10} = 0.030\pi \text{ A} \simeq 0.094 \text{ A}.$$

Joule heating is $p(t) = i^2(t)R = I_0^2 R \sin^2(\omega t)$. Although the current's signed average is zero, its square is positive, and the cycle average of $\sin^2(\omega t)$ is $1/2$. Therefore,

$$\begin{aligned} P_{\text{av}} &= \frac{I_0^2 R}{2} = \frac{e_0^2}{2R} \\ &= \frac{(0.30\pi)^2}{2(10)} \text{ W} \\ &= 0.0045\pi^2 \text{ W} \simeq 0.044 \text{ W}. \end{aligned}$$

Step 5: Conclusion The generator produces $e(t) = NBA\omega \sin(\omega t)$, with opposite polarities in successive half rotations. For the given coil, the maximum emf is 0.94 V ; the signed full-cycle average is zero, while the positive half-cycle average is 0.60 V . The maximum current is 0.094 A , and the average Joule-heating power is 0.044 W .

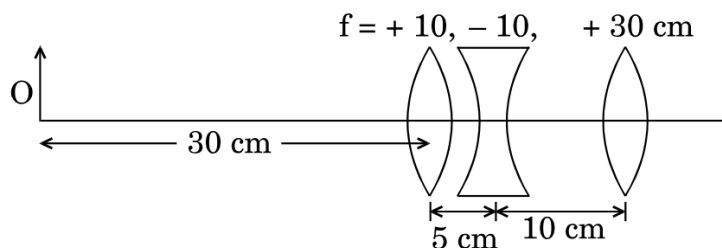
Quick Tip to Solve: Convert the radius to metres before using $A = \pi r^2$. For ac, specify the averaging interval: use $2e_0/\pi$ for the positive half cycle and $e_0^2/(2R)$ for mean heating power.

26. (a) Using the ray diagram for a system of two lenses of focal lengths f_1 and f_2 in contact with each other, show that the two lens system can be regarded as equivalent to a single lens of focal length f , where

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}.$$

Also write the relation for the equivalent power of the lens combination.

(b) Determine the position of the image formed by the lens combination given in the figure. [5 marks]



Solution:

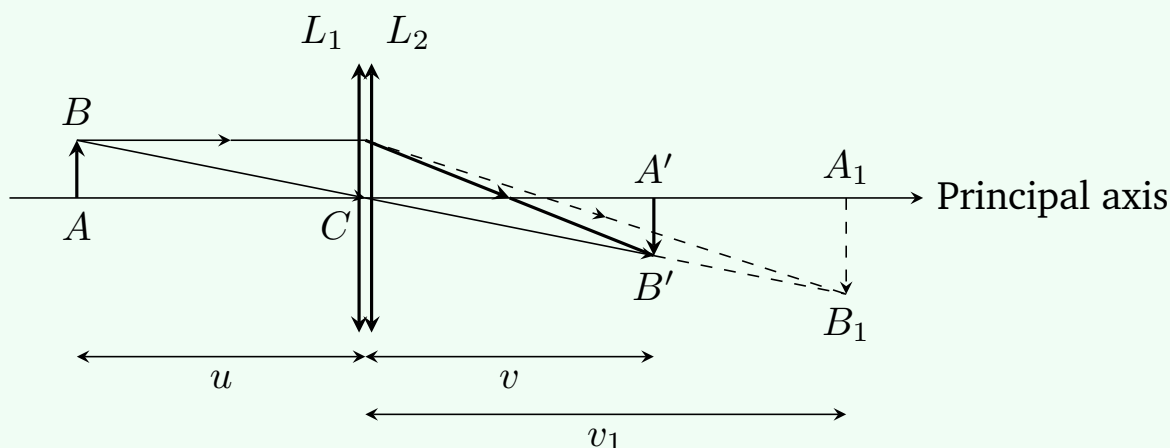
Concept:

- For a thin lens, the Cartesian lens formula is $1/v - 1/u = 1/f$. Distances are measured from the optical centre, positive in the direction of incident light; a real object on the left has $u < 0$.
- For thin lenses in contact, the image formed by the first lens acts as the object for the second lens at the same reference plane. Their powers add: $P = P_1 + P_2$, where $P_i = 1/f_i$ with focal lengths in metres and powers in dioptres.
- For separated lenses, use the lens formula successively, accounting for each separation. A converging incident beam gives a positive object distance, and parallel outgoing rays correspond to an image at infinity.

Step 1: (a) Ray diagram and the first lens Consider two thin lenses L_1 and L_2 in contact, with common optical centre C , and a paraxial object AB at distance u . The first lens alone would form the intermediate image A_1B_1 at distance v_1 . The second lens changes the convergence of these rays and produces the final image $A'B'$ at distance v . The ray through the common optical centre is undeviated. In the schematic below the dashed ray shows where the first lens alone would send the parallel incident ray; the solid refracted ray shows the actual final image after both lenses. Applying the lens formula to L_1 ,

$$\frac{1}{v_1} - \frac{1}{u} = \frac{1}{f_1}.$$

Quick Tip to Solve: At the second lens, first locate the prospective image of lens 1 relative to lens 2: it is on the right, so use a positive virtual-object distance.



Schematic paraxial rays for two thin converging lenses in contact; the dashed ray is the first lens's prospective ray.

Step 2: (a) Add the two lens equations Since the lenses are in contact, the signed object distance for L_2 is v_1 . When the first lens's prospective image lies to the right, it is a virtual object for the second lens, so this distance is positive. Thus

$$\frac{1}{v} - \frac{1}{v_1} = \frac{1}{f_2}.$$

Adding eliminates the intermediate image distance:

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f_1} + \frac{1}{f_2}.$$

An equivalent single lens at the same plane must produce the same v for the same u , and obeys $1/v - 1/u = 1/f$. Comparing gives

$$\boxed{\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}}, \quad \boxed{P = P_1 + P_2}.$$

The algebra uses signed focal lengths, so it applies to converging or diverging thin lenses, within the paraxial approximation and with negligible separation.

Step 3: (b) Image produced by the first lens The numerical figure has separated lenses, so the contact formula cannot be applied to the whole combination. Put the first lens at $x = 0$. The object is at $x = -30$ cm, the second lens at $x = 5$ cm, and the third at $x = 15$ cm. For the first lens, $u_1 = -30$ cm and $f_1 = +10$ cm:

$$\frac{1}{v_1} = \frac{1}{10} + \frac{1}{-30} = \frac{1}{15} \text{ cm}^{-1}, \quad v_1 = +15 \text{ cm}.$$

Without the other lenses, its rays would meet at $x = 15$ cm.

Step 4: (b) Effect of the diverging second lens At the second lens, the incident rays are converging towards a point $15 - 5 = 10$ cm to its right. This is a virtual object, hence $u_2 = +10$ cm, while $f_2 = -10$ cm. Therefore

$$\frac{1}{v_2} = \frac{1}{f_2} + \frac{1}{u_2} = -\frac{1}{10} + \frac{1}{10} = 0.$$

The second lens exactly cancels the incident convergence. Its outgoing rays are parallel, and the

Quick Tip to Solve: At the second lens, first locate the prospective image of lens 1 relative to lens 2: it is on the right, so use a positive virtual-object distance.

intermediate image is at infinity. The 10 cm gap before the third lens does not alter the parallel nature of this beam.

Step 5: (b) Focus of the third lens A parallel beam incident on the third converging lens forms a real image in its back focal plane. Equivalently, $u_3 = -\infty$, so

$$\frac{1}{v_3} = \frac{1}{30} + 0, \quad v_3 = +30 \text{ cm}.$$

Thus the final image is 30 cm to the right of the third lens. Its coordinate measured from the first lens is $x = 5 + 10 + 30 = 45 \text{ cm}$. The result follows from the virtual-object sign at the second lens, rather than treating all three lenses as touching.

Step 6: Conclusion (a) The equivalent focal length satisfies $\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$, and the equivalent power is $P = P_1 + P_2$. (b) The final image is real and lies **30 cm** to the right of the third lens, or 45 cm to the right of the first lens.

Quick Tip to Solve: At the second lens, first locate the prospective image of lens 1 relative to lens 2: it is on the right, so use a positive virtual-object distance.

OR

- 26. (a)** Explain, using a suitable diagram, how unpolarized light gets linearly polarized by scattering. **(b)** Describe briefly the variation of the intensity of transmitted light when a polaroid sheet kept between two crossed polaroids is rotated. Draw the graph depicting the variation of intensity with the angle of rotation. How many maxima and minima would be observed when θ varies from 0 to π ? [5 marks]

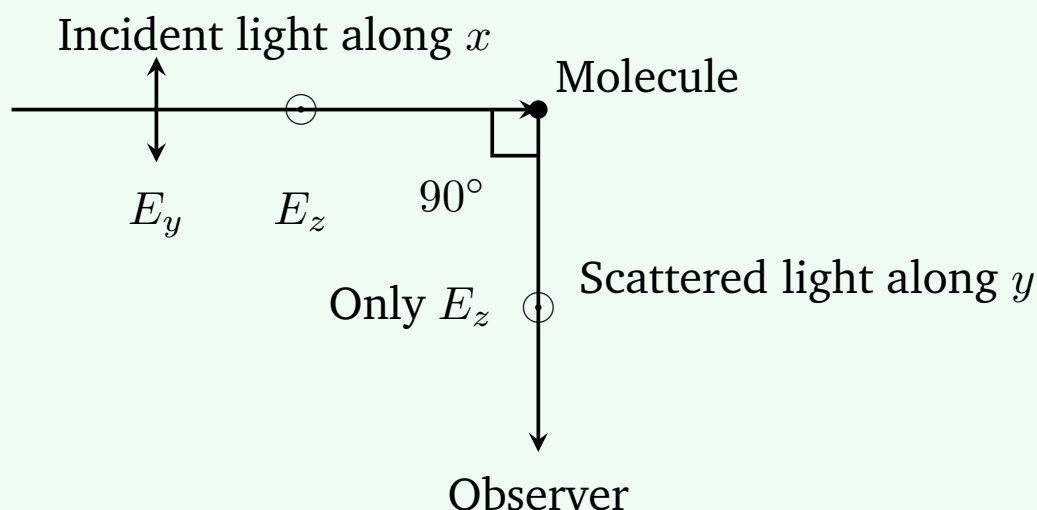
Solution:

Concept:

- Unpolarized light has electric-field vibrations in all directions perpendicular to its propagation. These can be resolved into two mutually perpendicular components.
- Scattering arises when the incident electric field drives oscillations of bound charges in molecules. An oscillating electric dipole radiates maximally perpendicular to its oscillation and does not radiate along its own axis.
- Malus's law is $I = I_{\text{in}} \cos^2 \alpha$, where α is the angle between the incident polarization and the transmission axis. An ideal first polaroid transmits half the intensity of an unpolarized beam.

Step 1: (a) Resolve the incident vibrations Let unpolarized light travel along the x -axis and observe the scattered light along the y -axis, at 90° to the incident direction. The xy -plane is the scattering plane. Resolve the incident electric field into its y and z components. These components drive molecular charges to oscillate along y and z , respectively. The diagram is a view of the scattering plane; dots denote electric-field vibrations perpendicular to that plane.

Quick Tip to Solve: Apply Malus's law twice, using the complementary angles at the two sheets; include both endpoints when counting the zero-intensity positions.



Right-angle scattering viewed in the scattering plane; circled dots indicate the surviving perpendicular electric-field component.

Step 2: (a) Select the polarization seen by the observer The charges oscillating along y cannot radiate towards the observer on their own oscillation axis. Charges oscillating along z do radiate in that direction. Thus only the z -component is present in the light scattered through 90° . Its electric field vibrates in one fixed direction, perpendicular to the scattering plane, so the observed scattered light is linearly polarized. This complete polarization refers to ideal single scattering at a right angle; scattering in general directions need not give complete polarization.

Step 3: (b) Apply Malus's law at the middle polaroid Call the fixed crossed polaroids P and A , with transmission axes at 0 and $\pi/2$. Let the middle sheet M make an angle θ with the axis of P . If the intensity after P is I_P , the intensity after M is

$$I_M = I_P \cos^2 \theta.$$

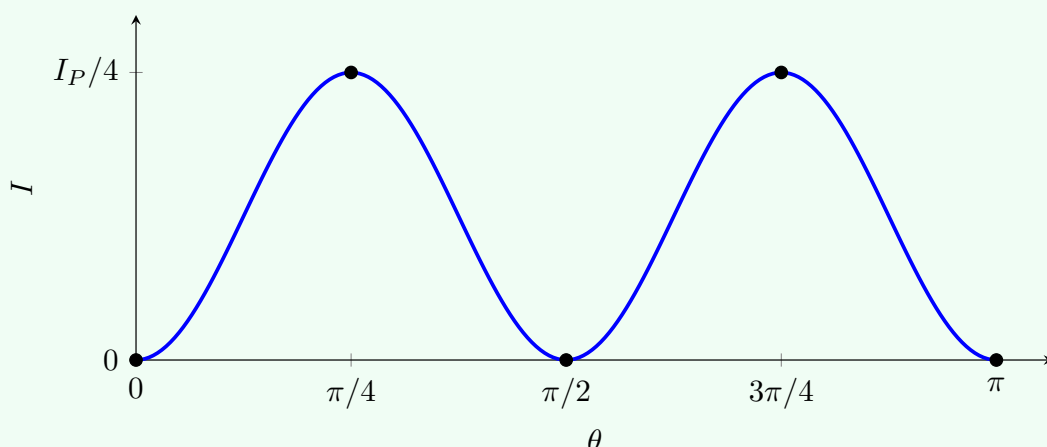
The emerging light is polarized along the axis of M , so its angle with the axis of A is $\pi/2 - \theta$.

Step 4: (b) Obtain and plot the final intensity Applying Malus's law once more gives

$$\begin{aligned} I(\theta) &= I_P \cos^2 \theta \cos^2(\pi/2 - \theta) \\ &= I_P \cos^2 \theta \sin^2 \theta = \frac{I_P}{4} \sin^2(2\theta). \end{aligned}$$

For unpolarized incident intensity I_0 , $I_P = I_0/2$, and hence $I(\theta) = I_0 \sin^2(2\theta)/8$. The middle sheet changes the polarization direction, allowing a component to pass through the final crossed sheet. The graph uses I_P as the intensity scale.

Quick Tip to Solve: Apply Malus's law twice, using the complementary angles at the two sheets; include both endpoints when counting the zero-intensity positions.



Transmitted intensity against middle-sheet rotation: two peaks and three zero-intensity positions on the closed interval.

Step 5: (b) Count the extrema over the stated rotation Maximum transmission occurs when $\sin^2(2\theta) = 1$, giving $\theta = \pi/4$ and $3\pi/4$. Minimum transmission occurs when $\sin(2\theta) = 0$, giving $\theta = 0, \pi/2, \pi$. Thus the closed interval from 0 to π contains two maxima and three minima, including the zero-intensity starting and ending positions. The peak intensity is $I_P/4$, or $I_0/8$ for unpolarized incident light.

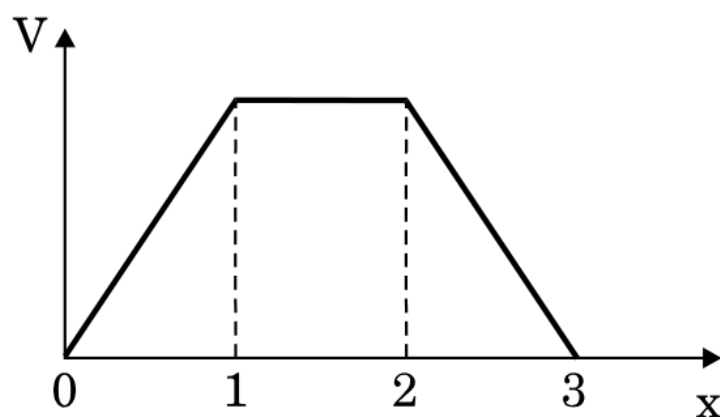
Step 6: Conclusion At 90° scattering, the scattered light is linearly polarized perpendicular to the scattering plane. With a rotating middle polaroid between crossed outer polaroids, $I = (I_P/4) \sin^2(2\theta)$: there are two maxima and three minima on $0 \leq \theta \leq \pi$.

Quick Tip to Solve: Apply Malus's law twice, using the complementary angles at the two sheets; include both endpoints when counting the zero-intensity positions.

27. (a) When a parallel plate capacitor is connected across a dc battery, explain briefly how the capacitor gets charged.

(b) A parallel plate capacitor of capacitance C is charged to V volt by a battery. After some time the battery is disconnected and the distance between the plates is doubled. Now a slab of dielectric constant $1 < k < 2$ is introduced to fill the space between the plates. How will the following be affected: (i) the electric field between the plates, (ii) the energy stored in the capacitor? Justify your answer in each case.

(c) The electric potential as a function of distance x is shown in a graph. Draw a graph of the electric field E as a function of x . [5 marks]



Solution:

Concept:

- A battery transfers charge between the capacitor plates until their potential difference equals the battery voltage. The plates carry equal and opposite free charges, with $Q = CV$.
- For an isolated capacitor, free charge remains constant. A parallel plate capacitor has $C = \epsilon_0 A/d$, and a completely filled dielectric gives $C' = k\epsilon_0 A/d'$, where A is the plate area and d' the new separation.
- The uniform field is $E = Q/(\epsilon A)$, stored energy is $U = Q^2/(2C)$, and the field along the x -direction is $E_x = -dV/dx$.

Step 1: (a) Explain charging by a dc battery When the plates are connected to the battery, its negative terminal supplies electrons to one plate, while its positive terminal removes electrons from the other. Thus the former becomes negatively charged and the latter positively charged, with equal charge magnitudes. Charge does not cross the insulating gap between the plates; it flows through the external connecting wires. Initially a charging current flows because the capacitor potential difference is below the battery voltage. As charge accumulates, the field and potential difference between the plates increase, opposing further charging. The charging current decreases and finally becomes zero when the capacitor voltage equals the battery voltage V . The final charges are $+CV$ and $-CV$, neglecting leakage.

Step 2: (b) Determine the new capacitance with the battery disconnected Let the original plate separation be d , so $C = \epsilon_0 A/d$. Immediately after disconnecting the battery, the free charge is $Q = CV$. Since there is no conducting path between the isolated plates, changing the separation and introducing the dielectric do not change this free charge. With the separation doubled and the dielectric filling the entire gap,

$$C' = \frac{k\epsilon_0 A}{2d} = \frac{kC}{2}, \quad V' = \frac{Q}{C'} = \frac{2V}{k}.$$

The dielectric becomes polarized, but polarization does not supply additional free charge to the plates.

Step 3: (b)(i) Find the change in electric field Initially $E_0 = V/d = Q/(\epsilon_0 A)$. Doubling the separation alone leaves the field unchanged for fixed free charge and plate area, in the ideal parallel plate approximation. After the dielectric is inserted, its polarization reduces the resultant field:

$$E' = \frac{Q}{k\epsilon_0 A} = \frac{E_0}{k}.$$

Equivalently, $E' = V'/(2d) = (2V/k)/(2d) = V/(kd)$. Since $1 < k < 2$, the final field is smaller than the initial field: $E_0/2 < E' < E_0$.

Step 4: (b)(ii) Find the change in stored energy Because the free charge remains fixed, use $U = Q^2/(2C)$, rather than treating the original battery voltage as fixed. Initially $U_0 = CV^2/2$. Hence

$$U' = \frac{Q^2}{2C'} = \frac{(CV)^2}{2(kC/2)} = \frac{CV^2}{k} = \frac{2}{k}U_0.$$

Therefore $U_0 < U' < 2U_0$: the final energy increases relative to the original charged capacitor.

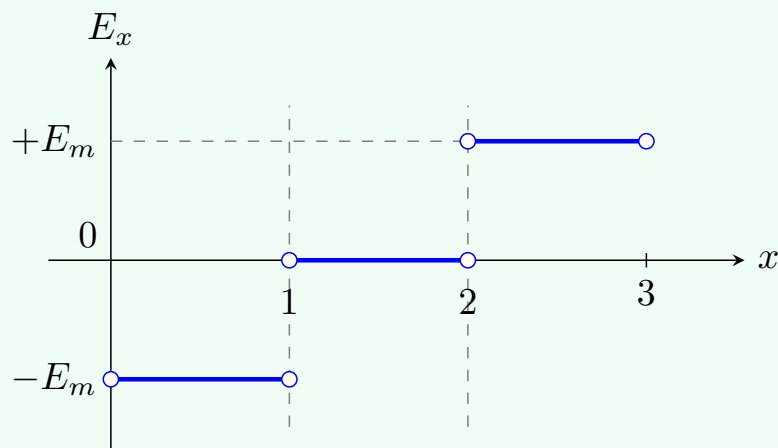
Quick Tip to Solve: First decide what stays fixed: battery disconnected means fixed charge. For the field graph, reverse the sign of each potential slope.

Doubling the gap first raises the energy to $2U_0$; inserting the dielectric then reduces that intermediate energy by a factor k . External mechanical work during these changes accounts for the energy change.

Step 5: (c) Obtain the electric field graph from the potential slopes Let $E_m > 0$ denote the magnitude of the slope of either inclined segment of the given potential graph. The rising segment from $x = 0$ to $x = 1$ has positive slope, the horizontal segment from $x = 1$ to $x = 2$ has zero slope, and the falling segment from $x = 2$ to $x = 3$ has the equal negative slope. Thus

$$E_x(x) = \begin{cases} -E_m, & 0 < x < 1, \\ 0, & 1 < x < 2, \\ +E_m, & 2 < x < 3. \end{cases}$$

The field is negative, zero, then positive on the three successive intervals. The abrupt corners of the ideal potential graph at $x = 1$ and $x = 2$ give discontinuous field values; the derivative at the corners themselves is not defined. No numerical field magnitude can be assigned because the potential scale is not given.



Electric field versus position; open endpoints indicate the interval limits at the ideal corners.

Step 6: Conclusion (a) Charging stops when the capacitor voltage reaches the battery voltage, leaving plate charges $\pm CV$. (b) With the battery disconnected, the final field is E_0/k , so it decreases, while the final energy is $(2/k)U_0 = CV^2/k$, so it increases relative to the original state. (c) The field graph consists of the levels $-E_m$, 0, and $+E_m$ on $(0, 1)$, $(1, 2)$, and $(2, 3)$, respectively.

Quick Tip to Solve: First decide what stays fixed: battery disconnected means fixed charge. For the field graph, reverse the sign of each potential slope.

27. (a) Derive an expression for the potential energy of an electric dipole in a uniform electric field. Explain conditions for stable and unstable equilibrium.

(b) Is the electrostatic potential necessarily zero at a point where the electric field is zero? Give an example to support your answer. [5 marks]

Solution:

Concept:

- An electric dipole consists of charges $-q$ and $+q$ separated by distance $2a$. Its dipole moment $\vec{p}$, of magnitude $p = q(2a)$, points from the negative charge to the positive charge.
- In a uniform electric field $\vec{E}$, the net force on the dipole vanishes, but a torque of magnitude $pE \sin \theta$ acts to align $\vec{p}$ with $\vec{E}$, where θ is the angle between them.
- Potential energy is the work done by an external agent in a slow rotation against the electric torque. Also, $\vec{E} = -\nabla V$: zero electric field means zero local potential gradient, not necessarily zero potential.

Step 1: (a) Calculate the work in rotating the dipole Take increasing θ as the positive sense of rotation. The field torque is $\tau_{\text{field}} = -pE \sin \theta$, so an external agent must apply $\tau_{\text{ext}} = pE \sin \theta$ to rotate the dipole slowly without changing its kinetic energy. For a small rotation, $dW_{\text{ext}} = pE \sin \theta d\theta$. Choose the potential energy to be zero when the dipole is perpendicular to the field, that is, at $\theta = \pi/2$. Then

$$\begin{aligned} U(\theta) - U(\pi/2) &= \int_{\pi/2}^{\theta} pE \sin \theta' d\theta' \\ &= [-pE \cos \theta']_{\pi/2}^{\theta} \\ &= -pE \cos \theta. \end{aligned}$$

Thus $U = -\vec{p} \cdot \vec{E}$. A different choice of zero adds only an arbitrary constant and does not change the torque or equilibrium conditions.

Step 2: (a) Identify stable and unstable equilibrium At equilibrium the torque must vanish: $pE \sin \theta = 0$. For nonzero p and E , this gives $\theta = 0$ or π , modulo full rotations. The nature of equilibrium follows from

$$\frac{dU}{d\theta} = pE \sin \theta, \quad \frac{d^2U}{d\theta^2} = pE \cos \theta.$$

At $\theta = 0$, the dipole is parallel to the field and $U = -pE$ is minimum. A small angular displacement produces a restoring torque; hence equilibrium is stable. At $\theta = \pi$, the dipole is antiparallel to the field and $U = +pE$ is maximum. A small displacement makes the torque rotate it farther away from this orientation; hence equilibrium is unstable. Zero torque by itself is therefore insufficient to establish stability.

Step 3: (b) Give a point with zero field but nonzero potential No. Place two identical positive point charges $+q$ at equal distances r on opposite sides of a midpoint O . At O , their electric fields have equal magnitudes $k_e q/r^2$ and opposite directions, so

$$\vec{E}_O = \vec{E}_1 + \vec{E}_2 = 0.$$

Quick Tip to Solve: Use the minimum of $-pE \cos \theta$ to identify stable alignment; when testing zero field versus zero potential, add fields as vectors and potentials as scalars.

With the conventional potential zero at infinity, their potentials are equal positive scalars and add:

$$V_O = \frac{k_e q}{r} + \frac{k_e q}{r} = \frac{2k_e q}{r} \neq 0, \quad k_e = \frac{1}{4\pi\epsilon_0}.$$

Electric field cancellation is a vector cancellation; it does not imply cancellation of the scalar potentials. The equation $\vec{E} = -\nabla V$ constrains the variation of potential near a point, not the value of potential at that point.

Step 4: Conclusion (a) Taking $U = 0$ at $\theta = \pi/2$, the dipole energy is $U = -pE \cos \theta = -\vec{p} \cdot \vec{E}$. Parallel alignment is stable and antiparallel alignment is unstable. (b) $E = 0$ does not require $V = 0$; the midpoint between two equal positive charges has zero field and nonzero positive potential relative to infinity.

Quick Tip to Solve: Use the minimum of $-pE \cos \theta$ to identify stable alignment; when testing zero field versus zero potential, add fields as vectors and potentials as scalars.