

CBSE Class 12 Physics Question Paper 2019 with Solution PDF-Set 55/5/1



Question Paper with Solutions

Conducted by the Central Board of Secondary Education (CBSE)

Time Allowed: 3 hours

Maximum Marks: 70

General Instructions

- (i) All questions are compulsory. There are 27 questions in all.
- (ii) This question paper has four sections : Section A, Section B, Section C and Section D.
- (iii) Section A contains five questions of one mark each, Section B contains seven questions of two marks each, Section C contains twelve questions of three marks each, Section D contains three questions of five marks each.
- (iv) There is no overall choice. However, an internal choice(s) has been provided in two questions of one mark, two questions of two marks, four questions of three marks and three questions of five marks weightage. You have to attempt only one of the choices in such questions.

You may use the following values of physical constants wherever necessary :

$$c = 3 \times 10^8 \text{ m/s}$$

$$h = 6.63 \times 10^{-34} \text{ Js}$$

$$e = 1.6 \times 10^{-19} \text{ C}$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ T m A}^{-1}$$

$$\epsilon_0 = 8.854 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$$

$$\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$$

$$\text{Mass of electron } (m_e) = 9.1 \times 10^{-31} \text{ kg}$$

$$\text{Mass of neutron} = 1.675 \times 10^{-27} \text{ kg}$$

$$\text{Mass of proton} = 1.673 \times 10^{-27} \text{ kg}$$

$$\text{Avogadro's number} = 6.023 \times 10^{23} \text{ per gram mole}$$

$$\text{Boltzmann constant} = 1.38 \times 10^{-23} \text{ JK}^{-1}$$

Section A (5 marks)

1. Why is the electrostatic potential inside a charged conducting shell constant throughout the volume of the conductor? [1 mark]

Solution:

Concept:

- In electrostatic equilibrium, the electric field inside conducting material is zero: $\mathbf{E} = 0$.
- The potential difference is $V_B - V_A = - \int_A^B \mathbf{E} \cdot d\mathbf{l}$, where A and B are two interior points.

Step 1: Use electrostatic equilibrium Free charges redistribute on the conducting shell until no electric field remains within its material. Otherwise they would keep moving, contradicting electrostatic equilibrium. For the empty cavity of a charged shell with no charge inside, the field also vanishes.

Step 2: Relate the field to potential For any path between two interior points, the integrand is zero. Thus

$$V_B - V_A = - \int_A^B 0 \, dl = 0.$$

The potential has no spatial variation, although its common value need not be zero.

Step 3: Conclusion The potential is constant throughout the interior because the electric field there is zero.

Quick Tip to Solve: Zero electric field means zero potential difference, not necessarily zero potential.

2. Write one important property of a paramagnetic material. [1 mark]

Solution:

Concept:

- A paramagnetic material has small positive magnetic susceptibility $\chi_m > 0$.
- Its magnetisation $\mathbf{M} = \chi_m \mathbf{H}$ is along the applied magnetic field $\mathbf{H}$.

Step 1: Consider the response to an applied field The atoms possess magnetic moments. In an external field, these moments have a weak tendency to align with the field; thermal motion opposes complete alignment.

Step 2: State the observable property The resulting magnetisation reinforces the applied field. Therefore a paramagnetic material is weakly attracted towards the stronger-field region of a nonuniform magnetic field. This attraction is weak because its positive susceptibility is small.

Step 3: Conclusion A paramagnetic material is weakly attracted by an external magnetic field.

Quick Tip to Solve: For paramagnetism, remember positive susceptibility and weak attraction towards stronger field.

2. Do the diamagnetic substances have resultant magnetic moment in an atom in the absence of external magnetic field? [1 mark]

Solution:

Concept:

- The resultant atomic magnetic moment is the vector sum of the magnetic moments of its electrons.
- In a diamagnetic atom, paired electronic contributions cancel in the absence of an applied magnetic field.

Step 1: Add the atomic contributions The orbital and spin contributions of the electrons combine so that the atom has no permanent resultant magnetic dipole moment. In particular, paired electrons have cancelling spin moments.

Step 2: Distinguish permanent and induced moments An external field can induce a small moment opposite to that field, which produces diamagnetism. Such an induced moment does not imply that a permanent atomic moment was present before the field was applied.

Step 3: Conclusion No. The resultant magnetic moment of a diamagnetic atom is zero in the absence of an external magnetic field.

Quick Tip to Solve: Diamagnetic atoms have zero permanent resultant moment; the opposing moment is induced by a field.

3. Plot a graph of the de-Broglie wavelength associated with a proton versus its momentum. [1 mark]

Solution:

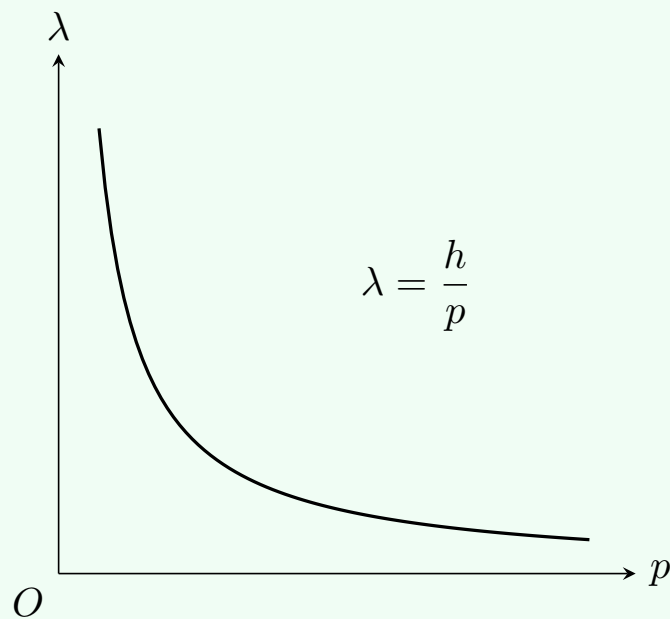
Concept:

- The de-Broglie relation is $\lambda = h/p$, where h is Planck's constant and p is the magnitude of proton momentum.
- For positive momentum, wavelength is inversely proportional to momentum: $\lambda p = h$.

Step 1: Choose axes and relation Put proton momentum p on the horizontal axis and de-Broglie wavelength λ on the vertical axis. Since h is constant, doubling momentum halves the wavelength.

Step 2: Sketch the inverse variation The curve decreases in the first quadrant and is a rectangular hyperbola. As $p \rightarrow 0^+$, $\lambda \rightarrow \infty$; as $p \rightarrow \infty$, $\lambda \rightarrow 0$. It approaches both axes asymptotically and does not intersect them for finite positive momentum.

Quick Tip to Solve: Plot wavelength against momentum as an inverse curve; against inverse momentum it would be a straight line.

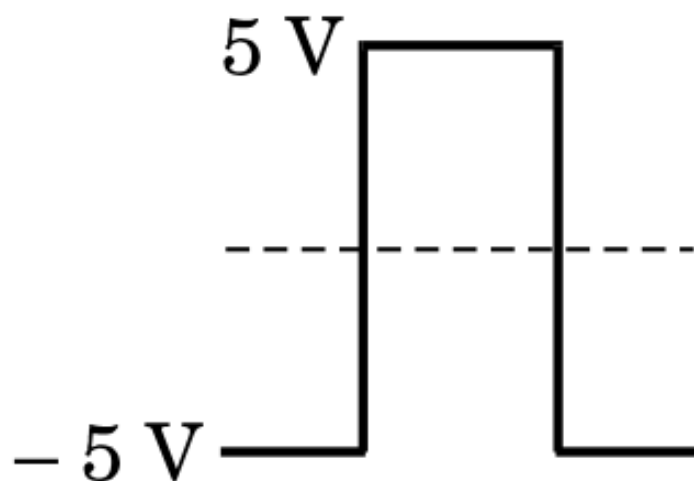


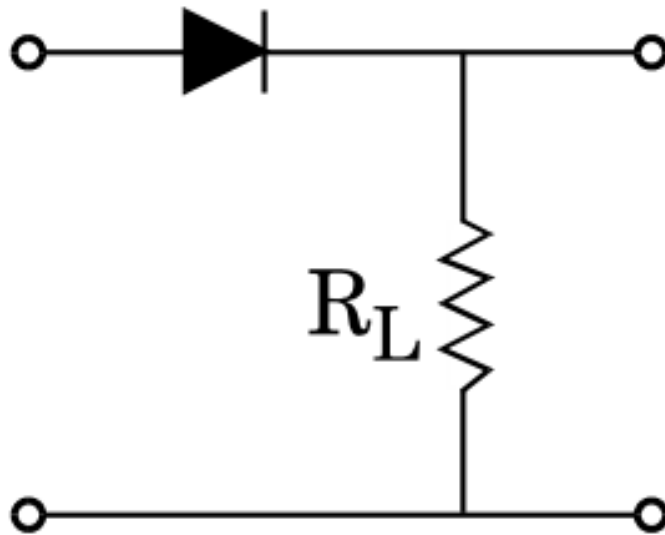
de-Broglie wavelength versus proton momentum

Step 3: Conclusion The required graph is the decreasing rectangular hyperbola $\lambda = h/p$.

Quick Tip to Solve: Plot wavelength against momentum as an inverse curve; against inverse momentum it would be a straight line.

4. Draw the output signal in a p-n junction diode when a square input signal of 10 V as shown in the figure is applied across it. [1 mark]





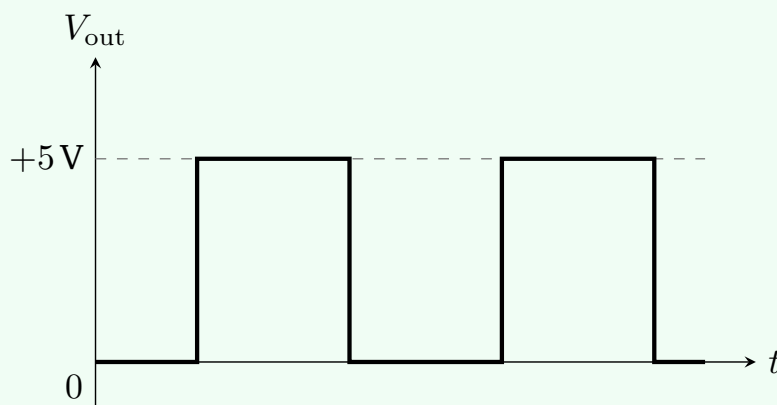
Solution:

Concept:

- A forward-biased ideal p-n junction diode conducts, whereas a reverse-biased diode blocks current.
- The output is measured across R_L . The printed input alternates between $+5\text{ V}$ and -5 V , a peak-to-peak swing of 10 V .

Step 1: Identify the conducting interval The diode anode is at the input and its cathode is connected to the upper end of the load. During the positive input level, the diode is forward biased. Neglecting its forward voltage drop, the load voltage equals the input level $+5\text{ V}$.

Step 2: Identify the blocked interval and draw the output During the negative input level, the diode is reverse biased. No current flows through the load, so $V_{\text{out}} = IR_L = 0$. The output therefore contains only positive rectangular pulses; its low level lies on the time axis. The ideal-diode approximation is appropriate because no forward drop is specified.

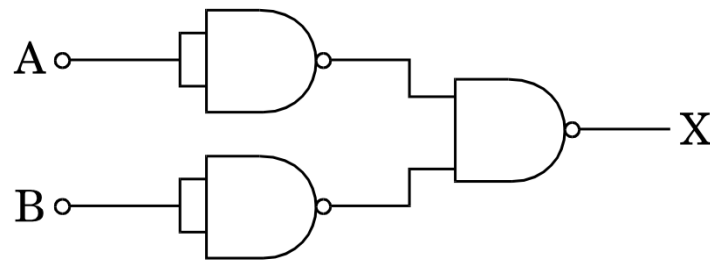


Output across the load for an ideal diode

Step 3: Conclusion The output alternates between 0 and $+5\text{ V}$, retaining the positive square pulses and suppressing the negative portions.

Quick Tip to Solve: The stated 10 V is peak to peak: the retained positive plateau is $+5\text{ V}$, not $+10\text{ V}$.

5. Identify the equivalent gate for the circuit of a combination of gates shown in the figure. Write its symbol. [1 mark]



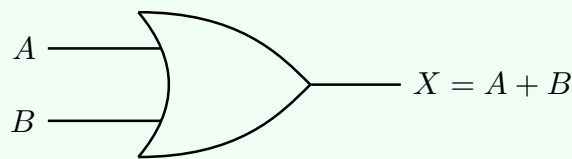
Solution:

Concept:

- A NAND gate gives the complement of the product of its inputs: $Y = \overline{AB}$. When its two inputs are joined, it acts as an inverter.
- De Morgan's law gives $\overline{\overline{A}\overline{B}} = A + B$, where multiplication denotes AND and addition denotes OR.

Step 1: Identify the first two outputs In the printed circuit, both inputs of the upper NAND gate are connected to A, so its output is $\overline{AA} = \overline{A}$. Both inputs of the lower NAND gate are connected to B, giving $\overline{BB} = \overline{B}$. The small circles on the outputs indicate inversion.

Step 2: Simplify the final NAND output The final NAND gate receives $\overline{A}$ and $\overline{B}$, hence $X = \overline{\overline{A}\overline{B}} = A + B$. For input pairs 00, 01, 10 and 11, its outputs are 0, 1, 1 and 1 respectively. Thus either high input makes the output high, which is the OR operation. The equivalent symbol has the curved OR outline and no output inversion bubble.

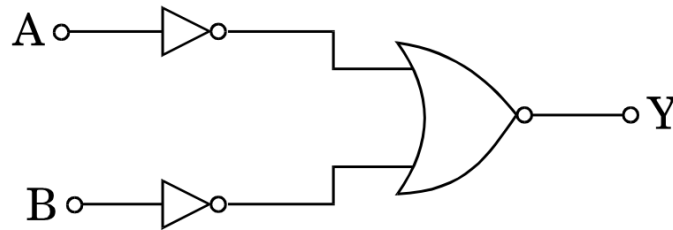


Equivalent two-input OR gate

Step 3: Conclusion The equivalent gate is an OR gate, with $X = A + B$. Its symbol is shown above.

Quick Tip to Solve: Treat each NAND with joined inputs as a NOT gate, then apply De Morgan's law to the final NAND.

5. Draw the logic symbol of the gate shown by the combination of gates and write its name. [1 mark]



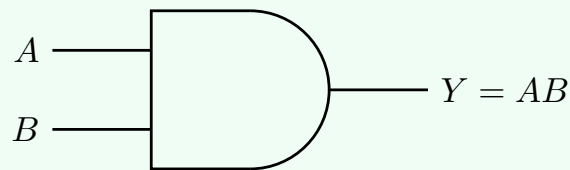
Solution:

Concept:

- A NOT gate complements its input: $A \mapsto \bar{A}$. A NOR gate complements the OR of its inputs.
- De Morgan's law states $\overline{\bar{A} + \bar{B}} = AB$, where addition denotes OR and multiplication denotes AND.

Step 1: Find the inputs of the final gate The two input inverters produce $\bar{A}$ and $\bar{B}$. The final gate has the OR outline and an output inversion bubble, so it is a NOR gate.

Step 2: Simplify the output Its output is $Y = \overline{\bar{A} + \bar{B}} = AB$. Thus the output is high only when both original inputs are high. For inputs 00, 01, 10 and 11, the outputs are respectively 0, 0, 0 and 1. The equivalent symbol therefore has two inputs and no output inversion bubble.



Equivalent two-input AND gate

Step 3: Conclusion The combination is an AND gate, with output $Y = AB$. Its logic symbol is shown above.

Quick Tip to Solve: Apply De Morgan to the whole NOR output; the two input inversions cancel into AND.

Section B (14 marks)

6. Derive an expression for the torque acting on an electric dipole of dipole moment $\vec{p}$ placed in a uniform electric field $\vec{E}$. Write the direction along which the torque acts. [2 marks]

Solution:

Concept:

- An electric dipole has charges $+q$ and $-q$ separated by distance a . Its dipole moment is $\vec{p} = q\vec{a}$, directed from negative to positive charge.
- Equal and opposite parallel forces form a couple whose torque magnitude is one force multiplied by the perpendicular separation of their lines of action.

Quick Tip to Solve: Use the perpendicular lever arm $a \sin(\theta)$; a uniform field gives zero net force but a couple.

Step 1: Find the forces in the uniform field Let the angle between $\vec{p}$ and $\vec{E}$ be θ . The positive charge experiences $\vec{F}_+ = q\vec{E}$, while the negative charge experiences $\vec{F}_- = -q\vec{E}$. Their resultant force is zero because the field is uniform, but their different points of application produce a turning effect.

Step 2: Evaluate the moment of the couple The perpendicular distance between the parallel force lines is $a \sin \theta$. Hence

$$\tau = (qE)(a \sin \theta) = pE \sin \theta.$$

Equivalently, taking moments about the dipole centre, each charge contributes $qE(a/2) \sin \theta$; adding them gives the same expression.

Step 3: Specify the direction The vector torque is $\vec{\tau} = \vec{p} \times \vec{E}$. It acts perpendicular to the plane containing $\vec{p}$ and $\vec{E}$, in the direction given by the right-hand rule from $\vec{p}$ towards $\vec{E}$. It tends to align the dipole with the field.

Step 4: Conclusion $\vec{\tau} = \vec{p} \times \vec{E}$, $\tau = pE \sin \theta$. The torque direction is perpendicular to the dipole-field plane, following the right-hand rule.

Quick Tip to Solve: Use the perpendicular lever arm $a \sin(\theta)$; a uniform field gives zero net force but a couple.

OR

6. Derive an expression for the electric field at a point on the axis of an electric dipole of dipole moment $\vec{p}$. Also write its expression when the distance $r \gg$ the length ' a ' of the dipole. [2 marks]

Solution:

Concept:

- Take the printed dipole length a as the full charge separation, so charges $\pm q$ lie at distances $a/2$ from the centre and $p = qa$.
- By Coulomb's law, the field of a point charge at distance d has magnitude kq/d^2 , where $k = 1/(4\pi\epsilon_0)$. Superpose the fields vectorially.

Step 1: Find the two fields at an external axial point Let point P be a distance $r > a/2$ from the centre on the side of $+q$. Its distances from $+q$ and $-q$ are $r - a/2$ and $r + a/2$. The positive-charge field points along $\vec{p}$, while the negative-charge field points opposite to $\vec{p}$.

Step 2: Subtract and simplify Writing $\hat{p}$ for the unit vector along the dipole moment,

$$\begin{aligned}\vec{E} &= kq \left[\frac{1}{(r - a/2)^2} - \frac{1}{(r + a/2)^2} \right] \hat{p} \\ &= kq \frac{2ra}{(r^2 - a^2/4)^2} \hat{p} \\ &= \frac{1}{4\pi\epsilon_0} \frac{2r\vec{p}}{(r^2 - a^2/4)^2}.\end{aligned}$$

This is the exact external axial field for the stated full-separation convention. The field at an external axial point on the opposite side also points along $\vec{p}$.

Quick Tip to Solve: Define whether a is full or half separation before writing the two charge-to-point distances.

Step 3: Use the distant-point approximation When $r \gg a$, the term $a^2/4$ is negligible compared with r^2 . Therefore the denominator becomes r^4 , giving $\vec{E} \simeq (1/(4\pi\epsilon_0)) 2\vec{p}/r^3$. The dipole field falls as the inverse cube of the distance.

Step 4: Conclusion $\vec{E}_{\text{axial}} = \frac{1}{4\pi\epsilon_0} \frac{2r\vec{p}}{(r^2 - a^2/4)^2}$; for $r \gg a$, $\vec{E}_{\text{axial}} \simeq \frac{1}{4\pi\epsilon_0} \frac{2\vec{p}}{r^3}$.

Quick Tip to Solve: Define whether a is full or half separation before writing the two charge-to-point distances.

7. Two identical capacitors of 12 pF each are connected in series across a 50 V battery. Calculate the electrostatic energy stored in the combination. If these were connected in parallel across the same battery, find out the value of the energy stored in this combination. [2 marks]

Solution:

Concept:

- The energy stored in a capacitor combination across voltage V is $U = \frac{1}{2}C_{\text{eq}}V^2$. Here V is the voltage across the entire combination.
- For two identical capacitances C , the series equivalent is $C/2$, whereas the parallel equivalent is $2C$. Also $1 \text{ pF} = 10^{-12} \text{ F}$.

Step 1: Calculate the energy for the series combination Each capacitor has $C = 12 \times 10^{-12} \text{ F}$. In series,

$$\frac{1}{C_s} = \frac{1}{C} + \frac{1}{C}, \quad C_s = 6 \times 10^{-12} \text{ F}.$$

The battery voltage across the pair is 50 V. Therefore

$$U_s = \frac{1}{2}(6 \times 10^{-12})(50)^2 = 7.5 \times 10^{-9} \text{ J}.$$

Each identical capacitor has half the battery voltage, consistent with this equivalent-capacitance calculation.

Step 2: Calculate the energy for the parallel combination In parallel both capacitors have the full battery voltage. Their equivalent capacitance is $C_p = C + C = 24 \times 10^{-12} \text{ F}$. Thus

$$U_p = \frac{1}{2}(24 \times 10^{-12})(50)^2 = 3.0 \times 10^{-8} \text{ J}.$$

Since the applied voltage is unchanged, the energy ratio equals the capacitance ratio: $U_p/U_s = C_p/C_s = 4$.

Step 3: Conclusion The stored energies are $U_s = 7.5 \times 10^{-9} \text{ J}$ in series and $U_p = 3.0 \times 10^{-8} \text{ J}$ in parallel.

Quick Tip to Solve: With the same battery attached, compare equivalent capacitances: the parallel energy is four times the series energy.

8. A set of ' n ' identical resistors, each of resistance ' R ' when connected in series have an effective resistance ' X '. When they are connected in parallel, their effective resistance becomes ' Y '. Find out the product of X and Y .

[2 marks]

Solution:

Concept:

- For series resistors the same current passes through each resistor and the voltage drops add, so the equivalent resistance is the sum of individual resistances.
- For parallel resistors the same voltage appears across every branch and the currents add, so reciprocal resistances add. Here n is the number of identical resistors of resistance R .

Step 1: Find the series effective resistance Using Ohm's law for the common current I , each resistor has voltage drop IR . The total voltage is $V = nIR$. Hence the series equivalent resistance is

$$X = \frac{V}{I} = nR.$$

Thus adding identical resistors in series increases the effective resistance by the factor n .

Step 2: Find the parallel effective resistance If voltage V is applied across the parallel set, each branch carries current V/R . The total current is $I_{\text{total}} = nV/R$. Consequently

$$Y = \frac{V}{I_{\text{total}}} = \frac{R}{n}.$$

Equivalently, $1/Y = n/R$.

Step 3: Multiply the two expressions The requested product is

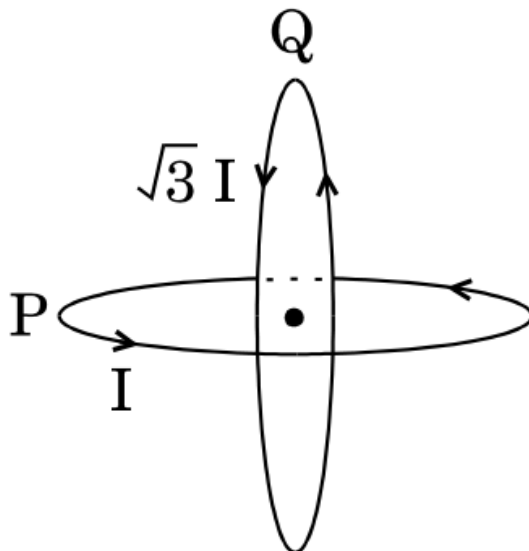
$$XY = (nR) \left(\frac{R}{n} \right) = R^2.$$

The factor n cancels. Thus the product is independent of the number of resistors, even though each individual equivalent resistance depends on that number. Since this is a product of resistances, its unit is Ω^2 .

Step 4: Conclusion $XY = R^2$, independent of n .

Quick Tip to Solve: For identical resistors, series multiplies R by n and parallel divides R by n ; the factors cancel in the product.

9. Two identical coils P and Q each of radius R are lying in perpendicular planes such that they have a common centre. Find the magnitude and direction of the magnetic field at the common centre when they carry currents equal to I and $\sqrt{3}I$ respectively. [2 marks]



Solution:

Concept:

- At the centre of a circular coil, $B = \mu_0 I / (2R)$, where R is radius and μ_0 is free-space permeability.
- The field is along the coil normal in the sense of the right-hand grip rule. Perpendicular fields add using vector components.

Step 1: Find the component fields Let $\hat{n}_P$ and $\hat{n}_Q$ be unit normals directed according to the right-hand grip rule for the currents shown. The perpendicular coil planes imply perpendicular normals. At their common centre,

$$B_P = \frac{\mu_0 I}{2R}, \quad B_Q = \frac{\mu_0 \sqrt{3}I}{2R} = \sqrt{3}B_P.$$

The identical coil geometry means the ratio of field strengths is the ratio of currents.

Step 2: Add the fields and find the angle The resultant is $\vec{B} = B_P \hat{n}_P + B_Q \hat{n}_Q$. Since its components are perpendicular,

$$B = \sqrt{B_P^2 + B_Q^2} = \frac{\mu_0 I}{2R} \sqrt{1 + 3} = \frac{\mu_0 I}{R}.$$

Let θ be its angle from the field direction of P towards that of Q. Then $\tan \theta = B_Q / B_P = \sqrt{3}$, so $\theta = 60^\circ$. It makes the complementary angle 30° with the field direction of Q. These angles specify the direction relative to the axes and the senses selected by the currents.

Step 3: Conclusion $B = \mu_0 I / R$; the field makes 60° with the axis of P and 30° with the axis of Q, towards their current-defined field directions.

Quick Tip to Solve: For perpendicular coil planes, form a right triangle from the two field strengths and use their ratio to find the angle.

10. (a) Obtain the conditions under which an electron does not suffer any deflection while passing through a magnetic field.

(b) Two protons P and Q moving with the same speed pass through the magnetic fields $\vec{B}_1$ and $\vec{B}_2$ respectively, at right angles to the field directions. If $|\vec{B}_2| > |\vec{B}_1|$, which of the two protons will describe the circular path of smaller radius? Explain. [2 marks]

Solution:

Concept:

- The magnetic force is $\vec{F} = q\vec{v} \times \vec{B}$, of magnitude $|q|vB \sin \theta$.
- For perpendicular motion, magnetic force supplies centripetal force, giving $r = mv/(|q|B)$.

Step 1: (a) Obtain the zero-force condition For an electron $q = -e$, $\vec{F} = -e(\vec{v} \times \vec{B})$. No magnetic deflection requires this cross product to vanish. For a moving electron in a nonzero field, $vB \sin \theta = 0$ gives $\theta = 0^\circ$ or 180° . Its velocity must therefore be parallel or antiparallel to the magnetic field. The negative charge reverses a nonzero force but does not alter the condition for zero force. The special cases $v = 0$ or $B = 0$ also give zero magnetic force.

Step 2: (b) Compare the two radii The two protons have identical mass m_p , charge e , and speed v . Each moves perpendicular to its respective field. Using $evB = m_p v^2/r$,

$$r_P = \frac{m_p v}{e|\vec{B}_1|}, \quad r_Q = \frac{m_p v}{e|\vec{B}_2|}.$$

The printed condition is $|\vec{B}_2| > |\vec{B}_1|$, hence $r_Q/r_P = |\vec{B}_1|/|\vec{B}_2| < 1$. At the same speed, the stronger field gives a greater transverse force and therefore tighter circular curvature.

Step 3: Conclusion (a) The moving electron is undeflected for velocity parallel or antiparallel to the field. (b) Proton Q describes the circle of smaller radius.

Quick Tip to Solve: At fixed mass, charge and speed, radius is inversely proportional to field strength: identify the stronger field first.

11. The frequencies of two side bands in an amplitude modulated wave are 640 kHz and 660 kHz respectively. Find the frequencies of the carrier and the modulating signals. Also obtain the value of the bandwidth required in amplitude modulation. [2 marks]

Solution:

Concept:

- AM sidebands satisfy $f_L = f_c - f_m$ and $f_U = f_c + f_m$, with carrier frequency f_c and message frequency f_m .
- The transmission bandwidth is the full sideband separation: $BW = f_U - f_L = 2f_m$.

Step 1: Find the carrier and message frequencies Here $f_L = 640$ kHz and $f_U = 660$ kHz. Adding the sideband equations eliminates the message frequency; subtracting them eliminates the carrier

Quick Tip to Solve: Use the average of the sidebands for the carrier, half their difference for the message, and the full difference for bandwidth.

frequency:

$$f_c = \frac{660 + 640}{2} \text{ kHz} = 650 \text{ kHz},$$
$$f_m = \frac{660 - 640}{2} \text{ kHz} = 10 \text{ kHz}.$$

The carrier lies at the midpoint of the two sidebands, and the message frequency is their offset from that midpoint.

Step 2: Calculate bandwidth and verify The required AM frequency interval extends from the lower sideband to the upper sideband because both must be included in transmission. Consequently,

$$\text{BW} = 660 - 640 = 20 \text{ kHz} = 2(10 \text{ kHz}).$$

As a check, the calculated frequencies give $650 - 10 = 640$ and $650 + 10 = 660$, in kHz, reproducing the two printed values. The separation of sidebands is twice the message frequency, rather than the message frequency itself.

Step 3: Conclusion $f_c = 650 \text{ kHz}, \quad f_m = 10 \text{ kHz}, \quad \text{BW} = 20 \text{ kHz}.$

Quick Tip to Solve: Use the average of the sidebands for the carrier, half their difference for the message, and the full difference for bandwidth.

OR

11. A sinusoidal carrier voltage is amplitude modulated by a sinusoidal voltage of 10 kHz with modulation index 0.3. If the carrier frequency is 10 MHz and its amplitude is 40 V, calculate the frequency and amplitude of the two sidebands. [2 marks]

Solution:

Concept:

- A sinusoidal AM voltage is $v(t) = A_c[1 + \mu \cos(2\pi f_m t)] \cos(2\pi f_c t)$. Here μ is the modulation index.
- The two sidebands have frequencies $f_c - f_m$ and $f_c + f_m$, and equal peak voltage amplitudes $\mu A_c/2$.

Step 1: Find the two sideband frequencies Use consistent units: $f_c = 10 \text{ MHz}$, while $f_m = 10 \text{ kHz} = 0.010 \text{ MHz}$. Therefore

$$f_L = f_c - f_m = 10 - 0.010 = 9.990 \text{ MHz},$$

$$f_U = f_c + f_m = 10 + 0.010 = 10.010 \text{ MHz}.$$

Each sideband lies only 10 kHz from the carrier; adding numerical values expressed in MHz and kHz directly would produce a wrong frequency.

Step 2: Expand the modulation term to obtain amplitudes The product term is

$$\mu A_c \cos(2\pi f_m t) \cos(2\pi f_c t).$$

Quick Tip to Solve: Convert the message frequency to MHz before finding offsets, and halve modulation-index times carrier amplitude for each sideband.

The product-to-sum identity converts it into

$$\frac{\mu A_c}{2} [\cos\{2\pi(f_c + f_m)t\} + \cos\{2\pi(f_c - f_m)t\}].$$

Thus each sideband has peak voltage amplitude

$$A_{SB} = \frac{0.3 \times 40}{2} = 6 \text{ V}.$$

The factor of one half is essential because the modulation product generates two cosine terms. The value $\mu A_c = 12 \text{ V}$ is not the amplitude of either individual sideband.

Step 3: Conclusion The lower and upper sidebands are 9.990 MHz and 10.010 MHz , respectively. Each has amplitude 6 V .

Quick Tip to Solve: Convert the message frequency to MHz before finding offsets, and halve modulation-index times carrier amplitude for each sideband.

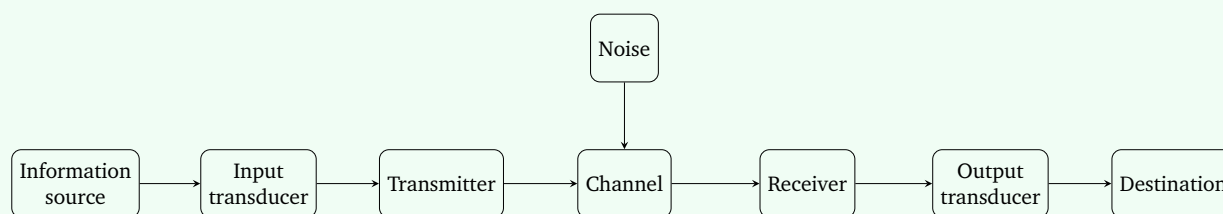
12. Draw a block diagram of a generalized communication system and write the functions of (i) a transmitter, and (ii) a receiver. [2 marks]

Solution:

Concept:

- Communication transfers information from a source to a destination through a physical channel.
- A transducer converts the message between a physical form, such as sound, and an electrical signal; the transmitter and receiver process that electrical signal.

Step 1: Arrange the communication blocks The information source supplies the message to an input transducer, which converts it into an electrical message signal. The signal passes through the transmitter, the communication channel and the receiver. An output transducer converts the recovered electrical signal into the required physical form for the user at the destination. Noise can enter the signal in the channel, so it is shown as a separate input to that block.



Generalized communication system; arrows show signal flow.

Step 2: State the transmitter function The transmitter converts the input message signal into a form suitable for transmission through the channel. It generally modulates a high-frequency carrier with the message and amplifies the resulting signal to provide sufficient power for transmission over the required distance.

Step 3: State the receiver function The receiver selects and amplifies the desired incoming signal and demodulates it to recover the original message signal. It supplies this recovered signal to the output

Quick Tip to Solve: Draw the blocks in the direction of information flow and connect the noise arrow to the channel; pair modulation at the transmitter with demodulation at the receiver.

transducer, such as a loudspeaker, so that the information is delivered in a usable form.

Step 4: Conclusion The transmitter prepares and sends a suitable signal through the channel; the receiver extracts the original message from the received signal and delivers it to the output transducer.

Quick Tip to Solve: Draw the blocks in the direction of information flow and connect the noise arrow to the channel; pair modulation at the transmitter with demodulation at the receiver.

Section C (36 marks)

13. A charge Q is distributed over the surfaces of two concentric hollow spheres of radii r and R ($R \gg r$), such that their surface charge densities are equal. Derive the expression for the potential at the common centre. [3 marks]

Solution:

Concept:

- The potential throughout the interior of a uniformly charged spherical shell of radius s and charge q is $V = kq/s$, where $k = 1/(4\pi\epsilon_0)$ and the potential at infinity is zero.
- Equal surface charge density means charge is proportional to surface area: $q = \sigma(4\pi s^2)$, where σ denotes charge per unit area.
- Electric potential is a scalar; the potentials due to the two shells add by superposition.

Step 1: Divide the total charge between the shells Let q_r and q_R be the charges on the shells of radii r and R . The given Q is their combined charge, rather than the charge on each shell. Since both surfaces have the same density,

$$\frac{q_r}{4\pi r^2} = \frac{q_R}{4\pi R^2} = \sigma, \quad Q = 4\pi\sigma(r^2 + R^2).$$

Thus the common density and the individual charges are

$$\sigma = \frac{Q}{4\pi(r^2 + R^2)}, \quad q_r = \frac{Qr^2}{r^2 + R^2}, \quad q_R = \frac{QR^2}{r^2 + R^2}.$$

The larger shell therefore carries more charge because its surface area is larger.

Step 2: Add the potentials at the common centre The centre lies inside both shells. Although the electric field of each shell is zero there, its potential relative to infinity is a nonzero constant, equal to its surface potential. Hence

$$\begin{aligned} V_O &= k \left(\frac{q_r}{r} + \frac{q_R}{R} \right) \\ &= \frac{1}{4\pi\epsilon_0} \left(\frac{Qr}{r^2 + R^2} + \frac{QR}{r^2 + R^2} \right) \\ &= \frac{Q(r + R)}{4\pi\epsilon_0(r^2 + R^2)}. \end{aligned}$$

This is exact for the specified charge distribution. For the printed condition $R \gg r$, divide numerator

Quick Tip to Solve: Equal surface densities imply charges in the ratio of squared radii; first divide the total charge using that ratio, then add each shell's constant interior potential.

and denominator by R and R^2 , respectively:

$$V_O = \frac{Q}{4\pi\epsilon_0 R} \frac{1 + r/R}{1 + (r/R)^2} \simeq \frac{Q}{4\pi\epsilon_0 R}.$$

This leading approximation is justified only when the ratio r/R is negligible.

Step 3: Conclusion Relative to infinity, the exact potential at the centre is

$$V_O = \frac{Q(r + R)}{4\pi\epsilon_0(r^2 + R^2)}.$$

In the limit $R \gg r$, $V_O \simeq Q/(4\pi\epsilon_0 R)$. Its sign is the sign of Q .

Quick Tip to Solve: Equal surface densities imply charges in the ratio of squared radii; first divide the total charge using that ratio, then add each shell's constant interior potential.

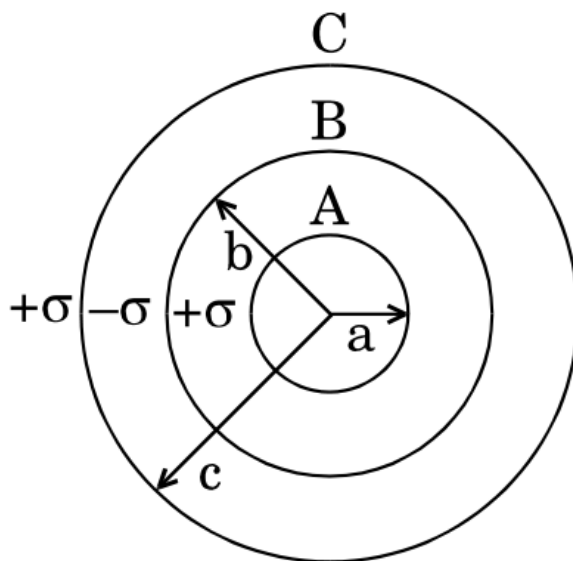
OR

13. Three concentric metallic shells A, B and C of radii a , b and c ($a < b < c$) have surface charge densities $+\sigma$, $-\sigma$ and $+\sigma$ respectively as shown.

(a) Obtain the expressions for the potential of three shells A, B and C.

(b) If shells A and C are at the same potential, obtain the relation between a , b and c .

[3 marks]



Solution:

Concept:

- A spherical shell of charge q and radius s produces potential kq/s at and inside its surface, and kq/x at an exterior distance x from its centre; $k = 1/(4\pi\epsilon_0)$.
- The charges of the shells are $q_A = 4\pi a^2\sigma$, $q_B = -4\pi b^2\sigma$, and $q_C = 4\pi c^2\sigma$, using their given surface charge densities.
- Superpose the three scalar potentials, taking zero potential at infinity. A negative shell charge contributes a negative potential.

Step 1: Evaluate the potential on shell A A point on A is at radius a , inside both B and C. Thus their contributions use their own shell radii, while A contributes its surface potential:

$$\begin{aligned} V_A &= k \left(\frac{q_A}{a} + \frac{q_B}{b} + \frac{q_C}{c} \right) \\ &= \frac{\sigma}{\epsilon_0} (a - b + c). \end{aligned}$$

Using the observation distance a for the outer shells would be incorrect because their potential is constant throughout their interiors.

Step 2: Evaluate the potentials on shells B and C On B, A acts as a point charge at the centre; B contributes its surface potential, and C contributes its constant interior potential. Therefore

$$\begin{aligned} V_B &= k \left(\frac{q_A}{b} + \frac{q_B}{b} + \frac{q_C}{c} \right) \\ &= \frac{\sigma}{\epsilon_0} \left(\frac{a^2}{b} - b + c \right). \end{aligned}$$

On the outermost shell C, all three contributions have the same denominator c , since A and B lie inside the observation point:

$$\begin{aligned} V_C &= k \frac{q_A + q_B + q_C}{c} \\ &= \frac{\sigma}{\epsilon_0} \left(\frac{a^2 - b^2}{c} + c \right). \end{aligned}$$

Step 3: Impose equal potentials on A and C For the given nonzero surface density, substitute the expressions in $V_A = V_C$ and cancel σ/ϵ_0 . This gives

$$\begin{aligned} a - b + c &= \frac{a^2 - b^2}{c} + c, \\ c(a - b) &= (a - b)(a + b). \end{aligned}$$

Since $a < b$, the factor $a - b$ is nonzero and can be cancelled. The required geometrical relation follows without assuming equal charges on the shells. If $\sigma = 0$, all potentials would vanish and no

Quick Tip to Solve: For each contribution, use the shell radius when the observation point is inside that shell and the observation radius when outside; retain B's negative charge throughout.

such radius constraint would follow, but that is not the charged configuration posed here.

Step 4: Conclusion The shell potentials are

$$\begin{aligned}V_A &= \frac{\sigma}{\epsilon_0}(a - b + c), \\V_B &= \frac{\sigma}{\epsilon_0}\left(\frac{a^2}{b} - b + c\right), \\V_C &= \frac{\sigma}{\epsilon_0}\left(\frac{a^2 - b^2}{c} + c\right).\end{aligned}$$

Equal potentials on A and C require $c = a + b$.

Quick Tip to Solve: For each contribution, use the shell radius when the observation point is inside that shell and the observation radius when outside; retain B's negative charge throughout.

14. State the principle of a moving coil galvanometer. Explain its working and obtain the expression for the deflection produced due to the current passed through the coil. Define current sensitivity.

[3 marks]

Solution:

Concept:

- A current-carrying coil in a magnetic field experiences a torque: $\tau_m = NIAB \sin \alpha$, where N is the number of turns, I the current, A the coil area and α the angle between its normal and the field.
- A radial magnetic field keeps $\alpha = 90^\circ$. The suspension provides a restoring torque $\tau_r = k\theta$, where k is the torsional constant and θ is angular deflection in radians.

Step 1: State the principle and explain the arrangement The principle is that a current-carrying coil placed in a magnetic field experiences a torque. A light rectangular coil with many turns is suspended or pivoted between the concave pole faces of a strong permanent magnet. A cylindrical soft-iron core strengthens the field and makes it radial. The suspension or spring both conducts current and provides the restoring torque. A pointer or mirror indicates the angular deflection.

Step 2: Obtain the equilibrium deflection When current flows, the magnetic torque rotates the coil. In the radial field, the plane of the coil remains parallel to the field, so its normal is perpendicular to the field at every deflection. Therefore $\tau_m = NIAB$. The restoring torque grows as the coil turns. At equilibrium,

$$NIAB = k\theta, \quad \theta = \frac{NAB}{k}I.$$

Hence deflection is directly proportional to current, giving a uniform scale. Reversing the current reverses the deflection.

Step 3: Define current sensitivity Current sensitivity is the angular deflection produced per unit current through the coil:

$$S_I = \frac{\theta}{I} = \frac{NAB}{k}.$$

Its SI unit is rad A^{-1} . More turns, a larger area or a stronger magnetic field increases sensitivity, while

Quick Tip to Solve: Use the radial field to set the torque factor to unity before balancing magnetic and restoring torques.

a stiffer suspension with a larger torsional constant reduces it.

Step 4: Conclusion The galvanometer works through magnetic torque balanced by restoring torque:

$$\theta = N I A B / k. \text{ Its current sensitivity is } S_I = N A B / k.$$

Quick Tip to Solve: Use the radial field to set the torque factor to unity before balancing magnetic and restoring torques.

OR

14. Explain how a galvanometer can be converted into an ammeter of a given range. Derive an expression for shunt resistance and current for full scale deflection. Find the effective resistance of the ammeter. [3 marks]

Solution:

Concept:

- A galvanometer of resistance G carries a maximum safe current I_g at full-scale deflection. An ammeter of range I , with $I > I_g$, must bypass the excess current through a shunt.
- Parallel branches have the same voltage, and Kirchhoff's current law gives $I = I_g + I_s$, where I_s is the shunt current.

Step 1: Connect the shunt to extend the range Connect a suitable low resistance S , called a shunt, in parallel with the galvanometer. Connect this parallel combination in series with the external circuit whose current is measured. At the desired upper range I , only I_g passes through the galvanometer and the remaining current $I - I_g$ passes through the shunt. The galvanometer scale is then calibrated to read the total current.

Step 2: Derive shunt resistance and full-scale current Equality of the voltage drops across the parallel branches gives

$$I_g G = (I - I_g) S.$$

Solving for the required resistance,

$$S = \frac{I_g G}{I - I_g}.$$

Conversely, for a selected shunt the total current for full-scale deflection is

$$I = I_g \left(1 + \frac{G}{S} \right).$$

The galvanometer current itself is $I_g = IS/(G + S)$. A small shunt resistance allows most of the current to bypass the sensitive coil.

Step 3: Find the effective ammeter resistance The ammeter resistance is the parallel equivalent of G and S :

$$R_A = \frac{GS}{G + S}.$$

Quick Tip to Solve: The shunt carries the excess current, so write its current as total range minus galvanometer full-scale current.

Substituting the required shunt, or dividing its full-scale voltage $I_g G$ by total current I , gives

$$R_A = \frac{I_g G}{I}.$$

Thus $R_A < G$. A low effective resistance reduces the voltage drop introduced by the meter and reduces its effect on the current being measured.

Step 4: Conclusion A parallel shunt converts the galvanometer into an ammeter: $S = I_g G / (I - I_g)$, with full-scale total current $I = I_g (1 + G/S)$ and resistance $R_A = GS / (G + S) = I_g G / I$.

Quick Tip to Solve: The shunt carries the excess current, so write its current as total range minus galvanometer full-scale current.

15. A voltage $v = v_m \sin \omega t$ **applied to a series LCR circuit, drives a current in the circuit given** $i = i_m \sin(\omega t + \phi)$. **Deduce the expression for the instantaneous power supplied by the source. Hence, obtain the expression for the average power.**

Define the terms ‘power factor’ and ‘wattless current’, giving the examples where power factor is maximum and the circuit where there is wattless current. [3 marks]

Solution:

Concept:

- Instantaneous electrical power is $p(t) = v(t)i(t)$. For sinusoidal voltage and current, $V_{\text{rms}} = v_m / \sqrt{2}$ and $I_{\text{rms}} = i_m / \sqrt{2}$.
- The identity $2 \sin a \sin b = \cos(a - b) - \cos(a + b)$ separates the constant and oscillating parts of power.

Step 1: Deduce instantaneous power Using the printed current phase convention, $v = v_m \sin \omega t$ and $i = i_m \sin(\omega t + \phi)$,

$$\begin{aligned} p(t) &= v_m i_m \sin \omega t \sin(\omega t + \phi) \\ &= \frac{v_m i_m}{2} [\cos \phi - \cos(2\omega t + \phi)]. \end{aligned}$$

The power has a constant term and an oscillating term at twice the source frequency. In a circuit with reactive elements, negative instantaneous power represents energy returned to the source.

Step 2: Obtain average power and define power factor Over one complete source cycle, the average of $\cos(2\omega t + \phi)$ vanishes. Therefore

$$P_{\text{av}} = \frac{v_m i_m}{2} \cos \phi = V_{\text{rms}} I_{\text{rms}} \cos \phi.$$

Power factor is the cosine of the phase difference between voltage and current, equivalently the ratio of average power to apparent power: $\cos \phi = P_{\text{av}} / (V_{\text{rms}} I_{\text{rms}})$. For a passive series LCR circuit it is R/Z , with $Z = \sqrt{R^2 + (\omega L - 1/\omega C)^2}$. Its maximum value is unity in a purely resistive circuit, or in a series LCR circuit at resonance where the reactances cancel.

Quick Tip to Solve: Keep the printed plus sign in the current phase; the oscillating power term is then cosine of twice omega t plus phi.

Step 3: Define wattless current and give examples Wattless current is an alternating current that produces no average power consumption, although current flows. This occurs when voltage and current differ in phase by 90° , so $\cos \phi = 0$. An ideal pure inductor or an ideal pure capacitor carries such current: energy is stored during part of the cycle and returned during another part. In a general AC circuit, the quadrature component $I_{\text{rms}} \sin \phi$ is the wattless component; the in-phase component supplies the average power.

Step 4: Conclusion $p(t) = \frac{1}{2} v_m i_m [\cos \phi - \cos(2\omega t + \phi)]$ and $P_{\text{av}} = V_{\text{rms}} I_{\text{rms}} \cos \phi$. Power factor reaches unity for a resistive or resonant circuit and is zero for ideal purely reactive circuits carrying wattless current.

Quick Tip to Solve: Keep the printed plus sign in the current phase; the oscillating power term is then cosine of twice omega t plus phi.

16. Define displacement current. What role does it play while charging a capacitor by dc source. Is the value of displacement current same as that of the conduction current? Explain. [3 marks]

Solution:

Concept:

- Displacement current is the current equivalent associated with a time-varying electric flux: $I_d = \epsilon_0 d\Phi_E/dt$ in vacuum, where $\Phi_E = \int \mathbf{E} \cdot d\mathbf{A}$.
- Maxwell's corrected circuital law is $\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0(I_c + I_d)$. Conduction current I_c involves transport of charge; displacement current need not do so.

Step 1: Define displacement current and identify it in a charging capacitor While a capacitor is being charged from a dc source, charge flows through the connecting wires and accumulates on the plates. For an ideal capacitor, no conduction current crosses the insulating gap. However, the increasing plate charge produces a changing electric field between the plates, and hence a changing electric flux. The associated displacement current supplies the current term in Maxwell's law for a surface passing through that gap.

Step 2: Demonstrate equality with conduction current Consider an ideal parallel-plate capacitor with vacuum or air between plates of area A , neglecting fringing. If the positive plate carries charge Q ,

$$E = \frac{Q}{\epsilon_0 A}, \quad \Phi_E = EA = \frac{Q}{\epsilon_0}.$$

Consequently,

$$I_d = \epsilon_0 \frac{d\Phi_E}{dt} = \frac{dQ}{dt} = I_c.$$

Thus the displacement current through the gap equals the simultaneous conduction current in the wire in magnitude, with a consistent orientation. If a dielectric is present, the general expression is $I_d = d(\int \mathbf{D} \cdot d\mathbf{A})/dt$, and the same equality follows from the free plate charge.

Step 3: Explain its role and the steady state Without displacement current, the calculated magnetic field around a charging wire would depend on whether the surface bounded by the same loop cuts the wire or passes between the capacitor plates. Including displacement current makes the two

Quick Tip to Solve: Express the flux through the capacitor gap as charge divided by epsilon zero, then differentiate to compare both currents.

calculations agree and preserves current continuity. It does not imply that free charges travel across the ideal insulating gap. Once charging is complete under a constant dc voltage, the electric flux becomes constant; both displacement current and conduction current fall to zero.

Step 4: Conclusion Displacement current is produced by changing electric flux. During ideal capacitor charging, $I_d = I_c = dQ/dt$; it ensures a consistent magnetic field and current continuity across the gap. Both currents vanish after charging is complete.

Quick Tip to Solve: Express the flux through the capacitor gap as charge divided by epsilon zero, then differentiate to compare both currents.

17. A screen is placed 90 cm from an object. The image of the object on the screen is formed by a convex lens at two different positions separated by 20 cm. Calculate the focal length of the lens. [3 marks]

Solution:

Concept:

- For a thin convex lens producing a real image, the positive object and image distances a and b satisfy $1/f = 1/a + 1/b$.
- In the displacement method, the fixed object-screen distance is $D = a + b$. The two lens positions interchange a and b , so their separation is $d = |b - a|$.

Step 1: Use the two conjugate positions The object and screen remain fixed, with $D = 90$ cm. At one lens position the object distance is smaller and the image distance larger; at the other position these distances exchange because the lens equation is symmetric in their positive magnitudes. Their difference therefore equals the measured lens displacement, $d = 20$ cm. Taking $b > a$, we have $a + b = 90$ and $b - a = 20$, giving $a = 35$ cm and $b = 55$ cm.

Step 2: Apply the lens equation Using Cartesian signs in the first position, $u = -35$ cm and $v = +55$ cm. Thus

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u} = \frac{1}{55} + \frac{1}{35} = \frac{90}{1925} \text{ cm}^{-1}.$$

Equivalently, eliminating the distances gives $f = (D^2 - d^2)/(4D)$. Substitution gives

$$f = \frac{90^2 - 20^2}{4(90)} \text{ cm} = \frac{7700}{360} \text{ cm}.$$

The distance condition $D > 4f$ is satisfied, consistently allowing two distinct real-image positions.

Step 3: Conclusion The focal length of the convex lens is $f = 21.39 \text{ cm} \approx 21.4 \text{ cm}$.

Quick Tip to Solve: For fixed object and screen, use $f = (D^2 - d^2)/(4D)$; the displacement is the difference of the conjugate distances, not either distance itself.

17. A convex lens of focal length 20 cm and a concave lens of focal length 15 cm are kept 30 cm apart with their principal axes coincident. When an object is placed 30 cm in front of the convex lens, calculate the position of the final image formed by the combination. Would this result change if the object were placed 30 cm in front of the concave lens? Give reason. [3 marks]

Solution:

Concept:

- Use the thin-lens equation $1/v - 1/u = 1/f$, taking distances positive along the incident-light direction for each arrangement.
- A convex lens has positive focal length and a concave lens negative focal length. The first lens image acts as the object for the second lens; a point on its outgoing side is a virtual object with $u > 0$.

Step 1: First arrangement: image produced by the convex lens Let light travel from the convex lens towards the concave lens. For the convex lens, $u_1 = -30$ cm and $f_1 = +20$ cm. Hence

$$\frac{1}{v_1} = \frac{1}{20} - \frac{1}{30} = \frac{1}{60}, \quad v_1 = +60 \text{ cm.}$$

The concave lens is only 30 cm beyond the convex lens, so the converging rays would meet 30 cm beyond the concave lens. Its object is therefore virtual: $u_2 = +30$ cm.

Step 2: Find the final image in the first arrangement For the concave lens $f_2 = -15$ cm. Therefore

$$\frac{1}{v_2} = \frac{1}{f_2} + \frac{1}{u_2} = -\frac{1}{15} + \frac{1}{30} = -\frac{1}{30}, \quad v_2 = -30 \text{ cm.}$$

The final image is virtual, on the incident side of the concave lens, at the position of the convex lens.

Step 3: Reverse the order encountered by the light Now take the positive direction from the concave lens towards the convex lens. An object 30 cm in front of the concave lens has $u_1 = -30$ cm, $f_1 = -15$ cm. Thus $1/v_1 = -1/15 - 1/30 = -1/10$, so $v_1 = -10$ cm. This virtual image is 10 cm before the concave lens and hence 40 cm before the convex lens. For the latter, $u_2 = -40$ cm and $f_2 = +20$ cm, giving

$$\frac{1}{v_2} = \frac{1}{20} - \frac{1}{40} = \frac{1}{40}, \quad v_2 = +40 \text{ cm.}$$

The second arrangement therefore produces a real image beyond the convex lens. Reversal changes the object distance presented to the second lens; the lenses are separated, so their successive imaging cannot be replaced by simply adding powers at a single plane.

Step 4: Conclusion Initially, the final image is virtual, 30 cm in front of the concave lens, at the convex lens. Yes, the result changes on reversal: the final image is real and 40 cm beyond the convex lens along the new light direction.

Quick Tip to Solve: Locate the first image relative to the second lens before using its lens equation; converging incident rays mean a positive virtual-object distance.

18. (a) Define a wavefront. Using Huygens' geometrical construction, explain with the help of a diagram how the plane wavefront travels from the instant t_1 to t_2 in air.

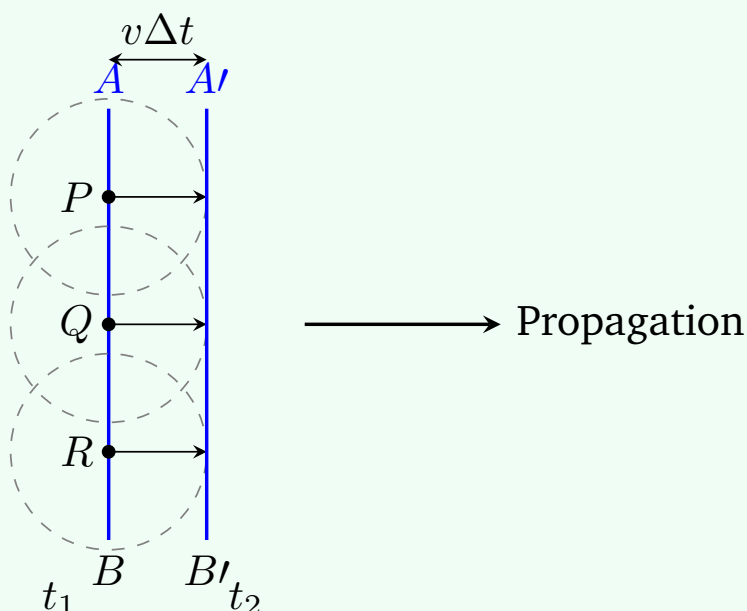
(b) A plane wavefront is incident on a convex lens. Explain, with the help of the diagram, the shape of the refracted wavefront formed. [3 marks]

Solution:

Concept:

- A wavefront is the locus of points of a wave that vibrate in the same phase at a given instant. Rays in an isotropic medium are normal to the wavefront.
- Huygens principle: each point on an existing wavefront acts as a source of secondary wavelets. Their forward common tangent or envelope at a later instant gives the new wavefront.

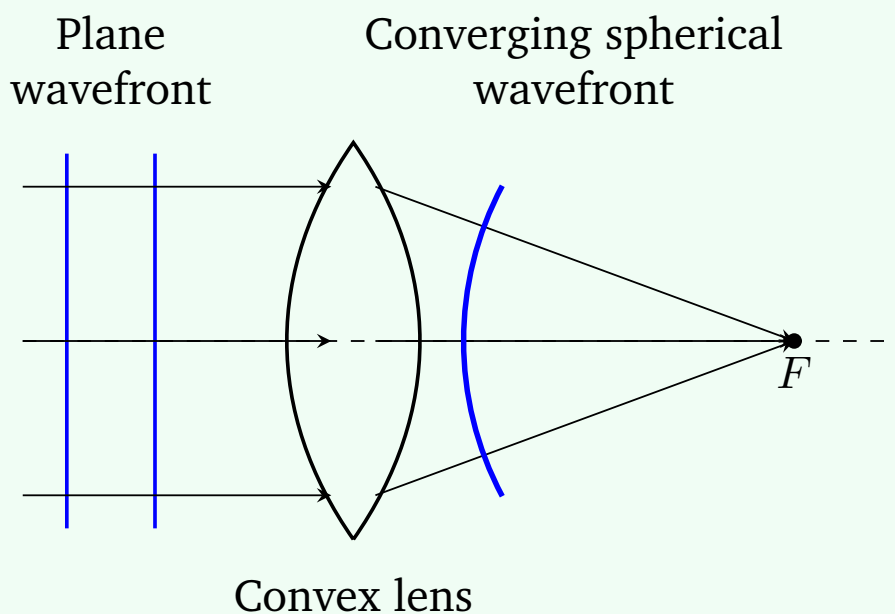
Step 1: (a) Construct the later plane wavefront Let AB represent a plane wavefront at t_1 . Choose points P, Q, R on it. In uniform air, all their secondary wavelets travel with the same speed v . After $\Delta t = t_2 - t_1$, each wavelet has radius $r = v\Delta t$. Draw these equal-radius wavelets and their common forward tangent $A'B'$. This tangent is the wavefront at t_2 . It is a plane parallel to AB , displaced by $v(t_2 - t_1)$ in the direction of propagation. In the diagram, the dashed circles show secondary wavelets and the right-hand vertical line shows their forward envelope.



Huygens construction of a plane wavefront in air

Step 2: (b) Wavefront refracted by a convex lens An incident plane wavefront corresponds to parallel rays. A convex lens is thicker near its principal axis, so the central portion of the wave experiences a larger optical delay than the outer portions. The emergent wavefront consequently becomes a converging spherical wavefront, whose centre is the principal focus F on the far side of the lens. In a two-dimensional section it appears as a circular arc concave towards F . The refracted rays are normals to this arc and meet at F . As the wave travels towards the focus the spherical wavefront radius decreases; after crossing the focus it becomes diverging.

Quick Tip to Solve: Draw wavefronts perpendicular to rays: parallel rays give a plane wavefront, while rays meeting at a focus give spherical wavefronts centred there.



A convex lens produces a converging spherical wavefront

Step 3: Conclusion (a) The forward envelope remains plane and advances by $v(t_2 - t_1)$. (b) A convex lens transforms the incident plane wavefront into a converging spherical wavefront centred at its principal focus.

Quick Tip to Solve: Draw wavefronts perpendicular to rays: parallel rays give a plane wavefront, while rays meeting at a focus give spherical wavefronts centred there.

19. (a) Good quality sunglasses made of polaroids are preferred over ordinary coloured glasses. Explain why.

(b) How is plane polarized light defined?

(c) A beam of plane polarised light is passed through a polaroid. Show graphically, variation of the intensity of the transmitted light with angle of rotation of the polaroid. [3 marks]

Solution:

Concept:

- Polarisation restricts the electric-field vibrations of light to one direction perpendicular to its direction of propagation.
- Malus' law is $I = I_0 \cos^2 \theta$, where I_0 is the intensity of incident plane polarised light and θ is the angle between its electric-field direction and the transmission axis of the analyser.

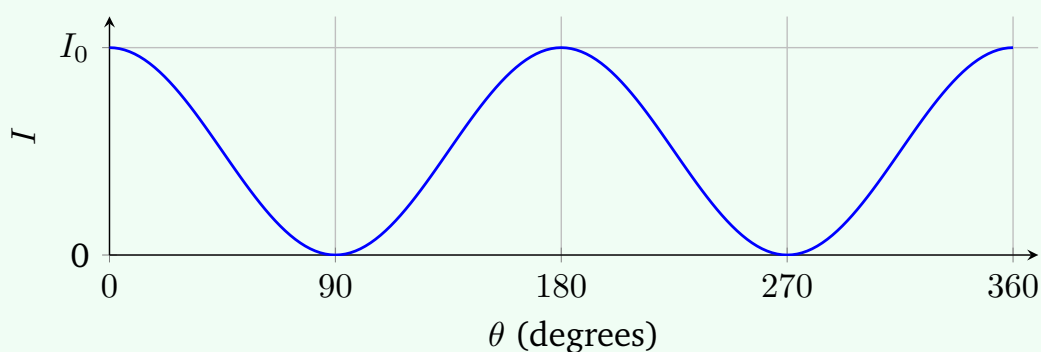
Step 1: (a) Explain the advantage of polaroid sunglasses Light reflected from a horizontal surface, such as a road or water, is predominantly polarised in the horizontal direction. This reflected light produces glare. A polaroid in good sunglasses has its transmission axis approximately vertical, so it suppresses the horizontally polarised component and greatly reduces glare. Ordinary coloured glasses absorb some light of all vibration directions; they reduce brightness but do not selectively remove the

Quick Tip to Solve: Set the angle to zero for parallel axes, then place zeros at crossed axes before drawing the squared-cosine curve.

polarised glare. Thus polaroid sunglasses give clearer, more comfortable vision while retaining useful light from the surroundings.

Step 2: (b) Define plane polarised light Light is plane polarised when the vibrations of its electric field are confined to a single fixed direction perpendicular to the direction of propagation. Equivalently, the electric-field vector and the direction of propagation define one fixed plane. The light remains transverse; polarisation specifies the orientation of its transverse vibration.

Step 3: (c) Plot the transmitted intensity Choose $\theta = 0^\circ$ when the transmission axis of the polaroid is parallel to the incident electric field. The transmitted electric-field amplitude is $E_0 \cos \theta$. Since intensity is proportional to the square of amplitude, the transmitted intensity follows $I = I_0 \cos^2 \theta$. It has maxima at $0^\circ, 180^\circ, 360^\circ$ and zero values at $90^\circ, 270^\circ$. The smooth curve has period 180° ; the intensity cannot be negative.



Malus' law for rotation of the analyser through one full turn.

Step 4: Conclusion Polaroid sunglasses selectively suppress polarised glare. Plane polarised light has a fixed transverse electric-field direction, and its transmitted intensity varies as $I_0 \cos^2 \theta$.

Quick Tip to Solve: Set the angle to zero for parallel axes, then place zeros at crossed axes before drawing the squared-cosine curve.

20. When a given photosensitive material is irradiated with light of frequency ν , the maximum speed of the emitted photoelectrons equals $v_{\max}$. The graph shown in the figure gives a plot of $v_{\max}^2$ varying with frequency ν .

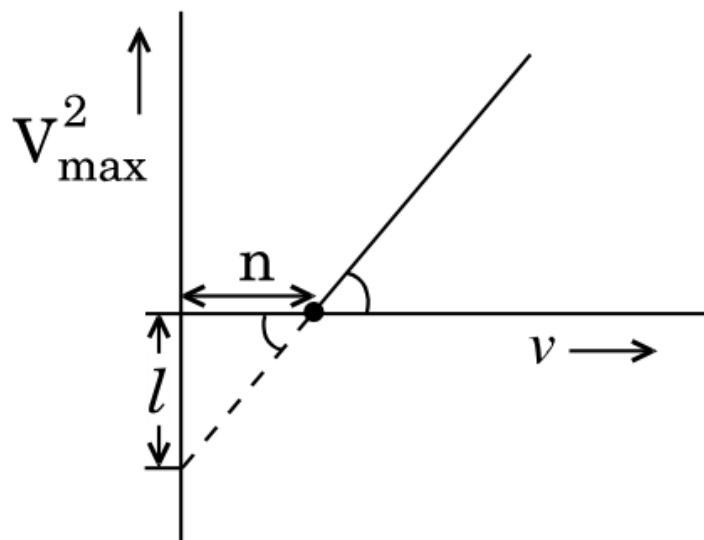
Obtain an expression for

(a) Planck's constant, and

(b) The work function of the given photosensitive material in terms of the parameters ' l ', ' n ' and the mass ' m ' of the electron.

(c) How is threshold frequency determined from the plot?

[3 marks]



Solution:

Concept:

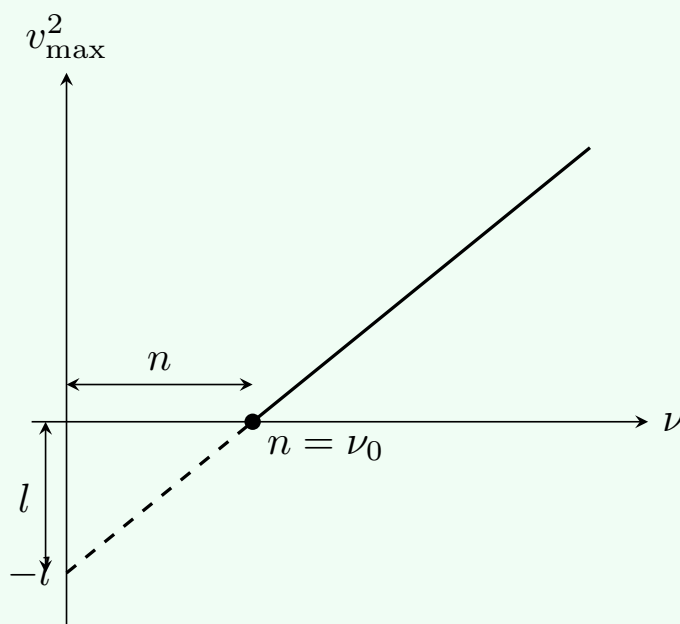
- Einstein's photoelectric equation is $h\nu = \phi + \frac{1}{2}mv_{\max}^2$, where ϕ is the work function and m is the electron mass.
- A straight-line plot of $v_{\max}^2$ against ν has slope $2h/m$ and extrapolated vertical intercept $-2\phi/m$. At the threshold frequency ν_0 , the maximum kinetic energy vanishes.

Step 1: Read the intercepts and write the line equation The printed graph shows a positive horizontal intercept n and a negative vertical intercept whose magnitude is l . Thus its intercepts are $(n, 0)$ and $(0, -l)$. Here l has the units of speed squared and n has the units of frequency. The slope is therefore l/n , giving

$$v_{\max}^2 = \frac{l}{n}\nu - l.$$

The dashed continuation below the frequency axis is an extrapolation used to read the intercept; a negative value of speed squared does not describe emitted electrons.

Quick Tip to Solve: Treat the downward intercept as minus l ; the positive slope is l divided by n , not n divided by l .



Intercepts of the speed-squared versus frequency plot.

Step 2: (a) Determine Planck's constant Rearranging Einstein's equation gives

$$v_{\max}^2 = \frac{2h}{m}\nu - \frac{2\phi}{m}.$$

Comparing the coefficient of frequency with the graph's slope yields $2h/m = l/n$. Hence $h = ml/(2n)$. Dimensionally, mass times speed squared divided by frequency has units of energy times time, as required for Planck's constant.

Step 3: (b) Determine the work function The negative vertical intercept equals $-2\phi/m = -l$. Consequently $\phi = ml/2$. The work function is positive because l denotes the magnitude of the downward intercept. This is the minimum photon energy needed to remove an electron from the surface; it is not the energy carried away by the electron.

Step 4: (c) Read the threshold frequency Set $v_{\max}^2 = 0$, corresponding to electrons just escaping with zero maximum kinetic energy. The straight line intersects the frequency axis at $\nu = n$, so $\nu_0 = n$. As a check, $h\nu_0 = [ml/(2n)]n = ml/2 = \phi$. Below this frequency there is no photoelectric emission, whatever the intensity of the incident light.

Step 5: Conclusion $h = \frac{ml}{2n}$, $\phi = \frac{ml}{2}$, $\nu_0 = n$.

Quick Tip to Solve: Treat the downward intercept as minus l ; the positive slope is l divided by n , not n divided by l .

21. Obtain the first Bohr's radius and the ground state energy of a muonic hydrogen atom i.e. an atom where the electron is replaced by a negatively charged muon (μ^-) of mass about $207 m_e$ that orbits around a proton.

(Given for hydrogen atom, radius of first orbit and ground state energy are 0.53×10^{-10} m and -13.6 eV respectively) [3 marks]

Solution:

Concept:

- For a hydrogen-like atom with nuclear charge $+e$, Bohr's first-orbit radius is $r_1 = \epsilon_0 h^2 / (\pi M e^2)$, where M is the mass of the orbiting negative particle in the fixed-nucleus approximation.
- The first-orbit total energy is $E_1 = -M e^4 / (8 \epsilon_0^2 h^2)$. Therefore r_1 varies inversely with particle mass whereas the binding-energy magnitude varies directly with it.

Step 1: Establish the mass dependence from Bohr's equations A muon has the same negative charge as an electron, so the Coulomb attraction remains $e^2 / (4\pi\epsilon_0 r^2)$. For an orbiting particle of mass M , use the force equation and angular-momentum quantisation:

$$\frac{Mv^2}{r} = \frac{e^2}{4\pi\epsilon_0 r^2}, \quad Mvr = \frac{h}{2\pi}.$$

Eliminating v gives $r_1 = \epsilon_0 h^2 / (\pi M e^2)$. The kinetic energy is $e^2 / (8\pi\epsilon_0 r_1)$ and the potential energy is $-e^2 / (4\pi\epsilon_0 r_1)$, so their sum is $E_1 = -e^2 / (8\pi\epsilon_0 r_1)$. A smaller radius thus gives a more negative ground-state energy.

Step 2: Calculate the first Bohr radius Using the same fixed-proton Bohr approximation as for the supplied hydrogen values, substitute $M = m_\mu = 207 m_e$. The radius ratio is $r_{1,\mu} / r_{1,H} = m_e / m_\mu = 1/207$. Therefore

$$r_{1,\mu} = \frac{0.53 \times 10^{-10}}{207} \text{ m} = 2.56 \times 10^{-13} \text{ m}.$$

The heavier negatively charged particle occupies a much smaller orbit even though its charge is unchanged.

Step 3: Calculate the ground-state energy Since E_1 is proportional to the orbiting mass, $E_{1,\mu} / E_{1,H} = 207$. Hence

$$E_{1,\mu} = 207(-13.6) \text{ eV} = -2815.2 \text{ eV} \simeq -2.82 \text{ keV}.$$

The negative sign means the muon is bound relative to a separated proton and muon at rest. Its ionisation energy would be the positive magnitude of this energy. These results use the standard mass-scaling approximation intended by the supplied data; a finite-proton-motion correction would require the reduced mass instead.

Step 4: Conclusion The first Bohr radius is 2.56×10^{-13} m, and the ground-state energy is -2.82 keV in the fixed-nucleus Bohr approximation.

Quick Tip to Solve: Multiply the energy by 207 but divide the radius by 207, keeping the ground-state energy negative.

22. (a) Distinguish between isotopes and isobars, giving one example for each.

(b) Why is the mass of a nucleus always less than the sum of the masses of its constituents? Write one example to justify your answer. [3 marks]

Solution:

Concept:

- A nuclide A_ZX has proton number Z , mass number A , and neutron number $N = A - Z$.
- The mass defect of a bound nucleus is $\Delta m = Zm_p + (A - Z)m_n - M_{\text{nucleus}}$. Its positive binding energy is $B = \Delta mc^2$.

Step 1: (a) Distinguish isotopes from isobars Isotopes are nuclei of the same element with the same atomic number Z but different mass numbers A . They have equal proton numbers and different neutron numbers. For example, ${}^{12}_6\text{C}$ and ${}^{14}_6\text{C}$ both have six protons, but have six and eight neutrons, respectively.

Isobars are nuclei of different elements with the same mass number A but different atomic numbers Z . Thus their total nucleon numbers agree, although their proton and neutron numbers differ. For example, ${}^{14}_6\text{C}$ and ${}^{14}_7\text{N}$ both have fourteen nucleons, but six and seven protons, respectively.

Step 2: (b) Relate the mass deficit to binding energy When free protons and neutrons combine into a stable bound nucleus, energy is released. The bound system therefore has less total energy than its separated constituents. By mass-energy equivalence, the released binding energy corresponds to a decrease in rest mass:

$$M_{\text{nucleus}} = Zm_p + (A - Z)m_n - \frac{B}{c^2}.$$

Energy equal to B must be supplied to separate that nucleus completely into free nucleons. The mass is not lost without accounting; it is the equivalent of the energy released during formation.

Step 3: Justify the mass defect with deuterium A deuteron consists of one proton and one neutron. Their separate masses sum to approximately $1.007276 + 1.008665 = 2.015941$ u, whereas the deuteron mass is about 2.013553 u. Hence

$$\Delta m \simeq 0.002388 \text{ u}, \quad B \simeq 0.002388 \times 931.5 = 2.22 \text{ MeV}.$$

These are nuclear masses, so no electron mass is included in this comparison.

Step 4: Conclusion Isotopes share Z ; isobars share A . A bound nucleus has less mass than its free nucleons because its binding energy B has been released, giving the mass defect B/c^2 , as illustrated by the deuteron.

Quick Tip to Solve: Compare the lower number for isotopes and the upper number for isobars; use nuclear masses consistently when calculating a mass defect.

22. (a) Classify the following six nuclides into (i) isotones, (ii) isotopes, and (iii) isobars: ${}^{12}_6\text{C}$, ${}^3_2\text{He}$, ${}^{198}_{80}\text{Hg}$, ${}^3_1\text{H}$, ${}^{197}_{79}\text{Au}$, ${}^{14}_6\text{C}$.

(b) How does the size of a nucleus depend on its mass number? Hence explain why the density of nuclear matter should be independent of the size of the nucleus. [3 marks]

Solution:

Concept:

- For A_ZX , the neutron number is $N = A - Z$. Isotones share N , isotopes share Z , and isobars share A .
- The empirical nuclear radius is $R = R_0 A^{1/3}$, where $R_0 \simeq 1.2 \times 10^{-15} \text{ m}$. Nuclear mass is approximately Am_N , with m_N the mean nucleon mass.

Step 1: (a) Calculate the neutron number of each nuclide Subtract the lower number from the upper number, keeping the printed nuclides unchanged:

$$\begin{aligned} {}^{12}_6\text{C} : N = 6, & \quad {}^3_2\text{He} : N = 1, \\ {}^{198}_{80}\text{Hg} : N = 118, & \quad {}^3_1\text{H} : N = 2, \\ {}^{197}_{79}\text{Au} : N = 118, & \quad {}^{14}_6\text{C} : N = 8. \end{aligned}$$

The lower number determines the element, while the upper number counts both protons and neutrons.

Step 2: Group the nuclei by the required equality (i) Isotones: ${}^{198}_{80}\text{Hg}$ and ${}^{197}_{79}\text{Au}$, because each contains 118 neutrons, although their proton numbers differ.

(ii) Isotopes: ${}^{12}_6\text{C}$ and ${}^{14}_6\text{C}$, because both have atomic number 6, but their mass numbers and neutron numbers differ.

(iii) Isobars: ${}^3_2\text{He}$ and ${}^3_1\text{H}$, because both have mass number 3, but different atomic numbers.

Step 3: (b) Derive the independence of nuclear density Nuclear radius, and hence diameter, varies as the cube root of mass number. Approximating a nucleus as a sphere, its volume is

$$V = \frac{4}{3}\pi R^3 = \frac{4}{3}\pi R_0^3 A.$$

Thus both mass and volume are proportional to A . Their ratio is

$$\rho = \frac{Am_N}{(4/3)\pi R_0^3 A} = \frac{3m_N}{4\pi R_0^3}.$$

The mass number cancels. Consequently nuclear matter has approximately the same density for small and large nuclei. The approximation neglects the small variations in binding energy per nucleon and surface effects.

Step 4: Conclusion The isotones are Hg-198 and Au-197, the isotopes are C-12 and C-14, and the isobars are He-3 and H-3. Since $R \propto A^{1/3}$, nuclear volume scales as A , so nuclear density is approximately independent of size.

Quick Tip to Solve: Write A , Z , and $A - Z$ beside every nuclide before pairing; cube the radius law before forming the density ratio.

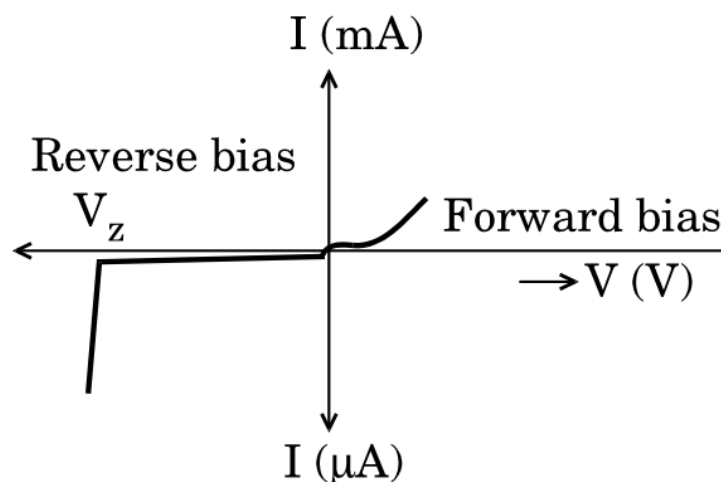
23. The figure shows the V-I characteristic of a semiconductor diode designed to operate under reverse bias.

(a) Identify the semiconductor diode used.

(b) Draw the circuit diagram to obtain the given characteristics of this device.

(c) Briefly explain one use of this device.

[3 marks]



Solution:

Concept:

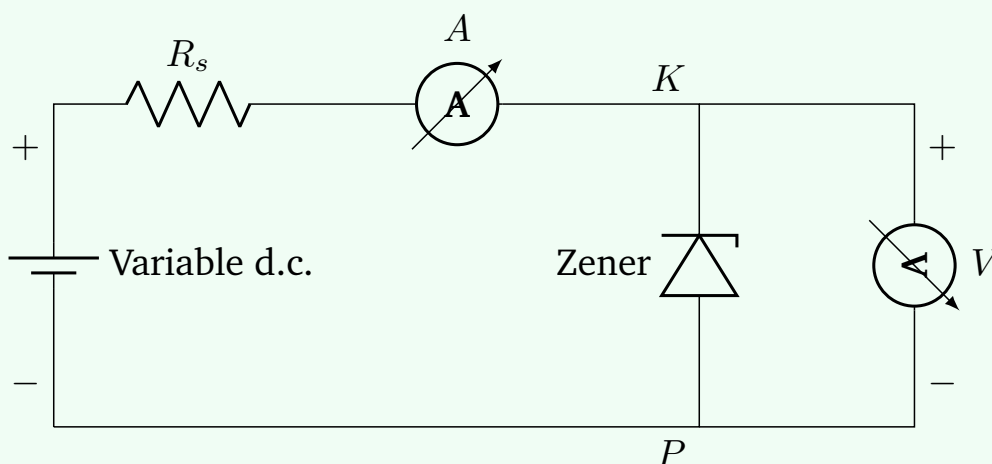
- A Zener diode is a heavily doped p-n junction designed to work safely in reverse breakdown. Beyond the breakdown voltage V_Z , the reverse current increases rapidly while voltage changes very little.
- To measure a diode characteristic, connect an ammeter in series and a voltmeter across the diode. A series resistance limits current and prevents damage.

Step 1: (a) Identify the device from its breakdown region The device is a Zener diode. On the reverse-bias side, the plotted current stays small until the reverse voltage reaches V_Z . It then rises sharply in magnitude at nearly constant voltage. This controlled reverse-breakdown operation distinguishes its intended use from that of an ordinary rectifier diode.

Step 2: (b) Connect the reverse-bias measuring circuit Connect the cathode K to the positive terminal of a variable d.c. supply through a protective series resistor R_s and an ammeter A . Connect the anode P to the negative terminal. Place the voltmeter V directly across the diode, with its positive terminal on the cathode side.

Increase the supply gradually and record the diode voltage and current. Below breakdown a sensitive current range measures the small leakage current; in breakdown a suitable higher range is used. The series resistor must remain present so the maximum permissible Zener current is never exceeded. To obtain the forward branch as well, reverse the diode connection and use the appropriate current range.

Quick Tip to Solve: In a reverse-bias Zener circuit, put the cathode toward supply positive; the ammeter is in series and the voltmeter is across the diode.



Reverse-bias circuit for measuring the Zener characteristic; the d.c. supply is variable.

Step 3: (c) Explain its use as a voltage regulator Connect the reverse-biased Zener diode in parallel with a load, fed through R_s . In breakdown the load voltage is approximately V_Z . If the input rises, the extra current flows mainly through the Zener, while its voltage changes very little. If load current changes, Zener current adjusts in the opposite sense:

$$I_s = I_L + I_Z, \quad I_s = \frac{V_{in} - V_Z}{R_s}.$$

Thus the output remains nearly constant, provided I_Z stays within the breakdown operating range and the diode's power rating is respected.

Step 4: Conclusion The device is a Zener diode. Its characteristic is measured with reverse bias, a series current meter and limiting resistor, and a voltmeter across it. It is used to maintain an approximately constant load voltage V_Z .

Quick Tip to Solve: In a reverse-bias Zener circuit, put the cathode toward supply positive; the ammeter is in series and the voltmeter is across the diode.

24. (a) Differentiate between three segments of an n-p-n transistor on the basis of their size and level of doping.

(b) Draw a plot of transfer characteristic and show which portion of the characteristic is used in amplification and why. [3 marks]

Solution:

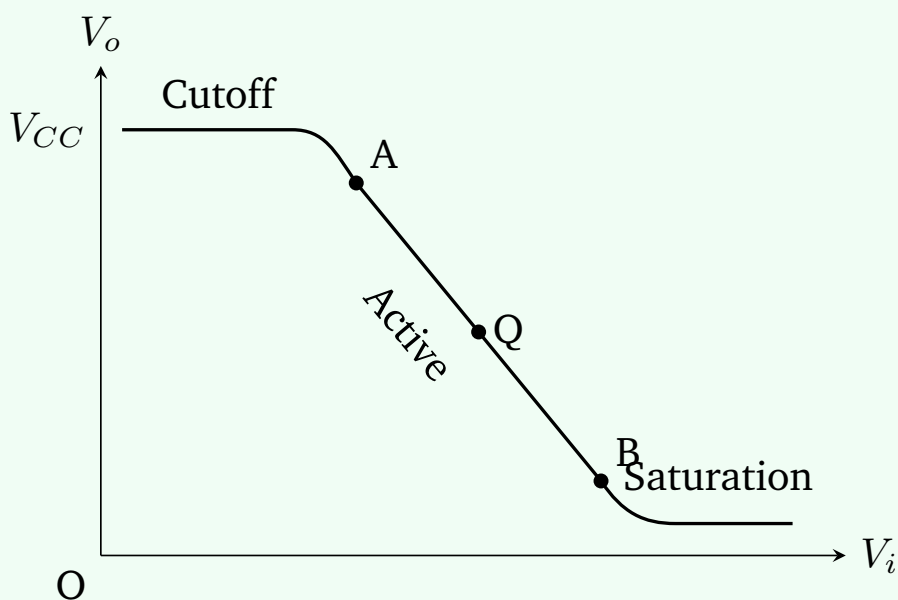
Concept:

- An n-p-n transistor consists of an n-type emitter, a thin p-type base and an n-type collector. Their different dimensions and doping levels support injection, transport and collection of electrons.
- In a common-emitter amplifier, the transfer characteristic plots output voltage V_o against input voltage V_i . The active portion has an approximately constant negative slope, so small input changes give proportional, inverted output changes.

Quick Tip to Solve: For the voltage transfer curve, label both voltage axes and mark the falling active segment; place Q near its midpoint to avoid clipping.

Step 1: (a) Compare the three regions The emitter is of moderate size and is heavily doped. Its purpose is to inject a large number of electrons into the base. The base is the thinnest region and is very lightly doped; consequently, only a small fraction of the injected electrons recombine there. The collector is the largest region and is moderately doped, less heavily than the emitter. Its larger size enables it to collect carriers and dissipate the heat produced in operation. Thus, the base is much thinner than either outer region, while the collector has the largest physical size.

Step 2: (b) Identify the amplification portion For the common-emitter circuit, $V_o = V_{CC} - I_C R_C$, where V_{CC} is the supply voltage, I_C the collector current and R_C the collector load resistance. At small input voltage the transistor is in cutoff, so the output remains close to V_{CC} . Increasing the input through the active region increases I_C , and therefore decreases V_o . At sufficiently large input the transistor saturates and the output becomes nearly constant at a low value. The approximately straight, falling portion AB is used for amplification. Choose the operating point Q near its middle, so both positive and negative input excursions remain within the active region. Here $\Delta V_o = A_v \Delta V_i$, with approximately constant $A_v < 0$; the signal is amplified with phase reversal and little distortion. The flat cutoff and saturation portions cannot reproduce the signal proportionally and would cause clipping.



Common-emitter voltage transfer characteristic; AB is the active region.

Step 3: Conclusion The emitter is heavily doped and of moderate size; the base is very thin and lightly doped; the collector is largest and moderately doped. Amplification uses the approximately linear active portion AB of the transfer characteristic, with the operating point away from its ends.

Quick Tip to Solve: For the voltage transfer curve, label both voltage axes and mark the falling active segment; place Q near its midpoint to avoid clipping.

Section D (15 marks)

25. (a) Derive a relation between the internal resistance, emf and terminal potential difference of a cell from which current I is drawn. Draw V vs I graph for a cell and explain its significance.

(b) A voltmeter of resistance $998\ \Omega$ is connected across a cell of emf 2 V and internal resistance $2\ \Omega$. Find the potential difference across the voltmeter and also across the terminals of the cell. Estimate the percentage error in the reading of the voltmeter. [5 marks]

Solution:

Concept:

- The emf $\mathcal{E}$ is the energy supplied per unit charge by the cell. When it supplies current I , its internal resistance r causes an internal potential drop Ir .
- The terminal potential difference V of a discharging cell satisfies $V = \mathcal{E} - Ir$. A voltmeter of finite resistance draws current and reduces the terminal voltage below the open-circuit emf.

Step 1: (a) Derive the terminal-voltage relation Let an external resistance R be connected across the cell. The same current I flows through R and the internal resistance r . Applying Kirchhoff's loop rule around the circuit gives

$$\mathcal{E} - IR - Ir = 0.$$

Since the external resistance is connected directly between the terminals, $V = IR$. Substitution gives

$$V = \mathcal{E} - Ir, \quad r = \frac{\mathcal{E} - V}{I} \quad (I \neq 0).$$

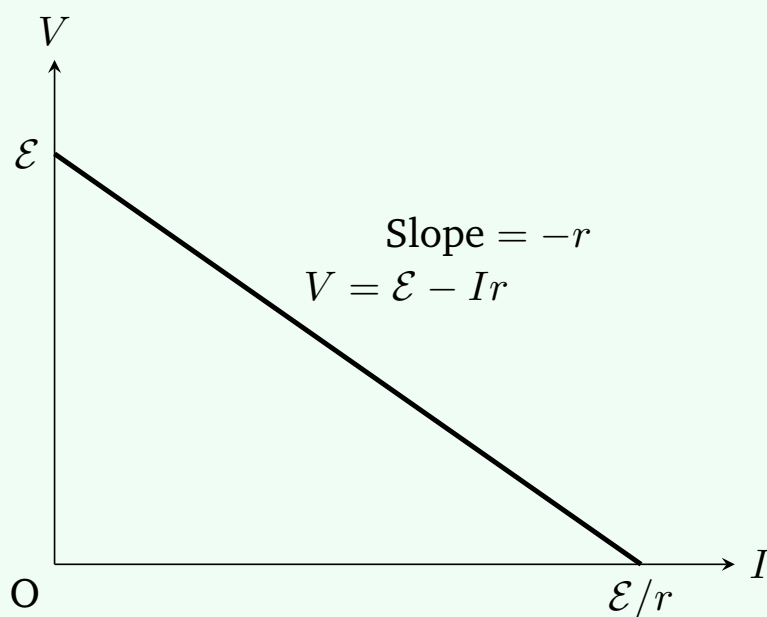
Thus part of the emf is lost inside the cell whenever current is drawn. With the circuit open, $I = 0$, there is no internal voltage drop and the terminal voltage equals the emf.

Step 2: Interpret the graph Plot V on the vertical axis and I on the horizontal axis. The equation $V = \mathcal{E} - rI$ is a straight line with vertical intercept $\mathcal{E}$ and slope $-r$. Therefore the emf can be read from the voltage intercept and the internal resistance from the magnitude of the gradient:

$$r = -\frac{\Delta V}{\Delta I}.$$

The horizontal intercept is $I = \mathcal{E}/r$, the idealised short-circuit current in this constant-internal-resistance model. The downward slope shows why the terminal voltage falls as the load draws more current.

Quick Tip to Solve: Treat the finite-resistance voltmeter as the load: first find current using $R_V + r$, then use IR_V for its reading.



Terminal voltage decreases linearly with current for constant internal resistance.

Step 3: (b) Calculate the current drawn by the voltmeter The voltmeter itself acts as the external resistance $R_V = 998 \Omega$. The total series resistance of the measuring circuit is $R_V + r = 998 + 2 = 1000 \Omega$. Hence

$$I = \frac{\mathcal{E}}{R_V + r} = \frac{2}{1000} = 0.002 \text{ A.}$$

The internal potential drop is $Ir = 0.002 \times 2 = 0.004 \text{ V}$. The finite current drawn is the source of the loading error.

Step 4: Find the measured voltage and percentage error The voltage across the voltmeter is

$$V_V = IR_V = 0.002 \times 998 = 1.996 \text{ V.}$$

Because its leads are connected across the cell terminals, the cell terminal potential difference is the same:

$$V_{\text{terminal}} = \mathcal{E} - Ir = 2 - 0.004 = 1.996 \text{ V.}$$

Relative to the true open-circuit emf, the percentage underestimate is

$$\frac{\mathcal{E} - V_V}{\mathcal{E}} \times 100 = \frac{2 - 1.996}{2} \times 100 = 0.2\%.$$

The meter correctly reads the loaded terminal voltage; its reading is smaller than the emf because connecting it draws current through the cell's internal resistance.

Step 5: Conclusion For a cell supplying current, $V = \mathcal{E} - Ir$. Its V -versus- I graph has intercept $\mathcal{E}$ and slope $-r$. Both the voltmeter potential difference and cell terminal potential difference are 1.996 V ; the reading underestimates the emf by 0.2% .

Quick Tip to Solve: Treat the finite-resistance voltmeter as the load: first find current using $R_V + r$, then use IR_V for its reading.

25. (a) Two cells of different emfs and internal resistances are connected in parallel with one another. Derive the expression for the equivalent emf and equivalent internal resistance of the combination.
- (b) Two identical cells of emf 1.5 V and internal resistance r are each connected in parallel providing a supply to an external circuit consisting of two resistances of $17\ \Omega$ each joined in parallel. A very high resistance voltmeter reads the terminal voltage of the cell to be 1.4 V. Calculate the internal resistance of each cell.

[5 marks]

Solution:**Concept:**

- For a cell of emf E , internal resistance r and outward current I , the terminal voltage is $V = E - Ir$.
- Parallel cells have a common terminal voltage; their branch currents add. Like polarities are connected together.

Step 1: (a) Apply the cell equation to each branch Let the cells have emfs E_1, E_2 , internal resistances r_1, r_2 , and common terminal voltage V . Take branch current as positive when supplied by the cell. Then

$$I_1 = \frac{E_1 - V}{r_1}, \quad I_2 = \frac{E_2 - V}{r_2}.$$

A negative branch current means that current enters that cell. Kirchhoff's junction rule remains valid in either case: the load current is $I = I_1 + I_2$.

Step 2: (a) Obtain the equivalent source parameters Adding and rearranging gives

$$I = \frac{E_1}{r_1} + \frac{E_2}{r_2} - V \left(\frac{1}{r_1} + \frac{1}{r_2} \right),$$

$$V = \frac{E_1 r_2 + E_2 r_1}{r_1 + r_2} - I \frac{r_1 r_2}{r_1 + r_2}.$$

Compare with $V = E_{\text{eq}} - Ir_{\text{eq}}$. The first term is the open-circuit voltage and the coefficient of load current is the equivalent internal resistance. The equivalent emf is weighted by the internal resistances; it is not the sum of the two emfs. In the identical-cell limit the emf remains equal to that of one cell and the internal resistance becomes half that of one cell.

Step 3: (b) Calculate each cell's internal resistance The load resistance is $R_L = 17 \parallel 17 = 8.5\ \Omega$. Since the voltmeter resistance is very high, its current is negligible. Hence

$$I = \frac{1.4}{8.5} = \frac{14}{85} \text{ A}.$$

The identical parallel cells have $E_{\text{eq}} = 1.5 \text{ V}$ and $r_{\text{eq}} = r/2$. Substitution in the terminal voltage relation gives

$$1.4 = 1.5 - \frac{14}{85} \frac{r}{2}, \quad r = \frac{2(0.1)(85)}{14} = \frac{17}{14} \Omega.$$

Each cell supplies $I/2$. Its internal drop is $(I/2)r = 0.1 \text{ V}$, confirming the measured common voltage of 1.4 V.

Quick Tip to Solve: Reduce both parallel combinations first: the load is 8.5 ohms and the cells' equivalent internal resistance is $r/2$.

Step 4: Conclusion

$$E_{\text{eq}} = \frac{E_1 r_2 + E_2 r_1}{r_1 + r_2}, \quad r_{\text{eq}} = \frac{r_1 r_2}{r_1 + r_2}.$$

Each cell has internal resistance $r = 17/14 \Omega \simeq 1.21 \Omega$.

Quick Tip to Solve: Reduce both parallel combinations first: the load is 8.5 ohms and the cells' equivalent internal resistance is $r/2$.

26. (a) A metallic rod of length ' l ' and resistance ' R ' is rotated with a frequency ' ν ' with one end hinged at the centre and the other end at the circumference of a circular metallic ring of radius ' l ', about an axis passing through the centre and perpendicular to the plane of the ring. A constant and uniform magnetic field ' B ' parallel to the axis is present everywhere.

(i) Derive the expression for the induced emf and the current in the rod.

(ii) Due to the presence of current in the rod and of the magnetic field, find the expression for the magnitude and direction of the force acting on this rod.

(iii) Hence, obtain an expression for the power required to rotate the rod.

(b) A copper coil is taken out of a magnetic field with a fixed velocity. Will it be easy to remove it from the same field if its ohmic resistance is increased?

[5 marks]

Solution:**Concept:**

- Motional emf along a conductor is $\varepsilon = \int (\mathbf{v} \times \mathbf{B}) \cdot d\mathbf{l}$. At radius x , the speed is $v = \omega x$, where $\omega = 2\pi\nu$.
- A current element experiences $d\mathbf{F} = I d\mathbf{l} \times \mathbf{B}$. Lenz's law makes the magnetic torque oppose the rotation.
- At constant angular speed the applied mechanical power is $P = \tau\omega$; with negligible ring and contact resistance it equals $I^2 R$.

Step 1: (a)(i) Integrate the motional emf Assume the conducting ring and contacts have negligible resistance, so the circuit resistance is the rod's R . A short radial element dx at distance x moves perpendicular to the field at speed ωx . Its emf magnitude is $d\varepsilon = B\omega x dx$. Therefore

$$\varepsilon = \int_0^l B\omega x dx = \frac{B\omega l^2}{2} = \pi B\nu l^2, \quad I = \frac{\varepsilon}{R} = \frac{\pi B\nu l^2}{R}.$$

For a field out of the plane and anticlockwise rotation, $\mathbf{v} \times \mathbf{B}$ points radially outward, making the rim positive relative to the centre. Current in the rod then flows outward. Reversing the field or rotation reverses its direction.

Step 2: (a)(ii) Determine the magnetic force Every element carries the same current and is perpendicular to $\mathbf{B}$, so $dF = IB dx$. The element forces are parallel, tangential, and oppose the instantaneous

Quick Tip to Solve: Integrate both the local emf and the torque: speed and lever arm each depend on distance from the pivot.

velocity of the rod. Integration gives

$$F = \int_0^l IB \, dx = IBl = \frac{\pi B^2 \nu l^3}{R}.$$

For the example above, outward current crossed with the outward field gives a clockwise tangential force. This is the braking force required by Lenz's law. Since the force per unit length is uniform, its resultant acts at the rod's midpoint, a distance $l/2$ from the pivot.

Step 3: (a)(iii) Calculate the required mechanical power The braking torque must be found with the varying lever arm, rather than by assigning the full length to every force element:

$$\tau = \int_0^l x IB \, dx = \frac{IBl^2}{2} = \frac{Fl}{2}.$$

The external torque balances this torque to maintain constant speed. Hence

$$P = \tau\omega = \frac{IBl^2\omega}{2} = \frac{B^2\omega^2 l^4}{4R} = \frac{\pi^2 B^2 \nu^2 l^4}{R}.$$

Equivalently, $P = I\varepsilon = I^2 R$: mechanical work is converted into Joule heat in the closed circuit. Frictional losses, if present, require additional mechanical power.

Step 4: (b) Explain the effect of increasing coil resistance At the same withdrawal speed in the same field and with unchanged coil geometry, the rate of flux change and induced emf remain the same. Increasing the ohmic resistance reduces $I = \varepsilon/R$. The current produces a magnetic force opposing removal, so a smaller induced current produces a smaller retarding force. Thus a smaller external pulling force is needed to maintain the prescribed speed, and the coil is easier to remove.

Step 5: Conclusion

$$\varepsilon = \pi B \nu l^2, \quad I = \frac{\pi B \nu l^2}{R}, \quad F = \frac{\pi B^2 \nu l^3}{R}, \quad P = \frac{\pi^2 B^2 \nu^2 l^4}{R}.$$

The magnetic force is tangential and opposes rotation. Increasing the copper coil's resistance makes withdrawal easier.

Quick Tip to Solve: Integrate both the local emf and the torque: speed and lever arm each depend on distance from the pivot.

26. (a) A rectangular coil rotates in a uniform magnetic field. Obtain an expression for induced emf and current at any instant. Also find their peak values. Show the variation of induced emf versus angle of rotation (ωt) on a graph.

(b) An iron bar falling through the hollow region of a thick cylindrical shell made of copper experiences a retarding force. What can you conclude about the nature of the iron bar? Explain. [5 marks]

Solution:

Concept:

- Magnetic flux linkage of a coil is $N\Phi = NBA \cos \theta$, where N is the number of turns, A is the area of each turn, B is the uniform magnetic field, and θ is the angle between the field and the coil's normal.
- Faraday's law gives $\varepsilon = -d(N\Phi)/dt$. For a closed coil of resistance R , neglecting self-inductance, Ohm's law gives $i = \varepsilon/R$.
- Eddy currents circulate in a bulk conductor when its magnetic flux changes. By Lenz's law, their magnetic effects oppose the relative motion responsible for the flux change.

Step 1: (a) Express the flux linkage during rotation Let the rectangular coil have sides l and b , so that $A = lb$. Suppose it rotates at constant angular speed ω about an axis in its plane perpendicular to the field. Choose $t = 0$ when the normal to the coil is parallel to the field. Then $\theta = \omega t$, and the flux linkage is

$$N\Phi(t) = NBA \cos(\omega t).$$

If the question's coil has one turn, set $N = 1$. Introducing N also gives the usual general generator expression. At the starting position the flux is maximum; a quarter-turn later the normal is perpendicular to the field and the flux is zero.

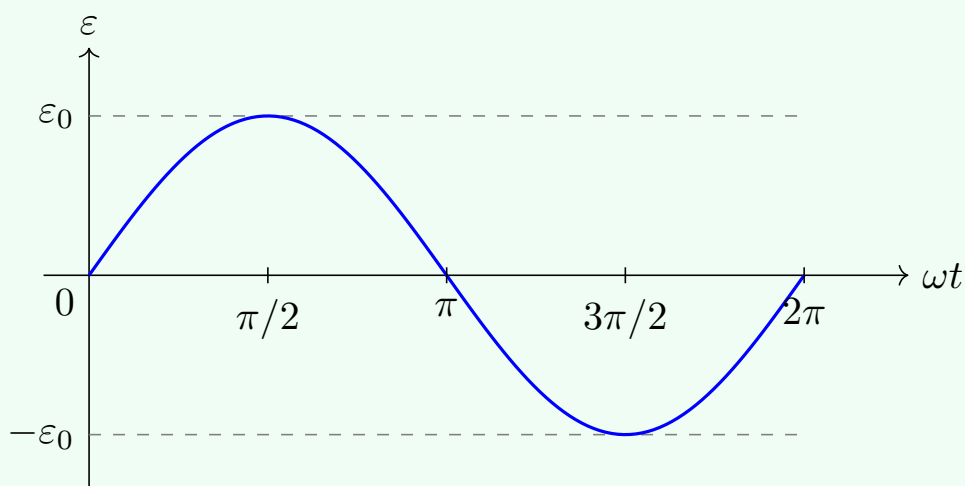
Step 2: (a) Derive instantaneous emf, current and their peak values Differentiate the flux linkage, retaining the minus sign in Faraday's law:

$$\begin{aligned}\varepsilon(t) &= -\frac{d}{dt} [NBA \cos(\omega t)] \\ &= NBA\omega \sin(\omega t) = \varepsilon_0 \sin(\omega t), \\ i(t) &= \frac{NBA\omega}{R} \sin(\omega t) = i_0 \sin(\omega t).\end{aligned}$$

Here R is the total resistance of the closed circuit; for a coil alone it is the coil resistance. The resistive-circuit assumption makes current and emf in phase. The positive sign corresponds to the chosen orientation of the coil and terminal polarity; reversing that convention reverses both signs without changing their amplitudes. Their magnitudes are greatest when $|\sin(\omega t)| = 1$, namely at odd multiples of $\pi/2$. The current reverses direction after every half revolution.

Step 3: (a) Plot the induced emf against rotation angle With $\theta = \omega t$, the emf follows a sine curve. It is zero at $0, \pi, 2\pi$, reaches $+\varepsilon_0$ at $\pi/2$, and reaches $-\varepsilon_0$ at $3\pi/2$. Thus emf is zero when the flux itself is extreme and greatest in magnitude when the flux changes most rapidly. The curve repeats after 2π , corresponding to a time period $T = 2\pi/\omega$.

Quick Tip to Solve: Choose the coil normal parallel to the field at zero angle: flux is cosine, so induced emf is sine. A conducting nonmagnetic tube can still brake a falling magnet through eddy currents.



Induced emf versus rotation angle for one complete revolution.

Step 4: (b) Explain the retarding force on the falling iron bar The iron bar must be magnetised: it behaves as a bar magnet with its own magnetic field. As it falls, the magnetic flux through successive conducting loops of the thick copper shell changes. This induces circulating eddy currents in the shell even though there is no externally connected circuit. The field of these currents opposes the change in flux and therefore opposes the motion of the magnet. Below the falling magnet the induced field resists its approach, while above it the induced field resists its departure; both effects produce an upward retarding force. Mechanical energy is transferred to thermal energy through Joule heating in the copper. Copper need not be ferromagnetic for this effect: its electrical conductivity permits the eddy currents.

Step 5: Conclusion For a resistive closed circuit, $\varepsilon(t) = NBA\omega \sin(\omega t)$ and $i(t) = (NBA\omega/R) \sin(\omega t)$, with peak values $\varepsilon_0 = NBA\omega$ and $i_0 = NBA\omega/R$. The emf-angle graph is sinusoidal. The falling iron bar is magnetised, and eddy currents in the copper shell produce the retarding force by Lenz's law.

Quick Tip to Solve: Choose the coil normal parallel to the field at zero angle: flux is cosine, so induced emf is sine. A conducting nonmagnetic tube can still brake a falling magnet through eddy currents.

27. (a) Draw a labelled ray diagram of compound microscope, when final image forms at the least distance of distinct vision.
- (b) Why is its objective of short focal length and of short aperture, compared to its eyepiece? Explain.
- (c) The focal length of the objective is 4 cm while that of eyepiece is 10 cm. The object is placed at a distance of 6 cm from the objective lens.
- (i) Calculate the magnifying power of the compound microscope, if its final image is formed at the near point.
- (ii) Also calculate length of the compound microscope. [5 marks]

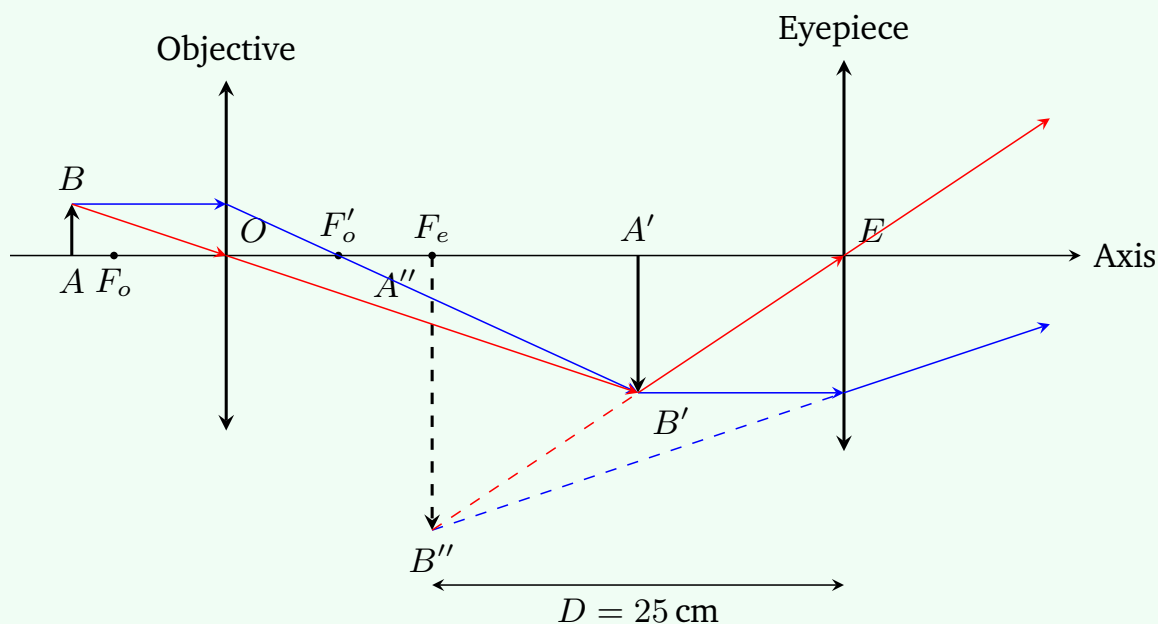
Solution:

Concept:

- A compound microscope uses a short focal length objective to form a real, inverted and enlarged intermediate image. The eyepiece acts as a simple microscope, producing an enlarged virtual final image.
- With Cartesian signs, the lens formula is $1/f = 1/v - 1/u$, where u and v are the object and image distances from the lens. Objective magnification is $m_o = v_o/u_o$.
- For a final image at the near point $D = 25$ cm, the eyepiece magnification is $m_e = 1 + D/f_e$. The microscope magnifying power is $M = m_o m_e$; its magnitude describes enlargement and its negative sign describes inversion.

Step 1: (a) Ray diagram for the final image at the near point Place the object AB just beyond the objective focus F_o . A ray from B parallel to the principal axis is refracted through F'_o , while a ray through the optical centre O continues undeviated. They meet at B' , forming the real, inverted, enlarged image $A'B'$. This intermediate image lies between the eyepiece and its first focus F_e . The eyepiece therefore sends out diverging rays whose backward extensions meet at B'' . The final image $A''B''$ is virtual, enlarged and inverted relative to the original object, and lies at distance D from the eyepiece. The diagram is schematic: dashed lines are backward extensions, not actual light paths.

Quick Tip to Solve: First find the actual objective image distance; then add the magnitude of the eyepiece object distance to obtain the lens separation.



Compound microscope with virtual final image at the near point (schematic).

Step 2: (b) Choice of objective focal length and aperture The objective must produce a strongly enlarged intermediate image. For an object placed close to its focus, its magnification is approximately $|m_o| \simeq L/f_o$, where L represents the image-side distance within the instrument. Thus a short objective focal length gives large magnification. Its focal length is chosen shorter than that of the eyepiece because the objective provides substantial linear enlargement before the eyepiece magnifies the intermediate image angularly. A small objective aperture restricts marginal rays and reduces spherical aberration, helping to obtain a sharp image of the small specimen. The eyepiece has a comparatively larger aperture to provide a useful field of view and allow the emerging rays to enter the eye conveniently.

Step 3: (c)(i) Objective image distance and magnification Here $f_o = +4$ cm and $u_o = -6$ cm. Applying the objective lens formula,

$$\frac{1}{v_o} = \frac{1}{f_o} + \frac{1}{u_o} = \frac{1}{4} - \frac{1}{6} = \frac{1}{12},$$

$$v_o = +12 \text{ cm}, \quad m_o = \frac{12}{-6} = -2.$$

The intermediate image is real, lies to the right of the objective and is twice the object height in magnitude. For the eyepiece, $f_e = 10$ cm and the final image is at the near point, so $m_e = 1 + 25/10 = 3.5$. Therefore $M = (-2)(3.5) = -7$. The angular enlargement is sevenfold; the minus sign records that the final image is inverted.

Step 4: (c)(ii) Separation of the objective and eyepiece The final image is virtual on the incident side of the eyepiece, hence $v_e = -25$ cm. Its object is the intermediate image. The eyepiece lens formula

Quick Tip to Solve: First find the actual objective image distance; then add the magnitude of the eyepiece object distance to obtain the lens separation.

gives

$$\frac{1}{u_e} = \frac{1}{v_e} - \frac{1}{f_e} = -\frac{1}{25} - \frac{1}{10} = -\frac{7}{50},$$
$$u_e = -\frac{50}{7} \text{ cm.}$$

Thus the intermediate image is $50/7$ cm to the left of the eyepiece. Since it is also 12 cm to the right of the objective, the distance between the lenses is

$$\ell = v_o + |u_e| = 12 + \frac{50}{7} = \frac{134}{7} \text{ cm} \simeq 19.14 \text{ cm.}$$

This exact separation uses the calculated image distances; replacing the objective image distance by a tube-length approximation would be inappropriate for the given numbers.

Step 5: Conclusion The near-point microscope has magnifying power $M = -7$, or magnitude 7, with an inverted final image. Its objective-eyepiece separation is $134/7$ cm $\simeq 19.14$ cm. A short objective focal length increases enlargement, and its small aperture reduces spherical aberration.

Quick Tip to Solve: First find the actual objective image distance; then add the magnitude of the eyepiece object distance to obtain the lens separation.

OR

27. (a) With the help of a labelled ray diagram, explain the construction and working of a Cassegrain reflecting telescope.

(b) An amateur astronomer wishes to estimate roughly the size of the Sun using his crude telescope consisting of an objective lens of focal length 200 cm and an eyepiece of focal length 10 cm. By adjusting the distance of the eyepiece from the objective, he obtains an image of the Sun on a screen 40 cm behind the eyepiece. The diameter of the Sun's image is measured to be 6.0 cm. Estimate the Sun's size, given that the average Earth-Sun distance is 1.5×10^{11} m. [5 marks]

Solution:

Concept:

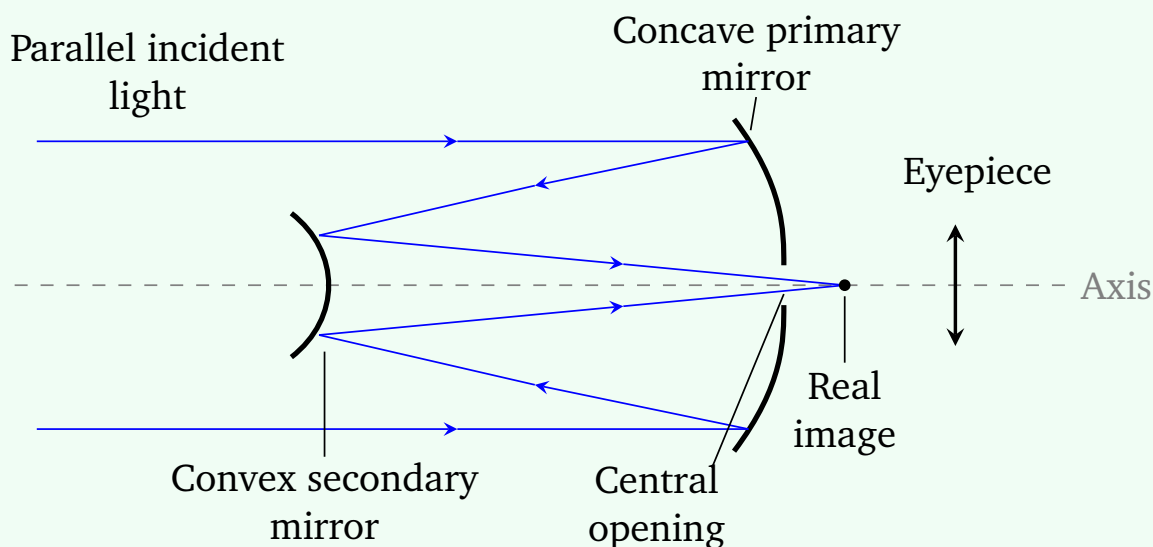
- A Cassegrain telescope uses a large concave primary mirror and a small convex secondary mirror. Two reflections bring light to a focus behind the primary, through its central opening.
- For a lens, the Cartesian sign convention gives $1/f = 1/v - 1/u$. The transverse magnification is $m = v/u$; its magnitude relates the image and object diameters.
- For a distant object of diameter D_s at distance R , its small angular diameter is $\theta \simeq D_s/R$. The objective forms an image of diameter $d_o \simeq f_o\theta$.

Step 1: (a) Construction of the Cassegrain telescope A large concave primary mirror is mounted at the rear of the telescope tube, with a circular opening at its centre. A small convex secondary mirror is placed on the common axis, in front of the primary and before the primary's focus. Its reflecting surface faces the primary. The eyepiece is placed behind the primary, along the axis of its central opening. The large primary collects light from distant objects; its diameter determines the

Quick Tip to Solve: For a projected telescope image, first remove the eyepiece's transverse magnification; then divide the intermediate diameter by the objective focal length to find the angular diameter.

light-gathering ability. The secondary intercepts the converging reflected beam and folds the optical path, giving a long effective focal length within a comparatively short tube.

Step 2: (a) Working and labelled ray diagram Light from a distant axial object arrives approximately parallel to the principal axis and is reflected by the primary into a converging beam. Before the rays reach the primary's original focus, they strike the convex secondary. The secondary reflects them back towards the primary, reducing their convergence so that they pass through its central hole and form a real image behind it. The eyepiece magnifies this real image; when the image lies in the eyepiece's focal plane, the final image is at infinity for relaxed viewing. The diagram shows representative axial rays and the folded path. Reflecting objectives avoid chromatic aberration because reflection does not separate the colours as refraction does.



Cassegrain telescope: two reflections bring an axial beam through the primary's central opening to the real image near the eyepiece.

Step 3: (b) Determine the object distance for the eyepiece Here the telescope projects a real image onto a screen; therefore the eyepiece is used as a projecting lens. Its focal length is $f_e = +10$ cm and the screen distance is $v_e = +40$ cm. The objective's intermediate image acts as a real object for the eyepiece. Thus

$$\frac{1}{10} = \frac{1}{40} - \frac{1}{u_e}, \quad u_e = -\frac{40}{3} \text{ cm.}$$

The negative object distance places the intermediate image in front of the eyepiece. It is beyond the eyepiece's focus, as required to produce a real screen image.

Step 4: (b) Recover the intermediate image diameter The eyepiece's transverse magnification is

$$m_e = \frac{v_e}{u_e} = \frac{40}{-40/3} = -3.$$

The screen image is therefore three times the intermediate image diameter, with reversed orientation

Quick Tip to Solve: For a projected telescope image, first remove the eyepiece's transverse magnification; then divide the intermediate diameter by the objective focal length to find the angular diameter.

relative to it. Since its measured diameter is 6.0 cm,

$$d_o = \frac{6.0}{|m_e|} = 2.0 \text{ cm.}$$

The screen diameter cannot be directly divided by the objective focal length: the eyepiece has enlarged the intermediate image.

Step 5: (b) Relate angular diameter to the Sun's diameter The Sun is so far away that the objective image forms approximately in its focal plane. With $f_o = 200 \text{ cm}$, its angular diameter is

$$\theta \simeq \frac{d_o}{f_o} = \frac{2.0}{200} = 0.010 \text{ rad.}$$

Using the Earth-Sun distance $R = 1.5 \times 10^{11} \text{ m}$, the small-angle relation gives

$$D_s \simeq R\theta = (1.5 \times 10^{11})(0.010) = 1.5 \times 10^9 \text{ m.}$$

The approximation is justified by the small angle and is appropriate for the requested rough estimate.

Step 6: Conclusion The Cassegrain telescope forms its real intermediate image behind the perforated primary mirror after reflections from the concave primary and convex secondary. The estimated diameter of the Sun is $1.5 \times 10^9 \text{ m}$, or $1.5 \times 10^6 \text{ km}$.

Quick Tip to Solve: For a projected telescope image, first remove the eyepiece's transverse magnification; then divide the intermediate diameter by the objective focal length to find the angular diameter.