

CBSE Class 12 Physics Question Paper 2019 with Solution PDF-Set 55/5/2

Question Paper with Solutions

Conducted by the Central Board of Secondary Education (CBSE)



Time Allowed: 3 hours

Maximum Marks: 70

General Instructions

- (i) All questions are compulsory. There are 27 questions in all.
- (ii) This question paper has four sections : Section A, Section B, Section C and Section D.
- (iii) Section A contains five questions of one mark each, Section B contains seven questions of two marks each, Section C contains twelve questions of three marks each, Section D contains three questions of five marks each.
- (iv) There is no overall choice. However, an internal choice(s) has been provided in two questions of one mark, two questions of two marks, four questions of three marks and three questions of five marks weightage. You have to attempt only one of the choices in such questions.

You may use the following values of physical constants wherever necessary :

$$c = 3 \times 10^8 \text{ m/s}$$

$$h = 6.63 \times 10^{-34} \text{ Js}$$

$$e = 1.6 \times 10^{-19} \text{ C}$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ T m A}^{-1}$$

$$\epsilon_0 = 8.854 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$$

$$\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$$

$$\text{Mass of electron (m}_e\text{)} = 9.1 \times 10^{-31} \text{ kg}$$

$$\text{Mass of neutron} = 1.675 \times 10^{-27} \text{ kg}$$

$$\text{Mass of proton} = 1.673 \times 10^{-27} \text{ kg}$$

$$\text{Avogadro's number} = 6.023 \times 10^{23} \text{ per gram mole}$$

$$\text{Boltzmann constant} = 1.38 \times 10^{-23} \text{ JK}^{-1}$$

Section A (5 marks)

1. Write one important property of a paramagnetic material.

[1 mark]

Solution:

Concept:

- A paramagnetic material has atoms with permanent magnetic dipole moments.
- Its magnetic susceptibility χ_m is small and positive; in an applied magnetic field, its magnetisation is along the field.

Step 1: Consider the applied field Without an external field, thermal motion leaves the atomic moments randomly oriented. An applied field produces a slight preference for alignment along its direction.

Step 2: Identify the observable property This alignment gives a weak magnetisation parallel to the applied field. In a non-uniform field, the material tends to move towards the region where the field is stronger.

Step 3: Conclusion A paramagnetic material is weakly attracted by an external magnetic field.

Quick Tip to Solve: Remember the positive susceptibility of paramagnets: their weak magnetisation follows the applied field.

OR

1. Do the diamagnetic substances have resultant magnetic moment in an atom in the absence of external magnetic field?

[1 mark]

Solution:

Concept:

- The resultant magnetic moment of an atom is the vector sum of its electronic orbital and spin magnetic moments.
- Diamagnetic atoms have no permanent resultant magnetic moment in the absence of an external field.

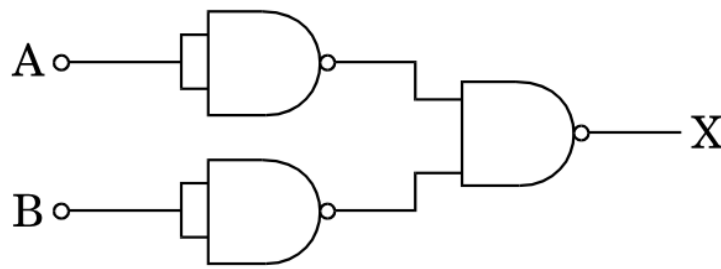
Step 1: Examine the atomic moments In a diamagnetic substance, electrons are paired so that their magnetic contributions cancel. Thus the atom does not carry a permanent net magnetic dipole moment.

Step 2: Apply the stated condition The question specifies that the external magnetic field is absent. Therefore there is also no field-induced magnetic moment. Diamagnetism appears only when an applied field induces a small moment opposing it.

Step 3: Conclusion No. The resultant magnetic moment of a diamagnetic atom is zero in the absence of an external magnetic field.

Quick Tip to Solve: Distinguish zero atomic moment in diamagnets from randomly oriented non-zero atomic moments in paramagnets.

2. Identify the equivalent gate for the circuit of a combination of gates shown in the figure. Write its symbol. [1 mark]



Solution:

Concept:

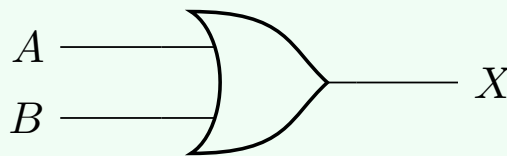
- A NAND gate gives the complement of the AND operation. When both its inputs are joined, it acts as a NOT gate: $\overline{A \cdot A} = \overline{A}$.
- De Morgan's theorem gives $\overline{\overline{A} \cdot \overline{B}} = A + B$, where $+$ denotes logical OR.

Step 1: Find the intermediate outputs Both inputs of the upper NAND gate receive A , while both inputs of the lower NAND gate receive B . Their outputs are therefore $\overline{A}$ and $\overline{B}$, respectively.

Step 2: Simplify the final output These intermediate outputs enter another NAND gate, so

$$X = \overline{\overline{A} \cdot \overline{B}} = A + B.$$

The output is low only when both inputs are low. The corresponding gate symbol has no output inversion bubble, as shown below.

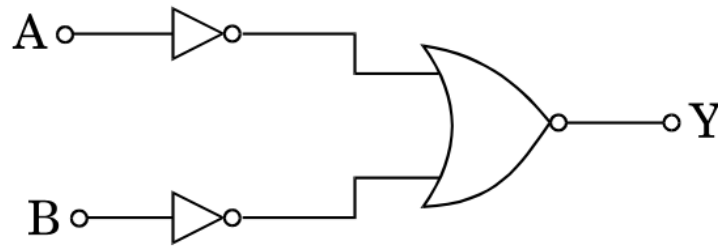


Equivalent OR gate

Step 3: Conclusion The combination is equivalent to an **OR gate**, with output $X = A + B$.

Quick Tip to Solve: First replace each NAND gate with joined inputs by a NOT gate, then apply De Morgan's theorem at the final gate.

2. Draw the logic symbol of the gate shown by the combination of gates and write its name. [1 mark]



Solution:

Concept:

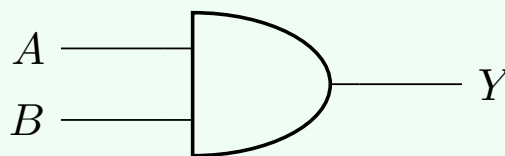
- A NOT gate complements its input, while a NOR gate gives the complement of the logical OR of its inputs.
- De Morgan's theorem states $\overline{\overline{A} + \overline{B}} = A \cdot B$, where $+$ means OR and $\cdot$ means AND.

Step 1: Follow the input inversions Each input passes through a NOT gate. Thus the two signals entering the final NOR gate are $\overline{A}$ and $\overline{B}$.

Step 2: Apply the NOR operation The output of the last gate is

$$Y = \overline{\overline{A} + \overline{B}} = A \cdot B.$$

It is high only when both original inputs are high. The equivalent symbol is the D-shaped gate below, with no inversion bubble at its output.



Equivalent AND gate

Step 3: Conclusion The combination is an **AND gate**, with output $Y = A \cdot B$.

Quick Tip to Solve: For inverted inputs entering a NOR gate, apply De Morgan's theorem to obtain the AND of the original inputs.

3. Can a transistor be used as a rectifier? [1 mark]

Solution:

Concept:

- Rectification relies on the unequal conduction of a p-n junction under forward and reverse bias.
- A transistor contains two p-n junctions: the emitter-base junction and the collector-base junction.

Step 1: Select one junction A transistor can be used as a two-terminal diode by using its base and

Quick Tip to Solve: Identify a single p-n junction between two transistor terminals; rectification uses its one-way conduction.

collector terminals, with the emitter left unconnected. The base-collector junction then supplies the required p-n junction.

Step 2: Examine the alternating voltage During the half-cycle that forward-biases the selected junction, an appreciable current flows. During the opposite half-cycle, reverse bias allows only a very small leakage current, provided breakdown is avoided. The load current is therefore predominantly unidirectional.

Step 3: Conclusion Yes. A transistor can be used as a rectifier by using one of its p-n junctions as a diode and leaving the third terminal unconnected.

Quick Tip to Solve: Identify a single p-n junction between two transistor terminals; rectification uses its one-way conduction.

4. Why is the direction of the electric field due to a charged conducting sphere at any point perpendicular to its surface?

[1 mark]

Solution:

Concept:

- In electrostatic equilibrium, free charges in a conductor have no sustained motion, and its surface is equipotential.
- A tangential electric field E_t would exert a tangential force $F_t = qE_t$ on a free charge q .

Step 1: Test a tangential field component Suppose the electric field at the sphere's surface had a component along the surface. It would push free electrons along the conducting surface, causing the charges to redistribute.

Step 2: Impose electrostatic equilibrium Charge redistribution continues until this tangential component vanishes. The electric field just outside the charged surface therefore has only a normal component; for a sphere this direction is radial.

Step 3: Conclusion The field is perpendicular to the conducting surface because any tangential component would move free charges and violate electrostatic equilibrium.

Quick Tip to Solve: At a conductor's surface, eliminate the tangential field by asking whether it would make free charges move.

5. Plot a graph of the de-Broglie wavelength associated with electron as a function of accelerating potential.

[1 mark]

Solution:

Concept:

- The de-Broglie wavelength is $\lambda = h/p$, where h is Planck's constant and p is the electron momentum.
- An electron accelerated from rest through a positive potential difference V gains kinetic energy eV .

Quick Tip to Solve: Quadrupling the accelerating potential halves the wavelength; use this ratio to check the graph's shape.

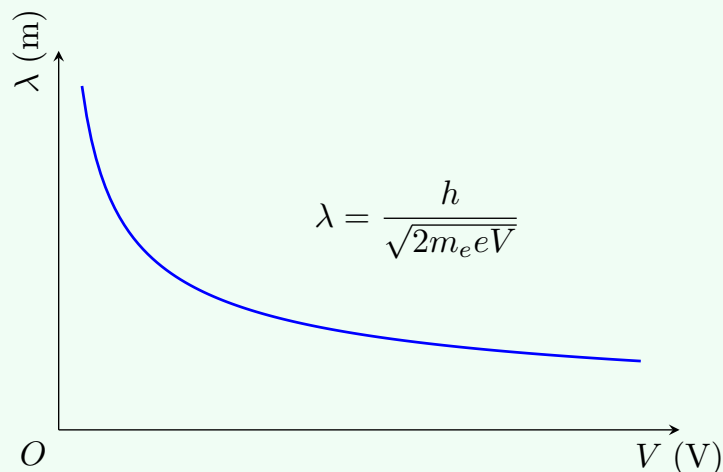
In the non-relativistic limit, $p^2/(2m_e) = eV$.

Step 1: Express wavelength in terms of potential Eliminate the momentum using the kinetic-energy relation:

$$p = \sqrt{2m_e eV}, \quad \lambda = \frac{h}{\sqrt{2m_e eV}}.$$

Thus increasing the accelerating potential increases momentum and decreases wavelength.

Step 2: Draw the inverse-square-root curve Put accelerating potential V on the horizontal axis and wavelength λ on the vertical axis. For $V > 0$, the graph decreases steeply at first and then becomes flatter. It remains above the horizontal axis; it is not a straight line. The sketch shows the usual non-relativistic relation.



de-Broglie wavelength versus accelerating potential (schematic)

Step 3: Conclusion The required curve is $\lambda \propto V^{-1/2}$, decreasing and convex upward in the first quadrant.

Quick Tip to Solve: Quadrupling the accelerating potential halves the wavelength; use this ratio to check the graph's shape.

Section B (14 marks)

6. (a) Obtain the conditions under which an electron does not suffer any deflection while passing through a magnetic field.

(b) Two protons P and Q moving with the same speed pass through the magnetic fields $\vec{B}_1$ and $\vec{B}_2$ respectively, at right angles to the field directions. If $|\vec{B}_2| > |\vec{B}_1|$, which of the two protons will describe the circular path of smaller radius? Explain. [2 marks]

Solution:

Concept:

- A charge q moving at velocity $\vec{v}$ in a magnetic field experiences $\vec{F} = q(\vec{v} \times \vec{B})$, whose magnitude is $|q|vB \sin \theta$.

Quick Tip to Solve: For identical particles with equal speeds entering normally, compare inverse field strengths: the stronger field produces the smaller radius.

- For motion perpendicular to a uniform magnetic field, the magnetic force provides centripetal force: $|q|vB = mv^2/r$. Here r is the circular orbit radius.

Step 1: (a) Set the magnetic force equal to zero For an electron passing through a nonzero magnetic field, its speed and charge magnitude are nonzero. No magnetic deflection requires

$$evB \sin \theta = 0 \Rightarrow \theta = 0^\circ \text{ or } 180^\circ.$$

Thus its velocity must be parallel or antiparallel to the magnetic field throughout its path. A stationary electron also experiences zero magnetic force, although it is not passing through the field. The negative sign of the electron's charge changes force direction when the force exists, but not these zero-force conditions.

Step 2: (b) Compare the two circular radii Both protons have the same mass m_p , charge e , and speed v . Since each enters perpendicular to its field,

$$r_P = \frac{m_p v}{e|\vec{B}_1|}, \quad r_Q = \frac{m_p v}{e|\vec{B}_2|}.$$

Dividing these expressions gives $r_Q/r_P = |\vec{B}_1|/|\vec{B}_2| < 1$, because the printed condition is $|\vec{B}_2| > |\vec{B}_1|$. The stronger field supplies a larger centripetal force at the same speed.

Step 3: Conclusion (a) A moving electron is undeflected when $\vec{v}$ is parallel or antiparallel to $\vec{B}$. (b) Proton Q describes the smaller circular path: $r_Q < r_P$.

Quick Tip to Solve: For identical particles with equal speeds entering normally, compare inverse field strengths: the stronger field produces the smaller radius.

7. A wire of resistance R , length l and area of cross-section A , is cut into two parts, having their lengths in the ratio 1 : 2. The shorter wire is now stretched till its length becomes equal to that of the longer wire. If they are now connected in parallel, find the net resistance of the combination. [2 marks]

Solution:

Concept:

- The resistance of a uniform wire is $R = \rho l/A$, where ρ is resistivity, l is length, and A is cross-sectional area.
- Stretching conserves the volume of the wire. At unchanged temperature, take resistivity as constant; then doubling its length halves its area and multiplies its resistance by four.

Step 1: Find the lengths and resistances after cutting The length ratio 1 : 2 divides the initial wire into lengths $l/3$ and $2l/3$. Both initially retain the area A , so their resistances are

$$R_s = \frac{\rho(l/3)}{A} = \frac{R}{3}, \quad R_L = \frac{\rho(2l/3)}{A} = \frac{2R}{3}.$$

Step 2: Account for the shorter wire's reduced area The shorter wire is stretched from $l/3$ to $2l/3$,

Quick Tip to Solve: First apply the square of the stretch factor to the shorter piece's resistance; only then use the parallel-resistance formula.

doubling its length. If its new area is A'_s , volume conservation gives

$$A \frac{l}{3} = A'_s \frac{2l}{3} \Rightarrow A'_s = \frac{A}{2}.$$

Therefore its new resistance is $R'_s = \rho(2l/3)/(A/2) = 4R/3$. The longer wire is unstretched and still has resistance $R_L = 2R/3$.

Step 3: Combine the final wires in parallel The potential difference is the same across the two branches. Their conductances therefore add:

$$\frac{1}{R_{eq}} = \frac{1}{4R/3} + \frac{1}{2R/3} = \frac{3}{4R} + \frac{3}{2R} = \frac{9}{4R}.$$

Step 4: Conclusion The net resistance of the parallel combination is $R_{eq} = 4R/9$.

Quick Tip to Solve: First apply the square of the stretch factor to the shorter piece's resistance; only then use the parallel-resistance formula.

8. Derive an expression for the torque acting on an electric dipole of dipole moment $\vec{p}$ placed in a uniform electric field $\vec{E}$. Write the direction along which the torque acts. [2 marks]

Solution:

Concept:

- An electric dipole consists of charges $-q$ and $+q$ separated by a distance d . Its dipole moment $\vec{p}$ has magnitude qd and points from the negative charge towards the positive charge.
- In a uniform electric field $\vec{E}$, each charge experiences $\vec{F} = q\vec{E}$, with the sign of the charge included. Equal and opposite forces with different lines of action form a couple.

Step 1: Identify the forces and their separation Let θ be the angle between $\vec{p}$ and $\vec{E}$. The positive charge experiences a force qE along the field, while the negative charge experiences a force qE opposite to it. Their resultant force is zero. The perpendicular distance between their parallel lines of action is $d \sin \theta$.

Step 2: Calculate the moment of the couple The torque magnitude equals either force multiplied by the perpendicular separation of the forces:

$$\tau = (qE)(d \sin \theta) = (qd)E \sin \theta = pE \sin \theta.$$

Equivalently, taking moments about the midpoint gives two torques of magnitude $qE(d/2) \sin \theta$ in the same rotational sense. The torque tends to turn the dipole towards alignment with the field.

Step 3: State the vector direction The direction is perpendicular to the plane containing $\vec{p}$ and $\vec{E}$, as given by the right-hand rule for $\vec{p} \times \vec{E}$: curl the fingers from $\vec{p}$ towards $\vec{E}$, and the thumb gives the torque direction. For parallel or antiparallel vectors the torque is zero.

Step 4: Conclusion $\vec{\tau} = \vec{p} \times \vec{E}$, with magnitude $\tau = pE \sin \theta$. Its nonzero direction is normal to the dipole-field plane by the right-hand rule.

Quick Tip to Solve: Use the perpendicular separation of the two force lines, not the full charge separation, when calculating the couple's torque.

8. Derive an expression for the electric field at a point on the axis of an electric dipole of dipole moment $\vec{p}$. Also write its expression when the distance $r \gg$ the length 'a' of the dipole. [2 marks]

Solution:

Concept:

- The electric field of a point charge at distance s has magnitude $k|q|/s^2$, where $k = 1/(4\pi\epsilon_0)$; fields add vectorially.
- Use the printed dipole length a as the full separation of charges $-q$ and $+q$. Then $p = qa$, and the charges lie at distances $a/2$ from the midpoint. Measure axial distance r from this midpoint.

Step 1: Write the two axial fields Take an exterior point P on the axis beyond the positive charge, so $r > a/2$. Its distances from $+q$ and $-q$ are $r - a/2$ and $r + a/2$, respectively. The positive charge's field is along $\vec{p}$, and the negative charge's field is opposite to $\vec{p}$. The former is larger because its source charge is nearer.

Step 2: Subtract and simplify The net field magnitude is

$$\begin{aligned} E_{\text{ax}} &= kq \left[\frac{1}{(r - a/2)^2} - \frac{1}{(r + a/2)^2} \right] \\ &= kq \frac{(r + a/2)^2 - (r - a/2)^2}{(r^2 - a^2/4)^2} \\ &= \frac{2kqar}{(r^2 - a^2/4)^2} = \frac{2kpr}{(r^2 - a^2/4)^2}. \end{aligned}$$

Its direction is along $\vec{p}$. An exterior axial point on the other side gives the same direction and magnitude at the same distance from the midpoint. If half-separation is denoted by $b = a/2$, the denominator is equivalently $(r^2 - b^2)^2$.

Step 3: Apply the short-dipole approximation When $r \gg a$, the term $a^2/4$ is negligible compared with r^2 . Hence the denominator approaches r^4 , and the axial field decreases as the inverse cube of distance.

Step 4: Conclusion For an exterior axial point, $\vec{E}_{\text{ax}} = \frac{1}{4\pi\epsilon_0} \frac{2r\vec{p}}{(r^2 - a^2/4)^2}$. For $r \gg a$, $\boxed{\vec{E}_{\text{ax}} \simeq \frac{1}{4\pi\epsilon_0} \frac{2\vec{p}}{r^3}}$, along the dipole moment.

Quick Tip to Solve: Define whether the symbol for dipole length denotes full separation or half-separation before writing the two charge-to-point distances.

9. Two identical capacitors of 10 pF each are connected in turn (i) in series, and (ii) in parallel across a 20 V battery. Calculate the potential difference across each capacitor in the first case and charge acquired by each capacitor in the second case. [2 marks]

Solution:

Concept:

- For a capacitor, $Q = CV$, where Q is the magnitude of charge on either plate, C is capacitance and V is potential difference.
- Series capacitors carry equal charge and their potential differences add; parallel capacitors have the same potential difference as the battery.

Step 1: Divide the voltage in the series connection Both capacitors have $C = 10 \text{ pF}$. Since their charges are equal in series and their capacitances are identical, $V_1 = Q/C = V_2$. The two potential differences together equal the battery voltage:

$$V_1 + V_2 = 20 \text{ V}, \quad 2V_1 = 20 \text{ V}.$$

Thus each capacitor has a potential difference of 10 V across it. The equal division follows from equal capacitances, not merely from the series connection.

Step 2: Calculate each charge in the parallel connection In parallel, each capacitor is directly connected across the battery, so each has $V = 20 \text{ V}$. Converting picofarads to farads and using $Q = CV$,

$$Q_1 = Q_2 = (10 \times 10^{-12})(20) = 2.0 \times 10^{-10} \text{ C}.$$

This is the charge magnitude on either plate of each capacitor; the two plates themselves carry equal and opposite charges.

Step 3: Conclusion (i) In series, the potential difference across each capacitor is 10 V. (ii) In parallel, each capacitor acquires charge of magnitude $2.0 \times 10^{-10} \text{ C} = 200 \text{ pC}$.

Quick Tip to Solve: Equal series capacitances split the battery voltage equally; in parallel use the full battery voltage in $Q = CV$ for each capacitor.

10. The frequencies of two side bands in an amplitude modulated wave are 640 kHz and 660 kHz respectively. Find the frequencies of the carrier and the modulating signals. Also obtain the value of the bandwidth required in amplitude modulation. [2 marks]

Solution:

Concept:

- For sinusoidal amplitude modulation, the lower and upper sideband frequencies are $f_L = f_c - f_m$ and $f_U = f_c + f_m$, where f_c and f_m are the carrier and modulating frequencies.
- The bandwidth covers the complete interval between the two sidebands: $B = f_U - f_L = 2f_m$.

Step 1: Find the carrier frequency from the sideband midpoint The given frequencies are $f_L = 640 \text{ kHz}$ and $f_U = 660 \text{ kHz}$. Adding the sideband equations cancels the modulating frequency, giving

Quick Tip to Solve: Take the average of the sideband frequencies for the carrier, half their difference for the message, and their full difference for the bandwidth.

$f_U + f_L = 2f_c$. Hence,

$$f_c = \frac{660 + 640}{2} = 650 \text{ kHz.}$$

The carrier lies midway between the two sidebands, as required by sinusoidal amplitude modulation.

Step 2: Obtain the modulating frequency and bandwidth Subtracting the sideband equations gives $f_U - f_L = 2f_m$. Therefore,

$$f_m = \frac{660 - 640}{2} = 10 \text{ kHz.}$$

The bandwidth is the entire sideband separation, not the distance from the carrier to one sideband:

$$B = f_U - f_L = 660 - 640 = 20 \text{ kHz.}$$

As a check, $650 - 10 = 640$ and $650 + 10 = 660$, both in kilohertz, reproduce the given frequencies.

Step 3: Conclusion The carrier frequency is 650 kHz, the modulating frequency is 10 kHz, and the required AM bandwidth is 20 kHz.

Quick Tip to Solve: Take the average of the sideband frequencies for the carrier, half their difference for the message, and their full difference for the bandwidth.

OR

10. A sinusoidal carrier voltage is amplitude modulated by a sinusoidal voltage of 10 kHz with modulation index 0.3. If the carrier frequency is 10 MHz and its amplitude is 40 V, calculate the frequency and amplitude of the two sidebands. [2 marks]

Solution:

Concept:

- A sinusoidal AM wave has sidebands at $f_c - f_m$ and $f_c + f_m$, where f_c is the carrier frequency and f_m is the modulating frequency.
- If the unmodulated carrier amplitude is A_c and the modulation index is μ , each sideband has voltage amplitude $A_{SB} = \mu A_c / 2$.

Step 1: Calculate the two sideband frequencies Here $f_c = 10 \text{ MHz}$, $f_m = 10 \text{ kHz} = 0.01 \text{ MHz}$, and $\mu = 0.3$. Expressing both frequencies in the same unit before adding and subtracting gives

$$f_L = f_c - f_m = 10 - 0.01 = 9.99 \text{ MHz,}$$

$$f_U = f_c + f_m = 10 + 0.01 = 10.01 \text{ MHz.}$$

Thus the sidebands lie symmetrically on either side of the carrier, each displaced from it by 10 kHz.

Step 2: Obtain the amplitude of each sideband The AM expression is

$$v(t) = A_c [1 + \mu \cos(2\pi f_m t)] \cos(2\pi f_c t).$$

Using the product-to-sum identity on the product of the two cosines produces two sideband terms,

Quick Tip to Solve: Convert kilohertz to megahertz before finding $f_c \pm f_m$; then divide μA_c by two to obtain each sideband's amplitude.

each with coefficient $\mu A_c/2$. With $A_c = 40\text{ V}$,

$$A_L = A_U = \frac{0.3 \times 40}{2} = 6\text{ V}.$$

The value $\mu A_c = 12\text{ V}$ is not the amplitude of either individual sideband; the factor of one-half is essential.

Step 3: Conclusion The lower sideband is at 9.99 MHz and the upper sideband is at 10.01 MHz. Each sideband has voltage amplitude 6 V.

Quick Tip to Solve: Convert kilohertz to megahertz before finding $f_c \pm f_m$; then divide μA_c by two to obtain each sideband's amplitude.

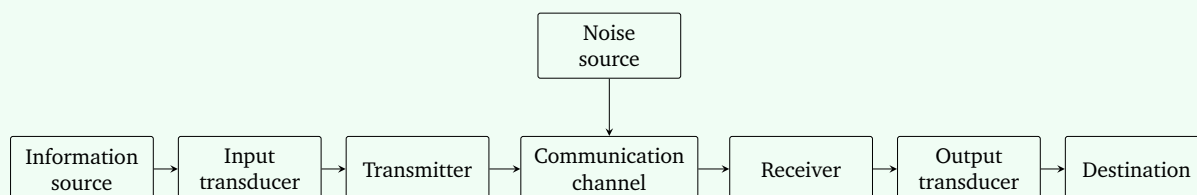
11. Draw a block diagram of a generalized communication system and write the functions of (i) a transmitter, and (ii) a receiver. [2 marks]

Solution:

Concept:

- A communication system transfers information from a source to a destination through a communication channel, such as a cable, optical fibre or free space.
- Transducers convert between physical information and electrical signals. The transmitter prepares the signal for transmission; the receiver recovers the message from the received signal.

Step 1: Draw the generalized communication chain Information passes through the blocks in the arrow direction. The input transducer converts the source message, for example sound, into an electrical signal. Noise represents unwanted disturbances added during transmission. After the receiver recovers the message, the output transducer converts it into a form suitable for the destination, for example sound from a loudspeaker.



Generalized communication system; arrows show information flow.

Step 2: State the functions of transmitter and receiver **(i) Transmitter:** It processes the electrical message signal into a form suitable for the chosen channel. In a carrier communication system this includes modulation, in which the message varies a property of a high-frequency carrier, and amplification to provide sufficient power for transmission.

(ii) Receiver: It selects and receives the desired transmitted signal, amplifies the weakened signal and demodulates it to recover the original message. The recovered electrical message is supplied to the output transducer for delivery to the destination.

Step 3: Conclusion The principal stages are transmitter, communication channel and receiver, connected between source and destination through suitable transducers. The transmitter prepares

Quick Tip to Solve: Draw arrows in the direction of information flow and add noise at the channel; pair transmitter modulation with receiver demodulation.

and sends the signal; the receiver selects it and recovers the information.

Quick Tip to Solve: Draw arrows in the direction of information flow and add noise at the channel; pair transmitter modulation with receiver demodulation.

12. Two coils P and Q of radius R and $2R$ respectively are lying in perpendicular planes having a common centre. Find the magnitude and direction of the resultant magnetic field at the common centre, if they carry the currents I and $2\sqrt{3}I$ respectively. [2 marks]

Solution:

Concept:

- The magnetic field at the centre of a circular coil of radius r , carrying current i , is $B = \mu_0 i / (2r)$, for one turn. Its direction is normal to the coil, given by the right-hand thumb rule.
- Magnetic fields add vectorially. The normals to two perpendicular coil planes are perpendicular, so their centre fields are perpendicular as well.

Step 1: Calculate the fields of the two coils For coil P, the radius is R and the current is I ; for Q, the radius is $2R$ and the current is $2\sqrt{3}I$. Thus,

$$B_P = \frac{\mu_0 I}{2R}, \quad B_Q = \frac{\mu_0 (2\sqrt{3}I)}{2(2R)} = \sqrt{3} \frac{\mu_0 I}{2R}.$$

Define $\hat{n}_P$ and $\hat{n}_Q$ as the respective field directions, each chosen by curling the right-hand fingers along that coil's current. Then $\vec{B} = B_P \hat{n}_P + B_Q \hat{n}_Q$.

Step 2: Combine the perpendicular fields Let θ be the angle of the resultant measured from $\vec{B}_P$ towards $\vec{B}_Q$. Since the two fields are perpendicular,

$$B = \sqrt{B_P^2 + B_Q^2} = \frac{\mu_0 I}{2R} \sqrt{1 + 3},$$
$$\tan \theta = \frac{B_Q}{B_P} = \sqrt{3}.$$

The resultant lies in the plane of the two normals, between the individual field vectors. The printed question does not specify current senses, so its absolute spatial orientation is determined by applying the right-hand rule to those senses.

Step 3: Conclusion The resultant magnitude is $B = \mu_0 I / R$. It makes 60° with P's field direction, towards Q's field, and 30° with Q's field direction.

Quick Tip to Solve: First compare current divided by radius: the field ratio is $1 : \sqrt{3}$. Perpendicular fields with this ratio give a 60° resultant angle from the smaller field.

Section C (36 marks)

13. Given three elements X, Y and Z to be connected across an ac source. With only X connected across the ac source, voltage and current are found to be in the same phase. With only element Y in the circuit, the voltage lags behind the current in phase by $\pi/2$ while with the element Z in the circuit, the voltage leads the current in phase by $\pi/2$.

(a) Identify the elements X, Y and Z.

(b) When all these elements are connected in series across the same source, (i) determine the power factor, and (ii) find out the condition when the circuit is in resonant state. [3 marks]

Solution:

Concept:

- In an ideal resistor, voltage and current are in phase. In a capacitor, current leads voltage by $\pi/2$; in an inductor, voltage leads current by $\pi/2$.
- A series RLC circuit has inductive reactance $X_L = \omega L$, capacitive reactance $X_C = 1/(\omega C)$, and impedance magnitude $Z_{\text{imp}} = \sqrt{R^2 + (X_L - X_C)^2}$. Here ω is the source angular frequency.
- The power factor is $\cos \phi$, where ϕ is the phase angle between the source voltage and current. Series resonance occurs when the net reactance vanishes.

Step 1: Identify the elements from their phase relations For part (a), X gives zero phase difference and is therefore a resistor of resistance R . With Y, the voltage lags the current by $\pi/2$, identifying a capacitor of capacitance C . With Z, the voltage leads the current by $\pi/2$, identifying an inductor of inductance L .

Step 2: Find the power factor of the series combination For part (b)(i), the same current flows through all three elements. Taking current as the phase reference, $V_R = IR$ is in phase, $V_L = IX_L$ leads by $\pi/2$, and $V_C = IX_C$ lags by $\pi/2$. The two reactive voltage phasors oppose each other, giving

$$V^2 = V_R^2 + (V_L - V_C)^2, \quad \tan \phi = \frac{X_L - X_C}{R}.$$

Hence the ratio of the in-phase voltage to the source voltage is

$$\cos \phi = \frac{V_R}{V} = \frac{R}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}}.$$

Step 3: Apply the resonance condition For part (b)(ii), the inductive and capacitive voltage phasors cancel at resonance. Thus,

$$\omega_0 L = \frac{1}{\omega_0 C} \Rightarrow \omega_0 = \frac{1}{\sqrt{LC}}, \quad f_0 = \frac{1}{2\pi\sqrt{LC}}.$$

The impedance then reduces to R , the source voltage and current are in phase, and the current is maximum for a fixed source voltage.

Step 4: Conclusion $X = \text{resistor, } Y = \text{capacitor, } Z = \text{inductor}$. The series power factor is $R/\sqrt{R^2 + (X_L - X_C)^2}$. Resonance requires $X_L = X_C$, or $\omega_0 = 1/\sqrt{LC}$, and the power factor then

Quick Tip to Solve: Read which quantity leads: current leads in a capacitor, voltage leads in an inductor. In series, subtract the two reactances before finding impedance and power factor.

equals unity.

Quick Tip to Solve: Read which quantity leads: current leads in a capacitor, voltage leads in an inductor. In series, subtract the two reactances before finding impedance and power factor.

14. (a) Good quality sunglasses made of polaroids are preferred over ordinary coloured glasses. Explain why.

(b) How is plane polarized light defined?

(c) A beam of plane polarised light is passed through a polaroid. Show graphically, variation of the intensity of the transmitted light with angle of rotation of the polaroid. [3 marks]

Solution:

Concept:

- A polaroid transmits the electric-field component parallel to its transmission axis. Glare reflected from horizontal surfaces is predominantly horizontally polarized.
- Malus's law is $I = I_0 \cos^2 \theta$, where I_0 is the incident plane polarized intensity and θ is the angle between its electric field and the analysing polaroid's transmission axis.

Step 1: Explain the advantage of polaroid sunglasses (a) Polaroid sunglasses have an approximately vertical transmission axis, so they preferentially absorb the horizontally polarized glare reflected from roads and water. Ordinary coloured glasses mainly reduce overall brightness without selectively blocking glare's polarization. Polaroids therefore improve visibility and contrast in reflected glare.

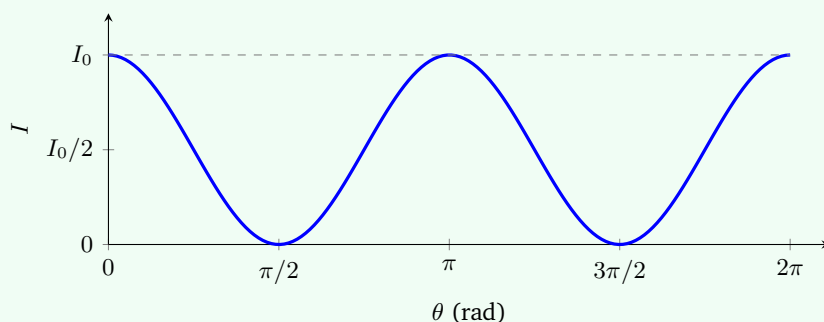
Step 2: Define plane polarized light (b) In plane polarized light, the electric-field vibrations are restricted to one fixed direction perpendicular to propagation. This vibration direction and the propagation direction define a fixed plane. In unpolarized light the transverse vibration direction varies randomly.

Step 3: Derive and plot the intensity variation (c) Set $\theta = 0$ when the transmission axis is parallel to the incident electric field. The transmitted field amplitude is $E_0 \cos \theta$. Since intensity is proportional to amplitude squared,

$$\frac{I}{I_0} = \frac{(E_0 \cos \theta)^2}{E_0^2} = \cos^2 \theta.$$

Thus maxima I_0 occur at $0, \pi, 2\pi$, zeros at $\pi/2, 3\pi/2$, and $I = I_0/2$ at $\pi/4$. The smooth, nonnegative graph repeats every π radians.

Quick Tip to Solve: Mark maxima for parallel axes and zeros for crossed axes; join them with a smooth nonnegative curve of period π .



Malus's law for a rotating analysing polaroid.

Step 4: Conclusion Polaroid sunglasses reduce polarized glare. Plane polarized light has a fixed transverse electric-field direction, and its transmission through a rotating polaroid follows $I = I_0 \cos^2 \theta$.

Quick Tip to Solve: Mark maxima for parallel axes and zeros for crossed axes; join them with a smooth nonnegative curve of period π .

15. A charge Q is distributed over the surfaces of two concentric hollow spheres of radii r and R ($R \gg r$), such that their surface charge densities are equal. Derive the expression for the potential at the common centre. [3 marks]

Solution:

Concept:

- For a uniformly charged spherical shell of radius s and charge q , the potential at every interior point is $V = q/(4\pi\epsilon_0 s)$, taking zero potential at infinity.
- Surface charge density is charge divided by surface area: $\sigma = q/(4\pi s^2)$. Potentials due to different shells add algebraically by superposition.

Step 1: Express the charge on each sphere Let the charges on the smaller and larger spheres be q_r and q_R . Their equal surface charge densities do not imply equal charges, because their surface areas differ. With common density σ ,

$$q_r = 4\pi r^2 \sigma, \quad q_R = 4\pi R^2 \sigma.$$

Since the total distributed charge is Q ,

$$Q = 4\pi\sigma(r^2 + R^2), \quad \sigma = \frac{Q}{4\pi(r^2 + R^2)}.$$

Step 2: Add the two potentials at the centre The common centre is inside both shells. Although the electric field there is zero, the potential need not be zero: each shell contributes its constant interior

Quick Tip to Solve: Equal surface densities give charges in the ratio $r^2 : R^2$; divide each charge by its own radius before adding potentials.

potential. Hence,

$$\begin{aligned} V_O &= \frac{1}{4\pi\epsilon_0} \left(\frac{q_r}{r} + \frac{q_R}{R} \right) \\ &= \frac{\sigma}{\epsilon_0} (r + R) \\ &= \frac{Q(r + R)}{4\pi\epsilon_0(r^2 + R^2)}. \end{aligned}$$

This exact expression retains the contribution of both spheres. For the stated limit $R \gg r$, the leading approximation follows by neglecting r/R compared with unity.

Step 3: Conclusion The potential at the common centre, relative to infinity, is

$$V_O = \frac{Q(r + R)}{4\pi\epsilon_0(r^2 + R^2)}.$$

In the limit $R \gg r$, $V_O \simeq Q/(4\pi\epsilon_0 R)$.

Quick Tip to Solve: Equal surface densities give charges in the ratio $r^2 : R^2$; divide each charge by its own radius before adding potentials.

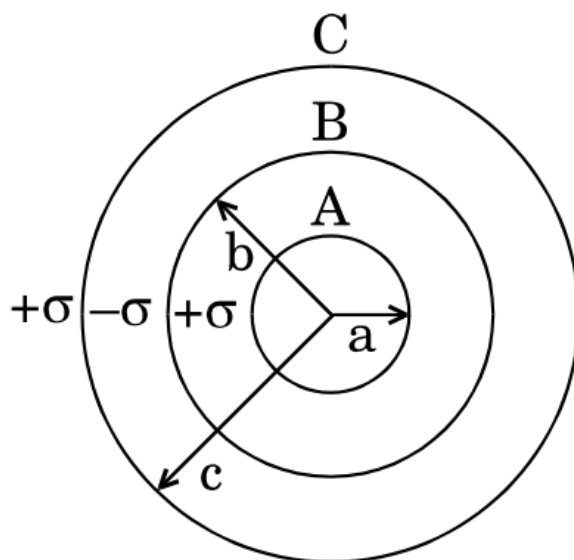
OR

15. Three concentric metallic shells A, B and C of radii a , b and c ($a < b < c$) have surface charge densities $+\sigma$, $-\sigma$ and $+\sigma$ respectively.

(a) Obtain the expressions for the potential of the three shells A, B and C.

(b) If shells A and C are at the same potential, obtain the relation between a , b and c .

[3 marks]



Solution:

Concept:

- A spherical shell of radius s and total charge q gives potential kq/s inside or on it, and kq/x at an exterior distance x , where $k = 1/(4\pi\epsilon_0)$.
- Potentials add algebraically. A shell's charge is its stated surface charge density multiplied by $4\pi s^2$; the reference potential is zero at infinity.

Step 1: Write the three shell charges Using the given signs of the surface densities, the charges on A, B and C are

$$Q_A = 4\pi a^2\sigma, \quad Q_B = -4\pi b^2\sigma, \quad Q_C = 4\pi c^2\sigma.$$

For any chosen shell, contributions from larger shells use those larger shells' radii, whereas contributions from smaller shells use the observation radius.

Step 2: Obtain the potentials of A, B and C A lies inside both B and C. B lies outside A but inside C. C lies outside both smaller shells. Therefore,

$$\begin{aligned} V_A &= k \left(\frac{Q_A}{a} + \frac{Q_B}{b} + \frac{Q_C}{c} \right) = \frac{\sigma}{\epsilon_0} (a - b + c), \\ V_B &= k \left(\frac{Q_A}{b} + \frac{Q_B}{b} + \frac{Q_C}{c} \right) = \frac{\sigma}{\epsilon_0} \left(\frac{a^2}{b} - b + c \right), \\ V_C &= \frac{k}{c} (Q_A + Q_B + Q_C) = \frac{\sigma}{\epsilon_0 c} (a^2 - b^2 + c^2). \end{aligned}$$

The negative sign in B's charge must be retained in every potential; potential is a signed scalar, not a magnitude.

Step 3: Apply equality of the potentials of A and C For the stated nonzero charge density, set $V_A = V_C$ and cancel σ/ϵ_0 :

$$\begin{aligned} a - b + c &= \frac{a^2 - b^2 + c^2}{c}, \\ c(a - b) &= (a - b)(a + b). \end{aligned}$$

Because the radii obey $a < b$, the factor $a - b$ is nonzero and can be cancelled. This gives the required relation without equating the individual charges.

Step 4: Conclusion The shell potentials are

$$\begin{aligned} V_A &= \frac{\sigma}{\epsilon_0} (a - b + c), \\ V_B &= \frac{\sigma}{\epsilon_0} \left(\frac{a^2}{b} - b + c \right), \\ V_C &= \frac{\sigma}{\epsilon_0 c} (a^2 - b^2 + c^2). \end{aligned}$$

Equal potentials of A and C require $c = a + b$.

Quick Tip to Solve: For each shell contribution use the larger of the shell radius and observation radius in the denominator, while retaining the charge sign.

16. When light of frequency ν_1 is incident on a photosensitive surface, the stopping potential is V_1 . If the frequency of incident radiation becomes $\nu_1/2$, the stopping potential changes to V_2 . Find out the expression for the threshold frequency for the surface in terms of V_1 and V_2 .

If the frequency of incident radiation is doubled, will the maximum kinetic energy of the photoelectrons also be doubled? Give reason.

[3 marks]

Solution:

Concept:

- Einstein's photoelectric equation is $K_{\max} = h\nu - \phi = h(\nu - \nu_0)$, where h is Planck's constant, ϕ is the work function and $\nu_0 = \phi/h$ is the threshold frequency.
- The positive magnitude V_s of the stopping potential satisfies $eV_s = K_{\max}$, where e is the magnitude of the electronic charge.

Step 1: Write the energy equations for both frequencies The same photosensitive surface has the same work function in both cases. Assuming emission occurs at both stated frequencies, the two stopping potentials give

$$eV_1 = h\nu_1 - h\nu_0, \quad eV_2 = \frac{h\nu_1}{2} - h\nu_0.$$

Halving the incident frequency halves the photon energy but does not halve the fixed work function.

Step 2: Eliminate the incident frequency Multiply the second equation by two and subtract it from the first. This removes the unknown incident photon energy:

$$\begin{aligned} eV_1 - 2eV_2 &= (h\nu_1 - h\nu_0) - (h\nu_1 - 2h\nu_0) \\ &= h\nu_0. \end{aligned}$$

Thus the threshold is determined using only the measured stopping potentials and constants. The physical work function is positive, so the given potentials must satisfy $V_1 > 2V_2$ for an ordinary photosensitive surface.

Step 3: Compare kinetic energies when the frequency doubles For any initial emitting frequency ν , let $K = h\nu - \phi$. After doubling that frequency,

$$K' = 2h\nu - \phi = 2(h\nu - \phi) + \phi = 2K + \phi.$$

Therefore the new maximum kinetic energy exceeds twice its original value by the work function. The straight-line dependence on frequency has a nonzero threshold, so kinetic energy is not directly proportional to frequency.

Step 4: Conclusion The threshold frequency is $\nu_0 = \frac{e}{h}(V_1 - 2V_2)$. Doubling the incident frequency does not double the maximum kinetic energy: $K' = 2K + \phi > 2K$ for $\phi > 0$.

Quick Tip to Solve: Eliminate ν_1 by taking the first stopping-potential equation minus twice the second; keep the work function fixed when changing frequency.

17. State the principle of a moving coil galvanometer. Explain its working and obtain the expression for the deflection produced due to the current passed through the coil. Define current sensitivity.

[3 marks]

Solution:

Concept:

- A current-carrying coil in a magnetic field experiences a torque. For N turns, current I , area A and magnetic field B , its magnitude is $\tau_m = NIAB \sin \alpha$, where α is the angle between the coil's normal and the field.
- A suspension or spring produces an opposing restoring torque $\tau_r = \kappa\theta$, where κ is its torsional constant and θ is the angular deflection in radians. At equilibrium the two torques balance.

Step 1: State the principle and describe the radial field A moving coil galvanometer works on the principle that a current-carrying coil placed in a magnetic field experiences a torque. A light rectangular coil with many turns is suspended between the concave pole pieces of a strong permanent magnet. A soft iron cylindrical core makes the field strong and radial. The current enters and leaves through the suspension and spring arrangement.

Step 2: Derive the equilibrium deflection When current passes through the coil, magnetic torque rotates it and twists the suspension. In the radial field the plane of the coil remains parallel to the magnetic field, so its normal is perpendicular to the field: $\alpha = 90^\circ$. Therefore the magnetic torque is $NIAB$, independent of the deflection. The spring's opposing torque grows until equilibrium is reached:

$$NIAB = \kappa\theta, \quad \theta = \frac{NAB}{\kappa}I.$$

With N , A , B and κ fixed, the deflection is proportional to current, allowing a uniform scale to measure small currents. Reversing the current reverses the direction of deflection.

Step 3: Define current sensitivity Current sensitivity is the angular deflection produced per unit current through the galvanometer. Hence,

$$S_I = \frac{\theta}{I} = \frac{NAB}{\kappa}.$$

Its SI unit is rad A^{-1} . More turns, a larger coil area or a stronger field increases current sensitivity; a smaller torsional constant also increases it.

Step 4: Conclusion The galvanometer uses the magnetic torque on a current-carrying coil. Its equilibrium deflection is $\theta = (NAB/\kappa)I$, and its current sensitivity is $S_I = NAB/\kappa$, in rad A^{-1} .

Quick Tip to Solve: In a radial field the coil's normal is always perpendicular to the field; set the magnetic torque equal to the spring torque to find both deflection and sensitivity.

17. Explain how a galvanometer can be converted into an ammeter of a given range. Derive an expression for shunt resistance and current for full scale deflection. Find the effective resistance of the ammeter. [3 marks]

Solution:

Concept:

- A galvanometer has resistance G and carries current I_g at full-scale deflection. An ammeter of range I must safely measure a total current up to I .
- A low resistance called the shunt, S , is connected in parallel with the galvanometer. Parallel branches have equal potential differences, while their currents add.

Step 1: Connect a shunt across the galvanometer Connect a small resistance S in parallel with the galvanometer and calibrate its scale in amperes. The resulting combination is connected in series with the circuit whose current is measured. At the desired upper limit I , only I_g passes through the sensitive galvanometer coil; the remaining current $I - I_g$ is diverted through the shunt. Thus the coil is protected while its deflection indicates the total current.

Step 2: Equate the branch potential differences At full scale, the voltage across the galvanometer is $I_g G$, while that across the shunt is $(I - I_g)S$. Since both are connected between the same two points,

$$I_g G = (I - I_g)S.$$

Solving for the required shunt resistance gives

$$S = \frac{I_g G}{I - I_g}.$$

Here $I > I_g$; extending the range greatly requires $S \ll G$.

Step 3: Obtain the full-scale current and effective resistance Rearranging the same equation gives the total current producing full-scale deflection of the converted instrument:

$$I = I_g \left(1 + \frac{G}{S} \right), \quad I_g = \frac{IS}{G + S}.$$

The effective ammeter resistance is the parallel equivalent of the coil and shunt:

$$R_A = \frac{GS}{G + S} = \frac{I_g G}{I}.$$

It is smaller than either branch resistance, so the ammeter introduces only a small extra resistance when connected in series. For a range multiplication factor $n = I/I_g$, the same results read $S = G/(n - 1)$ and $R_A = G/n$.

Step 4: Conclusion Connect a low-resistance shunt in parallel. The required shunt is $S = I_g G/(I - I_g)$, the full-scale total current is $I = I_g(1 + G/S)$, and the effective resistance is $R_A = GS/(G + S) = I_g G/I$.

Quick Tip to Solve: Put I_g through the coil and $I - I_g$ through the shunt before equating voltages; using I as the shunt current gives the wrong range.

18. A hydrogen atom in its excited state emits radiations of wavelengths $1218 \text{ \AA}$ and $974.3 \text{ \AA}$ when it finally comes to the ground state. Identify the energy levels from where transitions occur. Given Rydberg constant $R = 1.1 \times 10^7 \text{ m}^{-1}$. Also specify the spectral series to which these lines belong.

[3 marks]

Solution:

Concept:

- The hydrogen wavelengths obey the Rydberg formula $1/\lambda = R(1/n_f^2 - 1/n_i^2)$, where $n_i > n_f$ are the initial and final principal quantum numbers.
- The ground state has $n_f = 1$. All hydrogen spectral lines ending in this state belong to the Lyman series, in the ultraviolet region.

Step 1: Set the final level and convert the wavelengths Because the atom ends in the ground state, set $n_f = 1$. Use $1 \text{ \AA} = 10^{-10} \text{ m}$, so the wavelengths are

$$\lambda_1 = 1.218 \times 10^{-7} \text{ m}, \quad \lambda_2 = 9.743 \times 10^{-8} \text{ m}.$$

Rearranging the Rydberg equation to determine the upper level gives

$$\frac{1}{n_i^2} = 1 - \frac{1}{R\lambda}, \quad n_i = \left(1 - \frac{1}{R\lambda}\right)^{-1/2}.$$

Step 2: Identify the upper level for the first wavelength Substitute $R = 1.1 \times 10^7 \text{ m}^{-1}$ and the first wavelength:

$$\begin{aligned} \frac{1}{n_i^2} &= 1 - \frac{1}{(1.1 \times 10^7)(1.218 \times 10^{-7})} \\ &= 1 - \frac{1}{1.3398} \approx 0.2536, \\ n_i &\approx 1.986 \approx 2. \end{aligned}$$

Thus the $1218 \text{ \AA}$ line is associated with a transition from the first excited state, $n_i = 2$, to the ground state.

Step 3: Identify the upper level for the second wavelength For the second line, the corresponding calculation is

$$\begin{aligned} \frac{1}{n_i^2} &= 1 - \frac{1}{(1.1 \times 10^7)(9.743 \times 10^{-8})} \\ &= 1 - \frac{1}{1.07173} \approx 0.06693, \\ n_i &\approx 3.865 \approx 4. \end{aligned}$$

Quantum numbers must be integers. The small departures from integer values arise from the rounded numerical data, especially the two-significant-figure value of R ; the intended levels are therefore 2 and 4. The shorter wavelength has greater photon energy and consequently corresponds to the higher initial level. Both transitions terminate at $n_f = 1$.

Step 4: Conclusion The transitions are $n = 2 \rightarrow 1$ for $1218 \text{ \AA}$ and $n = 4 \rightarrow 1$ for $974.3 \text{ \AA}$. Both lines belong to the ultraviolet Lyman series.

Quick Tip to Solve: First identify the final state to fix the spectral series; then convert angstroms to metres before using R in m^{-1} .

19. (a) Differentiate between three segments of an n-p-n transistor on the basis of their size and level of doping.

(b) Draw a plot of transfer characteristic and show which portion of the characteristic is used in amplification and why. [3 marks]

Solution:

Concept:

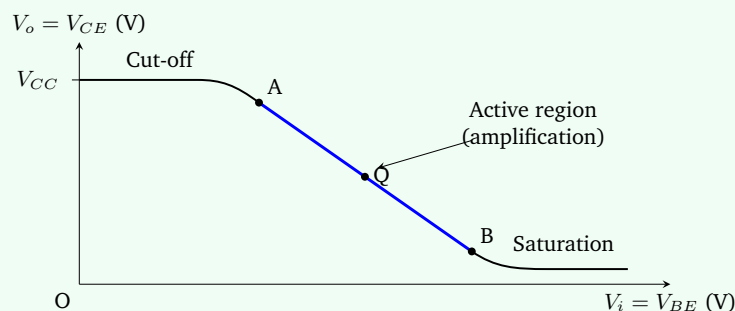
- An n-p-n transistor has n-type emitter and collector regions separated by a thin p-type base.
- The common-emitter transfer curve plots $V_o = V_{CE}$ against $V_i = V_{BE}$. For fixed supply V_{CC} and load R_C , $V_o = V_{CC} - I_C R_C$.

Step 1: Compare the three transistor regions (a) The three regions differ as follows:

Region	Relative size	Doping level
Emitter (n)	Moderate	Heavily doped
Base (p)	Very thin; smallest	Lightly doped
Collector (n)	Largest	Moderately doped

Heavy emitter doping supplies many electrons. The thin, lightly doped base minimizes recombination, so most electrons reach the collector. The larger collector dissipates more heat; its doping is lower than the emitter's but higher than the base's.

Step 2: Read the voltage transfer characteristic (b) In cut-off, $I_C \approx 0$ and $V_o \approx V_{CC}$. Increasing the input into the active region raises I_C , so the output falls along the approximately straight portion AB. At large input, saturation makes V_o nearly constant at a small value.



Schematic common-emitter voltage transfer characteristic; AB is the amplification region.

Step 3: Choose the region for amplification Bias at Q in the middle of AB: emitter-base forward biased and collector-base reverse biased. Small input variations give larger proportional output variations:

$$A_v = \frac{\Delta V_o}{\Delta V_i}, \quad \Delta V_o = -R_C \Delta I_C.$$

The negative slope indicates phase inversion. Keeping the signal within AB gives faithful amplification without clipping. The nearly horizontal cut-off and saturation regions cannot provide linear amplification.

Step 4: Conclusion The three regions differ as tabulated. The straight active portion AB, with Q near its middle, provides faithful voltage amplification and phase inversion.

Quick Tip to Solve: For the voltage transfer curve put V_i horizontally and V_o vertically; amplification is the falling active segment, so its negative slope shows phase inversion.

20. (a) Distinguish between isotopes and isobars, giving one example for each.

(b) Why is the mass of a nucleus always less than the sum of the masses of its constituents? Write one example to justify your answer. [3 marks]

Solution:

Concept:

- A nuclide A_ZX contains Z protons and $N = A - Z$ neutrons; A is its mass number.
- The binding energy of a nucleus is $B = \Delta mc^2$, where $\Delta m = Zm_p + (A - Z)m_n - M$ is its mass defect and M is the nuclear mass.

Step 1: (a) Distinguish the two kinds of nuclides Isotopes have the same atomic number Z but different mass numbers A . They are atoms of the same element with different neutron numbers; for example, ${}^{12}_6\text{C}$ and ${}^{14}_6\text{C}$ both contain six protons, but contain six and eight neutrons respectively.

Isobars have the same mass number A but different atomic numbers Z . They belong to different elements; for example, ${}^3_1\text{H}$ and ${}^3_2\text{He}$ both contain three nucleons, but have one and two protons respectively.

Step 2: (b) Relate nuclear mass to the energy released on binding When free protons and neutrons bind to form a nucleus, energy is released. The bound system has less total rest energy than its separated constituents. By mass-energy equivalence, the released energy corresponds to a decrease in mass:

$$M = Zm_p + (A - Z)m_n - \frac{B}{c^2}.$$

Since a bound nucleus has positive binding energy B , its mass is less than the sum of the masses of its free constituent nucleons. Conversely, this energy must be supplied to separate the nucleus completely.

Step 3: Justify the mass defect using deuterium A deuteron consists of one proton and one neutron. Using their approximate nuclear masses,

$$m_p + m_n = 1.007276 + 1.008665 = 2.015941 \text{ u},$$

$$M_d = 2.013553 \text{ u},$$

$$\Delta m = 0.002388 \text{ u}.$$

The corresponding binding energy is approximately $0.002388 \times 931.5 = 2.224 \text{ MeV}$. Thus the bound deuteron is lighter than the free proton-neutron pair.

Step 4: Conclusion Isotopes share Z , whereas isobars share A . The nuclear mass deficit is $\Delta m = B/c^2$, as illustrated by the deuteron's positive mass defect.

Quick Tip to Solve: Compare the lower numbers for isotopes and upper numbers for isobars; use nuclear masses consistently when calculating a binding-energy mass defect.

20. (a) Classify the following six nuclides into (i) isotones, (ii) isotopes, and (iii) isobars: ${}^{12}_6\text{C}$, ${}^3_2\text{He}$, ${}^{198}_{80}\text{Hg}$, ${}^3_1\text{H}$, ${}^{197}_{79}\text{Au}$, ${}^{14}_6\text{C}$.

(b) How does the size of a nucleus depend on its mass number? Hence explain why the density of nuclear matter should be independent of the size of the nucleus. [3 marks]

Solution:

Concept:

- For a nuclide A_ZX , the neutron number is $N = A - Z$. Isotones share N , isotopes share Z , and isobars share A .
- The empirical nuclear radius is $R = R_0 A^{1/3}$, where $R_0 \approx 1.2 \times 10^{-15}$ m. Nuclear mass is approximately Am_N , with m_N the mean nucleon mass.

Step 1: (a) Calculate the neutron numbers Subtract the lower number from the upper number for each printed nuclide:

Nuclide	A	Z	$N = A - Z$
${}^{12}_6\text{C}$	12	6	6
${}^3_2\text{He}$	3	2	1
${}^{198}_{80}\text{Hg}$	198	80	118
${}^3_1\text{H}$	3	1	2
${}^{197}_{79}\text{Au}$	197	79	118
${}^{14}_6\text{C}$	14	6	8

These values let us distinguish equal neutron number from equal mass number, which are different conditions.

Step 2: Form each requested pair (i) Isotones: ${}^{198}_{80}\text{Hg}$ and ${}^{197}_{79}\text{Au}$, since both have $N = 118$, although their proton numbers differ.

(ii) Isotopes: ${}^{12}_6\text{C}$ and ${}^{14}_6\text{C}$, since both have $Z = 6$, but their mass numbers are different.

(iii) Isobars: ${}^3_2\text{He}$ and ${}^3_1\text{H}$, since both have $A = 3$, but their atomic numbers differ.

Step 3: (b) Derive the independence of nuclear density Nuclear radius, and hence linear size, increases as the cube root of mass number:

$$R = R_0 A^{1/3}, \quad V = \frac{4}{3}\pi R^3 = \frac{4}{3}\pi R_0^3 A.$$

With nearly equal proton and neutron masses, the nuclear mass is approximately $M = Am_N$. Therefore,

$$\rho = \frac{M}{V} = \frac{Am_N}{(4/3)\pi R_0^3 A} = \frac{3m_N}{4\pi R_0^3}.$$

The factor A cancels because both mass and volume increase in the same proportion. Thus nuclear density is approximately constant for different nuclei, independent of their size, within this empirical nuclear-radius model.

Step 4: Conclusion The isotone pair is Hg–Au, the isotope pair is the two carbon nuclides, and the isobar pair is He–H. Nuclear size obeys $R \propto A^{1/3}$, giving $\rho = 3m_N/(4\pi R_0^3)$, independent of A .

Quick Tip to Solve: List A , Z , and $A - Z$ before pairing nuclides; cube the radius law before substituting it into the density formula.

21. (a) What do you understand by coherent sources?

(b) Derive the expression for the intensity distribution in the interference pattern on the screen when two harmonic waves having same amplitude (a) and phase difference (ϕ) are superposed on each other. Hence obtain the conditions for constructive and destructive interference. [3 marks]

Solution:

Concept:

- Coherent waves have the same frequency and a phase difference that remains constant with time at a given observation point.
- The superposition principle adds displacements. Intensity is proportional to the square of the resultant amplitude: $I = KA^2$, where K is the common proportionality constant.

Step 1: (a) Define coherent sources Coherent sources emit waves of the same frequency with a constant phase difference. Their phase difference need not be zero; its stability produces a stationary interference pattern.

Step 2: (b) Add the two harmonic displacements At a point on the screen, take the two displacements to be $y_1 = a \sin \omega t$ and $y_2 = a \sin(\omega t + \phi)$, where a is each wave's amplitude and ϕ is their phase difference. By the sine-sum identity,

$$\begin{aligned} y &= y_1 + y_2 \\ &= 2a \cos \frac{\phi}{2} \sin \left(\omega t + \frac{\phi}{2} \right). \end{aligned}$$

Hence the resultant amplitude magnitude is $A = 2a |\cos(\phi/2)|$.

Step 3: Obtain the intensity distribution Let the intensity due to either wave alone be $I_0 = Ka^2$. Squaring the resultant amplitude gives

$$I = KA^2 = 4I_0 \cos^2 \frac{\phi}{2} = 2I_0(1 + \cos \phi).$$

The phase varies across the screen. For initially in-phase sources and path difference Δ , $\phi = 2\pi\Delta/\lambda$, giving $I = 4I_0 \cos^2(\pi\Delta/\lambda)$.

Step 4: State the conditions for bright and dark fringes Constructive interference occurs when $\cos^2(\phi/2) = 1$:

$$\phi = 2n\pi, \quad \Delta = n\lambda, \quad I_{\max} = 4I_0.$$

Destructive interference occurs when $\cos^2(\phi/2) = 0$:

$$\phi = (2n+1)\pi, \quad \Delta = \frac{(2n+1)\lambda}{2}, \quad I_{\min} = 0,$$

where n is an integer and the path conditions assume initially in-phase sources. Equal amplitudes ensure complete cancellation at a dark fringe.

Step 5: Conclusion The intensity is $I = 4I_0 \cos^2(\phi/2)$, maximal at $\phi = 2n\pi$ and zero at $\phi = (2n+1)\pi$.

Quick Tip to Solve: Add the wave displacements first and then square their amplitude; adding the two individual intensities alone loses the interference term.

22. A screen is placed 90 cm from an object. The image of the object on the screen is formed by a convex lens at two different positions separated by 20 cm. Calculate the focal length of the lens.

[3 marks]

Solution:

Concept:

- For a convex lens forming a real image, let x and y be positive object and image distances. The lens formula is $1/f = 1/x + 1/y$, equivalent to the Cartesian formula with $u = -x$, $v = y$.
- With a fixed object and screen separated by D , $x + y = D$. The two sharp-image positions interchange the object and image distances, and their separation is $d = |y - x|$.

Step 1: Express the two conjugate distances The object-screen distance is $D = 90$ cm, and the separation of the two lens positions is $d = 20$ cm. Choose the position closer to the object, so $y > x$. Then

$$x + y = D, \quad y - x = d.$$

Adding and subtracting these relations gives

$$x = \frac{D - d}{2} = 35 \text{ cm}, \quad y = \frac{D + d}{2} = 55 \text{ cm}.$$

At the other position the distances become 55 cm and 35 cm. Their interchange leaves the lens formula unchanged, explaining why the same lens forms a sharp image at both positions.

Step 2: Apply the lens formula and simplify Using the positive distances for this real object and real image,

$$\frac{1}{f} = \frac{1}{x} + \frac{1}{y} = \frac{x + y}{xy}, \quad f = \frac{xy}{D}.$$

Substituting the distance expressions also gives the displacement-method formula,

$$f = \frac{(D - d)(D + d)}{4D} = \frac{D^2 - d^2}{4D}.$$

Thus

$$f = \frac{90^2 - 20^2}{4 \times 90} \text{ cm} = \frac{7700}{360} \text{ cm} = 21.39 \text{ cm}.$$

As a consistency check, $4f \approx 85.56 \text{ cm} < 90 \text{ cm}$, so two distinct real-image positions are indeed possible.

Step 3: Conclusion The convex lens has focal length $f \approx +21.4 \text{ cm}$.

Quick Tip to Solve: Use the full object-screen distance as D , and the separation of the two lens positions as d , in $f = (D^2 - d^2)/(4D)$.

22. A convex lens of focal length 20 cm and a concave lens of focal length 15 cm are kept 30 cm apart with their principal axes coincident. When an object is placed 30 cm in front of the convex lens, calculate the position of the final image formed by the combination. Would this result change if the object were placed 30 cm in front of the concave lens? Give reason. [3 marks]

Solution:

Concept:

- For a thin lens, $1/v - 1/u = 1/f$, where u , v and f are signed object distance, image distance and focal length, measured from that lens.
- Take the direction of incident light as positive. A converging lens has positive focal length; a diverging lens has negative focal length. The image of the first lens acts as the object for the second lens.

Step 1: Find the image formed by the convex lens first With light travelling from the object to the convex lens and then to the concave lens, $u_1 = -30$ cm and $f_1 = +20$ cm. Therefore,

$$\frac{1}{v_1} = \frac{1}{20} - \frac{1}{30} = \frac{1}{60}, \quad v_1 = +60 \text{ cm.}$$

These rays would meet 60 cm beyond the convex lens if the concave lens were absent. The concave lens lies only 30 cm beyond it, so its incident rays are converging towards a point 30 cm beyond the concave lens: this is a virtual object for that lens.

Step 2: Apply the lens formula to the concave lens For the second lens, $u_2 = +30$ cm and $f_2 = -15$ cm. Thus,

$$\frac{1}{v_2} = -\frac{1}{15} + \frac{1}{30} = -\frac{1}{30}, \quad v_2 = -30 \text{ cm.}$$

The negative sign means that the emergent rays diverge and their backward extensions intersect on the incident side of the concave lens, at the plane of the convex lens.

Step 3: Reverse the order encountered by the light Now place the object 30 cm in front of the concave lens so light meets it first and then the convex lens. Distances are again positive along this new direction of propagation. For the concave lens,

$$\frac{1}{v'_1} = -\frac{1}{15} - \frac{1}{30} = -\frac{1}{10}, \quad v'_1 = -10 \text{ cm.}$$

Its virtual image lies 10 cm before it, hence 40 cm before the convex lens. For the convex lens, $u'_2 = -40$ cm, giving

$$\frac{1}{v'_2} = \frac{1}{20} - \frac{1}{40} = \frac{1}{40}, \quad v'_2 = +40 \text{ cm.}$$

The separated lenses receive different incident vergences when their order is reversed: the intermediate image, and therefore the object distance for the second lens, changes.

Step 4: Conclusion With the convex lens first, the final image is virtual, 30 cm in front of the concave lens, at the convex-lens plane. With the concave lens first, it is real, 40 cm beyond the convex lens. Thus the result does change.

Quick Tip to Solve: Locate the first image relative to the second lens before choosing its object-distance sign; converging incident rays give a positive virtual-object distance.

23. Define displacement current. What role does it play while charging a capacitor by dc source. Is the value of displacement current same as that of the conduction current? Explain. [3 marks]

Solution:

Concept:

- Displacement current is the current equivalent associated with a time-varying electric flux. In vacuum, $I_d = \epsilon_0 d\Phi_E/dt$, where $\Phi_E = \int \mathbf{E} \cdot d\mathbf{A}$.
- Maxwell's corrected Ampere law is $\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0(I_c + I_d)$, where I_c is conduction current. Either contribution can produce a magnetic field.

Step 1: Explain its role during capacitor charging On connecting an initially uncharged capacitor to a dc source, conduction current flows in the connecting wires and deposits opposite charges on the two plates. No free charge crosses the insulating gap of an ideal capacitor, so there is no conduction current there. However, while charging proceeds, the electric field between the plates changes and therefore produces displacement current in the gap. This term maintains the continuity of total current and makes Ampere's law consistent for a surface cutting the wire and another passing between the plates with the same boundary.

Step 2: Compare the two currents quantitatively Consider parallel plates of area A with vacuum or air in the gap, neglecting fringing. If the instantaneous plate charge has magnitude $Q(t)$, then

$$E = \frac{Q}{\epsilon_0 A}, \quad \Phi_E = EA = \frac{Q}{\epsilon_0}.$$

Hence the displacement current through the gap is

$$I_d = \epsilon_0 \frac{d\Phi_E}{dt} = \epsilon_0 \frac{d}{dt} \left(\frac{Q}{\epsilon_0} \right) = \frac{dQ}{dt} = I_c.$$

Thus its instantaneous magnitude equals the conduction current in the wire; the compatible direction completes the current path. It is not the transport of free electrons through the dielectric. Once charging ends under steady dc conditions, the plate charge and field are constant, so both currents become zero. In a dielectric the general expression uses electric displacement, $I_d = d \int \mathbf{D} \cdot d\mathbf{A}/dt$, and the same continuity argument applies.

Step 3: Conclusion Displacement current is the contribution due to changing electric flux. It supplies the current term in the capacitor gap during charging, with $I_d = I_c = dQ/dt$. Both vanish after an ideal capacitor is fully charged by a dc source.

Quick Tip to Solve: Use $\Phi_E = Q/\epsilon_0$ between capacitor plates: differentiating immediately proves $I_d = I_c$, even though free charge does not cross the gap.

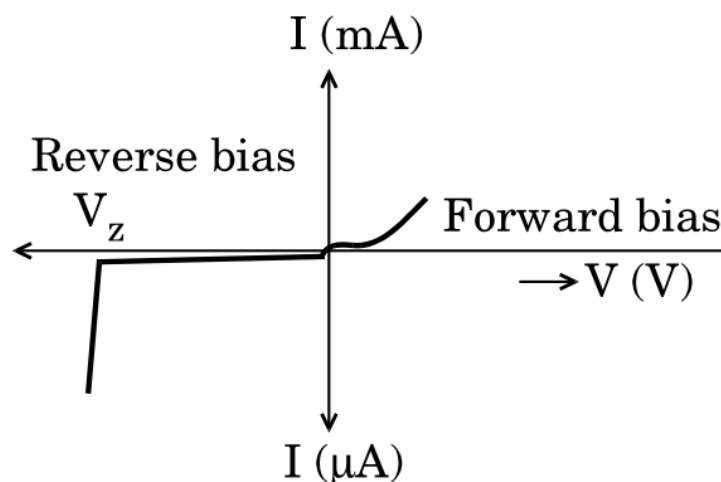
24. The figure shows the V-I characteristic of a semiconductor diode designed to operate under reverse bias.

(a) Identify the semiconductor diode used.

(b) Draw the circuit diagram to obtain the given characteristics of this device.

(c) Briefly explain one use of this device.

[3 marks]



Solution:

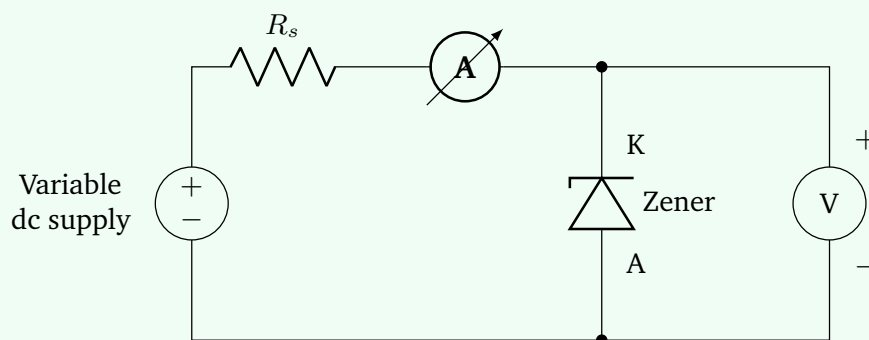
Concept:

- A Zener diode is a heavily doped p-n junction designed to operate safely in reverse breakdown. At its breakdown voltage V_Z , a large current change produces only a small change in diode voltage.
- To measure a V-I characteristic, connect an ammeter in series and a voltmeter in parallel with the diode. A series resistance limits the current and protects the junction.

Step 1: Identify the diode from its characteristic (a) The device is a **Zener diode**. The reverse current is very small before the knee. At a reverse-voltage magnitude V_Z , it rises sharply while the voltage remains almost constant. This controlled breakdown region distinguishes its intended use from that of an ordinary rectifying diode.

Step 2: Connect the measurement circuit (b) The diagram shows the reverse-bias arrangement: connect the cathode K to the positive terminal through the current-limiting resistor R_s and the ammeter A, and the anode A to the negative terminal. The voltmeter V is connected directly across the diode. Increase the variable dc supply gradually and record diode voltage and current, taking care to keep the breakdown current within the diode's rating. Use the sensitive current range for reverse leakage and an appropriate higher range in breakdown. To obtain the forward branch of the same characteristic, reverse the diode connections so that its anode is towards the positive terminal, keeping the voltmeter positive lead towards the positive supply. Use the milliampere current range, then increase the supply and record readings. Plot reverse readings with negative voltage and current, and forward readings with positive signs.

Quick Tip to Solve: For the reverse Zener test, put the cathode towards the positive supply; always measure the diode voltage across the diode, not across the diode plus resistor.



Reverse-bias test circuit. A is a multirange ammeter; V measures the voltage across the Zener diode.

Step 3: Explain voltage regulation (c) A Zener diode acts as a **voltage regulator** when connected in reverse bias across a load, with a resistor in series with the supply. In breakdown the load voltage is approximately V_Z . If the input voltage rises, the series current rises and the Zener takes the extra current, with only a small change in voltage. If the load current rises, the Zener current decreases correspondingly. Thus the load voltage stays nearly constant, provided the diode remains in breakdown and within its current and power limits. The series resistor drops the excess supply voltage and prevents excessive current.

Step 4: Conclusion The device is a Zener diode. Its characteristic is measured using a series ammeter and current-limiting resistor, with a voltmeter across it. Its nearly constant reverse-breakdown voltage makes it useful as a voltage regulator.

Quick Tip to Solve: For the reverse Zener test, put the cathode towards the positive supply; always measure the diode voltage across the diode, not across the diode plus resistor.

Section D (15 marks)

25. (a) Draw a labelled ray diagram of compound microscope, when final image forms at the least distance of distinct vision.

(b) Why is its objective of short focal length and of short aperture, compared to its eyepiece? Explain.

(c) The focal length of the objective is 4 cm while that of eyepiece is 10 cm. The object is placed at a distance of 6 cm from the objective lens.

(i) Calculate the magnifying power of the compound microscope, if its final image is formed at the near point.

(ii) Also calculate length of the compound microscope.

[5 marks]

Solution:

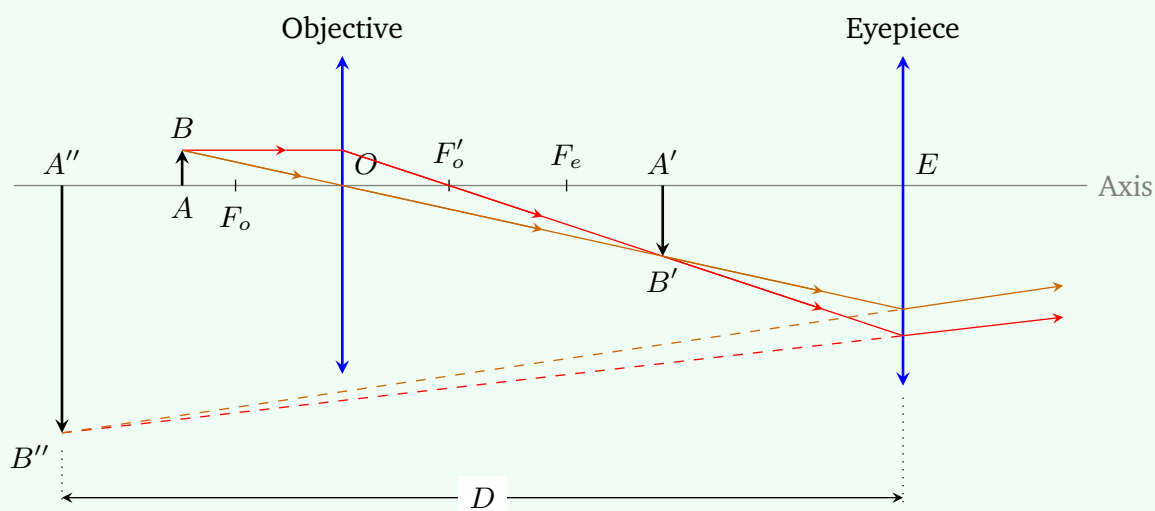
Concept:

- A compound microscope uses its objective to form a real, inverted, enlarged intermediate image; the eyepiece magnifies that image as a simple microscope.
- For a thin lens, $1/f = 1/v - 1/u$, with Cartesian signs. The objective's lateral magnification is $m_o = v_o/u_o$.

Quick Tip to Solve: First find the objective's real image. For near-point viewing put $v_e = -25$ cm, then add v_o and $|u_e|$ to obtain the lens separation.

- With the final image at the near point $D = 25$ cm, the eyepiece's angular magnification is $m_e = 1 + D/f_e$. The total magnifying power is $M = m_o m_e$.

Step 1: (a) Trace the rays for near-point viewing Place the small object AB just beyond the objective's first focus F_o . A ray from B parallel to the principal axis refracts through F'_o ; another through the objective's optical centre O is undeviated. Their intersection gives the inverted real image $A'B'$. Adjust the eyepiece so that $A'B'$ lies between its first focus F_e and its optical centre E . After refraction at the eyepiece the rays diverge; their backward extensions intersect at B'' . Thus $A''B''$ is virtual, enlarged and inverted relative to the original object, and lies a distance D to the left of the eyepiece. The schematic below shows solid actual rays and dashed backward extensions.



Compound microscope at the near point (schematic, not to scale). Dashed lines are virtual extensions; $EA'' = D$.

Step 2: (b) Explain the objective's focal length and aperture A short focal length gives the objective a large linear magnification: for the usual microscope arrangement $|m_o| \approx L/f_o$, where L is the optical tube length. The object is kept very close to this objective and just beyond its focus. Its small aperture limits marginal rays and hence reduces spherical aberration, giving a sharper intermediate image. The eyepiece has a larger aperture to admit the rays from that image and provide a useful field of view. This explains the stated school-level design comparison; a small aperture does not itself increase resolving power.

Step 3: (c)(i) Find the objective image and magnifying power Use $f_o = +4$ cm and $u_o = -6$ cm. The lens equation gives

$$\frac{1}{v_o} = \frac{1}{f_o} + \frac{1}{u_o} = \frac{1}{4} - \frac{1}{6} = \frac{1}{12}, \quad v_o = +12 \text{ cm}.$$

Hence $m_o = 12/(-6) = -2$; the negative sign records inversion. For the eyepiece, $f_e = 10$ cm, so

$$m_e = 1 + \frac{25}{10} = 3.5, \quad M = (-2)(3.5) = -7.$$

The magnitude of the angular magnification is therefore seven, with an inverted final image. The exact objective magnification is used because the given object distance is known; the approximate tube-length formula is unnecessary for this calculation.

Step 4: (c)(ii) Find the separation of the lenses For the final virtual image at the near point,

Quick Tip to Solve: First find the objective's real image. For near-point viewing put $v_e = -25$ cm, then add v_o and $|u_e|$ to obtain the lens separation.

$v_e = -25$ cm. Applying the lens equation to the eyepiece,

$$\frac{1}{u_e} = \frac{1}{v_e} - \frac{1}{f_e} = -\frac{1}{25} - \frac{1}{10} = -\frac{7}{50}, \quad u_e = -\frac{50}{7} \text{ cm.}$$

Thus the real intermediate image is $50/7$ cm to the left of the eyepiece and 12 cm to the right of the objective. Adding these positive distances gives the microscope's length (lens separation):

$$\ell = v_o + |u_e| = 12 + \frac{50}{7} = \frac{134}{7} \text{ cm} \approx 19.14 \text{ cm.}$$

Step 5: Conclusion The near-point arrangement produces a virtual, enlarged and inverted final image. The short-focus objective provides high magnification, and its small aperture reduces marginal-ray aberration. The magnifying power has magnitude $\boxed{7}$ (signed value $M = -7$); the microscope length is $\boxed{19.14 \text{ cm}}$.

Quick Tip to Solve: First find the objective's real image. For near-point viewing put $v_e = -25$ cm, then add v_o and $|u_e|$ to obtain the lens separation.

OR

25. (a) With the help of a labelled ray diagram, explain the construction and working of a Cassegrain reflecting telescope.

(b) An amateur astronomer wishes to estimate roughly the size of the Sun using his crude telescope consisting of an objective lens of focal length 200 cm and an eyepiece of focal length 10 cm. By adjusting the distance of the eyepiece from the objective, he obtains an image of the Sun on a screen 40 cm behind the eyepiece. The diameter of the Sun's image is measured to be 6.0 cm. Estimate the Sun's size, given that the average Earth-Sun distance is 1.5×10^{11} m. [5 marks]

Solution:

Concept:

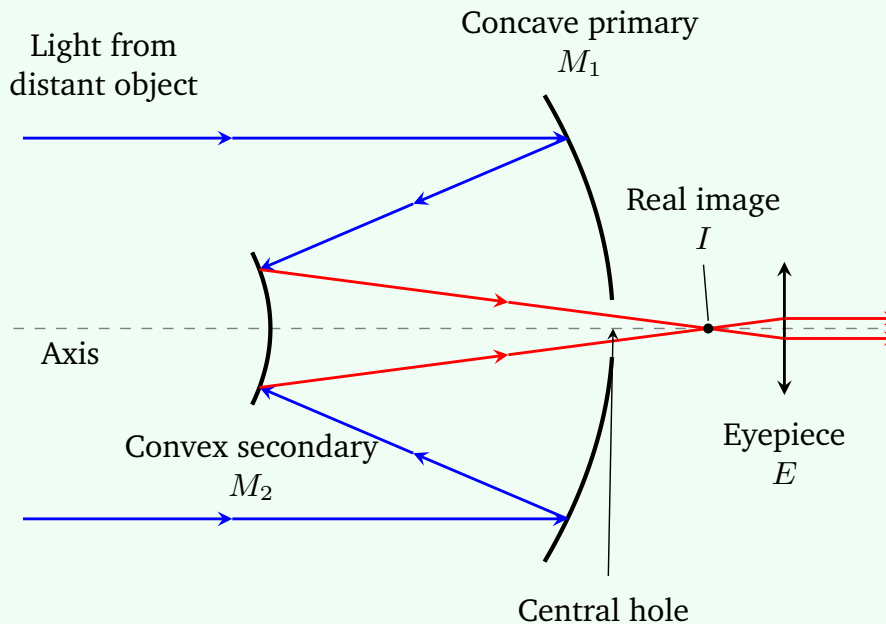
- A Cassegrain telescope uses a large concave primary mirror and a small convex secondary mirror to fold the optical path. Reflection is free from chromatic aberration.
- For a thin lens, $1/f = 1/v - 1/u$, with distances measured using the Cartesian sign convention. Its transverse magnification is $m = v/u$.
- A distant object of diameter D , at distance R , subtends a small angle $\theta \simeq D/R$. An objective of focal length f_o forms an image of diameter $d_o \simeq f_o \theta$.

Step 1: (a) Describe the construction The objective is a large concave primary mirror M_1 , usually parabolic, with a central hole. A small convex secondary mirror M_2 is mounted coaxially in front of it, between the primary and the primary's focus. An eyepiece E is placed behind the central hole of M_1 . The primary has a large aperture to collect light from faint distant objects; the folded path gives a long effective focal length within a comparatively short tube.

Step 2: Trace the light through the mirrors and eyepiece Nearly parallel light from a distant object falls on M_1 . After reflection it converges towards the primary focus, but M_2 intercepts the converging

Quick Tip to Solve: For a telescope projecting onto a screen, first undo the eyepiece's linear magnification. Then divide the intermediate image diameter by the objective focal length to find the angular diameter.

beam before it reaches that focus. The convex secondary reflects the beam back towards M_1 , reducing its convergence. The rays pass through the primary's central hole and meet to form a real image I behind the primary. The eyepiece magnifies this image for viewing. In normal adjustment, I lies in the eyepiece's first focal plane and the final image is at infinity. The schematic shows axial rays and their direction of travel.



Cassegrain telescope: two reflections fold the light path; the real image lies behind the perforated primary mirror (schematic, not to scale).

Step 3: (b) Find the object distance for the eyepiece Here the Sun is projected onto a screen, so the eyepiece forms a real image rather than a final virtual image for direct viewing. For the eyepiece, $f_e = +10$ cm and $v_e = +40$ cm. Its object is the real solar image already formed by the objective, on the incident side of the eyepiece.

$$\frac{1}{u_e} = \frac{1}{v_e} - \frac{1}{f_e} = \frac{1}{40} - \frac{1}{10} = -\frac{3}{40} \text{ cm}^{-1},$$

$$u_e = -\frac{40}{3} \text{ cm}.$$

Step 4: Recover the diameter of the objective's image The eyepiece's linear magnification is

$$m_e = \frac{v_e}{u_e} = \frac{40}{-40/3} = -3.$$

Its magnitude is 3; the negative sign denotes inversion relative to the intermediate image. Therefore the screen-image diameter of 6.0 cm corresponds to an intermediate diameter

$$d_o = \frac{6.0}{3} = 2.0 \text{ cm}.$$

We use linear magnification here because both the object and image distances for the eyepiece are finite. The usual angular magnification f_o/f_e for a telescope in normal adjustment does not describe

Quick Tip to Solve: For a telescope projecting onto a screen, first undo the eyepiece's linear magnification. Then divide the intermediate image diameter by the objective focal length to find the angular diameter.

this screen projection.

Step 5: Use the Sun's angular diameter The Sun is so far away that its intermediate image is effectively in the objective's focal plane. With $f_o = 200$ cm,

$$\theta \simeq \frac{d_o}{f_o} = \frac{2.0}{200} = 1.0 \times 10^{-2} \text{ rad.}$$

The diameters and distances form similar small-angle triangles. Using the average Earth–Sun distance $R = 1.5 \times 10^{11}$ m,

$$D \simeq R\theta = (1.5 \times 10^{11})(1.0 \times 10^{-2}) \text{ m.}$$

Step 6: Conclusion (a) The concave primary and convex secondary send the light through the primary's central hole to a real image behind it; the eyepiece magnifies that image. (b) The estimated diameter of the Sun is $D \simeq 1.5 \times 10^9 \text{ m} = 1.5 \times 10^6 \text{ km}$.

Quick Tip to Solve: For a telescope projecting onto a screen, first undo the eyepiece's linear magnification. Then divide the intermediate image diameter by the objective focal length to find the angular diameter.

26. (a) Derive a relation between the internal resistance, emf and terminal potential difference of a cell from which current I is drawn. Draw V vs I graph for a cell and explain its significance.

(b) A voltmeter of resistance 998Ω is connected across a cell of emf 2 V and internal resistance 2Ω . Find the potential difference across the voltmeter and also across the terminals of the cell. Estimate the percentage error in the reading of the voltmeter. [5 marks]

Solution:

Concept:

- The emf $\mathcal{E}$ is the energy supplied by a cell per unit charge. Its terminal potential difference V is the energy delivered per unit charge to the external circuit.
- If current I is drawn from a cell of internal resistance r , the internal potential drop is Ir . Thus $\mathcal{E} = V + Ir$.
- A voltmeter of finite resistance R_V draws current and loads the cell. It measures the loaded terminal voltage; an ideal voltmeter draws no current and would read the emf of an otherwise open cell.

Step 1: (a) Derive the relation for a cell supplying current Let charge $q = I\Delta t$ flow through the cell and the external circuit in time Δt . The cell supplies energy $\mathcal{E}q$. The external circuit receives energy Vq , while the internal resistance dissipates energy $I^2 r \Delta t$. Conservation of energy gives

$$\mathcal{E}I\Delta t = VI\Delta t + I^2 r \Delta t.$$

Dividing by $I\Delta t$, for nonzero current, yields

$$\mathcal{E} = V + Ir, \quad V = \mathcal{E} - Ir, \quad r = \frac{\mathcal{E} - V}{I}.$$

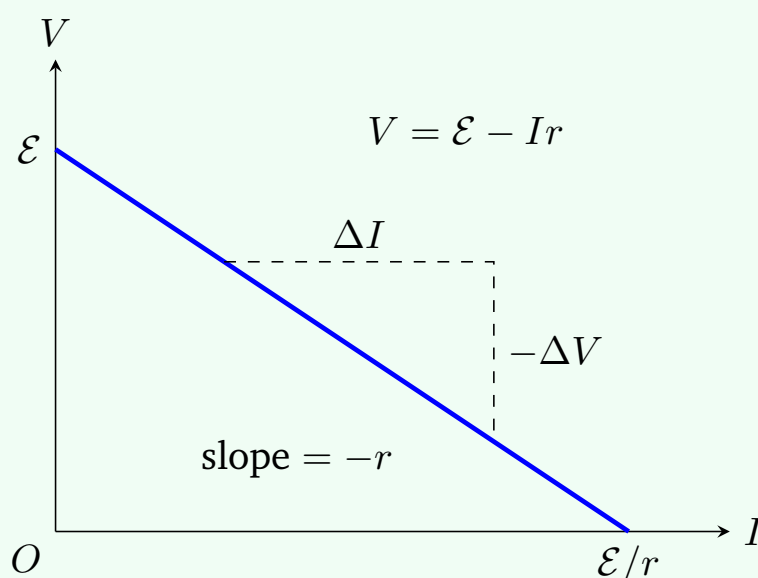
Quick Tip to Solve: For a voltmeter alone across a cell, use $V_V = \mathcal{E}R_V/(R_V + r)$; its percentage loading error is immediately $100r/(R_V + r)$.

The negative sign in $V = \mathcal{E} - Ir$ applies to a discharging cell, as specified by current being drawn from it. At open circuit, $I = 0$ and $V = \mathcal{E}$.

Step 2: (a) Interpret the voltage-current graph For constant emf and internal resistance, plotting V vertically against I horizontally gives a straight line. Comparing $V = \mathcal{E} - rI$ with $y = c + mx$, its vertical intercept is $\mathcal{E}$ and its slope is $-r$. Hence the magnitude of the slope determines the internal resistance:

$$r = -\frac{\Delta V}{\Delta I}.$$

The decrease of terminal voltage with increasing current represents the increasing internal drop. Extrapolating the line to $V = 0$ gives the idealized short-circuit current $I_{sc} = \mathcal{E}/r$. This intercept assumes r remains constant.



Terminal voltage against current for a cell of constant emf and internal resistance.

Step 3: (b) Calculate the voltmeter current and reading The voltmeter is the external load across the cell. Its resistance $R_V = 998 \Omega$ and the cell's internal resistance $r = 2 \Omega$ are in series in the complete circuit. Therefore,

$$I = \frac{\mathcal{E}}{R_V + r} = \frac{2}{998 + 2} = 0.002 \text{ A}.$$

The potential difference across the voltmeter is

$$V_V = IR_V = 0.002 \times 998 = 1.996 \text{ V}.$$

Since the voltmeter is connected directly to the cell terminals, both share the same potential difference. As an independent check,

$$V_{\text{cell}} = \mathcal{E} - Ir = 2 - (0.002)(2) = 1.996 \text{ V}.$$

Step 4: (b) Estimate the percentage loading error The relevant error is the shortfall of the reading from the unloaded cell voltage, which equals its emf. The voltmeter correctly reads the loaded

Quick Tip to Solve: For a voltmeter alone across a cell, use $V_V = \mathcal{E}R_V/(R_V + r)$; its percentage loading error is immediately $100r/(R_V + r)$.

terminal voltage, but its finite resistance lowers that voltage by $Ir = 0.004\text{ V}$. Thus,

$$\text{Percentage error} = \frac{\mathcal{E} - V_V}{\mathcal{E}} \times 100 = \frac{2 - 1.996}{2} \times 100 = 0.2\%.$$

The instrument under-reads the emf. Equivalently, the fractional loading error is $r/(R_V + r) = 2/1000$.

Step 5: Conclusion For a cell supplying current, $V = \mathcal{E} - Ir$. The V -versus- I graph has voltage intercept $\mathcal{E}$, slope $-r$, and idealized current intercept $\mathcal{E}/r$. In part (b), $V_V = V_{\text{cell}} = 1.996\text{ V}$; the percentage error relative to the emf is 0.2% , an under-reading.

Quick Tip to Solve: For a voltmeter alone across a cell, use $V_V = \mathcal{E}R_V/(R_V + r)$; its percentage loading error is immediately $100r/(R_V + r)$.

OR

26. (a) Two cells of different emfs and internal resistances are connected in parallel with one another. Derive the expression for the equivalent emf and equivalent internal resistance of the combination.
- (b) Two identical cells of emf 1.5 V and internal resistance r are each connected in parallel providing a supply to an external circuit consisting of two resistances of $17\ \Omega$ each joined in parallel. A very high resistance voltmeter reads the terminal voltage of the cell to be 1.4 V . Calculate the internal resistance of each cell.

[5 marks]

Solution:

Concept:

- For a cell supplying current I , terminal voltage is $V = E - Ir$, where E is emf and r is internal resistance.
- Cells connected with like terminals together have the same terminal voltage; their signed branch currents add to the load current by Kirchhoff's junction rule.
- Parallel resistors satisfy $1/R_{\text{eq}} = 1/R_1 + 1/R_2$. A very high resistance voltmeter draws negligible current.

Step 1: (a) Express the branch currents in terms of terminal voltage Let the two emfs be E_1, E_2 , their internal resistances be r_1, r_2 , and the common terminal voltage be V . Connect their positive terminals together and their negative terminals together. Take I_1, I_2 positive when each cell supplies the external circuit. Then

$$I_1 = \frac{E_1 - V}{r_1}, \quad I_2 = \frac{E_2 - V}{r_2}.$$

These are signed relations: if the stronger cell charges the weaker one, the corresponding branch current is negative, so the derivation remains valid.

Step 2: (a) Identify the equivalent emf and resistance The total current delivered to the load is

Quick Tip to Solve: With two identical parallel cells, keep the emf unchanged and halve the internal resistance before using $E - V = Ir_{\text{eq}}$; the requested resistance of each cell is twice r_{eq} .

$I = I_1 + I_2$. Hence

$$I = \frac{E_1}{r_1} + \frac{E_2}{r_2} - V \left(\frac{1}{r_1} + \frac{1}{r_2} \right),$$
$$V = \frac{E_1 r_2 + E_2 r_1}{r_1 + r_2} - I \frac{r_1 r_2}{r_1 + r_2}.$$

Comparing this expression with $V = E_{\text{eq}} - I r_{\text{eq}}$ identifies the open-circuit emf and the coefficient of the current:

$$E_{\text{eq}} = \frac{E_1 r_2 + E_2 r_1}{r_1 + r_2}, \quad r_{\text{eq}} = \frac{r_1 r_2}{r_1 + r_2}.$$

Thus the equivalent emf is weighted by the cell conductances; it is not the sum of the emfs.

Step 3: (b) Reduce the identical cells and external resistors For two identical parallel cells, substitute $E_1 = E_2 = 1.5 \text{ V}$ and $r_1 = r_2 = r$. The supply therefore has $E_{\text{eq}} = 1.5 \text{ V}$ and $r_{\text{eq}} = r/2$. The two external resistors give

$$R_L = \frac{17 \times 17}{17 + 17} = 8.5 \Omega.$$

The voltmeter measures the voltage across this load. Neglecting its current, the total load current is

$$I = \frac{V}{R_L} = \frac{1.4}{8.5} = \frac{14}{85} \text{ A}.$$

Step 4: (b) Use the internal voltage drop The voltage lost inside the equivalent source is $E_{\text{eq}} - V = 1.5 - 1.4 = 0.1 \text{ V}$. Therefore

$$0.1 = \frac{14}{85} \frac{r}{2},$$
$$r = \frac{2(0.1)(85)}{14} = \frac{17}{14} \Omega \simeq 1.21 \Omega.$$

Each identical cell supplies half the load current, so its own internal drop is also $(I/2)r = 0.1 \text{ V}$, which checks the result.

Step 5: Conclusion For like-polarity parallel cells, $E_{\text{eq}} = (E_1 r_2 + E_2 r_1)/(r_1 + r_2)$ and $r_{\text{eq}} = r_1 r_2/(r_1 + r_2)$. The internal resistance of each cell in (b) is $r \simeq 1.21 \Omega$.

Quick Tip to Solve: With two identical parallel cells, keep the emf unchanged and halve the internal resistance before using $E - V = I r_{\text{eq}}$; the requested resistance of each cell is twice r_{eq} .

27. (a) A metallic rod of length ' l ' and resistance ' R ' is rotated with a frequency ' ν ' with one end hinged at the centre and the other end at the circumference of a circular metallic ring of radius ' l ', about an axis passing through the centre and perpendicular to the plane of the ring. A constant and uniform magnetic field ' B ' parallel to the axis is present everywhere.

(i) Derive the expression for the induced emf and the current in the rod.

(ii) Due to the presence of current in the rod and of the magnetic field, find the expression for the magnitude and direction of the force acting on this rod.

(iii) Hence, obtain an expression for the power required to rotate the rod.

(b) A copper coil is taken out of a magnetic field with a fixed velocity. Will it be easy to remove it from the same field if its ohmic resistance is increased? [5 marks]

Solution:

Concept:

- Motional emf across a radial element is $d\varepsilon = Bv dx$. For a rod rotating with angular speed $\omega = 2\pi\nu$, an element at radius x moves at speed $v = \omega x$.
- A current-carrying element experiences $d\vec{F} = I d\vec{l} \times \vec{B}$. Mechanical power at constant angular speed is $P = \tau\omega$, where τ is the opposing torque magnitude.
- Faraday's law gives $\varepsilon = -d\Phi/dt$; Lenz's law makes the induced current oppose the change of magnetic flux that produces it.

Step 1: (a)(i) Integrate the motional emf along the rod Consider a small radial element of length dx , at a distance x from the hinge. Its velocity is tangential, and the magnetic field is perpendicular to the plane of rotation. Thus $\vec{v} \times \vec{B}$ lies along the rod and the contributions from all elements add:

$$\varepsilon = \int_0^l B\omega x dx = \frac{1}{2}B\omega l^2 = \pi B\nu l^2.$$

This is the magnitude of the potential difference between the rim end and the centre. For anticlockwise rotation viewed along a field pointing out of the page, positive charges are driven towards the rim; reversing either the rotation or the field reverses the polarity.

Step 2: (a)(i) Find the current in the closed circuit For the intended closed circuit between the central contact and the ring, take the connecting path and ring resistance as negligible, so the circuit resistance is the specified rod resistance R . Ohm's law then gives

$$I = \frac{\varepsilon}{R} = \frac{B\omega l^2}{2R} = \frac{\pi B\nu l^2}{R}.$$

The current in the rod follows the direction of the motional emf, from the centre to the rim for the orientation described above. A closed return path is necessary for this current; without one, only the open-circuit emf exists.

Step 3: (a)(ii) Calculate the magnetic force and its direction The same steady current flows through every element of the rod. Each element is perpendicular to the field, so $dF = IB dx$. The element

Quick Tip to Solve: Use $v = \omega x$ and integrate along a rotating rod. Its magnetic force acts effectively at $l/2$, so obtain power from $\tau\omega$, and check it against ε^2/R .

forces all point tangentially in the same direction at a given instant. Their resultant is

$$F = \int_0^l IB \, dx = IBl = \frac{B^2 \omega l^3}{2R} = \frac{\pi B^2 \nu l^3}{R}.$$

By $I d\vec{l} \times \vec{B}$, this force is perpendicular to the rod in its plane of rotation and opposes the local tangential motion. In the stated example it acts clockwise, opposing the anticlockwise rotation. The uniformly distributed magnetic force has its resultant at the midpoint of the rod.

Step 4: (a)(iii) Find the torque and mechanical power Each force element acts at its own lever arm x . Hence the retarding torque about the hinge is

$$\tau = \int_0^l xIB \, dx = \frac{IBl^2}{2} = \frac{Fl}{2}.$$

An external torque of equal magnitude is needed to maintain uniform rotation. Its power, neglecting friction, is

$$\begin{aligned} P = \tau\omega &= \frac{IB\omega l^2}{2} = I\varepsilon, \\ &= \frac{B^2 \omega^2 l^4}{4R} = \frac{\pi^2 B^2 \nu^2 l^4}{R}. \end{aligned}$$

This equals $I^2 R$, the electrical energy dissipated per second, as required by conservation of energy. Multiplying the total force by the tip speed would be incorrect because the force is distributed along the rod.

Step 5: (b) Compare the opposition when coil resistance increases At the same withdrawal speed through the same field boundary, the rate of change of flux, and hence the induced emf magnitude, is unchanged. If the coil resistance is increased, the induced current $I = |\varepsilon|/R_{\text{coil}}$ decreases. Lenz's law makes its magnetic force oppose withdrawal; the smaller current therefore gives a smaller retarding force. Consequently the required pulling force and mechanical power decrease, and removing the coil is easier.

Step 6: Conclusion The rod has $\varepsilon = \pi B \nu l^2$, $I = \pi B \nu l^2 / R$, and a tangential opposing force $F = \pi B^2 \nu l^3 / R$. The required power is $P = \pi^2 B^2 \nu^2 l^4 / R$. Increasing the coil's ohmic resistance makes withdrawal easier by reducing the induced current and magnetic drag.

Quick Tip to Solve: Use $v = \omega x$ and integrate along a rotating rod. Its magnetic force acts effectively at $l/2$, so obtain power from $\tau\omega$, and check it against ε^2/R .

27. (a) A rectangular coil rotates in a uniform magnetic field. Obtain an expression for induced emf and current at any instant. Also find their peak values. Show the variation of induced emf versus angle of rotation (ωt) on a graph.

(b) An iron bar falling through the hollow region of a thick cylindrical shell made of copper experiences a retarding force. What can you conclude about the nature of the iron bar? Explain. [5 marks]

Solution:

Concept:

- Faraday's law gives the induced emf as $\varepsilon = -d(N\Phi)/dt$, where $N\Phi$ is the flux linkage of an N -turn coil.
- For a plane coil of area A , the magnetic flux per turn is $\Phi = BA \cos \theta$, where θ is the angle between the magnetic field and the coil's normal.
- For total circuit resistance R , with inductance neglected, Ohm's law gives $i = \varepsilon/R$. Lenz's law states that the induced current opposes the change of flux producing it.

Step 1: (a) Express the changing flux linkage Let the rectangular coil have length l , breadth b , area $A = lb$, and N turns. It rotates uniformly with angular speed ω about an axis in its plane perpendicular to the uniform magnetic field B . Choose $t = 0$ when the coil's normal is parallel to the field. Thus $\theta = \omega t$, and the flux linkage is

$$N\Phi = NBA \cos(\omega t).$$

Although the field and area remain constant, the changing orientation changes the magnetic flux through the coil.

Step 2: (a) Differentiate to obtain the instantaneous emf Applying Faraday's law to the flux linkage,

$$\begin{aligned}\varepsilon(t) &= -\frac{d}{dt}[NBA \cos(\omega t)] \\ &= NBA\omega \sin(\omega t).\end{aligned}$$

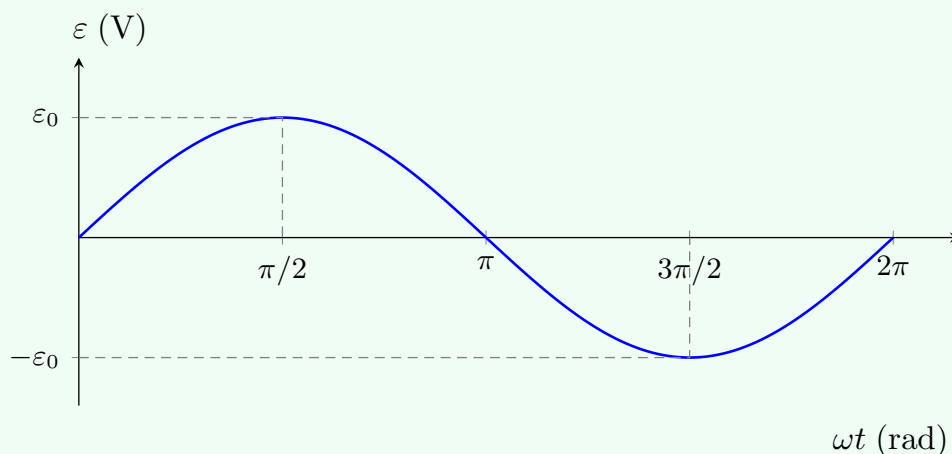
Hence the emf varies sinusoidally and reverses direction every half rotation. Its maximum magnitude occurs when $|\sin(\omega t)| = 1$, so the peak emf is $\varepsilon_0 = NBA\omega = NBlb\omega$. For a single-turn coil, take $N = 1$. Changing the initial orientation changes the phase, but not this peak value.

Step 3: (a) Obtain the current and plot the emf If the coil forms a closed resistive circuit of total resistance R , neglecting its self-inductance, the instantaneous current and its peak value are

$$i(t) = \frac{NBA\omega}{R} \sin(\omega t), \quad i_0 = \frac{NBA\omega}{R}.$$

The current is in phase with the emf for this resistive circuit. An open coil has the induced emf but no conduction current around a complete circuit. The requested graph starts at zero for our chosen initial orientation, reaches $+\varepsilon_0$ at $\omega t = \pi/2$, crosses zero at π , reaches $-\varepsilon_0$ at $3\pi/2$, and returns to zero at 2π . The emf is zero when flux is greatest in magnitude, and greatest in magnitude when flux passes through zero.

Quick Tip to Solve: Define the angle using the coil's normal before differentiating flux: maximum flux means zero emf. For the falling bar, apply Lenz's law to its motion to identify the upward braking force.



Induced emf versus angle of rotation, with the coil normal initially parallel to the field.

Step 4: (b) Explain the retarding force in the copper shell The iron bar must be magnetised: it behaves as a bar magnet. As it falls through the conducting copper shell, the magnetic flux through circular paths in the shell changes. This produces circulating eddy currents in the copper even though the shell is not ferromagnetic. By Lenz's law, the magnetic fields of these currents oppose the changing flux. Currents ahead of the falling magnet oppose its approach, while currents behind it oppose its recession. Both effects produce an upward magnetic force opposing the downward motion. Work done against this force converts mechanical energy into Joule heat in the copper. An ordinary unmagnetised iron bar would not produce the changing magnetic flux responsible for this observed magnetic braking.

Step 5: Conclusion For the stated orientation, $\varepsilon = NBA\omega \sin(\omega t)$ and $i = (NBA\omega/R) \sin(\omega t)$, with peaks $\varepsilon_0 = NBA\omega$ and $i_0 = NBA\omega/R$. The emf-angle graph is sinusoidal. The falling iron bar is magnetised; eddy currents induced in the copper shell exert the retarding force.

Quick Tip to Solve: Define the angle using the coil's normal before differentiating flux: maximum flux means zero emf. For the falling bar, apply Lenz's law to its motion to identify the upward braking force.