

CBSE Class 12 Physics Question Paper 2019 with Solution PDF-Set 55/5/3

Question Paper with Solutions

Conducted by the Central Board of Secondary Education (CBSE)



Time Allowed: 3 hours

Maximum Marks: 70

General Instructions

- (i) All questions are compulsory. There are 27 questions in all.
- (ii) This question paper has four sections : Section A, Section B, Section C and Section D.
- (iii) Section A contains five questions of one mark each, Section B contains seven questions of two marks each, Section C contains twelve questions of three marks each, Section D contains three questions of five marks each.
- (iv) There is no overall choice. However, an internal choice(s) has been provided in two questions of one mark, two questions of two marks, four questions of three marks and three questions of five marks weightage. You have to attempt only one of the choices in such questions.

You may use the following values of physical constants wherever necessary :

$$c = 3 \times 10^8 \text{ m/s}$$

$$h = 6.63 \times 10^{-34} \text{ Js}$$

$$e = 1.6 \times 10^{-19} \text{ C}$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ T m A}^{-1}$$

$$\epsilon_0 = 8.854 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$$

$$\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$$

$$\text{Mass of electron } (m_e) = 9.1 \times 10^{-31} \text{ kg}$$

$$\text{Mass of neutron} = 1.675 \times 10^{-27} \text{ kg}$$

$$\text{Mass of proton} = 1.673 \times 10^{-27} \text{ kg}$$

$$\text{Avogadro's number} = 6.023 \times 10^{23} \text{ per gram mole}$$

$$\text{Boltzmann constant} = 1.38 \times 10^{-23} \text{ JK}^{-1}$$

Section A (5 marks)

1. Plot a graph of de-Broglie wavelength associated with an electron as a function of its kinetic energy. [1 mark]

Solution:

Concept:

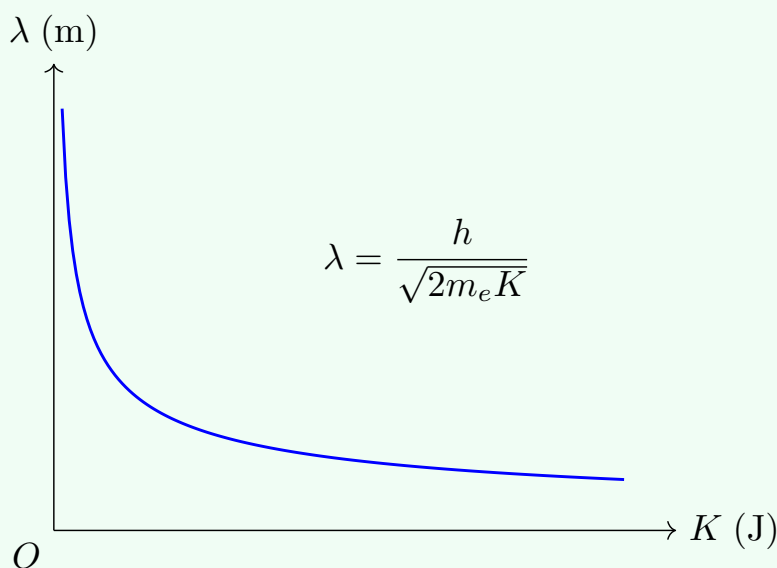
- The de-Broglie wavelength is $\lambda = h/p$, where h is Planck's constant and p is the electron's momentum.
- For non-relativistic kinetic energy K , $K = p^2/(2m_e)$, where m_e is the electron mass.

Step 1: Relate wavelength to kinetic energy Eliminating momentum gives the dependence to be plotted:

$$\lambda = \frac{h}{\sqrt{2m_e K}}.$$

Thus wavelength varies inversely as the square root of kinetic energy, rather than inversely as kinetic energy itself.

Step 2: Sketch the decreasing curve Place K on the horizontal axis and λ on the vertical axis. For $K > 0$, the curve falls steeply at first and then becomes flatter. It stays above the energy axis; formally $\lambda \rightarrow \infty$ as $K \rightarrow 0^+$. The sketch is qualitative and uses the usual non-relativistic relation.

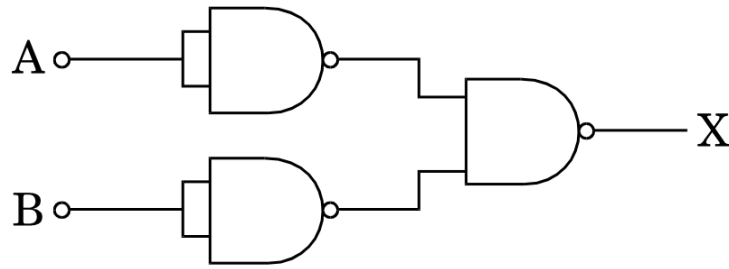


Qualitative de-Broglie wavelength versus electron kinetic energy

Step 3: Conclusion The graph is a decreasing, upward-curving $K^{-1/2}$ curve, as shown.

Quick Tip to Solve: If kinetic energy becomes four times its initial value, the electron wavelength halves; use this to check the shape.

2. Identify the equivalent gate for the circuit of a combination of gates shown in the figure. Write its symbol. [1 mark]



Solution:

Concept:

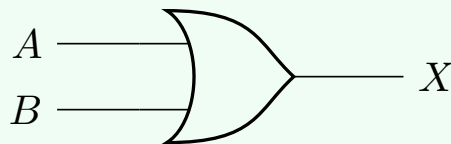
- A NAND gate has output $\overline{AB}$. With its inputs joined, it behaves as a NOT gate: $\overline{AA} = \overline{A}$.
- De Morgan's theorem gives $\overline{\overline{A}\overline{B}} = A + B$, where $+$ denotes the Boolean OR operation.

Step 1: Find the outputs of the first gates Both inputs of the upper NAND gate are connected to A , so its output is $\overline{A}$. Similarly, both inputs of the lower NAND gate are connected to B , so its output is $\overline{B}$. These signals feed the final NAND gate.

Step 2: Simplify the final gate output The final NAND gate complements the AND of its inputs:

$$X = \overline{\overline{A}\overline{B}} = A + B.$$

The output is zero only for $A = B = 0$, and is one whenever either input is one. The equivalent symbol therefore has the curved input side of an OR gate and no output inversion circle.

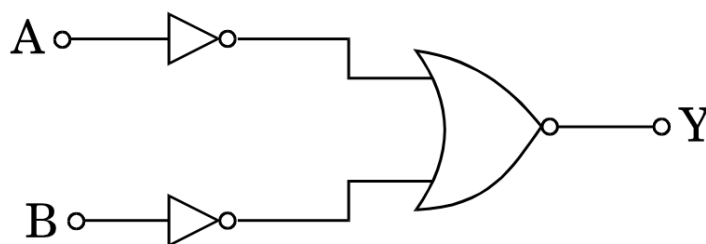


Equivalent OR gate

Step 3: Conclusion The combination is equivalent to an OR gate. Its logic symbol is shown.

Quick Tip to Solve: Read each output circle as a complement, then apply De Morgan's theorem to the complete expression.

2. Draw the logic symbol of the gate shown by the combination of gates and write its name. [1 mark]



Solution:

Concept:

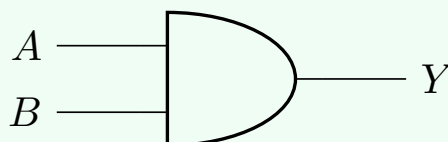
- Each NOT gate complements its input: $A \rightarrow \bar{A}$ and $B \rightarrow \bar{B}$.
- A NOR gate complements the OR of its inputs. De Morgan's theorem states $\overline{\bar{A} + \bar{B}} = AB$.

Step 1: Read the input and output inversions The two triangles with output circles are NOT gates, giving $\bar{A}$ and $\bar{B}$. The final gate has the OR outline and an output circle, so it is a NOR gate. Retain this final inversion when writing its Boolean expression.

Step 2: Apply De Morgan's theorem The circuit output is

$$Y = \overline{\bar{A} + \bar{B}} = AB.$$

It is one only when both A and B are one; for every other input pair it is zero. Draw the equivalent AND gate with a flat input edge and a rounded output edge, without an inversion circle.



Equivalent AND gate

Step 3: Conclusion The combination is equivalent to an AND gate. Its logic symbol is shown.

Quick Tip to Solve: Read each output circle as a complement, then apply De Morgan's theorem to the complete expression.

3. Write one important property of a paramagnetic material. [1 mark]

Solution:

Concept:

- Paramagnetic atoms or ions possess permanent magnetic dipole moments, but thermal motion makes their directions random without an applied field.
- A paramagnetic material has a small positive magnetic susceptibility χ_m ; its magnetisation obeys $M = \chi_m H$ in a weak field.

Quick Tip to Solve: Associate paramagnetism with weak attraction and positive susceptibility; random moments explain the absence of bulk magnetisation without a field.

Step 1: Consider the response to an external field An applied field tends to align the atomic magnetic moments along itself. Thermal agitation opposes complete alignment, so the material develops a small net magnetisation in the direction of the applied field.

Step 2: Identify the observable property In a non-uniform magnetic field, this induced net alignment makes the material move towards the region of stronger field. The attraction is weak because only a small fraction of the random orientations become aligned.

Step 3: Conclusion A paramagnetic material is weakly attracted by an external magnetic field.

Quick Tip to Solve: Associate paramagnetism with weak attraction and positive susceptibility; random moments explain the absence of bulk magnetisation without a field.

OR

3. Do the diamagnetic substances have resultant magnetic moment in an atom in the absence of external magnetic field? [1 mark]

Solution:

Concept:

- For a diamagnetic atom, the orbital and spin magnetic moments cancel, giving zero permanent resultant magnetic moment.
- An applied magnetic field can induce a moment opposite to the field; this induced response is distinct from a permanent atomic moment.

Step 1: Examine the atom without an applied field In the absence of an external field, the contributions of the electrons to the total magnetic dipole moment cancel. Electron pairing is the usual explanation for this cancellation in diamagnetic substances.

Step 2: Distinguish induced magnetism When an external field is applied, it changes the electronic orbital motion slightly and produces an induced moment opposing that field. This does not imply that a nonzero permanent resultant moment was present in the atom before the field was applied.

Step 3: Conclusion No. A diamagnetic atom has zero resultant magnetic moment in the absence of an external magnetic field.

Quick Tip to Solve: For this question discuss the moment of an individual atom, not merely the cancellation of randomly oriented moments in a whole specimen.

4. Why can the interior of a conductor have no excess charge in the static situation? [1 mark]

Solution:

Concept:

- In electrostatic equilibrium, the electric field inside the conducting material is zero; otherwise its free charges would continue moving.

Quick Tip to Solve: Use the chain: static free charges imply zero interior electric field, and zero flux through an interior Gaussian surface implies zero enclosed excess charge.

- Gauss's law is $\oint \vec{E} \cdot d\vec{S} = Q_{\text{enc}}/\epsilon_0$, where Q_{enc} is the charge enclosed by a closed surface.

Step 1: Use the equilibrium condition A conductor contains mobile charges. Any electric field in its bulk would exert a force on them and violate the assumed static situation. The free charges therefore redistribute until the electric field throughout the conducting interior vanishes.

Step 2: Apply Gauss's law inside the conductor Take a small closed Gaussian surface wholly within the conducting material. Since $\vec{E} = 0$ at every point of this surface, its electric flux is zero. Gauss's law then gives $Q_{\text{enc}} = 0$. The surface may enclose any small interior region, so no excess charge can remain in the bulk.

Step 3: Conclusion The conducting interior has no excess charge in electrostatic equilibrium; any excess charge resides on its surface.

Quick Tip to Solve: Use the chain: static free charges imply zero interior electric field, and zero flux through an interior Gaussian surface implies zero enclosed excess charge.

5. When a p-n junction diode is forward biased, how will its barrier potential be affected? [1 mark]

Solution:

Concept:

- The depletion layer of a p-n junction produces a built-in barrier potential V_b which opposes the diffusion of majority charge carriers.
- In forward bias, the p-side is connected to the positive terminal and the n-side to the negative terminal of the applied source.

Step 1: Compare the applied and junction fields The electric field produced by the forward voltage is opposite to the built-in electric field in the depletion region. It therefore opposes the field responsible for the potential barrier rather than reinforcing it.

Step 2: Identify the change in the barrier For an applied forward voltage V_f , the effective barrier is approximately $V_b - V_f$. The depletion layer becomes narrower, so majority carriers can cross the junction more easily and the forward current increases.

Step 3: Conclusion The barrier potential of the p-n junction decreases when it is forward biased.

Quick Tip to Solve: Remember that forward bias opposes the junction field: both the barrier potential and depletion-layer width decrease.

Section B (14 marks)

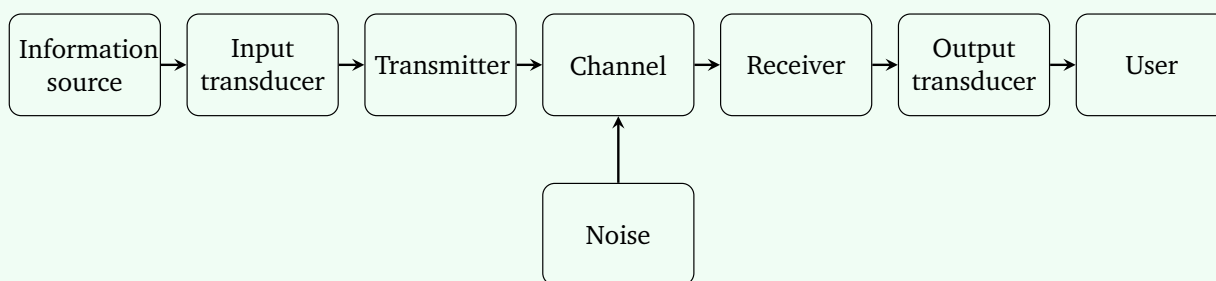
6. Draw a block diagram of a generalized communication system and write the functions of (i) a transmitter, and (ii) a receiver. [2 marks]

Solution:

Concept:

- A communication system transfers information from a source to a user through a transmitter, a communication channel and a receiver.
- Transducers convert a physical message into an electrical signal, or convert the recovered electrical signal into a form usable by the listener or observer.

Step 1: Draw the generalized signal path The information source supplies the message to the input transducer. Its electrical output passes successively through the transmitter, channel and receiver. The output transducer then produces the message for the user. Noise may enter the transmitted signal through the channel, as indicated by the additional arrow.



Generalized communication system with noise entering the channel.

Step 2: State the function of the transmitter The transmitter processes the electrical message signal into a form suitable for transmission through the chosen channel. It amplifies the signal and, where required, modulates a high-frequency carrier with the message signal before sending it into the channel.

Step 3: State the function of the receiver The receiver selects and amplifies the received signal and recovers the original message by demodulation or detection. It supplies the recovered electrical message to the output transducer so that the user can receive the information.

Step 4: Conclusion The transmitter prepares and sends the message signal through the channel; the receiver processes the received signal to recover the original information.

Quick Tip to Solve: Place transmitter before the channel and receiver after it; draw noise entering the channel, not as part of the intended message path.

7. (a) Obtain the conditions under which an electron does not suffer any deflection while passing through a magnetic field.

(b) Two protons P and Q moving with the same speed pass through the magnetic fields $\vec{B}_1$ and $\vec{B}_2$ respectively, at right angles to the field directions. If $|\vec{B}_2| > |\vec{B}_1|$, which of the two protons will describe the circular path of smaller radius? Explain. [2 marks]

Solution:

Concept:

- The magnetic force on a moving charge is $\vec{F} = q(\vec{v} \times \vec{B})$, with magnitude $F = |q|vB \sin \theta$, where θ is the angle between velocity and field.
- For motion perpendicular to a uniform magnetic field, the magnetic force supplies the centripetal force: $r = mv/(|q|B)$, where m is the particle mass.

Step 1: Find the condition for no electron deflection For an electron passing through a non-zero magnetic field, its speed is non-zero. Hence the force vanishes when $\sin \theta = 0$, that is, when $\theta = 0^\circ$ or 180° . The electron must move parallel or antiparallel to the magnetic field. A stationary electron also feels no magnetic force, although it is not passing through the field. The electron's negative charge reverses the force direction when a force exists, but does not change this zero-force condition.

Step 2: Compare the circular radii of the protons Each proton enters perpendicular to its field, so $evB = m_p v^2/r$. Their masses, charges and speeds are equal; only the field magnitudes differ. Thus

$$r_P = \frac{m_p v}{e|\vec{B}_1|}, \quad r_Q = \frac{m_p v}{e|\vec{B}_2|}, \quad \frac{r_Q}{r_P} = \frac{|\vec{B}_1|}{|\vec{B}_2|} < 1.$$

A stronger magnetic field provides a larger transverse force at the same speed, so it produces a tighter circular trajectory.

Step 3: Conclusion (a) A moving electron is undeflected when its velocity is parallel or antiparallel to the magnetic field. (b) Proton Q describes the smaller circular path because the printed condition is $|\vec{B}_2| > |\vec{B}_1|$.

Quick Tip to Solve: First use the printed field inequality, then apply $r \propto 1/B$ for equal particles at equal speed; the stronger field gives the smaller radius.

8. Two coils P and Q of radius R and 2R are lying in the same plane with their centres coinciding. Find the magnitude and direction of the resultant magnetic field at the common centre if they respectively carry currents $3I$ and $2I$ in opposite directions. [2 marks]

Solution:

Concept:

- The magnetic field at the centre of a circular single-turn coil of radius a , carrying current i , has magnitude $B = \mu_0 i/(2a)$.
- The field is perpendicular to the coil plane. Curl the fingers of the right hand in the current direction; the thumb gives its direction. Fields add vectorially by superposition.

Quick Tip to Solve: Compare i/a for the coils first: P's field is three times Q's, and opposite currents mean subtracting their centre fields.

Step 1: Calculate the field of each coil Coil P has radius R and current $3I$, while coil Q has radius $2R$ and current $2I$. Therefore,

$$B_P = \frac{\mu_0(3I)}{2R} = \frac{3\mu_0 I}{2R}, \quad B_Q = \frac{\mu_0(2I)}{2(2R)} = \frac{\mu_0 I}{2R}.$$

Both field vectors lie along the same line normal to the common plane, since the coils are coplanar and concentric.

Step 2: Subtract the opposite field vectors The two currents circulate in opposite directions. By the right-hand rule, their fields at the common centre are oppositely directed. Choose the positive normal along P's field. The algebraic sum is then

$$B_{\text{net}} = B_P - B_Q = \frac{3\mu_0 I}{2R} - \frac{\mu_0 I}{2R} = \frac{\mu_0 I}{R}.$$

The positive sign means that the resultant points in the direction of the stronger field, which is the field due to P.

Step 3: Conclusion The resultant magnetic field has magnitude $\mu_0 I/R$. It is normal to the plane of the coils and directed along the magnetic field due to coil P, as determined by the right-hand rule for its current.

Quick Tip to Solve: Compare i/a for the coils first: P's field is three times Q's, and opposite currents mean subtracting their centre fields.

9. A wire of resistance R , length l and uniform area of cross-section A is stretched till its length becomes double. It is then cut into three pieces of equal length which are connected in parallel. Find the net resistance of the combination in terms of the resistance R . [2 marks]

Solution:

Concept:

- A uniform wire has resistance $R = \rho l/A$, where ρ is its resistivity, l its length and A its cross-sectional area.
- Stretching conserves the volume of the wire, and its resistivity is assumed unchanged. For n identical resistances r in parallel, the equivalent resistance is r/n .

Step 1: Calculate the resistance after stretching The new length is $l' = 2l$. Since the volume is unchanged, $Al = A'l'$, so $A' = A/2$. Both the increased length and the decreased area raise the resistance:

$$R' = \frac{\rho(2l)}{A/2} = 4\frac{\rho l}{A} = 4R.$$

Merely doubling the length without accounting for the change in area would miss a factor of two.

Step 2: Find the resistance of each equal piece Cutting the stretched wire into three equal lengths leaves its area and resistivity unchanged. Each piece has length $2l/3$, so its resistance is one-third of

Quick Tip to Solve: At constant volume, doubling length multiplies resistance by four; three equal cuts and their parallel connection each contribute a division by three.

the stretched wire's resistance:

$$r = \frac{R'}{3} = \frac{4R}{3}.$$

Step 3: Combine the three pieces in parallel Each piece is connected across the same potential difference. Their conductances add, giving

$$\frac{1}{R_{\text{net}}} = \frac{1}{r} + \frac{1}{r} + \frac{1}{r} = \frac{3}{r} = \frac{9}{4R}.$$

Thus the parallel connection reduces the resistance of a single piece by a further factor of three.

Step 4: Conclusion The net resistance of the parallel combination is $R_{\text{net}} = 4R/9$.

Quick Tip to Solve: At constant volume, doubling length multiplies resistance by four; three equal cuts and their parallel connection each contribute a division by three.

10. Derive an expression for the torque acting on an electric dipole of dipole moment $\vec{p}$ placed in a uniform electric field $\vec{E}$. Write the direction along which the torque acts. [2 marks]

Solution:

Concept:

- An electric dipole consists of charges $+q$ and $-q$ separated by a distance a . Its dipole moment is $\vec{p} = q\vec{a}$, directed from negative to positive charge.
- In a uniform electric field, each charge experiences a force $\vec{F} = q\vec{E}$. Equal opposite forces form a couple, whose torque equals either force times the perpendicular separation of their lines of action.

Step 1: Identify the forces and their separation Let the dipole axis make an angle θ with $\vec{E}$. The force on $+q$ is $+q\vec{E}$, while that on $-q$ is $-q\vec{E}$. Thus the net force is zero, but these forces form a couple for a nonzero perpendicular separation. The distance between their parallel lines of action is $a \sin \theta$.

Step 2: Calculate the torque and specify its direction The magnitude of the couple is therefore

$$\tau = (qE)(a \sin \theta) = pE \sin \theta.$$

Equivalently, adding the moments of the two forces about the dipole centre gives $2(qE)(a/2) \sin \theta$. The vector form is $\vec{\tau} = \vec{p} \times \vec{E}$. It is perpendicular to the plane containing $\vec{p}$ and $\vec{E}$, along the right-hand-rule direction of their cross product. The torque tends to rotate the dipole towards alignment with the field.

Step 3: Conclusion $\vec{\tau} = \vec{p} \times \vec{E}$, $\tau = pE \sin \theta$. Its direction is normal to the plane of the dipole moment and electric field, as given by the right-hand rule.

Quick Tip to Solve: Use the perpendicular distance between the two force lines; the net force vanishes even when the torque is nonzero.

10. Derive an expression for the electric field at a point on the axis of an electric dipole of dipole moment $\vec{p}$. Also write its expression when the distance $r \gg$ the length ' a ' of the dipole. [2 marks]

Solution:

Concept:

- The dipole moment points from $-q$ towards $+q$; for the printed dipole length a , its magnitude is $p = qa$.
- Use Coulomb's law and superposition. At an exterior axial point, the two electric fields have opposite directions and unequal magnitudes.

Step 1: Define the axial distances and individual fields Place the charges at $x = -a/2$ and $x = +a/2$, with the centre at the origin. Consider the point P at $x = r > a/2$, on the positive axial side. Its distances from $+q$ and $-q$ are $r - a/2$ and $r + a/2$. The positive-charge field points along $\vec{p}$; the negative-charge field points oppositely. Hence

$$E = \frac{q}{4\pi\epsilon_0} \left[\frac{1}{(r - a/2)^2} - \frac{1}{(r + a/2)^2} \right].$$

Step 2: Simplify and take the far-field limit Using the difference of the two squared distances in the numerator,

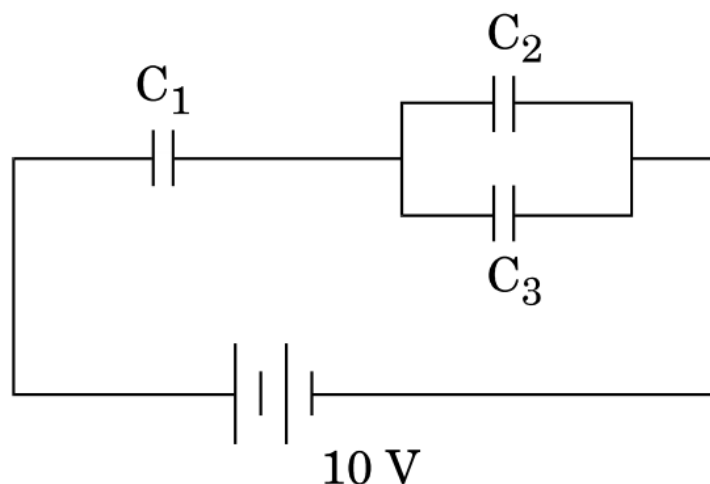
$$\begin{aligned} E &= \frac{q}{4\pi\epsilon_0} \frac{2ar}{(r^2 - a^2/4)^2} \\ &= \frac{1}{4\pi\epsilon_0} \frac{2pr}{(r^2 - a^2/4)^2}. \end{aligned}$$

The field points along $\vec{p}$ at either exterior axial end. If $r \gg a$, neglect $a^2/4$ compared with r^2 , so the denominator becomes r^4 . Here r is measured from the centre and a denotes the full dipole length, as in the question.

Step 3: Conclusion $\vec{E}_{\text{axial}} = \frac{1}{4\pi\epsilon_0} \frac{2r\vec{p}}{(r^2 - a^2/4)^2}$. For $r \gg a$, $\vec{E}_{\text{axial}} \simeq \frac{1}{4\pi\epsilon_0} \frac{2\vec{p}}{r^3}$, directed along the dipole moment.

Quick Tip to Solve: First state whether a denotes full separation or half separation; this avoids a factor-of-four error in the exact denominator.

11. The figure shows a network of three capacitors $C_1 = 2\ \mu\text{F}$; $C_2 = 6\ \mu\text{F}$ and $C_3 = 3\ \mu\text{F}$ connected across a battery of 10 V. If a charge of $6\ \mu\text{C}$ is acquired by the capacitor C_3 , calculate the charge acquired by C_1 . [2 marks]



Solution:

Concept:

- A capacitor obeys $Q = CV$. Capacitors connected in parallel have the same potential difference.
- The series capacitor C_1 supplies the combined parallel branch; conservation of charge at the isolated junction gives $Q_1 = Q_2 + Q_3$.

Step 1: Use the stated charge to find the parallel voltage The circuit shows C_2 and C_3 in parallel, with that combination in series with C_1 . Taking the explicitly supplied charge $Q_3 = 6\ \mu\text{C}$,

$$V_3 = \frac{Q_3}{C_3} = \frac{6\ \mu\text{C}}{3\ \mu\text{F}} = 2\text{ V}.$$

Thus $V_2 = V_3 = 2\text{ V}$, and $Q_2 = C_2 V_2 = 6\ \mu\text{F} \times 2\text{ V} = 12\ \mu\text{C}$.

Step 2: Apply junction charge conservation and check the printed data The charge on C_1 equals the sum of the charges on the parallel capacitors:

$$Q_1 = Q_2 + Q_3 = (12 + 6)\ \mu\text{C}.$$

However, the printed givens are internally inconsistent. This charge requires $V_1 = Q_1/C_1 = 9\text{ V}$, hence a total voltage $V_1 + V_3 = 11\text{ V}$, rather than the stated 10 V. If the battery voltage is enforced instead, $C_{23} = 9\ \mu\text{F}$ and $C_{\text{eq}} = 18/11\ \mu\text{F}$, giving $Q_1 = 180/11\ \mu\text{C}$ and $Q_3 = 60/11\ \mu\text{C}$. These cannot simultaneously satisfy the supplied $6\ \mu\text{C}$.

Step 3: Conclusion Using the explicitly given charge on C_3 , $Q_1 = 18\ \mu\text{C}$. The printed 10 V battery value is incompatible with this supplied charge and the stated capacitances.

Quick Tip to Solve: In a series-plus-parallel circuit, the series charge is the sum of the branch charges; check the total voltage if the data seem inconsistent.

12. The frequencies of two side bands in an amplitude modulated wave are 640 kHz and 660 kHz respectively. Find the frequencies of the carrier and the modulating signals. Also obtain the value of the bandwidth required in amplitude modulation. [2 marks]

Solution:

Concept:

- For sinusoidal amplitude modulation, the lower and upper sideband frequencies are $f_L = f_c - f_m$ and $f_U = f_c + f_m$, where f_c is the carrier frequency and f_m the modulating frequency.
- The bandwidth required extends from the lower sideband to the upper sideband: $B = f_U - f_L = 2f_m$.

Step 1: Find the carrier frequency from the midpoint The given frequencies are $f_L = 640$ kHz and $f_U = 660$ kHz. Adding their defining equations removes the modulating frequency:

$$\begin{aligned}f_U + f_L &= 2f_c, \\f_c &= \frac{660 + 640}{2} \text{ kHz} = 650 \text{ kHz}.\end{aligned}$$

The carrier lies midway between the two sidebands; it is not equal to either listed frequency.

Step 2: Find the modulating frequency and bandwidth Subtracting the sideband equations removes the carrier frequency:

$$f_m = \frac{f_U - f_L}{2} = \frac{660 - 640}{2} \text{ kHz} = 10 \text{ kHz}.$$

The required frequency range spans both sidebands, so

$$B = f_U - f_L = 660 - 640 = 20 \text{ kHz}.$$

As a consistency check, $650 - 10 = 640$ and $650 + 10 = 660$, in kilohertz. The sideband separation is twice the modulating frequency, which explains why the bandwidth is 20 kHz, rather than 10 kHz.

Step 3: Conclusion $f_c = 650 \text{ kHz}, \quad f_m = 10 \text{ kHz}, \quad B = 20 \text{ kHz}.$

Quick Tip to Solve: Take the mean for the carrier and half the difference for the signal; the whole sideband difference is the AM bandwidth.

OR

12. A sinusoidal carrier voltage is amplitude modulated by a sinusoidal voltage of 10 kHz with modulation index 0.3. If the carrier frequency is 10 MHz and its amplitude is 40 V, calculate the frequency and amplitude of the two sidebands. [2 marks]

Solution:

Concept:

- For single-tone amplitude modulation, $v(t) = A_c[1 + m \cos(2\pi f_m t)] \cos(2\pi f_c t)$, where m is the modulation index and A_c the carrier amplitude.

Quick Tip to Solve: Convert the modulating frequency to the carrier units before adding; each sideband amplitude is half of m times the carrier amplitude.

- The product-to-sum identity gives sidebands at $f_c - f_m$ and $f_c + f_m$, each having voltage amplitude $mA_c/2$.

Step 1: Calculate the two sideband frequencies Here $f_c = 10 \text{ MHz} = 10000 \text{ kHz}$ and $f_m = 10 \text{ kHz} = 0.010 \text{ MHz}$. Therefore

$$f_L = f_c - f_m = 10 - 0.010 = 9.990 \text{ MHz},$$

$$f_U = f_c + f_m = 10 + 0.010 = 10.010 \text{ MHz}.$$

Both frequencies are close to the carrier, one below it and the other above it by the same modulating frequency.

Step 2: Expand the modulated voltage to obtain amplitudes Expanding the voltage expression gives

$$\begin{aligned} v(t) &= A_c \cos(2\pi f_c t) \\ &+ \frac{mA_c}{2} \cos[2\pi(f_c + f_m)t] \\ &+ \frac{mA_c}{2} \cos[2\pi(f_c - f_m)t]. \end{aligned}$$

Thus both sidebands have equal voltage amplitudes. Substituting $m = 0.3$ and $A_c = 40 \text{ V}$,

$$A_L = A_U = \frac{0.3 \times 40}{2} = 6 \text{ V}.$$

The quantity $mA_c = 12 \text{ V}$ is the modulating envelope amplitude; the factor of one-half in the product-to-sum identity makes each individual sideband amplitude 6 V .

Step 3: Conclusion The lower sideband is 9.990 MHz , the upper sideband is 10.010 MHz , and the amplitude of each is 6 V .

Quick Tip to Solve: Convert the modulating frequency to the carrier units before adding; each sideband amplitude is half of m times the carrier amplitude.

Section C (36 marks)

13. A screen is placed 90 cm from an object. The image of the object on the screen is formed by a convex lens at two different positions separated by 20 cm . Calculate the focal length of the lens. [3 marks]

Solution:

Concept:

- A convex lens forms a real image on the screen. With positive distance magnitudes U and V , the lens formula is $1/f = 1/U + 1/V$, where f is the focal length.
- In the displacement method the object-screen distance is $D = U + V$. The two sharp-image lens positions interchange U and V , so their separation is $d = |V - U|$.

Step 1: Determine the object and image distances The object and screen stay fixed at a separation $D = 90 \text{ cm}$, while the lens can give a sharp image at two positions separated by $d = 20 \text{ cm}$. At one

Quick Tip to Solve: For two lens positions use $f = (D^2 - d^2)/(4D)$; here D is the full object-screen distance and d is the lens displacement.

position choose $V > U$. Then

$$U + V = 90 \text{ cm}, \quad V - U = 20 \text{ cm}.$$

Adding and subtracting these equations gives

$$V = 55 \text{ cm}, \quad U = 35 \text{ cm}.$$

At the other lens position these distances are interchanged, because the object-screen separation and the product UV remain the same. Thus both positions correspond to the same focal length.

Step 2: Apply the lens formula In the Cartesian convention $u = -U$ and $v = V$, so $1/f = 1/v - 1/u = 1/V + 1/U$. Consequently,

$$f = \frac{UV}{U + V} = \frac{35 \times 55}{90} \text{ cm} = 21.39 \text{ cm}.$$

Equivalently, substituting $U = (D - d)/2$ and $V = (D + d)/2$ gives the displacement formula

$$f = \frac{D^2 - d^2}{4D} = \frac{90^2 - 20^2}{4 \times 90} \text{ cm} = \frac{7700}{360} \text{ cm}.$$

The focal length is positive, as expected for the convex lens, and $D > 4f$, consistent with two distinct lens positions.

Step 3: Conclusion The focal length of the convex lens is $f \approx 21.4 \text{ cm}$.

Quick Tip to Solve: For two lens positions use $f = (D^2 - d^2)/(4D)$; here D is the full object-screen distance and d is the lens displacement.

OR

13. A convex lens of focal length 20 cm and a concave lens of focal length 15 cm are kept 30 cm apart with their principal axes coincident. When an object is placed 30 cm in front of the convex lens, calculate the position of the final image formed by the combination. Would this result change if the object were placed 30 cm in front of the concave lens? Give reason. [3 marks]

Solution:

Concept:

- For each thin lens, $1/v - 1/u = 1/f$. Measure distances from that lens, taking the incident-light direction as positive: a convex lens has positive f and a concave lens negative f .
- The image formed by the first lens acts as the object for the second. If the incoming rays converge towards a point beyond the second lens, its object is virtual and u is positive.

Step 1: Find the image due to the convex lens first Let light travel from the object through the convex lens to the concave lens. For the convex lens, $u_1 = -30 \text{ cm}$ and $f_1 = +20 \text{ cm}$. Therefore,

$$\frac{1}{v_1} = \frac{1}{f_1} + \frac{1}{u_1} = \frac{1}{20} - \frac{1}{30} = \frac{1}{60}, \quad v_1 = +60 \text{ cm}.$$

Quick Tip to Solve: Locate the first image on a common axis before using the second lens formula; a converging beam incident on the second lens gives a positive virtual-object distance.

Without the concave lens the rays would meet 60 cm beyond the convex lens. Since the concave lens is only 30 cm beyond it, the converging rays have a virtual object 30 cm beyond the concave lens: $u_2 = +30$ cm.

Step 2: Calculate the final image for this order For the concave lens $f_2 = -15$ cm. Applying the lens formula again,

$$\frac{1}{v_2} = \frac{1}{f_2} + \frac{1}{u_2} = -\frac{1}{15} + \frac{1}{30} = -\frac{1}{30}, \quad v_2 = -30 \text{ cm.}$$

The emerging rays diverge and their backward extensions meet 30 cm in front of the concave lens, towards the convex lens. This is a virtual final image, located in the plane of the convex lens.

Step 3: Reverse the order and compare Now place the object 30 cm in front of the concave lens, with light passing through the concave lens before the convex lens. For the first lens,

$$\frac{1}{v'_1} = -\frac{1}{15} - \frac{1}{30} = -\frac{1}{10}, \quad v'_1 = -10 \text{ cm.}$$

This virtual image lies 10 cm on the object side of the concave lens. The convex lens is 30 cm further along the light path, so its real object distance is $u'_2 = -(30 + 10) = -40$ cm. Hence,

$$\frac{1}{v'_2} = \frac{1}{20} - \frac{1}{40} = \frac{1}{40}, \quad v'_2 = +40 \text{ cm.}$$

The result changes because reversing separated lenses changes the intermediate image and hence the object distance seen by the second lens; their powers cannot simply be added as for lenses in contact.

Step 4: Conclusion With the convex lens first, the final image is virtual, 30 cm in front of the concave lens (at the convex-lens plane). With the concave lens first, it is real, 40 cm beyond the convex lens. Thus the result does change.

Quick Tip to Solve: Locate the first image on a common axis before using the second lens formula; a converging beam incident on the second lens gives a positive virtual-object distance.

14. (a) Good quality sunglasses made of polaroids are preferred over ordinary coloured glasses. Explain why.

(b) How is plane polarized light defined?

(c) A beam of plane polarised light is passed through a polaroid. Show graphically, variation of the intensity of the transmitted light with angle of rotation of the polaroid. [3 marks]

Solution:

Concept:

- Polarisation restricts the electric-field vibrations of light to one direction perpendicular to its propagation. A polaroid transmits the component parallel to its transmission axis.
- Malus's law is $I = I_0 \cos^2 \theta$, where I_0 is the incident plane-polarised intensity and θ is the angle between its electric-field direction and the polaroid's transmission axis.

Step 1: Explain why polaroid sunglasses reduce glare (a) Light reflected from roads, water and other

Quick Tip to Solve: Mark maxima at parallel axes and zeros at crossed axes, then join them with a smooth $\cos^2 \theta$ curve of period 180° .

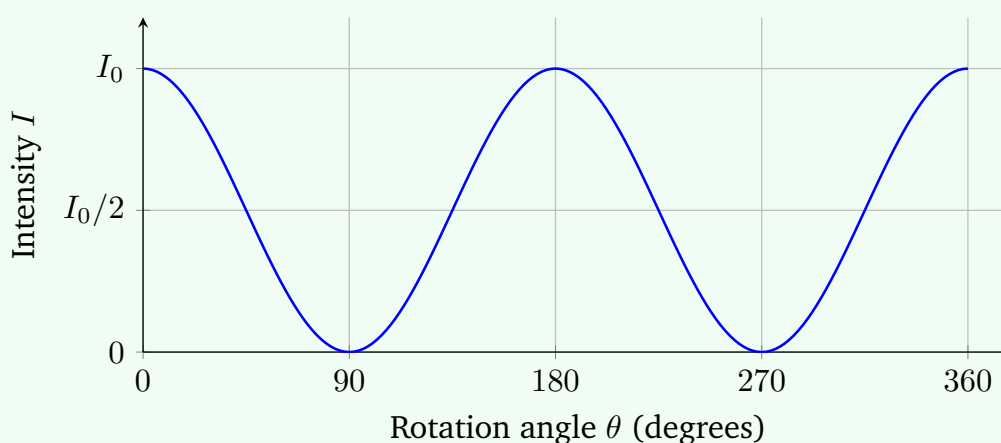
horizontal surfaces is often strongly plane polarised, predominantly with horizontal electric-field vibrations. Good polaroid sunglasses have a suitable, normally vertical, transmission axis and absorb much of this horizontally polarised glare. They therefore improve visibility and contrast as well as reducing intensity. Ordinary coloured glasses absorb light according to colour and reduce overall brightness, but do not selectively suppress the polarised component responsible for glare.

Step 2: Define plane-polarised light (b) Light is plane polarised when its electric-field vibrations are confined to a single fixed direction perpendicular to the direction of propagation, instead of occurring in all transverse directions as in unpolarised light. The propagation direction and that fixed electric-field direction define one plane.

Step 3: Plot the transmitted intensity during rotation (c) Take $\theta = 0^\circ$ when the polaroid axis is parallel to the incident electric-field direction. The transmitted field amplitude is $E = E_0 \cos \theta$. As intensity is proportional to the square of the amplitude,

$$I = I_0 \cos^2 \theta.$$

Thus the intensity is maximum I_0 at $0^\circ, 180^\circ, 360^\circ$ and zero at $90^\circ, 270^\circ$. At 45° it is $I_0/2$. The smooth non-negative curve repeats every 180° ; no factor of one-half is inserted because the incident beam is already plane polarised.



Transmitted intensity as a polaroid rotates relative to the incident polarisation direction.

Step 4: Conclusion Polaroid sunglasses selectively reduce polarised glare. Plane-polarised light has one fixed transverse electric-field direction, and the transmitted intensity follows the plotted law

$$I = I_0 \cos^2 \theta.$$

Quick Tip to Solve: Mark maxima at parallel axes and zeros at crossed axes, then join them with a smooth $\cos^2 \theta$ curve of period 180° .

15. State the principle of a moving coil galvanometer. Explain its working and obtain the expression for the deflection produced due to the current passed through the coil. Define current sensitivity. [3 marks]

Solution:

Concept:

- A current-carrying coil in a magnetic field experiences a torque. For N turns, current I , area A and field B , its magnitude is $\tau = NIAB \sin \alpha$, where α is the angle between the coil's normal and the field.
- In a radial magnetic field, the coil's normal remains perpendicular to the field, so the magnetic torque is $NIAB$ at every deflection.
- The suspension produces a restoring torque $k\theta$, where k is the torsional constant and θ is the angular deflection in radians.

Step 1: State the principle and explain the working The principle of a moving coil galvanometer is that a current-carrying coil placed in a magnetic field experiences a torque. A light rectangular coil of many turns is suspended between concave pole pieces of a strong permanent magnet. A soft iron cylindrical core makes the field strong and radial. When current passes through the coil, magnetic torque rotates it and a pointer or mirror indicates the deflection. As it rotates, the suspension twists and produces an opposing restoring torque. The coil settles when the two torques balance. Reversing the current reverses the direction of deflection.

Step 2: Obtain the deflection and define current sensitivity In the radial field, $\alpha = 90^\circ$. At equilibrium,

$$\begin{aligned}\tau_{\text{magnetic}} &= NIAB, \\ \tau_{\text{restoring}} &= k\theta, \\ NIAB &= k\theta, \\ \theta &= \frac{NAB}{k}I.\end{aligned}$$

Thus the deflection is directly proportional to the current, which gives a uniform scale. Current sensitivity is the angular deflection produced per unit current through the coil:

$$S_I = \frac{\theta}{I} = \frac{NAB}{k}.$$

Its SI unit is rad A^{-1} . More turns, a stronger field or a larger coil area increase sensitivity; a smaller torsional constant also increases it.

Step 3: Conclusion The radial-field magnetic torque balances the restoring torque, giving $\theta = (NAB/k)I$. The current sensitivity is $S_I = \theta/I = NAB/k$.

Quick Tip to Solve: Use the radial-field condition before balancing torques: it makes the magnetic torque independent of the coil's deflection.

15. Explain how a galvanometer can be converted into an ammeter of a given range. Derive an expression for shunt resistance and current for full scale deflection. Find the effective resistance of the ammeter. [3 marks]

Solution:

Concept:

- A galvanometer can carry only a small current. A low shunt resistance S connected in parallel diverts the excess current and converts it into an ammeter.
- Parallel branches have the same potential difference, so $I_g G = I_s S$, where G is the galvanometer resistance, I_g its full scale current and I_s the shunt current.
- The total current is the sum of the branch currents, and the effective resistance is the parallel combination of G and S .

Step 1: Connect a shunt to obtain the required range Let the desired ammeter range be 0 to I , with $I > I_g$. Connect a suitable low resistance S in parallel with the galvanometer. At the maximum reading, the galvanometer carries only its safe full scale current I_g , while the shunt carries $I - I_g$. The combination is connected in series with the circuit whose current is measured. Its low effective resistance limits the change in the measured current.

Step 2: Derive the shunt and full scale current Equating the voltage across the two parallel branches gives

$$I_g G = (I - I_g) S,$$

$$S = \frac{I_g G}{I - I_g}.$$

Conversely, for a chosen shunt, the total current giving full scale deflection is

$$I = I_g + \frac{I_g G}{S} = I_g \left(1 + \frac{G}{S} \right).$$

The galvanometer's own full scale current is $I_g = k\theta_{fs}/(NAB)$, where θ_{fs} is its full scale angular deflection.

Step 3: Find the effective resistance The ammeter resistance is

$$R_A = \frac{GS}{G + S}.$$

Using the derived shunt value, or dividing the common branch voltage $I_g G$ by the total current I , gives

$$R_A = \frac{I_g G}{I}.$$

This low resistance is suitable for a series ammeter.

Step 4: Conclusion Connect a shunt in parallel: $S = I_g G/(I - I_g)$. The total full scale current is $I = I_g(1 + G/S)$, and the effective ammeter resistance is $R_A = GS/(G + S) = I_g G/I$.

Quick Tip to Solve: Label the shunt current as total current minus galvanometer current before equating the two branch voltages.

16. Draw the phasor diagram of a series LCR connected across an ac source $V = V_0 \sin \omega t$. Hence, derive the expression for the impedance of the circuit. Obtain the conditions for the phase angle under which the current is (i) maximum, and (ii) minimum. [3 marks]

Solution:

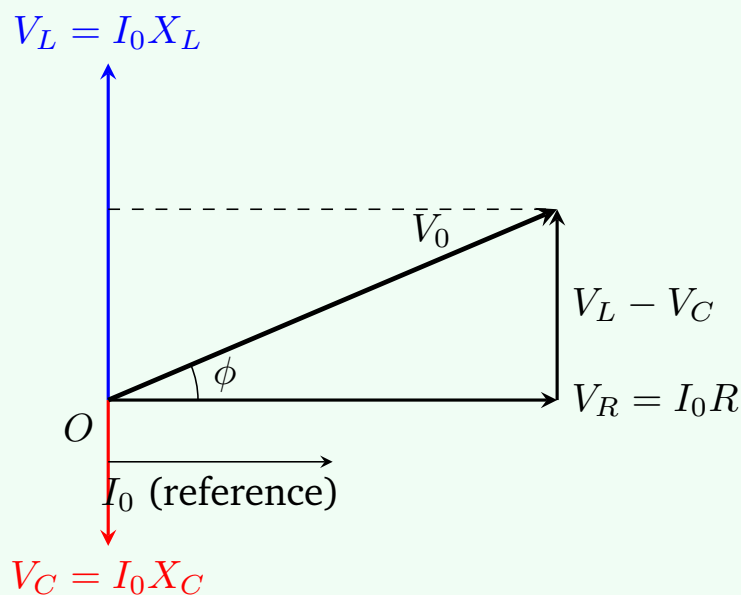
Concept:

- In a series LCR circuit the same alternating current flows through all three elements. Take its phasor as the reference direction.
- The voltage across R is in phase with current, that across L leads current by 90° , and that across C lags current by 90° .
- The reactances are $X_L = \omega L$ and $X_C = 1/(\omega C)$. The impedance is the ratio of the source voltage amplitude to the current amplitude.

Step 1: Construct the voltage phasors Let I_0 be the current amplitude and use amplitude phasors throughout. Draw $\vec{V}_R$ along the current, $\vec{V}_L$ vertically upwards and $\vec{V}_C$ vertically downwards. Their magnitudes are

$$V_R = I_0 R, \quad V_L = I_0 X_L, \quad V_C = I_0 X_C.$$

The source phasor is the vector sum $\vec{V}_0 = \vec{V}_R + \vec{V}_L + \vec{V}_C$. The diagram shows the inductive case $V_L > V_C$; the resultant's vertical component is $V_L - V_C$. For the capacitive case this component points downwards and the source voltage lags the current.



Voltage phasors for the inductive case, taking current as reference

Step 2: Derive impedance and the phase angle Since the resistive and net reactive components are perpendicular,

$$\begin{aligned} V_0^2 &= V_R^2 + (V_L - V_C)^2 \\ &= I_0^2 [R^2 + (X_L - X_C)^2]. \end{aligned}$$

Quick Tip to Solve: Add the inductor and capacitor voltages algebraically on the vertical axis, then use Pythagoras with the resistor voltage.

Hence

$$Z = \frac{V_0}{I_0} = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}.$$

If ϕ is the phase by which source voltage leads current,

$$\tan \phi = \frac{X_L - X_C}{R}, \quad \cos \phi = \frac{R}{Z}.$$

Thus, for the stated source $V = V_0 \sin \omega t$, the current is $i = I_0 \sin(\omega t - \phi)$, with $I_0 = V_0/Z$.

Step 3: Find the phase conditions for maximum and minimum current For fixed source amplitude V_0 and resistance $R > 0$, the current amplitude is $I_0 = (V_0/R) \cos \phi$. It is maximum when $\phi = 0$, so $X_L = X_C$. This is resonance: $\omega = 1/\sqrt{LC}$, $Z = R$, and $I_{0,\max} = V_0/R$. Voltage and current are then in phase.

The minimum-current phase condition is $|\phi| \rightarrow 90^\circ$, for which $\cos \phi \rightarrow 0$ and $I_0 \rightarrow 0$. This is a limiting condition for an LCR circuit with nonzero resistance: it occurs as $|X_L - X_C|/R \rightarrow \infty$, at very low or very high frequency. For finite nonzero frequency and finite component values, the current is positive and $|\phi| < 90^\circ$; zero current is approached as a limit.

Step 4: Conclusion The impedance is $Z = \sqrt{R^2 + (\omega L - 1/\omega C)^2}$. Maximum current occurs at $\phi = 0$ (resonance); minimum current approaches zero in the limiting phase conditions $\phi \rightarrow \pm 90^\circ$.

Quick Tip to Solve: Add the inductor and capacitor voltages algebraically on the vertical axis, then use Pythagoras with the resistor voltage.

17. Answer the following questions:

- (a) In a single slit diffraction experiment, the width of the slit is made double the original width. How does this affect the size and intensity of the central diffraction band?
- (b) How is the width of the central maximum changed when red light is replaced by blue?
- (c) In what way is diffraction from each slit related to the interference pattern in a double slit experiment?

[3 marks]

Solution:

Concept:

- For slit width a , the first diffraction minima satisfy $a \sin \theta = \pm \lambda$. At small angles, the central maximum has angular width $2\lambda/a$ and screen width $W = 2D\lambda/a$, where D is the screen distance.
- With unchanged incident illumination, all elementary wavelets across the slit arrive in phase at the centre. Central amplitude is proportional to a , so central peak intensity is proportional to a^2 .
- In a double slit experiment, interference fringes are modulated by the single slit diffraction envelope produced by the finite width of each slit.

Step 1: (a) Double the slit width Keeping wavelength and screen distance fixed, replace a by $2a$. The new central width is

$$W' = \frac{2D\lambda}{2a} = \frac{W}{2}.$$

Quick Tip to Solve: Keep width and intensity laws separate: central width varies as λ/a , while central peak intensity varies as a^2 for fixed incident illumination.

The central amplitude doubles because the illuminated slit width doubles. Since intensity is proportional to the square of amplitude, $I'_0 = (2)^2 I_0 = 4I_0$. Thus the band becomes half as wide and its peak intensity becomes four times the original value. This comparison assumes the same incident intensity, not a fixed total power passing through the slit.

Step 2: (b) Replace red light by blue light The wavelength of blue light is smaller than that of red light. With the apparatus unchanged,

$$\frac{W_{\text{blue}}}{W_{\text{red}}} = \frac{\lambda_{\text{blue}}}{\lambda_{\text{red}}} < 1.$$

Hence the central maximum becomes narrower; its width decreases in the same ratio as the wavelength.

Step 3: (c) Relate diffraction and interference Each finite slit diffracts the light entering it. The two diffracted waves overlap and interfere, producing closely spaced bright and dark fringes. Their brightness is governed by the broad single slit diffraction envelope: fringes are strongest near the central direction and become weaker towards the diffraction minima. At a diffraction minimum, even an otherwise bright interference fringe is absent. Thus the intensity of the interference maxima is not uniform when slit width is taken into account.

Step 4: Conclusion (a) The central band has half its original width and four times its original peak intensity. (b) Blue light produces a narrower central maximum. (c) Single slit diffraction provides the intensity envelope for the double slit interference fringes.

Quick Tip to Solve: Keep width and intensity laws separate: central width varies as λ/a , while central peak intensity varies as a^2 for fixed incident illumination.

18. A charge Q is distributed over the surfaces of two concentric hollow spheres of radii r and R ($R \gg r$), such that their surface charge densities are equal. Derive the expression for the potential at the common centre. [3 marks]

Solution:

Concept:

- A uniformly charged spherical shell of radius s and charge q produces constant potential kq/s everywhere inside it, taking zero potential at infinity; $k = 1/(4\pi\epsilon_0)$.
- Surface charge density is charge divided by surface area, $\sigma = q/(4\pi s^2)$. Equal densities therefore imply charges proportional to the squares of the radii.
- Electric potential is a scalar, so contributions from the two shells add algebraically at their common centre.

Step 1: Find the charges on the two shells Let q_r and q_R be the charges on the inner and outer shells, respectively. The total charge is $q_r + q_R = Q$. Since the surface charge densities are equal,

$$\frac{q_r}{4\pi r^2} = \frac{q_R}{4\pi R^2} = \sigma.$$

Quick Tip to Solve: First divide the total charge in the ratio $r^2 : R^2$, then add kq_r/r and kq_R/R ; a shell has constant, nonzero potential inside.

Therefore $Q = 4\pi\sigma(r^2 + R^2)$, giving

$$q_r = \frac{Qr^2}{r^2 + R^2}, \quad q_R = \frac{QR^2}{r^2 + R^2}.$$

In particular, the total charge must not be divided equally: the larger shell has a greater area and carries more charge at the same density.

Step 2: Add the potentials at the common centre The centre lies inside both shells. Each shell's potential there equals its own surface potential even though its electric field inside is zero. Superposition gives

$$\begin{aligned} V_O &= k \left(\frac{q_r}{r} + \frac{q_R}{R} \right) \\ &= \frac{Q}{4\pi\epsilon_0(r^2 + R^2)}(r + R). \end{aligned}$$

The potential is not zero merely because the field at the centre is zero: a zero field makes the potential spatially constant, while its value is fixed relative to infinity.

Step 3: Use the stated large-radius limit For $R \gg r$, divide numerator and denominator by suitable powers of R :

$$V_O = \frac{Q}{4\pi\epsilon_0 R} \frac{1 + r/R}{1 + (r/R)^2}.$$

Since $r/R \ll 1$, the leading approximation is $V_O \simeq Q/(4\pi\epsilon_0 R)$. The exact expression above retains the small contribution of the inner shell and applies for any two positive radii.

Step 4: Conclusion The potential at the common centre, relative to infinity, is $V_O = \frac{Q(r + R)}{4\pi\epsilon_0(r^2 + R^2)}$.

For $R \gg r$, $V_O \simeq \frac{Q}{4\pi\epsilon_0 R}$.

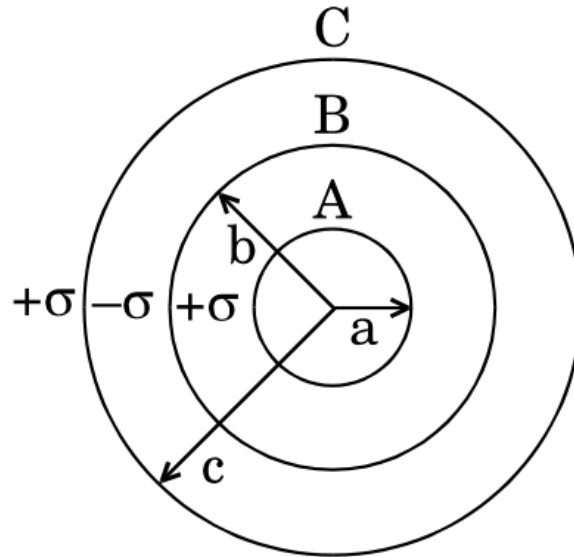
Quick Tip to Solve: First divide the total charge in the ratio $r^2 : R^2$, then add kq_r/r and kq_R/R ; a shell has constant, nonzero potential inside.

18. Three concentric metallic shells A, B and C of radii a , b and c ($a < b < c$) have surface charge densities $+\sigma$, $-\sigma$ and $+\sigma$ respectively.

(a) Obtain the expressions for the potential of the three shells A, B and C.

(b) If shells A and C are at the same potential, obtain the relation between a , b and c .

[3 marks]



Solution:

Concept:

- A spherical shell of radius s carrying charge q produces potential kq/s at or inside it and kq/x at an exterior distance x , where $k = 1/(4\pi\epsilon_0)$ and the potential at infinity is zero.
- The specified shell charges are $Q_A = 4\pi a^2\sigma$, $Q_B = -4\pi b^2\sigma$, and $Q_C = 4\pi c^2\sigma$. Add all three contributions algebraically to obtain each shell's potential.

Step 1: (a) Potential of the innermost shell A A point on A is on the surface of shell A and inside shells B and C. The distances used for the potentials of B and C are therefore their own radii, not a . Hence

$$\begin{aligned} V_A &= k \left(\frac{Q_A}{a} + \frac{Q_B}{b} + \frac{Q_C}{c} \right) \\ &= \frac{\sigma}{\epsilon_0} (a - b + c). \end{aligned}$$

The negative sign of the middle shell's charge is essential in the sum.

Step 2: (a) Potential of the middle shell B At radius b , the point lies outside A, on B, and inside C. Thus A behaves as a point charge at the common centre, while C still contributes its constant interior potential:

$$\begin{aligned} V_B &= k \left(\frac{Q_A}{b} + \frac{Q_B}{b} + \frac{Q_C}{c} \right) \\ &= \frac{\sigma}{\epsilon_0} \left(\frac{a^2}{b} - b + c \right). \end{aligned}$$

Quick Tip to Solve: For each contribution use $kq/\max(s, x)$, where s is that shell's radius and x is the observation radius; retain the negative charge of B.

Step 3: (a) Potential of the outermost shell C At radius c , both inner shells are viewed from outside and C is at its own surface. Every contribution now has denominator c :

$$V_C = k \frac{Q_A + Q_B + Q_C}{c}$$

$$= \frac{\sigma}{\epsilon_0 c} (a^2 - b^2 + c^2).$$

These expressions include the influence of all shells rather than just the self-potential of each one.

Step 4: (b) Equate the potentials of A and C For the given nonzero charge density, setting $V_A = V_C$ and cancelling σ/ϵ_0 gives

$$a - b + c = \frac{a^2 - b^2 + c^2}{c},$$

$$c(a - b) = a^2 - b^2 = (a - b)(a + b).$$

Since the printed condition $a < b$ ensures $a - b \neq 0$, division by this factor is permitted. It yields the required relation among the radii.

Step 5: Conclusion $V_A = \frac{\sigma}{\epsilon_0} (a - b + c)$, $V_B = \frac{\sigma}{\epsilon_0} \left(\frac{a^2}{b} - b + c \right)$, and $V_C = \frac{\sigma}{\epsilon_0 c} (a^2 - b^2 + c^2)$. Equal potentials of A and C require $\boxed{c = a + b}$.

Quick Tip to Solve: For each contribution use $kq/\max(s, x)$, where s is that shell's radius and x is the observation radius; retain the negative charge of B.

19. Write Einstein's photoelectric equation using the quantum picture of radiation. Explain briefly the three characteristic features observed in photoelectric effect which can be explained by Einstein's equation and not by the wave theory of light. [3 marks]

Solution:

Concept:

- Radiation consists of photons, each of energy $h\nu$, where h is Planck's constant and ν is the light frequency. In the usual photoelectric effect, one electron absorbs the energy of one photon.
- The work function ϕ_0 is the minimum energy needed to remove an electron from the metal. Energy conservation gives $h\nu = \phi_0 + K_{\max}$, with $K_{\max} = \frac{1}{2}mv_{\max}^2 = eV_s$, where V_s is the stopping potential magnitude.

Step 1: State Einstein's equation in the quantum picture A photon transfers its energy to an electron in a single absorption event. A minimum amount ϕ_0 is used to overcome the attraction binding an electron in the metal; the remainder is its maximum possible kinetic energy. Hence

$$h\nu = \phi_0 + \frac{1}{2}mv_{\max}^2, \quad K_{\max} = h\nu - \phi_0 = h(\nu - \nu_0),$$

where the threshold frequency is $\nu_0 = \phi_0/h$. Electrons that lose energy on their way to the surface emerge with less kinetic energy, which is why the equation specifies the maximum value.

Step 2: Explain the threshold frequency If $\nu < \nu_0$, a single photon's energy $h\nu$ is less than ϕ_0 , so it

Quick Tip to Solve: Frequency changes the energy of each photon; intensity changes how many photons arrive. Use this distinction to explain threshold, kinetic energy and photocurrent.

cannot eject an electron, regardless of intensity. Increasing intensity supplies more photons without increasing each photon's energy. Classical wave theory instead allows continuous accumulation of energy and would predict emission after sufficient exposure even at a low frequency; it does not account for the observed threshold.

Step 3: Explain maximum kinetic energy and intensity For a fixed metal, $K_{\max} = h\nu - \phi_0$ increases linearly with frequency, with slope h , and is independent of intensity. Above threshold, increasing intensity increases the number of photons incident per second and hence the number of emitted electrons and saturation photocurrent, but not their maximum kinetic energy. Classical wave theory would associate a larger wave intensity with a larger electron energy, contrary to this observation.

Step 4: Explain the absence of a measurable time lag For light above threshold, an electron receives sufficient energy in one photon absorption. Emission can therefore begin essentially immediately even at very low intensity; the electron need not accumulate energy over many wave cycles. The instantaneous nature follows from the quantum energy-transfer picture underlying Einstein's equation. Classical wave theory predicts a finite energy-accumulation delay at low intensity, which is not observed.

Step 5: Conclusion Einstein's photoelectric equation is $K_{\max} = h\nu - \phi_0 = eV_s$. The photon picture explains a definite threshold frequency, a maximum kinetic energy that depends on frequency but not intensity, and emission without a measurable time lag.

Quick Tip to Solve: Frequency changes the energy of each photon; intensity changes how many photons arrive. Use this distinction to explain threshold, kinetic energy and photocurrent.

20. (a) Distinguish between isotopes and isobars, giving one example for each.

(b) Why is the mass of a nucleus always less than the sum of the masses of its constituents? Write one example to justify your answer. [3 marks]

Solution:

Concept:

- In the notation ${}_Z^AX$, Z is the proton number and A is the total number of protons and neutrons; the neutron number is $N = A - Z$.
- The binding energy B of a bound nucleus is related to its mass defect by $B = \Delta mc^2$, where c is the speed of light.

Step 1: (a) Distinguish isotopes and isobars Isotopes are nuclei of the same element with the same atomic number Z but different mass numbers A . They contain equal numbers of protons but different numbers of neutrons. For example, ${}_6^{12}\text{C}$ and ${}_6^{14}\text{C}$ both have six protons, but have six and eight neutrons respectively.

Isobars are nuclei of different elements with the same mass number A but different atomic numbers Z . For example, ${}_1^3\text{H}$ and ${}_2^3\text{He}$ both contain three nucleons, but their proton numbers are one and two respectively. Thus equal mass number means equal total nucleon count, not equal proton count or exactly equal nuclear mass.

Step 2: (b) Explain the nuclear mass defect When free protons and neutrons combine to form a

Quick Tip to Solve: Compare Z for isotopes and A for isobars; use nuclear masses consistently when calculating a mass defect.

bound nucleus, energy is released. The bound system therefore has less total rest energy than its separated constituents. By mass-energy equivalence, this decrease in energy produces a decrease in mass:

$$\Delta m = Zm_p + (A - Z)m_n - M_{\text{nucleus}}, \quad B = \Delta mc^2 > 0.$$

Hence $M_{\text{nucleus}} < Zm_p + (A - Z)m_n$. This missing mass is the mass defect; it represents the energy required to separate the nucleus completely into free nucleons.

For example, a deuteron contains one proton and one neutron. Its mass is approximately 2.01355 u, whereas $m_p + m_n \approx 1.00728 + 1.00866 = 2.01594$ u. The difference is about 0.00239 u, corresponding to a binding energy of approximately 2.22 MeV. Here nuclear masses, rather than neutral atomic masses, are compared consistently.

Step 3: Conclusion Isotopes have equal Z and different A ; isobars have equal A and different Z . A bound nucleus has a positive mass defect because binding releases energy, as illustrated by the deuteron.

Quick Tip to Solve: Compare Z for isotopes and A for isobars; use nuclear masses consistently when calculating a mass defect.

OR

20. (a) Classify the following six nuclides into (i) isotones, (ii) isotopes, and (iii) isobars: ${}^{12}_6\text{C}$, ${}^3_2\text{He}$, ${}^{198}_{80}\text{Hg}$, ${}^3_1\text{H}$, ${}^{197}_{79}\text{Au}$, ${}^{14}_6\text{C}$.

(b) How does the size of a nucleus depend on its mass number? Hence explain why the density of nuclear matter should be independent of the size of the nucleus. [3 marks]

Solution:

Concept:

- The neutron number of a nuclide A_ZX is $N = A - Z$. Isotones have equal N , isotopes have equal Z , and isobars have equal A .
- Nuclear radius obeys $R = R_0 A^{1/3}$, where $R_0 \approx 1.2 \times 10^{-15}$ m; nuclear mass is approximately Am_N , with m_N the mean nucleon mass.

Step 1: (a) Count protons and neutrons Calculate the neutron numbers before grouping the nuclei:

Nuclide	A	Z	$N = A - Z$
${}^{12}_6\text{C}$	12	6	6
${}^3_2\text{He}$	3	2	1
${}^{198}_{80}\text{Hg}$	198	80	118
${}^3_1\text{H}$	3	1	2
${}^{197}_{79}\text{Au}$	197	79	118
${}^{14}_6\text{C}$	14	6	8

(i) The isotones are ${}^{198}_{80}\text{Hg}$ and ${}^{197}_{79}\text{Au}$: both contain 118 neutrons, although their atomic numbers differ.

Quick Tip to Solve: Make an A, Z, N table first; for density substitute R cubed so that the factor A cancels visibly.

- (ii) The isotopes are ${}^{12}_6\text{C}$ and ${}^{14}_6\text{C}$: both have atomic number six, but different mass numbers.
 (iii) The isobars are ${}^3_2\text{He}$ and ${}^3_1\text{H}$: both have mass number three, but different atomic numbers.

Step 2: (b) Derive the dependence of nuclear density on size The nuclear radius increases as the cube root of the mass number:

$$R = R_0 A^{1/3}, \quad R^3 = R_0^3 A.$$

Thus the volume of a spherical nucleus is proportional to A , while its mass is also approximately proportional to A . Its mean density is

$$\begin{aligned} \rho &= \frac{M}{V} = \frac{Am_N}{(4\pi/3)R^3} \\ &= \frac{Am_N}{(4\pi/3)R_0^3 A} = \frac{3m_N}{4\pi R_0^3}. \end{aligned}$$

The factor A cancels. Consequently, the mean density of nuclear matter is approximately the same for all nuclei, independent of their radius or mass number. Small binding-energy differences do not change this standard approximate result.

Step 3: Conclusion The required pairs are Hg-198/Au-197 (isotones), C-12/C-14 (isotopes), and He-3/H-3 (isobars). Nuclear radius varies as $A^{1/3}$, giving $\rho = 3m_N/(4\pi R_0^3)$, independent of nuclear size.

Quick Tip to Solve: Make an A, Z, N table first; for density substitute R cubed so that the factor A cancels visibly.

21. (a) Differentiate between three segments of an n-p-n transistor on the basis of their size and level of doping.

(b) Draw a plot of transfer characteristic and show which portion of the characteristic is used in amplification and why. [3 marks]

Solution:

Concept:

- An n-p-n transistor contains an n-type emitter, a p-type base, and an n-type collector. Their dimensions and doping serve different functions: injection, control, and collection of charge carriers.
- For a common-emitter circuit, the voltage transfer characteristic plots output voltage $V_o = V_{CE}$ against input voltage $V_i = V_{BE}$, with the supply and collector resistance fixed.

Step 1: (a) Compare the three transistor segments The emitter is moderately sized and heavily doped, so that it supplies a large number of majority carriers (electrons in an n-p-n transistor).

The base is very thin and lightly doped. Its small width and low hole concentration reduce recombination, allowing most injected electrons to reach the collector.

The collector is the largest segment and is moderately doped, less heavily than the emitter. Its larger size allows it to collect carriers and dissipate the heat generated during operation. Thus the base is much thinner than either outer region; the collector is larger than the emitter.

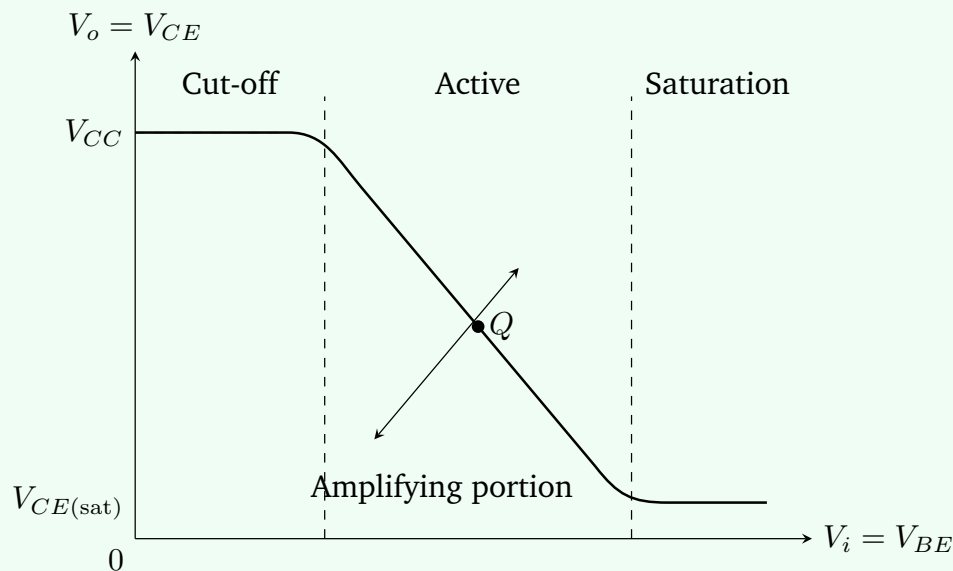
Quick Tip to Solve: On the voltage transfer plot, mark Q on the descending straight portion; its negative slope explains the inverted output.

Step 2: (b) Explain the voltage transfer characteristic For a common-emitter circuit with collector resistance R_C , the output is $V_o = V_{CC} - I_C R_C$. At low input voltage the transistor is in cut-off: the collector current is negligible and the output remains close to V_{CC} . As the input rises beyond the turn-on region, the collector current increases and the output falls. Eventually the transistor saturates and the output remains close to the small saturation voltage $V_{CE(sat)}$.

The middle, approximately straight descending portion is the active region used for amplification. Here the emitter-base junction is forward biased and the collector-base junction is reverse biased. Choose the quiescent operating point Q near the middle of this portion, so that both positive and negative signal variations remain in the active region. A small input change produces a larger output-voltage change:

$$A_v = \frac{\Delta V_o}{\Delta V_i} < 0.$$

The negative slope indicates phase reversal. The nearly constant slope preserves the waveform for a sufficiently small input signal. Cut-off and saturation would flatten parts of the output signal and cause clipping, so these regions are unsuitable for faithful linear amplification.



Common-emitter voltage transfer characteristic (schematic). The active portion around Q is used for amplification.

Step 3: Conclusion The emitter is heavily doped, the base is very thin and lightly doped, and the collector is largest and moderately doped. Amplification uses the approximately linear active portion of the transfer curve, with the operating point near its middle.

Quick Tip to Solve: On the voltage transfer plot, mark Q on the descending straight portion; its negative slope explains the inverted output.

22. Define displacement current. What role does it play while charging a capacitor by dc source. Is the value of displacement current same as that of the conduction current? Explain. [3 marks]

Solution:

Concept:

- Displacement current is the current equivalent associated with a changing electric flux: $I_d = \epsilon_0 d\Phi_E/dt$, where Φ_E is electric flux through a surface in vacuum.
- Conduction current is due to actual motion of charges through a conductor, with $I_c = dQ/dt$. No conduction charge crosses the insulating gap of an ideal capacitor.
- Maxwell's correction to Ampere's law is $\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0(I_c + I_d)$; it makes the magnetic field consistent for different surfaces bounded by the same loop.

Step 1: Describe the current during charging When a capacitor is connected to a dc source, electrons move through the connecting wires and equal, opposite charges build up on its plates. The plate charge and the electric field between the plates therefore change with time during charging. Although the dielectric prevents conduction current across the gap, the changing field produces displacement current there. It supplies the missing contribution in Ampere's law: a surface cutting the wire encloses conduction current, whereas a surface passing between the plates encloses displacement current, and both give the same magnetic field around the circuit.

Step 2: Compare its value with the conduction current For a parallel-plate capacitor of plate area A , neglect fringing and take the gap as vacuum or air. If the charge magnitude on a plate is Q , the uniform field is $E = Q/(\epsilon_0 A)$. Hence the flux through the gap is

$$\Phi_E = EA = \frac{Q}{\epsilon_0}.$$

Differentiating gives

$$I_d = \epsilon_0 \frac{d\Phi_E}{dt} = \epsilon_0 \frac{d}{dt} \left(\frac{Q}{\epsilon_0} \right) = \frac{dQ}{dt} = I_c.$$

Thus their instantaneous magnitudes are equal during charging, even though the physical mechanisms differ. In a dielectric the corresponding expression is $I_d = d \int \mathbf{D} \cdot d\mathbf{A}/dt$, giving the same equality. Once the capacitor is fully charged by a steady dc source, Q and E become constant, so both currents vanish.

Step 3: Conclusion Displacement current is the contribution of changing electric flux to the magnetic field. It maintains current continuity across the capacitor gap while charging, and its instantaneous value equals the conduction current in the wires: $I_d = I_c$.

Quick Tip to Solve: Start with $EA = Q/\epsilon_0$ and differentiate: this immediately proves the equality of the two currents without assuming a constant charging current.

23. Hydrogen atom in third excited state de-excites to the first excited state. Obtain the expressions for the frequency of radiation emitted in this process.

Also determine the ratio of the wavelengths of the emitted radiations when the atom de-excites from the third excited state to the second excited state and from the third excited state to the first excited state. [3 marks]

Solution:

Concept:

- The hydrogen energy levels are $E_n = -13.6/n^2$ eV, where $n = 1$ is the ground state. The first, second and third excited states have $n = 2, 3, 4$, respectively.
- For emission from n_i to $n_f < n_i$, the photon energy is $h\nu = E_{n_i} - E_{n_f}$, with Planck constant h . Its wavelength is $\lambda = c/\nu$.
- The Rydberg relation is $1/\lambda = R_H(1/n_f^2 - 1/n_i^2)$, where R_H is the hydrogen Rydberg constant and c is the speed of light.

Step 1: Identify the levels and calculate the emitted frequency Third excited state means $n_i = 4$, not $n_i = 3$; first excited state means $n_f = 2$. For the specified direct transition $4 \rightarrow 2$, the decrease in atomic energy becomes the energy of the emitted photon:

$$\begin{aligned}h\nu_{42} &= E_4 - E_2 \\&= 13.6 \left(\frac{1}{2^2} - \frac{1}{4^2} \right) \text{ eV} \\&= 13.6 \left(\frac{3}{16} \right) \text{ eV} = 2.55 \text{ eV}.\end{aligned}$$

Therefore the required equivalent expressions for the frequency are

$$\nu_{42} = cR_H \left(\frac{1}{4} - \frac{1}{16} \right) = \frac{3cR_H}{16} = \frac{2.55(1.602 \times 10^{-19})}{6.626 \times 10^{-34}} \text{ Hz}.$$

This is approximately 6.17×10^{14} Hz. The positive energy difference is required for emission; the atomic energy levels themselves are negative.

Step 2: Form the ratio of the two wavelengths The second excited state has $n = 3$. For the direct transition $4 \rightarrow 3$, and for $4 \rightarrow 2$, respectively,

$$\begin{aligned}\frac{1}{\lambda_{43}} &= R_H \left(\frac{1}{9} - \frac{1}{16} \right) = \frac{7R_H}{144}, \\ \frac{1}{\lambda_{42}} &= R_H \left(\frac{1}{4} - \frac{1}{16} \right) = \frac{3R_H}{16}.\end{aligned}$$

Wavelength is inversely proportional to transition energy. Thus, taking the wavelengths in the order asked,

$$\frac{\lambda_{43}}{\lambda_{42}} = \frac{144/(7R_H)}{16/(3R_H)} = \frac{27}{7}.$$

The $4 \rightarrow 3$ transition has the smaller energy gap, so its wavelength is longer, consistent with this ratio being greater than one.

Quick Tip to Solve: Add one to the excited-state number to obtain n , then invert the ratio of the Rydberg brackets when finding a wavelength ratio.

Step 3: Conclusion For the transition from the third to the first excited state, $\nu_{42} = 3cR_H/16 \approx 6.17 \times 10^{14}$ Hz. In the order third-to-second and third-to-first excited states, $\lambda_{43} : \lambda_{42} = 27 : 7$.

Quick Tip to Solve: Add one to the excited-state number to obtain n , then invert the ratio of the Rydberg brackets when finding a wavelength ratio.

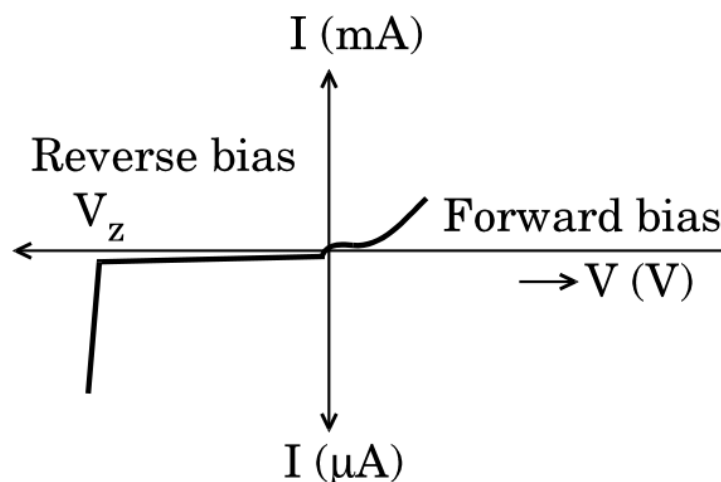
24. The figure shows the V-I characteristic of a semiconductor diode designed to operate under reverse bias.

(a) Identify the semiconductor diode used.

(b) Draw the circuit diagram to obtain the given characteristics of this device.

(c) Briefly explain one use of this device.

[3 marks]



Solution:

Concept:

- A Zener diode is a heavily doped p-n junction designed to operate safely in reverse breakdown. Its breakdown voltage magnitude is V_Z .
- To measure a diode characteristic, connect an ammeter in series and a voltmeter across the diode; a series resistor limits the current and prevents damage.
- In the breakdown region the reverse current changes greatly while the voltage across a Zener diode remains approximately V_Z , allowing voltage regulation.

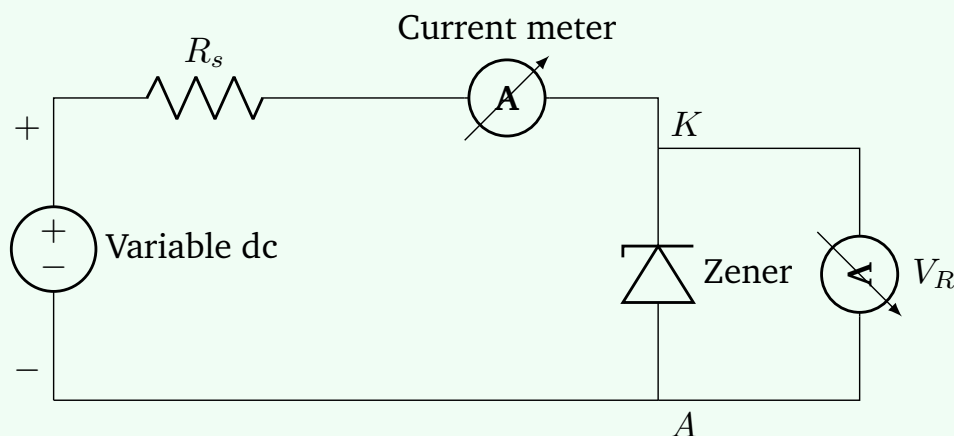
Step 1: Identify the device from the reverse characteristic (a) The device is a Zener diode. The reverse current is initially very small, but it increases sharply when the reverse voltage reaches the knee at $-V_Z$. This nearly vertical breakdown portion identifies a diode made to operate under reverse bias at an almost constant voltage. In forward bias, it conducts like an ordinary p-n junction diode.

Step 2: Connect the circuit for both bias directions (b) Use a variable dc supply, protective series resistor R_s , series ammeter and parallel voltmeter, as shown. In the reverse-bias circuit the cathode K goes towards the positive supply terminal and the anode A towards the negative terminal. Increase the supply gradually, recording the reverse-voltage and reverse-current magnitudes; use a microammeter before breakdown and a suitable higher current range after the knee. Keep the current within the

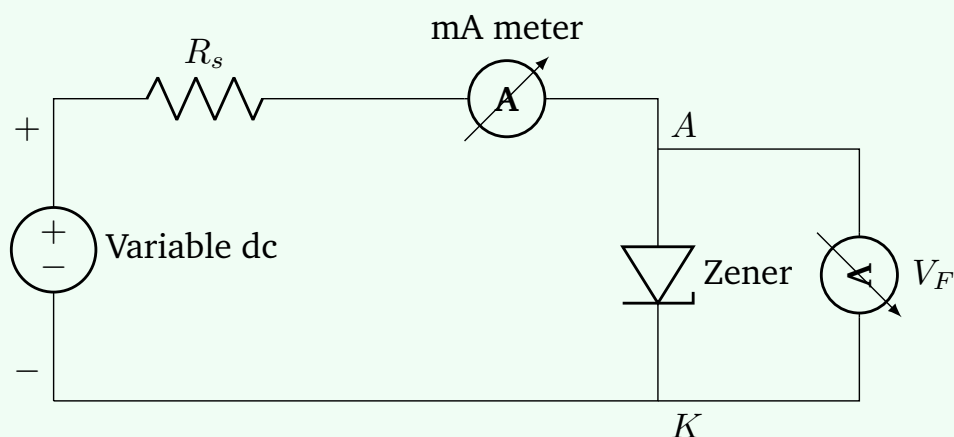
Quick Tip to Solve: For reverse bias, connect the Zener cathode to the positive side and place the voltmeter across the diode; never omit the protective series resistor.

diode's rated limit.

To record the forward part of the same characteristic, reverse the diode connections: its anode goes towards the positive terminal and its cathode towards the negative terminal. Use the milliampere range for forward current. In each circuit, the ammeter measures only the diode current, while the voltmeter measures the voltage across the diode. Plot forward readings in the first quadrant and reverse readings with negative voltage and current in the third quadrant.



Reverse-bias characteristic circuit; use an appropriate current-meter range near breakdown.



Forward-bias characteristic circuit, with the diode connections reversed.

Step 3: Explain voltage regulation (c) One use is as a voltage regulator. Connect the Zener diode in reverse bias across the load, with a resistor in series with the input supply. Once the diode operates in breakdown, the load voltage is approximately V_Z . If the supply voltage rises, the extra current mainly flows through the Zener diode and the series resistor drops the extra voltage. If the supply falls, the Zener current decreases. The output remains nearly constant provided the diode stays in its breakdown operating range and below its maximum power rating. Similarly, a change in load current is compensated by an opposite change in Zener current.

Step 4: Conclusion The device is a Zener diode. Its characteristic is measured with a series current meter, parallel voltmeter and current-limiting resistor, reversing the diode for forward readings. Its nearly constant breakdown voltage makes it useful as a voltage regulator.

Quick Tip to Solve: For reverse bias, connect the Zener cathode to the positive side and place the voltmeter across the diode; never omit the protective series resistor.

Section D (15 marks)

25. (a) A metallic rod of length ' l ' and resistance ' R ' is rotated with a frequency ' ν ' with one end hinged at the centre and the other end at the circumference of a circular metallic ring of radius ' l ', about an axis passing through the centre and perpendicular to the plane of the ring. A constant and uniform magnetic field ' B ' parallel to the axis is present everywhere.

(i) Derive the expression for the induced emf and the current in the rod.

(ii) Due to the presence of current in the rod and of the magnetic field, find the expression for the magnitude and direction of the force acting on this rod.

(iii) Hence, obtain an expression for the power required to rotate the rod.

(b) A copper coil is taken out of a magnetic field with a fixed velocity. Will it be easy to remove it from the same field if its ohmic resistance is increased? [5 marks]

Solution:

Concept:

- A moving conductor has motional emf: for a small radial element, $d\varepsilon = Bv dr$. Here B is the uniform axial field, r the distance from the hinge, and $v = \omega r$, where $\omega = 2\pi\nu$.
- In a completed circuit, Ohm's law gives $I = \varepsilon/R$. Taking the ring and connecting return path to have negligible resistance, the circuit resistance is the rod resistance R .
- A current element experiences $d\vec{F} = I d\vec{l} \times \vec{B}$. Lenz's law makes its torque oppose the rotation. Mechanical input power is $P = \tau\omega = I\varepsilon$.

Step 1: (a)(i) Integrate the motional emf along the rotating rod Speeds along the rod are not equal: the hinge is stationary and the free end moves at ωl . The magnetic force on mobile charges separates them along the rod. Since the field is perpendicular to the rod's plane, the emf of a radial element is

$$d\varepsilon = B\omega r dr.$$

Integrating from the centre to the circumference gives the magnitude

$$\varepsilon = \int_0^l B\omega r dr = \frac{B\omega l^2}{2} = \pi B\nu l^2.$$

Thus the induced current in the completed rod-and-return circuit is

$$I = \frac{B\omega l^2}{2R} = \frac{\pi B\nu l^2}{R}.$$

Its radial direction follows $\vec{v} \times \vec{B}$: if the field and angular velocity point in the same axial direction, conventional current in the rod flows outward. Reversing either reverses the current. Without a closed return path there is an emf but no steady current; the current expression uses the intended closed-circuit arrangement.

Step 2: (a)(ii) Find the resultant magnetic force The same current flows through every radial element,

Quick Tip to Solve: Use the rod's average speed $\omega l/2$ for emf, and the force's lever arm $l/2$ for torque; using the tip value twice overestimates the power.

and each is perpendicular to $\vec{B}$. Therefore

$$dF = IB \, dr, \quad F = \int_0^l IB \, dr = IBl = \frac{\pi B^2 \nu l^3}{R}.$$

All element forces point along the same tangent, opposite the instantaneous motion of the rod. This is the magnetic braking force. For example, with counterclockwise rotation and a field out of the plane, the current is outward and $I \vec{dl} \times \vec{B}$ points clockwise. Its distribution is uniform along the rod, so its resultant acts at $l/2$, not at the free end.

Step 3: (a)(iii) Obtain the input power from the braking torque Because the force acts along the whole rod, integrate its moment about the hinge:

$$\tau = \int_0^l r IB \, dr = \frac{IBl^2}{2} = \frac{B^2 \omega l^4}{4R}.$$

To maintain constant frequency, the external driving torque must equal this magnitude and act in the direction of rotation. Neglecting friction, the required power is

$$\begin{aligned} P &= \tau \omega = \frac{B^2 \omega^2 l^4}{4R} \\ &= \frac{\pi^2 B^2 \nu^2 l^4}{R} = I \varepsilon = I^2 R. \end{aligned}$$

Thus the mechanical energy supplied is converted into Joule heat, consistent with energy conservation.

Step 4: (b) Explain the effect of increasing the coil resistance Yes, it becomes easier to withdraw the copper coil. For the same field, geometry and fixed withdrawal velocity, the rate of change of flux and hence $|\varepsilon| = |d\Phi/dt|$ are unchanged. Increasing the ohmic resistance reduces the induced current, $I = |\varepsilon|/R$. By Lenz's law this current produces a magnetic force opposing withdrawal; its smaller magnitude therefore gives a smaller retarding force. Less external force, and less mechanical power at the same velocity, is required.

Step 5: Conclusion The rod has $\varepsilon = \pi B \nu l^2$, $I = \pi B \nu l^2 / R$, and tangential braking force $F = \pi B^2 \nu l^3 / R$. The required driving power is $P = \pi^2 B^2 \nu^2 l^4 / R$. Increasing the copper coil's resistance makes withdrawal easier by reducing its induced current and magnetic opposition.

Quick Tip to Solve: Use the rod's average speed $\omega l/2$ for emf, and the force's lever arm $l/2$ for torque; using the tip value twice overestimates the power.

25. (a) A rectangular coil rotates in a uniform magnetic field. Obtain an expression for induced emf and current at any instant. Also find their peak values. Show the variation of induced emf versus angle of rotation (ωt) on a graph.

(b) An iron bar falling through the hollow region of a thick cylindrical shell made of copper experiences a retarding force. What can you conclude about the nature of the iron bar? Explain. [5 marks]

Solution:

Concept:

- Magnetic flux linkage for an N -turn coil is $N\Phi = NBA \cos \theta$, where B is the uniform field, A is the area of each rectangular turn, and θ is the angle between the coil's normal and the field.
- Faraday's law gives $\varepsilon = -d(N\Phi)/dt$. For uniform rotation, $\theta = \omega t$, with angular speed ω ; Lenz's law determines the sign and direction.
- For a closed circuit of total resistance R , neglecting self-inductance, $i = \varepsilon/R$. Eddy currents are circulating currents induced in a bulk conductor by a changing magnetic flux and produce magnetic opposition to that change.

Step 1: (a) Set the flux and differentiate Let the rectangular coil have sides a, b , so $A = ab$. Its axis of rotation must allow the angle between its normal and $\vec{B}$ to vary uniformly; the usual generator axis is perpendicular to the field. Choose $t = 0$ with the coil's normal along the field, so that the initial flux is maximum. For N turns,

$$N\Phi(t) = NBA \cos(\omega t).$$

Therefore the instantaneous induced emf is

$$\begin{aligned}\varepsilon(t) &= -\frac{d}{dt}\{NBA \cos(\omega t)\} \\ &= NBA\omega \sin(\omega t).\end{aligned}$$

A single rectangular turn corresponds to $N = 1$, giving $\varepsilon = Bab\omega \sin(\omega t)$. The chosen sign convention specifies one terminal as positive during the first half-cycle; reversing rotation or the field reverses the polarity.

Step 2: (a) Find the instantaneous current and peak values If the rotating coil supplies a closed resistive circuit of total resistance R , its current is

$$i(t) = \frac{NBA\omega}{R} \sin(\omega t) = i_0 \sin(\omega t).$$

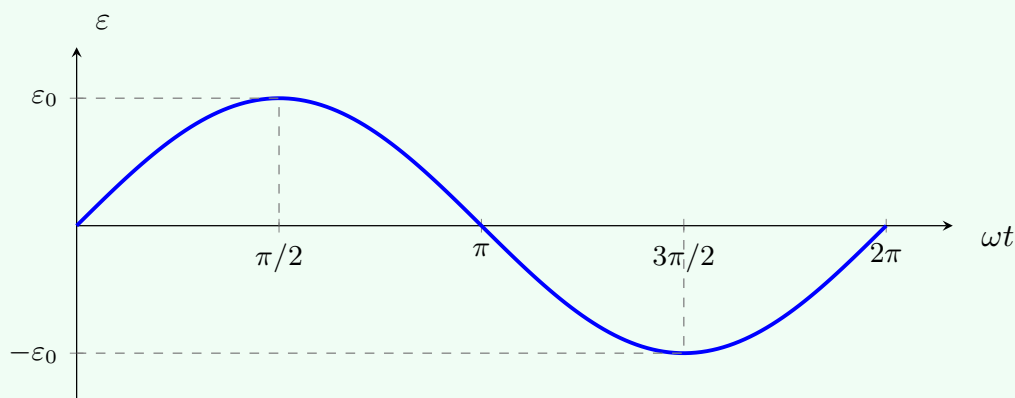
Since the greatest magnitude of the sine is unity, the peak values are

$$\varepsilon_0 = NBA\omega, \quad i_0 = \frac{NBA\omega}{R}.$$

For one turn these reduce to $BA\omega$ and $BA\omega/R$. Current and emf are in phase under the stated resistive-circuit approximation. With the terminals open there is still the generated emf, but no delivered current. If appreciable inductance is present, the current's magnitude and phase require the circuit impedance instead of R .

Quick Tip to Solve: Choose the coil's normal parallel to the field at zero angle: flux then starts at its maximum, so the emf graph starts at zero and rises as a sine wave.

Step 3: (a) Plot induced emf against rotation angle With $\theta = \omega t$, the emf follows $\varepsilon = \varepsilon_0 \sin \theta$. At $\theta = 0, \pi, 2\pi$, the flux is at an extremum and its instantaneous rate of change is zero, so the emf vanishes. At $\pi/2$ the emf is $+\varepsilon_0$, while at $3\pi/2$ it is $-\varepsilon_0$. The two halves therefore show opposite polarities. After one full rotation, the pattern repeats. The graph uses the angle of the normal from the field and the maximum-flux initial condition already specified.



Induced emf versus rotation angle for maximum flux at $\theta = 0$.

Step 4: (b) Infer the nature of the falling iron bar The iron bar is magnetised: it behaves as a magnet. As its poles move past each section of the copper shell, the magnetic flux through that section changes. Although copper is nonmagnetic, it conducts electricity; therefore circulating eddy currents are induced in the shell. By Lenz's law their fields oppose the relative motion of the magnet. Ahead of the falling bar the induced magnetic effect opposes its approach, and behind it the induced effect opposes its departure, so both contributions retard the fall. The magnetic interaction does not require contact with the shell. Some gravitational potential energy is dissipated as Joule heat in copper, accounting for the reduction in mechanical energy. An unmagnetised iron bar would not by itself supply this moving magnetic flux in the stated arrangement.

Step 5: Conclusion For uniform rotation, $\varepsilon = NBA\omega \sin(\omega t)$ and $i = (NBA\omega/R) \sin(\omega t)$, with peaks $NBA\omega$ and $NBA\omega/R$. The emf-angle graph is sinusoidal. The retarded iron bar is magnetised; its changing flux induces opposing eddy currents in the copper shell.

Quick Tip to Solve: Choose the coil's normal parallel to the field at zero angle: flux then starts at its maximum, so the emf graph starts at zero and rises as a sine wave.

26. (a) Draw a labelled ray diagram of compound microscope, when final image forms at the least distance of distinct vision.
- (b) Why is its objective of short focal length and of short aperture, compared to its eyepiece? Explain.
- (c) The focal length of the objective is 4 cm while that of eyepiece is 10 cm. The object is placed at a distance of 6 cm from the objective lens.
- (i) Calculate the magnifying power of the compound microscope, if its final image is formed at the near point.
- (ii) Also calculate length of the compound microscope. [5 marks]

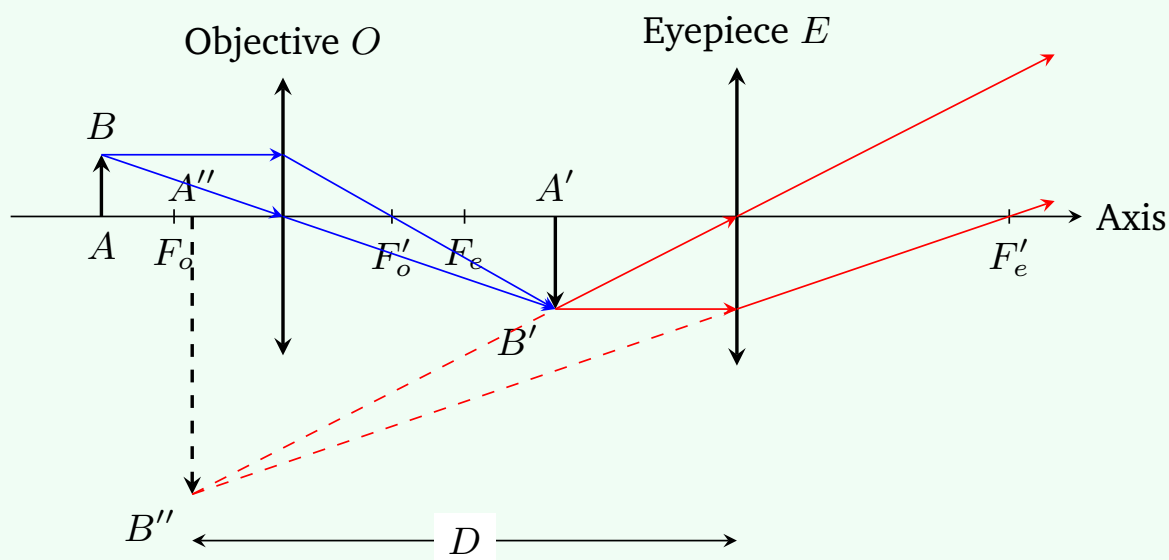
Solution:

Concept:

- A compound microscope uses two converging lenses. The objective forms a real, inverted, enlarged intermediate image; the eyepiece acts as a simple microscope and gives an enlarged virtual final image.
- Using the Cartesian sign convention, the thin-lens formula is $1/f = 1/v - 1/u$, and linear magnification is $m = v/u$. Distances are measured from the optical centre of the relevant lens.
- For a final image at the near point, $D = 25$ cm, the eyepiece angular magnification is $M_e = 1 + D/f_e$. The total magnifying power in magnitude is $M = |m_o|M_e$.

Step 1: (a) Ray diagram and image formation at the near point Place the small object AB just beyond the first focus F_o of the objective O . A ray from B parallel to the principal axis refracts through F'_o ; a ray through the optical centre of O continues undeviated. They meet at B' , giving the real, inverted intermediate image $A'B'$. Position the eyepiece E so that $A'B'$ lies between its first focus F_e and its optical centre. Rays leaving the eyepiece diverge; their backward extensions intersect at B'' . The final image $A''B''$ is virtual, enlarged and inverted relative to AB , and is located at the least distance of distinct vision from the eyepiece. The diagram is schematic; D is measured from E in the thin-lens approximation.

Quick Tip to Solve: First locate the objective's real image. Then use a negative near-point image distance for the eyepiece, and add the two positive distances to that intermediate image to obtain the lens separation.



Compound microscope adjusted for a virtual final image at the near point; dashed lines are backward extensions (schematic).

Step 2: (b) Why the objective has a short focal length and small aperture The objective must give a large linear enlargement of a very small nearby object. A short objective focal length permits the object to be placed close to its focus and gives a large objective magnification for a given tube length; approximately, $|m_o| \simeq L/f_o$ when the usual long-tube approximation applies. Its aperture is small compared with that of the eyepiece to restrict marginal rays and reduce spherical aberration, giving a sharper intermediate image. The larger eyepiece aperture permits convenient viewing of this enlarged image. A small aperture should not be confused with an improvement in diffraction-limited resolving power: its purpose here is to reduce aberration in the elementary microscope model.

Step 3: (c)(i) Find the objective image distance and magnification The given distances are $f_o = +4$ cm and $u_o = -6$ cm. Thus

$$\frac{1}{v_o} = \frac{1}{f_o} + \frac{1}{u_o} = \frac{1}{4} - \frac{1}{6} = \frac{1}{12},$$

$$v_o = +12 \text{ cm}, \quad m_o = \frac{v_o}{u_o} = \frac{12}{-6} = -2.$$

The intermediate image is therefore real and inverted, with twice the height of the object. With $f_e = 10$ cm, the near-point eyepiece magnification is

$$M_e = 1 + \frac{25}{10} = 3.5, \quad M = 2 \times 3.5 = 7.$$

With an orientation sign included, the total magnification is -7 ; the reported magnifying power is its magnitude.

Step 4: (c)(ii) Calculate the microscope length The final eyepiece image is virtual, so $v_e = -25$ cm.

Quick Tip to Solve: First locate the objective's real image. Then use a negative near-point image distance for the eyepiece, and add the two positive distances to that intermediate image to obtain the lens separation.

Apply the lens formula to the eyepiece:

$$\begin{aligned}\frac{1}{10} &= -\frac{1}{25} - \frac{1}{u_e}, \\ -\frac{1}{u_e} &= \frac{1}{10} + \frac{1}{25} = \frac{7}{50}, \\ u_e &= -\frac{50}{7} \text{ cm}.\end{aligned}$$

Thus the intermediate image is $50/7$ cm to the left of the eyepiece. It is also 12 cm to the right of the objective, so the separation of the two lenses is

$$\ell = v_o + |u_e| = 12 + \frac{50}{7} = \frac{134}{7} \text{ cm} \simeq 19.14 \text{ cm}.$$

Use this exact lens separation, since the given object distance does not justify replacing the objective magnification by a long-tube approximation in the numerical calculation.

Step 5: Conclusion The objective uses a short focal length for high enlargement and a smaller aperture to limit spherical aberration. For the given microscope, the magnifying power at the near point is 7 (final image inverted), and the objective-to-eyepiece separation is 19.14 cm .

Quick Tip to Solve: First locate the objective's real image. Then use a negative near-point image distance for the eyepiece, and add the two positive distances to that intermediate image to obtain the lens separation.

OR

26. (a) With the help of a labelled ray diagram, explain the construction and working of a Cassegrain reflecting telescope.

(b) An amateur astronomer wishes to estimate roughly the size of the Sun using his crude telescope consisting of an objective lens of focal length 200 cm and an eyepiece of focal length 10 cm. By adjusting the distance of the eyepiece from the objective, he obtains an image of the Sun on a screen 40 cm behind the eyepiece. The diameter of the Sun's image is measured to be 6.0 cm. Estimate the Sun's size, given that the average Earth-Sun distance is $1.5 \times 10^{11} \text{ m}$. [5 marks]

Solution:

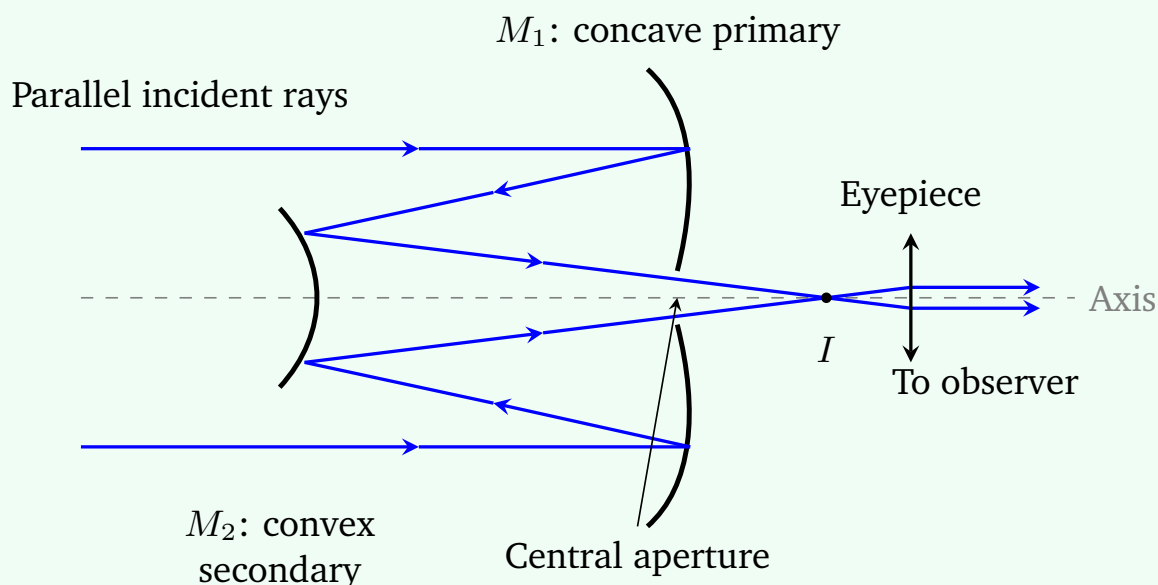
Concept:

- A Cassegrain telescope uses a large concave primary mirror and a small convex secondary mirror. The folded light path gives a long effective focal length in a compact tube.
- A distant object forms its first image approximately in the focal plane of the objective. For a thin eyepiece, $1/f_e = 1/v_e - 1/u_e$, using the Cartesian sign convention, and its transverse magnification is $m_e = v_e/u_e$.
- For a distant source of diameter D_s at distance R , its small angular diameter is $\theta \simeq D_s/R$. An objective of focal length f_o forms an image of diameter $d_o \simeq f_o\theta$.

Step 1: (a) Construction of the reflecting telescope The primary mirror M_1 is a large concave,

Quick Tip to Solve: For projection onto a screen, first use the eyepiece lens formula to undo its transverse enlargement; only then use objective-image diameter divided by objective focal length to obtain the solar angle.

approximately paraboloidal mirror with a central aperture. A smaller convex secondary mirror M_2 is mounted coaxially in front of it, between the primary and the focus at which the primary alone would concentrate the incoming light. An eyepiece is placed behind the primary mirror, along its central aperture. The primary collects light over a large area, while the secondary redirects the converging beam back through the aperture.



Cassegrain reflecting telescope: axial rays reflect from the concave primary to the convex secondary and return through the primary's aperture to image I .

Step 2: (a) Working and the folded light path Nearly parallel rays from a distant celestial object enter the tube and strike M_1 . After reflection from this concave mirror, the rays converge towards its original focus. Before they reach that focus, they strike M_2 . Reflection from the convex secondary redirects them towards the back of the telescope and reduces their rate of convergence. They pass through the central aperture in M_1 and form a real image I behind it. The eyepiece magnifies this image for the observer; in normal adjustment, I lies in the eyepiece's first focal plane, so the emerging rays are parallel and the final image is at infinity. Reflection avoids chromatic aberration in the objective.

Step 3: (b) Locate the image that acts as the eyepiece's object The Sun is extremely distant, so the objective first forms a real solar image approximately 200 cm behind it. This image is then enlarged by the eyepiece to produce the specified real image on the screen. For the eyepiece, the screen is on its outgoing-light side, so $v_e = +40$ cm and $f_e = +10$ cm. Hence

$$\frac{1}{u_e} = \frac{1}{v_e} - \frac{1}{f_e} = \frac{1}{40} - \frac{1}{10} = -\frac{3}{40} \text{ cm}^{-1},$$

$$u_e = -\frac{40}{3} \text{ cm}.$$

Thus the first image lies 13.33 cm in front of the eyepiece.

Step 4: (b) Recover the diameter of the objective's solar image The eyepiece's transverse magnification

Quick Tip to Solve: For projection onto a screen, first use the eyepiece lens formula to undo its transverse enlargement; only then use objective-image diameter divided by objective focal length to obtain the solar angle.

is

$$m_e = \frac{v_e}{u_e} = \frac{40}{-40/3} = -3.$$

Its magnitude is 3; the minus sign indicates inversion. Therefore the screen image of diameter $d = 6.0$ cm corresponds to an objective image of diameter

$$d_o = \frac{d}{|m_e|} = \frac{6.0}{3} = 2.0 \text{ cm}.$$

The ordinary telescope angular magnification f_o/f_e is inappropriate here because the eyepiece projects a real image onto a screen, rather than forming a final image at infinity.

Step 5: (b) Convert angular diameter to the Sun's physical diameter Using similar triangles and the small-angle approximation,

$$\theta \simeq \frac{d_o}{f_o} = \frac{2.0}{200} = 1.0 \times 10^{-2} \text{ rad}.$$

The Sun's diameter is its angular diameter multiplied by the Earth-Sun distance:

$$D_s \simeq R\theta = (1.5 \times 10^{11})(1.0 \times 10^{-2}) = 1.5 \times 10^9 \text{ m}.$$

The ratio d_o/f_o is dimensionless because both lengths are in centimetres. Using the screen diameter here would overestimate the result threefold.

Step 6: Conclusion (a) The concave primary and convex secondary form a folded reflecting system: light is collected, reflected twice, and brought to a real image behind the primary's central aperture for magnification by the eyepiece. (b) The estimated diameter of the Sun is $1.5 \times 10^9 \text{ m}$, or $1.5 \times 10^6 \text{ km}$.

Quick Tip to Solve: For projection onto a screen, first use the eyepiece lens formula to undo its transverse enlargement; only then use objective-image diameter divided by objective focal length to obtain the solar angle.

27. (a) Derive a relation between the internal resistance, emf and terminal potential difference of a cell from which current I is drawn. Draw V vs I graph for a cell and explain its significance.

(b) A voltmeter of resistance 998Ω is connected across a cell of emf 2 V and internal resistance 2Ω . Find the potential difference across the voltmeter and also across the terminals of the cell. Estimate the percentage error in the reading of the voltmeter. [5 marks]

Solution:

Concept:

- The emf $\mathcal{E}$ of a cell is its work done per unit charge. Its terminal potential difference V is the energy supplied per unit charge to the external circuit.
- If a cell supplies current I and has internal resistance r , the potential drop inside it is Ir . Kirchhoff's loop rule relates this internal drop to the emf and terminal voltage.
- A real voltmeter of resistance R_v draws current from the cell. Its reading equals the loaded terminal voltage, while the open-circuit voltage equals the emf.

Quick Tip to Solve: For a voltmeter across a cell, use $V_v = \mathcal{E}R_v/(R_v + r)$; the fractional loading error is $r/(R_v + r)$.

Step 1: (a) Derive the terminal-voltage relation Let the cell supply current I through an external resistance R . The same current passes through its internal resistance r . On traversing the closed circuit, the emf rise is balanced by the external and internal potential drops:

$$\mathcal{E} - IR - Ir = 0.$$

Since the terminal potential difference across the external circuit is $V = IR$, this gives

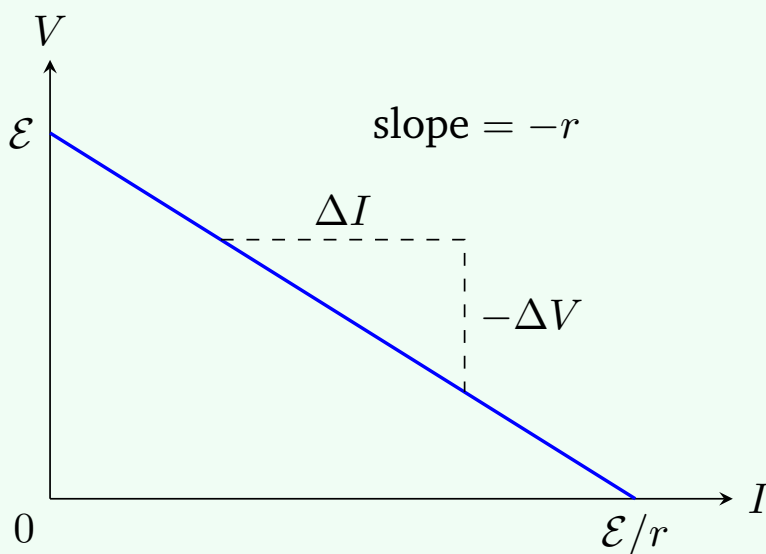
$$\boxed{V = \mathcal{E} - Ir}, \quad \boxed{r = \frac{\mathcal{E} - V}{I}} \quad (I \neq 0).$$

Thus, when current is drawn from a cell, its terminal voltage is less than its emf because some energy per unit charge is dissipated inside the cell. The relation assumes that the emf and internal resistance remain constant during the measurement.

Step 2: (a) Interpret the graph of terminal voltage against current Plot V on the vertical axis and I on the horizontal axis. Comparing $V = \mathcal{E} - rI$ with a straight-line equation shows that the graph has negative slope. At $I = 0$, there is no internal drop and the vertical intercept is $\mathcal{E}$. For two points on the line,

$$\frac{\Delta V}{\Delta I} = -r.$$

Therefore the magnitude of the gradient gives the internal resistance. Extrapolating the line to $V = 0$ gives the current-axis intercept $I_{sc} = \mathcal{E}/r$, the short-circuit current in this constant-resistance model. Increasing the current causes a proportionate reduction in terminal voltage.



Terminal voltage against current for a cell with constant emf and internal resistance.

Step 3: (b) Find the current drawn by the voltmeter The voltmeter is the external resistance connected across the cell, so its resistance and the internal resistance form the complete current path. With

Quick Tip to Solve: For a voltmeter across a cell, use $V_v = \mathcal{E}R_v/(R_v + r)$; the fractional loading error is $r/(R_v + r)$.

$\mathcal{E} = 2\text{ V}$, $R_v = 998\ \Omega$ and $r = 2\ \Omega$,

$$I = \frac{\mathcal{E}}{R_v + r} = \frac{2}{998 + 2} = 0.002\text{ A}.$$

The voltmeter therefore draws 2 mA; its finite resistance produces a small but nonzero internal voltage drop.

Step 4: (b) Calculate both potential differences and the reading error The voltage across the voltmeter is

$$V_v = IR_v = 0.002 \times 998 = 1.996\text{ V}.$$

The terminal voltage of the cell is independently obtained as

$$V_{\text{term}} = \mathcal{E} - Ir = 2 - (0.002 \times 2) = 1.996\text{ V}.$$

These are equal because the voltmeter is connected directly across the cell terminals. Compared with the emf that an ideal, infinite-resistance voltmeter would read, the underestimate is $2 - 1.996 = 0.004\text{ V}$. Hence the percentage error in measuring the emf is

$$\frac{\mathcal{E} - V_v}{\mathcal{E}} \times 100 = \frac{0.004}{2} \times 100 = 0.2\%.$$

The error is due to loading of the cell by the meter; the meter correctly reads the loaded terminal voltage.

Step 5: Conclusion The required relation is $V = \mathcal{E} - Ir$. The V -against- I graph is a falling straight line with vertical intercept $\mathcal{E}$ and slope $-r$. Both the voltmeter voltage and the cell's terminal voltage are 1.996 V; the voltmeter reading underestimates the emf by 0.2%.

Quick Tip to Solve: For a voltmeter across a cell, use $V_v = \mathcal{E}R_v/(R_v + r)$; the fractional loading error is $r/(R_v + r)$.

OR

- 27. (a)** Two cells of different emfs and internal resistances are connected in parallel with one another. Derive the expression for the equivalent emf and equivalent internal resistance of the combination.
- (b)** Two identical cells of emf 1.5 V and internal resistance r are each connected in parallel providing a supply to an external circuit consisting of two resistances of $17\ \Omega$ each joined in parallel. A very high resistance voltmeter reads the terminal voltage of the cell to be 1.4 V. Calculate the internal resistance of each cell.
- [5 marks]

Solution:

Concept:

- For a cell of emf E and internal resistance r , delivering current I , the terminal potential difference is $V = E - Ir$. Emf is the energy supplied per unit charge; Ir is the internal voltage drop.
- Cells in parallel have a common terminal potential difference. With like terminals joined, the

Quick Tip to Solve: For two identical cells in parallel, retain the single-cell emf and halve the internal resistance; use the measured terminal voltage to calculate the load current.

branch currents add algebraically: $I = I_1 + I_2$. A negative branch current means that the other cell charges that branch.

- The equivalent cell is defined by $V = E_{\text{eq}} - Ir_{\text{eq}}$ for the same external current I . For parallel external resistances, $1/R = 1/R_1 + 1/R_2$.

Step 1: (a) Write the equations for the two parallel cells Let the emfs be E_1, E_2 , and the positive internal resistances be r_1, r_2 . Join their positive terminals together and their negative terminals together, so both emfs have the same polarity. Let V be the common terminal voltage, and take I_1, I_2 as positive when delivered by the respective cells to the external circuit. Then

$$V = E_1 - I_1 r_1 = E_2 - I_2 r_2, \quad I_1 = \frac{E_1 - V}{r_1}, \quad I_2 = \frac{E_2 - V}{r_2}.$$

These equations remain valid even if unequal emfs cause one branch current to reverse. At the junction, charge conservation gives the external current $I = I_1 + I_2$.

Step 2: (a) Identify the equivalent emf and internal resistance Adding the branch currents and rearranging gives

$$I = \frac{E_1}{r_1} + \frac{E_2}{r_2} - V \left(\frac{1}{r_1} + \frac{1}{r_2} \right),$$

$$V = \frac{E_1/r_1 + E_2/r_2}{1/r_1 + 1/r_2} - I \frac{1}{1/r_1 + 1/r_2}.$$

Compare this relation with $V = E_{\text{eq}} - Ir_{\text{eq}}$. Thus

$$E_{\text{eq}} = \frac{E_1 r_2 + E_2 r_1}{r_1 + r_2}, \quad r_{\text{eq}} = \frac{r_1 r_2}{r_1 + r_2}.$$

The equivalent emf is the open-circuit terminal voltage: setting the external current to zero in the derived relation gives $V = E_{\text{eq}}$. Unequal emfs may still cause a circulating current between the cells at open circuit. The equivalent internal resistance is the parallel combination of their internal resistances. For identical cells, substitution gives $E_{\text{eq}} = E$ and $r_{\text{eq}} = r/2$.

Step 3: (b) Calculate the external load current The two 17Ω resistances are in parallel, so their equivalent resistance is

$$R = \frac{17 \times 17}{17 + 17} = 8.5\Omega.$$

The voltmeter has very high resistance, so its current is negligible. Its reading $V = 1.4\text{V}$ is therefore the voltage across this load. The total external current is

$$I = \frac{V}{R} = \frac{1.4}{8.5} = \frac{14}{85}\text{A} \approx 0.1647\text{A}.$$

Because the two cells are identical and connected in parallel, they share this current equally; each delivers $I/2$.

Step 4: (b) Determine the resistance of each cell The equivalent supply has emf 1.5V and internal

Quick Tip to Solve: For two identical cells in parallel, retain the single-cell emf and halve the internal resistance; use the measured terminal voltage to calculate the load current.

resistance $r/2$. Apply its terminal-voltage equation:

$$E_{\text{eq}} - V = I \frac{r}{2},$$
$$r = \frac{2(1.5 - 1.4)}{14/85} = \frac{17}{14} \Omega \approx 1.214 \Omega.$$

As a check, the current supplied by each cell is $7/85$ A. Its internal drop is $(7/85)(17/14) = 0.1$ V, leaving exactly the observed 1.4 V at its terminals.

Step 5: Conclusion (a) For like terminals joined in parallel,

$$E_{\text{eq}} = \frac{E_1 r_2 + E_2 r_1}{r_1 + r_2}, \quad r_{\text{eq}} = \frac{r_1 r_2}{r_1 + r_2}.$$

(b) The internal resistance of each cell is $r = 17/14 \Omega \approx 1.21 \Omega$.

Quick Tip to Solve: For two identical cells in parallel, retain the single-cell emf and halve the internal resistance; use the measured terminal voltage to calculate the load current.