

CBSE Class 12 Physics Question Paper 2019 with Solution PDF-Set 55/2/3

Question Paper with Solutions

Conducted by the Central Board of Secondary Education (CBSE)



Time Allowed: 3 hours

Maximum Marks: 70

General Instructions

- (i) All questions are compulsory. There are 27 questions in all.
- (ii) This question paper has four sections : Section A, Section B, Section C and Section D.
- (iii) Section A contains five questions of one mark each, Section B contains seven questions of two marks each, Section C contains twelve questions of three marks each, Section D contains three questions of five marks each.
- (iv) There is no overall choice. However, an internal choice(s) has been provided in two questions of one mark, two questions of two marks, four questions of three marks and three questions of five marks weightage. You have to attempt only one of the choices in such questions.

You may use the following values of physical constants wherever necessary :

$$c = 3 \times 10^8 \text{ m/s}$$

$$h = 6.63 \times 10^{-34} \text{ Js}$$

$$e = 1.6 \times 10^{-19} \text{ C}$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ T m A}^{-1}$$

$$\epsilon_0 = 8.854 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$$

$$\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$$

$$\text{Mass of electron } (m_e) = 9.1 \times 10^{-31} \text{ kg}$$

$$\text{Mass of neutron} = 1.675 \times 10^{-27} \text{ kg}$$

$$\text{Mass of proton} = 1.673 \times 10^{-27} \text{ kg}$$

$$\text{Avogadro's number} = 6.023 \times 10^{23} \text{ per gram mole}$$

$$\text{Boltzmann constant} = 1.38 \times 10^{-23} \text{ JK}^{-1}$$

Section A (5 marks)

1. The magnetic susceptibility of magnesium at 300 K is 1.2×10^5 . At what temperature will its magnetic susceptibility become 1.44×10^5 ? [1 mark]

Solution:

Concept:

- Curie's law for a paramagnetic material is $\chi = C/T$, where C is the Curie constant and T is absolute temperature.
- Thus χT remains constant for the given material.

Step 1: Relate the two states Using the values printed in the question,

$$\chi_1 T_1 = \chi_2 T_2, \quad T_2 = \frac{\chi_1}{\chi_2} T_1.$$

Susceptibility is dimensionless, while temperature must be expressed in kelvin.

Step 2: Substitute the given values

$$T_2 = \frac{1.2 \times 10^5}{1.44 \times 10^5} \times 300 \text{ K} = 250 \text{ K}.$$

The common power of ten cancels. Since susceptibility increases by a factor of 1.2, the temperature decreases by the same factor, as required by the inverse relation.

Step 3: Conclusion The required temperature is 250 K.

Quick Tip to Solve: Use the ratio of susceptibilities first: the common power of ten cancels immediately.

OR

1. The magnetic susceptibility χ of a given material is -0.5 . Identify the magnetic material. [1 mark]

Solution:

Concept:

- Magnetic susceptibility relates magnetisation to the applied magnetising field: $M = \chi H$.
- Diamagnetic materials have negative susceptibility; paramagnetic and ferromagnetic materials have positive susceptibility.

Step 1: Check the sign The given susceptibility is $\chi = -0.5$. Its negative sign means that the induced magnetisation is opposite to the applied field, rather than along it.

Step 2: Identify the magnetic behaviour A material whose induced magnetisation opposes the applied field is diamagnetic. Such a material is weakly repelled by an external magnet and tends to move from stronger to weaker field regions. Identification here follows directly from the sign of χ ; no temperature calculation is needed.

Step 3: Conclusion The given material is diamagnetic.

Quick Tip to Solve: For classification from susceptibility, check its sign before considering its magnitude.

2. Draw a pattern of electric field lines due to two positive charges placed a distance d apart. [1 mark]

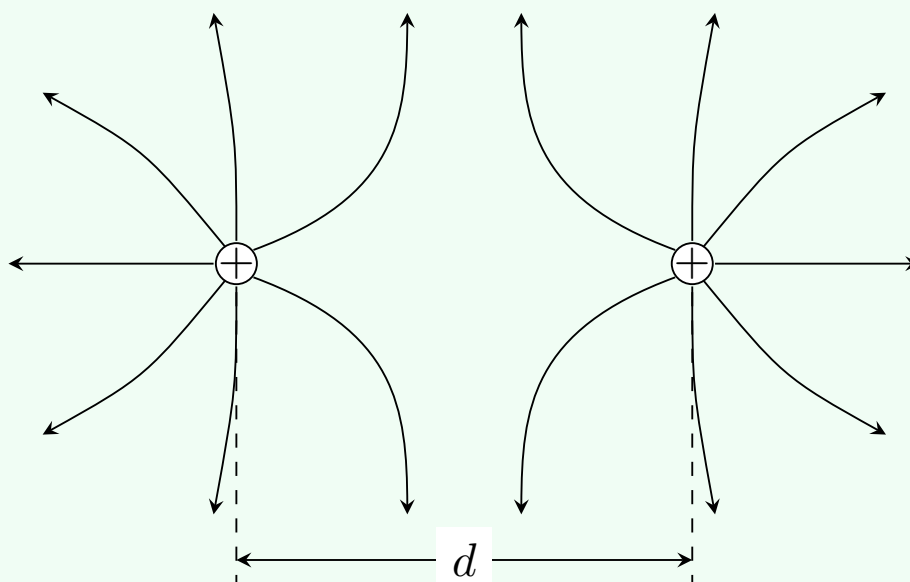
Solution:

Concept:

- Electric field lines originate on positive charges and their arrows point away from each charge.
- Field lines never intersect; for two like charges, lines in the region between them bend away from one another.

Step 1: Set the charge positions Place the two positive charges on a horizontal line with separation d . Draw outward-directed lines from each charge. The symmetric sketch represents equal positive charges, which is a valid pattern for the two positive charges specified.

Step 2: Draw the field-line pattern The lines on the outer sides spread outward. Lines directed towards the region between the charges curve upward or downward, because both charges repel a positive test charge. No line joins the two positive charges. For equal charges the fields cancel at the midpoint; lines do not pass through this zero-field point.



Electric field lines of two positive charges separated by d

Step 3: Conclusion The diagram shows outward field lines from both positive charges, bending away in the region between them.

Quick Tip to Solve: Never join two positive charges with a field line: both are starting points for outward arrows.

3. Which part of the electromagnetic spectrum is used in RADAR? Give its frequency range. [1 mark]

Solution:

Concept:

- RADAR transmits electromagnetic waves and detects their echoes from distant objects.
- Microwaves have short wavelengths and can be directed into narrow beams suitable for locating objects.

Step 1: Identify the spectral region RADAR uses microwaves. Their directional beams provide information about the position of an object, while the time delay of the reflected signal is used to determine its distance.

Step 2: State the frequency range The microwave region is approximately 10^9 to 3×10^{11} Hz, corresponding to wavelengths of about 0.3 m to 1 mm using $c = f\lambda$. A RADAR system operates within this band; it need not transmit over the whole band simultaneously.

Step 3: Conclusion RADAR uses microwaves, with frequency range approximately $10^9 - 3 \times 10^{11}$ Hz.

Quick Tip to Solve: Associate RADAR with directional microwave beams, then state the microwave frequency band.

OR

3. How are electromagnetic waves produced by accelerating charges? [1 mark]

Solution:

Concept:

- An accelerating electric charge produces a time-dependent electric field and a time-dependent magnetic field.
- These changing fields propagate through space as an electromagnetic wave.

Step 1: Explain the changing fields When a charge accelerates, its electric field changes with time. This changing electric field is associated with a changing magnetic field; the changing magnetic field in turn is associated with a changing electric field.

Step 2: Describe propagation The coupled field disturbance travels outward from the accelerating charge with the speed of light in vacuum. The electric and magnetic fields of the radiation are mutually perpendicular and also perpendicular to the direction of propagation. They carry energy away from the source; a stationary charge alone produces no electromagnetic radiation.

Step 3: Conclusion Accelerating charges radiate electromagnetic waves through the propagation of coupled, time-varying electric and magnetic fields.

Quick Tip to Solve: Connect acceleration to changing fields and then to the outward propagation of radiation.

4. When a charge q is moving in the presence of electric (E) and magnetic (B) fields which are perpendicular to each other and also perpendicular to the velocity v of the particle, write the relation expressing v in terms of E and B . [1 mark]

Solution:

Concept:

- The electric force has magnitude $|q|E$, while the magnetic force has magnitude $|q|vB$ when velocity and magnetic field are perpendicular.
- For undeflected motion in crossed fields, the electric and magnetic forces must be opposite and equal.

Step 1: Balance the transverse forces The stated relation is the velocity-selector condition. Taking the transverse electric and magnetic forces to cancel,

$$|q|E = |q|vB.$$

Their directions must be opposite; perpendicular fields by themselves do not ensure cancellation for every particle speed.

Step 2: Solve for the speed Cancelling the nonzero charge magnitude gives

$$v = \frac{E}{B}.$$

The result is independent of the sign and magnitude of the charge. Its units are $(\text{V m}^{-1})/\text{T} = \text{m s}^{-1}$, as expected for a speed. Particles with another speed experience a net transverse force.

Step 3: Conclusion For undeflected motion, the required relation is $v = E/B$.

Quick Tip to Solve: Write equality of the force magnitudes first; the charge cancels from the speed condition.

5. Draw the I-V characteristics of a Zener diode. [1 mark]

Solution:

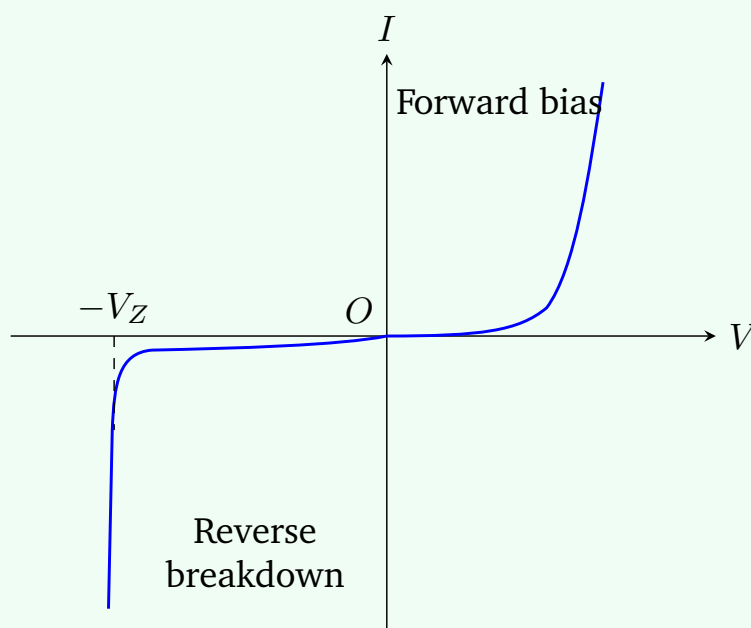
Concept:

- A Zener diode conducts like an ordinary diode in forward bias. In reverse bias, breakdown occurs at the Zener voltage V_Z .
- Take voltage V along the horizontal axis and current I along the vertical axis; reverse-bias quantities are negative.

Step 1: Draw the forward characteristic In the first quadrant, the forward current is initially small and then rises rapidly after the forward knee voltage.

Step 2: Draw the reverse characteristic In the third quadrant, a small reverse leakage current flows until $V = -V_Z$. At breakdown the reverse current increases sharply in magnitude while the voltage remains nearly constant. Label this breakdown voltage on the voltage axis.

Quick Tip to Solve: Put Zener breakdown in the third quadrant and label its voltage on the horizontal axis.



I-V characteristic of a Zener diode (schematic)

Step 3: Conclusion The required I-V characteristic has the usual forward knee and a sharp reverse breakdown at $-V_Z$, as shown.

Quick Tip to Solve: Put Zener breakdown in the third quadrant and label its voltage on the horizontal axis.

Section B (14 marks)

6. State, with the help of a ray diagram, the working principle of optical fibres. Write one important use of optical fibres. [2 marks]

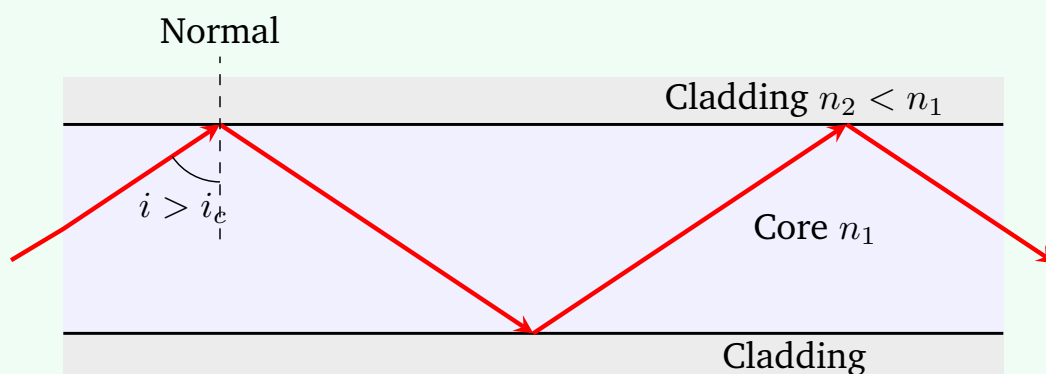
Solution:

Concept:

- An optical fibre guides light by repeated total internal reflection at the boundary between its core and cladding.
- The core refractive index n_1 exceeds the cladding index n_2 . The critical angle satisfies $\sin i_c = n_2/n_1$.

Step 1: Explain the working principle Light enters the fibre through one end and travels inside the denser core. When it strikes the core-cladding boundary at an incidence angle $i > i_c$, measured from the normal, it undergoes total internal reflection instead of passing into the cladding. Successive reflections confine the ray to the core and carry it to the other end. The fibre therefore guides light even along a gently curved path.

Quick Tip to Solve: Always compare the incidence angle with the critical angle using the normal, not the fibre surface.



Repeated total internal reflection in an optical fibre

Step 2: State an important use Optical fibres are used in medical endoscopy to illuminate and view internal organs. Light is transmitted through the fibre bundle, allowing examination through a narrow flexible instrument. They are also used for transmitting optical signals in communication systems.

Step 3: Conclusion Optical fibres work by repeated total internal reflection in a higher-index core surrounded by lower-index cladding. One important use is medical endoscopy.

Quick Tip to Solve: Always compare the incidence angle with the critical angle using the normal, not the fibre surface.

7. A photon emitted during the de-excitation of electron from a state n to the first excited state in a hydrogen atom, irradiates a metallic cathode of work function 2 eV, in a photo cell, with a stopping potential of 0.55 V. Obtain the value of the quantum number of the state n . [2 marks]

Solution:

Concept:

- Einstein's photoelectric equation is $E_\gamma = \phi + K_{\max}$, where ϕ is the work function and $K_{\max} = eV_s$ is the maximum photoelectron kinetic energy.
- Hydrogen energy levels are $E_n = -13.6/n^2$ eV. The first excited state is $n_f = 2$, not the ground state.

Step 1: Find the emitted photon energy The stopping potential is $V_s = 0.55$ V, so $K_{\max} = 0.55$ eV. Adding the work function gives

$$E_\gamma = 2.00 + 0.55 = 2.55 \text{ eV.}$$

The photon must supply both the energy needed to escape from the cathode and the maximum kinetic energy of the emitted electron.

Step 2: Use the hydrogen transition energy For de-excitation from n to 2, the positive emitted energy is

$$2.55 = 13.6 \left(\frac{1}{2^2} - \frac{1}{n^2} \right).$$

Thus

$$\frac{1}{n^2} = \frac{1}{4} - \frac{2.55}{13.6} = 0.25 - 0.1875 = 0.0625 = \frac{1}{16}.$$

Quick Tip to Solve: Add work function to stopping energy first; use final level 2 for the first excited state.

Taking the positive allowed principal quantum number yields the initial level. As a check, its energy is -0.85 eV and the final energy is -3.40 eV , whose difference is 2.55 eV .

Step 3: Conclusion The initial principal quantum number is $n = 4$.

Quick Tip to Solve: Add work function to stopping energy first; use final level 2 for the first excited state.

OR

7. A hydrogen atom in the ground state is excited by an electron beam of 12.5 eV energy. Find out the maximum number of lines emitted by the atom from its excited state. [2 marks]

Solution:

Concept:

- The energy needed to excite ground-state hydrogen to level n is $\Delta E_n = 13.6(1 - 1/n^2) \text{ eV}$.
- If levels up to n are populated, the maximum number of distinct downward-transition lines is $N = n(n - 1)/2$.

Step 1: Find the highest accessible excited level An incident electron can transfer part of its energy in a collision; its entire 12.5 eV need not be absorbed. For $n = 3$,

$$\Delta E_3 = 13.6 \left(1 - \frac{1}{9} \right) = 12.09 \text{ eV}.$$

For $n = 4$,

$$\Delta E_4 = 13.6 \left(1 - \frac{1}{16} \right) = 12.75 \text{ eV}.$$

Therefore the beam can excite hydrogen to level 3, but has insufficient energy to reach level 4 or any higher level.

Step 2: Count all possible spectral lines The available levels are 1, 2 and 3. The distinct radiative transitions are $3 \rightarrow 2$, $3 \rightarrow 1$ and $2 \rightarrow 1$. Across excited atoms and their possible decay paths, each transition supplies a different photon energy. Equivalently,

$$N = \frac{3(3 - 1)}{2} = 3.$$

The count concerns the maximum set of distinct lines, including both direct and cascade transitions.

Step 3: Conclusion The highest accessible level is $n = 3$, giving a maximum of 3 spectral lines .

Quick Tip to Solve: Compare the beam energy with adjacent excitation thresholds before counting level pairs.

8. How are electromagnetic waves produced by oscillating charges? What is the source of the energy associated with the em waves? [2 marks]

Solution:

Concept:

- An accelerating charge radiates electromagnetic energy. For an oscillating charge, its acceleration and the electric field around it change periodically.
- A time-varying electric field is associated with a magnetic field, and a time-varying magnetic field produces an electric field. In vacuum, the disturbance travels at $c = 1/\sqrt{\mu_0\epsilon_0}$.

Step 1: Production of the wave When a charge oscillates about its mean position, it undergoes acceleration. Its changing electric field is accompanied by a changing magnetic field. The coupled electric and magnetic disturbances propagate outwards as an electromagnetic wave. In the radiation region, $\vec{E}$ and $\vec{B}$ are mutually perpendicular and also perpendicular to the direction of propagation; the wave is transverse.

Step 2: Identify the energy source The wave carries energy away from the charge. This energy is supplied by the external agency that does work to maintain the oscillations, such as the alternating-voltage source driving charges in a transmitting antenna. If no external energy is supplied, radiation reduces the energy of the oscillator and its oscillations decay.

Step 3: Conclusion Electromagnetic waves are emitted by the accelerating, oscillating charge. Their energy comes from the work done by the source maintaining the oscillations (or from the oscillator's stored energy if it is undriven).

Quick Tip to Solve: Connect radiation to acceleration, then use energy conservation to identify the driving source.

9. The wavelength of light from the spectral emission line of sodium is 590 nm. Find the kinetic energy at which the electron would have the same de-Broglie wavelength. [2 marks]

Solution:

Concept:

- The de Broglie relation is $\lambda = h/p$, where h is Planck's constant and p is the electron momentum.
- For a non-relativistic electron, $K = p^2/(2m_e)$, so $K = h^2/(2m_e\lambda^2)$. The photon formula hc/λ does not give the electron's kinetic energy.

Step 1: Convert the wavelength and obtain momentum The required electron wavelength equals the sodium wavelength:

$$\lambda = 590 \text{ nm} = 5.90 \times 10^{-7} \text{ m}.$$

Hence

$$p = \frac{6.626 \times 10^{-34}}{5.90 \times 10^{-7}} = 1.123 \times 10^{-27} \text{ kg m s}^{-1}.$$

Quick Tip to Solve: For an electron with a known wavelength, use $h^2/(2m_e\lambda^2)$; square the wavelength after converting nm to m.

Step 2: Calculate the electron kinetic energy Using $m_e = 9.11 \times 10^{-31} \text{ kg}$,

$$\begin{aligned} K &= \frac{h^2}{2m_e\lambda^2} \\ &= \frac{(6.626 \times 10^{-34})^2}{2(9.11 \times 10^{-31})(5.90 \times 10^{-7})^2} \\ &= 6.92 \times 10^{-25} \text{ J}. \end{aligned}$$

Dividing by $1.602 \times 10^{-19} \text{ J/eV}$ gives $K = 4.32 \times 10^{-6} \text{ eV}$. As a check, $v = p/m_e \approx 1.23 \times 10^3 \text{ m s}^{-1} \ll c$, so the non-relativistic expression is valid.

Step 3: Conclusion The required kinetic energy is $K \approx 6.92 \times 10^{-25} \text{ J} = 4.32 \times 10^{-6} \text{ eV}$.

Quick Tip to Solve: For an electron with a known wavelength, use $h^2/(2m_e\lambda^2)$; square the wavelength after converting nm to m.

10. Draw the ray diagram of an astronomical telescope showing image formation in the normal adjustment position. Write the expression for its magnifying power. [2 marks]

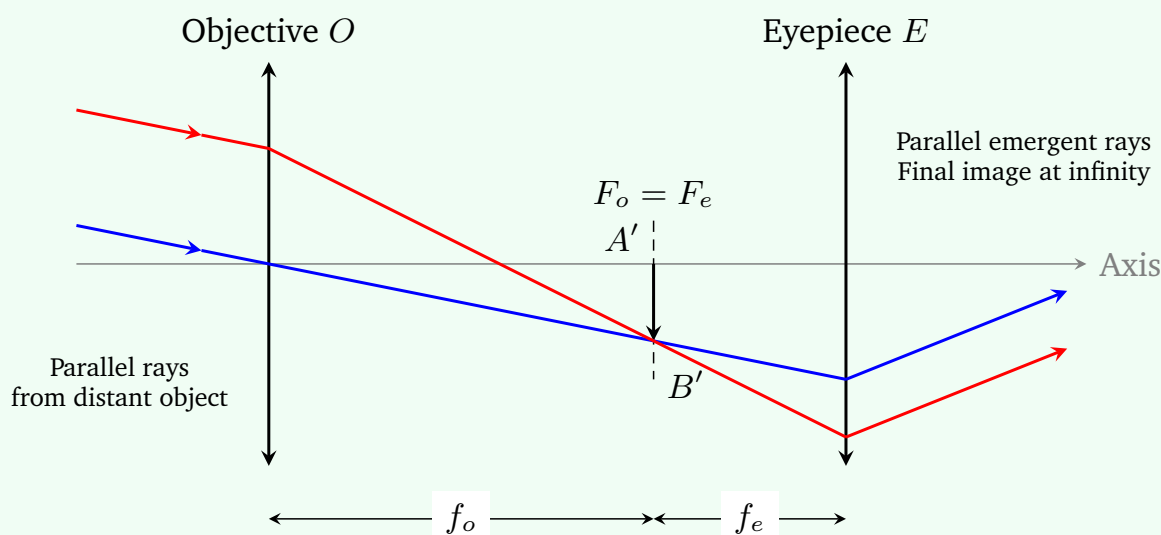
Solution:

Concept:

- In normal adjustment, both the distant object and the final image are at infinity. The objective forms a real image at its focal plane, which coincides with the first focal plane of the eyepiece.
- Angular magnifying power is $M = \beta/\alpha$, where α is the angle subtended by the distant object at the unaided eye and β is the angle subtended by the final image.

Step 1: Construct the image in normal adjustment Parallel rays from a distant off-axis object are brought to a focus by the objective O , producing the real, inverted intermediate image $A'B'$. Place the eyepiece E so that this image lies in its first focal plane. Rays leaving the eyepiece are then parallel, and the eye observes a final image at infinity. Thus the separation of the lenses is $f_o + f_e$.

Quick Tip to Solve: Place the intermediate image at the common focal plane: this immediately gives parallel output rays and lens separation $f_o + f_e$.



Astronomical telescope in normal adjustment; the common focal plane gives the final image at infinity.

Step 2: Express the angular magnification For intermediate image height h and small angles, the magnitudes obey $|\alpha| \simeq h/f_o$ and $|\beta| \simeq h/f_e$. Their ratio is therefore f_o/f_e . The final image is inverted relative to the distant object, so the signed angular magnification carries a negative sign.

Step 3: Conclusion In normal adjustment, $M = -f_o/f_e$, or magnifying power in magnitude $|M| = f_o/f_e$, where f_o and f_e are the focal lengths of the objective and eyepiece.

Quick Tip to Solve: Place the intermediate image at the common focal plane: this immediately gives parallel output rays and lens separation $f_o + f_e$.

OR

10. Draw a labelled ray diagram to show image formation by a compound microscope and write the expression for its resolving power. [2 marks]

Solution:

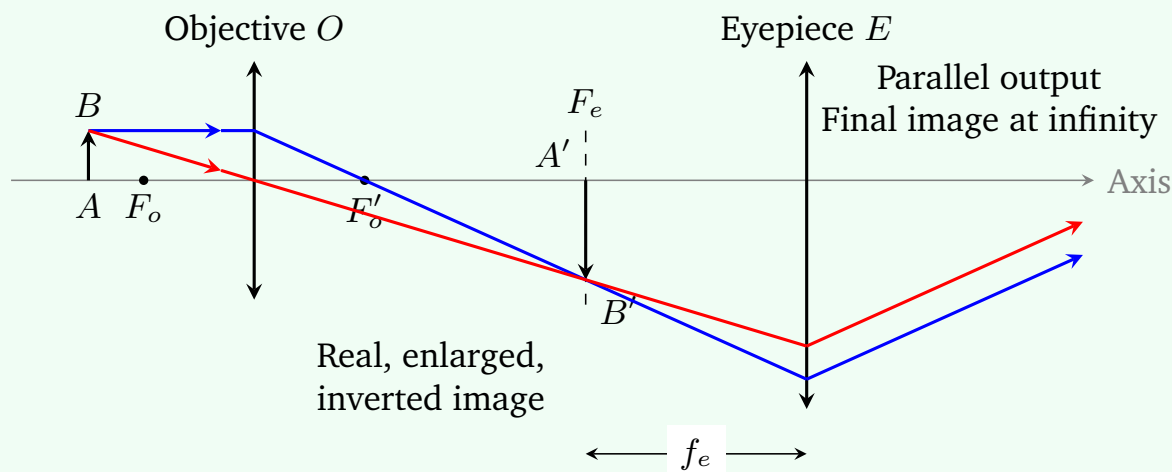
Concept:

- The objective of a compound microscope forms a real, enlarged, inverted intermediate image. The eyepiece acts as a magnifier; in normal adjustment the intermediate image is at its first focal plane and the final image is at infinity.
- Resolving power is the reciprocal of the minimum separation $d_{\min}$ of two distinguishable object points. The numerical aperture is $NA = n \sin \theta$, where n is the refractive index of the object-side medium and θ is the half-angle accepted by the objective.

Step 1: Draw and explain the image formation Place the small object AB just beyond the first focus F_o of the objective. One ray from B , parallel to the axis, refracts through F_o ; another passes through the optical centre of the objective undeviated. They meet at B' , giving the real intermediate image $A'B'$. In the normal-adjustment arrangement drawn below, this image is in the first focal plane F_e of

Quick Tip to Solve: Remember $RP = NA/(0.61\lambda)$: increase aperture or immersion index, or reduce wavelength, to improve resolution.

the eyepiece, so the emerging rays are parallel and the final magnified image is viewed at infinity.



Compound microscope in normal adjustment. The object lies just beyond the objective focus; the final image is at infinity.

Step 2: Write the diffraction-limited resolving power Using the Rayleigh criterion for a circular objective,

$$d_{\min} = \frac{1.22\lambda}{2n \sin \theta} = \frac{0.61\lambda}{n \sin \theta}.$$

Here λ is the wavelength in vacuum (approximately the wavelength in air). Thus a larger numerical aperture or a shorter wavelength permits smaller details to be distinguished. Resolving power concerns separation of details, whereas magnification concerns apparent image size.

Step 3: Conclusion The compound microscope has resolving power $\text{RP} = \frac{1}{d_{\min}} = \frac{2n \sin \theta}{1.22\lambda}$. The diagram shows a real intermediate image and the final image at infinity.

Quick Tip to Solve: Remember $\text{RP} = \text{NA}/(0.61\lambda)$: increase aperture or immersion index, or reduce wavelength, to improve resolution.

11. Explain the following:

- Sky appears blue.
- The Sun appears reddish at (i) sunset, (ii) sunrise.

[2 marks]

Solution:

Concept:

- Rayleigh scattering by atmospheric molecules is stronger for shorter wavelengths: scattered intensity $I_s \propto 1/\lambda^4$, where λ is the wavelength.
- The colour of scattered sunlight and the colour of directly transmitted sunlight need to be distinguished.

Step 1: (a) Why the sky appears blue Sunlight contains all visible colours. When it passes through the atmosphere, small air molecules scatter its shorter wavelengths much more strongly than its longer

Quick Tip to Solve: For sky colour consider scattered light; for the Sun near the horizon consider the light left in the direct beam.

wavelengths. Thus, blue light is scattered strongly into different directions and reaches our eyes from the sky. Red light is scattered much less. Although violet also scatters strongly, the solar spectrum and the lower sensitivity of our eyes to violet make the observed sky predominantly blue.

Step 2: (b) Why the Sun appears reddish at sunset and sunrise At both sunset and sunrise, the Sun is close to the horizon. Its rays travel through a much longer atmospheric path than when it is overhead. Most of the shorter-wavelength blue and violet light is scattered out of the direct beam before reaching the observer. The longer-wavelength red and orange components are scattered less and therefore dominate the light transmitted directly from the Sun to our eyes.

Step 3: Conclusion The sky appears blue because short-wavelength light is strongly scattered. At both sunset and sunrise, the long atmospheric path removes much of this light from the direct beam, making the Sun appear reddish.

Quick Tip to Solve: For sky colour consider scattered light; for the Sun near the horizon consider the light left in the direct beam.

12. Write the relation between the height of a TV antenna and the maximum range up to which signals transmitted by the antenna can be received. How is this expression modified in the case of line of sight communication by space waves? In which range of frequencies, is this mode of communication used? [2 marks]

Solution:

Concept:

- Space waves propagate approximately along straight paths, so Earth's curvature limits the line-of-sight range.
- Let R_E be Earth's radius and h_T, h_R the heights of transmitting and receiving antennas, with both heights small compared with R_E .

Step 1: Range of one TV transmitting antenna For an antenna of height h , the limiting ray just touches the Earth at the horizon. Right-triangle geometry gives $d^2 = (R_E + h)^2 - R_E^2 = 2R_E h + h^2$. Since $h \ll R_E$, the small h^2 term can be neglected. The maximum horizon range is therefore

$$d = \sqrt{2R_E h}.$$

Thus, increasing antenna height increases the range as its square root.

Step 2: Range with transmitting and receiving antennas Both elevated antennas extend the line of sight. The maximum separation is the sum of their individual horizon distances:

$$d_{\max} = \sqrt{2R_E h_T} + \sqrt{2R_E h_R}.$$

The second term accounts for the receiving antenna's horizon. The heights, radius and calculated distance must be expressed in consistent units.

Step 3: Frequency range Space-wave communication is used at frequencies above approximately 30 MHz, including VHF, UHF and microwave frequencies. At these frequencies, ionospheric reflection is generally ineffective for communication, while line-of-sight propagation is suitable for television

Quick Tip to Solve: Add the two horizon distances, not the two antenna heights inside a single square root.

and related links.

Step 4: Conclusion For one antenna, $d = \sqrt{2R_E h}$; for two elevated antennas, $d_{\max} = \sqrt{2R_E h_T} + \sqrt{2R_E h_R}$. This mode is used above approximately 30 MHz.

Quick Tip to Solve: Add the two horizon distances, not the two antenna heights inside a single square root.

Section C (36 marks)

13. (a) Draw the energy level diagram for the line spectra representing Lyman series and Balmer series in the spectrum of hydrogen atom.

(b) Using the Rydberg formula for the spectrum of hydrogen atom, calculate the largest and shortest wavelengths of the emission lines of the Balmer series in the spectrum of hydrogen atom. (Use the value of Rydberg constant $R = 1.1 \times 10^7 \text{ m}^{-1}$) [3 marks]

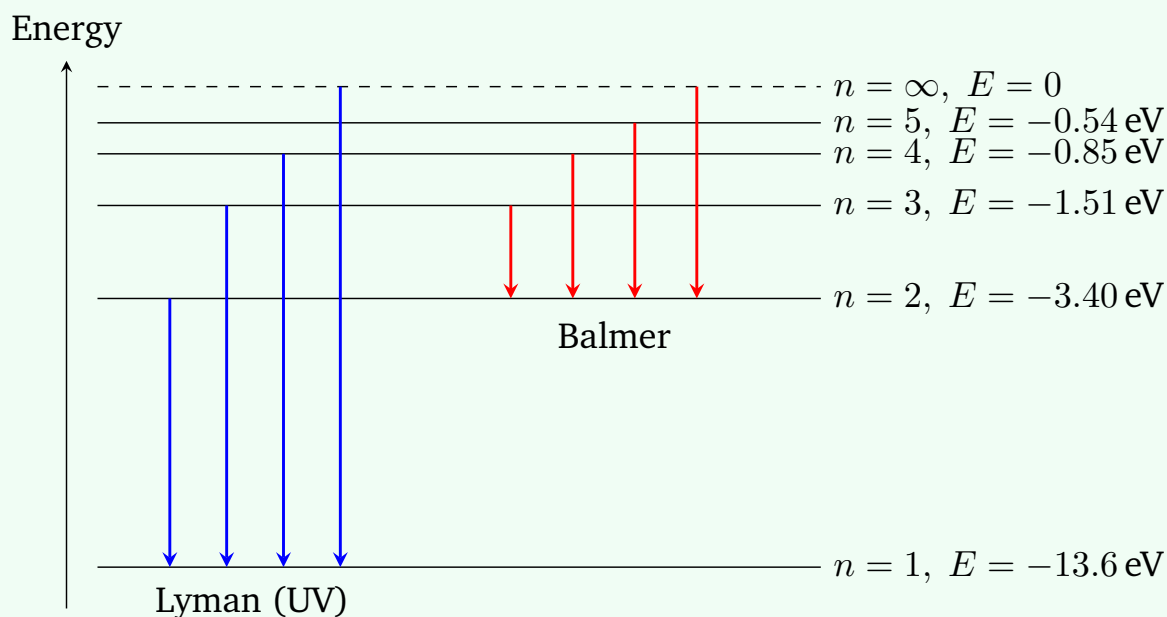
Solution:

Concept:

- Hydrogen energy levels obey $E_n = -13.6/n^2 \text{ eV}$, where n is the principal quantum number. Emission occurs when an electron falls to a lower level.
- The Rydberg relation is $1/\lambda = R(1/n_f^2 - 1/n_i^2)$, with $n_i > n_f$. Lyman lines end at $n_f = 1$, whereas Balmer lines end at $n_f = 2$.

Step 1: (a) Energy levels and transitions The diagram shows downward emission transitions. Lyman transitions begin at $n = 2, 3, 4, \dots$ and terminate at the ground level $n = 1$; these lie in the ultraviolet region. Balmer transitions begin at $n = 3, 4, 5, \dots$ and terminate at $n = 2$. The first Balmer lines lie in the visible region, and the series approaches an ultraviolet limit. The dashed zero-energy level represents the ionisation limit $n \rightarrow \infty$.

Quick Tip to Solve: Fix the final level at $n = 2$: use initial level 3 for the longest wavelength and infinity for the shortest.



Hydrogen energy levels and representative Lyman and Balmer emission transitions (vertical spacing schematic).

Step 2: (b) Largest Balmer wavelength The largest wavelength corresponds to the smallest photon energy and hence the smallest downward energy gap in the series: $n_i = 3 \rightarrow n_f = 2$. Using the supplied value $R = 1.1 \times 10^7 \text{ m}^{-1}$,

$$\frac{1}{\lambda_{\max}} = R \left(\frac{1}{4} - \frac{1}{9} \right) = \frac{5R}{36}.$$

Therefore,

$$\lambda_{\max} = \frac{36}{5(1.1 \times 10^7)} = 6.545 \times 10^{-7} \text{ m} \approx 655 \text{ nm}.$$

Step 3: (b) Shortest Balmer wavelength The shortest wavelength is the limiting value for the greatest energy gap to $n_f = 2$, obtained as $n_i \rightarrow \infty$. Since $1/n_i^2 \rightarrow 0$,

$$\frac{1}{\lambda_{\min}} = \frac{R}{4}, \quad \lambda_{\min} = \frac{4}{1.1 \times 10^7} = 3.636 \times 10^{-7} \text{ m} \approx 364 \text{ nm}.$$

This is the series limit; the wavelengths of the discrete Balmer emission lines approach it from above.

Step 4: Conclusion Lyman transitions terminate at $n = 1$, and Balmer transitions terminate at $n = 2$. For Balmer, the largest wavelength is approximately 655 nm and the shortest limiting wavelength is approximately 364 nm.

Quick Tip to Solve: Fix the final level at $n = 2$: use initial level 3 for the longest wavelength and infinity for the shortest.

14. A capacitor (C) and resistor (R) are connected in series with an ac source of voltage of frequency 50 Hz. The potential difference across C and R are respectively 120 V, 90 V, and the current in the circuit is 3 A. Calculate (i) the impedance of the circuit (ii) the value of the inductance, which when connected in series with C and R will make the power factor of the circuit unity. [3 marks]

Solution:

Concept:

- The resistor voltage is in phase with current, while the capacitor voltage lags current by 90° . Their rms voltages therefore add as perpendicular phasors.
- For a series RLC circuit, $Z = \sqrt{R^2 + (X_L - X_C)^2}$, where $X_L = \omega L$, $X_C = 1/(\omega C)$ and $\omega = 2\pi f$.
- The power factor is $\cos \phi = R/Z$. It equals unity when the net reactance vanishes: $X_L = X_C$.

Step 1: (i) Find the source voltage and impedance The stated values are $V_C = 120$ V, $V_R = 90$ V, $I = 3$ A and $f = 50$ Hz. Because the capacitor and resistor voltages are perpendicular in phase, the rms source voltage is

$$V = \sqrt{V_R^2 + V_C^2} = \sqrt{90^2 + 120^2} = 150 \text{ V.}$$

Hence the impedance magnitude of the original RC circuit is

$$Z = \frac{V}{I} = \frac{150}{3} = 50 \Omega.$$

Adding the voltages arithmetically would give an incorrect source voltage.

Step 2: (ii) Determine the capacitive reactance The same current flows through both components in series. Therefore,

$$R = \frac{V_R}{I} = \frac{90}{3} = 30 \Omega, \quad X_C = \frac{V_C}{I} = \frac{120}{3} = 40 \Omega.$$

As a consistency check, $\sqrt{R^2 + X_C^2} = \sqrt{30^2 + 40^2} = 50 \Omega$, agreeing with the impedance found above.

Step 3: (ii) Choose the inductance for unity power factor On adding an ideal inductor in series at the same source frequency, unity power factor requires cancellation of inductive and capacitive reactances. Thus,

$$\omega L = X_C, \quad L = \frac{X_C}{2\pi f} = \frac{40}{2\pi(50)} = \frac{0.4}{\pi} \text{ H} \approx 0.127 \text{ H.}$$

The resulting net reactance is zero and the circuit impedance becomes $R = 30 \Omega$; voltage and current are then in phase. The original 3 A current was used to determine the fixed capacitor reactance, not asserted to remain the current after the inductor is added.

Step 4: Conclusion The original circuit impedance is 50Ω . An ideal inductance $L \approx 0.127$ H connected in series gives unity power factor at 50 Hz.

Quick Tip to Solve: Use the 90-120-150 voltage triangle, then set the added inductive reactance equal to the original capacitor reactance.

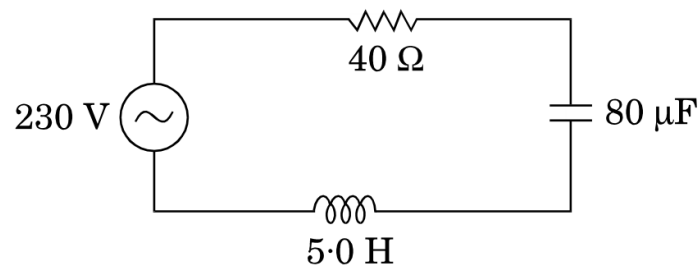
14. The figure shows a series LCR circuit connected to a variable frequency 230 V source.

(a) Determine the source frequency which drives the circuit in resonance.

(b) Calculate the impedance of the circuit and amplitude of current at resonance.

(c) Show that potential drop across LC combination is zero at resonating frequency.

[3 marks]



Solution:

Concept:

- In a series LCR circuit, resonance occurs when the inductive and capacitive reactances cancel: $X_L = \omega_0 L = X_C = 1/(\omega_0 C)$. Thus $\omega_0 = 1/\sqrt{LC}$ and $f_0 = \omega_0/(2\pi)$.
- The impedance is $Z = \sqrt{R^2 + (X_L - X_C)^2}$. The stated source voltage is its rms value; the current amplitude is $I_0 = \sqrt{2}V_{\text{rms}}/Z$.

Step 1: (a) Determine the resonant frequency The printed circuit has $L = 5.0 \text{ H}$, $C = 80 \mu\text{F} = 80 \times 10^{-6} \text{ F}$ and $R = 40 \Omega$. Therefore

$$\omega_0 = \frac{1}{\sqrt{5.0 \times 80 \times 10^{-6}}} = 50 \text{ rad s}^{-1}, \quad f_0 = \frac{50}{2\pi} = 7.96 \text{ Hz}.$$

Frequency in hertz is obtained by dividing angular frequency by 2π .

Step 2: (b) Find impedance and current amplitude At resonance, the net reactance is zero, so the circuit behaves as a pure resistance and voltage and current are in phase. Hence

$$Z_0 = R = 40 \Omega, \quad I_{\text{rms}} = \frac{230}{40} = 5.75 \text{ A}.$$

The requested amplitude is the peak current, rather than the rms current:

$$I_0 = \sqrt{2} I_{\text{rms}} = 5.75\sqrt{2} = 8.13 \text{ A}.$$

Step 3: (c) Show cancellation across LC At the resonant frequency, $X_L = 50 \times 5.0 = 250 \Omega$ and $X_C = 1/(50 \times 80 \times 10^{-6}) = 250 \Omega$. The inductor voltage leads the current by 90° , whereas the capacitor voltage lags by 90° ; their phasors are opposite, not in the same direction. Consequently

$$\tilde{V}_{LC} = jI(X_L - X_C) = 0.$$

The individual voltages need not vanish: it is their equal and opposite phasor sum across the combination that vanishes.

Quick Tip to Solve: At resonance set the net reactance to zero first, then distinguish rms current from the requested current amplitude by multiplying by $\sqrt{2}$.

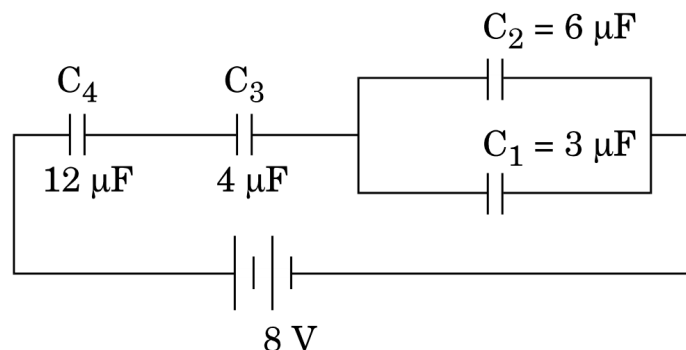
Step 4: Conclusion $f_0 = 7.96 \text{ Hz}$, $Z_0 = 40 \Omega$, $I_0 = 8.13 \text{ A}$, $V_{LC} = 0$.

Quick Tip to Solve: At resonance set the net reactance to zero first, then distinguish rms current from the requested current amplitude by multiplying by $\sqrt{2}$.

15. In a network, four capacitors C_1 , C_2 , C_3 and C_4 are connected as shown in the figure.

(a) Calculate the net capacitance in the circuit.

(b) If the charge on the capacitor C_1 is $6 \mu\text{C}$, (i) calculate the charge on the capacitors C_3 and C_4 , and (ii) net energy stored in the capacitors C_3 and C_4 connected in series. [3 marks]



Solution:

Concept:

- Parallel capacitors share a common potential difference and have equivalent capacitance equal to their sum. Series capacitors carry equal charge magnitudes; their reciprocal capacitances add.
- For a capacitor, $Q = CV$ and stored energy $U = Q^2/(2C)$, where Q is charge magnitude, V is its potential difference and C is capacitance.

Step 1: (a) Reduce the network The figure gives $C_1 = 3 \mu\text{F}$, $C_2 = 6 \mu\text{F}$, $C_3 = 4 \mu\text{F}$, and $C_4 = 12 \mu\text{F}$. Since C_1 and C_2 connect to the same two junctions, their parallel equivalent is $C_{12} = 3 + 6 = 9 \mu\text{F}$. This equivalent is in series with C_3 and C_4 :

$$\frac{1}{C_{\text{net}}} = \frac{1}{9} + \frac{1}{4} + \frac{1}{12} = \frac{4 + 9 + 3}{36} = \frac{4}{9} \quad (\mu\text{F})^{-1}.$$

Thus the equivalent capacitance is obtained by taking the reciprocal.

Step 2: (b)(i) Determine the series charges Given $Q_1 = 6 \mu\text{C}$, the voltage across the parallel pair is

$$V_{12} = \frac{Q_1}{C_1} = \frac{6}{3} = 2 \text{ V}.$$

The other parallel capacitor carries $Q_2 = C_2 V_{12} = 6 \times 2 = 12 \mu\text{C}$. The total charge entering this parallel combination is $Q_{12} = Q_1 + Q_2 = 18 \mu\text{C}$. Each of the series capacitors therefore carries this same magnitude: $Q_3 = Q_4 = 18 \mu\text{C}$. This also agrees with the supplied 8V battery, since $V_3 = 18/4 = 4.5 \text{ V}$, $V_4 = 18/12 = 1.5 \text{ V}$, and $V_{12} + V_3 + V_4 = 8 \text{ V}$.

Quick Tip to Solve: For the series charge, add the charges on the parallel capacitors first; do not set the charge on each series capacitor equal to Q_1 alone.

Step 3: (b)(ii) Add the stored energies Only the energies of C_3 and C_4 are requested. Their equal charge magnitudes give

$$\begin{aligned}U_{34} &= \frac{Q_3^2}{2C_3} + \frac{Q_4^2}{2C_4} \\&= \frac{(18 \times 10^{-6})^2}{2} \left(\frac{1}{4 \times 10^{-6}} + \frac{1}{12 \times 10^{-6}} \right) \\&= 40.5 \times 10^{-6} + 13.5 \times 10^{-6} = 54 \times 10^{-6} \text{ J}.\end{aligned}$$

Energies add as positive scalar quantities, unlike signed capacitor plate charges.

Step 4: Conclusion $C_{\text{net}} = 2.25 \mu\text{F}, \quad Q_3 = Q_4 = 18 \mu\text{C}, \quad U_{34} = 54 \mu\text{J}.$

Quick Tip to Solve: For the series charge, add the charges on the parallel capacitors first; do not set the charge on each series capacitor equal to Q_1 alone.

16. Draw a labelled diagram of cyclotron. Explain its working principle. Show that cyclotron frequency is independent of the speed and radius of the orbit. [3 marks]

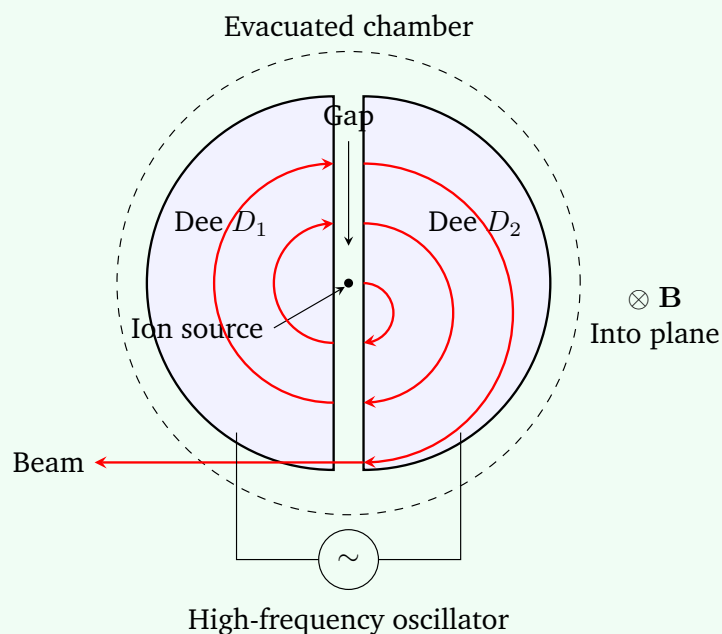
Solution:

Concept:

- A cyclotron accelerates charged particles using an alternating electric field across the gap between two hollow D-shaped electrodes (dees). A uniform magnetic field perpendicular to their plane supplies the centripetal force.
- For nonrelativistic motion, $|q|vB = mv^2/r$, where m is particle mass, q is charge, v is speed and r is the orbit radius. The magnetic force changes direction but does no work.

Step 1: Draw the arrangement and state the principle The dees lie in an evacuated chamber between the poles of an electromagnet. The uniform magnetic field is normal to the plane of the diagram. A high-frequency alternating voltage is applied between the dees, producing an electric field mainly in their narrow gap. The ion source is near the centre. The diagram labels the dees, accelerating gap, source, magnetic field and outgoing beam.

Quick Tip to Solve: Use the magnetic-force equation to replace r/v in the orbital period; this immediately removes both speed and radius.



Cyclotron arrangement viewed along the magnetic field

Step 2: Explain the repeated acceleration A charged particle starting near the centre gains kinetic energy while crossing the gap. Inside a hollow dee the electric field is screened; the magnetic force bends its path into a semicircle at constant speed. Before its next gap crossing, the alternating voltage reverses polarity, so the electric field again accelerates the particle. This repeats each half-turn. As its speed increases, $r = mv/(|q|B)$ increases, so successive semicircles form an outward spiral. At the outer edge the energetic particles are extracted as a beam. The alternating source must match the orbital frequency to maintain this repeated acceleration.

Step 3: Derive the cyclotron frequency Equating the magnetic force to the centripetal force gives

$$|q|vB = \frac{mv^2}{r}, \quad \frac{r}{v} = \frac{m}{|q|B}.$$

The time for a full revolution and the corresponding frequency are

$$T = \frac{2\pi r}{v} = \frac{2\pi m}{|q|B}, \quad f_c = \frac{1}{T} = \frac{|q|B}{2\pi m}.$$

Thus every half-revolution takes $T/2 = \pi m/(|q|B)$, allowing the gap voltage to reverse at the correct time. Neither v nor r appears in the frequency: their ratio stays constant as the particle gains speed. This result assumes nonrelativistic speeds, so that the particle mass is constant.

Step 4: Conclusion The cyclotron works by repeated electric acceleration in the gap and magnetic bending inside the dees. Its frequency is $f_c = |q|B/(2\pi m)$, independent of orbit speed and radius for nonrelativistic motion.

Quick Tip to Solve: Use the magnetic-force equation to replace r/v in the orbital period; this immediately removes both speed and radius.

16. (a) Derive, with the help of a diagram, the expression for the magnetic field inside a very long solenoid having n turns per unit length carrying a current I .

(b) How is a toroid different from a solenoid?

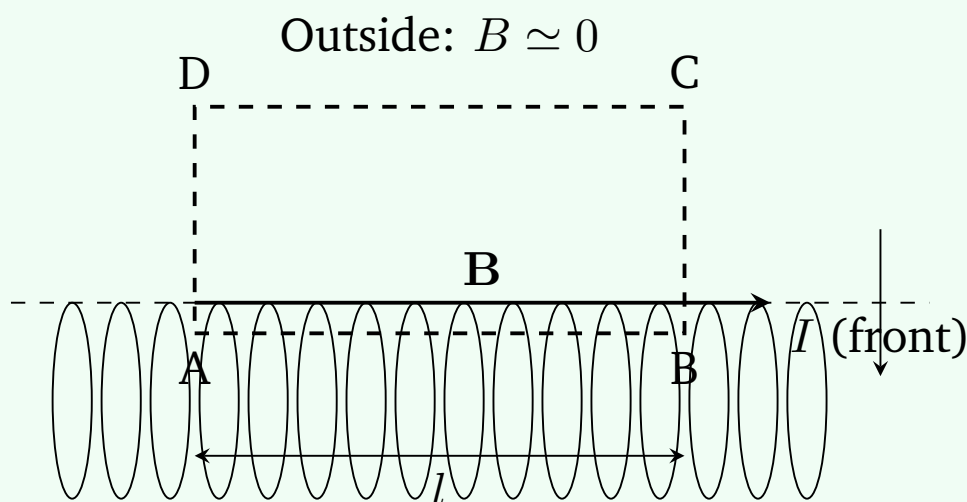
[3 marks]

Solution:

Concept:

- Ampere's circuital law is $\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I_{\text{enc}}$, where μ_0 is the permeability of free space and I_{enc} is the enclosed current.
- A very long air-core solenoid has an approximately uniform axial field inside and negligible field outside, away from its ends. Here n is the number of turns per unit length and I the current per turn.

Step 1: Choose an Amperian rectangle Consider a rectangular loop ABCD whose side AB of length l lies inside the solenoid parallel to its axis. Side CD lies outside. The other two sides are perpendicular to the field. The direction shown follows the right-hand grip rule: the curled fingers follow the winding current and the thumb gives the axial field.



Long solenoid and rectangular Amperian loop ABCD

Step 2: Apply the circuital law The contributions on BC and DA vanish because $\mathbf{B} \perp d\mathbf{l}$. The contribution on CD is negligible for a very long solenoid. The enclosed number of turns is nl , and each carries current I . Consequently,

$$\oint \mathbf{B} \cdot d\mathbf{l} = Bl = \mu_0(nl)I.$$

Cancelling the arbitrary nonzero length l gives the magnetic field. This ideal expression is valid well inside, sufficiently far from the end regions.

Step 3: Compare the two windings A solenoid is a long straight cylindrical coil; its field lines run along its axis inside and return through the exterior, with end effects near its open ends. A toroid is formed by bending such a coil into a closed circular ring. It has no free ends and the magnetic field

Quick Tip to Solve: Choose one long Amperian side inside and the other outside so only the inside side contributes.

follows circular paths within the ring. For an ideal toroid with N turns, $B(r) = \mu_0 NI / (2\pi r)$ in its core, whereas the field in the central empty hole and outside is negligible. Thus the toroidal field varies with radial distance r , unlike the approximately uniform field in a long straight solenoid.

Step 4: Conclusion The field inside a very long air-core solenoid is $B = \mu_0 nI$, directed along its axis. A toroid is a closed ring-shaped solenoid with circular core field lines and no free ends.

Quick Tip to Solve: Choose one long Amperian side inside and the other outside so only the inside side contributes.

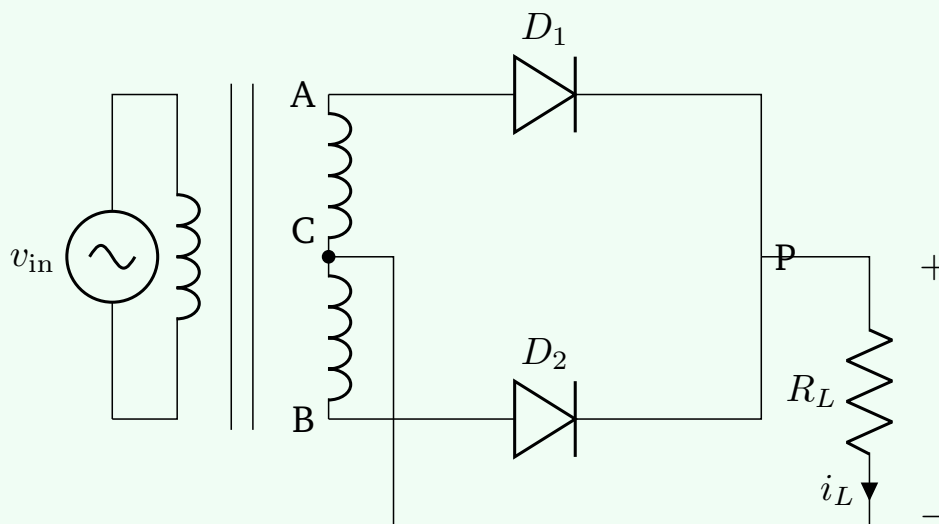
17. Draw the circuit diagram of a full wave rectifier. Explain its working principle. Show the input waveforms given to the diodes D_1 and D_2 and the corresponding output waveforms obtained at the load connected to the circuit. [3 marks]

Solution:

Concept:

- A p-n junction diode conducts readily when forward biased and blocks current when reverse biased. Rectification uses this directional conduction to convert alternating input into unidirectional output.
- In a centre-tapped full wave rectifier, the two equal secondary halves supply opposite voltages relative to the centre tap C: $v_A = V_m \sin \omega t$ and $v_B = -V_m \sin \omega t$. Ideal diodes are assumed.

Step 1: Connect the full wave rectifier The centre-tapped transformer secondary ends A and B connect to the anodes of D_1 and D_2 , respectively. Their cathodes join at the positive output node P. The load R_L connects from P to centre tap C. The transformer primary is connected to the alternating supply. Thus either secondary half can supply the load through its associated diode.



Centre-tapped transformer and two-diode full wave rectifier

Step 2: Positive half cycle When A is positive relative to C, B is negative. Diode D_1 is forward biased and conducts, while D_2 is reverse biased and blocks. The current path is A, D_1 , P, R_L , C and the upper secondary half back to A. The load current flows from P to C, making the upper end of the load

Quick Tip to Solve: Trace each half-cycle path through the load: both must pass from the joined cathodes towards the centre tap.

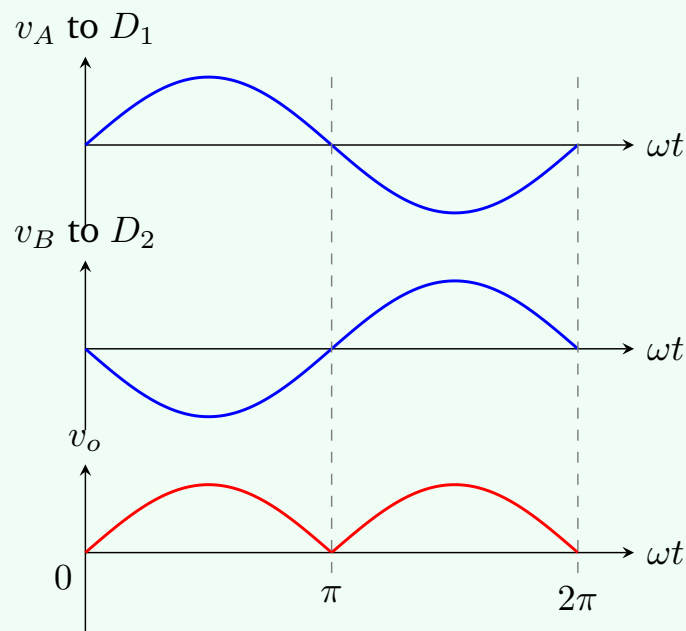
positive.

Step 3: Negative half cycle In the next half cycle B is positive relative to C and A is negative. Now D_2 conducts and D_1 blocks. The current path is B, D_2 , P, R_L , C and the lower secondary half back to B. The direction through the load is again P to C. Both halves of the alternating input therefore deliver current in the same direction through the load.

Step 4: Relate the waveforms The two diode input waveforms, measured at their anodes relative to C, differ in phase by π . The output follows the positive half of either secondary voltage:

$$v_o(t) = V_m |\sin \omega t|, \quad i_L(t) = \frac{V_m}{R_L} |\sin \omega t|.$$

The waveform is pulsating unidirectional voltage, with two positive pulses per input cycle, so its ripple frequency is $2f$ for supply frequency f . Without a smoothing filter it is not a constant voltage.



Opposite-phase diode inputs and full-wave load output

Step 5: Conclusion The centre-tapped full wave rectifier uses D_1 and D_2 alternately in successive half cycles; its load voltage is $v_o = V_m |\sin \omega t|$, with load current always in the same direction.

Quick Tip to Solve: Trace each half-cycle path through the load: both must pass from the joined cathodes towards the centre tap.

18. Two large charged plane sheets of charge densities σ and -2σ C/m² are arranged vertically with a separation of d between them. Deduce expressions for the electric field at points (i) to the left of the first sheet, (ii) to the right of the second sheet, and (iii) between the two sheets. [3 marks]

Solution:

Concept:

- An infinite plane sheet of surface charge density σ_s produces a field of magnitude $|\sigma_s|/(2\epsilon_0)$ on each side. It points away from a positive sheet and towards a negative sheet.
- The net field is the vector sum of the two sheet fields. Let the first sheet be at $x = 0$, the second at $x = d$, and take $+\hat{x}$ to the right. Assume $\sigma > 0$.

Step 1: Fields to the left of the first sheet For $x < 0$, the positive sheet contributes $-\sigma\hat{x}/(2\epsilon_0)$, away from itself. The negative sheet attracts a positive test charge towards the right and contributes $+2\sigma\hat{x}/(2\epsilon_0)$. Thus

$$\mathbf{E}_{\text{left}} = \left(-\frac{\sigma}{2\epsilon_0} + \frac{2\sigma}{2\epsilon_0} \right) \hat{x} = \frac{\sigma}{2\epsilon_0} \hat{x}.$$

Step 2: Fields to the right of the second sheet For $x > d$, the positive sheet field points right and the negative sheet field points left. Their magnitudes remain unchanged because a very large plane sheet has a distance-independent field. Therefore

$$\mathbf{E}_{\text{right}} = \left(\frac{\sigma}{2\epsilon_0} - \frac{2\sigma}{2\epsilon_0} \right) \hat{x} = -\frac{\sigma}{2\epsilon_0} \hat{x}.$$

Step 3: Fields between the sheets For $0 < x < d$, the field of the positive sheet points right, away from the sheet, and the field of the negative sheet also points right, towards it. Hence the contributions add:

$$\mathbf{E}_{\text{between}} = \left(\frac{\sigma}{2\epsilon_0} + \frac{2\sigma}{2\epsilon_0} \right) \hat{x} = \frac{3\sigma}{2\epsilon_0} \hat{x}.$$

The separation d specifies the extent of the middle region; it does not enter these field magnitudes when edge effects are neglected.

Step 4: Conclusion (i) Left: $\sigma/(2\epsilon_0)$ to the right. (ii) Right: $\sigma/(2\epsilon_0)$ to the left. (iii) Between: $3\sigma/(2\epsilon_0)$, from the positive sheet towards the negative sheet.

Quick Tip to Solve: Draw an away arrow for the positive sheet and a towards arrow for the negative sheet before adding.

18. A spherical conducting shell of inner radius r_1 and outer radius r_2 has a charge Q .

(a) A charge q is placed at the centre of the shell. Find out the surface charge density on the inner and outer surfaces of the shell.

(b) Is the electric field inside a cavity (with no charge) zero, independent of the fact whether the shell is spherical or not? Explain. [3 marks]

Solution:

Concept:

- In electrostatic equilibrium, the electric field in the conducting material is zero. Gauss's law is $\oint \vec{E} \cdot d\vec{S} = Q_{\text{enc}}/\epsilon_0$.
- The shell retains its total charge Q ; induced charges redistribute between its surfaces. For a spherical surface with uniform charge, $\sigma = Q_{\text{surface}}/(4\pi r^2)$.

Step 1: (a) Determine the charge on the inner surface Choose a spherical Gaussian surface within the conducting material, between radii r_1 and r_2 . Its electric flux is zero because $\vec{E} = 0$ there. If the inner surface charge is Q_{in} , Gauss's law gives

$$q + Q_{\text{in}} = 0, \quad Q_{\text{in}} = -q.$$

The central position of the charge gives spherical symmetry, so the induced charge is uniformly distributed over the inner surface.

Step 2: (a) Use conservation of the shell charge The charge q inside the cavity is separate from the charge Q supplied to the conductor. Therefore

$$Q_{\text{in}} + Q_{\text{out}} = Q, \quad Q_{\text{out}} = Q + q.$$

Spherical symmetry also makes the outer surface distribution uniform. Divide these charges by the corresponding areas, $4\pi r_1^2$ and $4\pi r_2^2$, respectively.

Step 3: (b) Explain the field in an empty cavity Yes, an empty closed cavity has zero electrostatic field, whatever its shape. The whole conductor, including the cavity wall, is at one potential V_0 . With no charge inside, the cavity potential obeys Laplace's equation. The constant solution $V = V_0$ satisfies the boundary condition and is unique. Thus $\vec{E} = -\nabla V = 0$ everywhere in the empty cavity. Zero net enclosed charge alone would not prove this: the equipotential boundary is essential.

Step 4: Conclusion The surface charge densities are

$$\sigma_{\text{in}} = -\frac{q}{4\pi r_1^2}, \quad \sigma_{\text{out}} = \frac{Q + q}{4\pi r_2^2}.$$

An empty closed cavity in a conductor has $\boxed{E = 0}$ in electrostatic equilibrium, independently of its shape.

Quick Tip to Solve: First set the enclosed charge inside the metal to zero; then conserve the conductor's own charge to find the outer-surface charge.

19. Prove that the magnetic moment of the electron revolving around a nucleus in an orbit of radius r with orbital speed v is equal to $evr/2$. Hence using Bohr's postulate of quantization of angular momentum, deduce the expression for the magnetic moment of hydrogen atom in the ground state.

[3 marks]

Solution:

Concept:

- An electron revolving periodically constitutes a current loop. The magnitude of the current is $I = e/T$, where e is the magnitude of the electron charge and T its orbital period.
- The magnetic moment magnitude of a planar current loop is $\mu = IA$. Bohr quantization gives $m_e v r = nh/(2\pi) = n\hbar$.

Step 1: Find the equivalent orbital current For a circular orbit of radius r , one revolution covers distance $2\pi r$. At speed v , the period and equivalent current magnitude are

$$T = \frac{2\pi r}{v}, \quad I = \frac{e}{T} = \frac{ev}{2\pi r}.$$

Conventional current flows opposite to the electron's motion because the electron has negative charge. The current magnitude uses the positive charge magnitude e .

Step 2: Obtain the magnetic moment The area enclosed by the circular orbit is $A = \pi r^2$. Hence the current-loop formula yields

$$\mu = IA = \frac{ev}{2\pi r} \pi r^2 = \frac{evr}{2}.$$

The orbital angular momentum has magnitude $L = m_e v r$, so equivalently $\mu = eL/(2m_e)$. In vector form, $\vec{\mu} = -e\vec{L}/(2m_e)$: the magnetic moment is opposite to the angular momentum.

Step 3: Apply Bohr quantization to the ground state Bohr's postulate gives $m_e v r = nh/(2\pi)$. Substituting $vr = nh/(2\pi m_e)$ into the moment yields

$$\mu_n = \frac{e}{2} \frac{nh}{2\pi m_e} = \frac{neh}{4\pi m_e}.$$

The ground state has $n = 1$. Using $e = 1.60 \times 10^{-19}$ C, $h = 6.63 \times 10^{-34}$ J s and $m_e = 9.11 \times 10^{-31}$ kg gives the Bohr magneton, approximately 9.27×10^{-24} J T⁻¹.

Step 4: Conclusion Thus $\mu = evr/2$, and the ground-state magnetic moment in Bohr's orbital model is

$$\mu_1 = \frac{eh}{4\pi m_e} = \frac{e\hbar}{2m_e} = 9.27 \times 10^{-24} \text{ J T}^{-1}.$$

Quick Tip to Solve: After deriving the loop moment, replace the product vr directly using Bohr quantization; calculating r and v separately is unnecessary.

20. Prove that in a common-emitter amplifier, the output and input differ in phase by 180° .

In a transistor, the change of base current by $30\text{ }\mu\text{A}$ produces change of 0.02 V in the base-emitter voltage and a change of 4 mA in the collector current. Calculate the current amplification factor and the load resistance used, if the voltage gain of the amplifier is 400. [3 marks]

Solution:

Concept:

- In a common-emitter amplifier, the input is across base and emitter and the output is across collector and emitter. The collector voltage is $V_{CE} = V_{CC} - I_C R_L$.
- The small-signal current gain is $\beta = \Delta I_C / \Delta I_B$, the input resistance is $r_i = \Delta V_{BE} / \Delta I_B$, and $|A_v| = \beta R_L / r_i$.

Step 1: Show the phase reversal A positive input variation increases V_{BE} , which increases the base current and consequently the collector current in the active region. The voltage drop $I_C R_L$ across the collector load increases. Since V_{CC} is constant, the collector-emitter output voltage decreases. A negative input variation produces the reverse changes. Algebraically,

$$\Delta V_{CE} = -R_L \Delta I_C = -\beta R_L \Delta I_B, \quad \Delta V_{BE} = r_i \Delta I_B.$$

Thus $A_v = \Delta V_{CE} / \Delta V_{BE} = -\beta R_L / r_i$. For sinusoidal signals, the negative sign means a phase difference of 180° .

Step 2: Calculate the current amplification factor Convert both current changes to amperes before dividing:

$$\beta = \frac{4 \times 10^{-3}}{30 \times 10^{-6}} = \frac{400}{3} \simeq 133.3.$$

This gain is dimensionless. The input resistance obtained from the given voltage and base-current changes is

$$r_i = \frac{0.02}{30 \times 10^{-6}} = 666.7\text{ }\Omega.$$

Step 3: Determine the collector load resistance The quoted voltage gain 400 is its magnitude; the signed common-emitter gain is negative. The output-voltage change therefore has magnitude

$$|\Delta V_{CE}| = 400(0.02) = 8\text{ V}.$$

Since $|\Delta V_{CE}| = R_L |\Delta I_C|$,

$$R_L = \frac{8}{4 \times 10^{-3}} = 2000\text{ }\Omega.$$

Equivalently, $R_L = |A_v| r_i / \beta$ gives the same result.

Step 4: Conclusion The output is 180° out of phase with the input. The current amplification factor is $\beta \simeq 133.3$, and the load resistance is $R_L = 2.0\text{ k}\Omega$.

Quick Tip to Solve: Find the output-voltage change as gain times input-voltage change, then divide by the collector-current change to obtain the load directly.

21. (a) When a convex lens of focal length 30 cm is in contact with a concave lens of focal length 20 cm, find out if the system is converging or diverging.

(b) Obtain the expression for the angle of incidence of a ray of light which is incident on the face of a prism of refracting angle A so that it suffers total internal reflection at the other face. (Given the refractive index of the glass of the prism is μ .) [3 marks]

Solution:

Concept:

- For thin lenses in contact, $1/F = 1/f_1 + 1/f_2$. A convex lens has positive focal length and a concave lens has negative focal length.
- In a prism, the internal angles with the face normals obey $r_1 + r_2 = A$. For a glass-air boundary the critical angle is $C = \sin^{-1}(1/\mu)$, and total internal reflection requires $r_2 > C$.

Step 1: (a) Combine the signed focal lengths Take $f_1 = +30$ cm and $f_2 = -20$ cm. Therefore

$$\frac{1}{F} = \frac{1}{30} - \frac{1}{20} = \frac{2-3}{60} = -\frac{1}{60} \text{ cm}^{-1}, \quad F = -60 \text{ cm}.$$

The negative equivalent focal length means that parallel incident rays emerge diverging. The concave lens has the larger power magnitude, so it determines the nature of the combination.

Step 2: (b) Express the internal incidence at the second face Let i be the external angle of incidence at the first face and r_1 its refracted angle, both measured from the corresponding normal. The incidence angle inside the prism at the second face is $r_2 = A - r_1$. Snell's law for entry from air gives

$$\sin i = \mu \sin r_1, \quad r_1 = \sin^{-1}\left(\frac{\sin i}{\mu}\right).$$

Total internal reflection is possible only from the denser glass into the rarer air, with the internal incidence strictly greater than the critical angle.

Step 3: (b) Derive the limiting incident angle and condition At the limiting condition, $r_2 = C$, so $r_1 = A - C$. Substitution into Snell's law gives

$$\sin i_{\text{lim}} = \mu \sin(A - C), \quad i_{\text{lim}} = \sin^{-1}\left[\mu \sin\left(A - \sin^{-1}\frac{1}{\mu}\right)\right].$$

For an accessible limiting angle, total internal reflection requires $i < i_{\text{lim}}$, because increasing i increases r_1 and decreases r_2 . Equality gives critical refraction along the second face, rather than total internal reflection. The general condition, also covering cases where no physical limiting angle exists, is

$$A - \sin^{-1}\left(\frac{\sin i}{\mu}\right) > \sin^{-1}\left(\frac{1}{\mu}\right).$$

Step 4: Conclusion The lens combination is diverging, $F = -60$ cm. For the prism, with $C = \sin^{-1}(1/\mu)$, the limiting angle is $i_{\text{lim}} = \sin^{-1}[\mu \sin(A - C)]$; total internal reflection occurs when $r_2 > C$, or $i < i_{\text{lim}}$ when this limiting angle is physically accessible.

Quick Tip to Solve: Use signed focal lengths for the lenses. For the prism first set the second-face incidence to the critical angle, then apply Snell's law at entry.

22. Why is it difficult to detect the presence of an anti-neutrino during beta-decay? Define the term decay constant of a radioactive nucleus and derive the expression for its mean life in terms of the decay constant. [3 marks]

Solution:

Concept:

- An anti-neutrino is electrically neutral and interacts extremely weakly with matter, so it produces almost no observable ionisation while passing through a detector.
- Radioactive decay obeys $dN/dt = -\lambda N$, where N is the number of undecayed nuclei and λ is the decay constant.
- The mean life is the average lifetime of all nuclei in an initially radioactive sample, weighted by the number decaying in each time interval.

Step 1: Explain the difficulty of detecting an anti-neutrino In beta-minus decay an electron and an anti-neutrino are emitted along with the daughter nucleus. The anti-neutrino has no electric charge, and its interaction with matter is extremely weak. It therefore passes through a large thickness of matter with a very small probability of interaction. Unlike the emitted beta particle, it does not readily leave an ionisation track, which makes direct detection difficult.

Step 2: Define the decay constant The decay constant is the probability per unit time that an undecayed nucleus will decay. Equivalently, it is the fractional decay rate of a sample:

$$\lambda = -\frac{1}{N} \frac{dN}{dt}.$$

Its SI unit is s^{-1} . Integrating the decay law for $N(0) = N_0$ gives $N(t) = N_0 e^{-\lambda t}$.

Step 3: Average the lifetimes of the nuclei The positive number of nuclei decaying between t and $t + dt$ is $dN_{\text{dec}} = \lambda N_0 e^{-\lambda t} dt$. These nuclei each have lifetime t . Thus

$$\begin{aligned} \tau &= \frac{1}{N_0} \int_0^\infty t dN_{\text{dec}} = \lambda \int_0^\infty t e^{-\lambda t} dt, \\ \int_0^\infty t e^{-\lambda t} dt &= \left[-\frac{t e^{-\lambda t}}{\lambda} \right]_0^\infty + \frac{1}{\lambda} \int_0^\infty e^{-\lambda t} dt = \frac{1}{\lambda^2}. \end{aligned}$$

The boundary term vanishes because the exponential tends to zero faster than t grows.

Step 4: Conclusion Anti-neutrinos are difficult to detect because they are neutral and interact extremely weakly with matter. The decay constant is the decay probability per unit time, and the mean life is $\tau = 1/\lambda$.

Quick Tip to Solve: In the mean-life integral use the positive number decaying, $-dN$, rather than the negative change in the surviving population.

22. (a) State two distinguishing features of nuclear force.

(b) Draw a plot showing the variation of potential energy of a pair of nucleons as a function of their separation. Mark the regions on the graph where the force is (i) attractive, and (ii) repulsive.

[3 marks]

Solution:

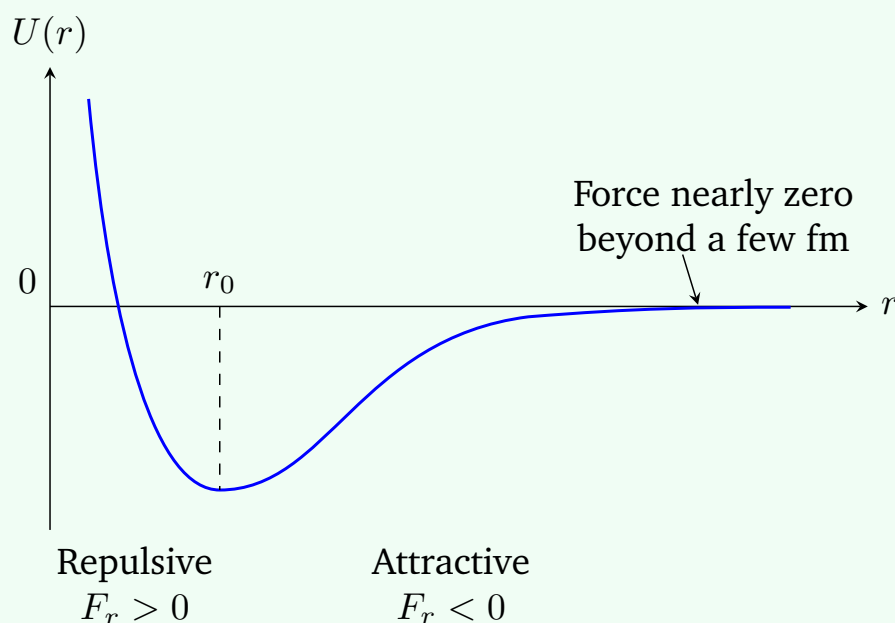
Concept:

- Nuclear force binds nucleons over distances comparable to nuclear dimensions. It is very strong but has a short range of a few femtometres.
- For a pair potential $U(r)$, the radial force is $F_r = -dU/dr$. A positive radial force is repulsive; a negative radial force is attractive.

Step 1: State two distinguishing features of nuclear force First, nuclear force is a very strong, short-range interaction: it is significant only when the separation of nucleons is of the order of a few femtometres and becomes negligible at larger separations. Its strong attraction over nuclear distances can bind protons despite their electrostatic repulsion. Second, it is approximately charge independent. After allowing for the electrostatic interaction of protons, the nuclear interaction is approximately the same for proton-proton, neutron-neutron and proton-neutron pairs in comparable states. Thus electric charge does not determine the nuclear attraction.

Step 2: Draw and interpret the potential-energy curve Taking the potential energy at infinite separation as zero, the curve has a steep positive core at very small separation, a negative minimum at $r = r_0$, and then approaches zero from below. The diagram is schematic; r_0 is of the order of 1 fm. To the left of the minimum, $dU/dr < 0$, so $F_r > 0$: the force pushes the nucleons apart and is repulsive. To the right of the minimum, where the potential rises towards zero, $dU/dr > 0$, so $F_r < 0$: the force pulls the nucleons together and is attractive. At the minimum the slope and hence the force are zero. At separations beyond a few femtometres both the potential and force are approximately zero.

Quick Tip to Solve: Identify attraction from the slope of the potential, using $F_r = -dU/dr$; the sign of potential energy alone does not decide the force.



Schematic nucleon potential: repulsive core and attractive outer region.

Step 3: Conclusion Nuclear force is strong and short ranged, and approximately charge independent. The potential-energy graph has a repulsive inner region ($r < r_0$) and an attractive outer region ($r > r_0$) within its short range.

Quick Tip to Solve: Identify attraction from the slope of the potential, using $F_r = -dU/dr$; the sign of potential energy alone does not decide the force.

23. Show, on a plot, variation of resistivity of (i) a conductor, and (ii) a typical semiconductor as a function of temperature.

Using the expression for the resistivity in terms of number density and relaxation time between the collisions, explain how resistivity in the case of a conductor increases while it decreases in a semiconductor, with the rise of temperature. [3 marks]

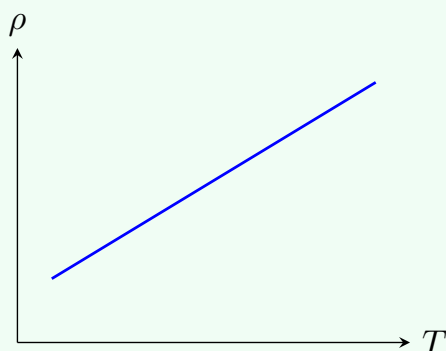
Solution:

Concept:

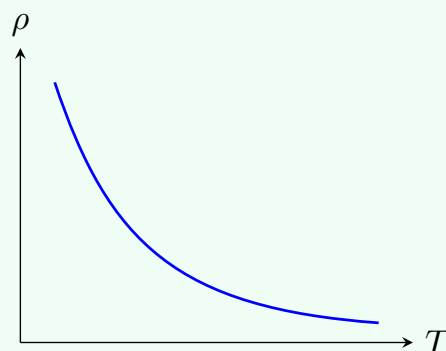
- The drift model gives $\rho = m/(ne^2\tau)$, where m is carrier mass, n is carrier number density, e is the magnitude of carrier charge and τ is the mean relaxation time between collisions.
- A metal has an approximately fixed free-electron density; a semiconductor gains charge carriers when heating excites electrons across its energy gap.

Step 1: Sketch the two temperature dependences On each graph the vertical axis is resistivity ρ and the horizontal axis is temperature T . Over an ordinary temperature interval the resistivity of a metallic conductor rises approximately linearly, so it has a positive temperature coefficient. A typical semiconductor has a falling, nonlinear resistivity curve and a negative temperature coefficient. These are qualitative plots, with separate vertical scales because the magnitudes need not be comparable.

Quick Tip to Solve: In $\rho = m/(ne^2\tau)$, check both n and τ : heating changes mainly τ in a metal but greatly increases n in a semiconductor.



(i) Conductor



(ii) Semiconductor

Qualitative resistivity-temperature plots, on independent vertical scales.

Step 2: Explain the increase for a conductor From the drift-speed relation $v_d = eE\tau/m$, the current density is $J = nev_d = ne^2\tau E/m$. Since $\rho = E/J$,

$$\rho = \frac{m}{ne^2\tau}.$$

In a metal, the number density of conduction electrons remains nearly constant as temperature rises. Stronger lattice vibrations cause more frequent scattering, reducing the relaxation time τ . Hence $1/\tau$, and therefore ρ , increase. Over a moderate interval this is represented by $\rho(T) \simeq \rho(T_0)[1 + \alpha(T - T_0)]$, with $\alpha > 0$.

Step 3: Explain the decrease for a semiconductor Heating a typical semiconductor excites more electrons from the valence band into the conduction band, creating electron-hole pairs. Its carrier density therefore increases strongly with temperature. Although lattice scattering can reduce the relaxation time, the rise in carrier density dominates; the effective product $n\tau$ increases, so the resistivity decreases. More precisely, both electrons and holes conduct, giving $\sigma = e(n_e\mu_e + n_h\mu_h)$ and $\rho = 1/\sigma$, where μ_e and μ_h are their mobilities. The increase in carrier numbers outweighs the mobility decrease in the typical semiconductor range shown.

Step 4: Conclusion A metallic conductor has increasing resistivity with temperature because its relaxation time decreases while carrier density is nearly fixed. A typical semiconductor has decreasing resistivity because the increase in carrier density dominates the effect of scattering.

Quick Tip to Solve: In $\rho = m/(ne^2\tau)$, check both n and τ : heating changes mainly τ in a metal but greatly increases n in a semiconductor.

24. A signal of low frequency f_m is to be transmitted using a carrier wave of frequency f_c . Derive the expression for the amplitude modulated wave and deduce expressions for the lower and upper sidebands produced. Hence, obtain the expression for modulation index. [3 marks]

Solution:

Concept:

- In amplitude modulation, the carrier amplitude varies in proportion to the instantaneous message signal; its carrier frequency remains fixed. Write the message as $m(t) = A_m \cos \omega_m t$, where $\omega_m = 2\pi f_m$.
- The unmodulated carrier is $c(t) = A_c \cos \omega_c t$, where $\omega_c = 2\pi f_c$ and $f_c \gg f_m$. Taking both amplitudes in the same effective units, the modulation index is $\mu = A_m/A_c$.

Step 1: Form the amplitude modulated wave The instantaneous carrier amplitude becomes $A_c + A_m \cos \omega_m t$. Hence the modulated voltage is

$$v(t) = [A_c + A_m \cos \omega_m t] \cos \omega_c t = A_c [1 + \mu \cos \omega_m t] \cos \omega_c t.$$

The slowly varying factor in square brackets forms the envelope of the rapid carrier oscillations. Normal undistorted modulation requires $0 \leq \mu \leq 1$, so that the envelope does not reverse sign.

Step 2: Resolve the signal into carrier and sidebands Using $\cos x \cos y = \frac{1}{2} [\cos(x+y) + \cos(x-y)]$,

$$\begin{aligned} v(t) &= A_c \cos \omega_c t \\ &\quad + \frac{A_m}{2} \cos[(\omega_c + \omega_m)t] \\ &\quad + \frac{A_m}{2} \cos[(\omega_c - \omega_m)t]. \end{aligned}$$

The upper sideband is $v_U(t) = \frac{A_m}{2} \cos[2\pi(f_c + f_m)t]$, and the lower sideband is $v_L(t) = \frac{A_m}{2} \cos[2\pi(f_c - f_m)t]$. Each has amplitude $A_m/2 = \mu A_c/2$. Their frequency separation is $2f_m$, which is the required transmission bandwidth for this single tone.

Step 3: Obtain the modulation index from the envelope At the message crest and trough the envelope amplitudes are $A_{\max} = A_c + A_m$ and $A_{\min} = A_c - A_m$. Subtraction gives $A_{\max} - A_{\min} = 2A_m$, while addition gives $A_{\max} + A_{\min} = 2A_c$. Dividing therefore gives the modulation index directly from measured envelope amplitudes.

Step 4: Conclusion $v(t) = A_c [1 + \mu \cos(2\pi f_m t)] \cos(2\pi f_c t)$, with sidebands at $f_c - f_m$ and $f_c + f_m$. The modulation index is

$$\mu = \frac{A_m}{A_c} = \frac{A_{\max} - A_{\min}}{A_{\max} + A_{\min}}.$$

Quick Tip to Solve: Expand the product of the message and carrier cosines: the sum and difference frequencies immediately identify the two sidebands.

Section D (15 marks)

25. Explain, with the help of a diagram, how plane polarized light can be produced by scattering of light from the Sun.

Two polaroids P_1 and P_2 are placed with their pass axes perpendicular to each other. Unpolarised light of intensity I is incident on P_1 . A third polaroid P_3 is kept between P_1 and P_2 such that its pass axis makes an angle of 45° with that of P_1 . Calculate the intensity of light transmitted through P_1 , P_2 and P_3 .

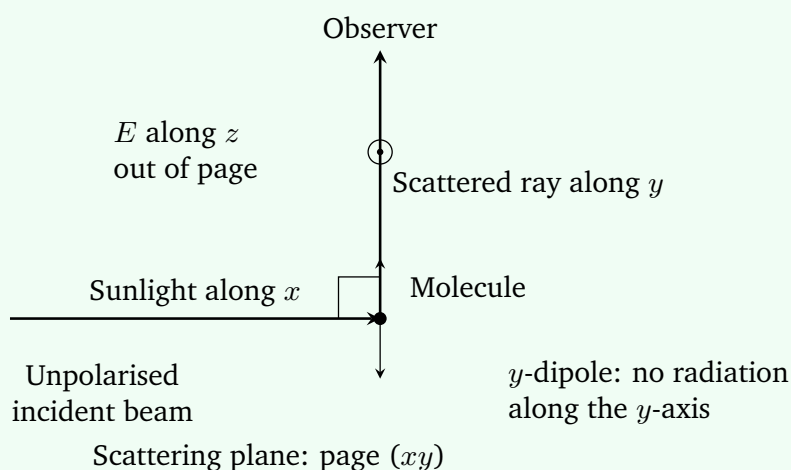
[5 marks]

Solution:

Concept:

- Unpolarised sunlight contains electric field vibrations in all directions perpendicular to its propagation. Atmospheric molecules act as oscillating electric dipoles when driven by this field.
- An oscillating dipole does not radiate along its vibration axis. Light scattered at 90° to the incident beam is therefore plane polarised with its electric field perpendicular to the scattering plane.
- An ideal polaroid transmits half the intensity of unpolarised light. For already plane polarised light, Malus's law is $I_{\text{out}} = I_{\text{in}} \cos^2 \theta$, where θ is the angle between the incident electric field and the pass axis.

Step 1: Explain polarisation of scattered sunlight Let sunlight travel along the x -direction, and observe the scattered light along the perpendicular y -direction. The incident electric field can be resolved into y and z components. These drive molecular dipoles along y and z , respectively. A dipole vibrating along y cannot radiate toward the observer on its own axis. The z -directed dipole does radiate toward that observer, so the observed scattered wave contains only the z -directed electric field. Its vibrations are confined to one direction and the light is plane polarised. The xy -plane containing the incident and scattered rays is the scattering plane; the transmitted electric field is perpendicular to it. This explains polarisation of skylight viewed approximately at right angles to the direction of the Sun, in the single scattering model.



Scattering at a right angle: only the dipole component perpendicular to the scattering plane radiates toward the observer. The circle and dot denote the electric field directed out of the page.

Step 2: Find the intensity immediately after the first polaroid The actual order in which light

Quick Tip to Solve: Write the physical plate order before using Malus's law: halve the unpolarised input once, then multiply by the cosine squared at each later plate.

encounters the plates is P_1 , then P_3 , then P_2 . The initial light of intensity I is unpolarised, so the first ideal polaroid selects one electric field component and transmits

$$I_1 = \frac{I}{2}.$$

This emerging light is polarised along the pass axis of P_1 . The factor of one half is the average of $\cos^2 \theta$ over the randomly oriented incident vibrations.

Step 3: Apply Malus's law at the inserted polaroid The pass axis of P_3 makes 45° with the polarisation emerging from P_1 . Consequently,

$$I_3 = I_1 \cos^2 45^\circ = \frac{I}{2} \times \frac{1}{2} = \frac{I}{4}.$$

The light now vibrates along the pass axis of P_3 ; its direction of polarisation has changed even though the intensity has decreased.

Step 4: Calculate transmission through the last polaroid The axes of P_1 and P_2 differ by 90° . Since P_3 is at 45° to P_1 , its acute angle to P_2 is also 45° . Applying Malus's law to the light incident on P_2 ,

$$I_2 = I_3 \cos^2 45^\circ = \frac{I}{4} \times \frac{1}{2} = \frac{I}{8}.$$

Without the intermediate plate, the crossed pair would transmit zero intensity. The inserted plate changes the polarisation direction before the light reaches the final analyser, permitting nonzero transmission.

Step 5: Conclusion Sunlight scattered at 90° is plane polarised perpendicular to the scattering plane. The intensities transmitted through the named plates are $I_1 = I/2$, $I_2 = I/8$, $I_3 = I/4$; the propagation order is $P_1 \rightarrow P_3 \rightarrow P_2$.

Quick Tip to Solve: Write the physical plate order before using Malus's law: halve the unpolarised input once, then multiply by the cosine squared at each later plate.

25. (a) Why cannot the phenomenon of interference be observed by illuminating two pin holes with two sodium lamps?

(b) Two monochromatic waves having displacements $y_1 = a \cos \omega t$ and $y_2 = a \cos(\omega t + \phi)$ from two coherent sources interfere to produce an interference pattern. Derive the expression for the resultant intensity and obtain the conditions for constructive and destructive interference.

(c) Two wavelengths of sodium light of 590 nm and 596 nm are used in turn to study the diffraction taking place at a single slit of aperture 2×10^{-6} m. If the distance between the slit and the screen is 1.5 m, calculate the separation between the positions of the second maxima of diffraction pattern obtained in the two cases. [5 marks]

Solution:

Concept:

- Coherent sources maintain a constant phase difference; independent lamps have rapidly fluctuating relative phases, so their interference term averages to zero.
- Superposition adds displacements. For a wave of amplitude A , intensity is $I = KA^2$, where K is the same proportionality constant for both waves.
- For slit width b , the secondary diffraction maxima approximately satisfy $b \sin \theta \simeq (n + \frac{1}{2})\lambda$, where $n = 1, 2, \dots$. In the usual screen-position approximation, $y_n \simeq (n + \frac{1}{2})\lambda D/b$.

Step 1: (a) Explain why two sodium lamps do not give stable fringes Light emitted by different atoms in the two independent sodium lamps consists of separate wave trains with unrelated initial phases. Even though the lamps emit approximately the same wavelengths, their relative phase changes randomly and rapidly. Hence the two pin holes, each illuminated by a different lamp, do not act as coherent sources. Bright and dark positions fluctuate so rapidly that an observer records only the time-averaged sum of the two intensities. A sustained interference pattern therefore cannot be observed. Equal wavelength alone is insufficient: a constant phase difference is also required.

Step 2: (b) Add the two coherent displacements Using $\cos u + \cos v = 2 \cos[(u - v)/2] \cos[(u + v)/2]$,

$$\begin{aligned} y &= a \cos \omega t + a \cos(\omega t + \phi) \\ &= 2a \cos(\phi/2) \cos(\omega t + \phi/2). \end{aligned}$$

The resultant amplitude has magnitude $A = |2a \cos(\phi/2)|$. If either incident wave alone has intensity $I_0 = Ka^2$, the resultant intensity is

$$I = KA^2 = 4I_0 \cos^2(\phi/2) = 2I_0(1 + \cos \phi).$$

Equivalently, the general interference expression $I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \phi$ reduces to this result when $I_1 = I_2 = I_0$. The phase difference determines whether the displacements reinforce or cancel.

Step 3: (b) Obtain the constructive and destructive conditions For constructive interference, $\cos \phi = 1$, so $\phi = 2m\pi$ and the path difference is $\Delta = m\lambda$, with m an integer. The amplitudes add and $I_{\max} = 4I_0$. For destructive interference, $\cos \phi = -1$, so $\phi = (2m + 1)\pi$ and $\Delta = (m + \frac{1}{2})\lambda$. The equal amplitudes cancel and $I_{\min} = 0$. Complete darkness follows here specifically because the given waves have equal

Quick Tip to Solve: For the second secondary diffraction maximum use the factor $5/2$, then substitute the wavelength difference directly to find the separation.

amplitudes.

Step 4: (c) Calculate the separation of the second secondary maxima The central maximum is excluded when numbering secondary maxima. The first secondary maximum lies approximately between the first and second minima, and the second between the second and third minima. Thus the standard approximate position of the second secondary maximum on either side is $y_2 \simeq 5\lambda D/(2b)$. Here

$$b = 2 \times 10^{-6} \text{ m}, \quad D = 1.5 \text{ m}, \quad \Delta\lambda = (596 - 590) \times 10^{-9} = 6 \times 10^{-9} \text{ m}.$$

Comparing maxima on the same side of the centre,

$$\begin{aligned} \Delta y_2 &\simeq \frac{5D}{2b} \Delta\lambda \\ &= \frac{5 \times 1.5 \times 6 \times 10^{-9}}{2 \times 2 \times 10^{-6}} \text{ m} \\ &= 1.125 \times 10^{-2} \text{ m} = 1.125 \text{ cm}. \end{aligned}$$

The longer wavelength gives the maximum farther from the centre. This numerical result uses the conventional approximate secondary-maximum and screen-position formula; secondary maxima are not exactly midway between successive minima.

Step 5: Conclusion Independent sodium lamps lack a fixed relative phase. The coherent waves give $I = 4I_0 \cos^2(\phi/2)$, with constructive interference at $\phi = 2m\pi$ and destructive interference at $\phi = (2m + 1)\pi$. The separation of the second secondary maxima is approximately 1.125 cm.

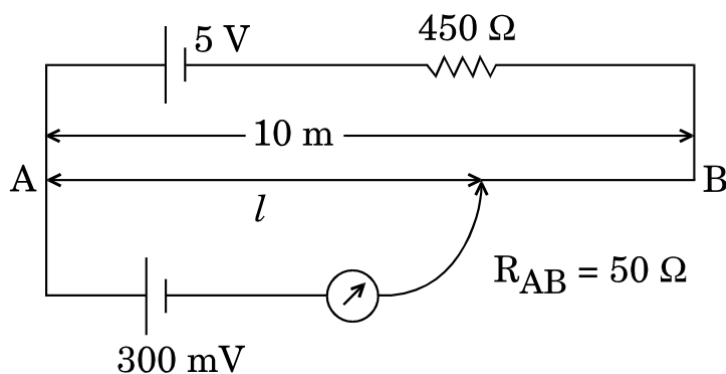
Quick Tip to Solve: For the second secondary diffraction maximum use the factor $5/2$, then substitute the wavelength difference directly to find the separation.

26. (a) Describe briefly, with the help of a circuit diagram, the method of measuring the internal resistance of a cell.

(b) Give reason why a potentiometer is preferred over a voltmeter for the measurement of emf of a cell.

(c) In a potentiometer circuit, calculate the balancing length l . Give reason, whether the circuit will work, if the driver cell of emf 5 V is replaced with a cell of 2 V, keeping all other factors constant.

[5 marks]

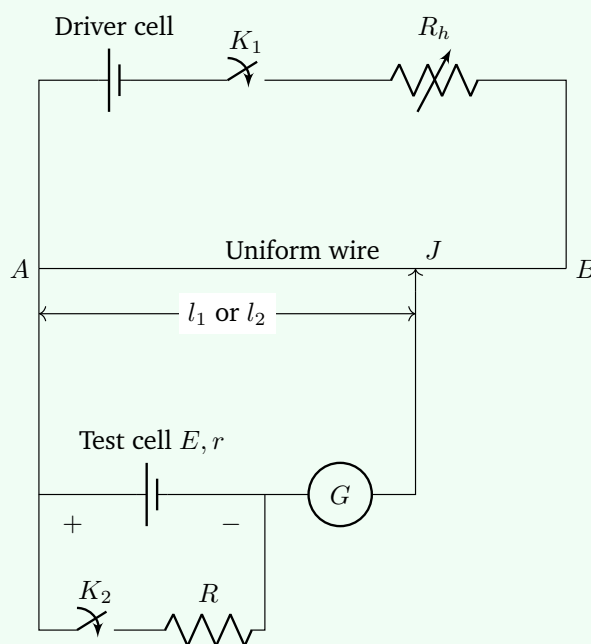


Solution:

Concept:

- A uniform potentiometer wire carrying steady current has a constant potential gradient k . At balance, the potential difference across length l equals the emf or terminal voltage being measured: $E = kl$.
- For a cell of emf E and internal resistance r supplying a known external resistance R , its terminal voltage is $V = ER/(R + r)$. Comparing two null measurements eliminates the unknown potential gradient.
- A null measurement draws no current from the test cell at balance, so the measured emf is unaffected by the voltage drop across its internal resistance.

Step 1: (a) Arrange the internal-resistance measurement circuit Connect a driver cell, key K_1 and rheostat R_h in series with the uniform wire AB , keeping A at the higher potential. Connect the positive terminal of the test cell to A ; connect its negative terminal through a galvanometer G to the jockey J . A known resistance R , controlled by key K_2 , is connected across the test cell. The driver must provide a potential drop along the whole wire greater than the test-cell emf. Close K_1 and adjust the rheostat; keep this adjustment unchanged for both readings.



Potentiometer circuit for measuring internal resistance; close K_2 to load the test cell through R .

Step 2: (a) Obtain the two balance lengths and derive the formula With K_2 open, the test cell supplies no external current at balance. Find the null point and record $AJ = l_1$, giving $E = kl_1$. Close K_2 , so current flows through R , and find the new null point $AJ = l_2$. This reading measures the terminal voltage $V = kl_2$. The galvanometer branch still draws no current at null, although the cell supplies the load.

$$\frac{E}{V} = \frac{l_1}{l_2} = \frac{R + r}{R},$$
$$r = R \left(\frac{l_1}{l_2} - 1 \right) = R \frac{l_1 - l_2}{l_2}.$$

Quick Tip to Solve: Calculate the voltage across the potentiometer wire before finding its gradient; the driver's full emf is not necessarily across the wire.

Thus, measure l_1 , l_2 and the known load R . Ordinarily $l_2 < l_1$, because the terminal voltage falls when the cell delivers current.

Step 3: (b) Explain why the potentiometer is preferred At the potentiometer balance point, no current passes through the test cell in the measuring branch. With the external load disconnected, there is consequently no internal voltage drop and the instrument measures the true emf. A voltmeter has finite resistance and draws current I ; it then measures the terminal voltage $E - Ir$, which is smaller than the emf. The potentiometer avoids this loading error and therefore provides a more accurate emf measurement.

Step 4: (c) Calculate the balance length with the 5 V driver The printed circuit has a $450\ \Omega$ series resistor and a $50\ \Omega$, 10 m potentiometer wire. Hence the primary-circuit current and wire potential gradient are

$$\begin{aligned} I &= \frac{5}{450 + 50} = 0.010\text{ A}, \\ V_{AB} &= IR_{AB} = 0.010 \times 50 = 0.50\text{ V}, \\ k &= \frac{0.50}{10} = 0.050\text{ V m}^{-1}, \\ l &= \frac{0.300}{0.050} = 6.0\text{ m}. \end{aligned}$$

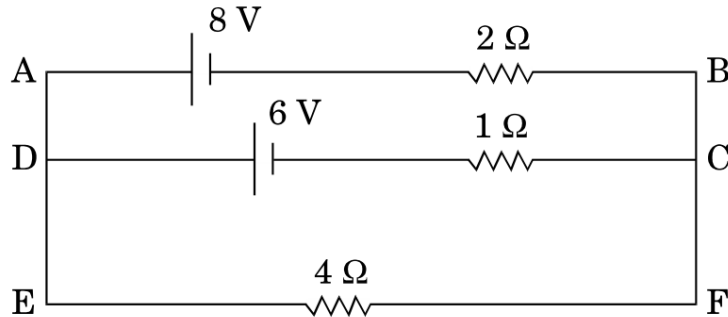
The 300 mV test-cell emf can be balanced because it is smaller than the total potential drop across AB .

Step 5: (c) Test the replacement 2 V driver With all resistances unchanged, the new primary current is $2/500 = 0.004\text{ A}$. The wire drop becomes $0.004 \times 50 = 0.20\text{ V}$ and the potential gradient becomes 0.020 V m^{-1} . The required length would be $0.300/0.020 = 15\text{ m}$, exceeding the available 10 m. Therefore no null point exists on the wire. Although the replacement driver's emf exceeds the test-cell emf, most of its voltage drops across the series resistor; it is the available wire drop that determines whether balance is possible.

Step 6: Conclusion The internal resistance is $r = R(l_1 - l_2)/l_2$. A potentiometer measures emf without drawing current at balance. For the given 5 V driver, $l = 6.0\text{ m}$. The 2 V replacement cannot balance the cell because the whole wire provides only $0.20\text{ V} < 0.300\text{ V}$.

Quick Tip to Solve: Calculate the voltage across the potentiometer wire before finding its gradient; the driver's full emf is not necessarily across the wire.

26. (a) State the working principle of a metre bridge used to measure an unknown resistance.
 (b) Give reason (i) why the connections between the resistors in a metre bridge are made of thick copper strips, (ii) why it is generally preferred to obtain the balance length near the mid-point of the bridge wire.
 (c) Calculate the potential difference across the 4Ω resistor in the given electrical circuit, using Kirchhoff's rules. [5 marks]



Solution:

Concept:

- A metre bridge is a practical Wheatstone bridge. At balance the galvanometer current is zero and the ratios of the resistances in the two arms are equal.
- For a uniform wire, resistance is proportional to length. If unknown resistance X is in the left gap and known resistance R in the right gap, $X/R = l/(100 - l)$, where l is in centimetres.
- Kirchhoff's junction rule expresses charge conservation; the algebraic sum of potential changes around any closed loop is zero by energy conservation.

Step 1: (a) State the balance principle Move the jockey along the uniform bridge wire until the galvanometer shows no deflection. The junction between the two gap resistances and the contact point on the wire then have equal potentials. Therefore,

$$\frac{X}{R} = \frac{R_{AJ}}{R_{JB}} = \frac{l}{100 - l}, \quad X = R \frac{l}{100 - l}.$$

Thus the unknown resistance is determined from a known resistance and the measured balance length. The resistance per unit length cancels because the same uniform wire forms both arms.

Step 2: (b)(i) Explain the thick copper connections Copper has low resistivity, and a thick strip has a large cross-sectional area. Since $R = \rho L/A$, its resistance is very small. Hence the resistance of the connecting strips is negligible compared with the gap resistances; unwanted connection resistance and associated potential drops introduce minimal error in the balance relation.

Step 3: (b)(ii) Explain why a central balance is preferred Choose the known resistance comparable to the unknown resistance so that l is close to 50 cm. For a small uncertainty δl , the relative uncertainty

Quick Tip to Solve: Collapse each vertical ideal wire into one node, then express all three branch currents using the single node voltage; keep the cell polarities explicit.

in the measured ratio is approximately

$$\frac{\delta X}{X} = \left(\frac{1}{l} + \frac{1}{100-l} \right) \delta l = \frac{100 \delta l}{l(100-l)}.$$

The denominator is largest at the midpoint, so the relative error is smallest there. Neither wire segment is very short, and end/contact resistance errors also have less influence on the result.

Step 4: (c) Assign currents and apply Kirchhoff's rules The ideal connecting wires make A, D, E one node and B, C, F another. Both cells have their positive terminals on the left. Let $V = V_E - V_F$, and initially take currents I_1, I_2, I_3 from left to right through the upper, middle and lower branches respectively. The loop rule for the upper branch with the lower return path gives $V = 8 + 2I_1$; the middle branch gives $V = 6 + I_2$; and the lower resistor gives $V = 4I_3$, with volts, amperes and ohms understood. The junction rule is $I_1 + I_2 + I_3 = 0$. Hence,

$$\begin{aligned} \frac{V-8}{2} + \frac{V-6}{1} + \frac{V}{4} &= 0, \\ 2V - 16 + 4V - 24 + V &= 0, \\ 7V &= 40. \end{aligned}$$

As a consistency check, the branch currents are $I_1 = -8/7$ A, $I_2 = -2/7$ A and $I_3 = 10/7$ A. Their sum vanishes; the negative signs mean that the cell branches actually carry current from right to left.

Step 5: Conclusion The bridge works on the Wheatstone balance condition. Thick copper strips minimise connection resistance, and a balance near 50 cm minimises measurement error. The potential difference across the 4Ω resistor is $V_E - V_F = 40/7 \text{ V} \approx 5.71 \text{ V}$, with E at the higher potential.

Quick Tip to Solve: Collapse each vertical ideal wire into one node, then express all three branch currents using the single node voltage; keep the cell polarities explicit.

27. (a) Derive an expression for the induced emf developed when a coil of N turns, and area of cross-section A , is rotated at a constant angular speed ω in a uniform magnetic field B .

(b) A wheel with 100 metallic spokes each 0.5 m long is rotated with a speed of 120 rev/min in a plane normal to the horizontal component of the Earth's magnetic field. If the resultant magnetic field at that place is 4×10^{-4} T and the angle of dip at the place is 30° , find the emf induced between the axle and the rim of the wheel. [5 marks]

Solution:

Concept:

- Magnetic flux through one turn of area A is $\Phi = BA \cos \theta$, where θ is the angle between the field and the normal to the coil.
- Faraday's law gives $\varepsilon = -d(N\Phi)/dt$; the negative sign expresses Lenz's law.
- For a radial conductor rotating with angular speed ω , a small segment at radius r has speed $v = \omega r$. In a field $B_\perp$ normal to the rotation plane, $d\varepsilon = B_\perp \omega r dr$.

Step 1: (a) Write the time-dependent flux linkage Consider the usual generator arrangement in which

Quick Tip to Solve: Use the field component normal to the wheel plane and integrate the changing spoke speed; parallel spokes do not multiply the axle-to-rim emf.

the coil rotates about an axis in its plane and perpendicular to the uniform magnetic field. Choose $t = 0$ when the normal to the coil is parallel to B . After time t , the angle between this normal and the field is $\theta = \omega t$, because the angular speed is constant. All N turns have the same area and orientation, so the total flux linkage is

$$N\Phi(t) = NBA \cos(\omega t).$$

A different initial orientation would simply add an initial phase to ωt .

Step 2: (a) Differentiate to obtain the induced emf Applying Faraday's law to the entire coil,

$$\begin{aligned}\varepsilon(t) &= -\frac{d}{dt} [NBA \cos(\omega t)] \\ &= NBA\omega \sin(\omega t).\end{aligned}$$

Thus the emf is sinusoidal, with amplitude $\varepsilon_0 = NBA\omega$ and frequency $f = \omega/(2\pi)$. The emf is zero when the coil normal is parallel to the field because the flux is momentarily stationary. Its magnitude is maximum when the normal is perpendicular to the field because the flux changes most rapidly. The sign reverses after every half rotation.

Step 3: (b) Find the field component and angular speed The wheel rotates in a plane normal to the horizontal component of the Earth's field. Therefore the component normal to the wheel plane is

$$B_H = B \cos \delta = (4 \times 10^{-4}) \cos 30^\circ = 2\sqrt{3} \times 10^{-4} \text{ T}.$$

The vertical component lies in the wheel plane and produces no radial axle-to-rim emf. The rotation frequency and angular speed are

$$f = \frac{120}{60} = 2 \text{ s}^{-1}, \quad \omega = 2\pi f = 4\pi \text{ rad s}^{-1}.$$

Each spoke extends radially from the axle to radius $R = 0.5 \text{ m}$.

Step 4: (b) Integrate the motional emf along a spoke At a distance r from the axle, the tangential speed is ωr , so the magnitude of the radial motional emf over a segment dr is $B_H \omega r dr$. Integrating from axle to rim gives

$$\begin{aligned}|\varepsilon| &= \int_0^R B_H \omega r dr = \frac{1}{2} B_H \omega R^2 \\ &= \frac{1}{2} (2\sqrt{3} \times 10^{-4}) (4\pi) (0.5)^2 \text{ V} \\ &= \pi\sqrt{3} \times 10^{-4} \text{ V} \approx 5.44 \times 10^{-4} \text{ V}.\end{aligned}$$

All 100 spokes connect the same axle and rim, so they act as parallel emf-producing paths. Their emfs are not added in series; the number of spokes therefore does not multiply the potential difference. The axle/rim polarity depends on the rotation sense relative to the field, which is not specified.

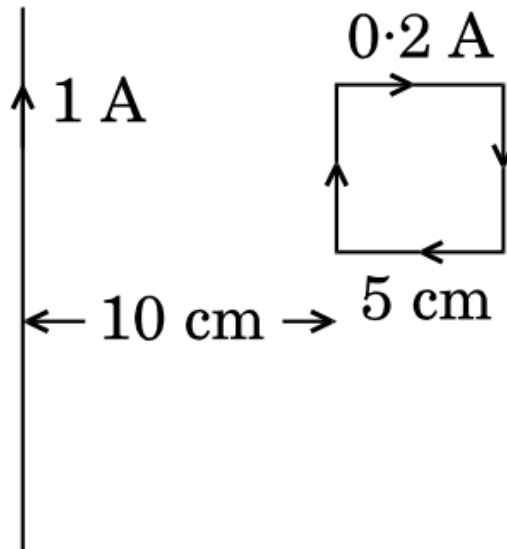
Step 5: Conclusion The rotating coil develops $\varepsilon(t) = NBA\omega \sin(\omega t)$, with peak emf $NBA\omega$, for the chosen zero phase. The magnitude of the wheel's axle-to-rim emf is $5.44 \times 10^{-4} \text{ V}$, or 0.544 mV .

Quick Tip to Solve: Use the field component normal to the wheel plane and integrate the changing spoke speed; parallel spokes do not multiply the axle-to-rim emf.

OR

27. (a) Derive the expression for the magnetic energy stored in an inductor when a current I develops in it. Hence, obtain the expression for the magnetic energy density.

(b) A square loop of sides 5 cm carrying a current of 0.2 A in the clockwise direction is placed at a distance of 10 cm from an infinitely long wire carrying a current of 1 A as shown. Calculate (i) the resultant magnetic force, and (ii) the torque, if any, acting on the loop. [5 marks]



Solution:

Concept:

- An inductor opposes a changing current by the self-induced emf $\varepsilon = -L di/dt$. For constant inductance L , the work supplied against this emf is stored in its magnetic field.
- A long air-core solenoid with N turns, length ℓ and cross-sectional area A has $L = \mu_0 N^2 A/\ell$, field $B = \mu_0 NI/\ell$ and field volume Al .
- The field at distance r from a straight wire carrying current I_w is $B(r) = \mu_0 I_w/(2\pi r)$. A straight element of a loop experiences $d\mathbf{F} = I_l d\ell \times \mathbf{B}$.

Step 1: (a) Work done in establishing the current Let the current increase from zero to I . At an intermediate current i , the external source must provide the potential difference $L di/dt$ against the back emf. Its power supplied to the magnetic field is therefore $P = iL di/dt$, so the infinitesimal stored energy is $dU = Li di$. For a linear inductor of constant L , integration gives

$$U = \int_0^I Li di = \frac{1}{2} LI^2.$$

This is the magnetic energy; any heat dissipated in the resistance of a real coil is separate from the energy stored in its field.

Step 2: (a) Obtain the magnetic energy per unit volume Use a long air-core solenoid, whose field is approximately uniform inside and negligible outside. Substituting its inductance into the energy

Quick Tip to Solve: Measure the two vertical-side distances separately: the farther side is at 15 cm, and subtract its repulsion from the nearer side's attraction before checking torque.

expression,

$$U = \frac{1}{2} \frac{\mu_0 N^2 A}{\ell} I^2.$$

Divide by the field volume $A\ell$, and use $NI/\ell = B/\mu_0$:

$$u_B = \frac{U}{A\ell} = \frac{1}{2} \mu_0 \left(\frac{NI}{\ell} \right)^2 = \frac{B^2}{2\mu_0}.$$

Thus magnetic energy density is measured in J m^{-3} . In a uniform linear magnetic medium of permeability μ , the corresponding expression is $u_B = B^2/(2\mu)$.

Step 3: (b)(i) Forces on the two vertical sides The figure gives the distance to the nearer side as $r_1 = 0.10$ m. The side length is $a = 0.05$ m, so the farther side is at $r_2 = 0.15$ m. The wire carries $I_w = 1$ A upward and the loop carries $I_l = 0.2$ A clockwise. To the right of the wire its field is into the page. The nearer vertical side carries upward current, parallel to the wire, and is attracted towards it; the farther side carries downward current and is repelled away from it.

$$F_1 = \frac{\mu_0 I_w I_l a}{2\pi r_1} = \frac{2 \times 10^{-7} \times 1 \times 0.2 \times 0.05}{0.10} = 2.0 \times 10^{-8} \text{ N},$$

$$F_2 = \frac{\mu_0 I_w I_l a}{2\pi r_2} = \frac{2 \times 10^{-9}}{0.15} = 1.33 \times 10^{-8} \text{ N}.$$

The closer side feels the larger force because the field decreases with distance from the wire.

Step 4: (b)(i) Cancellation of horizontal-side forces On the top side the current runs rightward and the magnetic force is upward; on the bottom side the current runs leftward and the force is downward. At each horizontal position these sides are at the same distance from the wire, so their force elements have equal magnitudes and opposite directions. Their total forces cancel. Therefore the resultant comes only from the difference between the two vertical-side forces:

$$F_{\text{net}} = F_1 - F_2 = 6.67 \times 10^{-9} \text{ N},$$

directed towards the long wire.

Step 5: (b)(ii) Check the torque despite the nonuniform field The magnetic moment of the clockwise loop points into the page, parallel to the wire's field throughout the loop. To check torque without assuming a uniform field, take moments about the loop centre. Each vertical side has uniform horizontal force along its length; contributions above and below the centre cancel. The top and bottom horizontal sides have equal and opposite vertical force elements at every common horizontal position, and their torques also cancel pairwise. Hence the total magnetic torque is zero: the loop translates towards the wire without a turning tendency.

Step 6: Conclusion (a) The stored magnetic energy is $U = \frac{1}{2} LI^2$, and the magnetic energy density in air or vacuum is $u_B = B^2/(2\mu_0)$. (b) The resultant force is $6.67 \times 10^{-9} \text{ N}$ towards the long wire, while the torque is 0 N m .

Quick Tip to Solve: Measure the two vertical-side distances separately: the farther side is at 15 cm, and subtract its repulsion from the nearer side's attraction before checking torque.