

CBSE Class 12 Physics Question Paper 2019 with Solution PDF-Set 55/3/2

Question Paper with Solutions

Conducted by the Central Board of Secondary Education (CBSE)



Time Allowed: 3 hours

Maximum Marks: 70

General Instructions

- (i) All questions are compulsory. There are 27 questions in all.
- (ii) This question paper has four sections : Section A, Section B, Section C and Section D.
- (iii) Section A contains five questions of one mark each, Section B contains seven questions of two marks each, Section C contains twelve questions of three marks each, Section D contains three questions of five marks each.
- (iv) There is no overall choice. However, an internal choice(s) has been provided in two questions of one mark, two questions of two marks, four questions of three marks and three questions of five marks weightage. You have to attempt only one of the choices in such questions.

You may use the following values of physical constants wherever necessary :

$$c = 3 \times 10^8 \text{ m/s}$$

$$h = 6.63 \times 10^{-34} \text{ Js}$$

$$e = 1.6 \times 10^{-19} \text{ C}$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ T m A}^{-1}$$

$$\epsilon_0 = 8.854 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$$

$$\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$$

$$\text{Mass of electron } (m_e) = 9.1 \times 10^{-31} \text{ kg}$$

$$\text{Mass of neutron} = 1.675 \times 10^{-27} \text{ kg}$$

$$\text{Mass of proton} = 1.673 \times 10^{-27} \text{ kg}$$

$$\text{Avogadro's number} = 6.023 \times 10^{23} \text{ per gram mole}$$

$$\text{Boltzmann constant} = 1.38 \times 10^{-23} \text{ JK}^{-1}$$

Section A (5 marks)

1. Draw a plot of resistivity versus temperature for a typical semiconductor.

[1 mark]

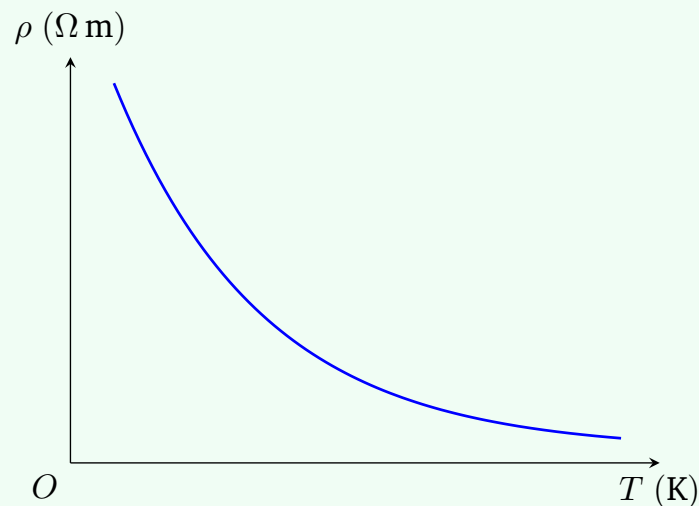
Solution:

Concept:

- Resistivity ρ measures opposition to current and is the reciprocal of conductivity: $\rho = 1/\sigma$.
- In a typical semiconductor, heating creates more mobile electrons and holes; the increase in carrier concentration makes resistivity decrease.

Step 1: Choose the axes Put absolute temperature T , in kelvin, on the horizontal axis and resistivity ρ , in $\Omega \text{ m}$, on the vertical axis. The graph is schematic because no numerical data are specified.

Step 2: Sketch the temperature dependence Draw a falling, non-linear curve: resistivity is relatively high at low temperature and falls as temperature rises. This represents the negative temperature coefficient of a typical semiconductor over the usual temperature range.



Schematic resistivity-temperature curve for a typical semiconductor.

Step 3: Conclusion The required ρ -versus- T plot is a decreasing curve, as shown.

Quick Tip to Solve: For a semiconductor, recall that heating produces more charge carriers, so draw resistivity falling as temperature rises.

2. Define the term 'attenuation' in communication system.

[1 mark]

Solution:

Concept:

- A communication channel carries a signal from a transmitter towards a receiver.
- Signal strength can be described by its amplitude or power, and it generally decreases during transmission through a medium.

Quick Tip to Solve: In the definition, include both ideas: loss of signal strength and propagation through a medium.

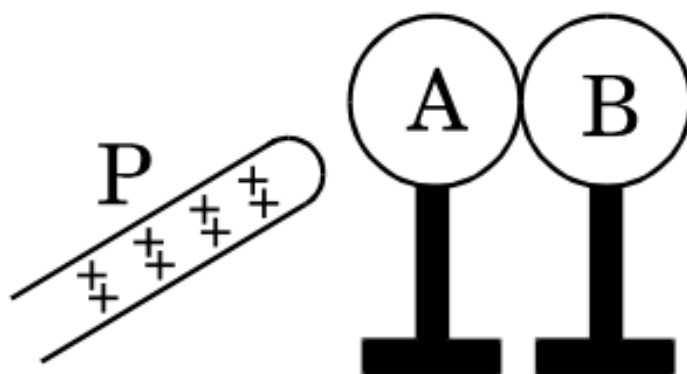
Step 1: Identify the change in the signal Compare the strength of the signal at the start of the transmission path with its strength after propagation. Some signal energy is lost in the medium, so the received signal is weaker.

Step 2: Name the loss in strength This reduction in signal strength is called attenuation. It refers to weakening of the signal while it propagates through the communication channel; it does not mean amplification or a change in the information being transmitted.

Step 3: Conclusion Attenuation is the loss of strength of a signal while it propagates through a medium.

Quick Tip to Solve: In the definition, include both ideas: loss of signal strength and propagation through a medium.

3. Two metallic spheres A and B kept on insulating stands are in contact with each other. A positively charged rod P is brought near the sphere A as shown in the figure. The two spheres are separated from each other, and the rod P is removed. What will be the nature of charges on spheres A and B? [1 mark]



Solution:

Concept:

- Electrostatic induction redistributes mobile electrons in a conductor when a nearby charged body produces an electric field.
- Contacting metallic spheres allow electrons to pass between them, while insulating stands prevent exchange of charge with the ground.

Step 1: Follow the electrons while the spheres touch The positively charged rod attracts electrons towards the nearer sphere A. Electrons move from B to A through the contact, leaving A with an excess of electrons and B with an equal electron deficiency. No contact with the rod is required.

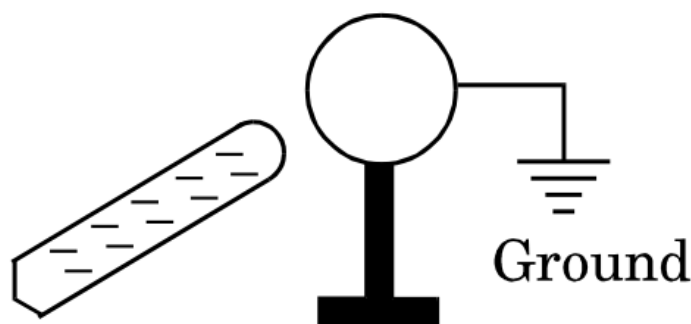
Step 2: Separate the spheres before removing the rod Separating the spheres interrupts the conducting path, so electrons cannot flow back from A to B. After the rod is removed, charge redistributes over each individual sphere, but each sphere retains its net charge.

Step 3: Conclusion Sphere A is negatively charged and sphere B is positively charged.

Quick Tip to Solve: Track electron motion first: a positive inducing rod draws electrons towards the nearer conductor, which becomes negative.

OR

3. A metal sphere is kept on an insulating stand. A negatively charged rod is brought near it, then the sphere is earthed as shown. On removing the earthing, and taking the negatively charged rod away, what will be the nature of charge on the sphere? Give reason for your answer. [1 mark]



Solution:

Concept:

- A negatively charged rod repels the mobile electrons in a nearby metal sphere.
- Earthing provides a conducting path between the sphere and the Earth, allowing electrons to leave the sphere.

Step 1: Earth the sphere with the rod nearby The negative rod repels electrons from the sphere into the Earth through the earthing connection. The sphere is therefore left with a deficiency of electrons. Its positive ions remain bound in the metal; they do not flow into the sphere.

Step 2: Disconnect the Earth before taking away the rod Removing the earthing connection first prevents electrons from returning from the Earth. When the rod is subsequently taken away, the remaining charge spreads over the sphere, but the electron deficiency persists because the sphere is isolated on an insulating stand.

Step 3: Conclusion The sphere becomes positively charged because electrons are repelled into the Earth and cannot return after the earthing connection is removed.

Quick Tip to Solve: For induction with earthing, disconnect the Earth before removing the rod; the sphere retains charge opposite to the rod.

4. In photoelectric emission, when the frequency of incident radiation is doubled, will the maximum kinetic energy of photoelectrons also be doubled? Justify your answer. [1 mark]

Solution:

Concept:

- Einstein's photoelectric equation is $K_{\max} = h\nu - \phi$, where h is Planck's constant, ν is incident frequency and ϕ is the metal's work function.
- The work function is fixed for a given metal; only photon energy $h\nu$ is directly proportional to frequency.

Quick Tip to Solve: Write $h\nu - \phi$ before changing the frequency; retain the same work function when ν is doubled.

Step 1: Write the original kinetic energy Since photoelectric emission occurs, the incident photon has sufficient energy to overcome the work function. The excess energy becomes the maximum kinetic energy:

$$K_{\max} = h\nu - \phi.$$

Step 2: Replace the frequency by twice its value At frequency 2ν , the work function is unchanged. Therefore,

$$K'_{\max} = 2h\nu - \phi = 2(h\nu - \phi) + \phi = 2K_{\max} + \phi.$$

The same fixed energy ϕ is subtracted from each photon, so doubling photon energy does not double the electron's kinetic energy.

Step 3: Conclusion No. The new maximum kinetic energy is $2K_{\max} + \phi$, which is greater than twice the initial value for a metal with positive work function.

Quick Tip to Solve: Write $h\nu - \phi$ before changing the frequency; retain the same work function when ν is doubled.

5. State the condition under which a large magnification can be achieved in an astronomical telescope.

[1 mark]

Solution:

Concept:

- In normal adjustment, an astronomical telescope has angular magnification $M = -f_o/f_e$, where f_o and f_e are the objective and eyepiece focal lengths.
- The magnitude $|M| = f_o/f_e$ gives the angular enlargement; the negative sign indicates an inverted final image.

Step 1: Compare the two focal lengths The objective forms an intermediate image of the distant object, and the eyepiece magnifies its angular size. For normal adjustment, the focal-length ratio determines the magnitude of magnification.

Step 2: Make the focal-length ratio large Choose an objective with a large focal length and an eyepiece with a small focal length, so that f_o/f_e is large. A large objective aperture helps light collection and resolving power, but it is the focal-length ratio that supplies this condition for magnification.

Step 3: Conclusion A large magnification is achieved when the objective focal length is much larger than the eyepiece focal length: $f_o \gg f_e$.

Quick Tip to Solve: Remember telescope magnification as objective focal length divided by eyepiece focal length: long objective, short eyepiece.

5. How does the angle of minimum deviation of a glass prism vary if the incident violet light is replaced by red light? [1 mark]

Solution:

Concept:

- For normal dispersion in glass, the refractive index for violet light is greater than that for red light: $\mu_v > \mu_r$.
- At minimum deviation, $\mu = \frac{\sin[(A + \delta_m)/2]}{\sin(A/2)}$, where A is the prism angle and δ_m is its minimum deviation.

Step 1: Compare the refractive indices Red light has a longer wavelength than violet light. For a glass prism in the visible region, replacing violet light by red light therefore decreases the refractive index experienced by the incident light.

Step 2: Relate the index to minimum deviation For the same prism, A stays fixed. The prism relation can be written as

$$\delta_m = 2 \sin^{-1}[\mu \sin(A/2)] - A.$$

A smaller value of μ gives a smaller value of δ_m , since the inverse sine increases with its argument in the physical range.

Step 3: Conclusion The angle of minimum deviation decreases when violet light is replaced by red light: $\delta_{m,r} < \delta_{m,v}$.

Quick Tip to Solve: For the same glass prism, remember that violet deviates most and red least; compare their refractive indices first.

Section B (14 marks)

6. Show that the image formed by a convex mirror of an object is always virtual. [2 marks]

Solution:

Concept:

- The mirror formula is $\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$, where u , v and f are the object distance, image distance and focal length measured from the pole.
- With the Cartesian sign convention, a real object in front of the mirror has $u < 0$; a convex mirror has its focus behind the mirror, so $f > 0$.

Step 1: Apply the mirror formula for any real-object position Let the object distance be $u = -a$, where $a > 0$, and let the focal length be $f = b$, where $b > 0$. Rearranging the mirror formula gives

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u} = \frac{1}{b} + \frac{1}{a} = \frac{a+b}{ab}, \quad v = \frac{ab}{a+b}.$$

Both numerator and denominator are positive for every real-object position.

Quick Tip to Solve: Write $u = -a$ and $f = +b$: the resulting $v = ab/(a+b) > 0$ proves virtuality immediately.

Step 2: Interpret the sign and position of the image Thus $v > 0$, so the image lies behind the convex mirror. Reflected rays diverge in front of the mirror; their backward extensions intersect at this point behind it. Consequently, rays do not actually meet at the image and it cannot be obtained on a screen. Also, $v/b = a/(a + b) < 1$, locating the image between the pole and focus. For an object at infinity, the image approaches the focus from the pole side.

Step 3: Conclusion Therefore, for every real object in front of a convex mirror, $v > 0$ and the image is virtual, lying behind the mirror between its pole and focus (at the focus for an object at infinity).

Quick Tip to Solve: Write $u = -a$ and $f = +b$: the resulting $v = ab/(a + b) > 0$ proves virtuality immediately.

7. Distinguish between nuclear fusion and nuclear fission. Give one example for each.

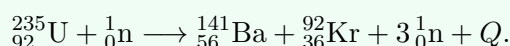
[2 marks]

Solution:

Concept:

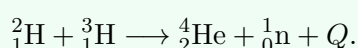
- Nuclear fission splits a heavy nucleus into lighter nuclei, whereas nuclear fusion combines light nuclei to form a heavier nucleus.
- In both processes, the final products can have a larger binding energy per nucleon. The decrease in total rest mass releases energy according to $Q = \Delta mc^2$.

Step 1: Explain fission and give an example In fission, a heavy nucleus such as uranium-235 absorbs a neutron and splits into two medium-mass nuclei, releasing energy and additional neutrons. One possible reaction is



The nucleon totals are 236 on both sides and the atomic-number totals are 92. Released neutrons can initiate further fissions, permitting a chain reaction.

Step 2: Explain fusion and give an example In fusion, two light nuclei combine to produce a heavier nucleus. They must approach closely enough for the nuclear force to act, overcoming their electrostatic repulsion; hence very high temperature is generally required. For example,



Here both nucleon number and charge are conserved: $2 + 3 = 4 + 1$ and $1 + 1 = 2 + 0$. This deuterium-tritium reaction is an example of fusion rather than the splitting of a heavy nucleus.

Step 3: Conclusion Fission is the splitting of a heavy nucleus, as in neutron-induced uranium-235 fission; fusion is the combination of light nuclei, as in deuterium-tritium fusion. Both release nuclear energy.

Quick Tip to Solve: Identify the reactants first: one heavy nucleus splitting indicates fission; two light nuclei combining indicates fusion.

8. An electron in a hydrogen atom in the second excited state jumps to the first and ground state of the atom. Find the ratio of the wavelengths emitted during this process. [2 marks]

Solution:

Concept:

- For hydrogen, $E_n = -13.6/n^2$ eV. The ground, first excited and second excited states correspond to $n = 1, 2, 3$, respectively.
- For an emission transition from n_i to $n_f < n_i$, the Rydberg formula is $1/\lambda = R_H(1/n_f^2 - 1/n_i^2)$, where R_H is the hydrogen Rydberg constant.

Step 1: Calculate the wavelength for the transition to the first excited state The electron begins in the second excited state, $n_i = 3$. For its transition to the first excited state, $n_f = 2$, denote the wavelength by λ_{32} . Then

$$\frac{1}{\lambda_{32}} = R_H \left(\frac{1}{2^2} - \frac{1}{3^2} \right) = R_H \frac{5}{36}, \quad \lambda_{32} = \frac{36}{5R_H}.$$

This transition releases the smaller energy difference.

Step 2: Calculate the direct transition to the ground state and compare For the transition from the same initial state $n_i = 3$ to the ground state $n_f = 1$, denote the wavelength by λ_{31} . The Rydberg formula gives

$$\frac{1}{\lambda_{31}} = R_H \left(1 - \frac{1}{9} \right) = \frac{8R_H}{9}, \quad \lambda_{31} = \frac{9}{8R_H}.$$

Taking the wavelengths in the order stated in the question,

$$\frac{\lambda_{32}}{\lambda_{31}} = \frac{36}{5R_H} \frac{8R_H}{9} = \frac{32}{5}.$$

The larger wavelength for the first transition is consistent with its smaller emitted photon energy, since $E = hc/\lambda$.

Step 3: Conclusion The wavelengths emitted in the $3 \rightarrow 2$ and $3 \rightarrow 1$ transitions are in the ratio $\lambda_{32} : \lambda_{31} = 32 : 5$.

Quick Tip to Solve: Convert excited-state names to n first, and remember that the wavelength ratio is the inverse photon-energy ratio.

9. Five point charges, each of charge $+q$ are placed on five vertices of a regular hexagon of side l . Find the magnitude of the resultant force on a charge $-q$ placed at the centre of the hexagon. [2 marks]

Solution:

Concept:

- In a regular hexagon, the distance from its centre to any vertex equals its side length l .
- Coulomb's law gives the magnitude $F_0 = kq^2/l^2$, where $k = 1/(4\pi\epsilon_0)$. Forces add vectorially, and opposite vertex pairs cancel by symmetry.

Step 1: Complete the hexagon to use symmetry Imagine placing a sixth charge $+q$ at the empty

Quick Tip to Solve: Add the missing sixth charge mentally: the complete hexagon has zero force, so the five-charge force is its negative.

vertex. The central charge $-q$ is attracted towards each of the six vertices. Every charge has an identical charge at the diametrically opposite vertex, at the same distance l from the centre. Thus each opposite pair produces equal and opposite forces on the central charge, giving zero net force for the completed arrangement.

Step 2: Subtract the force due to the imagined sixth charge Let $\mathbf{F}_5$ be the force from the actual five charges and $\mathbf{F}_{\text{missing}}$ the force that the imagined sixth charge would exert. Superposition gives

$$\mathbf{F}_5 + \mathbf{F}_{\text{missing}} = \mathbf{0}, \quad \mathbf{F}_5 = -\mathbf{F}_{\text{missing}}.$$

The latter force has magnitude kq^2/l^2 , directed from the centre towards the empty vertex. Hence the actual resultant has the same magnitude but points away from the empty vertex, along the line joining the centre to the opposite occupied vertex. The negative central charge changes the force direction to attraction but not its magnitude.

Step 3: Conclusion The magnitude of the resultant force is $F = \frac{q^2}{4\pi\epsilon_0 l^2}$, directed away from the empty vertex towards the diametrically opposite occupied vertex.

Quick Tip to Solve: Add the missing sixth charge mentally: the complete hexagon has zero force, so the five-charge force is its negative.

OR

9. A simple pendulum consists of a small sphere of mass m suspended by a thread of length l . The sphere carries a positive charge q . The pendulum is placed in a uniform electric field of strength E directed vertically downwards. Find the period of oscillation of the pendulum due to the electrostatic force acting on the sphere, neglecting the effect of the gravitational force. [2 marks]

Solution:

Concept:

- The positive bob experiences a constant downward electric force $F_e = qE$ in the uniform electric field; gravity is neglected as instructed.
- For a small angular displacement θ , $\sin \theta \simeq \theta$. A restoring torque proportional to $-\theta$ produces simple harmonic motion.

Step 1: Find the restoring torque about the point of suspension The equilibrium thread points vertically downward, parallel to the electric field. Displace the bob by an angle θ from this direction. The downward electric force has a tangential restoring component $-qE \sin \theta$. With thread length l , its torque about the suspension point is

$$\tau = -qEl \sin \theta.$$

Thread tension acts along the radius and has no torque about the suspension point. Since the bob is a small sphere, it can be treated as a point mass with moment of inertia $I = ml^2$.

Quick Tip to Solve: Replace g in the pendulum period by qE/m ; do not add gravitational acceleration when it is explicitly neglected.

Step 2: Obtain the small-oscillation period Using $I\ddot{\theta} = \tau$ gives

$$ml^2\ddot{\theta} = -qEl \sin \theta.$$

For small oscillations, substitute $\sin \theta \simeq \theta$:

$$\ddot{\theta} + \frac{qE}{ml}\theta = 0.$$

Comparison with the SHM equation $\ddot{\theta} + \omega^2\theta = 0$ identifies $\omega^2 = qE/(ml)$. Therefore the period is $2\pi/\omega$. The role of downward acceleration is played by qE/m , entirely due to the electric force, because the gravitational effect is excluded.

Step 3: Conclusion The period of small oscillations is $T = 2\pi\sqrt{\frac{ml}{qE}}$.

Quick Tip to Solve: Replace g in the pendulum period by qE/m ; do not add gravitational acceleration when it is explicitly neglected.

10. Plot a graph showing the variation of undecayed nuclei N versus time t . From the graph, find out how one can determine the half-life and average life of the radioactive nuclei. [2 marks]

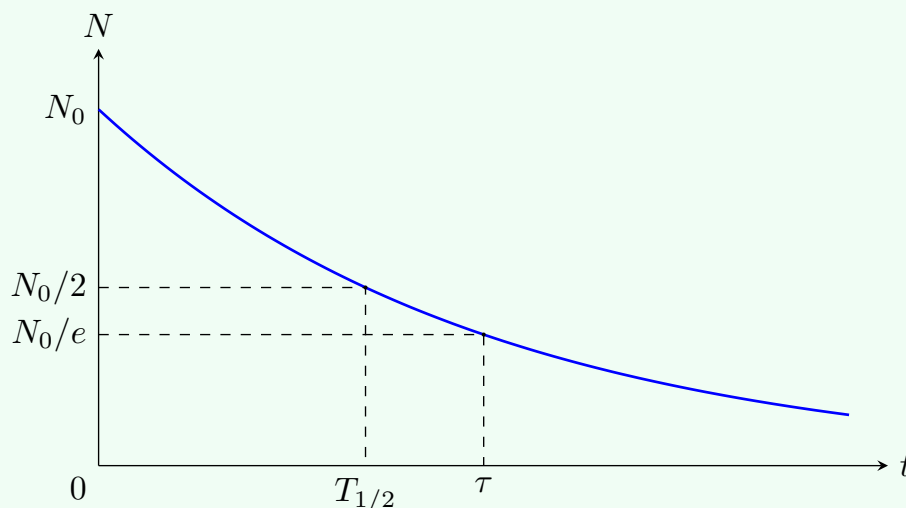
Solution:

Concept:

- The radioactive decay law is $N = N_0 e^{-\lambda t}$, where N_0 is the initial number of undecayed nuclei and λ is the decay constant.
- Half-life $T_{1/2}$ is the time for N to fall to $N_0/2$; mean or average life is $\tau = 1/\lambda$.

Step 1: Draw and read the decay curve Plot N vertically and time t horizontally. The curve starts at $(0, N_0)$, decreases exponentially and approaches zero asymptotically. Draw a horizontal line from $N_0/2$ to the curve and then a vertical line to the time axis. The corresponding time gives $T_{1/2}$. Setting $N = N_0/2$ in the decay law gives $e^{-\lambda T_{1/2}} = 1/2$, hence $T_{1/2} = \ln 2/\lambda$.

Quick Tip to Solve: On the same decay curve, use the levels $0.5N_0$ for half-life and $0.368N_0$ for mean life.



Exponential decay curve and graphical readings of half-life and mean life

Step 2: Determine the average life graphically At $t = \tau = 1/\lambda$, the decay law gives $N = N_0/e \simeq 0.368N_0$. Thus draw a horizontal line from N_0/e to the curve and project the intersection onto the time axis to read τ . Equivalently, the tangent at $t = 0$, whose slope is $-\lambda N_0$, cuts the time axis at $t = 1/\lambda = \tau$.

Step 3: Conclusion The graph yields $T_{1/2} = \ln 2/\lambda$ at $N = N_0/2$, and $\tau = 1/\lambda$ at $N = N_0/e$. Thus $T_{1/2} = 0.693\tau$.

Quick Tip to Solve: On the same decay curve, use the levels $0.5N_0$ for half-life and $0.368N_0$ for mean life.

11. A 0.5 m long solenoid of 10 turns/cm has area of cross-section 1 cm^2 . Calculate the voltage induced across its ends if the current in the solenoid is changed from 1 A to 2 A in 0.1 s. [2 marks]

Solution:

Concept:

- For an air-core solenoid, self-inductance is $L = \mu_0 n^2 A \ell$, where n is turns per metre, A the cross-sectional area and ℓ the length.
- The average self-induced emf is $\varepsilon = -L\Delta I/\Delta t$; its magnitude is the voltage across the ends.

Step 1: Convert the data and find the inductance The turn density is $n = 10 \text{ turns/cm} = 1000 \text{ m}^{-1}$, the area is $A = 1 \text{ cm}^2 = 10^{-4} \text{ m}^2$, and $\ell = 0.5 \text{ m}$. Taking $\mu_0 = 4\pi \times 10^{-7} \text{ H m}^{-1}$,

$$L = (4\pi \times 10^{-7})(1000)^2(10^{-4})(0.5) = 2\pi \times 10^{-5} \text{ H}.$$

The total number of turns is $n\ell = 500$, consistent with the stated turn density.

Step 2: Apply the self-induction law The current changes by $\Delta I = 2 - 1 = 1 \text{ A}$ in $\Delta t = 0.1 \text{ s}$, giving

Quick Tip to Solve: Convert turns per centimetre to turns per metre before squaring n in the inductance formula.

$\Delta I/\Delta t = 10 \text{ A s}^{-1}$. Therefore

$$|\varepsilon| = L \frac{\Delta I}{\Delta t} = (2\pi \times 10^{-5})(10) = 2\pi \times 10^{-4} \text{ V}.$$

The induced emf opposes the increase in current, as required by Lenz's law; the magnitude is positive.

Step 3: Conclusion The voltage induced across the solenoid is $6.28 \times 10^{-4} \text{ V} = 0.628 \text{ mV}$, opposing the increasing current.

Quick Tip to Solve: Convert turns per centimetre to turns per metre before squaring n in the inductance formula.

OR

11. A small flat search coil of area 5 cm^2 with 140 closely wound turns is placed between the poles of a powerful magnet producing magnetic field 0.09 T and then quickly removed out of the field region. Calculate

(a) change of magnetic flux through the coil, and

(b) emf induced in the coil.

[2 marks]

Solution:

Concept:

- Magnetic flux through one turn of a flat coil is $\Phi = BA \cos \theta$, where θ is the angle between the field and the normal to its plane.
- Faraday's law gives $\varepsilon = -d(n\Phi)/dt$, where $n = 140$ is the number of closely wound turns.

Step 1: Calculate the change of flux For the standard search-coil arrangement, the coil plane is perpendicular to the magnetic field, so the initial flux per turn is $\Phi_i = BA$. Convert $A = 5 \text{ cm}^2 = 5 \times 10^{-4} \text{ m}^2$. Then

$$\Phi_i = (0.09)(5 \times 10^{-4}) = 4.5 \times 10^{-5} \text{ Wb}.$$

After removal, the flux is zero, so $\Delta\Phi = -4.5 \times 10^{-5} \text{ Wb}$ per turn. The decrease in total flux linkage has magnitude $nBA = 140(4.5 \times 10^{-5}) = 6.3 \times 10^{-3} \text{ Wb-turn}$.

Step 2: Express the induced emf using the removal time If the coil is removed in a time Δt , its average induced emf has magnitude

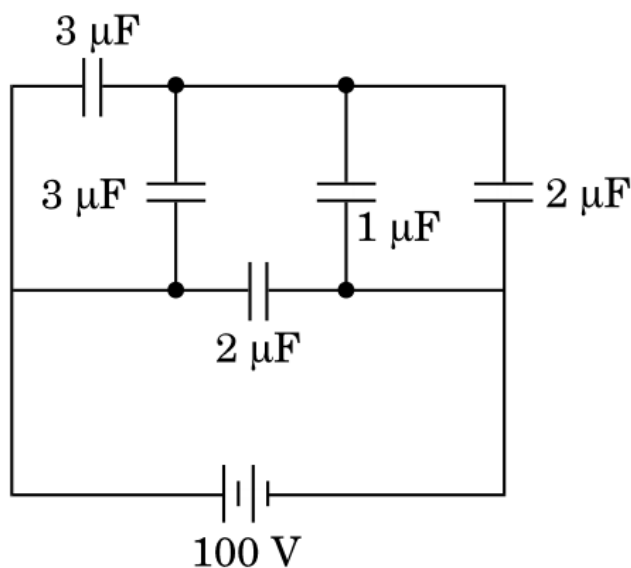
$$|\varepsilon_{\text{av}}| = \frac{n|\Delta\Phi|}{\Delta t} = \frac{6.3 \times 10^{-3}}{\Delta t} \text{ V},$$

with Δt in seconds. Its direction opposes the loss of flux. The printed question gives no removal time; the word "quickly" does not specify a numerical rate of change. Therefore a unique numerical emf cannot be calculated from the supplied data.

Step 3: Conclusion (a) Flux decreases by $4.5 \times 10^{-5} \text{ Wb}$ per turn; total flux linkage decreases by $6.3 \times 10^{-3} \text{ Wb-turn}$. (b) $|\varepsilon_{\text{av}}| = 6.3 \times 10^{-3}/\Delta t \text{ V}$; a numerical emf requires the removal time.

Quick Tip to Solve: Distinguish flux through one turn from total flux linkage; multiply by 140 only when applying Faraday's law.

12. The figure shows a network of five capacitors connected to a 100 V supply. Calculate the total energy stored in the network. [2 marks]



Solution:

Concept:

- Capacitances in parallel add; for two capacitors in series, $C_s = C_1 C_2 / (C_1 + C_2)$. Capacitors are parallel only when both their terminals share the same two nodes.
- Total electrostatic energy at supply voltage V is $U = \frac{1}{2} C_{eq} V^2$, with capacitance expressed in farads.

Step 1: Identify the three nodes and reduce the network Let the left supply node be A , the upper joining wire be B , and the right supply node be C . The upper-left $3 \mu\text{F}$ and the vertical-left $3 \mu\text{F}$ capacitors both join A to B , so $C_{AB} = 3 + 3 = 6 \mu\text{F}$. The vertical $1 \mu\text{F}$ and right-hand $2 \mu\text{F}$ capacitors both join B to C , so $C_{BC} = 1 + 2 = 3 \mu\text{F}$. These two groups form a series branch across the supply:

$$C_s = \frac{6 \times 3}{6 + 3} = 2 \mu\text{F}.$$

Step 2: Include the direct branch and calculate the energy The middle horizontal $2 \mu\text{F}$ capacitor joins A directly to C ; it is in parallel with the series branch. Hence $C_{eq} = 2 + 2 = 4 \mu\text{F} = 4 \times 10^{-6} \text{ F}$. With $V = 100 \text{ V}$,

$$U = \frac{1}{2} (4 \times 10^{-6}) (100)^2 = 2.0 \times 10^{-2} \text{ J}.$$

Using the equivalent capacitance accounts for the energy in all five capacitors, whose individual voltages need not equal the supply voltage.

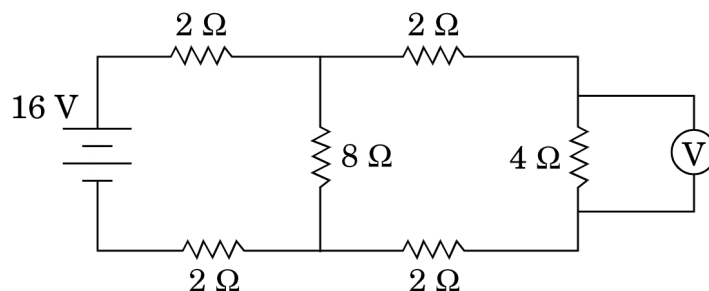
Step 3: Conclusion The total energy stored in the network is $2.0 \times 10^{-2} \text{ J}$.

Quick Tip to Solve: Mark common wire nodes first: the two left 3-microfarad capacitors are parallel, despite their different orientations.

Section C (36 marks)

13. In the circuit given below, find the voltmeter reading across a $4\ \Omega$ resistor.

[3 marks]



Solution:

Concept:

- Resistors in series carry the same current and their resistances add; parallel branches have the same potential difference.
- Ohm's law is $V = IR$, where V is potential difference, I is current and R is resistance. An ideal voltmeter has infinite resistance and draws no current.

Step 1: Identify the branches between the central junctions Call the upper central junction A and the lower central junction B. The middle branch between A and B contains the $8\ \Omega$ resistor. The right-hand route from A to B contains the upper $2\ \Omega$, the $4\ \Omega$ resistor and the lower $2\ \Omega$ resistor in series. Since the voltmeter draws negligible current, it does not change this branch resistance:

$$R_{\text{right}} = 2 + 4 + 2 = 8\ \Omega.$$

Thus the two A–B branches have equal resistance and carry equal currents.

Step 2: Find the total resistance and supply current The two $8\ \Omega$ branches are in parallel. Their equivalent resistance lies in series with the two left-hand $2\ \Omega$ resistors:

$$R_{AB} = \frac{8 \times 8}{8 + 8} = 4\ \Omega,$$

$$R_{\text{total}} = 2 + 4 + 2 = 8\ \Omega,$$

$$I_{\text{total}} = \frac{16}{8} = 2\ \text{A}.$$

Both left-hand resistors carry this total current before and after the current divides at the junctions.

Step 3: Calculate the voltage across the measured resistor The potential difference between A and B is $I_{\text{total}} R_{AB} = 2 \times 4 = 8\ \text{V}$. Hence the current through the complete right-hand branch is $8/8 = 1\ \text{A}$. The voltmeter is connected only across its $4\ \Omega$ resistor, rather than across the whole branch, so its reading is obtained from

$$V_{4\Omega} = I_{\text{right}} \times 4 = (1)(4) = 4\ \text{V}.$$

Step 4: Conclusion The voltmeter reads 4 V.

Quick Tip to Solve: Combine the right-hand $2 + 4 + 2\ \Omega$ route first: it equals the middle $8\ \Omega$ branch, so the supply current divides equally.

14. When a parallel plate capacitor is being charged by a dc battery, why does a galvanometer show a momentary deflection?

Obtain the expression for the displacement current and show how it is related to conduction current in a steady state. [3 marks]

Solution:

Concept:

- Conduction current is the flow of free charges in the connecting wires: $I_c = dq/dt$, where q is the charge accumulated on a capacitor plate.
- A changing electric flux produces displacement current $I_d = \epsilon_0 d\Phi_E/dt$, where ϵ_0 is the permittivity of free space and Φ_E is electric flux through the plate gap.

Step 1: Explain the momentary galvanometer deflection Immediately after connection to the dc battery, charges flow through the galvanometer onto the capacitor plates. This charging conduction current produces a deflection. As the capacitor charges, its potential difference rises towards the battery voltage and the charging current decreases. When the capacitor is fully charged, the potential difference is constant and no further charge flows; the galvanometer returns to zero. Free charges do not cross the insulating space between the plates. During charging, however, the changing electric field in this space produces displacement current.

Step 2: Derive the displacement current between the plates Consider parallel plates of area A , carrying charges $+q$ and $-q$, separated by air or vacuum. Neglecting edge effects, the electric field is uniform and perpendicular to the plates:

$$E = \frac{q}{\epsilon_0 A}, \quad \Phi_E = EA = \frac{q}{\epsilon_0}.$$

Differentiating with respect to time and using Maxwell's expression gives

$$I_d = \epsilon_0 \frac{d\Phi_E}{dt} = \epsilon_0 \frac{d}{dt} \left(\frac{q}{\epsilon_0} \right) = \frac{dq}{dt}.$$

Equivalently, for constant plate area, $I_d = \epsilon_0 A dE/dt$. Its direction corresponds to the increasing electric field during charging.

Step 3: Relate displacement current to conduction current The current in a connecting wire is exactly the rate at which charge accumulates on its connected plate. Therefore, during charging, $I_c = dq/dt = I_d$ in magnitude. This equality preserves current continuity even though there is no conduction across the plate gap. In the final dc steady state the charge, electric field and electric flux are all constant:

$$\frac{dq}{dt} = 0, \quad \frac{d\Phi_E}{dt} = 0, \quad I_c = I_d = 0.$$

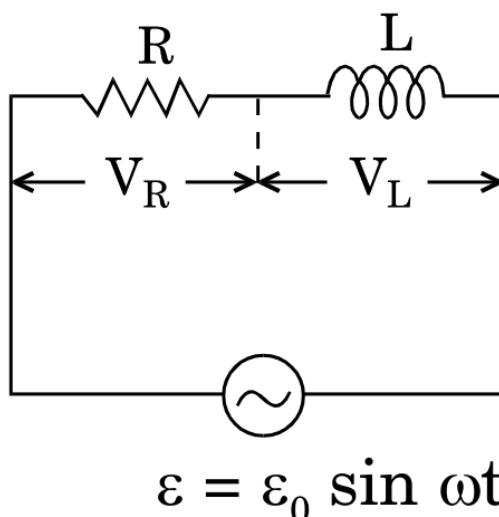
Step 4: Conclusion The galvanometer deflects only while the capacitor charges. The displacement current is $I_d = \epsilon_0 d\Phi_E/dt = I_c$; in the final dc steady state both currents are zero.

Quick Tip to Solve: Use $\Phi_E = q/\epsilon_0$ to prove current continuity directly, and then set the time derivatives to zero for a fully charged dc capacitor.

15. (a) An ac circuit as shown in the figure has an inductor of inductance L and a resistor of resistance R connected in series. Using the phasor diagram, explain why the voltage in the circuit will lead the current in phase.

(b) The potential difference across the resistor is 160 V and that across the inductor is 120 V. Find the effective value of the applied voltage. If the effective current in the circuit be 1.0 A, calculate the total impedance of the circuit.

(c) What will be the potential difference in the circuit when direct current is passed through the circuit? [3 marks]



Solution:

Concept:

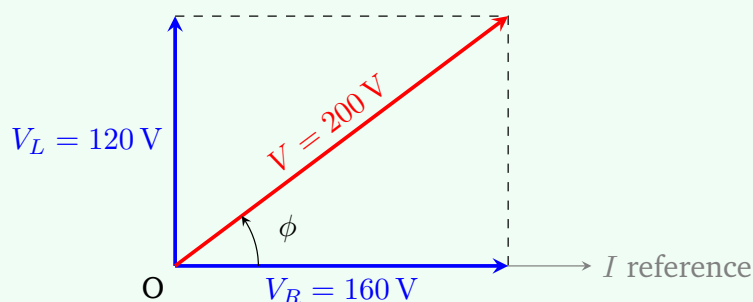
- In a resistor, voltage $V_R = IR$ is in phase with current I . In an ideal inductor, voltage $V_L = IX_L$ leads the current by 90° , with inductive reactance $X_L = \omega L$.
- Series RL voltages add as phasors: $V = \sqrt{V_R^2 + V_L^2}$, $Z = \sqrt{R^2 + X_L^2} = V/I$, and $\tan \phi = X_L/R$. The symbols V and I denote effective (rms) values.

Step 1: Explain the phase relation using a phasor diagram (a) Take the current as the reference along the horizontal axis. The resistor voltage $\vec{V}_R$ points along the same axis. The inductor voltage $\vec{V}_L$ points vertically upwards because it leads the current by 90° . The applied voltage is their vector sum $\vec{V} = \vec{V}_R + \vec{V}_L$, drawn as the diagonal of the right triangle. Thus it lies ahead of the current by an angle ϕ , where

$$\tan \phi = \frac{V_L}{V_R} = \frac{\omega L}{R}.$$

Since $L > 0$, this angle is positive: the applied voltage leads and the current lags. The two component voltages cannot be added as ordinary scalar magnitudes because they are perpendicular phasors.

Quick Tip to Solve: The component voltages 120, 160 form a scaled 3 : 4 : 5 triangle, so the ac total is 200 V; with steady dc, keep only the resistive drop.



Voltage phasors for the series RL circuit, with current as reference

Step 2: Calculate the effective voltage and impedance (b) Apply Pythagoras to the voltage triangle using the given rms component voltages:

$$V = \sqrt{(160)^2 + (120)^2} = \sqrt{40000} = 200 \text{ V.}$$

With effective current $I = 1.0 \text{ A}$, the resistance and inductive reactance are

$$R = \frac{160}{1.0} = 160 \Omega, \quad X_L = \frac{120}{1.0} = 120 \Omega.$$

Hence the impedance is

$$Z = \frac{V}{I} = \frac{200}{1.0} = 200 \Omega.$$

This also agrees with $\sqrt{160^2 + 120^2} \Omega$. For these values, $\phi = \tan^{-1}(120/160) \approx 36.9^\circ$.

Step 3: Examine the circuit with direct current (c) For steady direct current, $dI/dt = 0$, so an ideal inductor has no potential difference across it: $V_L = L dI/dt = 0$. Therefore the entire circuit potential difference is across the resistor and equals $V_{dc} = I_{dc}R$. For the same current of 1.0 A used in part (b),

$$V_{dc} = (1.0)(160) = 160 \text{ V.}$$

Thus only the resistive drop remains. If a different dc current is supplied, the corresponding voltage is $160I_{dc}$ volts, with I_{dc} expressed in amperes; the numerical value 160 V refers to retaining the stated current. During switching, the inductor can have a transient voltage until the current becomes steady.

Step 4: Conclusion (a) Voltage leads current by $\phi = \tan^{-1}(\omega L/R)$. (b) The effective applied voltage is $\boxed{200 \text{ V}}$ and total impedance is $\boxed{200 \Omega}$. (c) With steady dc, $V_L = 0$ and $V = I_{dc}R$, giving $\boxed{160 \text{ V}}$ for the same 1.0 A current.

Quick Tip to Solve: The component voltages 120, 160 form a scaled 3 : 4 : 5 triangle, so the ac total is 200 V ; with steady dc, keep only the resistive drop.

15. An ac circuit consists of a series combination of circuit elements X and Y . The current is ahead of the voltage in phase by $\frac{\pi}{4}$. If element X is a pure resistor of $100\ \Omega$,

(a) name the circuit element Y .

(b) calculate the rms value of current, if rms value of voltage is 141 V .

(c) what will happen if the ac source is replaced by a dc source?

[3 marks]

Solution:

Concept:

- In a series resistor-capacitor circuit, the current leads the applied voltage. The capacitive reactance is $X_C = 1/(\omega C)$, where C is capacitance and ω is angular frequency.
- The phase lead satisfies $\tan \phi = X_C/R$. The impedance magnitude is $Z = \sqrt{R^2 + X_C^2}$, and $I_{\text{rms}} = V_{\text{rms}}/Z$.

Step 1: (a) Identify element Y The resistor voltage is in phase with current, whereas the capacitor voltage lags current by $\pi/2$. Their resultant therefore lags current. Since the given current leads the applied voltage, element Y must be a capacitor. An inductor would make current lag; another resistor would give no phase difference.

Step 2: (b) Determine the reactance and rms current For the specified phase lead $\phi = \pi/4$,

$$X_C = R \tan \phi = 100 \tan(\pi/4) = 100\ \Omega.$$

Hence the series impedance is

$$Z = \sqrt{100^2 + 100^2} = 100\sqrt{2}\ \Omega.$$

Using rms voltage and rms current consistently,

$$I_{\text{rms}} = \frac{141}{100\sqrt{2}}\text{ A} \simeq 0.997\text{ A}.$$

The voltage is specified approximately, so the current may be rounded to 1.0 A . There is no extra factor of $\sqrt{2}$ to convert this given rms voltage to an amplitude.

Step 3: (c) Replace the ac source by a dc source Immediately after switching to dc, a transient charging current can flow through the resistor. As charge accumulates, the capacitor voltage rises and the current decays. Once the ideal capacitor is fully charged, it blocks steady dc and acts as an open circuit: the steady current is zero. The applied dc voltage then appears across the capacitor, while the resistor has zero steady voltage drop.

Step 4: Conclusion Y is a capacitor; $I_{\text{rms}} \simeq 1.0\text{ A}$. With a dc source, only a charging transient occurs and the steady current becomes zero.

Quick Tip to Solve: A current lead identifies capacitance; at a lead of 45° , set capacitive reactance equal to resistance before finding impedance.

16. (a) Explain briefly, using a proper diagram, the difference in behaviour of a conductor and a dielectric in the presence of external electric field.

(b) Define the term polarization of a dielectric and write the expression for a linear isotropic dielectric in terms of electric field. [3 marks]

Solution:

Concept:

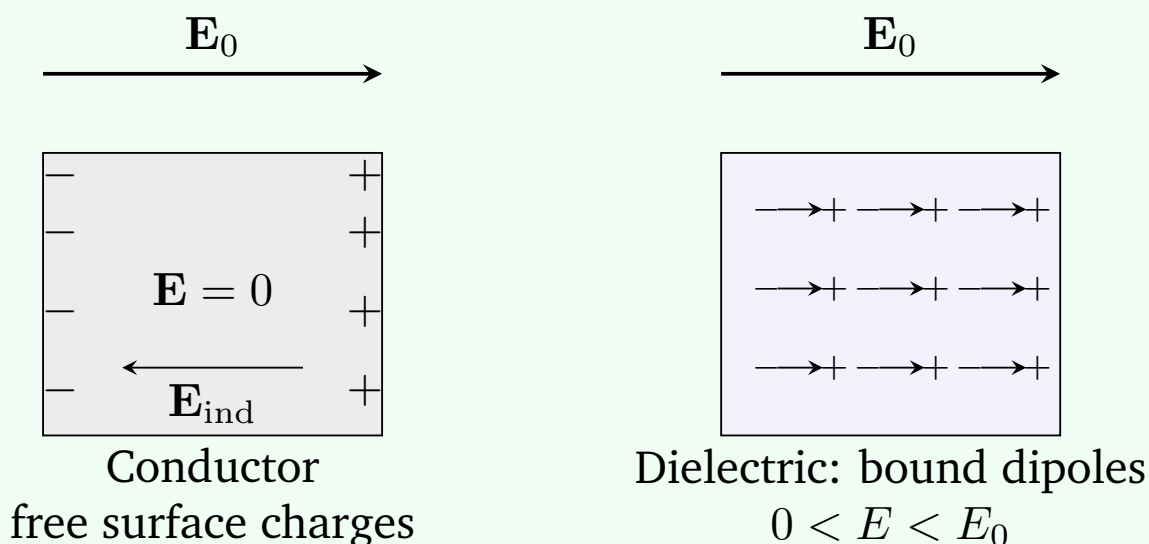
- Free charges in a conductor move throughout its bulk; bound charges in a dielectric undergo small relative displacements or dipoles reorient.
- Polarization $\mathbf{P}$ is the electric dipole moment per unit volume. For a linear isotropic dielectric, $\mathbf{P} = \epsilon_0 \chi_e \mathbf{E}$, where χ_e is electric susceptibility and $\mathbf{E}$ is the total internal field.

Step 1: (a) Behaviour of a conductor Apply an external field $\mathbf{E}_0$ to an initially neutral conductor. Its free electrons move opposite to this field, producing negative surface charge on the left and positive surface charge on the right in the diagram. The induced field $\mathbf{E}_{\text{ind}}$ opposes $\mathbf{E}_0$. Charge rearrangement continues until electrostatic equilibrium is reached:

$$\mathbf{E}_{\text{inside}} = \mathbf{E}_0 + \mathbf{E}_{\text{ind}} = 0.$$

Thus the conductor interior is equipotential, and the induced excess charge lies on its surface.

Step 2: (a) Behaviour of a dielectric In a dielectric there are no freely mobile charges that can cross the entire material. The external field slightly separates the centres of positive and negative bound charge in nonpolar molecules, or tends to align existing dipoles in polar molecules. Dipoles point along the applied field, producing negative bound surface charge on the left and positive bound surface charge on the right. Their field opposes the applied field but does not ordinarily cancel it completely. The total field inside the dielectric is reduced and remains nonzero. The diagram contrasts free surface-charge redistribution with polarization of bound dipoles.



Charge redistribution in a conductor and polarization in a dielectric under an external field to the right.

Step 3: (b) Define polarization and give its constitutive relation Polarization is the net electric dipole

Quick Tip to Solve: Use the total field inside the dielectric in the polarization formula, and distinguish bound-charge displacement from free electron motion.

moment per unit volume of dielectric:

$$\mathbf{P} = \frac{\sum \mathbf{p}}{\Delta V},$$

where the sum runs over dipoles in a small representative volume ΔV . Its SI unit is C m^{-2} . For a linear isotropic dielectric,

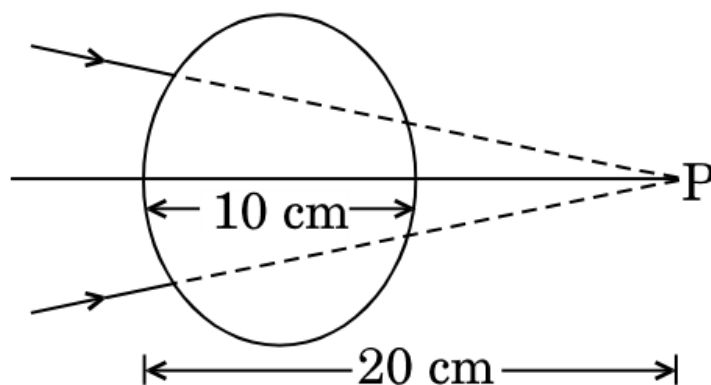
$$\mathbf{P} = \varepsilon_0 \chi_e \mathbf{E} = \varepsilon_0 (\varepsilon_r - 1) \mathbf{E}.$$

Here ε_0 is vacuum permittivity, χ_e is a dimensionless scalar susceptibility, and ε_r is relative permittivity. Linearity gives proportionality to the total internal field; isotropy makes $\mathbf{P}$ parallel to $\mathbf{E}$.

Step 4: Conclusion A conductor cancels the internal electrostatic field by free-charge redistribution; a dielectric polarizes and reduces the internal field. Polarization is dipole moment per unit volume, with $\mathbf{P} = \varepsilon_0 \chi_e \mathbf{E}$.

Quick Tip to Solve: Use the total field inside the dielectric in the polarization formula, and distinguish bound-charge displacement from free electron motion.

17. A converging beam of light travelling in air converges at a point P as shown in the figure. When a glass sphere of refractive index 1.5 is introduced in between the path of the beam, calculate the new position of the image. Also draw the ray diagram for the image formed. [3 marks]



Solution:

Concept:

- For paraxial refraction at a spherical surface, $n_2/v - n_1/u = (n_2 - n_1)/R$. Here u , v and R are measured from that surface, positive in the direction of incident light.
- A beam already converging toward a point beyond the refracting surface has a virtual object there, so its object distance is positive. The given sphere has diameter 10 cm, hence radius 5 cm.

Step 1: Identify the virtual object for the first surface Let A be the front vertex and B the rear vertex of the glass sphere. Light travels from left to right. The original beam would converge at P , which the printed figure places 20 cm to the right of A . Thus P is a virtual object for refraction at the front surface: $u_1 = +20$ cm. The centre of curvature is to the right of A , so $R_1 = +5$ cm. The refractive indices are $n_1 = 1$ for air and $n_2 = 1.5$ for glass.

Quick Tip to Solve: For an initially converging beam, take the original focus as a virtual object with positive u ; compare the first image distance with the sphere diameter.

Step 2: Find where the refracted beam meets Substitute into the spherical-refraction relation:

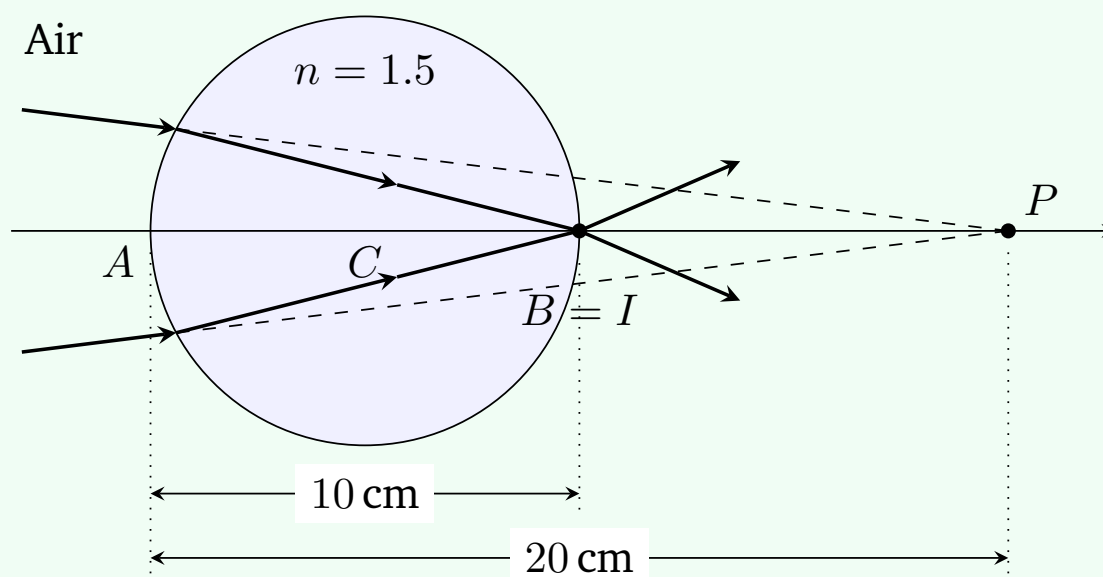
$$\frac{1.5}{v_1} - \frac{1}{20} = \frac{1.5 - 1}{5}.$$

Therefore, with distances in centimetres,

$$\frac{1.5}{v_1} = \frac{1}{20} + \frac{0.5}{5} = 0.15, \quad v_1 = +10 \text{ cm}.$$

The positive sign means a real convergence point to the right of the first surface. Since $AB = 10 \text{ cm}$, this convergence point is exactly at the rear vertex B , rather than beyond the sphere.

Step 3: Account for the rear surface and locate the final image The paraxial rays meet at the rear surface itself, at B . Refraction at that surface changes their directions after the meeting point but leaves that meeting point at the surface. Hence the final image is at B . One must not substitute an ordinary finite object distance for the second surface: the intermediate image is at that surface, so $u_2 = 0$. Equivalently, in the limit $u_2 \rightarrow 0$, the second-surface relation gives $v_2 \rightarrow 0$. The solid rays in the schematic converge at B , whereas the dashed continuations show the original focus P .



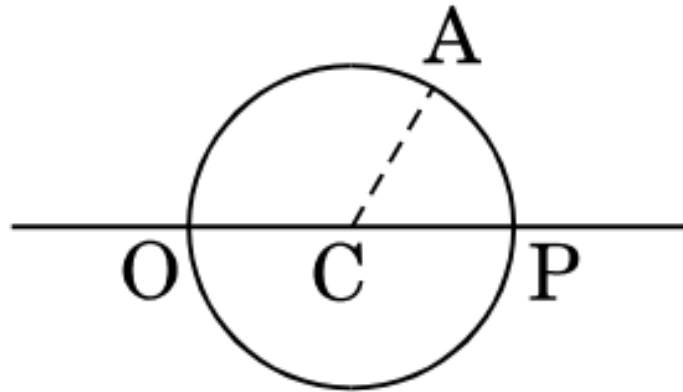
Paraxial schematic: insertion of the glass sphere shifts the convergence from P to the rear vertex B .

Step 4: Conclusion The new image is at the rear surface of the sphere: 10 cm from the front surface, or 10 cm towards the sphere from the original point P .

Quick Tip to Solve: For an initially converging beam, take the original focus as a virtual object with positive u ; compare the first image distance with the sphere diameter.

OR

17. A point 'O' marked on the surface of a glass sphere of diameter 20 cm is viewed through glass from the position directly opposite to the point O. If the refractive index of the glass is 1.5, find the position of the image formed. Also, draw the ray diagram for the formation of the image. [3 marks]



Solution:

Concept:

- For paraxial refraction at a spherical surface, $\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$. Here n_1 and n_2 are the incident and emergent refractive indices, and u, v, R are signed distances from the pole of the refracting surface.
- Use the Cartesian sign convention: measure all distances from the pole, taking the direction of incident light as positive. A negative image distance means a virtual image on the incident side.

Step 1: Identify the surface and signed distances The observer looks from the side opposite O , so rays travel inside the glass from O towards the opposite surface at P , then emerge into air. The marked point is already on the glass surface; the relevant refraction is at the far glass-air surface. Take the direction from O to P as positive. The sphere has radius 10 cm, and both O and the centre C lie to the left of P . Thus

$$u = -20 \text{ cm}, \quad R = -10 \text{ cm}, \quad n_1 = 1.5, \quad n_2 = 1.$$

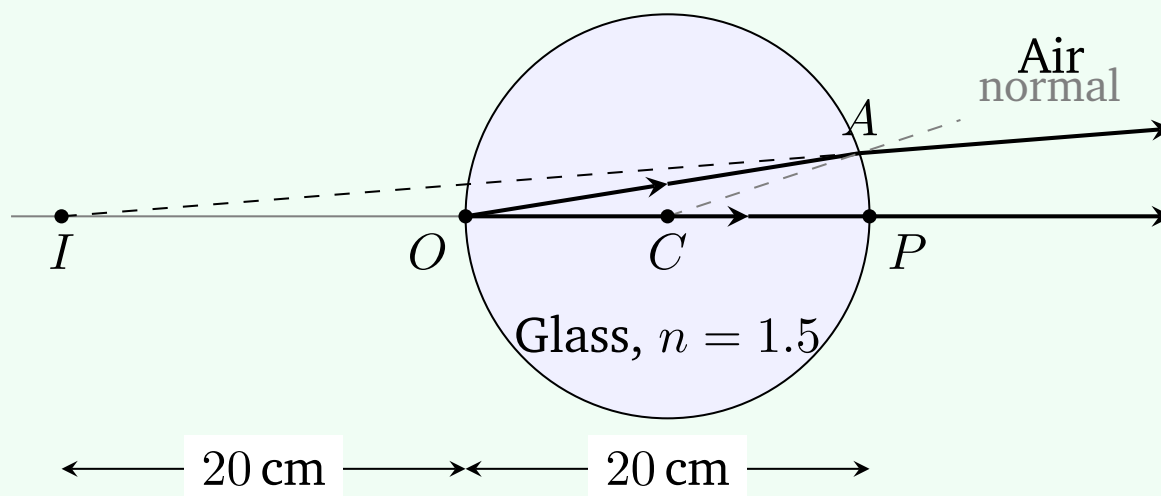
Step 2: Apply refraction at the far spherical surface Substitution gives

$$\begin{aligned} \frac{1}{v} - \frac{1.5}{-20} &= \frac{1 - 1.5}{-10}, \\ \frac{1}{v} + 0.075 &= 0.050, \\ \frac{1}{v} &= -0.025 \text{ cm}^{-1}, \quad v = -40 \text{ cm}. \end{aligned}$$

The emerging rays diverge. Their backward extensions meet at I on the object side, so the observer sees a virtual image. Since O is 20 cm from P , the image is another 20 cm beyond O , outside the sphere.

Quick Tip to Solve: Set the origin at the far surface P : both the object and centre are behind it, so u and R are negative.

Step 3: Interpret the ray diagram The axial ray passes normally through P without deviation. An oblique paraxial ray from O meets the far surface at A , whose normal is CA . On going from glass to air it bends away from this normal. The dashed backward extension of the emergent ray meets the axis at I . The diagram represents the paraxial image used in the spherical-refraction formula.



Paraxial rays emerging from glass into air; backward extensions meet at the virtual image I .

Step 4: Conclusion The image is virtual and lies 40 cm from the opposite surface P on the object side: 20 cm beyond O , outside the glass sphere.

Quick Tip to Solve: Set the origin at the far surface P : both the object and centre are behind it, so u and R are negative.

18. (a) Use Kirchhoff's rules to obtain the balance condition in a Wheatstone bridge.

(b) Give one practical application that is based on this principle.

[3 marks]

Solution:

Concept:

- Kirchhoff's junction rule expresses conservation of charge: the total current entering a junction equals the total current leaving it.
- Kirchhoff's loop rule expresses conservation of energy: the algebraic sum of potential changes around a closed loop is zero. Across a resistor in the direction of current the potential change is $-IR$.
- A Wheatstone bridge is balanced when the galvanometer connecting its two intermediate junctions carries zero current, so these junctions have equal potential.

Step 1: (a) Define the bridge and use the junction rule Let the four arms of the bridge be $AB = P$, $BC = Q$, $AD = R$ and $DC = S$. Connect the battery across A and C , with A at higher potential, and a galvanometer between B and D . At balance, $I_g = 0$ and $V_B = V_D$. Kirchhoff's junction rule then shows that the same current I_1 flows through P and Q , and the same current I_2 flows through R and S . The battery current is $I = I_1 + I_2$.

Quick Tip to Solve: At null deflection write equal potential drops from the battery junction to the two middle junctions, then divide the two loop equations.

Step 2: Apply the loop rule to the two galvanometer loops In loop $A \rightarrow B \rightarrow D \rightarrow A$, the galvanometer drop vanishes at balance, so

$$-I_1P + I_2R = 0 \Rightarrow I_1P = I_2R.$$

Similarly, loop $B \rightarrow C \rightarrow D \rightarrow B$ gives

$$-I_1Q + I_2S = 0 \Rightarrow I_1Q = I_2S.$$

Dividing the first equality by the second eliminates the branch currents:

$$\frac{P}{Q} = \frac{R}{S}, \quad \text{or equivalently} \quad PS = QR.$$

Thus the balance condition depends on the ratio of arm resistances, rather than the battery voltage. It is essential that the galvanometer current is zero before treating each pair of resistors as carrying the same branch current.

Step 3: (b) State a practical application A metre bridge uses the Wheatstone-bridge principle to measure an unknown resistance X . Put X and a known resistance R_0 in its two gaps and obtain a null deflection with the jockey. If the wire length adjacent to X is l cm, the other length is $(100 - l)$ cm. For a uniform wire, resistance is proportional to length, giving

$$\frac{X}{R_0} = \frac{l}{100 - l}, \quad X = R_0 \frac{l}{100 - l}.$$

Step 4: Conclusion The Wheatstone-bridge balance condition is $P/Q = R/S$, or $PS = QR$. A metre bridge applies this principle to determine an unknown resistance.

Quick Tip to Solve: At null deflection write equal potential drops from the battery junction to the two middle junctions, then divide the two loop equations.

19. (a) Plot a graph to show the variation of stopping potential with frequency of incident radiation in relation to photoelectric effect.

(b) Use Einstein's photoelectric equation to show how from this graph, (i) Threshold frequency, and (ii) Planck's constant can be determined. [3 marks]

Solution:

Concept:

- Einstein's photoelectric equation is $h\nu = \phi + K_{\max}$, where h is Planck's constant, ν is incident frequency, ϕ is the metal's work function and $K_{\max}$ is the maximum photoelectron kinetic energy.
- The magnitude of stopping potential V_s satisfies $eV_s = K_{\max}$, where e is the positive elementary charge. The threshold frequency ν_0 satisfies $\phi = h\nu_0$.

Step 1: (a) Obtain and plot the straight-line relation Combine Einstein's equation with the definition

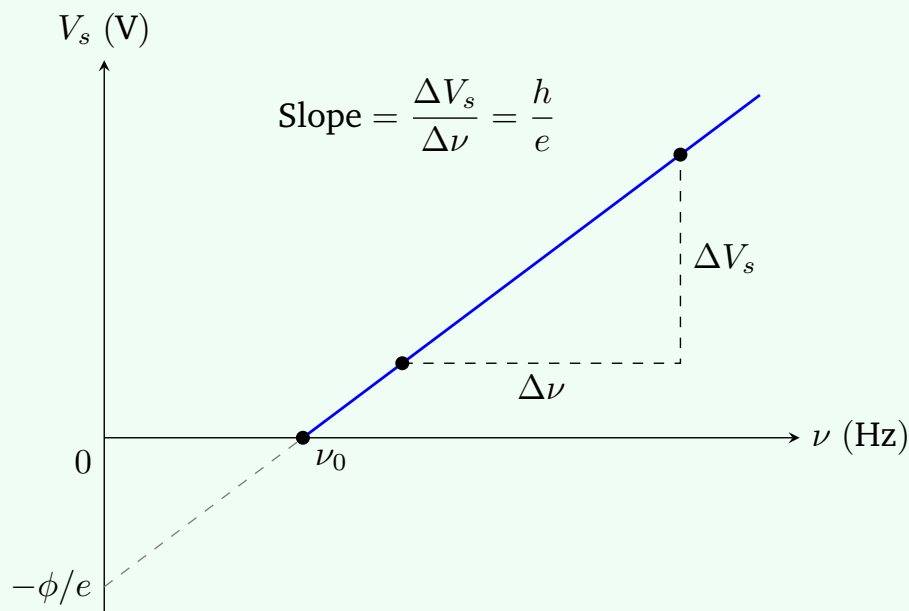
Quick Tip to Solve: Read threshold frequency from the horizontal intercept; multiply the stopping-potential versus frequency slope by e to obtain h .

of stopping potential:

$$eV_s = h\nu - \phi,$$

$$V_s = \frac{h}{e}\nu - \frac{\phi}{e} = \frac{h}{e}(\nu - \nu_0).$$

Plot V_s on the vertical axis and ν on the horizontal axis for a fixed photosensitive material. The graph is a straight line of positive slope h/e , beginning at $(\nu_0, 0)$. For $\nu < \nu_0$, no photoelectrons are emitted and hence there is no measurable stopping potential. The dashed extension below the frequency axis is only a mathematical extrapolation of the straight line.



Stopping potential versus incident frequency; the dashed line is an extrapolation only.

Step 2: (b)(i) Read the threshold frequency At the point where the line meets the frequency axis, $V_s = 0$, so the emitted electrons have zero maximum kinetic energy. Einstein's equation becomes

$$0 = h\nu_0 - \phi \Rightarrow \nu_0 = \frac{\phi}{h}.$$

Therefore the frequency-axis intercept gives the threshold frequency directly. It is the minimum photon frequency needed to overcome the work function, independent of incident intensity.

Step 3: (b)(ii) Determine Planck's constant from the slope Select two points (ν_1, V_1) and (ν_2, V_2) on the straight line, with frequencies expressed in hertz and stopping potentials in volts. Its slope is

$$m = \frac{V_2 - V_1}{\nu_2 - \nu_1} = \frac{h}{e}.$$

Consequently, multiply the measured slope by the elementary charge:

$$h = e m = e \frac{\Delta V_s}{\Delta \nu}, \quad e = 1.602 \times 10^{-19} \text{ C}.$$

The slope has units Vs; multiplication by charge gives Js, the required unit of Planck's constant. Changing the metal changes the intercept through its work function, but not the slope h/e .

Quick Tip to Solve: Read threshold frequency from the horizontal intercept; multiply the stopping-potential versus frequency slope by e to obtain h .

Step 4: Conclusion The stopping-potential graph is a straight line: $V_s = (h/e)(\nu - \nu_0)$. Its frequency-axis intercept gives ν_0 , and Planck's constant is $h = e \times \text{slope}$.

Quick Tip to Solve: Read threshold frequency from the horizontal intercept; multiply the stopping-potential versus frequency slope by e to obtain h .

OR

19. (a) How does one explain the emission of electrons from a photosensitive surface with the help of Einstein's photoelectric equation?

(b) Work function of aluminium is 4.2 eV. If two photons each of energy 2.5 eV are incident on its surface, will the emission of electrons take place? Justify your answer.

(c) The stopping potential in an experiment on photoelectric effect is 1.5 V. What is the maximum kinetic energy of the photoelectrons emitted? Calculate in Joules. [3 marks]

Solution:

Concept:

- A photon of frequency ν carries energy $h\nu$, where h is Planck's constant. The work function ϕ is the minimum energy needed to remove an electron from the surface.
- Einstein's equation is $h\nu = \phi + K_{\max}$. At stopping potential V_s , the maximum kinetic energy is $K_{\max} = eV_s$, where e is the magnitude of electronic charge.

Step 1: (a) Explain emission using energy conservation In the photoelectric interaction, one electron absorbs the energy of one incident photon. Part of this energy supplies the work function, and the remainder appears as kinetic energy. For an electron escaping with the greatest kinetic energy,

$$K_{\max} = h\nu - \phi.$$

Emission is possible only when $h\nu \geq \phi$, or $\nu \geq \nu_0 = \phi/h$. Below threshold, increasing intensity cannot cause emission; above threshold, increasing frequency raises maximum kinetic energy.

Step 2: (b) Compare each photon with the aluminium work function Each incident photon has energy 2.5 eV, whereas the work function is 4.2 eV. Thus an individual photon has insufficient energy:

$$2.5 \text{ eV} < 4.2 \text{ eV}.$$

The energies of the two photons are not added in the ordinary one-photon photoelectric process described by Einstein's equation. Neither photon can therefore eject an electron.

Step 3: (c) Convert the stopping potential into kinetic energy At the stopping potential, the retarding electric field just stops even the fastest emitted electrons. Its opposing work is eV_s . With $e = 1.6 \times 10^{-19} \text{ C}$ and $V_s = 1.5 \text{ V}$,

$$K_{\max} = eV_s = (1.6 \times 10^{-19})(1.5) = 2.4 \times 10^{-19} \text{ J}.$$

The same energy is 1.5 eV.

Quick Tip to Solve: Compare the energy of each photon with the work function; for stopping potential, multiply its value in volts by electronic charge to obtain joules.

Step 4: Conclusion Einstein's equation explains emission through $h\nu = \phi + K_{\max}$ with a threshold $h\nu \geq \phi$. The two 2.5 eV photons do not cause emission from aluminium of work function 4.2 eV. The maximum kinetic energy at a stopping potential of 1.5 V is $2.4 \times 10^{-19} \text{ J}$.

Quick Tip to Solve: Compare the energy of each photon with the work function; for stopping potential, multiply its value in volts by electronic charge to obtain joules.

20. (a) What do you mean by bandwidth of a signal? Give its importance.

(b) Differentiate between Analog and Digital communication.

(c) Write the functions of transducer and repeaters.

[3 marks]

Solution:

Concept:

- A message signal contains frequency components. Its bandwidth is the range of frequencies needed to represent and transmit the message faithfully.
- Communication converts information to a suitable signal, sends it through a channel and recovers it at the receiver. The form of the signal determines whether communication is analog or digital.

Step 1: (a) Define bandwidth and explain its importance The bandwidth of a signal is the difference between the highest and lowest frequencies present in it:

$$\Delta f = f_{\max} - f_{\min}.$$

It is measured in hertz. For a baseband signal extending from zero to $f_{\max}$, its bandwidth is $f_{\max}$. Knowing this range lets us select a communication channel and equipment with sufficient bandwidth to carry the information. If essential frequency components lie outside the available passband, the received signal loses information or becomes distorted. Different messages, such as speech and television pictures, consequently require different bandwidths.

Step 2: (b) Differentiate analog and digital communication In analog communication, the information is represented by a signal that varies continuously and can take a continuous range of amplitudes. An audio voltage from a microphone is an example. Noise added during transmission changes this continuous waveform and is difficult to remove completely.

In digital communication, information is represented by discrete signal levels, usually binary 0 and 1, transmitted as a sequence of pulses. The receiver can identify these levels and regenerate the original pulse pattern, making digital transmission more resistant to moderate noise. Converting an analog message to digital form involves sampling, quantisation and coding before transmission.

Step 3: (c) State the functions of a transducer and a repeater A transducer converts one form of energy or physical variation into another. In a communication transmitter, an input transducer converts the message into an electrical signal; for example, a microphone converts sound variations into an electrical voltage. At reception, a loudspeaker converts electrical signals into sound.

A repeater receives a weakened signal, amplifies or regenerates it, and retransmits it towards the destination. It acts as a receiver and transmitter together and increases the effective communication

Quick Tip to Solve: Link bandwidth to the frequencies needed by the message, and remember the repeater's sequence: receive, strengthen or regenerate, retransmit.

distance by overcoming attenuation along the transmission path.

Step 4: Conclusion Signal bandwidth is $f_{\max} - f_{\min}$ and determines the required transmission range. Analog communication uses continuously varying signals, while digital communication uses discrete coded levels. A transducer converts the message between physical and electrical forms, and a repeater receives, strengthens or regenerates, and retransmits a signal.

Quick Tip to Solve: Link bandwidth to the frequencies needed by the message, and remember the repeater's sequence: receive, strengthen or regenerate, retransmit.

21. Answer the following:

- (a) Magnetic field lines can be entirely confined within the core of a toroid, but not within a straight solenoid. Why?
- (b) Does a bar magnet exert a torque on itself due to its own field? Justify your answer.
- (c) When an electron revolves around a nucleus, obtain the expression for the magnetic moment associated with it.

[3 marks]

Solution:

Concept:

- Magnetic field lines form closed loops. A toroid is a solenoid bent into a closed ring; a straight solenoid has two distinct ends.
- A magnetic dipole in an external magnetic field experiences torque $\tau = \mu \times \mathbf{B}_{\text{ext}}$. Orbital motion of a charge produces an equivalent current loop of magnetic moment $\mu = IA$.

Step 1: (a) Compare the field paths of a toroid and a solenoid In an ideal closely wound toroid, the field lines are closed circular loops within the ring-shaped core. There are no open ends, and the windings produce cancelling fields outside the toroid, making its ideal external field zero.

A finite straight solenoid has two ends that behave as magnetic poles. Its nearly uniform field inside cannot terminate at those ends, because magnetic field lines must be closed. The lines therefore leave one end and return to the other through the outside region. Thus the field cannot be entirely confined within a finite straight solenoid.

Step 2: (b) Explain why a magnet has no net self-torque A bar magnet does not exert a net torque on itself due to its own field. Torque that changes the orientation of the magnet requires an external magnetic field. Internal magnetic interactions do not produce an overall torque rotating the magnet as a whole. The dipole-torque formula uses the applied external field, not the dipole's self-field.

Step 3: (c) Derive the orbital magnetic moment Let an electron of charge $-e$ move with speed v in a circular orbit of radius r . The orbital period is $T = 2\pi r/v$. The magnitude of the equivalent current is

$$I = \frac{e}{T} = \frac{ev}{2\pi r}.$$

Since the loop area is $A = \pi r^2$, the magnetic-moment magnitude is

$$\mu = IA = \frac{ev}{2\pi r} \pi r^2 = \frac{evr}{2}.$$

Quick Tip to Solve: Derive orbital moment from current times area, then reverse its direction relative to angular momentum because the electron has negative charge.

The magnitude of orbital angular momentum is $L = m_e v r$, where m_e is the electron mass. Therefore $\mu = eL/(2m_e)$. Conventional current is opposite to the electron's motion because the electron is negatively charged; the magnetic moment is consequently opposite to its orbital angular momentum:

$$\boldsymbol{\mu} = -\frac{e}{2m_e} \mathbf{L}.$$

Step 4: Conclusion A toroid's closed geometry confines its ideal field, whereas a finite straight solenoid needs an external return path. A bar magnet has no net torque from its own field. The electron's orbital magnetic moment is $\mu = evr/2$, or in vector form $\boldsymbol{\mu} = -(e/2m_e)\mathbf{L}$.

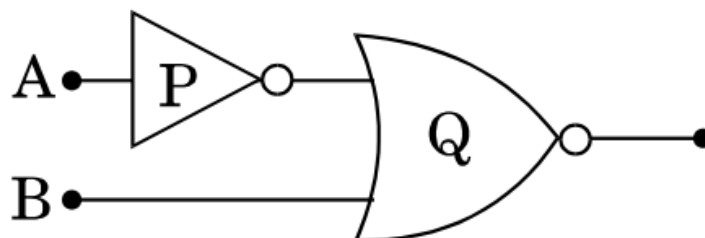
Quick Tip to Solve: Derive orbital moment from current times area, then reverse its direction relative to angular momentum because the electron has negative charge.

22. (a)(i) Write the truth tables of the logic gates marked P and Q in the given circuit.

(ii) Write the truth table for the circuit.

(b) Why are NOR gates considered as universal gates?

[3 marks]



Solution:

Concept:

- A NOT gate complements its single input: $X = \bar{A}$. A NOR gate complements the OR of its two inputs: $Y = \overline{X + B}$.
- A universal gate can be used alone, in suitable combinations, to implement NOT, OR and AND operations and hence any Boolean logic function.

Step 1: (a)(i) Identify P and Q and give their truth tables Gate P is the triangular NOT gate with an inversion bubble at its output. Its output X , which enters Q, is $\bar{A}$:

A	$X = \bar{A}$
0	1
1	0

Gate Q has the OR-gate outline with an inversion bubble at the output, so Q is a NOR gate. Its

Quick Tip to Solve: Name the intermediate output of P before making the circuit truth table; Q receives the complement of A, rather than A itself.

independent input truth table is

X	B	$Y = \overline{X + B}$
0	0	1
0	1	0
1	0	0
1	1	0

Thus Q gives a high output only when both of its inputs are low.

Step 2: (a)(ii) Determine the complete circuit's truth table The first input to Q is the inverted input A, while B reaches Q directly. Substituting $X = \overline{A}$, and using De Morgan's law,

$$Y = \overline{\overline{A} + B} = A\overline{B}.$$

Compute the intermediate value for each pair of original inputs:

A	B	$X = \overline{A}$	Y
0	0	1	0
0	1	1	0
1	0	0	1
1	1	0	0

In particular, $A = 0$ makes $X = 1$, forcing the NOR output to zero regardless of B. For $A = 1$, the first NOR input becomes zero, so the final output is the complement of B.

Step 3: (b) Show why NOR is universal Denote the NOR operation by $A \downarrow B = \overline{A + B}$. Tying the two inputs together implements NOT:

$$A \downarrow A = \overline{A}.$$

Inverting the output of a NOR gate with a second NOR gate implements OR:

$$(A \downarrow B) \downarrow (A \downarrow B) = A + B.$$

Inverting A and B separately and then applying NOR implements AND, by De Morgan's law:

$$(A \downarrow A) \downarrow (B \downarrow B) = \overline{\overline{A} + \overline{B}} = AB.$$

Since the basic NOT, OR and AND operations can all be built using NOR gates alone, all Boolean functions and logic circuits can be realised with NOR gates. This is why NOR is called a universal gate.

Step 4: Conclusion P is a NOT gate and Q is a NOR gate. The combined output is $Y = A\overline{B}$, with outputs 0, 0, 1, 0 for inputs 00, 01, 10, 11, respectively. NOR is universal because it alone can realise NOT, OR and AND.

Quick Tip to Solve: Name the intermediate output of P before making the circuit truth table; Q receives the complement of A, rather than A itself.

22. (a) Explain how a potential barrier is developed in a p-n junction diode.

(b) Draw the circuit arrangement for studying the V-I characteristics of a p-n junction diode in reverse bias. Plot the V-I characteristics in this case. [3 marks]

Solution:

Concept:

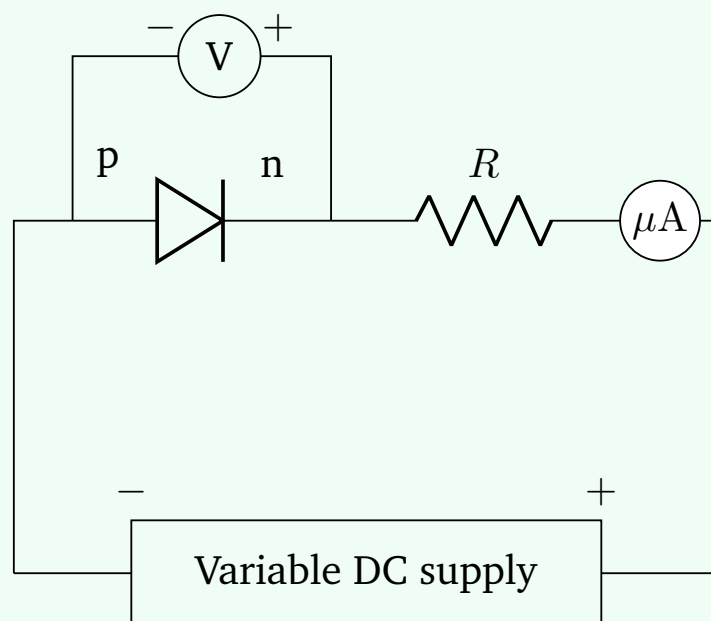
- Majority carriers diffuse across a newly formed junction because their concentrations differ on its two sides. Immobile ions then form a depletion layer and an opposing electric field.
- Reverse bias connects the p side to the negative terminal and the n side to the positive terminal. The barrier and depletion width increase; minority carriers produce a small reverse saturation current.

Step 1: (a) Formation of the potential barrier On joining p-type and n-type semiconductors, electrons diffuse from the n side to the p side and holes diffuse from the p side to the n side. Near the junction they recombine, removing mobile charge carriers. The n side is left with immobile positive donor ions and the p side with immobile negative acceptor ions. This region, depleted of free carriers, is the depletion layer.

The exposed ions produce an electric field directed from the n side to the p side. It opposes further diffusion of majority carriers and creates a potential barrier across the junction. As the field grows, the drift current increases until it balances the diffusion current. At equilibrium, the net current is zero although the potential barrier remains.

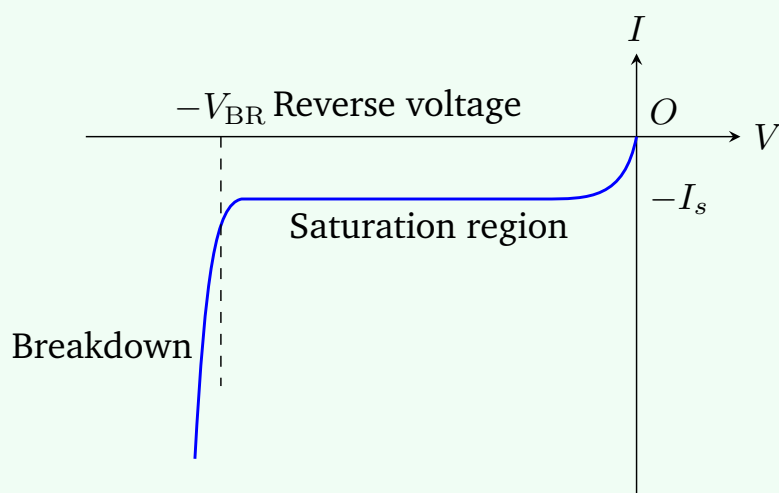
Step 2: (b) Reverse-bias circuit arrangement Connect the p terminal of the diode to the negative terminal of a variable DC supply and its n terminal to the positive terminal. A series resistance limits the current, and a microammeter in series measures the very small reverse current. Connect a voltmeter across the diode, with its positive terminal on the n side. Increase the reverse voltage in small steps and record the corresponding current. The series resistance is especially necessary near breakdown to prevent damage.

Quick Tip to Solve: For reverse bias, put supply positive on n and negative on p; use a microammeter, and show a nearly horizontal reverse-current curve before breakdown.



Reverse-bias arrangement: supply positive to n, negative to p.

Step 3: (b) Reverse V-I characteristic With signed diode voltage $V = V_p - V_n$ and current positive from p to n, reverse voltage and reverse current are negative. After a short initial rise in magnitude, the reverse current is nearly constant at the reverse saturation value $-I_s$, owing to thermally generated minority carriers. At the breakdown voltage $V = -V_{BR}$, the magnitude of reverse current rises sharply. The graph below shows the reverse characteristic in the third quadrant; it is schematic and not to scale.



Reverse characteristic, using signed diode voltage and current.

Step 4: Conclusion Diffusion and recombination leave fixed ions that establish the depletion layer and potential barrier. In reverse bias, the diode carries a small, nearly constant minority-carrier current until breakdown, when the reverse current increases rapidly.

Quick Tip to Solve: For reverse bias, put supply positive on n and negative on p; use a microammeter, and show a nearly horizontal reverse-current curve before breakdown.

23. (a) How are eddy currents generated in a conductor which is subjected to a magnetic field?

(b) Write two examples of their useful applications.

(c) How can the disadvantages of eddy currents be minimized?

[3 marks]

Solution:

Concept:

- Faraday's law gives the induced emf around a closed conducting path: $\varepsilon = -d\Phi_B/dt$, where Φ_B is magnetic flux through that path.
- Eddy currents are circulating currents within the bulk of a conductor. By Lenz's law, their magnetic effect opposes the change that produces them.

Step 1: (a) Generation of eddy currents If the magnetic flux through a conducting body changes, an emf is induced along closed paths within the conductor. Because the metal offers conducting paths, the induced emf drives circulating currents, called eddy currents. Their paths resemble eddies in a liquid. Flux can change when the magnetic field varies with time or when the conductor moves through a nonuniform field. Merely placing a stationary conductor in a steady field does not continuously generate eddy currents: a change in flux is necessary.

Step 2: (b) Two useful applications **Induction furnace:** A rapidly alternating magnetic field induces large eddy currents in a metal charge. Their Joule heating raises its temperature and can melt the metal without direct electrical contact.

Magnetic braking: A moving conducting disc or plate passes through a magnetic field. The resulting eddy currents interact with the field to produce a force opposing the motion, providing contactless braking. The opposing force follows Lenz's law, and the lost mechanical energy becomes heat.

Step 3: (c) Reducing unwanted effects Eddy currents in transformer and motor cores cause unwanted heating and energy loss. Make the core from thin laminations insulated from each other, rather than a solid metal block. These sheets interrupt large current loops and increase the resistance of the remaining paths, reducing eddy currents and their heating. Orient the laminations so that the useful magnetic flux travels along the sheets. High-resistivity core materials can reduce the losses further.

Step 4: Conclusion Changing magnetic flux generates circulating eddy currents. They are useful in induction furnaces and magnetic brakes, while insulated laminated cores reduce their unwanted heating and energy losses.

Quick Tip to Solve: State the changing-flux condition first; for reducing losses, mention thin insulated laminations rather than merely a thinner solid core.

24. (a) Explain how an unpolarised light gets polarised when incident on the interface separating the two transparent media.

(b) Green light is incident at the polarising angle on a certain transparent medium. The angle of refraction is 30° . Find

(i) polarising angle, and

(ii) refractive index of the medium.

[3 marks]

Solution:

Concept:

- Unpolarised light has electric-field vibrations in all directions perpendicular to its propagation. Reflection at a dielectric interface can favour one vibration direction.
- At the polarising angle i_p , the reflected light is completely plane polarised and the reflected and refracted rays are perpendicular: $i_p + r = 90^\circ$.
- Brewster's law is $n_2/n_1 = \tan i_p$, where n_1 and n_2 are the refractive indices of the incident and transmitting media.

Step 1: (a) Polarisation by reflection Resolve the electric-field vibrations of the incident unpolarised light into components parallel and perpendicular to the plane of incidence. The plane of incidence contains the incident ray and the normal to the interface. In general, the interface reflects these two components by different amounts, so the reflected beam is partially polarised.

At a particular angle of incidence, called the polarising or Brewster angle i_p , the component parallel to the plane of incidence is not reflected. The reflected light therefore has its electric field vibrating only perpendicular to that plane: it is completely plane polarised. At this angle, the reflected ray and refracted ray are mutually perpendicular. The transmitted beam generally remains partially polarised.

Step 2: (b)(i) Calculate the polarising angle The given refraction angle for green light is $r = 30^\circ$, measured from the normal. At the polarising angle,

$$i_p + r = 90^\circ, \quad i_p = 90^\circ - 30^\circ = 60^\circ.$$

Step 3: (b)(ii) Calculate the refractive index For incidence from air, take $n_1 \simeq 1$. Brewster's law gives the refractive index of the medium for green light:

$$n = \tan i_p = \tan 60^\circ = \sqrt{3} \simeq 1.73.$$

As a check, Snell's law yields the same value:

$$n = \frac{\sin i_p}{\sin r} = \frac{\sin 60^\circ}{\sin 30^\circ} = \frac{\sqrt{3}/2}{1/2} = \sqrt{3}.$$

The refractive index is dimensionless.

Step 4: Conclusion At the Brewster angle the reflected light is completely plane polarised, with its electric field perpendicular to the plane of incidence. For the given green light, $i_p = 60^\circ$ and the medium's refractive index relative to air is $n = \sqrt{3} \simeq 1.73$.

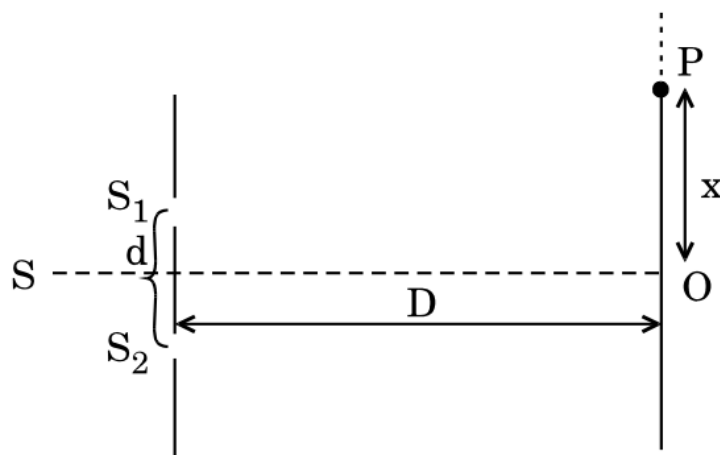
Quick Tip to Solve: When a refraction angle is given at polarisation, first use its complement to find the Brewster angle, then apply the tangent law.

Section D (15 marks)

25. (a) Can the interference pattern be produced by two independent monochromatic sources of light? Explain.

(b) The intensity at the central maximum (O) in a Young's double slit experimental set-up shown in the figure is I_0 . If the distance OP equals one-third of the fringe width of the pattern, show that the intensity at point P , would equal $\frac{I_0}{4}$.

(c) In Young's double slit experiment, the slits are separated by 0.5 mm and screen is placed 1.0 m away from the slit. It is found that the 5^{th} bright fringe is at a distance of 4.13 mm from the 2^{nd} dark fringe. Find the wavelength of light used. [5 marks]



Solution:

Concept:

- A sustained interference pattern needs coherent sources: they have the same frequency and a constant phase difference.
- For two equal slit intensities I_s , the resultant intensity is $I = 4I_s \cos^2(\phi/2)$, where ϕ is the phase difference. At the central maximum, $I_0 = 4I_s$.
- In Young's experiment, $\beta = \lambda D/d$, where β is fringe width, λ is wavelength, D is slit-to-screen distance and d is slit separation. At transverse position y , $\phi = 2\pi y/\beta$.

Step 1: (a) Explain why independent sources do not give a stable pattern Two independent monochromatic light sources cannot produce a sustained, observable interference pattern. Even if their frequencies are equal, their emissions arise from independent atomic transitions, so their relative phase changes randomly and rapidly. They therefore do not maintain the constant phase difference required for coherence. At a given point, constructive and destructive interference alternate irregularly; the interference term averages to zero over the observation time. The observed intensity is consequently the sum of the individual intensities, rather than a stationary series of bright and dark fringes. In Young's arrangement, both slits are illuminated from one source to obtain coherent secondary sources.

Step 2: (b) Calculate the phase difference at P The two slits in the usual Young's arrangement supply equal amplitudes. For a point at distance $y = OP$ from the central maximum, the path difference is

Quick Tip to Solve: Write bright positions as $n\beta$ and dark positions as $(n - \frac{1}{2})\beta$; for these two fringes subtract 1.5β from 5β before using $\lambda = \beta d/D$.

approximately $\Delta = dy/D$. Hence

$$\phi = \frac{2\pi\Delta}{\lambda} = \frac{2\pi dy}{\lambda D} = \frac{2\pi y}{\beta}.$$

Substituting the specified displacement $y = \beta/3$ gives $\phi = 2\pi/3$. At O the waves are in phase and their intensity is $I_0 = 4I_s$. Thus, at P,

$$I_P = I_0 \cos^2\left(\frac{\phi}{2}\right) = I_0 \cos^2\left(\frac{\pi}{3}\right) = I_0 \left(\frac{1}{2}\right)^2.$$

The factor involves the square of the resultant amplitude; the displacement is one-third of a fringe width, but intensity is not proportional to displacement.

Step 3: (c) Locate the fifth bright and second dark fringes Counting the central maximum as order zero, the fifth bright fringe is at $y_{B5} = 5\beta$. The dark fringes occur midway between consecutive bright fringes; the second dark fringe is therefore at $y_{D2} = 3\beta/2$. Using the corresponding fringes on the same side of the central maximum, their separation is

$$y_{B5} - y_{D2} = \left(5 - \frac{3}{2}\right)\beta = \frac{7}{2}\beta = 4.13 \text{ mm},$$

$$\beta = \frac{4.13}{3.5} \text{ mm} = 1.18 \times 10^{-3} \text{ m}.$$

Now use $d = 0.5 \text{ mm} = 5.0 \times 10^{-4} \text{ m}$ and $D = 1.0 \text{ m}$:

$$\lambda = \frac{\beta d}{D} = \frac{(1.18 \times 10^{-3})(5.0 \times 10^{-4})}{1.0} \text{ m} = 5.90 \times 10^{-7} \text{ m}.$$

Step 4: Conclusion (a) Independent monochromatic sources do not yield a sustained interference pattern because their phase difference is not constant. (b) $I_P = I_0/4$. (c) The wavelength is $5.90 \times 10^{-7} \text{ m} = 590 \text{ nm}$.

Quick Tip to Solve: Write bright positions as $n\beta$ and dark positions as $(n - \frac{1}{2})\beta$; for these two fringes subtract 1.5β from 5β before using $\lambda = \beta d/D$.

25. (a) Derive the relation $a \sin \theta = \lambda$ for the first minimum of the diffraction pattern produced due to a single slit of width ' a ' using light of wavelength λ .

(b) State with reason, how the linear width of central maximum will be affected if (i) monochromatic yellow light is replaced with red light, and (ii) distance between the slit and the screen is increased.

(c) Using the monochromatic light of same wavelength in the experimental set-up of the diffraction pattern as well as in the interference pattern where the slit separation is 1 mm, 10 interference fringes are found to be within the central maximum of the diffraction pattern. Determine the width of the single slit, if the screen is kept at the same distance from the slit in the two cases. [5 marks]

Solution:

Concept:

- In single-slit diffraction, each point of the illuminated slit acts as a coherent secondary source. The path difference between rays from two points separated by s is $s \sin \theta$.
- A path difference $\lambda/2$ corresponds to a phase difference π , so equal secondary wave amplitudes cancel pairwise.
- The first diffraction minima satisfy $a \sin \theta = \lambda$. For small angles, the full linear width of the central maximum is $W = 2D\lambda/a$, whereas Young's interference fringe width is $\beta = D\lambda/d$.

Step 1: (a) Pair secondary sources in the two halves of the slit Consider a uniformly illuminated slit of width a . For observation at an angle θ to the normal, divide the slit into two equal halves. Pair every point in the upper half with the corresponding point in the lower half, a distance $a/2$ away. In the far-field arrangement, rays from each such pair travel in essentially the same direction, so their path difference is

$$\Delta_{\text{pair}} = \frac{a}{2} \sin \theta.$$

When this path difference is $\lambda/2$, the two contributions in each pair have a phase difference of π . Their amplitudes are equal for uniform illumination, and they cancel. Every contribution from one half is paired with a cancelling contribution from the other half, so the resultant amplitude is zero. Therefore,

$$\frac{a}{2} \sin \theta = \frac{\lambda}{2} \Rightarrow a \sin \theta = \lambda.$$

This is the first minimum on the positive-angle side. A corresponding minimum occurs at negative angle; the bright region between the two is the central maximum.

Step 2: (b) Express the full linear width and examine both changes If D is the distance between slit and screen, the first minimum is at $y_1 = D \tan \theta$. Under the usual small-angle approximation, $\tan \theta \simeq \sin \theta = \lambda/a$. The minima on the two sides are at $\pm D\lambda/a$, hence the full central width is

$$W = 2y_1 = \frac{2D\lambda}{a}.$$

(i) Red light has a longer wavelength than yellow light. With D and a fixed, W is directly proportional to λ , so replacing yellow light with red light increases the central maximum's linear width. The diffraction angles also increase.

Quick Tip to Solve: Compare the full central diffraction width $2D\lambda/a$ with N interference fringe widths $ND\lambda/d$; cancellation gives $a = 2d/N$ directly.

(ii) With λ and a fixed, increasing the slit-to-screen distance D increases W in the same proportion. The angular positions of the minima remain fixed, but their linear distances from the centre increase as the screen moves farther away.

Step 3: (c) Compare the central diffraction width with interference fringe width Let $d = 1 \text{ mm}$ be the separation of the two slits in the interference experiment, and a the width of the single slit in the diffraction experiment. Because the same wavelength and screen distance are used, ten interference fringe widths occupy the central diffraction maximum:

$$\begin{aligned} W &= 10\beta, \\ \frac{2D\lambda}{a} &= 10 \frac{D\lambda}{d}, \\ \frac{2}{a} &= \frac{10}{d}, \\ a &= \frac{2d}{10} = \frac{2 \times 1 \text{ mm}}{10} = 0.20 \text{ mm}. \end{aligned}$$

Both D and λ cancel because they are identical in the two arrangements. The factor of two must be retained: the central diffraction maximum extends between the first minima on both sides, whereas $D\lambda/a$ is only its half-width.

Step 4: Conclusion (a) Pairwise cancellation gives $a \sin \theta = \lambda$ for the first minimum. (b) The central maximum becomes wider both when yellow light is replaced by red light and when the screen distance increases. (c) The single-slit width is $a = 0.20 \text{ mm} = 2.0 \times 10^{-4} \text{ m}$.

Quick Tip to Solve: Compare the full central diffraction width $2D\lambda/a$ with N interference fringe widths $ND\lambda/d$; cancellation gives $a = 2d/N$ directly.

26. (a) Draw a circuit diagram of an n-p-n transistor with its emitter-base junction forward biased and base-collector junction reverse biased. Briefly describe its working. Explain how a transistor in its active state exhibits a low resistance at its emitter-base junction and high resistance at its base-collector junction.

(b) Derive the expression for the voltage gain of a transistor amplifier in CE configuration in terms of the load resistance R_L , current gain β_a and input resistance. Explain why input and output voltages are in opposite phase. [5 marks]

Solution:

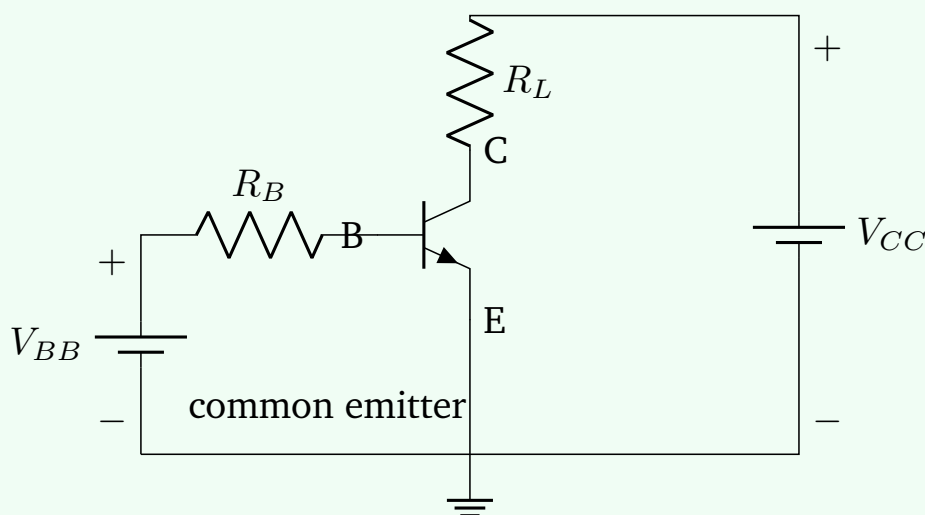
Concept:

- In the active state of an n-p-n transistor, the emitter-base junction is forward biased and the collector-base junction is reverse biased: $V_C > V_B > V_E$.
- The emitter is heavily doped and supplies electrons; the base is very thin and lightly doped. Most injected electrons reach the collector, so $I_E = I_B + I_C$.
- For small alternating signals in a common-emitter (CE) amplifier, $\beta_a = \Delta I_C / \Delta I_B$ and the input resistance is $r_i = \Delta V_{BE} / \Delta I_B$.
- With collector load resistance R_L , the output voltage is $V_{CE} = V_{CC} - I_C R_L$, where V_{CC} is the

Quick Tip to Solve: Check $V_C > V_B > V_E$ for an active n-p-n transistor. For the CE gain sign, write $V_{CE} = V_{CC} - I_C R_L$ before taking the small change.

constant collector supply voltage.

Step 1: (a) Apply the correct junction biases Connect the positive terminal of the base supply V_{BB} , through a current-limiting resistor R_B , to the p-type base and its negative terminal to the n-type emitter. The base is then positive relative to the emitter, making the emitter-base junction forward biased. Connect the collector through the load R_L to the positive terminal of a sufficiently larger supply V_{CC} , with its negative terminal also connected to the emitter. Adjust the supplies so that the n-type collector is more positive than the p-type base. This reverse biases the collector-base junction. The emitter is common to the input and output circuits; the outward arrow on its lead identifies an n-p-n transistor.



Active-state n-p-n bias circuit with a common emitter; $V_C > V_B > V_E$.

Step 2: Describe the carrier motion and junction resistances Forward bias lowers the potential barrier at the emitter-base junction. A large number of electrons from the heavily doped n-type emitter enter the base. Because the base is very thin and lightly doped, only a small fraction recombine with holes in it, producing the small base current I_B . Most electrons diffuse to the collector-base depletion region. Its reverse-bias electric field sweeps them into the collector, producing I_C . Thus a small change of base current controls a much larger change of collector current; normally I_C is nearly equal to I_E .

The forward-biased emitter-base junction conducts readily: a small increase in its voltage produces a substantial increase in current, so its incremental resistance is low. Reverse bias widens the collector-base depletion region. This junction presents a high resistance, and the collector current changes little with collector voltage in the active region when the base drive is fixed. The large collector current is supplied by emitter-injected carriers; it does not mean the reverse-biased junction has become a low-resistance junction.

Step 3: (b) Relate the input signal to the collector-current change Superpose a small alternating input voltage v_i on the forward base bias, keeping the transistor in its active region throughout the cycle. The emitter remains at the common reference potential. The corresponding small base-current change is

$$v_i = \Delta V_{BE} = r_i \Delta I_B.$$

Quick Tip to Solve: Check $V_C > V_B > V_E$ for an active n-p-n transistor. For the CE gain sign, write $V_{CE} = V_{CC} - I_C R_L$ before taking the small change.

By the definition of alternating current gain,

$$\Delta I_C = \beta_a \Delta I_B.$$

Here r_i is the small-signal input resistance and β_a is the CE current gain about the operating point, rather than an unrelated junction resistance.

Step 4: Derive the voltage gain and explain the phase reversal Apply Kirchhoff's voltage law to the collector circuit. Since the collector supply is constant, increasing collector current increases the drop across R_L and decreases the collector-emitter voltage:

$$\begin{aligned} V_{CE} &= V_{CC} - I_C R_L, \\ v_o &= \Delta V_{CE} = -R_L \Delta I_C = -\beta_a R_L \Delta I_B. \end{aligned}$$

Dividing the output signal by the input signal gives

$$A_v = \frac{v_o}{v_i} = \frac{-\beta_a R_L \Delta I_B}{r_i \Delta I_B} = -\frac{\beta_a R_L}{r_i}.$$

Therefore the magnitude of voltage gain is $\beta_a R_L / r_i$. During the positive half-cycle of the input, V_{BE} and I_B increase, hence I_C and the load voltage drop increase. The collector voltage falls, giving a negative output excursion. During the negative input half-cycle the collector voltage rises. Thus input and output are in opposite phase, with a phase difference of 180° .

Step 5: Conclusion The active n-p-n transistor has a forward-biased emitter-base junction of low resistance and a reverse-biased collector-base junction of high resistance. Its CE voltage gain is $A_v = -\beta_a R_L / r_i$, with magnitude $\beta_a R_L / r_i$; the negative sign represents a 180° phase reversal.

Quick Tip to Solve: Check $V_C > V_B > V_E$ for an active n-p-n transistor. For the CE gain sign, write $V_{CE} = V_{CC} - I_C R_L$ before taking the small change.

OR

26. (a) Write the important considerations which are to be taken into account while fabricating a p-n junction diode to be used as a Light Emitting Diode (LED). What should be the order of band gap of an LED, if it is required to emit light in the visible range? Draw a circuit diagram and explain its action.

(b) Draw the V-I characteristics of an LED. State two advantages of LED lamps over conventional incandescent lamps. [5 marks]

Solution:

Concept:

- An LED is a heavily doped p-n junction made from a suitable direct-band-gap semiconductor. Electron-hole recombination near the junction releases energy as photons when the junction is forward biased.
- The emitted photon energy is approximately the band gap: $E_g = h\nu = hc/\lambda$, where h is Planck's

Quick Tip to Solve: Use $E_g(\text{eV}) = 1240/\lambda(\text{nm})$ to check the colour, and always draw the LED forward biased with a series resistor and two outward light arrows.

constant, c is the speed of light, and λ is the wavelength.

- Forward bias lowers the junction barrier and injects electrons and holes. A series resistor limits the current; the light intensity increases with forward current over the normal operating range.

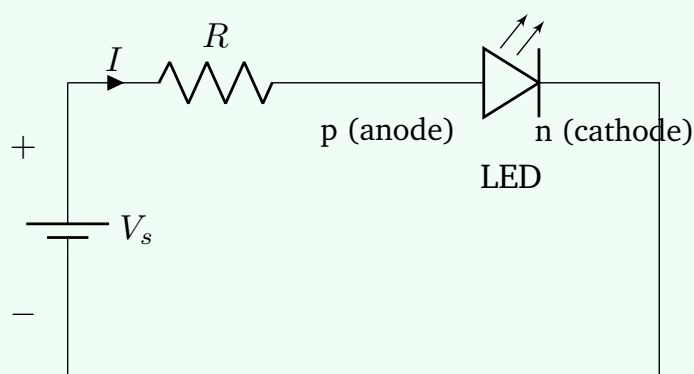
Step 1: (a) Choose the material and fabricate the junction Use a direct-band-gap semiconductor so that a substantial fraction of electron-hole recombinations emits photons. Suitable compound semiconductors include GaAsP for visible emission; ordinary silicon and germanium are inefficient light emitters because their indirect band gaps favour non-radiative recombination. The junction is heavily doped to provide sufficient carriers for strong light emission. Choose the composition and hence the band gap for the required colour, and arrange the junction and contacts so that the emitted light can escape without strong absorption or obstruction. Enclose the diode in a transparent cover to let the emitted light escape.

Step 2: (a) Determine the band gap for visible light Taking visible wavelengths approximately from 400 nm to 700 nm, use $hc \simeq 1240 \text{ eV nm}$:

$$E_g \simeq \frac{1240}{\lambda(\text{nm})} \text{ eV}.$$

Thus the band gap must be approximately 1.8–3.1 eV, of the order of 2 eV. For example, $E_g = 2.0 \text{ eV}$ gives $\lambda \simeq 620 \text{ nm}$, in the visible range.

Step 3: (a) Connect the LED and explain its action Connect the p-side (anode) to the positive terminal of a dc source through a current-limiting resistor R , and the n-side (cathode) to the negative terminal, as in the circuit. This forward biases the junction. Electrons from the n-region and holes from the p-region are injected across the junction and recombine in the active region. An electron falling from the conduction band to the valence band releases energy approximately equal to E_g as a photon. If the source voltage is V_s and the LED forward drop is V_F , the operating current is approximately $I = (V_s - V_F)/R$; the resistor prevents excessive current. Avoid reverse bias beyond the permitted limit.

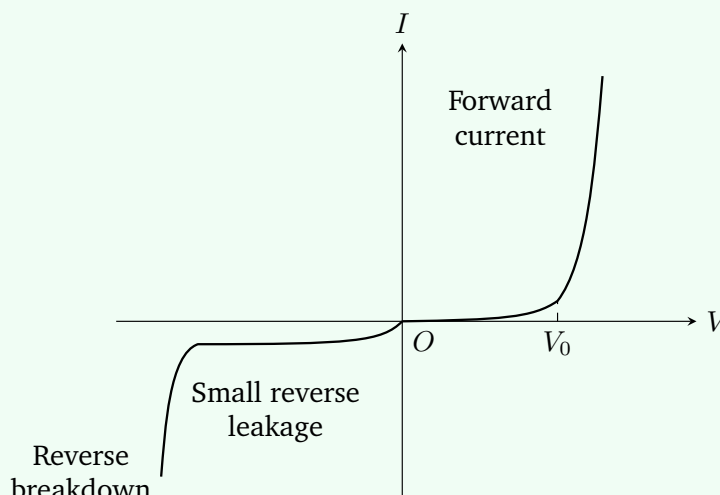


Forward-biased LED with a current-limiting resistor; arrows represent emitted light.

Step 4: (b) Draw and interpret the V-I characteristic The qualitative characteristic has the diode form shown below. Below the forward turn-on voltage V_0 , the current is very small. Above V_0 , the forward current rises rapidly, and the LED emits appreciable light. The value of V_0 depends on the material and colour. Under reverse bias, only a very small leakage current flows until reverse breakdown. An LED is normally operated on the forward branch; reverse breakdown can damage it. The reverse

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leakage is exaggerated in the sketch to make it visible; no numerical scale is implied.



Qualitative LED characteristic, including the reverse branch (leakage exaggerated).

Step 5: (b) State two advantages over incandescent lamps First, LED lamps have much greater luminous efficiency: they use less electrical energy for the same useful light output and dissipate less energy as heat. Second, they have a much longer operating life, because light production does not require a fragile filament to be heated to a high temperature.

Step 6: Conclusion A visible LED needs a suitable heavily doped direct-band-gap p-n junction with E_g of the order of 2 eV, approximately 1.8–3.1 eV. Forward bias produces light by electron-hole recombination; a series resistor limits its current. Its V-I curve is diode-like. LED lamps are more energy efficient and longer lived than incandescent lamps.

Quick Tip to Solve: Use $E_g(\text{eV}) = 1240/\lambda(\text{nm})$ to check the colour, and always draw the LED forward biased with a series resistor and two outward light arrows.

27. (a) State and explain the law used to determine magnetic field at a point due to a current element. Derive the expression for the magnetic field due to a circular current carrying loop of radius r at its centre.

(b) A long wire with a small current element of length 1 cm is placed at the origin and carries a current of 10 A along the X-axis. Find the magnitude and direction of the magnetic field due to the element on the Y-axis at a distance 0.5 m from it. [5 marks]

Solution:

Concept:

- Biot–Savart law gives the magnetic field of a steady current element: $d\vec{B} = \frac{\mu_0}{4\pi} \frac{I d\vec{\ell} \times \hat{R}}{R^2}$. Here I is the current, $d\vec{\ell}$ points along it, R is the element-to-observation-point distance and $\hat{R}$ points from the element to that point.
- The magnitude is $dB = \frac{\mu_0}{4\pi} \frac{I d\ell \sin \theta}{R^2}$, where θ is the angle between $d\vec{\ell}$ and $\vec{R}$. Fields from different

Quick Tip to Solve: Use the length of the small element in Biot–Savart law; the infinite-wire formula would include contributions the question has not asked for.

elements add vectorially.

- The direction is perpendicular to the plane of $d\vec{\ell}$ and $\vec{R}$, as given by their cross product. In vacuum, $\mu_0 = 4\pi \times 10^{-7} \text{ T m A}^{-1}$.

Step 1: (a) State and explain Biot–Savart law A small length $d\ell$ of wire carrying a steady current I produces a magnetic field at a point whose magnitude is directly proportional to I , $d\ell$ and $\sin \theta$, and inversely proportional to the square of its distance R . Thus

$$dB = \frac{\mu_0 I d\ell \sin \theta}{4\pi R^2}.$$

The vector form fixes the direction as $d\vec{\ell} \times \hat{R}$. A point along the line of the element has $\sin \theta = 0$ and receives zero contribution; a point perpendicular to the element has the largest contribution at the same distance. The law gives the field of an element; the field of an extended wire is found by integrating over the wire.

Step 2: (a) Integrate the field at the centre of the loop Let a circular loop of radius r carry current I . At its centre, every current element is at distance r , and its tangent is perpendicular to the inward radius, so $\theta = 90^\circ$. Every contribution is normal to the plane of the loop and points in the same sense. Consequently, the magnitudes may be added:

$$\begin{aligned} B &= \oint dB = \frac{\mu_0 I}{4\pi r^2} \oint d\ell \\ &= \frac{\mu_0 I}{4\pi r^2} (2\pi r) = \frac{\mu_0 I}{2r}. \end{aligned}$$

Curling the right-hand fingers in the direction of current gives the normal direction of the field with the thumb. An anticlockwise current, viewed from one side, produces a field towards that observer; a clockwise current produces a field away from that observer.

Step 3: (b) Calculate the field of the specified element Here the element, rather than the entire long wire, is the source whose contribution is required. Use $\ell = 1 \text{ cm} = 10^{-2} \text{ m}$, $I = 10 \text{ A}$ and $R = 0.5 \text{ m}$. At the point on the positive Y-axis the displacement is perpendicular to the element along the positive X-axis, so $\sin \theta = 1$. Treating the short element by Biot–Savart law gives

$$\begin{aligned} B &= \frac{\mu_0}{4\pi} \frac{I\ell}{R^2} \\ &= 10^{-7} \frac{10 \times 10^{-2}}{(0.5)^2} = 4.0 \times 10^{-8} \text{ T}. \end{aligned}$$

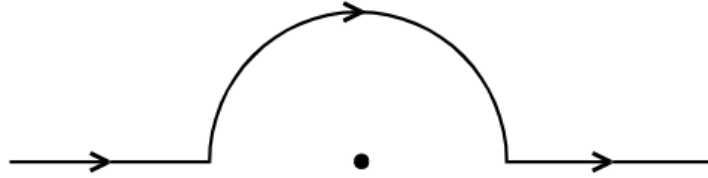
Since $\hat{x} \times \hat{y} = \hat{z}$, the field is along the positive Z-axis. If the observation point is on the negative Y-axis instead, its magnitude is unchanged and its direction is the negative Z-axis.

Step 4: Conclusion (a) The required law is Biot–Savart law, and a single circular loop has central field $B = \mu_0 I / (2r)$, normal to its plane. (b) The specified element produces $4.0 \times 10^{-8} \text{ T}$, along $+Z$ at the point on $+Y$.

Quick Tip to Solve: Use the length of the small element in Biot–Savart law; the infinite-wire formula would include contributions the question has not asked for.

27. (a) Derive the expression for the magnetic field due to a current carrying coil of radius r at a distance x from the centre along the X-axis.

(b) A straight wire carrying a current of 5 A is bent into a semicircular arc of radius 2 cm as shown in the figure. Find the magnitude and direction of the magnetic field at the centre of the arc. [5 marks]



Solution:

Concept:

- Biot–Savart law is $d\vec{B} = \frac{\mu_0 I}{4\pi} \frac{d\vec{\ell} \times \hat{R}}{R^2}$, where $\vec{R}$ points from the current element to the observation point and $d\vec{\ell}$ follows the current.
- On the axis of a circular coil, transverse field components cancel in pairs. The axial components reinforce each other, and the direction is fixed by the right-hand grip rule.
- An arc of radius r subtending angle ϕ in radians produces $B = \mu_0 I \phi / (4\pi r)$ at its centre. Collinear straight portions give zero contribution there.

Step 1: (a) Set up the geometry of a circular turn Place a circular turn of radius r in the YZ-plane with its centre at the origin. Let P lie on its X-axis at distance x . The distance from any current element to P is

$$R = \sqrt{r^2 + x^2}.$$

The element is tangent to the circle and perpendicular to both the local radius and the X-axis. Hence it is perpendicular to the vector joining it to P , giving $\sin \theta = 1$. Biot–Savart law therefore gives

$$dB = \frac{\mu_0 I d\ell}{4\pi(r^2 + x^2)}.$$

The field from this element is oblique to the X-axis. If α is its angle with that axis, the geometry of the cross product gives $\cos \alpha = r/R$.

Step 2: (a) Add the axial components around the coil Diametrically opposite elements have equal and opposite components perpendicular to the X-axis. These cancel, while their axial components point in the same direction. Thus

$$dB_x = dB \cos \alpha = \frac{\mu_0 I r d\ell}{4\pi(r^2 + x^2)^{3/2}}.$$

Quick Tip to Solve: First check whether each wire segment is collinear with the centre: those straight portions give zero field, so only the semicircle enters the calculation.

Since r and x are constant around the circular turn, integration gives

$$\begin{aligned} B_x &= \frac{\mu_0 I r}{4\pi(r^2 + x^2)^{3/2}} \oint d\ell \\ &= \frac{\mu_0 I r}{4\pi(r^2 + x^2)^{3/2}} (2\pi r) \\ &= \frac{\mu_0 I r^2}{2(r^2 + x^2)^{3/2}}. \end{aligned}$$

For a coil with N closely wound turns of the same radius, the contributions multiply by N : $B = \mu_0 N I r^2 / [2(r^2 + x^2)^{3/2}]$. Its direction is along the axis in the sense indicated by the right-hand grip rule. At $x = 0$, the expression reduces to $\mu_0 N I / (2r)$, the central field.

Step 3: (b) Evaluate the contributions of the straight portions and arc In the printed figure, the current flows from left to right along the straight portions and clockwise around the upper semicircle. For each element of either straight portion, the vector to the centre is collinear with the element. Therefore $d\vec{\ell} \times \hat{R} = 0$: the straight portions contribute no magnetic field at the centre. For the curved portion, each tangent is perpendicular to the radius to the centre. All its field contributions are perpendicular to the page in the same sense. Its length is πr , so

$$\begin{aligned} B_{\text{arc}} &= \frac{\mu_0 I}{4\pi r^2} (\pi r) = \frac{\mu_0 I}{4r} \\ &= \frac{(4\pi \times 10^{-7})(5)}{4(2 \times 10^{-2})} \\ &= 2.5\pi \times 10^{-5} \text{ T} \simeq 7.85 \times 10^{-5} \text{ T}. \end{aligned}$$

Here $2 \text{ cm} = 2 \times 10^{-2} \text{ m}$. By curling the right-hand fingers clockwise with the current around the arc, the thumb points into the page. This is the direction of the total field.

Step 4: Conclusion (a) The axial field is $B = \mu_0 N I r^2 / [2(r^2 + x^2)^{3/2}]$, with $N = 1$ for a single turn.
(b) The total field at the centre is $7.85 \times 10^{-5} \text{ T}$, directed into the page.

Quick Tip to Solve: First check whether each wire segment is collinear with the centre: those straight portions give zero field, so only the semicircle enters the calculation.