

CBSE Class 12 Physics Question Paper 2019 with Solution PDF-Set 55/2/1



Question Paper with Solutions

Conducted by the Central Board of Secondary Education (CBSE)

Time Allowed: 3 hours

Maximum Marks: 70

General Instructions

- (i) All questions are compulsory. There are 27 questions in all.
- (ii) This question paper has four sections : Section A, Section B, Section C and Section D.
- (iii) Section A contains five questions of one mark each, Section B contains seven questions of two marks each, Section C contains twelve questions of three marks each, Section D contains three questions of five marks each.
- (iv) There is no overall choice. However, an internal choice(s) has been provided in two questions of one mark, two questions of two marks, four questions of three marks and three questions of five marks weightage. You have to attempt only one of the choices in such questions.

You may use the following values of physical constants wherever necessary :

$$c = 3 \times 10^8 \text{ m/s}$$

$$h = 6.63 \times 10^{-34} \text{ Js}$$

$$e = 1.6 \times 10^{-19} \text{ C}$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ T m A}^{-1}$$

$$\epsilon_0 = 8.854 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$$

$$\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$$

$$\text{Mass of electron } (m_e) = 9.1 \times 10^{-31} \text{ kg}$$

$$\text{Mass of neutron} = 1.675 \times 10^{-27} \text{ kg}$$

$$\text{Mass of proton} = 1.673 \times 10^{-27} \text{ kg}$$

$$\text{Avogadro's number} = 6.023 \times 10^{23} \text{ per gram mole}$$

$$\text{Boltzmann constant} = 1.38 \times 10^{-23} \text{ JK}^{-1}$$

Section A (5 marks)

1. Draw equipotential surfaces for an electric dipole.

[1 mark]

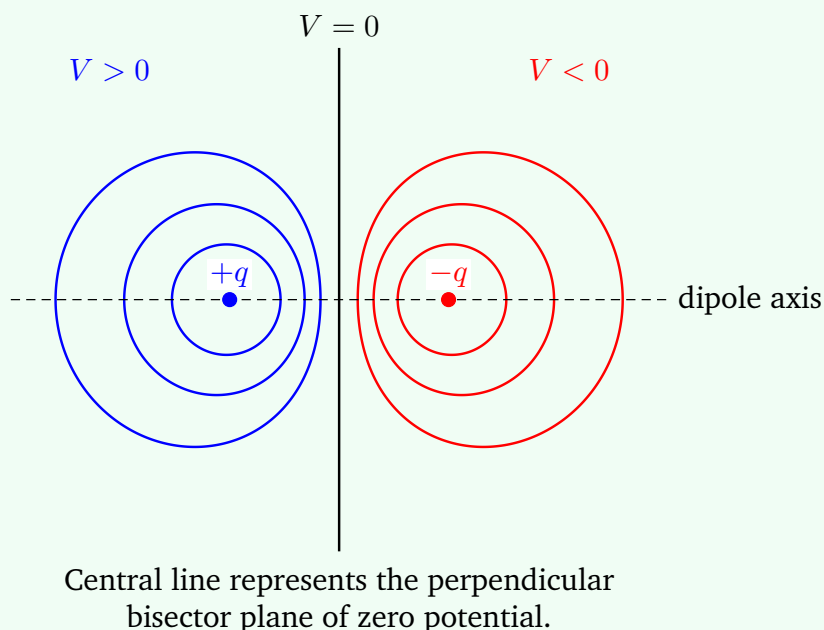
Solution:

Concept:

- An equipotential surface contains points having the same electric potential; no work is done in moving a test charge along it.
- For a dipole, $V = \frac{q}{4\pi\epsilon_0} (1/r_+ - 1/r_-)$, where r_+ and r_- are distances from $+q$ and $-q$.

Step 1: Locate the zero-potential surface On the perpendicular bisector of the dipole, $r_+ = r_-$. Thus the two charge potentials cancel at every point of this plane.

Step 2: Draw the positive and negative surfaces The diagram shows cross-sections of nested equipotential surfaces around each charge. Their values are positive near $+q$ and negative near $-q$. Rotating each contour about the dipole axis produces its three-dimensional surface; the central straight line generates the zero-potential plane.



Cross-section of dipole equipotential surfaces; rotate about the dipole axis.

Step 3: Conclusion The nested surfaces and the central $V = 0$ plane shown above are the equipotential surfaces of the electric dipole.

Quick Tip to Solve: Sketch equal but opposite families around the charges, and include the perpendicular bisector plane with $V = 0$.

2. A proton is accelerated through a potential difference V , subjected to a uniform magnetic field acting normal to the velocity of the proton. If the potential difference is doubled, how will the radius of the circular path described by the proton in the magnetic field change? [1 mark]

Solution:

Concept:

- Acceleration through a potential difference supplies kinetic energy $eV = \frac{1}{2}m_p v^2$, with e and m_p the proton charge and mass.
- For motion perpendicular to a uniform magnetic field, $evB = m_p v^2/r$, so the orbit radius is $r = m_p v/(eB)$.

Step 1: Express radius in terms of potential Eliminate the speed between the energy equation and magnetic-force equation:

$$r = \frac{m_p}{eB} \sqrt{\frac{2eV}{m_p}} = \frac{1}{B} \sqrt{\frac{2m_p V}{e}}.$$

For the same proton and unchanged field, $r \propto \sqrt{V}$.

Step 2: Compare the two radii Put $V_2 = 2V_1$ in the ratio:

$$\frac{r_2}{r_1} = \sqrt{\frac{V_2}{V_1}} = \sqrt{2}.$$

Doubling the accelerating potential doubles kinetic energy; speed and radius both acquire only its square-root factor.

Step 3: Conclusion The radius becomes $r_2 = \sqrt{2}r_1$, approximately 1.414 times its original value.

Quick Tip to Solve: At fixed magnetic field use $r \propto \sqrt{V}$: apply the square root to the potential multiplier.

3. The magnetic susceptibility of magnesium at 300 K is 1.2×10^5 . At what temperature will its magnetic susceptibility become 1.44×10^5 ? [1 mark]

Solution:

Concept:

- For a paramagnetic material obeying Curie's law, susceptibility $\chi = C/T$, where C is Curie's constant and T is absolute temperature.
- For the same sample, $\chi_1 T_1 = \chi_2 T_2$; an increase in susceptibility corresponds to a decrease in temperature.

Step 1: Apply Curie's law to the sample Use $T_1 = 300$ K, $\chi_1 = 1.2 \times 10^5$ and $\chi_2 = 1.44 \times 10^5$. Rearranging the constant-product relation gives $T_2 = T_1 \chi_1 / \chi_2$.

Step 2: Substitute and simplify Both printed susceptibilities contain the same power of ten, which cancels in their ratio:

$$T_2 = 300 \times \frac{1.2 \times 10^5}{1.44 \times 10^5} = 300 \times \frac{5}{6} \text{ K}.$$

The decrease in temperature is consistent with the stated increase in susceptibility.

Quick Tip to Solve: For Curie's law keep χT constant, and cancel any common power of ten before calculating.

Step 3: Conclusion The required temperature is 250 K .

Quick Tip to Solve: For Curie's law keep χT constant, and cancel any common power of ten before calculating.

OR

3. The magnetic susceptibility χ of a given material is -0.5 . Identify the magnetic material. [1 mark]

Solution:

Concept:

- Magnetic susceptibility is defined by $\vec{M} = \chi \vec{H}$, where $\vec{M}$ is the magnetisation and $\vec{H}$ is the applied magnetic field intensity.
- Diamagnetic materials have negative susceptibility; paramagnetic and ferromagnetic materials have positive susceptibility.

Step 1: Inspect the sign of susceptibility The given value is $\chi = -0.5$. The negative sign means the induced magnetisation points opposite to the applied field and therefore opposes it.

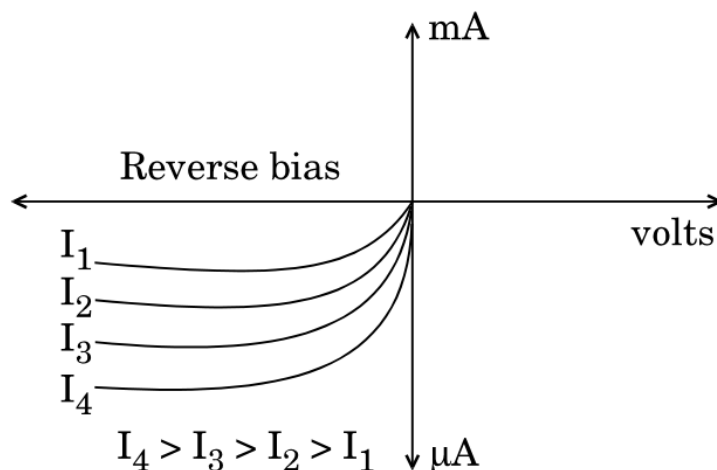
Step 2: Use the magnetic classification This opposition is the characteristic response of diamagnetism. A positive susceptibility would indicate magnetisation along the applied field instead. The relative permeability also satisfies $\mu_r = 1 + \chi = 0.5 < 1$, consistent with the negative response.

Step 3: Conclusion The material is diamagnetic .

Quick Tip to Solve: Classify by the sign first: $\chi < 0$ identifies diamagnetism without needing the material's chemical name.

4. Identify the semiconductor diode whose V-I characteristics are as shown.

[1 mark]



Solution:

Concept:

- A photodiode is a light-sensitive semiconductor junction normally operated under reverse bias.
- Incident photons create electron-hole pairs; the junction field separates them, producing a reverse photocurrent that increases with illumination.

Step 1: Read the reverse-bias curves The graph lies in the reverse-bias region and shows several nearly saturated reverse currents. Their magnitudes are ordered $I_4 > I_3 > I_2 > I_1$, so the reverse current changes between the curves.

Step 2: Relate the current changes to illumination For a photodiode, increasing the intensity of incident light generates more charge carriers per second and increases the reverse current. This family of reverse characteristics is therefore associated with different illumination intensities. It is not the sharp single breakdown knee used to identify a Zener diode.

Step 3: Conclusion The semiconductor device is a photodiode.

Quick Tip to Solve: A family of nearly flat reverse-current curves at different illumination levels is the signature of a photodiode.

5. Which part of the electromagnetic spectrum is used in RADAR? Give its frequency range. [1 mark]

Solution:

Concept:

- RADAR detects objects using transmitted electromagnetic waves and the reflected signal received from them.
- Microwaves are short-wavelength radio waves; their approximate frequency band is 10^9 to 10^{12} Hz.

Step 1: Choose the suitable spectral region Radar needs waves that can be transmitted as a narrow directed beam and reflected by distant objects. The comparatively short wavelengths of microwaves

Quick Tip to Solve: Remember the radar-microwave pairing; write the frequency range in hertz, not the wavelength range.

permit strong directionality with practical antenna sizes.

Step 2: Specify the frequency band The microwave region extends approximately from 1 GHz to 1000 GHz. Using $\lambda = c/f$, these limits correspond roughly to wavelengths from 30 cm down to 0.3 mm. A particular radar operates within a selected part of this band.

Step 3: Conclusion RADAR uses microwaves, with an approximate frequency range 10^9 – 10^{12} Hz.

Quick Tip to Solve: Remember the radar-microwave pairing; write the frequency range in hertz, not the wavelength range.

OR

5. How are electromagnetic waves produced by accelerating charges?

[1 mark]

Solution:

Concept:

- An accelerating electric charge produces time-dependent electric and magnetic fields.
- A changing electric field is associated with a magnetic field, and a changing magnetic field induces an electric field; this coupled disturbance can propagate through space.

Step 1: Identify the effect of acceleration When a charge accelerates, its changing motion produces a disturbance in its electromagnetic field. An oscillating charge is a standard example because its velocity and acceleration change continuously.

Step 2: Describe the travelling disturbance The time-varying electric and magnetic fields propagate away from the charge and carry energy as electromagnetic radiation. Far from the source, the electric field, magnetic field and propagation direction are mutually perpendicular. No material medium is needed for this propagation.

Step 3: Conclusion Accelerating charges produce electromagnetic waves by generating coupled, time-varying electric and magnetic fields that travel outward through space.

Quick Tip to Solve: Use an oscillating charge as the example: acceleration generates radiation, whereas a stationary charge has only a static electric field.

Section B (14 marks)

6. A capacitor made of two parallel plates, each of area 'A' and separation 'd' is charged by an external dc source. Show that during charging, the displacement current inside the capacitor is the same as the current charging the capacitor. [2 marks]

Solution:

Concept:

- The conduction current charging a capacitor is $I = dq/dt$, where q is the instantaneous charge on its positive plate.
- Maxwell's displacement current is $I_d = \epsilon_0 d\Phi_E/dt$, where Φ_E is the electric flux through the gap between the plates.

Step 1: Express the electric flux in terms of plate charge Neglecting edge effects, the field between the parallel plates is uniform and normal to them. With plate area A , the surface charge density is q/A , so

$$E = \frac{q}{\epsilon_0 A}, \quad \Phi_E = EA = \frac{q}{\epsilon_0}.$$

Equivalently, $C = \epsilon_0 A/d$ and $E = V/d$ give the same expression. Although the plates have fixed area and separation, their charge and the electric field change during charging.

Step 2: Differentiate the flux during charging Substitute this flux into Maxwell's expression. Since ϵ_0 is constant,

$$I_d = \epsilon_0 \frac{d}{dt} \left(\frac{q}{\epsilon_0} \right) = \frac{dq}{dt}.$$

The rate dq/dt is precisely the current delivered to the plate by the connecting wire. No conduction charge crosses the ideal insulating gap; the changing electric field supplies the displacement-current contribution there.

Step 3: Conclusion During charging, the displacement current between the plates equals the charging current at every instant: $I_d = I$.

Quick Tip to Solve: First write $\Phi_E = q/\epsilon_0$; differentiating immediately converts displacement current into the charging rate dq/dt .

7. A photon and a proton have the same de-Broglie wavelength λ . Prove that the energy of the photon is $(2m\lambda c/h)$ times the kinetic energy of the proton. [2 marks]

Solution:

Concept:

- The de-Broglie relation is $p = h/\lambda$, where p is momentum and h is Planck's constant. Equal wavelengths therefore imply equal momenta.
- A photon has energy $E_\gamma = pc = hc/\lambda$. For a non-relativistic proton of mass m , the kinetic energy is $K_p = p^2/(2m)$.

Quick Tip to Solve: For equal wavelengths, set the momenta equal first; then use pc for the photon and $p^2/(2m)$ for the proton.

Step 1: Write both energies using the common wavelength The photon and proton have the same momentum magnitude h/λ , but their energy-momentum relations differ. The photon travels at speed c , whereas the proton is a massive particle. Using the non-relativistic kinetic-energy expression implicit in the required result,

$$E_\gamma = \frac{hc}{\lambda}, \quad K_p = \frac{1}{2m} \left(\frac{h}{\lambda} \right)^2 = \frac{h^2}{2m\lambda^2}.$$

Here K_p excludes the proton's rest energy.

Step 2: Form the ratio and simplify Divide the photon energy by the proton kinetic energy. The common factors of h and λ cancel once:

$$\frac{E_\gamma}{K_p} = \frac{hc}{\lambda} \frac{2m\lambda^2}{h^2} = \frac{2m\lambda c}{h}.$$

This ratio is dimensionless, since both $m\lambda c$ and h have the dimensions of action. Equal momentum does not imply equal energy for a photon and a proton.

Step 3: Conclusion Hence, $E_\gamma = (2m\lambda c/h)K_p$: the photon energy is $2m\lambda c/h$ times the proton's kinetic energy.

Quick Tip to Solve: For equal wavelengths, set the momenta equal first; then use pc for the photon and $p^2/(2m)$ for the proton.

8. A photon emitted during the de-excitation of electron from a state n to the first excited state in a hydrogen atom, irradiates a metallic cathode of work function 2 eV, in a photo cell, with a stopping potential of 0.55 V. Obtain the value of the quantum number of the state n . [2 marks]

Solution:

Concept:

- Einstein's photoelectric equation is $E_\gamma = \phi + K_{\max}$, where ϕ is the work function and $K_{\max} = eV_s$ for stopping potential V_s .
- Hydrogen energy levels are $E_n = -13.6/n^2$ eV. A photon emitted in a downward transition carries the difference between the initial and final atomic energies.

Step 1: Calculate the energy of the emitted photon The stopping potential removes the maximum kinetic energy of the emitted photoelectrons. Thus $V_s = 0.55$ V corresponds to $K_{\max} = 0.55$ eV. Including the energy needed to escape the metal gives

$$E_\gamma = \phi + eV_s = 2 + 0.55 = 2.55 \text{ eV}.$$

The work function must be added to the maximum kinetic energy, because both are supplied by the incident photon.

Step 2: Use the transition to the first excited state The first excited state of hydrogen is $n_f = 2$, so the initial level n satisfies

$$13.6 \left(\frac{1}{2^2} - \frac{1}{n^2} \right) = 2.55.$$

Quick Tip to Solve: Convert the stopping potential directly to photoelectron energy in eV, add the work function, and remember that the first excited hydrogen level is $n = 2$.

Rearranging in electron-volt units,

$$\frac{13.6}{n^2} = 3.40 - 2.55 = 0.85, \quad n^2 = \frac{13.6}{0.85} = 16.$$

As a check, the initial level has energy -0.85 eV, and the final level has energy -3.40 eV; their difference is the required photon energy.

Step 3: Conclusion Taking the positive integer quantum number, $n = 4$. The photon is emitted in the $4 \rightarrow 2$ transition.

Quick Tip to Solve: Convert the stopping potential directly to photoelectron energy in eV, add the work function, and remember that the first excited hydrogen level is $n = 2$.

OR

8. A hydrogen atom in the ground state is excited by an electron beam of 12.5 eV energy. Find out the maximum number of lines emitted by the atom from its excited state. [2 marks]

Solution:

Concept:

- Hydrogen energy levels are $E_n = -13.6/n^2$ eV. Excitation from the ground state to level n requires $\Delta E = 13.6(1 - 1/n^2)$ eV.
- The maximum number of distinct emission lines available among levels 1 through n is $N = n(n-1)/2$, since each downward pair of levels gives a possible photon energy.

Step 1: Identify the highest level the beam can excite An incident electron can transfer part of its 12.5 eV kinetic energy in a collision; the transferred excitation energy must match an allowed atomic energy gap. Compare the next relevant levels:

$$\begin{aligned}\Delta E_{1 \rightarrow 3} &= 13.6 \left(1 - \frac{1}{9}\right) \simeq 12.09 \text{ eV}, \\ \Delta E_{1 \rightarrow 4} &= 13.6 \left(1 - \frac{1}{16}\right) = 12.75 \text{ eV}.\end{aligned}$$

The beam has enough energy to excite $n = 3$, but not $n = 4$. Its surplus energy after an excitation remains with the scattered electron.

Step 2: Count the possible downward transitions With the highest excited level $n = 3$, the available levels are 3, 2, 1. All possible spectral transitions are $3 \rightarrow 2$, $3 \rightarrow 1$, and $2 \rightarrow 1$, giving

$$N = \frac{3(3-1)}{2} = 3.$$

These are the distinct possible lines in the emission spectrum over repeated excitations. A particular decay can be direct or proceed through an intermediate level; it need not emit all the possible lines in one cascade.

Step 3: Conclusion The highest accessible excited state is $n = 3$, so the maximum number of distinct

Quick Tip to Solve: For electron-impact excitation, choose the highest integer level whose excitation energy does not exceed the beam energy, then use $n(n-1)/2$.

spectral lines is $\boxed{3}$.

Quick Tip to Solve: For electron-impact excitation, choose the highest integer level whose excitation energy does not exceed the beam energy, then use $n(n-1)/2$.

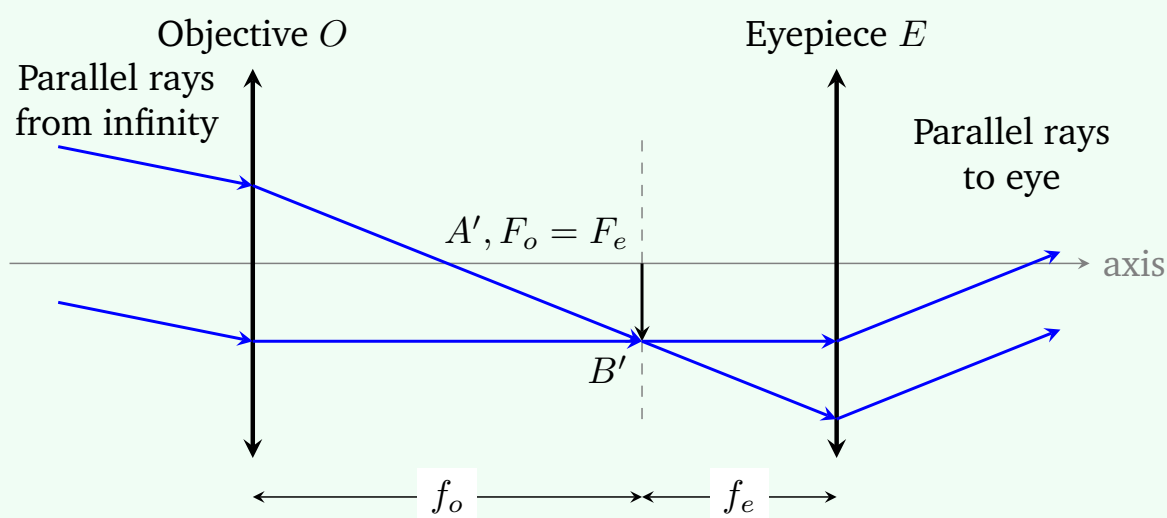
9. Draw the ray diagram of an astronomical telescope showing image formation in the normal adjustment position. Write the expression for its magnifying power. [2 marks]

Solution:

Concept:

- An astronomical telescope uses a convex objective of long focal length f_o and a convex eyepiece of short focal length f_e .
- In normal adjustment the final image is at infinity: the objective's rear focal plane coincides with the eyepiece's front focal plane.

Step 1: Form the intermediate image and trace the emergent rays Parallel rays from one point of a distant object enter the objective O and converge at B' . Thus $A'B'$ is a real, inverted image in the common focal plane $F_o = F_e$. The eyepiece E receives rays diverging from B' in its focal plane and makes them parallel again. The eye views the final inverted image at infinity without accommodation. The lens separation is $f_o + f_e$.



Astronomical telescope in normal adjustment; arrows on rays show light travelling towards the eyepiece.

Step 2: Relate the angular sizes If the intermediate image has height h , the small angles subtended by the distant object and final image have magnitudes $\alpha \simeq h/f_o$ and $\beta \simeq h/f_e$. Their ratio gives the angular magnification; the negative sign records inversion relative to the object.

$$M = -\frac{\beta}{\alpha} = -\frac{f_o}{f_e}.$$

Step 3: Conclusion For a telescope in normal adjustment, $\boxed{M = -f_o/f_e}$, or magnifying power in

Quick Tip to Solve: Put the intermediate image in the common focal plane; parallel emergent rays then verify normal adjustment immediately.

magnitude $|M| = f_o/f_e$. The final image is inverted and at infinity.

Quick Tip to Solve: Put the intermediate image in the common focal plane; parallel emergent rays then verify normal adjustment immediately.

OR

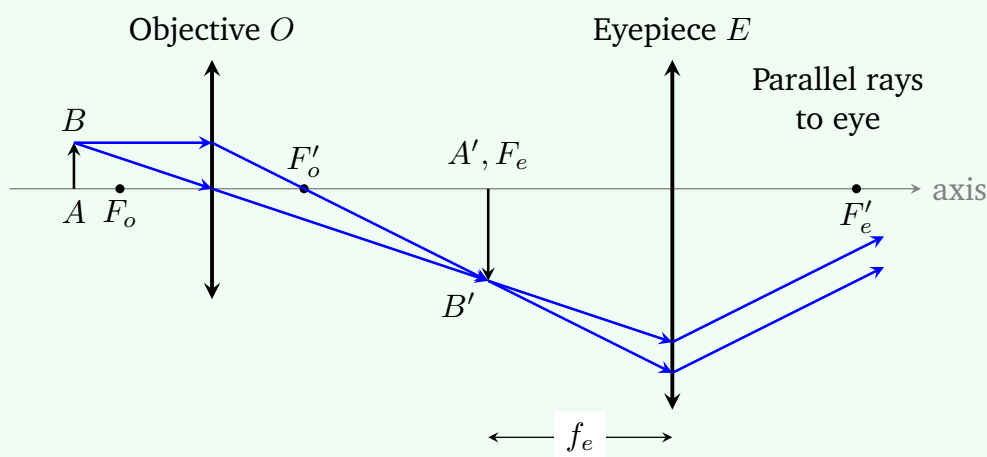
9. Draw a labelled ray diagram to show image formation by a compound microscope and write the expression for its resolving power. [2 marks]

Solution:

Concept:

- A compound microscope uses a short-focal-length objective to form a real, enlarged intermediate image, which is magnified by the eyepiece.
- Resolving power is the reciprocal of the minimum resolvable separation $d_{\min}$ of two object points. The numerical aperture is $NA = n \sin \theta$, where n is the refractive index of the medium and θ is the objective's angular semi-aperture.

Step 1: Construct the image with the two lenses Place the small object AB just beyond the objective's first focus F_o . One ray from B , parallel to the axis, refracts through the objective's second focus F'_o ; another through its optical centre passes undeviated. They meet at B' , giving the real, enlarged, inverted image $A'B'$. In the normal-adjustment arrangement drawn, this intermediate image lies at the first focus F_e of the eyepiece. The emerging rays are parallel and the final virtual, enlarged image is at infinity, inverted relative to AB .



Compound microscope adjusted for a final virtual image at infinity; $A'B'$ is the intermediate image.

Step 2: Apply the diffraction limit If λ is the wavelength in vacuum (approximately the air wavelength), the Rayleigh criterion for the microscope gives

$$d_{\min} = \frac{1.22\lambda}{2n \sin \theta}.$$

Quick Tip to Solve: For resolving power use numerical aperture and wavelength; do not substitute the microscope's magnifying-power formula.

A larger numerical aperture or a shorter wavelength permits closer points to be distinguished.

Step 3: Conclusion The labelled diagram shows image formation by the compound microscope. Its resolving power is
$$\text{RP} = \frac{1}{d_{\min}} = \frac{2n \sin \theta}{1.22\lambda}.$$

Quick Tip to Solve: For resolving power use numerical aperture and wavelength; do not substitute the microscope's magnifying-power formula.

10. Write the relation between the height of a TV antenna and the maximum range up to which signals transmitted by the antenna can be received. How is this expression modified in the case of line of sight communication by space waves? In which range of frequencies, is this mode of communication used? [2 marks]

Solution:

Concept:

- Space waves travel essentially in straight lines, so Earth's curvature limits the direct line-of-sight range of an antenna.
- For antenna height $h \ll R$, the horizon distance is $d \simeq \sqrt{2Rh}$, where R is Earth's radius and all lengths use the same unit.

Step 1: Find the range of one transmitting antenna At the limiting distance the ray from the antenna top is tangent to Earth's surface. The radius to the tangent point is perpendicular to that ray. Thus the right triangle gives

$$(R + h)^2 = R^2 + d^2 \Rightarrow d = \sqrt{2Rh + h^2} \simeq \sqrt{2Rh}.$$

The h^2 term is negligible compared with $2Rh$. This is the maximum range to a receiving point at ground level; at small heights the straight-line and surface distances agree to this approximation.

Step 2: Include the receiving antenna height and state the frequency band If the transmitting and receiving antenna heights are h_T and h_R , respectively, each has its own horizon. Communication is possible over the sum of these distances:

$$d_{\max} = \sqrt{2Rh_T} + \sqrt{2Rh_R}.$$

Space-wave communication is used at frequencies above about 30 MHz, including VHF, UHF and microwave bands. These frequencies generally pass through the ionosphere, so reliable long-distance sky-wave return is unavailable and direct visibility becomes important.

Step 3: Conclusion For one antenna, $d \simeq \sqrt{2Rh}$. For two elevated antennas,

$$d_{\max} = \sqrt{2Rh_T} + \sqrt{2Rh_R}.$$

The usual space-wave frequency range is above 30 MHz.

Quick Tip to Solve: Find each antenna's horizon separately and add the two distances; do not put the sum of the heights inside a single square root.

11. Under which conditions can a rainbow be observed? Distinguish between a primary and a secondary rainbow. [2 marks]

Solution:

Concept:

- A rainbow forms when sunlight undergoes refraction and dispersion on entering water droplets, internal reflection inside them, and refraction again on emerging.
- Different colours emerge at different angles because the refractive index of water depends on wavelength; violet is deviated more strongly than red.

Step 1: State the viewing conditions There must be sunlit water droplets, such as rain or fine spray, in front of the observer, while the Sun is behind the observer. For an ordinary rainbow viewed from ground level, the Sun should be low in the sky and the observer should look towards the sky opposite the Sun. The droplets send different colours into the eye along different directions, producing the coloured arc centred on the direction opposite the Sun.

Step 2: Compare the two rainbows In a **primary rainbow**, light undergoes one internal reflection within each droplet. The arc is brighter, with red on its outer edge and violet on its inner edge; its angular radius is approximately 42° for red.

In a **secondary rainbow**, light undergoes two internal reflections. It appears outside the primary arc, at an angular radius of roughly 51° to 54° , and is fainter because less light survives the additional reflection. Its colour order is reversed: red lies on the inner edge and violet on the outer edge. Both types also involve refraction at entry and exit.

Step 3: Conclusion With the Sun behind and illuminated droplets ahead, a rainbow can be seen. The primary bow is brighter, results from one internal reflection and has red outside; the secondary is fainter, results from two internal reflections and has violet outside.

Quick Tip to Solve: Link one internal reflection with red outside in the primary bow; the second reflection reverses the colour order in the secondary bow.

12. Explain the following:

(a) Sky appears blue.

(b) The Sun appears reddish at (i) sunset, (ii) sunrise.

[2 marks]

Solution:

Concept:

- Rayleigh scattering by air molecules is stronger for shorter wavelengths: the scattered intensity obeys $I_s \propto 1/\lambda^4$, where λ is the wavelength of light.
- The colour of the sky is determined by scattered sunlight, whereas the colour of the solar disc is determined by sunlight transmitted directly through the atmosphere.

Step 1: (a) Explain the blue sky Sunlight contains a range of visible wavelengths. Molecules of the atmosphere, whose sizes are much smaller than visible wavelengths, scatter the shorter-wavelength blue component much more strongly than the longer-wavelength red component. This scattered light

Quick Tip to Solve: Distinguish the light scattered towards your eye (blue sky) from the direct beam left after scattering (reddish Sun).

reaches our eyes from directions away from the Sun, so the sky appears blue. Although violet is scattered still more strongly, the spectral distribution of sunlight and the lower sensitivity of our eyes to violet make the perceived sky predominantly blue.

Step 2: (b) Explain the reddish Sun at sunset and sunrise At both sunset and sunrise the Sun is close to the horizon, so its light traverses a much longer path through the atmosphere than when it is overhead. Along this path, much of the shorter-wavelength blue light is scattered out of the direct beam. The longer-wavelength red and orange components are scattered less and form a larger fraction of the light reaching our eyes directly from the Sun. Thus the same scattering mechanism accounts for the reddish solar disc at both times.

Step 3: Conclusion The sky appears blue because shorter wavelengths are preferentially scattered towards us. The Sun appears reddish at both sunrise and sunset because its long atmospheric path removes much of the blue light from the direct beam.

Quick Tip to Solve: Distinguish the light scattered towards your eye (blue sky) from the direct beam left after scattering (reddish Sun).

Section C (36 marks)

13. A capacitor (C) and resistor (R) are connected in series with an ac source of voltage of frequency 50 Hz. The potential difference across C and R are respectively 120 V, 90 V, and the current in the circuit is 3 A. Calculate (i) the impedance of the circuit (ii) the value of the inductance, which when connected in series with C and R will make the power factor of the circuit unity. [3 marks]

Solution:

Concept:

- In a series RC circuit the same current I flows through both elements. The resistance and capacitive reactance are $R = V_R/I$ and $X_C = V_C/I$, respectively.
- The resistor voltage is in phase with current while the capacitor voltage lags it by 90° . Therefore the impedance magnitude is $Z = \sqrt{R^2 + X_C^2}$.
- For a series LCR circuit, $\cos \phi = R/\sqrt{R^2 + (X_L - X_C)^2}$, with $X_L = 2\pi fL$. Unity power factor requires $X_L = X_C$.

Step 1: Obtain the resistance and capacitive reactance The given component voltages are $V_R = 90$ V and $V_C = 120$ V, with current $I = 3$ A. Hence

$$R = \frac{90}{3} = 30 \, \Omega, \quad X_C = \frac{120}{3} = 40 \, \Omega.$$

The two component voltages are perpendicular phasors, so their magnitudes must not be added arithmetically.

Step 2: (i) Calculate the impedance Taking the magnitude of the complex impedance $R - jX_C$,

$$Z = \sqrt{30^2 + 40^2} = \sqrt{2500} = 50 \, \Omega.$$

Quick Tip to Solve: Use the perpendicular voltage triangle first; for unity power factor, set the added inductive reactance equal to the existing capacitive reactance.

As a check, the source voltage is $V = \sqrt{90^2 + 120^2} = 150 \text{ V}$, giving the same value $Z = V/I = 150/3$.

Step 3: (ii) Find the inductance for unity power factor Add an ideal inductor in series. At the unchanged frequency $f = 50 \text{ Hz}$, its inductive reactance must cancel the capacitive reactance. Thus

$$2\pi fL = X_C, \quad L = \frac{40}{2\pi \times 50} = \frac{0.4}{\pi} \text{ H} \simeq 0.127 \text{ H}.$$

With this inductance the net reactance is zero, the total impedance is purely resistive, and the supply voltage and current are in phase. The original 3 A is used to determine component properties; it need not remain the current after adding the inductor.

Step 4: Conclusion (i) The original circuit impedance is 50Ω . (ii) The required series inductance is $L \simeq 0.127 \text{ H}$, or approximately 127 mH.

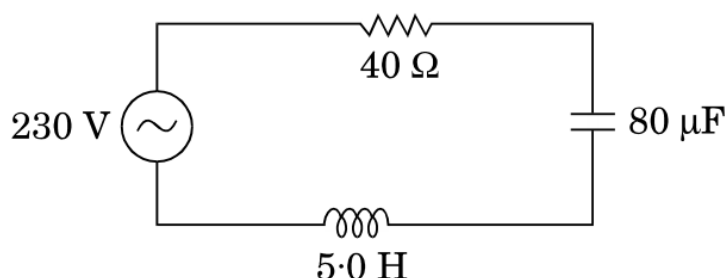
Quick Tip to Solve: Use the perpendicular voltage triangle first; for unity power factor, set the added inductive reactance equal to the existing capacitive reactance.

OR

13. The figure shows a series LCR circuit connected to a variable frequency 230 V source.

- Determine the source frequency which drives the circuit in resonance.
- Calculate the impedance of the circuit and amplitude of current at resonance.
- Show that potential drop across LC combination is zero at resonating frequency.

[3 marks]



Solution:

Concept:

- Series resonance occurs when the inductive and capacitive reactances cancel: $\omega_0 L = 1/(\omega_0 C)$, so $f_0 = 1/(2\pi\sqrt{LC})$.
- The impedance magnitude is $Z = \sqrt{R^2 + (X_L - X_C)^2}$. At resonance, it equals R , and $I_{\text{rms}} = V_{\text{rms}}/R$.
- For a sinusoidal current its amplitude is $I_0 = \sqrt{2}I_{\text{rms}}$. Inductor and capacitor voltages are opposite phasors, so their combined voltage is their phasor sum.

Step 1: (a) Determine the resonant frequency The figure gives $L = 5.0 \text{ H}$, $C = 80 \mu\text{F} = 80 \times 10^{-6} \text{ F}$,

Quick Tip to Solve: At resonance cancel reactances before finding current, then multiply the rms current by $\sqrt{2}$ if amplitude is requested.

and $R = 40\ \Omega$. Therefore

$$LC = 5.0 \times 80 \times 10^{-6} = 4.0 \times 10^{-4}\ \text{s}^2,$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = 50\ \text{rad s}^{-1},$$

$$f_0 = \frac{50}{2\pi} \simeq 7.96\ \text{Hz}.$$

The angular frequency ω_0 and ordinary frequency f_0 differ by the factor 2π .

Step 2: (b) Calculate impedance and current amplitude At resonance $X_L - X_C = 0$, so $Z = R = 40\ \Omega$. The stated 230 V ac supply is its rms voltage. Thus

$$I_{\text{rms}} = \frac{230}{40} = 5.75\ \text{A}, \quad I_0 = \sqrt{2} \times 5.75 \simeq 8.13\ \text{A}.$$

The question asks for amplitude, which is the peak current rather than its rms value.

Step 3: (c) Show cancellation of the LC voltage At the calculated frequency,

$$X_L = 50 \times 5.0 = 250\ \Omega, \quad X_C = \frac{1}{50 \times 80 \times 10^{-6}} = 250\ \Omega.$$

Relative to the current, the voltage across L leads by 90° , while that across C lags by 90° . They have equal magnitudes and a phase difference of 180° . Consequently their phasor sum is

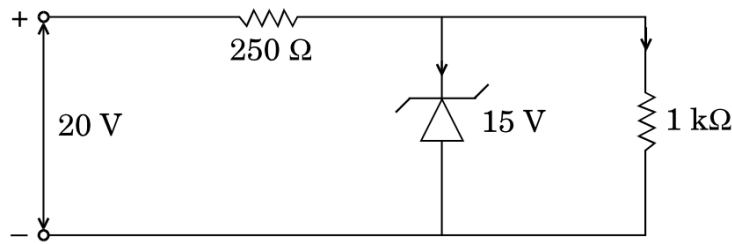
$$\underline{V}_{LC} = jI X_L - jI X_C = jI(X_L - X_C) = 0.$$

Each component has a nonzero voltage, but their instantaneous voltages cancel throughout the sinusoidal cycle.

Step 4: Conclusion At resonance, $f_0 \simeq 7.96\ \text{Hz}$, $Z = 40\ \Omega$, and the current amplitude is $I_0 \simeq 8.13\ \text{A}$. The net potential difference across the series LC combination is $0\ \text{V}$.

Quick Tip to Solve: At resonance cancel reactances before finding current, then multiply the rms current by $\sqrt{2}$ if amplitude is requested.

14. Give reason to explain why n and p regions of a Zener diode are heavily doped. Find the current through the Zener diode in the circuit given below: (Zener breakdown voltage is 15 V) [3 marks]



Solution:

Concept:

- Heavy doping on both sides of a Zener junction makes its depletion layer very thin. A strong electric field can therefore be established at a comparatively small reverse voltage, producing a sharp, well-defined breakdown.
- In breakdown an ideal Zener diode maintains an approximately constant voltage V_Z across itself and a parallel load. The series resistor limits the total supply current.
- At the branch junction, Kirchhoff's current law gives $I_S = I_L + I_Z$, where I_S is the series-resistor current, I_L is the load current and I_Z is the Zener current.

Step 1: Explain the need for heavy doping The high donor and acceptor concentrations in the n and p regions reduce the depletion-layer width. Since the reverse voltage is applied across this narrow region, the junction electric field becomes very large even at a relatively low reverse bias. This allows controlled reverse breakdown at the specified Zener voltage. Over its operating range the voltage then changes very little when reverse current changes, which makes the device useful for voltage regulation.

Step 2: Establish the regulated output voltage The diagram has a 20 V supply, a series resistance $R_S = 250 \Omega$, and a load $R_L = 1 \text{ k}\Omega = 1000 \Omega$. With the Zener omitted, the divider would produce

$$V_L = 20 \frac{1000}{250 + 1000} = 16 \text{ V}.$$

This exceeds the reverse breakdown voltage 15 V, so the correctly reverse-biased Zener conducts and holds the load voltage at $V_Z = 15 \text{ V}$.

Step 3: Calculate and separate the branch currents The voltage across the series resistor is $20 - 15 = 5 \text{ V}$. Its current and the load current are

$$I_S = \frac{20 - 15}{250} = 0.020 \text{ A} = 20 \text{ mA}, \quad I_L = \frac{15}{1000} = 0.015 \text{ A} = 15 \text{ mA}.$$

Applying the junction rule, the remaining current flows through the Zener:

$$I_Z = I_S - I_L = 20 - 15 = 5 \text{ mA}.$$

The positive result confirms that the assumed breakdown state is consistent. This current flows downward through the Zener branch, from its cathode at the upper junction towards its anode at the lower junction.

Quick Tip to Solve: Find the series current and load current separately; only their difference flows through the regulating Zener diode.

Step 4: Conclusion Heavy doping produces a thin depletion layer and a large junction field, enabling sharp reverse breakdown at a well-defined voltage. The Zener current is $I_Z = 5 \text{ mA}$.

Quick Tip to Solve: Find the series current and load current separately; only their difference flows through the regulating Zener diode.

15. Draw a labelled diagram of cyclotron. Explain its working principle. Show that cyclotron frequency is independent of the speed and radius of the orbit. [3 marks]

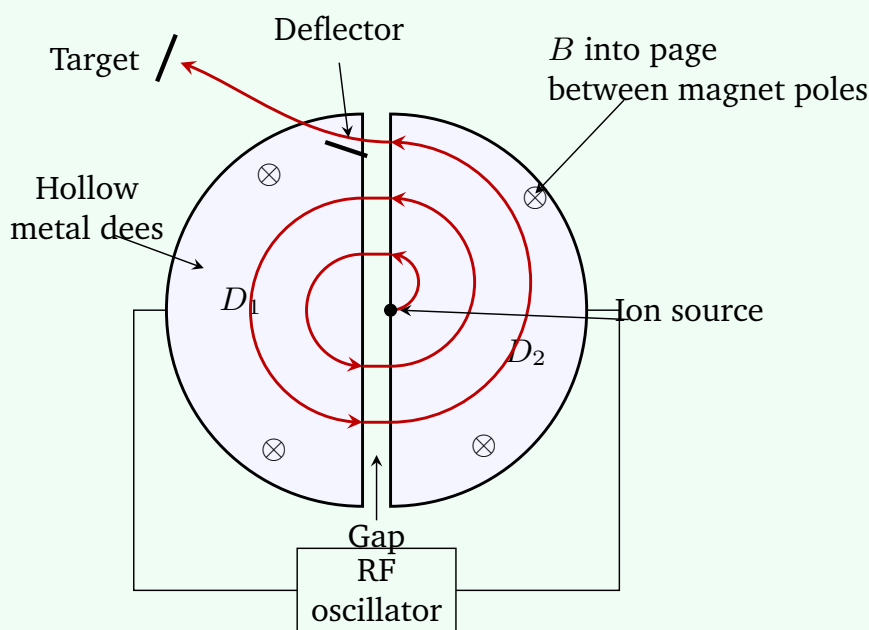
Solution:

Concept:

- A cyclotron accelerates charged particles by an alternating electric field across the gap between two hollow metal dees. A uniform magnetic field perpendicular to the dees bends the path inside them.
- For a non-relativistic particle of mass m , charge magnitude q , and speed v , the magnetic force supplies the centripetal force: $qvB = mv^2/r$, where r is the orbit radius.
- Resonance requires the frequency of the applied alternating voltage to equal the particle's revolution frequency, so that the electric force accelerates it at every gap crossing.

Step 1: Identify the arrangement Two hollow semicircular metal electrodes, D_1 and D_2 , are enclosed in an evacuated chamber between the poles of a strong electromagnet. The magnetic field is normal to their plane. A high-frequency oscillator supplies an alternating potential difference between the dees. Positive ions are released near the centre; the diagram shows their path for a magnetic field into the page.

Quick Tip to Solve: Eliminate the ratio r/v using the magnetic-force equation before calculating the period; this immediately shows why increasing the orbit radius does not change the frequency.



Cyclotron, viewed normal to the dees; the chamber is evacuated and the magnetic field is into the page.

Step 2: Explain successive acceleration In the gap, the electric field accelerates the ion toward the oppositely charged dee. Inside a dee the electric field is shielded, so the magnetic force changes the direction of velocity without doing work; the ion follows a semicircle at constant speed. During this half-revolution, the oscillator reverses the polarity of the dees. On reaching the gap again, the ion is therefore accelerated once more. Its kinetic energy and speed increase at successive crossings, and $r = mv/(qB)$ increases. The outward-growing path finally reaches a deflector, which directs the energetic ion beam toward a target.

Step 3: Derive the cyclotron frequency The magnetic force and the period of circular motion give

$$qvB = \frac{mv^2}{r}, \quad \frac{v}{r} = \frac{qB}{m}.$$

Hence the revolution period and frequency are

$$T = \frac{2\pi r}{v} = \frac{2\pi m}{qB}, \quad f_c = \frac{1}{T} = \frac{qB}{2\pi m}.$$

The half-revolution time is $T/2 = \pi m/(qB)$, independent of both v and r . Thus a fixed oscillator frequency can reverse the field at every crossing even as the orbit expands. This result assumes non-relativistic speeds, for which the particle mass remains constant.

Step 4: Conclusion The cyclotron operates by resonant acceleration across the dee gap, with magnetic bending inside the dees. Its frequency is $f_c = qB/(2\pi m)$, independent of orbit speed and radius in

Quick Tip to Solve: Eliminate the ratio r/v using the magnetic-force equation before calculating the period; this immediately shows why increasing the orbit radius does not change the frequency.

the non-relativistic regime.

Quick Tip to Solve: Eliminate the ratio r/v using the magnetic-force equation before calculating the period; this immediately shows why increasing the orbit radius does not change the frequency.

OR

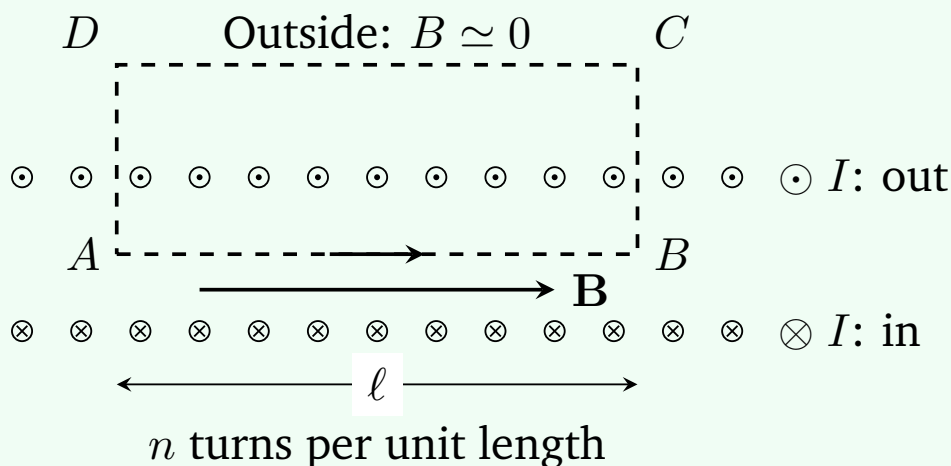
15(a). Derive, with the help of a diagram, the expression for the magnetic field inside a very long solenoid having n turns per unit length carrying a current I . [3 marks total]

Solution:

Concept:

- Ampere's circuital law is $\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I_{\text{enc}}$, where I_{enc} is the net current through the chosen loop.
- For a very long, closely wound air-core solenoid, the field is approximately uniform and axial inside; the external field is negligible away from the ends. The winding density is n turns per unit length.

Step 1: Choose an Amperian rectangle Consider the rectangular path $ABCD$, with $AB = \ell$ inside the solenoid parallel to its axis, and CD outside. The sides BC and DA are perpendicular to the internal field. The sketch is a longitudinal section: dots indicate current out of the page and crosses current into the page. For the current directions shown, the field inside points from A to B .



Longitudinal section of a long solenoid and rectangular Amperian path

Step 2: Evaluate the circulation and enclosed current On AB , the field is parallel to the path, so its contribution is $B\ell$. The contribution on CD vanishes because the external field is negligible. The transverse sides contribute zero since $\mathbf{B} \cdot d\mathbf{l} = 0$ there. The rectangle encloses $n\ell$ turns, each carrying current I . Hence Ampere's law gives

$$\oint \mathbf{B} \cdot d\mathbf{l} = B\ell = \mu_0(n\ell)I.$$

The factors of ℓ cancel, showing that the interior field does not depend on the particular length of

Quick Tip to Solve: Put one long side of the Amperian rectangle inside and the other outside; only the inside side contributes to the circulation.

rectangle chosen.

Step 3: Conclusion The magnetic field inside a very long air-core solenoid is $B = \mu_0 n I$, directed along its axis according to the right-hand grip rule. End effects are neglected in this result.

Quick Tip to Solve: Put one long side of the Amperian rectangle inside and the other outside; only the inside side contributes to the circulation.

15(b). How is a toroid different from a solenoid?

Solution:

Concept:

- A solenoid consists of many closely wound turns along a cylindrical surface. Its magnetic field inside a long coil is nearly uniform and parallel to the cylinder's axis.
- A toroid is a solenoid bent into a closed circular ring. Its field lines run around the ring inside the winding, so the magnetic path is closed.

Step 1: Compare the shapes and ends A straight solenoid has a long cylindrical shape and two distinct ends. It behaves like a bar magnet, with a north end and a south end determined by the direction of current. A toroid has a doughnut-like shape and no free ends; consequently it has no separate north and south end poles.

Step 2: Compare the magnetic fields In an air-core long solenoid the interior field is approximately axial and uniform, $B = \mu_0 n I$, apart from end effects. A finite solenoid also has a weak external return field. In an ideal closely wound toroid, the magnetic field is confined to the ring-shaped region occupied by its windings; it is negligible both outside and in the central hole. At radius r within that region,

$$B(r) = \frac{\mu_0 N I}{2\pi r},$$

where N is the total number of turns and I is the current. Thus its field is tangential around the ring and varies with radius; it is nearly uniform only for a thin ring.

Step 3: Conclusion A toroid is a closed ring-shaped solenoid without free ends or distinct end poles. It confines its field within the winding region, whereas a straight finite solenoid has two ends and an external return field.

Quick Tip to Solve: Compare shape, free ends and field confinement: bending a solenoid into a closed ring gives a toroid, with no end poles.

16. Prove that the magnetic moment of the electron revolving around a nucleus in an orbit of radius r with orbital speed v is equal to $evr/2$. Hence using Bohr's postulate of quantization of angular momentum, deduce the expression for the magnetic moment of hydrogen atom in the ground state.

[3 marks]

Solution:

Concept:

- An electron moving in a circular orbit is equivalent to a current loop. The current magnitude is charge passing a point per period: $I = e/T$, where e is the magnitude of the electron charge.
- A planar current loop has magnetic moment magnitude $\mu = IA$, where A is the enclosed area. Its direction follows the right-hand rule for conventional current.
- Bohr's angular-momentum condition is $L = m_e v r = nh/(2\pi)$, where m_e is the electron mass, h is Planck's constant and $n = 1, 2, \dots$ is the orbit number.

Step 1: Express the orbital motion as a current In one revolution the electron covers the circumference $2\pi r$ at speed v . One electronic charge passes a fixed point on the orbit during this period, so

$$T = \frac{2\pi r}{v}, \quad I = \frac{e}{T} = \frac{ev}{2\pi r}.$$

Because the electron is negatively charged, conventional current circulates opposite to its direction of motion.

Step 2: Calculate the magnetic moment of the orbit The circular orbit encloses area $A = \pi r^2$. Thus the current-loop expression gives

$$\mu = IA = \frac{ev}{2\pi r} \pi r^2 = \frac{evr}{2}.$$

Since $L = m_e v r$, the same relation can be written as $\mu = eL/(2m_e)$ for magnitudes. The magnetic moment vector is opposite to the orbital angular momentum vector because of the negative electron charge: $\boldsymbol{\mu} = -\frac{e}{2m_e} \mathbf{L}$.

Step 3: Apply Bohr's condition to the ground state Substitute the allowed orbital angular momentum into the moment formula:

$$\mu_n = \frac{e}{2m_e} \frac{nh}{2\pi} = \frac{neh}{4\pi m_e}.$$

The ground state in Bohr's model has $n = 1$, so

$$\mu_1 = \frac{eh}{4\pi m_e} = \frac{e\hbar}{2m_e}.$$

This magnitude is the Bohr magneton, denoted by μ_B , with SI unit A m^2 or J T^{-1} .

Step 4: Conclusion The orbital magnetic moment has magnitude $\boxed{\mu = evr/2}$. For the hydrogen ground-state orbit in Bohr's model, $\boxed{\mu_1 = eh/(4\pi m_e) = \mu_B}$, directed opposite to the orbital angular momentum.

Quick Tip to Solve: Start with $I = e/T$ and $\mu = IA$, then replace $m_e v r$ using Bohr's condition; this avoids calculating the ground-state speed and radius separately.

17. Two large charged plane sheets of charge densities σ and -2σ C/m² are arranged vertically with a separation of d between them. Deduce expressions for the electric field at points (i) to the left of the first sheet, (ii) to the right of the second sheet, and (iii) between the two sheets. [3 marks]

Solution:

Concept:

- An infinite plane sheet with uniform surface charge density s produces a field of magnitude $|s|/(2\epsilon_0)$ on either side, where ϵ_0 is the permittivity of free space.
- The field points away from a positive sheet and towards a negative sheet. The resultant field is the vector sum of the fields of the two sheets.

Step 1: Establish the field of each sheet Neglect edge effects for these large plane sheets. Place the first sheet, of density $+\sigma$, at $x = 0$ and the second, of density -2σ , at $x = d$. Take $\sigma > 0$ and $+\hat{x}$ towards the right. A Gaussian pillbox of face area A crossing a single sheet gives $2EA = |s|A/\epsilon_0$. Thus the first sheet contributes magnitude $\sigma/(2\epsilon_0)$, and the second contributes magnitude $2\sigma/(2\epsilon_0)$.

Step 2: (i) Field to the left of the first sheet For $x < 0$, the positive sheet's field points left whereas the negative sheet's field points right. Their signed contributions are

$$\mathbf{E}_I = \left(-\frac{\sigma}{2\epsilon_0} + \frac{2\sigma}{2\epsilon_0} \right) \hat{x} = \frac{\sigma}{2\epsilon_0} \hat{x}.$$

The stronger negative sheet therefore determines the rightward direction here.

Step 3: (ii) Field to the right of the second sheet For $x > d$, the positive sheet's field points right and the negative sheet's field points left. Consequently,

$$\mathbf{E}_{II} = \left(\frac{\sigma}{2\epsilon_0} - \frac{2\sigma}{2\epsilon_0} \right) \hat{x} = -\frac{\sigma}{2\epsilon_0} \hat{x}.$$

Step 4: (iii) Field between the sheets For $0 < x < d$, both fields point right, away from the first sheet and towards the second. Hence

$$\mathbf{E}_{III} = \left(\frac{\sigma}{2\epsilon_0} + \frac{2\sigma}{2\epsilon_0} \right) \hat{x} = \frac{3\sigma}{2\epsilon_0} \hat{x}.$$

The field of an ideal infinite sheet does not decrease with distance, so the separation d does not enter these expressions.

Step 5: Conclusion The fields are (i) $\sigma/(2\epsilon_0)$ rightward, (ii) $\sigma/(2\epsilon_0)$ leftward, and (iii) $3\sigma/(2\epsilon_0)$ from the positive sheet towards the negative sheet.

Quick Tip to Solve: Draw one field arrow from each sheet in each region before adding: the fields add between opposite charges and oppose outside.

17. A spherical conducting shell of inner radius r_1 and outer radius r_2 has a charge Q .

(a) A charge q is placed at the centre of the shell. Find out the surface charge density on the inner and outer surfaces of the shell.

(b) Is the electric field inside a cavity (with no charge) zero; independent of the fact whether the shell is spherical or not? Explain. [3 marks]

Solution:

Concept:

- In electrostatic equilibrium the electric field within conducting material is zero. Gauss's law states $\oint \mathbf{E} \cdot d\mathbf{A} = Q_{\text{enclosed}}/\epsilon_0$.
- The net charge of the conducting shell remains Q . A point charge at the common centre preserves spherical symmetry, so the charge on each spherical surface is uniform.
- An empty closed cavity inside a conductor has an equipotential boundary. The unique solution of Laplace's equation inside that cavity is constant potential.

Step 1: (a) Determine the inner-surface charge Let Q_{in} and Q_{out} denote the charges on the inner and outer surfaces. Choose a concentric Gaussian sphere lying entirely in the metal, with $r_1 < r < r_2$. Since the electric field on this surface is zero, its flux vanishes. Gauss's law then requires

$$0 = \frac{q + Q_{\text{in}}}{\epsilon_0}, \quad Q_{\text{in}} = -q.$$

Spherical symmetry makes this charge uniform over area $4\pi r_1^2$, giving

$$\sigma_{\text{in}} = -\frac{q}{4\pi r_1^2}.$$

Step 2: (a) Determine the outer-surface charge The shell retains its net charge Q ; thus

$$Q_{\text{in}} + Q_{\text{out}} = Q, \quad Q_{\text{out}} = Q + q.$$

This charge is uniform over the outer area $4\pi r_2^2$. Thus

$$\sigma_{\text{out}} = \frac{Q + q}{4\pi r_2^2}.$$

Step 3: (b) Explain the field in an empty cavity In electrostatic equilibrium the cavity wall has constant potential V_0 , because the conductor is equipotential. Inside the empty cavity, $\nabla^2 V = 0$. The unique solution with this boundary potential is $V = V_0$ everywhere. Therefore $\mathbf{E} = -\nabla V = 0$. This holds for any shape. The empty cavity here differs from the charged cavity of part (a).

Step 4: Conclusion (a) $\sigma_{\text{in}} = -q/(4\pi r_1^2)$ and $\sigma_{\text{out}} = (Q + q)/(4\pi r_2^2)$. (b) The field in a closed charge-free cavity is zero for any conductor shape in electrostatic equilibrium.

Quick Tip to Solve: Use a Gaussian surface inside the metal to fix the inner charge first, then conserve the shell's charge to find the outer charge.

18. A signal of low frequency f_m is to be transmitted using a carrier wave of frequency f_c . Derive the expression for the amplitude modulated wave and deduce expressions for the lower and upper sidebands produced. Hence, obtain the expression for modulation index. [3 marks]

Solution:

Concept:

- In AM the carrier amplitude varies with the message signal while its frequency remains fixed.
- The modulation index $\mu = A_m/A_c$ compares the amplitude variation A_m to the unmodulated carrier amplitude A_c .
- Multiplying two sinusoids creates frequency components at their sum and difference: $2 \cos a \cos b = \cos(a + b) + \cos(a - b)$.

Step 1: Write the message, carrier and modulated signal Take the message and carrier as:

$$m(t) = A_m \cos(2\pi f_m t), \quad c(t) = A_c \cos(2\pi f_c t),$$

Here $f_c \gg f_m$, and modulator sensitivity is absorbed into A_m . Varying the carrier amplitude gives

$$\begin{aligned} s(t) &= [A_c + A_m \cos(2\pi f_m t)] \cos(2\pi f_c t) \\ &= A_c [1 + \mu \cos(2\pi f_m t)] \cos(2\pi f_c t). \end{aligned}$$

Step 2: Expand to identify the sidebands Apply the product-to-sum identity to the second term:

$$\begin{aligned} s(t) &= A_c \cos(2\pi f_c t) \\ &\quad + \frac{\mu A_c}{2} \cos[2\pi(f_c + f_m)t] \\ &\quad + \frac{\mu A_c}{2} \cos[2\pi(f_c - f_m)t]. \end{aligned}$$

The second term is the upper sideband at $f_U = f_c + f_m$; the third is the lower sideband at $f_L = f_c - f_m$. Each has amplitude $\mu A_c/2 = A_m/2$.

Step 3: Obtain the modulation index from the envelope For $0 \leq \mu \leq 1$, the envelope extrema are:

$$A_{\max} = A_c + A_m, \quad A_{\min} = A_c - A_m.$$

Subtracting and adding these equations gives $A_{\max} - A_{\min} = 2A_m$ and $A_{\max} + A_{\min} = 2A_c$. Their ratio gives μ . For $\mu > 1$, overmodulation occurs.

Step 4: Conclusion The AM wave is $s(t) = A_c [1 + \mu \cos(2\pi f_m t)] \cos(2\pi f_c t)$, with lower and upper sideband frequencies $f_c - f_m$ and $f_c + f_m$. The modulation index is

$$\mu = \frac{A_m}{A_c} = \frac{A_{\max} - A_{\min}}{A_{\max} + A_{\min}}.$$

Quick Tip to Solve: Expand the product of message and carrier cosines: the two new frequencies are immediately the sum and difference of their frequencies.

19. Draw a plot of α -particle scattering by a thin foil of gold to show the variation of the number of the scattered particles with scattering angle. Describe briefly how the large angle scattering explains the existence of the nucleus inside the atom. Explain with the help of impact parameter picture, how Rutherford scattering serves a powerful way to determine an upper limit on the size of the nucleus.

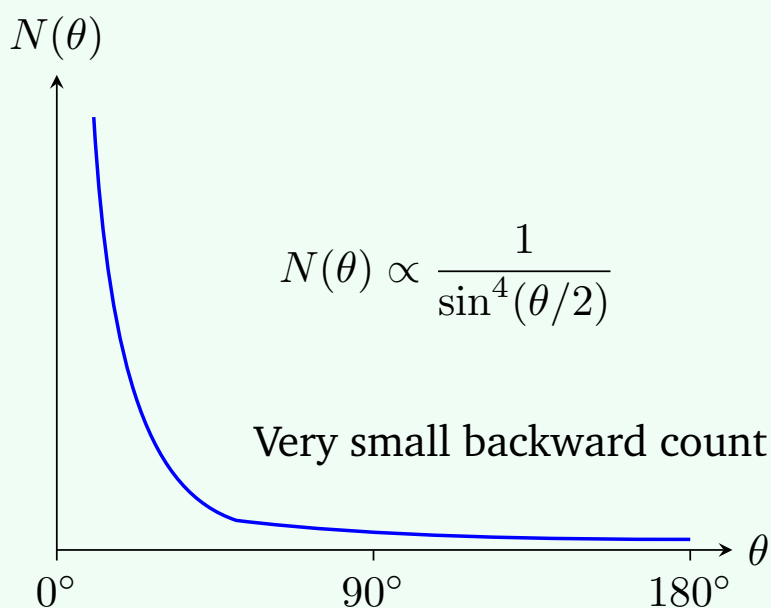
[3 marks]

Solution:

Concept:

- Rutherford scattering is the Coulomb repulsion of a positive α -particle of charge $2e$ by a heavy nucleus of charge Ze .
- For the same incident beam and equal detector solid angles, the scattered count obeys $N(\theta) \propto 1/\sin^4(\theta/2)$, where θ is the scattering angle.
- The impact parameter b is the perpendicular distance between the initial straight-line path and the nuclear centre; smaller b produces larger deflection.

Step 1: Plot the angular distribution The count is very large at small angles, decreases rapidly as θ increases, and is very small but nonzero for backward scattering. The sketch shows the Rutherford trend: most particles suffer little deflection, while only a very small fraction are scattered through large angles. The ideal point-charge formula diverges at zero angle, so the plotted curve starts at a small positive angle.



Qualitative scattered count for equal detector solid angles.

Step 2: Infer a compact nucleus from large-angle scattering Reversing the direction of a fast, massive α -particle requires a strong repulsive force in a close encounter. A diffuse distribution of positive charge cannot provide this large, concentrated impulse, and the light electrons cannot turn back such a heavy particle. Thus the positive charge and almost all the atomic mass must be concentrated in a tiny, massive nucleus. The rarity of these encounters shows that the nucleus occupies a very small fraction of the atom, while most of the atomic volume is empty space.

Quick Tip to Solve: For the tightest size bound use a head-on encounter: set the incident kinetic energy equal to the electrostatic potential energy at closest approach.

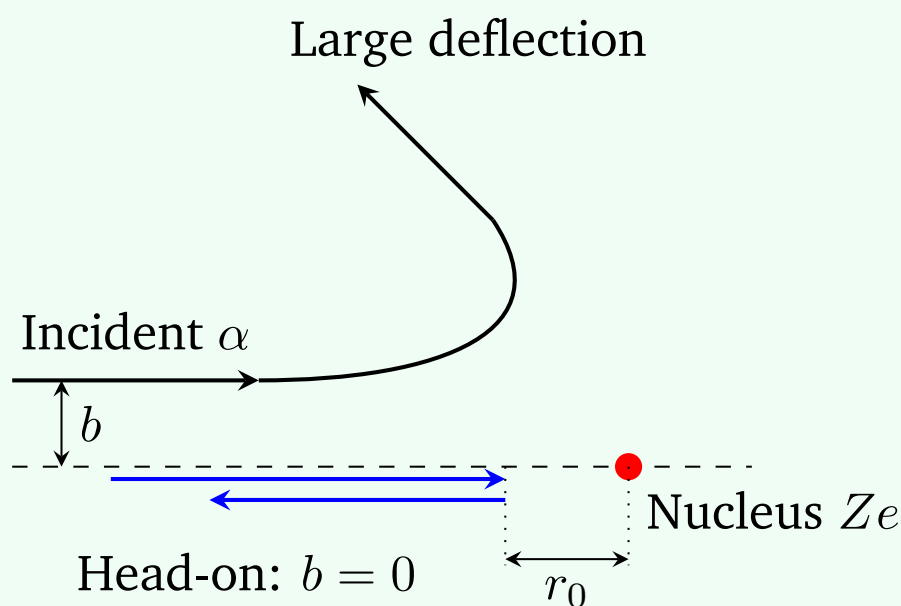
Step 3: Relate impact parameter to closest approach For incident kinetic energy K , Coulomb scattering gives

$$b = \frac{1}{4\pi\epsilon_0} \frac{Ze^2}{K} \cot \frac{\theta}{2}.$$

Thus $b \rightarrow 0$ gives $\theta \rightarrow 180^\circ$. In this head-on encounter, the particle momentarily stops at the closest distance r_0 before retracing its path. Neglecting recoil of the heavy gold nucleus, conservation of energy gives

$$K = \frac{1}{4\pi\epsilon_0} \frac{2Ze^2}{r_0}, \quad r_0 = \frac{1}{4\pi\epsilon_0} \frac{2Ze^2}{K}.$$

If Coulomb scattering remains valid at this energy, the particle turns around outside the nucleus, so its radius must be smaller than this distance. Increasing the incident energy probes a smaller closest distance and hence tightens the upper bound; for gold, $Z = 79$. This is an upper limit, not a direct equality for the nuclear radius.



Small impact parameter causes strong deflection; a head-on particle turns at r_0 .

Step 4: Conclusion The scattered count decreases sharply with angle. Rare large-angle events establish a small, massive, positively charged nucleus. Head-on scattering sets the bound $R_{\text{nucleus}} < r_0 = 2Ze^2/(4\pi\epsilon_0 K)$.

Quick Tip to Solve: For the tightest size bound use a head-on encounter: set the incident kinetic energy equal to the electrostatic potential energy at closest approach.

20. A $200\ \mu\text{F}$ parallel plate capacitor having plate separation of $5\ \text{mm}$ is charged by a $100\ \text{V}$ dc source. It remains connected to the source. Using an insulated handle, the distance between the plates is doubled and a dielectric slab of thickness $5\ \text{mm}$ and dielectric constant 10 is introduced between the plates. Explain with reason, how the (i) capacitance, (ii) electric field between the plates, (iii) energy density of the capacitor will change? [3 marks]

Solution:

Concept:

- A connected ideal source keeps the potential difference V constant. Layers normal to the field behave as capacitors in series.
- For air thickness a and dielectric thickness t , $C = \varepsilon_0 A / (a + t/\kappa)$, where A is plate area and κ is the dielectric constant.
- With no free charge at the air–dielectric interface, the normal displacement is continuous: $E_{\text{air}} = \kappa E_{\text{diel}}$. Local energy density is $u = \frac{1}{2} \varepsilon E^2$.

Step 1: (i) Calculate the effective separation and capacitance Initially $C_0 = 200\ \mu\text{F}$, $d = 5\ \text{mm}$ and $V = 100\ \text{V}$. The new separation is $2d = 10\ \text{mm}$, of which the dielectric occupies $t = d$, leaving air thickness $a = d$. With $\kappa = 10$,

$$C' = \frac{\varepsilon_0 A}{d + d/10} = \frac{10}{11} C_0 = 181.8\ \mu\text{F}.$$

The added air gap outweighs the slab's enhancement: the effective separation increases from 5 to $5.5\ \text{mm}$, so capacitance decreases.

Step 2: (ii) Distinguish the fields in the two layers The initial uniform field is

$$E_0 = \frac{V}{d} = 2.00 \times 10^4\ \text{V m}^{-1}.$$

The final field is not uniform throughout the gap. Applying $V = E_{\text{air}}d + E_{\text{diel}}d$ with $E_{\text{air}} = 10E_{\text{diel}}$,

$$E_{\text{air}} = \frac{10}{11} E_0 = 1.818 \times 10^4\ \text{V m}^{-1},$$

$$E_{\text{diel}} = \frac{1}{11} E_0 = 1.818 \times 10^3\ \text{V m}^{-1}.$$

Both local fields decrease. The potential drop per total separation, or spatially averaged field, is $V/(2d) = E_0/2 = 1.00 \times 10^4\ \text{V m}^{-1}$; this average must not be used as either local field.

Step 3: (iii) Calculate local and average energy densities Initially $u_0 = \frac{1}{2} \varepsilon_0 E_0^2 = 1.77 \times 10^{-3}\ \text{J m}^{-3}$. Using the appropriate permittivity in each region,

$$u_{\text{air}} = \frac{1}{2} \varepsilon_0 E_{\text{air}}^2 = \frac{100}{121} u_0,$$

$$u_{\text{diel}} = \frac{1}{2} (10\varepsilon_0) E_{\text{diel}}^2 = \frac{10}{121} u_0.$$

Thus both local energy densities decrease. Since the two layers have equal volumes, the energy per

Quick Tip to Solve: For a slab that only partly fills the gap, first add the air thickness to the dielectric thickness divided by its dielectric constant; then calculate each layer's field separately.

total gap volume is

$$\bar{u}' = \frac{u_{\text{air}} + u_{\text{diel}}}{2} = \frac{5}{11}u_0 = 8.05 \times 10^{-4} \text{ J m}^{-3}.$$

Equivalently, stored energy $U = \frac{1}{2}CV^2$ becomes $(10/11)U_0$, while the gap volume doubles, giving the same average-density ratio.

Step 4: Conclusion Capacitance decreases to $181.8 \mu\text{F}$. The air and dielectric fields become $(10/11)E_0$ and $E_0/11$, respectively. Their energy densities become $(100/121)u_0$ and $(10/121)u_0$; the average over the whole capacitor gap becomes $(5/11)u_0$.

Quick Tip to Solve: For a slab that only partly fills the gap, first add the air thickness to the dielectric thickness divided by its dielectric constant; then calculate each layer's field separately.

21. Why is it difficult to detect the presence of an anti-neutrino during β -decay? Define the term decay constant of a radioactive nucleus and derive the expression for its mean life in terms of the decay constant. [3 marks]

Solution:

Concept:

- In β^- -decay, a neutron changes into a proton, electron and electron antineutrino: $n \rightarrow p + e^- + \bar{\nu}_e$.
- The decay constant λ is the probability per unit time that an undecayed nucleus decays; equivalently, $\lambda = -\frac{1}{N} \frac{dN}{dt}$. Its SI unit is s^{-1} .
- For identical radioactive nuclei with constant λ , the survival law is $N(t) = N_0 e^{-\lambda t}$, with N_0 the initial number of nuclei.

Step 1: Explain why an antineutrino is difficult to detect An antineutrino has no electric charge and interacts with matter extremely weakly. It therefore does not leave the readily observable ionisation track made by a charged beta particle. Its chance of interacting in an ordinary detector is very small, so most antineutrinos pass through large amounts of matter without being detected. Its very small mass does not mean its energy is zero; it carries away part of the decay energy and momentum. Rare weak interactions, rather than direct ionisation, provide the means of detection.

Step 2: Define the decay constant and obtain the survival law For a short interval dt , the number of decays is $-dN = \lambda N dt$. Thus the decay constant measures the fractional decay rate of the nuclei still present, not the total activity of the sample. Separating variables and using $N = N_0$ at $t = 0$,

$$\int_{N_0}^N \frac{dN}{N} = -\lambda \int_0^t dt, \quad N(t) = N_0 e^{-\lambda t}.$$

Step 3: Derive the mean life by averaging the decay times Mean life τ is the average time a nucleus survives before decaying, measured from the initial instant. The positive number of nuclei decaying between t and $t + dt$ is $\lambda N_0 e^{-\lambda t} dt$. Weighting each decay time by this number and dividing by N_0 gives

$$\tau = \frac{1}{N_0} \int_0^\infty t \lambda N_0 e^{-\lambda t} dt = \lambda \int_0^\infty t e^{-\lambda t} dt.$$

Quick Tip to Solve: To derive mean life, average the decay time using the positive decay count $-dN$, rather than the number $N(t)$ still surviving at that time.

Integration by parts yields

$$\begin{aligned}\tau &= \left[-te^{-\lambda t} \right]_0^\infty + \int_0^\infty e^{-\lambda t} dt \\ &= 0 + \left[-\frac{e^{-\lambda t}}{\lambda} \right]_0^\infty = \frac{1}{\lambda}.\end{aligned}$$

The boundary term vanishes because exponential decay dominates the factor t . The reciprocal of a decay rate has units of time, as a mean life must.

Step 4: Conclusion Antineutrinos are difficult to detect because they are electrically neutral and interact only very weakly with matter. The decay constant is the probability of decay per unit time for an undecayed nucleus, and its mean life is $\tau = 1/\lambda$.

Quick Tip to Solve: To derive mean life, average the decay time using the positive decay count $-dN$, rather than the number $N(t)$ still surviving at that time.

OR

21. (a) State two distinguishing features of nuclear force.

(b) Draw a plot showing the variation of potential energy of a pair of nucleons as a function of their separation. Mark the regions on the graph where the force is (i) attractive, and (ii) repulsive.

[3 marks]

Solution:

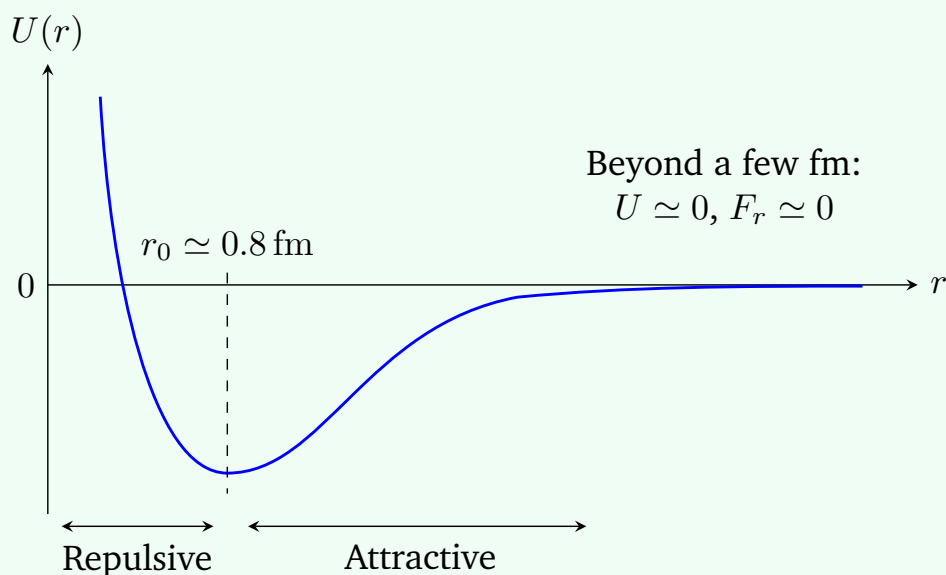
Concept:

- Nuclear force is the strong interaction binding protons and neutrons in a nucleus; its effective range is only a few femtometres, where $1 \text{ fm} = 10^{-15} \text{ m}$.
- For a pair of nucleons with separation r , the radial force follows from the potential energy $U(r)$ as $F_r = -dU/dr$. A positive radial force increases the separation.

Step 1: State two distinguishing features First, the nuclear force is extremely strong but short-ranged: it binds neighbouring nucleons strongly, while becoming negligible when their separation exceeds a few femtometres. It therefore cannot act over macroscopic distances like gravitational or electrostatic forces. Second, it is approximately charge independent: after excluding the Coulomb interaction, the nuclear interaction between proton–proton, neutron–neutron and proton–neutron pairs is approximately the same in corresponding states. Thus nuclear attraction is not an electrical attraction between opposite charges.

Step 2: Sketch and interpret the potential-energy curve Choose $U = 0$ for widely separated nucleons. As they approach, attraction lowers their potential energy below zero; the curve has a negative minimum at a separation r_0 , roughly 0.8 fm . At much smaller separation, the repulsive core makes the potential rise steeply. To identify the force, use the slope of the curve rather than the sign of U . For $r < r_0$, the slope $dU/dr < 0$, so $F_r > 0$: the force is repulsive. For $r > r_0$ within the nuclear range, $dU/dr > 0$, so $F_r < 0$: the force is attractive. At the minimum the slope and net radial force are zero; far away, both the potential and nuclear force approach zero.

Quick Tip to Solve: Mark attraction and repulsion using $F_r = -dU/dr$; a negative potential alone does not tell you the direction of the force.



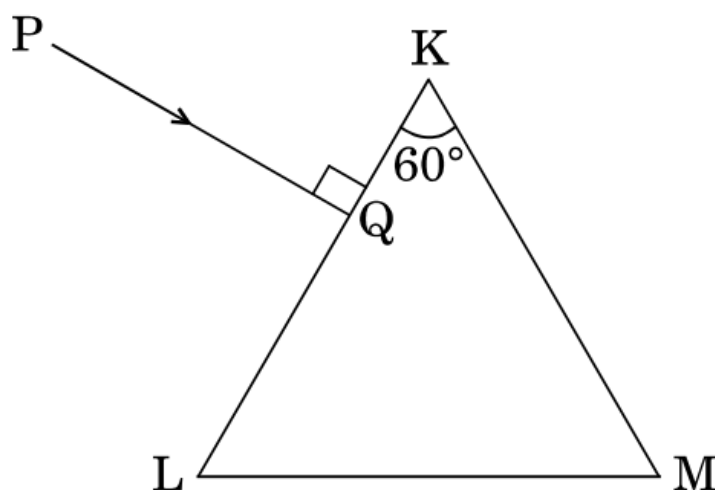
Schematic nucleon potential: force is set by the negative slope.

Step 3: Conclusion Nuclear force is strong and short-ranged, and approximately charge independent. The potential has a negative well, with a repulsive region to the left of its minimum and an attractive region to the right within the nuclear range.

Quick Tip to Solve: Mark attraction and repulsion using $F_r = -dU/dr$; a negative potential alone does not tell you the direction of the force.

22. A triangular prism of refracting angle 60° is made of a transparent material of refractive index $2/\sqrt{3}$. A ray of light is incident normally on the face KL as shown in the figure. Trace the path of the ray as it passes through the prism and calculate the angle of emergence and angle of deviation.

[3 marks]



Solution:

Concept:

- Snell's law is $n_1 \sin i = n_2 \sin r$, where the angles are measured from the normal. A normally incident ray suffers no change in direction.
- In a prism, the internal refraction angles obey $r_1 + r_2 = A$, and the deviation is $\delta = i + e - A$. Here A is the prism angle and e is the emergence angle relative to the exit-face normal.

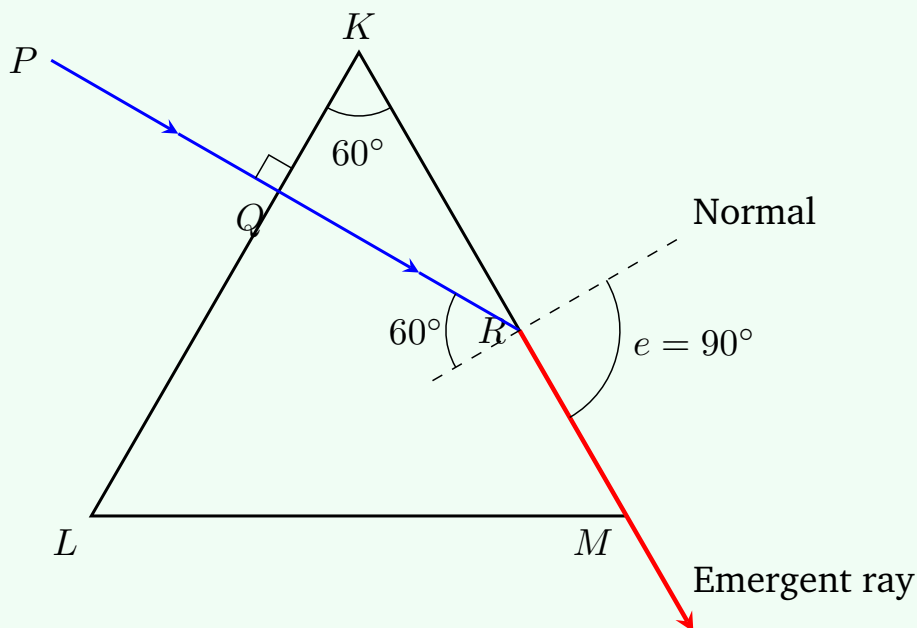
Step 1: Trace the ray through the first face The ray PQ strikes face KL normally, so its incidence angle is $i = 0^\circ$. Applying Snell's law at the air–prism boundary gives $\sin r_1 = 0$, hence $r_1 = 0^\circ$. The ray therefore continues straight along QR inside the prism and meets the second face KM at R . The angle between the faces is $A = 60^\circ$, so the internal incidence angle at the second face is

$$r_2 = A - r_1 = 60^\circ - 0^\circ = 60^\circ.$$

Step 2: Apply refraction at the second face Taking the refractive index of air as unity, Snell's law at KM gives

$$\sin e = \mu \sin r_2 = \frac{2}{\sqrt{3}} \sin 60^\circ = \frac{2}{\sqrt{3}} \frac{\sqrt{3}}{2} = 1.$$

This is the critical-incidence condition because $\sin C = 1/\mu = \sqrt{3}/2$, so $C = 60^\circ = r_2$. The refracted ray therefore grazes face KM towards M . Total internal reflection would require an incidence angle greater than C ; equality gives emergence along the surface. In the diagram the dashed line at R is the normal and the outgoing ray lies along the face.



Normal entry at Q and grazing emergence at R .

Step 3: Calculate the deviation Emergence is measured from the normal, so the grazing emergent ray has $e = 90^\circ$. Substituting in the prism deviation relation,

$$\delta = i + e - A = 0^\circ + 90^\circ - 60^\circ = 30^\circ.$$

Quick Tip to Solve: If Snell's law gives $\sin e = 1$, draw a grazing ray along the exit face and use $e = 90^\circ$ in the deviation formula.

There is no deviation at entry; the entire change in direction occurs at the second face, where the ray bends away from the normal on entering air.

Step 4: Conclusion The ray enters KL undeviated and emerges grazing KM towards M . Its angle of emergence is $e = 90^\circ$ and its angle of deviation is $\delta = 30^\circ$.

Quick Tip to Solve: If Snell's law gives $\sin e = 1$, draw a grazing ray along the exit face and use $e = 90^\circ$ in the deviation formula.

23. Prove that in a common-emitter amplifier, the output and input differ in phase by 180° .

In a transistor, the change of base current by $30 \mu\text{A}$ produces change of 0.02 V in the base-emitter voltage and a change of 4 mA in the collector current. Calculate the current amplification factor and the load resistance used, if the voltage gain of the amplifier is 400. [3 marks]

Solution:

Concept:

- For a common-emitter amplifier, the small-signal current gain is $\beta_{ac} = \Delta I_C / \Delta I_B$, where I_C and I_B are collector and base currents.
- The input resistance is $r_i = \Delta V_{BE} / \Delta I_B$. For an emitter at a fixed potential, $V_{CE} = V_{CC} - I_C R_L$, giving $A_v = \Delta V_{CE} / \Delta V_{BE} = -\beta_{ac} R_L / r_i$.

Step 1: Establish the phase reversal Consider a transistor biased in its active region, with the emitter common to the input and output circuits and a constant collector supply V_{CC} . During the positive half-cycle of the input signal, V_{BE} increases. The base current increases, and the collector current increases by $\Delta I_C = \beta_{ac} \Delta I_B$. The larger collector current increases the voltage drop across R_L . Applying the collector-loop equation,

$$V_{CE} = V_{CC} - I_C R_L, \quad \Delta V_{CE} = -R_L \Delta I_C.$$

Thus the collector output voltage decreases when the input voltage increases. During the negative half-cycle the collector current decreases and the output voltage increases. For a sinusoidal signal in the usual small-signal operating range, the output is proportional to the negative of the input; since $-\sin \omega t = \sin(\omega t + \pi)$, their phase difference is 180° .

Step 2: Calculate the current amplification factor Convert both current changes to amperes before dividing:

$$\beta_{ac} = \frac{4 \times 10^{-3}}{30 \times 10^{-6}} = \frac{400}{3} \simeq 133.3.$$

This is a dimensionless gain because it is a ratio of currents. The corresponding input resistance is

$$r_i = \frac{0.02}{30 \times 10^{-6}} = \frac{2000}{3} \Omega \simeq 666.7 \Omega.$$

Step 3: Determine the load resistance The stated voltage gain of 400 is its magnitude; the signed

Quick Tip to Solve: Find the load directly from $R_L = |A_v| \Delta V_{BE} / \Delta I_C$; use the negative gain sign separately to explain phase reversal.

common-emitter gain is negative because of the phase reversal. Using the magnitude relation,

$$|A_v| = \frac{\beta_{ac} R_L}{r_i}, \quad R_L = \frac{|A_v| r_i}{\beta_{ac}} = \frac{400(2000/3)}{400/3} = 2000 \Omega.$$

An independent direct check uses the output-voltage change: $|\Delta V_{CE}| = 400(0.02) = 8 \text{ V}$, so $R_L = 8/(4 \times 10^{-3}) = 2000 \Omega$, consistent with the gain formula.

Step 4: Conclusion The common-emitter output is 180° out of phase with the input. The current amplification factor is $\beta_{ac} \simeq 133.3$, and the load resistance is $R_L = 2.0 \text{ k}\Omega$.

Quick Tip to Solve: Find the load directly from $R_L = |A_v| \Delta V_{BE} / \Delta I_C$; use the negative gain sign separately to explain phase reversal.

24. Show, on a plot, variation of resistivity of (i) a conductor, and (ii) a typical semiconductor as a function of temperature.

Using the expression for the resistivity in terms of number density and relaxation time between the collisions, explain how resistivity in the case of a conductor increases while it decreases in a semiconductor, with the rise of temperature. [3 marks]

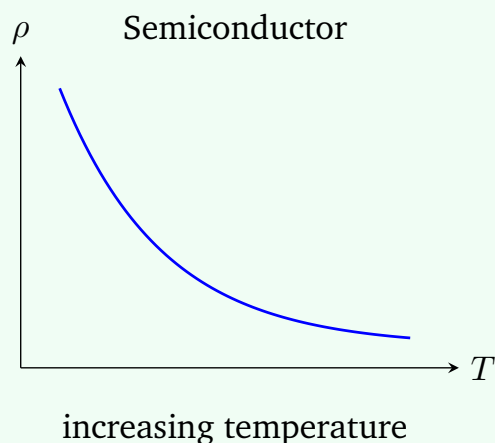
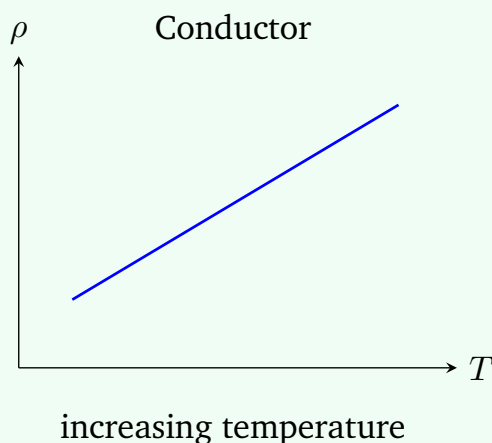
Solution:

Concept:

- For carriers of number density n , charge magnitude e , effective mass m and mean relaxation time τ , the Drude expression is $\rho = m/(ne^2\tau)$. Thus resistivity depends on both the carrier population and the collision rate.
- A metallic conductor has an approximately fixed density of free electrons. A typical semiconductor acquires many additional mobile electrons and holes as its temperature rises.

Step 1: Sketch the temperature dependence The qualitative plots below show the usual temperature ranges: metallic resistivity rises approximately linearly, whereas semiconductor resistivity falls nonlinearly. The axes are schematic; the two materials need not have comparable numerical resistivities. For a metal over a moderate temperature interval, $\rho(T) \simeq \rho(T_0)[1 + \alpha(T - T_0)]$, where $\alpha > 0$. The decreasing semiconductor curve corresponds to a negative temperature coefficient.

Quick Tip to Solve: In $\rho = m/(ne^2\tau)$, hold n nearly fixed for a metal; for a semiconductor, explicitly include the strong thermal rise in carrier density.



Qualitative resistivity-temperature plots over ordinary temperature ranges

Step 2: Explain the increase for a conductor As temperature rises, the ions of the metallic lattice vibrate more vigorously, causing more frequent scattering of conduction electrons. The mean time τ between collisions therefore decreases. Since n is practically independent of temperature for a metal, the relation

$$\rho = \frac{m}{ne^2\tau} \propto \frac{1}{\tau}$$

shows that its resistivity increases. The effect is due to reduced mobility, rather than a substantial loss of free electrons.

Step 3: Explain the decrease for a semiconductor Heating thermally excites electrons across the energy gap, creating electron-hole pairs. The number density of mobile carriers increases rapidly. Although increased scattering tends to reduce relaxation times and mobility, the rise in carrier density dominates for the typical semiconductor considered here. Hence the effective product $n\tau$ increases and $\rho = m/(ne^2\tau)$ decreases. More generally, accounting for both carrier types,

$$\sigma = e(n_e\mu_e + n_h\mu_h), \quad \rho = \frac{1}{\sigma},$$

where n_e, n_h are electron and hole densities and μ_e, μ_h their mobilities. Thermal carrier generation raises the total conductivity despite the reduction in mobility.

Step 4: Conclusion A conductor has $d\rho/dT > 0$ because its relaxation time decreases while carrier density remains nearly constant. A typical semiconductor has $d\rho/dT < 0$ because the increase in mobile carrier density outweighs the decrease in relaxation time.

Quick Tip to Solve: In $\rho = m/(ne^2\tau)$, hold n nearly fixed for a metal; for a semiconductor, explicitly include the strong thermal rise in carrier density.

Section D (15 marks)

25. (a) Derive an expression for the induced emf developed when a coil of N turns, and area of cross-section A , is rotated at a constant angular speed ω in a uniform magnetic field B .

(b) A wheel with 100 metallic spokes each 0.5 m long is rotated with a speed of 120 rev/min in a plane normal to the horizontal component of the Earth's magnetic field. If the resultant magnetic field at that place is 4×10^{-4} T and the angle of dip at the place is 30° , find the emf induced between the axle and the rim of the wheel. [5 marks]

Solution:

Concept:

- Faraday's law gives the induced emf as $\mathcal{E} = -d\Lambda/dt$, where $\Lambda = N\Phi$ is the flux linkage of a coil of N turns. The negative sign expresses Lenz's law.
- Magnetic flux through a planar turn is $\Phi = BA \cos \theta$, with θ the angle between the magnetic field and the normal to the coil. A rotating radial conductor develops motional emf $d\mathcal{E} = B_\perp v dr$.

Step 1: (a) Express the changing flux linkage Let the coil rotate about an axis in its plane perpendicular to the uniform magnetic field. Take $t = 0$ when the coil's normal is parallel to $\vec{B}$, so that $\theta = \omega t$. Each turn has area A , and all N turns link the same flux. Therefore

$$\Phi(t) = BA \cos(\omega t), \quad \Lambda(t) = NBA \cos(\omega t).$$

The flux varies even though B and A remain constant, because the coil's orientation changes continuously.

Step 2: (a) Apply Faraday's law Differentiate the flux linkage with respect to time:

$$\begin{aligned} \mathcal{E}(t) &= -\frac{d}{dt}[NBA \cos(\omega t)] \\ &= NBA\omega \sin(\omega t). \end{aligned}$$

Thus the induced emf is sinusoidal, with peak magnitude $\mathcal{E}_0 = NBA\omega$. It is zero when the flux is instantaneously maximum or minimum, and has its greatest magnitude when the coil's normal is perpendicular to the field. The sign reverses every half-turn. Choosing another initial orientation introduces a phase constant: $\mathcal{E} = \mathcal{E}_0 \sin(\omega t + \phi)$.

Step 3: (b) Find the relevant field and angular speed The wheel rotates in a plane normal to Earth's horizontal magnetic component. Therefore only B_H , the component perpendicular to the wheel's plane, contributes to the axle-to-rim emf. With resultant field B_E and angle of dip δ ,

$$B_H = B_E \cos \delta = 4 \times 10^{-4} \cos 30^\circ = 2\sqrt{3} \times 10^{-4} \text{ T}.$$

The vertical field component lies in the wheel's plane and makes no contribution to the radial motional emf. The rotation frequency and angular speed are

$$f = \frac{120}{60} = 2 \text{ s}^{-1}, \quad \omega = 2\pi f = 4\pi \text{ rad s}^{-1}.$$

Quick Tip to Solve: For a rotating spoke use $B_\perp \omega l^2/2$; convert rev/min to rad/s and remember that spokes sharing an axle and rim are in parallel.

Step 4: (b) Integrate the motional emf along a spoke At distance r from the axle, a point on the spoke has tangential speed $v = \omega r$. The magnetic force separates charges radially, so an element of length dr contributes emf magnitude $d\mathcal{E} = B_H \omega r dr$. Integrating from the axle to the rim, with spoke length $l = 0.5$ m, gives

$$\begin{aligned} |\mathcal{E}| &= \int_0^l B_H \omega r dr = \frac{1}{2} B_H \omega l^2 \\ &= \frac{1}{2} (2\sqrt{3} \times 10^{-4}) (4\pi) (0.5)^2 \text{ V} \\ &= \pi\sqrt{3} \times 10^{-4} \text{ V}. \end{aligned}$$

All 100 spokes join the same axle and rim, so they act as parallel emf sources of the same polarity. Their emfs are not added; the number of spokes does not multiply the potential difference. Since the direction of rotation is unspecified, only the magnitude, rather than which terminal is positive, is determined.

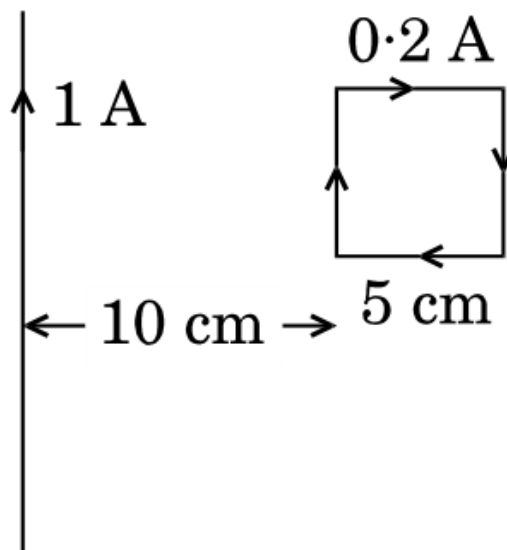
Step 5: Conclusion (a) The induced emf is $\mathcal{E}(t) = NBA\omega \sin(\omega t)$, for the chosen initial orientation, with peak emf $NBA\omega$. (b) The magnitude of the axle-to-rim emf is $5.44 \times 10^{-4} \text{ V} = 0.544 \text{ mV}$.

Quick Tip to Solve: For a rotating spoke use $B_{\perp} \omega l^2 / 2$; convert rev/min to rad/s and remember that spokes sharing an axle and rim are in parallel.

OR

25. (a) Derive the expression for the magnetic energy stored in an inductor when a current I develops in it. Hence, obtain the expression for the magnetic energy density.

(b) A square loop of sides 5 cm carrying a current of 0.2 A in the clockwise direction is placed at a distance of 10 cm from an infinitely long wire carrying a current of 1 A as shown. Calculate (i) the resultant magnetic force, and (ii) the torque, if any, acting on the loop. [5 marks]



Solution:

Concept:

- An inductor of constant self-inductance L opposes a changing current i by the back emf $\varepsilon = -L di/dt$. Work done against this emf is stored as magnetic energy.
- For a long air-core solenoid, $L = \mu_0 n^2 A \ell$ and $B = \mu_0 n I$, where n is turns per unit length, A the cross-sectional area and ℓ the length.
- A long straight wire carrying current I_w produces $B(r) = \mu_0 I_w / (2\pi r)$. A loop element carrying current I_s experiences $d\vec{F} = I_s d\vec{\ell} \times \vec{B}$.

Step 1: (a) Integrate the work done against the back emf Let the current grow from zero to I . The external source must supply an opposing voltage $L di/dt$ across the ideal inductive part. The rate of energy storage is therefore

$$\frac{dU}{dt} = iL \frac{di}{dt}, \quad dU = Li di.$$

Taking zero stored energy at zero current and integrating for constant L ,

$$U = \int_0^I Li di = \frac{1}{2} LI^2.$$

This is the energy stored in the field; winding resistance, if present, dissipates additional energy as heat.

Step 2: (a) Express the energy per unit volume in terms of the field Consider the approximately uniform field inside a long air-core solenoid, neglecting its end effects. Its field-filled volume is $A\ell$. Substituting its inductance in the energy expression gives

$$U = \frac{1}{2} \mu_0 n^2 A \ell I^2.$$

Hence the energy density is

$$u_B = \frac{U}{A\ell} = \frac{1}{2} \mu_0 n^2 I^2 = \frac{B^2}{2\mu_0},$$

measured in J m^{-3} , for vacuum or approximately for air.

Step 3: (b)(i) Determine the forces on the two vertical sides From the printed figure, the near side is at $a = 0.10 \text{ m}$ from the wire and the far side is at $a + s = 0.15 \text{ m}$, where $s = 0.05 \text{ m}$. The wire current is $I_w = 1 \text{ A}$ upward and the clockwise loop current is $I_s = 0.2 \text{ A}$. The field at the loop points into the page.

The current in the near vertical side is upward, parallel to the wire, so this side is attracted towards the wire. The far-side current is downward, so that side is repelled. Their force magnitudes are

$$F_{\text{near}} = \frac{\mu_0 I_w I_s s}{2\pi a} = \frac{2 \times 10^{-7} (1)(0.2)(0.05)}{0.10} = 2.0 \times 10^{-8} \text{ N},$$
$$F_{\text{far}} = \frac{\mu_0 I_w I_s s}{2\pi (a + s)} = \frac{2 \times 10^{-7} (1)(0.2)(0.05)}{0.15} = 1.33 \times 10^{-8} \text{ N}.$$

Step 4: (b)(i) Combine all four side forces The top horizontal side experiences an upward force and

Quick Tip to Solve: Use the distances to the near and far sides, 0.10 m and 0.15 m , separately. Cancel the horizontal-side forces pairwise before subtracting the vertical-side forces.

the bottom horizontal side experiences a downward force. At each distance r from the wire their current elements have equal lengths and opposite directions in the same magnetic field. These forces cancel pairwise, even though the field varies along each horizontal side.

Thus the resultant comes from the unequal forces on the vertical sides:

$$F_{\text{net}} = \frac{\mu_0 I_w I_s s}{2\pi} \left(\frac{1}{a} - \frac{1}{a+s} \right)$$

$$= 2 \times 10^{-9} \left(10 - \frac{1}{0.15} \right) = 6.67 \times 10^{-9} \text{ N}.$$

The near-side attraction is stronger, so the resultant is directed towards the straight wire.

Step 5: (b)(ii) Check the torque about the loop's centre Each vertical side has a uniform force per unit length because its distance from the wire is fixed. Its resultant acts at the side's midpoint along a horizontal line through the loop's centre, giving zero moment about that centre.

Corresponding force elements on the top and bottom sides are equal, opposite and collinear along the same vertical line. Their moments cancel pairwise. Therefore the total torque about the centre is zero. This also agrees with the clockwise loop's magnetic moment pointing into the page, parallel to the external field; the explicit force check remains valid despite the field being nonuniform.

Step 6: Conclusion The magnetic energy is $U = \frac{1}{2} LI^2$, and its density in air is $u_B = B^2/(2\mu_0)$. The square loop experiences $F = 6.67 \times 10^{-9} \text{ N}$ towards the straight wire and $\tau = 0 \text{ N m}$ about its centre.

Quick Tip to Solve: Use the distances to the near and far sides, 0.10 m and 0.15 m, separately. Cancel the horizontal-side forces pairwise before subtracting the vertical-side forces.

26. Explain, with the help of a diagram, how plane polarized light can be produced by scattering of light from the Sun.

Two polaroids P_1 and P_2 are placed with their pass axes perpendicular to each other. Unpolarised light of intensity I is incident on P_1 . A third polaroid P_3 is kept between P_1 and P_2 such that its pass axis makes an angle of 45° with that of P_1 . Calculate the intensity of light transmitted through P_1 , P_2 and P_3 . [5 marks]

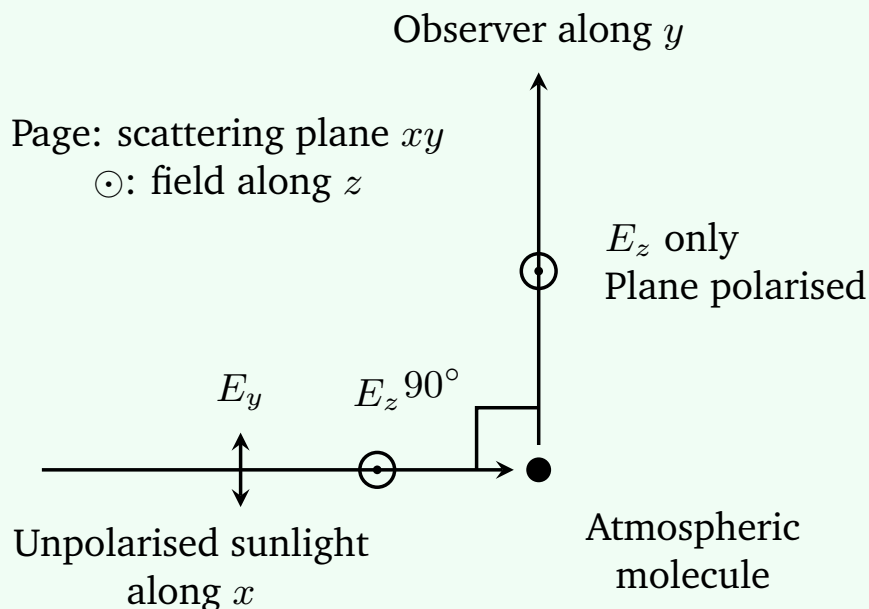
Solution:

Concept:

- Sunlight is unpolarised: its electric field has all possible directions perpendicular to its direction of propagation. It can be resolved into two mutually perpendicular transverse components.
- In scattering, the incident electric field makes the bound electrons of an atmospheric molecule oscillate. An oscillating electric dipole does not radiate along its own axis, but radiates most strongly perpendicular to that axis.
- An ideal polaroid transmits half the intensity of unpolarised incident light. For linearly polarised light, Malus's law is $I_{\text{out}} = I_{\text{in}} \cos^2 \theta$, where θ is the angle between its electric field and the transmission axis.

Quick Tip to Solve: Write the physical order as $P_1 \rightarrow P_3 \rightarrow P_2$ before applying Malus's law; the labels in the question's final list are not the order of passage.

Step 1: Resolve the incident sunlight into two components Let sunlight travel along the x -axis and let an observer look at the scattered light along the y -axis, perpendicular to the incident beam. The xy -plane is the scattering plane. The incident electric field has components along y and z , so the electrons in an atmospheric molecule are driven to oscillate in these directions. The diagram shows the scattering plane as the plane of the page; z is perpendicular to the page.



Scattering through 90° : only the electric field perpendicular to the scattering plane reaches the observer.

Step 2: Identify the polarisation of the light scattered at a right angle The electron oscillation along y cannot send radiation to the observer along y , because a dipole does not radiate along its axis. The oscillation along z , however, radiates towards this observer, with its electric field along z . Thus the light scattered through 90° has only the electric-field component perpendicular to the scattering plane and is plane polarised. This explains the polarisation of sunlight scattered by atmospheric molecules when viewed at right angles to the incident sunlight. Scattering in other directions generally gives partially polarised light.

Step 3: Find the intensity immediately after the first polaroid The incident light of intensity I is unpolarised. Averaging the squared electric-field projection onto the pass axis of P_1 gives $\langle \cos^2 \theta \rangle = 1/2$. Therefore the intensity emerging from P_1 is

$$I_1 = \frac{I}{2}.$$

This light is now polarised along the pass axis of P_1 . The actual order of passage is P_1 , then P_3 , then P_2 .

Step 4: Apply Malus's law at the intermediate polaroid The pass axis of P_3 is at 45° to the polarisation direction of the light arriving from P_1 . Hence

$$I_3 = I_1 \cos^2 45^\circ = \frac{I}{2} \times \frac{1}{2} = \frac{I}{4}.$$

Quick Tip to Solve: Write the physical order as $P_1 \rightarrow P_3 \rightarrow P_2$ before applying Malus's law; the labels in the question's final list are not the order of passage.

The transmitted light is polarised along the pass axis of P_3 , so this becomes the incident polarisation direction for the final polaroid.

Step 5: Apply Malus's law at the final polaroid The axes of P_1 and P_2 are perpendicular. Consequently the angle between the axes of P_3 and P_2 is also 45° . Using the intensity that actually reaches P_2 ,

$$I_2 = I_3 \cos^2 45^\circ = \frac{I}{4} \times \frac{1}{2} = \frac{I}{8}.$$

The intermediate polaroid changes the polarisation direction, allowing nonzero transmission through the final polaroid even though the outer two polaroids have perpendicular pass axes.

Step 6: Conclusion Sunlight scattered through 90° is plane polarised with its electric field perpendicular to the scattering plane. In the order requested, the intensities transmitted through P_1 , P_2 and P_3 are respectively $I/2$, $I/8$, $I/4$.

Quick Tip to Solve: Write the physical order as $P_1 \rightarrow P_3 \rightarrow P_2$ before applying Malus's law; the labels in the question's final list are not the order of passage.

OR

26. (a) Why cannot the phenomenon of interference be observed by illuminating two pin holes with two sodium lamps?

(b) Two monochromatic waves having displacements $y_1 = a \cos \omega t$ and $y_2 = a \cos(\omega t + \phi)$ from two coherent sources interfere to produce an interference pattern. Derive the expression for the resultant intensity and obtain the conditions for constructive and destructive interference.

(c) Two wavelengths of sodium light of 590 nm and 596 nm are used in turn to study the diffraction taking place at a single slit of aperture 2×10^{-6} m. If the distance between the slit and the screen is 1.5 m, calculate the separation between the positions of the second maxima of diffraction pattern obtained in the two cases.

[5 marks]

Solution:

Concept:

- A sustained interference pattern requires coherent sources: their frequency is the same and their phase difference remains constant during observation.
- Superposition adds displacements. For the same medium, intensity is proportional to the square of amplitude; write $I_0 = K a^2$ for either incident wave, with K the common proportionality constant.
- In the usual school-level approximation, the second secondary maximum of single-slit diffraction is near $x_2 = 5\lambda D/(2b)$, where b is slit width and D is screen distance.

Step 1: (a) Examine the coherence of the two lamps The two sodium lamps emit light through independent atomic emission events. Even if their dominant frequencies are equal, their relative phase changes randomly and rapidly. Thus the two illuminated pin holes do not act as coherent sources. The bright and dark positions change too rapidly to produce a stationary visible pattern. On time averaging, the interference term vanishes and the observed intensity is simply the sum of

Quick Tip to Solve: For the second secondary diffraction maximum use the factor $5/2$, and substitute $\Delta\lambda$ directly to find the separation of corresponding maxima.

the individual intensities. Using two pin holes illuminated by the same narrow source would instead maintain the required phase relationship.

Step 2: (b) Add the two wave displacements Apply the trigonometric identity for the sum of two cosines to the given waves:

$$\begin{aligned} y &= a \cos \omega t + a \cos(\omega t + \phi) \\ &= 2a \cos \frac{\phi}{2} \cos \left(\omega t + \frac{\phi}{2} \right). \end{aligned}$$

Hence the magnitude of the resultant amplitude is $A = 2a |\cos(\phi/2)|$. Squaring the amplitude gives

$$\begin{aligned} I &= KA^2 = 4Ka^2 \cos^2 \frac{\phi}{2} \\ &= 4I_0 \cos^2 \frac{\phi}{2} = 2I_0(1 + \cos \phi). \end{aligned}$$

The last expression contains the sum $2I_0$ of the individual intensities and the interference term $2I_0 \cos \phi$.

Step 3: (b) Obtain the bright and dark fringe conditions Constructive interference occurs when the two displacements are in phase, so $\cos \phi = 1$. Therefore

$$\phi = 2m\pi, \quad \Delta = m\lambda, \quad I_{\max} = 4I_0.$$

Destructive interference occurs when the two equal-amplitude displacements are opposite in phase, so $\cos \phi = -1$. Therefore

$$\phi = (2m + 1)\pi, \quad \Delta = \frac{(2m + 1)\lambda}{2}, \quad I_{\min} = 0.$$

Here m is an integer and Δ is the equivalent path difference, with $\phi = 2\pi\Delta/\lambda$. The zero minimum follows from the equal amplitudes specified in the question.

Step 4: (c) Compare the second secondary maxima The second secondary maximum lies approximately midway between the second and third minima, giving $b \sin \theta_2 \simeq 5\lambda/2$. With the usual paraxial relation $x \simeq D \sin \theta$, its position is $x_2 \simeq 5D\lambda/(2b)$. Use corresponding maxima on the same side of the central maximum. Their separation depends on the wavelength difference:

$$\begin{aligned} \Delta\lambda &= (596 - 590) \text{ nm} = 6 \times 10^{-9} \text{ m}, \\ \Delta x_2 &\simeq \frac{5D\Delta\lambda}{2b} \\ &= \frac{5(1.5)(6 \times 10^{-9})}{2(2 \times 10^{-6})} \text{ m} \\ &= 1.125 \times 10^{-2} \text{ m}. \end{aligned}$$

The individual approximate positions are 1.10625 m and 1.11750 m; the longer wavelength places its maximum farther from the centre. This numerical result uses the conventional approximate secondary-maximum and small-angle formulae.

Step 5: Conclusion (a) Independent sodium lamps are incoherent, so no sustained interference

Quick Tip to Solve: For the second secondary diffraction maximum use the factor $5/2$, and substitute $\Delta\lambda$ directly to find the separation of corresponding maxima.

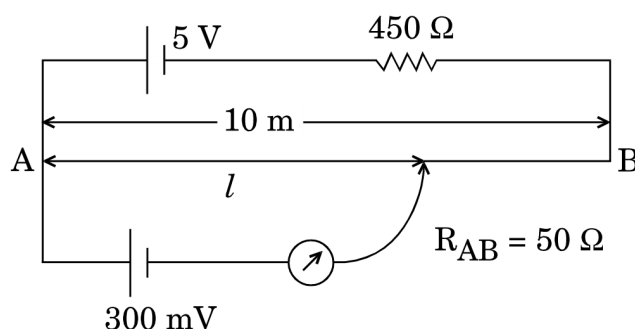
fringes are observed. (b) $I = 4I_0 \cos^2(\phi/2)$, with maxima at $\phi = 2m\pi$ and minima at $\phi = (2m + 1)\pi$.
 (c) The conventional diffraction approximation gives a separation of $1.125 \times 10^{-2} \text{ m} = 1.125 \text{ cm}$.

Quick Tip to Solve: For the second secondary diffraction maximum use the factor $5/2$, and substitute $\Delta\lambda$ directly to find the separation of corresponding maxima.

27. (a) Describe briefly, with the help of a circuit diagram, the method of measuring the internal resistance of a cell.

(b) Give reason why a potentiometer is preferred over a voltmeter for the measurement of emf of a cell.

(c) In the potentiometer circuit given below, calculate the balancing length l . Give reason, whether the circuit will work, if the driver cell of emf 5 V is replaced with a cell of 2 V , keeping all other factors constant. [5 marks]



Solution:

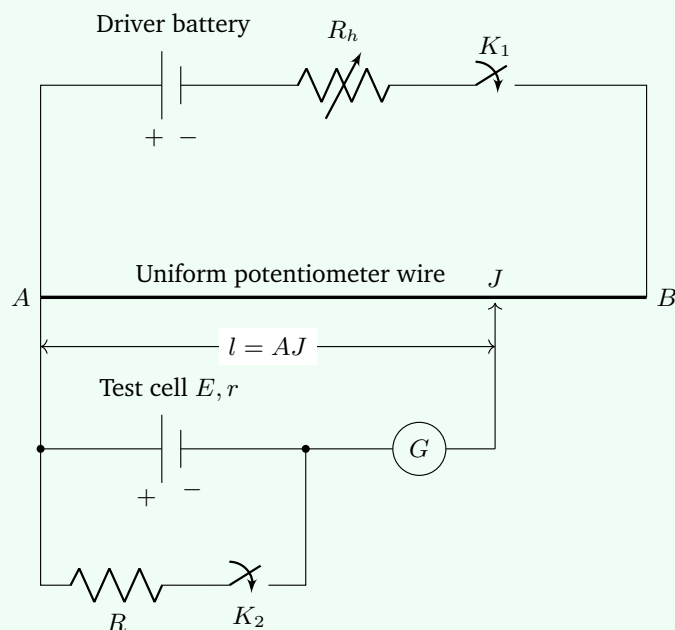
Concept:

- A uniform potentiometer wire carrying a steady current has a constant potential gradient $k = V_{AB}/L$. At null deflection the potential difference along the balancing length equals the voltage being measured.
- A cell of emf E and internal resistance r , supplying a known external resistance R , has terminal voltage $V = ER/(R + r)$.
- At balance the potentiometer draws no current from the cell under test. An ordinary voltmeter draws current and therefore reads the loaded terminal voltage.

Step 1: (a) Arrange the circuit and measure the open-circuit emf Connect a driver battery, key K_1 and rheostat R_h in series with the uniform potentiometer wire AB . Connect the positive terminal of the test cell to the higher-potential end A , and its negative terminal through a galvanometer G to jockey J . A known resistance R and key K_2 form a separate branch across the test cell, as shown. Close K_1 , adjust the rheostat to obtain a suitable steady potential gradient, and keep that setting unchanged throughout the experiment. With K_2 open, locate the null point and measure $AJ = l_1$. Since no current is drawn from the test cell at null, its internal voltage drop vanishes and

$$E = kl_1.$$

Quick Tip to Solve: Compare the test emf with the voltage across the potentiometer wire, not simply with the driver-cell emf: the series resistance takes part of the supply voltage.



Potentiometer method: open K_2 for l_1 , close K_2 for l_2 .

Step 2: (a) Load the cell and obtain its internal resistance Close K_2 , so the cell supplies current through the known external resistance R . Find the new null point without changing the primary current and record $AJ = l_2$. The measured terminal voltage is $V = kl_2$, while the external current is $I = V/R$. Using $E = V + Ir$,

$$\frac{E}{V} = \frac{l_1}{l_2}, \quad r = R \left(\frac{E}{V} - 1 \right) = R \frac{l_1 - l_2}{l_2}.$$

Repeat the measurements with suitable known resistances to improve reliability. Both lengths must be measured from the same end A , and the primary current must remain constant for the potential gradient to cancel.

Step 3: (b) Explain the preference over a voltmeter A potentiometer is a null instrument. At the balance point no current passes through the test-cell and galvanometer measurement branch, so there is no loss Ir inside the cell when its emf is measured with K_2 open. It therefore measures the emf directly. An ordinary voltmeter has finite resistance and draws some current; its reading is the terminal voltage $E - Ir$, which is less than the emf for a cell with nonzero internal resistance.

Step 4: (c) Calculate the potential gradient and balancing length In the given circuit the $450\ \Omega$ resistance is in series with the $50\ \Omega$ potentiometer wire. Thus the current driven by the 5 V cell and the voltage across the 10 m wire are

$$I = \frac{5}{450 + 50} = 0.010\text{ A}, \quad V_{AB} = 0.010 \times 50 = 0.50\text{ V}.$$

The potential gradient is $k = 0.50/10 = 0.050\text{ V m}^{-1}$. The test emf is $300\text{ mV} = 0.300\text{ V}$, so the null condition gives

$$kl = 0.300\text{ V}, \quad l = \frac{0.300}{0.050} = 6.0\text{ m}.$$

Quick Tip to Solve: Compare the test emf with the voltage across the potentiometer wire, not simply with the driver-cell emf: the series resistance takes part of the supply voltage.

Step 5: (c) Check operation with the replacement driver cell With the 2 V driver and unchanged resistances,

$$I' = \frac{2}{500} = 0.004 \text{ A}, \quad V'_{AB} = 0.004 \times 50 = 0.20 \text{ V}.$$

The greatest potential difference available on the wire is therefore only 0.20 V, below the test-cell emf 0.300 V. Equivalently, $k' = 0.020 \text{ V m}^{-1}$ would require $l' = 15 \text{ m}$, beyond the available 10 m wire. Hence no null point can be obtained with this replacement.

Step 6: Conclusion The internal resistance is $r = R(l_1 - l_2)/l_2$. A potentiometer measures emf without drawing current at balance. For the given circuit, $l = 6.0 \text{ m}$; with a 2 V driver the circuit cannot balance because $V_{AB} = 0.20 \text{ V} < 0.300 \text{ V}$.

Quick Tip to Solve: Compare the test emf with the voltage across the potentiometer wire, not simply with the driver-cell emf: the series resistance takes part of the supply voltage.

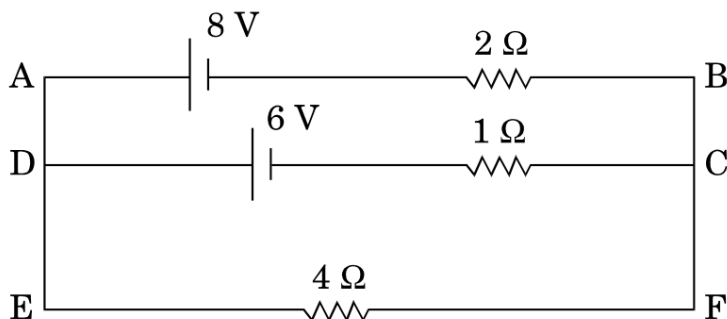
OR

27. (a) State the working principle of a meter bridge used to measure an unknown resistance.

(b) Give reason

- (i) why the connections between the resistors in a metre bridge are made of thick copper strips,
- (ii) why is it generally preferred to obtain the balance length near the mid-point of the bridge wire.

(c) Calculate the potential difference across the 4Ω resistor in the given electrical circuit, using Kirchhoff's rules. [5 marks]



Solution:

Concept:

- A metre bridge works on the Wheatstone bridge principle: at balance, the galvanometer current is zero and the ratios of the resistances in the two arms are equal.
- For a uniform wire, resistance is proportional to length. Kirchhoff's junction rule expresses conservation of charge, while the loop rule expresses conservation of energy.

Step 1: (a) State the balance principle Let the unknown resistance X and the known resistance R occupy the left and right gaps respectively. Let the balance point be at a length l , in centimetres, from the left end of the uniform bridge wire. When the galvanometer shows zero deflection, the potentials

Quick Tip to Solve: First identify the two common nodes and the positive terminal of each cell; then use one junction equation and a loop equation for each cell branch.

at its two ends are equal. The Wheatstone bridge balance condition gives

$$\frac{X}{R} = \frac{l}{100-l}, \quad X = R \frac{l}{100-l}.$$

Thus the unknown resistance is measured by comparing the two lengths of a uniform wire with a known resistance.

Step 2: (b)(i) Explain the use of thick copper strips Copper has low resistivity and a thick strip has a large cross-sectional area. From $R_{\text{strip}} = \rho L/A$, its resistance is consequently very small. The connecting strips therefore introduce negligible additional resistance into the bridge arms, reducing the error in the measured resistance.

Step 3: (b)(ii) Explain the preferred balance position Near the midpoint, the two wire segments have comparable resistances and the bridge gives high sensitivity to a small departure from balance. The fractional error due to uncertainty in locating the jockey is also least there. For a small length uncertainty $|\delta l|$, the balance formula gives

$$\frac{|\delta X|}{X} \simeq \left(\frac{1}{l} + \frac{1}{100-l} \right) |\delta l| = \frac{100|\delta l|}{l(100-l)}.$$

The product $l(100-l)$ is greatest at $l = 50$ cm, making this fractional uncertainty smallest. In practice, the known resistance is chosen close to the unknown resistance.

Step 4: (c) Apply Kirchhoff's rules to the circuit Points A, D, E form one equipotential node, and B, C, F form the other. Both cells have their positive terminals on the left. Take current I_1 from B to A in the upper branch, current I_2 from C to D in the middle branch, and current I from E to F through the $4\ \Omega$ resistor. The junction rule gives $I = I_1 + I_2$. Traversing each cell branch from right to left and returning through the bottom resistor, the loop rule gives

$$8 - 2I_1 - 4I = 0,$$

$$6 - I_2 - 4I = 0.$$

With emf in volts, resistance in ohms and current in amperes, these imply $I_1 = 4 - 2I$ and $I_2 = 6 - 4I$. Substitution into the junction equation yields

$$I = (4 - 2I) + (6 - 4I), \quad 7I = 10, \quad I = \frac{10}{7} \text{ A}.$$

The resulting branch currents are $I_1 = 8/7$ A and $I_2 = 2/7$ A, both positive in the chosen directions. Hence current in the bottom resistor flows from E to F , so E is at the higher potential.

Step 5: Conclusion The metre bridge uses the Wheatstone balance condition $X/R = l/(100-l)$. Thick copper strips reduce connecting resistance, and a balance near 50 cm improves sensitivity and reduces fractional length error. The required potential difference is $V_E - V_F = 4(10/7) = \boxed{40/7 \text{ V} \simeq 5.71 \text{ V}}$, with E positive relative to F .

Quick Tip to Solve: First identify the two common nodes and the positive terminal of each cell; then use one junction equation and a loop equation for each cell branch.