

CBSE Class 12 Mathematics 2019 - Set 65-1-3 - Compartment

Question Paper

Conducted by the Central Board of Secondary Education (CBSE)



Time Allowed: 3 Hours

Maximum Marks: 100

General Instructions

- (i) All questions are compulsory.
- (ii) The question paper consists of 29 questions divided into four sections A, B, C and D. Section A comprises of 4 questions of one mark each, Section B comprises of 8 questions of two marks each, Section C comprises of 11 questions of four marks each and Section D comprises of 6 questions of six marks each.
- (iii) All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.
- (iv) There is no overall choice. However, internal choice has been provided in 1 question of Section A, 3 questions of Section B, 3 questions of Section C and 3 questions of Section D. You have to attempt only one of the alternatives in all such questions.
- (v) Use of calculators is not permitted. You may ask for logarithmic tables, if required.

Section A

1. Find the cofactor of the element a_{23} of the

determinant $\begin{vmatrix} 5 & 3 & 8 \\ 2 & 0 & 1 \\ 1 & 2 & 3 \end{vmatrix}$.

- 2(a). Find the length of the perpendicular drawn from the point $P(3, -4, 5)$ on the z -axis.

OR

- 2(b). Find the vector equation of a plane, which is at a distance of 5 units from the origin and whose normal vector is $2\hat{i} - \hat{j} + 2\hat{k}$.

3. Form the differential equation representing the family of curves $ax + by = 0$, where a and b are arbitrary constants.

4. Find $\frac{dy}{dx}$, if $\sin^2 x + \cos^2 y = 1$.

Section B

5(a). Find $\int \frac{1}{\cos^2 x (1 - \tan x)^2} dx$.

OR

5(b). Evaluate $\int_0^1 x(1-x)^n dx$.

6. Form the differential equation representing the family of curves $y = b \cos(x + a)$, where a and b are arbitrary constants.

7(a). Find a unit vector perpendicular to both $\vec{a}$ and $\vec{b}$, where $\vec{a} = 4\hat{i} - \hat{j} + 8\hat{k}$, $\vec{b} = -\hat{j} + \hat{k}$.

OR

7(b). If $\vec{a} = 2\hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$ and $\vec{c} = 3\hat{i} + \hat{j}$ are such that $\vec{a} + \lambda\vec{b}$ is perpendicular to $\vec{c}$, then find the value of λ .

8(a). Given that A and B are two independent events such that $P(A) = 0.3$ and $P(B) = 0.5$. Find $P(A|B)$.

OR

8(b). A bag contains 3 white and 2 red balls, another bag contains 4 white and 3 red balls. One ball is drawn at random from each bag. Find the probability that the balls drawn are one white and one red.

9. The probabilities of A , B and C solving a problem independently are $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$ respectively. If all the three try to solve the problem independently, find the probability that the problem is solved.

10. If $f: \mathbb{R} \rightarrow \mathbb{R}$, $f(x) = (3 - x^3)^{\frac{1}{3}}$, find $f \circ f(x)$.

11. For what value of x is

$$\begin{bmatrix} 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 0 \\ 2 & 0 & 1 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix} = 0?$$

12. Find $\int \frac{dx}{\sqrt{5 - 4x - x^2}}$.

13. Find $\int \frac{\cos x}{(1 + \sin x)(2 + \sin x)} dx$.

14. Evaluate $\int_0^\pi \frac{x \sin x}{1 + \cos^2 x} dx$.

15(a). Find the particular solution of the differential equation $x dx - y e^y \sqrt{1 + x^2} dy = 0$, given that $y = 1$ when $x = 0$.

OR

15(b). Solve the differential equation $x \cos\left(\frac{y}{x}\right) \frac{dy}{dx} = y \cos\left(\frac{y}{x}\right) + x$.

16. Find the vector equation of the line passing through the point $(2, 3, -1)$ and parallel to the planes $\vec{r} \cdot (3\hat{i} + 4\hat{j} + 2\hat{k}) = 5$ and $\vec{r} \cdot (3\hat{i} - 2\hat{j} - 2\hat{k}) = 4$.

17(a). Let $A = \mathbb{N} \times \mathbb{N}$ be the set of all ordered pairs of natural numbers and R be the relation on the set A defined by $(a, b) R (c, d)$ iff $ad = bc$. Show that R is an equivalence relation.

OR

17(b). Show that $f : \mathbb{R} - \{2\} \rightarrow \mathbb{R} - \{1\}$ defined by $f(x) = \frac{x}{x-2}$ is one-one. Also, if $g : \mathbb{R} - \{1\} \rightarrow \mathbb{R} - \{2\}$ is defined as $g(x) = \frac{2x}{x-1}$, find $gof(x)$.

18. If $y = (\sin^{-1} x)^2$, prove that $(1 - x^2) \frac{d^2 y}{dx^2} - x \frac{dy}{dx} - 2 = 0$.

19. Prove that: $\tan^{-1} \left(\frac{\sqrt{1+x} + \sqrt{1-x}}{\sqrt{1+x} - \sqrt{1-x}} \right) = \frac{\pi}{4} + \frac{1}{2} \cos^{-1} x$; $\frac{-1}{\sqrt{2}} \leq x \leq 1$.

20(a). Find $\frac{dy}{dx}$, if $x^y \cdot y^x = x^x$.

OR

20(b). If $x = a \sec^3 \theta$ and $y = a \tan^3 \theta$, find $\frac{d^2 y}{dx^2}$.

21. Find x such that the four points $A(5, x, 4)$, $B(4, 4, 0)$, $C(5, 4, -3)$ and $D(7, 7, -2)$ are coplanar.

22. Find the equations of the tangent and normal to the parabola $y^2 = 4ax$ at the point $(at^2, 2at)$.

23. Prove the following, using properties of determinants:

$$\begin{vmatrix} \alpha & \alpha^2 & \beta + \gamma \\ \beta & \beta^2 & \gamma + \alpha \\ \gamma & \gamma^2 & \alpha + \beta \end{vmatrix} = (\beta - \gamma)(\gamma - \alpha)(\alpha - \beta)(\alpha + \beta + \gamma).$$

Section D

24(a). Find the coordinates of the point where the line $\frac{x-8}{4} = \frac{y-1}{1} = \frac{z-3}{8}$ intersects the plane $2x + 2y + z = 3$. Also find the angle between the line and the plane.

OR

24(b). A line with direction ratios $\langle 2, 2, 1 \rangle$ intersects the lines $\frac{x-7}{3} = \frac{y-5}{2} = \frac{z-3}{1}$ and $\frac{x-1}{2} = \frac{y+1}{4} = \frac{z+1}{3}$ at the points P and Q respectively. Find the length and the equation of the intercept PQ .

25. A small firm manufactures chairs and tables. Market demand and available resources indicate that the combined production of chairs and tables should not exceed 50 units per day. It takes 30 minutes to manufacture a chair and 1 hour to manufacture a table. A maximum of 40 man-hours per day are available. The profit on each chair is Rs.40 and profit on each table is Rs.60. Determine how many each of chairs and tables should be manufactured per day in order to maximize the profit. What is the maximum profit? Formulate the LPP and solve it graphically.

26. Bag I contains 4 red and 2 green balls and Bag II contains 3 red and 5 green balls. One ball is transferred at random from Bag I to Bag II and then a ball is drawn at random from Bag II. The ball so drawn is found to be green in colour. Find the probability that the transferred ball is also green.

27(a). If $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$, find A^{-1} . Using A^{-1} , solve the system of equations:

$$y + 2z = 5$$

$$x + 2y + 3z = 10$$

$$3x + y + z = 9$$

OR

27(b). Obtain the inverse of the following matrix using elementary operations: $A =$

$$\begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}.$$

28. A helicopter of the enemy is flying along the curve given by $y = x^2 + 7$. A soldier, placed at $(3, 7)$, wants to shoot down the helicopter when it is nearest to him. Find the nearest distance.

29(a). Using integration, find the area lying above x -axis and included between the circle $x^2 + y^2 = 4x$ and the parabola $y^2 = 2x$.

OR

29(b). Using integration, find the area of $\triangle ABC$ whose vertices are $A(3, 2)$, $B(5, 7)$ and $C(7, 5)$.

- End of Question Paper -