

Question Paper

Conducted by the Central Board of Secondary Education (CBSE)



Time Allowed: 3 Hours

Maximum Marks: 80

General Instructions

- (i) This question paper contains **36 questions**. All questions are compulsory.
- (ii) Section A: Questions 1 to 20 carry 1 mark each.
- (iii) Section B: Questions 21 to 26 carry 2 marks each.
- (iv) Section C: Questions 27 to 32 carry 4 marks each.
- (v) Section D: Questions 33 to 36 carry 6 marks each.
- (vi) Attempt one alternative where an internal choice is provided. Follow the instructions within each question.
- (vii) Use of calculators is not allowed.

Section A

1. The matrix $\begin{bmatrix} 2 & -1 & 3 \\ \lambda & 0 & 7 \\ -1 & 1 & 4 \end{bmatrix}$ is not invertible for

- (A) $\lambda = -1$
- (B) $\lambda = 0$
- (C) $\lambda = 1$
- (D) $\lambda \in \mathbb{R} - \{1\}$

2. The number of arbitrary constants in the particular solution of a differential equation of second order is (are)

- (A) 0
- (B) 1
- (C) 2
- (D) 3

3. The value of $\tan^{-1}\left(\tan \frac{7\pi}{6}\right)$ is

- (A) $\frac{\pi}{6}$
- (B) $\frac{\pi}{2}$
- (C) $\frac{3}{2}\pi$
- (D) $\frac{7\pi}{6}$

4. The corner points of the feasible region determined by the system of linear inequalities are $(0, 0)$, $(4, 0)$, $(2, 4)$ and $(0, 5)$. If the maximum value of $z = ax + by$, where $a, b > 0$ occurs at both $(2, 4)$ and $(4, 0)$, then

- (A) $a = 2b$
- (B) $2a = b$
- (C) $a = b$
- (D) $3a = b$

5. If A and B are two independent events with $P(A) = \frac{1}{3}$ and $P(B) = \frac{1}{4}$, then $P(B' | A)$ is equal to

- (A) $\frac{1}{4}$
- (B) $\frac{1}{3}$
- (C) $\frac{3}{4}$
- (D) 1

6. If A is a square matrix such that $A^2 = A$, then $(I - A)^3 + A$ is equal to

- (A) I
- (B) 0
- (C) $I - A$
- (D) $I + A$

7. $\int_1^e \frac{\log x}{x} dx$ is equal to

- (A) $\frac{e^2}{2}$
- (B) 1
- (C) $\frac{1}{2}$
- (D) $-\frac{1}{2}$

8. A point P lies on the line segment joining the points $(-1, 3, 2)$ and $(5, 0, 6)$. If x -coordinate of P is 2, then its z -coordinate is

- (A) -1
- (B) 4
- (C) $\frac{3}{2}$
- (D) 8

9. If the projection of $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$ on $\vec{b} = 2\hat{i} + \lambda\hat{k}$ is zero, then the value of λ is

- (A) 0
(B) 1
(C) $\frac{-2}{3}$
(D) $\frac{-3}{2}$

10. The vector equation of the line passing through the point $(-1, 5, 4)$ and perpendicular to the plane $z = 0$ is

- (A) $\vec{r} = -\hat{i} + 5\hat{j} + 4\hat{k} + \lambda(\hat{i} + \hat{j})$
(B) $\vec{r} = -\hat{i} + 5\hat{j} + (4 + \lambda)\hat{k}$
(C) $\vec{r} = \hat{i} - 5\hat{j} - 4\hat{k} + \lambda\hat{k}$
(D) $\vec{r} = \lambda\hat{k}$

11. The position vectors of two points A and B are $\vec{OA} = 2\hat{i} - \hat{j} - \hat{k}$ and $\vec{OB} = 2\hat{i} - \hat{j} + 2\hat{k}$, respectively. The position vector of a point P which divides the line segment joining A and B in the ratio $2 : 1$ is _____.

12. The equation of the normal to the curve $y^2 = 8x$ at the origin is _____.

OR

The radius of a circle is increasing at the uniform rate of 3 cm/sec. At the instant when the radius of the circle is 2 cm, its area increases at the rate of _____ cm^2/s .

13. If A is a square matrix of order 3 and A_{ij} is the cofactor of the element a_{ij} , then value of $a_{21}A_{11} + a_{22}A_{12} + a_{23}A_{13}$ is _____.

OR

If the matrix A is both symmetric and skew symmetric, then A is a _____.

14. A relation R in a set A is called _____, if $(a_1, a_2) \in R$ implies $(a_2, a_1) \in R$, for all $a_1, a_2 \in A$.

15. The greatest integer function defined by $f(x) = [x]$, $0 < x < 2$ is not differentiable at $x =$ _____.

16. If A is a square matrix of order 3 and $|A| = 2$, then find the value of $|-AA'|$.

17. Two cards are drawn at random and one-by-one without replacement from a well-shuffled pack of 52 playing cards. Find the probability that one card is red and the other is black.

18. Evaluate: $\int_1^3 |2x - 1| dx$

19. Find: $\int \frac{dx}{\sqrt{9 - 4x^2}}$

20. Find: $\int x^4 \log x dx$

OR

Find: $\int \frac{2x}{\sqrt[3]{x^2 + 1}} dx$

Section B

21. Find a unit vector perpendicular to each of the vectors $\vec{a}$ and $\vec{b}$ where $\vec{a} = 5\hat{i} + 6\hat{j} - 2\hat{k}$ and $\vec{b} = 7\hat{i} + 6\hat{j} + 2\hat{k}$.

OR

Find the volume of the parallelopiped whose adjacent edges are represented by $2\vec{a}$, $-\vec{b}$ and $3\vec{c}$, where

$$\vec{a} = \hat{i} - \hat{j} + 2\hat{k}, \vec{b} = 3\hat{i} + 4\hat{j} - 5\hat{k}, \text{ and } \vec{c} = 2\hat{i} - \hat{j} + 3\hat{k}.$$

22. If $f(x) = \sqrt{\tan \sqrt{x}}$, then find $f'(x)$.

23. Using differentials, find the approximate value of $\sqrt{25.3}$ up to two places of decimals.

24. The probability of finding a green signal on a busy crossing X is 30%. What is the probability of finding a green signal on X on two consecutive days out of three?

25. Prove that $\sin^{-1}(2x\sqrt{1-x^2}) = 2\cos^{-1}x$, $\frac{1}{\sqrt{2}} \leq x \leq 1$.

OR

Consider a bijective function $f : \mathbb{R}_+ \rightarrow (7, \infty)$ given by $f(x) = 16x^2 + 24x + 7$, where $\mathbb{R}_+$ is the set of all positive real numbers. Find the inverse function of f .

26. Find the value of k so that the lines $x = -y = kz$ and $x - 2 = 2y + 1 = -z + 1$ are perpendicular to each other.

Section C

27. A furniture trader deals in only two items – chairs and tables. He has Rs. 50,000 to invest and a space to store at most 35 items. A chair costs him Rs. 1,000 and a table costs him Rs. 2,000. The trader earns a profit of Rs. 150 and Rs. 250 on a chair and table, respectively. Formulate the above problem as an LPP to maximise the profit and solve it graphically.

28. If $x = a(\theta - \sin \theta)$, $y = a(1 - \cos \theta)$, $a > 0$, then find $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{2}$.

29. Evaluate $\int_1^3 e^x dx$ as limit of the sums.

30. There are two bags, I and II. Bag I contains 3 red and 5 black balls and Bag II contains 4 red and 3 black balls. One ball is transferred randomly from Bag I to Bag II and then a ball is drawn randomly from Bag II. If the ball so drawn is found to be black in colour, then find the probability that the transferred ball is also black.

OR

An urn contains 5 red, 2 white and 3 black balls. Three balls are drawn, one-by-one, at random without replacement. Find the probability distribution of the number of white balls. Also, find the mean and the variance of the number of white balls drawn.

31. Find the general solution of the differential equation $y e^y dx = (y^3 + 2x e^y) dy$.

OR

Find the particular solution of the differential equation $x \frac{dy}{dx} = y - x \tan\left(\frac{y}{x}\right)$, given that $y = \frac{\pi}{4}$ at $x = 1$.

32. Let N be the set of natural numbers and R be the relation on $N \times N$ defined by $(a, b)R(c, d)$ iff $ad = bc$ for all $a, b, c, d \in N$. Show that R is an equivalence relation.

Section D

33. Show that the height of the right circular cylinder of greatest volume which can be inscribed in a right circular cone of height h and radius r is one-third of the height of the cone, and the greatest volume of the cylinder is $\frac{4}{9}$ times the volume of the cone.

34. Using integration, find the area of the region enclosed by the parabola $y = 3x^2$ and the line $3x - y + 6 = 0$.

35. Find the equation of the plane that contains the point $A(2, 1, -1)$ and is perpendicular to the line of intersection of the planes $2x + y - z = 3$ and $x + 2y + z = 2$. Also find the angle between the plane thus obtained and the y -axis.

OR

Find the distance of the point $P(-2, -4, 7)$ from the point of intersection Q of the line $\vec{r} = (3\hat{i} - 2\hat{j} + 6\hat{k}) + \lambda(2\hat{i} - \hat{j} + 2\hat{k})$ and the plane $\vec{r} \cdot (\hat{i} - \hat{j} + \hat{k}) = 6$. Also write the vector equation of the line PQ .

36. If $A = \begin{bmatrix} 1 & 2 & -3 \\ 3 & 2 & -2 \\ 2 & -1 & 1 \end{bmatrix}$, then find A^{-1} and use

it to solve the following system of equations:

$$x + 2y - 3z = 6$$

$$3x + 2y - 2z = 3$$

$$2x - y + z = 2$$

OR

Using properties of determinants, prove that

$$\begin{vmatrix} (b+c)^2 & a^2 & bc \\ (c+a)^2 & b^2 & ca \\ (a+b)^2 & c^2 & ab \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)(a^2+b^2+c^2).$$

- End of Question Paper -