

Question Paper

Conducted by the Central Board of Secondary Education (CBSE)



Time Allowed: 3 Hours

Maximum Marks: 80

General Instructions

- (i) This question paper contains **36 questions**. All questions are compulsory.
- (ii) Section A: Questions 1 to 20 carry 1 mark each.
- (iii) Section B: Questions 21 to 26 carry 2 marks each.
- (iv) Section C: Questions 27 to 32 carry 4 marks each.
- (v) Section D: Questions 33 to 36 carry 6 marks each.
- (vi) Attempt one alternative where an internal choice is provided. Follow the instructions within each question.
- (vii) Use of calculators is not allowed.

Section A

1. The relation R in the set $\{1, 2, 3\}$ given by $R = \{(1, 2), (2, 1), (1, 1)\}$ is

- (A) symmetric and transitive, but not reflexive
- (B) reflexive and symmetric, but not transitive
- (C) symmetric, but neither reflexive nor transitive
- (D) an equivalence relation

2. $\tan^{-1} 3 + \tan^{-1} \lambda = \tan^{-1} \left(\frac{3 + \lambda}{1 - 3\lambda} \right)$ is valid for what values of λ ?

- (A) $\lambda \in \left(-\frac{1}{3}, \frac{1}{3} \right)$
- (B) $\lambda > \frac{1}{3}$
- (C) $\lambda < \frac{1}{3}$
- (D) All real values of λ

3. If A is a non-singular square matrix of order 3 such that $A^2 = 3A$, then the value of $|A|$ is

- (A) -3
- (B) 3
- (C) 9
- (D) 27

4. The function $f : \mathbb{R} \rightarrow \mathbb{R}$ given by $f(x) = -|x - 1|$ is

- (A) continuous as well as differentiable at $x = 1$
- (B) not continuous but differentiable at $x = 1$
- (C) continuous but not differentiable at $x = 1$
- (D) neither continuous nor differentiable at $x = 1$

5. Let $A = \{1, 3, 5\}$. Then the number of equivalence relations in A containing $(1, 3)$ is

- (A) 1
- (B) 2
- (C) 3
- (D) 4

6. The interval in which the function f given by $f(x) = x^2 e^{-x}$ is strictly increasing, is

- (A) $(-\infty, \infty)$
- (B) $(-\infty, 0)$
- (C) $(2, \infty)$
- (D) $(0, 2)$

7. If $|\vec{a}| = 4$ and $-3 \leq \lambda \leq 2$, then $|\lambda \vec{a}|$ lies in

- (A) $[0, 12]$
- (B) $[2, 3]$
- (C) $[8, 12]$
- (D) $[-12, 8]$

8. The vectors $3\hat{i} - \hat{j} + 2\hat{k}$, $2\hat{i} + \hat{j} + 3\hat{k}$ and $\hat{i} + \lambda\hat{j} - \hat{k}$ are coplanar if the value of λ is

- (A) -2
- (B) 0
- (C) 2
- (D) Any real number

9. The area of a triangle formed by vertices O , A and B , where $\overrightarrow{OA} = \hat{i} + 2\hat{j} + 3\hat{k}$ and $\overrightarrow{OB} = -3\hat{i} - 2\hat{j} + \hat{k}$ is

- (A) $3\sqrt{5}$ sq. units
- (B) $5\sqrt{5}$ sq. units
- (C) $6\sqrt{5}$ sq. units
- (D) 4 sq. units

10. The coordinates of the foot of the perpendicular drawn from the point $(2, -3, 4)$ on the y -axis is

- (A) $(2, 3, 4)$
- (B) $(-2, -3, -4)$
- (C) $(0, -3, 0)$
- (D) $(2, 0, 4)$

11. The range of the principal value branch of the function $y = \sec^{-1} x$ is _____.

OR

The principal value of $\cos^{-1}\left(-\frac{1}{2}\right)$ is _____.

12. Given a skew-symmetric matrix $A =$

$$\begin{bmatrix} 0 & a & 1 \\ -1 & b & 1 \\ -1 & c & 0 \end{bmatrix}, \text{ the value of } (a + b + c)^2 \text{ is}$$

_____.

13. The distance between parallel planes $2x + y - 2z - 6 = 0$ and $4x + 2y - 4z = 0$ is _____ units.

OR

If $P(1, 0, -3)$ is the foot of the perpendicular from the origin to the plane, then the cartesian equation of the plane is _____.

14. If the radius of the circle is increasing at the rate of 0.5 cm/s , then the rate of increase of its circumference is _____.

15. The corner points of the feasible region of an LPP are $(0, 0)$, $(0, 8)$, $(2, 7)$, $(5, 4)$ and $(6, 0)$. The maximum profit $P = 3x + 2y$ occurs at the point _____.

16. Differentiate $\sec^2(x^2)$ with respect to x^2 .

OR

If $y = f(x^2)$ and $f'(x) = e^{\sqrt{x}}$, then find $\frac{dy}{dx}$.

17. Find the value of k , so that the function

$$f(x) = \begin{cases} kx^2 + 5 & \text{if } x \leq 1 \\ 2 & \text{if } x > 1 \end{cases} \text{ is continuous at } x = 1.$$

18. Evaluate: $\int_{-\pi/2}^{\pi/2} x \cos^2 x \, dx$.

19. Find the general solution of the differential equation $e^{y-x} \frac{dy}{dx} = 1$.

20. Find the coordinates of the point where the line $\frac{x-1}{3} = \frac{y+4}{7} = \frac{z+4}{2}$ cuts the xy -plane.

Section B

21. If $A = \begin{bmatrix} -3 & 2 \\ 1 & -1 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, find scalar k so that $A^2 + I = kA$.

22. If $f(x) = \sqrt{\frac{\sec x - 1}{\sec x + 1}}$, find $f'\left(\frac{\pi}{3}\right)$.

OR

Find $f'(x)$ if $f(x) = (\tan x)^{\tan x}$.

23. Find: $\int \frac{\tan^3 x}{\cos^3 x} dx$.

24. Find a vector $\vec{r}$ equally inclined to the three axes and whose magnitude is $3\sqrt{3}$ units.

OR

Find the angle between unit vectors $\vec{a}$ and $\vec{b}$ so that $\sqrt{3}\vec{a} - \vec{b}$ is also a unit vector.

25. Find the points of intersection of the line $\vec{r} = 2\hat{i} - \hat{j} + 2\hat{k} + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k})$ and the plane $\vec{r} \cdot (\hat{i} - \hat{j} + \hat{k}) = 5$.

26. A purse contains 3 silver and 6 copper coins and a second purse contains 4 silver and 3 copper coins. If a coin is drawn at random from one of the two purses, find the probability that it is a silver coin.

27. Check whether the relation R in the set N of natural numbers given by $R = \{(a, b) : a \text{ is divisor of } b\}$ is reflexive, symmetric or transitive. Also determine whether R is an equivalence relation.

OR

Prove that $\tan^{-1} \frac{1}{4} + \tan^{-1} \frac{2}{9} = \frac{1}{2} \sin^{-1} \left(\frac{4}{5} \right)$.

28. If $\tan^{-1} \left(\frac{y}{x} \right) = \log \sqrt{x^2 + y^2}$, prove that $\frac{dy}{dx} = \frac{x+y}{x-y}$.

OR

If $y = e^{a \cos^{-1} x}$, $-1 \leq x \leq 1$, then show that $(1 - x^2) \frac{d^2 y}{dx^2} - x \frac{dy}{dx} - a^2 y = 0$.

29. Find: $\int \frac{x^3 + 1}{x^3 - x} dx$.

30. Solve the following differential equation: $(1 + e^{y/x}) dy + e^{y/x} \left(1 - \frac{y}{x} \right) dx = 0$ ($x \neq 0$).

31. Find the shortest distance between the lines

$$\vec{r} = 2\hat{i} - \hat{j} + \hat{k} + \lambda(3\hat{i} - 2\hat{j} + 5\hat{k})$$

$$\vec{r} = 3\hat{i} + 2\hat{j} - 4\hat{k} + \mu(4\hat{i} - \hat{j} + 3\hat{k}).$$

32. A cottage industry manufactures pedestal lamps and wooden shades. Both the products require machine time as well as craftsman time in the making. The number of hour(s) required for producing 1 unit of each and the corresponding profit is given in the following table:

Pedestal lamp: Machine Time 1.5 hours, Craftsman time 3 hours, Profit Rs. 30.

Wooden shades: Machine Time 3 hours, Craftsman time 1 hour, Profit Rs. 20.

In a day, the factory has availability of not more than 42 hours of machine time and 24 hours of craftsman time.

Assuming that all items manufactured are sold, how should the manufacturer schedule his daily production in order to maximise the profit? Formulate it as an LPP and solve it graphically.

Section D

33. If $A = \begin{bmatrix} 5 & -1 & 4 \\ 2 & 3 & 5 \\ 5 & -2 & 6 \end{bmatrix}$, find A^{-1} and use it to

solve the following system of equations:

$$5x - y + 4z = 5$$

$$2x + 3y + 5z = 2$$

$$5x - 2y + 6z = -1$$

OR

If x, y, z are different and $\begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix} = 0$,

then using properties of determinants show that $1 + xyz = 0$.

34. Amongst all open (from the top) right circular cylindrical boxes of volume $125\pi \text{ cm}^3$, find the dimensions of the box which has the least surface area.

35. Using integration, find the area lying above x -axis and included between the circle $x^2 + y^2 = 8x$ and inside the parabola $y^2 = 4x$.

OR

Using the method of integration, find the area of the triangle ABC , coordinates of whose vertices are $A(2, 0)$, $B(4, 5)$ and $C(6, 3)$.

36. Find the probability distribution of the random variable X , which denotes the number of doublets in four throws of a pair of dice. Hence, find the mean of the number of doublets (X).

- End of Question Paper -