

Question Paper

Conducted by the Central Board of Secondary Education (CBSE)



Time Allowed: 3 Hours

Maximum Marks: 80

General Instructions

- (i) This question paper contains **36 questions**. All questions are compulsory.
- (ii) Section A: Questions 1 to 20 carry 1 mark each.
- (iii) Section B: Questions 21 to 26 carry 2 marks each.
- (iv) Section C: Questions 27 to 32 carry 4 marks each.
- (v) Section D: Questions 33 to 36 carry 6 marks each.
- (vi) Attempt one alternative where an internal choice is provided. Follow the instructions within each question.
- (vii) Use of calculators is not allowed.

Section A

1. The area of a triangle formed by vertices O , A and B , where $\overrightarrow{OA} = \hat{i} + 2\hat{j} + 3\hat{k}$ and $\overrightarrow{OB} = -3\hat{i} - 2\hat{j} + \hat{k}$ is

- (A) $3\sqrt{5}$ sq. units
- (B) $5\sqrt{5}$ sq. units
- (C) $6\sqrt{5}$ sq. units
- (D) 4 sq. units

2. The domain of the function $f(x) = \sin^{-1}(2x)$ is

- (A) $[0, 1]$
- (B) $[-1, 1]$
- (C) $\left[-\frac{1}{2}, \frac{1}{2}\right]$
- (D) $[-2, 2]$

3. The interval in which the function f given by $f(x) = x^2 e^{-x}$ is strictly increasing, is

- (A) $(-\infty, \infty)$
- (B) $(-\infty, 0)$
- (C) $(2, \infty)$
- (D) $(0, 2)$

4. The value of k so that f defined by $f(x) = \begin{cases} x \sin\left(\frac{1}{x}\right) & \text{if } x \neq 0 \\ k & \text{if } x = 0 \end{cases}$ is continuous at $x = 0$ is

- (A) 0
- (B) $\frac{1}{2}$
- (C) 1
- (D) 2

5. The derivative of $\log x$ with respect to $\frac{1}{x}$ is

- (A) $-\frac{1}{x^2}$
- (B) $-\frac{1}{x}$
- (C) $-x$
- (D) $\frac{1}{x}$

6. The coordinates of the foot of the perpendicular drawn from the point $(2, -3, 4)$ on the y -axis is

- (A) $(2, 3, 4)$
- (B) $(-2, -3, -4)$
- (C) $(0, -3, 0)$
- (D) $(2, 0, 4)$

7. The relation R in the set $\{1, 2, 3\}$ given by $R = \{(1, 2), (2, 1), (1, 1)\}$ is

- (A) symmetric and transitive, but not reflexive
- (B) reflexive and symmetric, but not transitive
- (C) symmetric, but neither reflexive nor transitive
- (D) an equivalence relation

8. If $|\vec{a}| = 3$, $|\vec{b}| = 4$ and $|\vec{a} \times \vec{b}| = 6$, then the value of $\vec{a} \cdot \vec{b}$ is

- (A) 12
- (B) 6
- (C) $3\sqrt{3}$
- (D) $6\sqrt{3}$

9. If A is a non-singular square matrix of order 3 such that $A^2 = 3A$, then the value of $|A|$ is

- (A) -3
- (B) 3
- (C) 9
- (D) 27

10. If $|\vec{a}| = 4$ and $-3 \leq \lambda \leq 2$, then $|\lambda\vec{a}|$ lies in

- (A) $[0, 12]$
- (B) $[2, 3]$
- (C) $[8, 12]$
- (D) $[-12, 8]$

11. If the radius of the circle is increasing at the rate of 0.5 cm/s, then the rate of increase of its circumference is _____.

12. If $\begin{bmatrix} 3 & -2 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ k \\ -5 \end{bmatrix} = O$, where O is the null matrix, then the value of k is _____.

13. The corner points of the feasible region of an LPP are $(0, 0)$, $(0, 8)$, $(2, 7)$, $(5, 4)$ and $(6, 0)$. The maximum profit $P = 3x + 2y$ occurs at the point _____.

14. The range of the principal value branch of the function $y = \sec^{-1} x$ is _____.

OR

The principal value of $\cos^{-1}\left(-\frac{1}{2}\right)$ is _____.

15. The distance between parallel planes $2x + y - 2z - 6 = 0$ and $4x + 2y - 4z = 0$ is _____ units.

OR

If $P(1, 0, -3)$ is the foot of the perpendicular from the origin to the plane, then the cartesian equation of the plane is _____.

16. Evaluate: $\int_{-\pi/2}^{\pi/2} x \cos^2 x \, dx$.

17. Find the coordinates of the point where the line $\frac{x-1}{3} = \frac{y+4}{7} = \frac{z+4}{2}$ cuts the xy -plane.

18. Find the value of k , so that the function $f(x) = \begin{cases} kx^2 + 5 & \text{if } x \leq 1 \\ 2 & \text{if } x > 1 \end{cases}$ is continuous at $x = 1$.

19. Form the differential equation representing the family of curves $y = mx$, where m is an arbitrary constant.

20. Differentiate $\sec^2(x^2)$ with respect to x^2 .

OR

If $y = f(x^2)$ and $f'(x) = e^{\sqrt{x}}$, then find $\frac{dy}{dx}$.

Section B

21. Find a vector $\vec{r}$ equally inclined to the three axes and whose magnitude is $3\sqrt{3}$ units.

OR

Find the angle between unit vectors $\vec{a}$ and $\vec{b}$ so that $\sqrt{3}\vec{a} - \vec{b}$ is also a unit vector.

22. If $A = \begin{bmatrix} -3 & 2 \\ 1 & -1 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, find scalar k so that $A^2 + I = kA$.

23. If $f(x) = \sqrt{\frac{\sec x - 1}{\sec x + 1}}$, find $f'\left(\frac{\pi}{3}\right)$.

OR

Find $f'(x)$ if $f(x) = (\tan x)^{\tan x}$.

24. Find: $\int \frac{\tan^3 x}{\cos^3 x} dx$.

25. Find the angle between the line $\vec{r} = 2\hat{i} - 3\hat{j} + \lambda\hat{k}$ and the plane $\vec{r} \cdot (\hat{j} - \hat{k}) = 7$.

26. A and B throw a pair of dice alternately till one of them gets the sum of the numbers as multiples of 6 and wins the game. If A starts first, find the probability of B winning the game.

Section C

27. Solve the following differential equation:
 $(1 + e^{y/x}) dy + e^{y/x} \left(1 - \frac{y}{x}\right) dx = 0 \quad (x \neq 0).$

28. A cottage industry manufactures pedestal lamps and wooden shades. Both the products require machine time as well as craftsman time in the making. The number of hour(s) required for producing 1 unit of each and the corresponding profit is given in the following table:

Pedestal lamp: Machine Time 1.5 hours,
 Craftsman time 3 hours, Profit Rs. 30.

Wooden shades: Machine Time 3 hours,
 Craftsman time 1 hour, Profit Rs. 20.

In a day, the factory has availability of not more than 42 hours of machine time and 24 hours of craftsman time.

Assuming that all items manufactured are sold, how should the manufacturer schedule his daily production in order to maximise the profit? Formulate it as an LPP and solve it graphically.

29. Find: $\int \left(\sqrt{\cot x} + \frac{1}{\sqrt{\cot x}} \right) dx.$

30. Check whether the relation R in the set N of natural numbers given by $R = \{(a, b) : a \text{ is divisor of } b\}$ is reflexive, symmetric or transitive. Also determine whether R is an equivalence relation.

OR

Prove that $\tan^{-1} \frac{1}{4} + \tan^{-1} \frac{2}{9} = \frac{1}{2} \sin^{-1} \left(\frac{4}{5} \right).$

31. Find the cartesian equation of the plane passing through the intersection of the planes $x + 2y - 3 = 0$ and $2x - y + z = 1$ and the origin i.e. $(0, 0, 0)$. Also write the equation of the plane so obtained in vector form.

32. If $\tan^{-1} \left(\frac{y}{x} \right) = \log \sqrt{x^2 + y^2}$, prove that $\frac{dy}{dx} = \frac{x + y}{x - y}.$

OR

If $y = e^{a \cos^{-1} x}$, $-1 \leq x \leq 1$, then show that $(1 - x^2) \frac{d^2 y}{dx^2} - x \frac{dy}{dx} - a^2 y = 0.$

Section D

33. Amongst all open (from the top) right circular cylindrical boxes of volume $125\pi \text{ cm}^3$, find the dimensions of the box which has the least surface area.

34. Using integration, find the area lying above x -axis and included between the circle $x^2 + y^2 = 8x$ and inside the parabola $y^2 = 4x$.

OR

Using the method of integration, find the area of the triangle ABC , coordinates of whose vertices are $A(2, 0)$, $B(4, 5)$ and $C(6, 3)$.

35. If $A = \begin{bmatrix} 5 & -1 & 4 \\ 2 & 3 & 5 \\ 5 & -2 & 6 \end{bmatrix}$, find A^{-1} and use it to

solve the following system of equations:

$$5x - y + 4z = 5$$

$$2x + 3y + 5z = 2$$

$$5x - 2y + 6z = -1$$

OR

If x, y, z are different and $\begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix} = 0$,

then using properties of determinants show that $1 + xyz = 0$.

36. A discrete random variable X has the following probability distribution:

X	0	1	2	3	4	5
$P(X)$	$4C^2$	$3C^2$	$2C^2$	C^2	C	$2C$

(a) Find the value of C .

(b) Find the mean of the distribution.

(c) Given $\sum x_i^2 p_i = 14$, find the variance of the distribution.

- End of Question Paper -