

Question Paper

Conducted by the Central Board of Secondary Education (CBSE)



Time Allowed: 3 Hours

Maximum Marks: 80

General Instructions

- (i) This question paper contains **36 questions**. All questions are compulsory.
- (ii) Section A: Questions 1 to 20 carry 1 mark each.
- (iii) Section B: Questions 21 to 26 carry 2 marks each.
- (iv) Section C: Questions 27 to 32 carry 4 marks each.
- (v) Section D: Questions 33 to 36 carry 6 marks each.
- (vi) Attempt one alternative where an internal choice is provided. Follow the instructions within each question.
- (vii) Use of calculators is not allowed.

Section A

1. If f and g are two functions from $\mathbb{R}$ to $\mathbb{R}$ defined as $f(x) = |x| + x$ and $g(x) = |x| - x$, then $f \circ g(x)$ for $x < 0$ is

- (A) $4x$
- (B) $2x$
- (C) 0
- (D) $-4x$

2. The principal value of $\cot^{-1}(-\sqrt{3})$ is

- (A) $-\frac{\pi}{6}$
- (B) $\frac{\pi}{6}$
- (C) $\frac{2\pi}{3}$
- (D) $\frac{5\pi}{6}$

3. If $A = \begin{bmatrix} -2 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -2 \end{bmatrix}$, then the value of $|\text{adj } A|$ is

- (A) 64
- (B) 16
- (C) 0
- (D) -8

4. The maximum value of slope of the curve $y = -x^3 + 3x^2 + 12x - 5$ is

- (A) 15
- (B) 12
- (C) 9
- (D) 0

5. $\int \frac{x \cdot e^x(1+x)}{\cos^2(xe^x)} dx$ is equal to

- (A) $\tan(xe^x) + c$
- (B) $\cot(e^x) + c$
- (C) $\cot(e^x) + c$
- (D) $\tan[e^x(1+x)] + c$

6. The degree of the differential equation

$$x^2 \frac{d^2 y}{dx^2} = \left(x \frac{dy}{dx} - y \right)^3 \text{ is}$$

- (A) 1
- (B) 2
- (C) 3
- (D) 6

7. The value of p for which $p(\hat{i} + \hat{j} + \hat{k})$ is a unit vector is

- (A) 0
- (B) $\frac{1}{\sqrt{3}}$
- (C) 1
- (D) $\sqrt{3}$

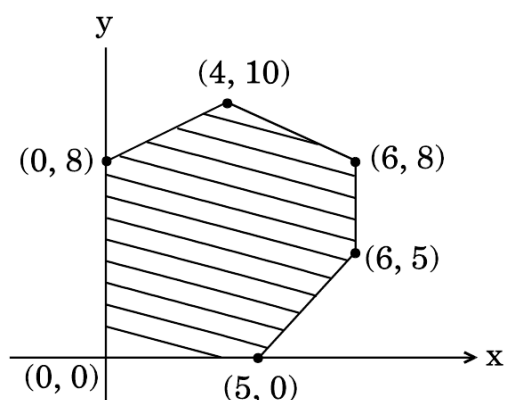
8. The coordinates of the foot of the perpendicular drawn from the point $(-2, 8, 7)$ on the XZ-plane is

- (A) $(2, -8, 7)$
- (B) $(2, 8, -7)$
- (C) $(-2, 0, 7)$
- (D) $(0, 8, 0)$

9. The vector equation of XY-plane is

- (A) $\vec{r} \cdot \hat{k} = 0$
- (B) $\vec{r} \cdot \hat{j} = 0$
- (C) $\vec{r} \cdot \hat{i} = 0$
- (D) $\vec{r} \cdot \hat{n} = 1$

10. The feasible region for an LPP is shown below:



Let $z = 3x - 4y$ be the objective function.
Minimum of z occurs at

- (A) (0, 0)
- (B) (0, 8)
- (C) (5, 0)
- (D) (4, 10)

11. If $y = \tan^{-1} x + \cot^{-1} x$, $x \in \mathbb{R}$, then $\frac{dy}{dx}$ is equal to _____.

OR

If $\cos(xy) = k$, where k is a constant and $xy \neq n\pi$, $n \in \mathbb{Z}$, then $\frac{dy}{dx}$ is equal to _____.

12. The value of λ so that the function f defined by

$$f(x) = \begin{cases} \lambda x, & \text{if } x \leq \pi \\ \cos x, & \text{if } x > \pi \end{cases}$$

is continuous at $x = \pi$ is _____.

13. The equation of the tangent to the curve $y = \sec x$ at the point (0, 1) is _____.

14. The area of the parallelogram whose diagonals are $2\hat{i}$ and $-3\hat{k}$ is _____ square units.

OR

The value of λ for which the vectors $2\hat{i} - \lambda\hat{j} + \hat{k}$ and $\hat{i} + 2\hat{j} - \hat{k}$ are orthogonal is _____.

15. A bag contains 3 black, 4 red and 2 green balls. If three balls are drawn simultaneously at random, then the probability that the balls are of different colours is _____.

16. Construct a 2×2 matrix $A = [a_{ij}]$ whose elements are given by $a_{ij} = |(i)^2 - j|$.

17. Differentiate $\sin^2(\sqrt{x})$ with respect to x .

18. Find the interval in which the function f given by $f(x) = 7 - 4x - x^2$ is strictly increasing.

19. Evaluate: $\int_{-2}^2 x^3 dx$.

OR

Find: $\int \frac{dx}{9 + 4x^2}$.

20. An unbiased coin is tossed 4 times. Find the probability of getting at least one head.

Section B

21. Solve for x : $\sin^{-1} 4x + \sin^{-1} 3x = -\frac{\pi}{2}$.

OR

Express $\tan^{-1}\left(\frac{\cos x}{1 - \sin x}\right)$, $-\frac{3\pi}{2} < x < \frac{\pi}{2}$ in the simplest form.

22. Express $A = \begin{bmatrix} 4 & -3 \\ 1 & 2 \end{bmatrix}$ as a sum of a symmetric and a skew symmetric matrix.

23. If $y^2 \cos\left(\frac{1}{x}\right) = a^2$, then find $\frac{dy}{dx}$.

24. Show that for any two non-zero vectors $\vec{a}$ and $\vec{b}$, $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$ if and only if $\vec{a}$ and $\vec{b}$ are perpendicular vectors.

OR

Show that the vectors $2\hat{i} - \hat{j} + \hat{k}$, $3\hat{i} + 7\hat{j} + \hat{k}$ and $5\hat{i} + 6\hat{j} + 2\hat{k}$ form the sides of a right-angled triangle.

25. Find the coordinates of the point where the line through $(-1, 1, -8)$ and $(5, -2, 10)$ crosses the ZX-plane.

26. If A and B are two events such that $P(A) = 0.4$, $P(B) = 0.3$ and $P(A \cup B) = 0.6$, then find $P(B' \cap A)$.

Section C

27. Show that the function $f : (-\infty, 0) \rightarrow (-1, 0)$ defined by $f(x) = \frac{x}{1+|x|}$, $x \in (-\infty, 0)$ is one-one and onto.

OR

Show that the relation R in the set $A = \{1, 2, 3, 4, 5, 6\}$ given by $R = \{(a, b) : |a - b| \text{ is divisible by } 2\}$ is an equivalence relation.

28. If $y = x^3(\cos x)^x + \sin^{-1} \sqrt{x}$, then find $\frac{dy}{dx}$.

29. Evaluate: $\int_{-1}^5 (|x| + |x+1| + |x-5|) dx$.

30. Find the general solution of the differential equation $x^2y dx - (x^3 + y^3) dy = 0$.

31. Solve the following LPP graphically:

Minimise $z = 5x + 7y$

subject to the constraints

$$2x + y \geq 8$$

$$x + 2y \geq 10$$

$$x, y \geq 0$$

32. A bag contains two coins, one biased and the other unbiased. When tossed, the biased coin has a 60% chance of showing heads. One of the coins is selected at random and on tossing it shows tails. What is the probability it was an unbiased coin?

OR

The probability distribution of a random variable X , where k is a constant is given below:

$$P(X = x) = \begin{cases} 0.1, & \text{if } x = 0 \\ kx^2, & \text{if } x = 1 \\ kx, & \text{if } x = 2 \text{ or } 3 \\ 0, & \text{otherwise} \end{cases}$$

Determine: (a) the value of k ; (b) $P(X \leq 2)$; (c) Mean of the variable X .

Section D

33. Solve the following system of equations by matrix method:

$$x - y + 2z = 7$$

$$2x - y + 3z = 12$$

$$3x + 2y - z = 8$$

OR

Obtain the inverse of the following matrix using elementary operations, where

$$A = \begin{bmatrix} 2 & 1 & -3 \\ -1 & -1 & 4 \\ 3 & 0 & 2 \end{bmatrix}.$$

34. Find the points on the curve $9y^2 = x^3$, where the normal to the curve makes equal intercepts with both the axes. Also find the equation of the normals.

35. Find the area of the following region using integration:

$$\{(x, y) : y \leq |x| + 2, y \geq x^2\}$$

OR

Using integration, find the area of a triangle whose vertices are (1, 0), (2, 2) and (3, 1).

36. Show that the lines

$$\frac{x-2}{\frac{1}{2}} = \frac{y-2}{3} = \frac{z-3}{1} \text{ and } \frac{x-2}{1} = \frac{y-3}{4} = \frac{z-\frac{1}{2}}{2} \text{ intersect.}$$

Also, find the coordinates of the point of intersection. Find the equation of the plane containing the two lines.

- End of Question Paper -