

Question Paper

Conducted by the Central Board of Secondary Education (CBSE)



Time Allowed: 3 Hours

Maximum Marks: 80

General Instructions

- (i) This question paper contains **36 questions**. All questions are compulsory.
- (ii) Section A: Questions 1 to 20 carry 1 mark each.
- (iii) Section B: Questions 21 to 26 carry 2 marks each.
- (iv) Section C: Questions 27 to 32 carry 4 marks each.
- (v) Section D: Questions 33 to 36 carry 6 marks each.
- (vi) Attempt one alternative where an internal choice is provided. Follow the instructions within each question.
- (vii) Use of calculators is not allowed.

Section A

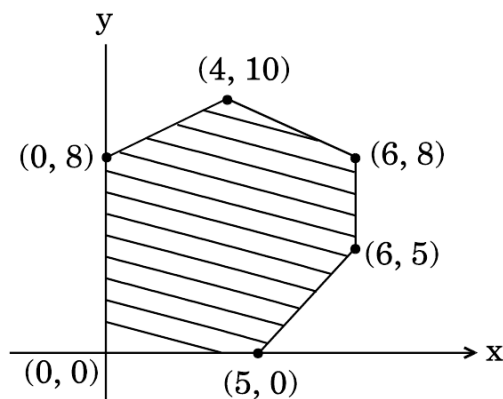
1. The value of p for which $p(\hat{i} + \hat{j} + \hat{k})$ is a unit vector is

- (A) 0
- (B) $\frac{1}{\sqrt{3}}$
- (C) 1
- (D) $\sqrt{3}$

2. $\left(\tan^{-1} \frac{7}{9} + \tan^{-1} \frac{1}{8}\right)$ is equal to

- (A) $\tan^{-1} \left(\frac{65}{72}\right)$
- (B) $\tan^{-1} \left(\frac{63}{65}\right)$
- (C) $\frac{\pi}{4}$
- (D) $\frac{\pi}{2}$

3. The feasible region for an LPP is shown below:



Let $z = 3x - 4y$ be the objective function.
Minimum of z occurs at

- (A) (0, 0)
- (B) (0, 8)
- (C) (5, 0)
- (D) (4, 10)

4. If f and g are two functions from $\mathbb{R}$ to $\mathbb{R}$ defined as $f(x) = |x| + x$ and $g(x) = |x| - x$, then $f \circ g(x)$ for $x < 0$ is

- (A) $4x$
- (B) $2x$
- (C) 0
- (D) $-4x$

5. $\int e^x \left(\frac{x \log x + 1}{x} \right) dx$ is equal to

- (A) $\log(e^x \log x) + c$
- (B) $\frac{e^x}{x} + c$
- (C) $x \log x + e^x + c$
- (D) $e^x \log x + c$

6. The integrating factor of the differential equation $(x + 3y^2) \frac{dy}{dx} = y$ is

- (A) y
- (B) $-y$
- (C) $\frac{1}{y}$
- (D) $-\frac{1}{y}$

7. If $A = \begin{bmatrix} -2 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -2 \end{bmatrix}$, then the value of $|\text{adj } A|$ is

- (A) 64
- (B) 16
- (C) 0
- (D) -8

8. The distance of the point $(2, 3, 4)$ from the plane $\vec{r} \cdot (3\hat{i} - 6\hat{j} + 2\hat{k}) = -11$ is

- (A) 0 unit
- (B) 1 unit
- (C) 2 units
- (D) $\frac{15}{7}$ units

9. The maximum value of slope of the curve $y = -x^3 + 3x^2 + 12x - 5$ is

- (A) 15
- (B) 12
- (C) 9
- (D) 0

10. The vector equation of XY-plane is

- (A) $\vec{r} \cdot \hat{k} = 0$
- (B) $\vec{r} \cdot \hat{j} = 0$
- (C) $\vec{r} \cdot \hat{i} = 0$
- (D) $\vec{r} \cdot \hat{n} = 1$

11. The area of the parallelogram whose diagonals are $2\hat{i}$ and $-3\hat{k}$ is _____ square units.

OR

The value of λ for which the vectors $2\hat{i} - \lambda\hat{j} + \hat{k}$ and $\hat{i} + 2\hat{j} - \hat{k}$ are orthogonal is _____.

12. A bag contains 3 black, 4 red and 2 green balls. If three balls are drawn simultaneously at random, then the probability that the balls are of different colours is _____.

13. The absolute minimum value of $f(x) = 2 \sin x$ in $\left[\frac{\pi}{2}, \pi\right]$ is _____.

14. If $y = \tan^{-1} x + \cot^{-1} x$, $x \in \mathbb{R}$, then $\frac{dy}{dx}$ is equal to _____.

OR

If $\cos(xy) = k$, where k is a constant and $xy \neq n\pi$, $n \in \mathbb{Z}$, then $\frac{dy}{dx}$ is equal to _____.

15. The value of λ so that the function f defined by

$$f(x) = \begin{cases} \lambda x, & \text{if } x \leq \pi \\ \cos x, & \text{if } x > \pi \end{cases}$$

is continuous at $x = \pi$ is _____.

16. Evaluate: $\int_{-2}^2 x^3 dx$.

OR

Find: $\int \frac{dx}{9 + 4x^2}$.

17. Find the interval in which the function f given by $f(x) = 7 - 4x - x^2$ is strictly increasing.

18. Differentiate $\sin^2(\sqrt{x})$ with respect to x .

19. Construct a 2×2 matrix $A = [a_{ij}]$ whose elements are given by $a_{ij} = |(i)^2 - j|$.

20. A problem is given to three students whose probabilities of solving it are $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$ respectively. If the events of solving the problem are independent, find the probability that at least one of them solves it.

Section B

21. Show that for any two non-zero vectors $\vec{a}$ and $\vec{b}$, $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$ if and only if $\vec{a}$ and $\vec{b}$ are perpendicular vectors.

OR

Show that the vectors $2\hat{i} - \hat{j} + \hat{k}$, $3\hat{i} + 7\hat{j} + \hat{k}$ and $5\hat{i} + 6\hat{j} + 2\hat{k}$ form the sides of a right-angled triangle.

22. Find $(AB)^{-1}$ if $A = \begin{bmatrix} 1 & 0 & 1 \\ -4 & 2 & 5 \end{bmatrix}$ and $B^{-1} = \begin{bmatrix} 3 & 1 \\ 1 & 0 \\ 2 & 2 \end{bmatrix}$.

23. If $x = a \sec \theta$, $y = b \tan \theta$, then find $\frac{dy}{dx}$ at $\theta = \frac{\pi}{4}$.

24. If A and B are two events such that $P(A) = 0.4$, $P(B) = 0.3$ and $P(A \cup B) = 0.6$, then find $P(B' \cap A)$.

25. Solve for x : $\sin^{-1} 4x + \sin^{-1} 3x = -\frac{\pi}{2}$.

OR

Express $\tan^{-1}\left(\frac{\cos x}{1 - \sin x}\right)$, $-\frac{3\pi}{2} < x < \frac{\pi}{2}$ in the simplest form.

26. Find the coordinates of the point where the line through $(-1, 1, -8)$ and $(5, -2, 10)$ crosses the ZX-plane.

Section C

27. Solve the following LPP graphically:

Minimise $z = 5x + 7y$

subject to the constraints

$$2x + y \geq 8$$

$$x + 2y \geq 10$$

$$x, y \geq 0$$

28. Evaluate: $\int_{-1}^2 |x^2 - x| dx$.

29. A bag contains two coins, one biased and the other unbiased. When tossed, the biased coin has a 60% chance of showing heads. One of the coins is selected at random and on tossing it shows tails. What is the probability it was an unbiased coin?

OR

The probability distribution of a random variable X , where k is a constant is given below:

$$P(X = x) = \begin{cases} 0.1, & \text{if } x = 0 \\ kx^2, & \text{if } x = 1 \\ kx, & \text{if } x = 2 \text{ or } 3 \\ 0, & \text{otherwise} \end{cases}$$

Determine: (a) the value of k ; (b) $P(X < 2)$; (c) Mean of the variable X .

30. Find the particular solution of the differential equation

$$\cos y dx + (1 + e^{-x}) \sin y dy = 0$$

given that $y = \frac{\pi}{4}$ when $x = 0$.

31. Show that the function $f : (-\infty, 0) \rightarrow (-1, 0)$ defined by $f(x) = \frac{x}{1 + |x|}$, $x \in (-\infty, 0)$ is one-one and onto.

OR

Show that the relation R in the set $A = \{1, 2, 3, 4, 5, 6\}$ given by $R = \{(a, b) : |a - b| \text{ is divisible by } 2\}$ is an equivalence relation.

32. If $y = x^3(\cos x)^x + \sin^{-1} \sqrt{x}$, then find $\frac{dy}{dx}$.

Section D

33. Find the points on the curve $9y^2 = x^3$, where the normal to the curve makes equal intercepts with both the axes. Also find the equation of the normals.

34. Show that the lines

$$\frac{x-2}{\frac{1}{2}} = \frac{y-2}{3} = \frac{z-3}{1} \text{ and } \frac{x-2}{1} = \frac{y-3}{4} = \frac{z-4}{2} \text{ intersect.}$$

Also, find the coordinates of the point of intersection. Find the equation of the plane containing the two lines.

35. Using integration, find the area of the region bounded by the lines $x - y = 0$, $3x - y = 0$ and $x + y = 12$.

OR

Using integration, find the smaller area enclosed by the circle $x^2 + y^2 = 4$ and the line $x + y = 2$.

36. Solve the following system of equations by matrix method:

$$x - y + 2z = 7$$

$$2x - y + 3z = 12$$

$$3x + 2y - z = 8$$

OR

Obtain the inverse of the following matrix using elementary operations, where

$$A = \begin{bmatrix} 2 & 1 & -3 \\ -1 & -1 & 4 \\ 3 & 0 & 2 \end{bmatrix}.$$

- End of Question Paper -