

Question Paper

Conducted by the Central Board of Secondary Education (CBSE)



Time Allowed: 3 Hours

Maximum Marks: 80

General Instructions

- (i) This question paper contains **36 questions**. All questions are compulsory.
- (ii) Section A: Questions 1 to 20 carry 1 mark each.
- (iii) Section B: Questions 21 to 26 carry 2 marks each.
- (iv) Section C: Questions 27 to 32 carry 4 marks each.
- (v) Section D: Questions 33 to 36 carry 6 marks each.
- (vi) Attempt one alternative where an internal choice is provided. Follow the instructions within each question.
- (vii) Use of calculators is not allowed.

Section A

1. The two lines $x = ay + b$, $z = cy + d$ and $x = a'y + b'$, $z = c'y + d'$ are perpendicular to each other, if

- (A) $\frac{a}{a'} + \frac{c}{c'} = 1$
- (B) $\frac{a}{a'} + \frac{c}{c'} = -1$
- (C) $aa' + cc' = 1$
- (D) $aa' + cc' = -1$

2. If $\begin{vmatrix} 2 & 3 & 2 \\ x & x & x \\ 4 & 9 & 1 \end{vmatrix} + 3 = 0$, then the value of x is

- (A) 3
- (B) 0
- (C) -1
- (D) 1

3. In an LPP, if the objective function $z = ax + by$ has the same maximum value on two corner points of the feasible region, then the number of points at which $z_{\max}$ occurs is

- (A) 0
- (B) 2
- (C) finite
- (D) infinite

4. From the set $\{1, 2, 3, 4, 5\}$, two numbers a and b ($a \neq b$) are chosen at random. The probability that $\frac{a}{b}$ is an integer is:

- (A) $\frac{1}{3}$
- (B) $\frac{1}{4}$
- (C) $\frac{1}{2}$
- (D) $\frac{3}{5}$

5. $\int_0^{\pi/8} \tan^2(2x) dx$ is equal to

- (A) $\frac{4 - \pi}{8}$
- (B) $\frac{4 + \pi}{8}$
- (C) $\frac{4 - \pi}{4}$
- (D) $\frac{4 - \pi}{2}$

6. If $\vec{a} \cdot \vec{b} = \frac{1}{2}|\vec{a}||\vec{b}|$, then the angle between $\vec{a}$ and $\vec{b}$ is

- (A) 0°
- (B) 30°
- (C) 60°
- (D) 90°

7. A bag contains 3 white, 4 black and 2 red balls. If 2 balls are drawn at random (without replacement), then the probability that both the balls are white is

- (A) $\frac{1}{18}$
- (B) $\frac{1}{36}$
- (C) $\frac{1}{12}$
- (D) $\frac{1}{24}$

8. The value of $\tan^{-1} \left[\frac{1}{2} \cos^{-1} \left(\frac{\sqrt{5}}{3} \right) \right]$ is

- (A) $\frac{3 + \sqrt{5}}{2}$
(B) $\frac{3 - \sqrt{5}}{2}$
(C) $\frac{-3 + \sqrt{5}}{2}$
(D) $\frac{-3 - \sqrt{5}}{2}$

9. If $A = \begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}$, then $\det(\text{adj } A)$ equals

- (A) a^{27}
(B) a^9
(C) a^6
(D) a^2

10. The line $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$ is parallel to the plane

- (A) $2x + 3y + 4z = 0$
(B) $3x + 4y - 5z = 7$
(C) $2x + y - 2z = 0$
(D) $x - y + z = 2$

11. The slope of the tangent to the curve $y = x^3 - x$ at the point $(2, 6)$ is _____.

OR

The rate of change of the area of a circle with respect to its radius r , when $r = 3$ cm, is _____.

12. If $f: \mathbb{R} \rightarrow \mathbb{R}$ be given by $f(x) = (3 - x^3)^{1/3}$, then $f \circ f(x) =$ _____.

13. If $\vec{a}$ is a non-zero vector, then $(\vec{a} \cdot \hat{i})\hat{i} + (\vec{a} \cdot \hat{j})\hat{j} + (\vec{a} \cdot \hat{k})\hat{k}$ equals _____.

OR

The projection of the vector $\hat{i} - \hat{j}$ on the vector $\hat{i} + \hat{j}$ is _____.

14. If $\begin{bmatrix} x+y & 7 \\ 9 & x-y \end{bmatrix} = \begin{bmatrix} 2 & 7 \\ 9 & 4 \end{bmatrix}$, then $x \cdot y =$ _____.

15. If $f(x) = x|x|$, then $f'(x) =$ _____.

16. Show that the function $y = ax + 2a^2$ is a solution of the differential equation $2 \left(\frac{dy}{dx} \right)^2 + x \left(\frac{dy}{dx} \right) - y = 0$.

17. Find $\text{adj } A$, if $A = \begin{bmatrix} 2 & -1 \\ 4 & 3 \end{bmatrix}$.

18. If $\int_0^a \frac{dx}{1+4x^2} = \frac{\pi}{8}$, then find the value of a .

OR

Find $\int \frac{dx}{\sqrt{x+x}}$.

19. Find $\int \frac{1}{x(1+x^2)} dx$.

20. If $[x]$ denotes the greatest integer function, then find $\int_0^{3/2} [x^2] dx$.

Section B

21. Find $|\vec{a}|$ and $|\vec{b}|$, if $|\vec{a}| = 2|\vec{b}|$ and $(\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b}) = 12$.

OR

Find the unit vector perpendicular to each of the vectors $\vec{a} = 4\hat{i} + 3\hat{j} + \hat{k}$ and $\vec{b} = 2\hat{i} - \hat{j} + 2\hat{k}$.

22. Find the value of $\frac{dy}{dx}$ at $\theta = \frac{\pi}{3}$, if $x = \cos \theta - \cos 2\theta$, $y = \sin \theta - \sin 2\theta$.

23. Find the equation of the plane with intercept 3 on the y -axis and parallel to xz -plane.

24. Check if the relation R on the set $A = \{1, 2, 3, 4, 5, 6\}$ defined as $R = \{(x, y) : y \text{ is divisible by } x\}$ is (i) symmetric (ii) transitive.

OR

Prove that: $\frac{9\pi}{8} - \frac{9}{4} \sin^{-1}\left(\frac{1}{3}\right) = \frac{9}{4} \sin^{-1}\left(\frac{2\sqrt{2}}{3}\right)$.

25. Show that the function $f(x) = \frac{x}{3} + \frac{3}{x}$ decreases in the intervals $(-3, 0) \cup (0, 3)$.

26. Three distinct numbers are chosen randomly from the first 50 natural numbers. Find the probability that all the three numbers are divisible by both 2 and 3.

27. A manufacturer has three machines I, II and III installed in his factory. Machines I and II are capable of being operated for at most 12 hours whereas machine III must be operated for at least 5 hours a day. He produces only two items M and N each requiring the use of all the three machines.

The number of hours required for producing 1 unit of M and N on three machines are given in the following table:

Items	I	II	III
M	1	2	1
N	2	1	1.25

He makes a profit of Rs. 600 and Rs. 400 on one unit of items M and N respectively. How many units of each item should he produce so as to maximize his profit assuming that he can sell all the items that he produced? What will be the maximum profit?

28. Prove that the relation R on $\mathbb{Z}$, defined by $R = \{(x, y) : (x - y) \text{ is divisible by } 5\}$, is an equivalence relation.

29. A coin is biased so that the head is three times as likely to occur as tail. If the coin is tossed twice, find the probability distribution of number of tails. Hence find the mean of the number of tails.

OR

Suppose that 5 men out of 100 and 25 women out of 1000 are good orators. Assuming that there are equal number of men and women, find the probability of choosing a good orator.

30. If $y = \sin^{-1}\left(\frac{\sqrt{1+x} + \sqrt{1-x}}{2}\right)$, then show that $\frac{dy}{dx} = \frac{-1}{2\sqrt{1-x^2}}$.

OR

Verify the Rolle's Theorem for the function $f(x) = e^x \cos x$ in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

31. Evaluate $\int_0^1 \sqrt{3-2x-x^2} dx$.

32. Find the general solution of the differential equation $\frac{dy}{dx} + \frac{y}{x} = \frac{e^y}{x}$.

Section D

33. Find the area of the region lying in the first quadrant and enclosed by the x -axis, the line $y = x$ and the circle $x^2 + y^2 = 32$.

34. Using properties of determinants, prove that:

$$\begin{vmatrix} a-b & b+c & a \\ b-c & c+a & b \\ c-a & a+b & c \end{vmatrix} = a^3 + b^3 + c^3 - 3abc.$$

OR

If $A = \begin{bmatrix} 1 & 3 & 2 \\ 2 & 0 & -1 \\ 1 & 2 & 3 \end{bmatrix}$, then show that $A^3 - 4A^2 - 3A + 11I = O$. Hence find A^{-1} .

35. Find the intervals on which the function $f(x) = (x-1)^3(x-2)^2$ is (a) strictly increasing (b) strictly decreasing.

OR

Find the dimensions of the rectangle of perimeter 36 cm which will sweep out a volume as large as possible, when revolved about one of its side. Also, find the maximum volume.

36. Find the image of the point $(-1, 3, 4)$ in the plane $x - 2y = 0$.

- End of Question Paper -