

Question Paper

Conducted by the Central Board of Secondary Education (CBSE)



Time Allowed: 3 Hours

Maximum Marks: 80

General Instructions

- (i) This question paper contains **36 questions**. All questions are compulsory.
- (ii) Section A: Questions 1 to 20 carry 1 mark each.
- (iii) Section B: Questions 21 to 26 carry 2 marks each.
- (iv) Section C: Questions 27 to 32 carry 4 marks each.
- (v) Section D: Questions 33 to 36 carry 6 marks each.
- (vi) Attempt one alternative where an internal choice is provided. Follow the instructions within each question.
- (vii) Use of calculators is not allowed.

Section A

1. If $\begin{bmatrix} x & 1 \\ -2 & 0 \end{bmatrix} = O$, then x equals

- (a) 0 (b) -2 (c) -1 (d) 2

2. $\int 4^x 3^x dx$ equals

- (a) $\frac{12^x}{\log 12} + C$ (b) $\frac{4^x}{\log 4} + C$ (c) $\left(\frac{4^x \cdot 3^x}{\log 4 \cdot \log 3}\right) + C$ (d) $\frac{3^x}{\log 3} + C$

3. A number is chosen randomly from numbers 1 to 60. The probability that the chosen number is a multiple of 2 or 5 is

- (a) $\frac{2}{3}$ (b) $\frac{3}{5}$ (c) $\frac{7}{10}$ (d) $\frac{9}{16}$

4. $ABCD$ is a rhombus whose diagonals intersect at E . Then $\vec{EA} + \vec{EB} + \vec{EC} + \vec{ED}$ equals

- (a) $\vec{0}$ (b) $\vec{AD}$ (c) $2\vec{BC}$ (d) $2\vec{AD}$

5. If A is a square matrix of order 3, such that $A(\text{adj } A) = 10I$, then $|\text{adj } A|$ is equal to

- (a) 1 (b) 10 (c) 100 (d) 101

6. A card is picked at random from a pack of 52 playing cards. Given that the picked card is a queen, the probability of this card to be a card of spade is

- (a) $\frac{1}{3}$ (b) $\frac{4}{13}$ (c) $\frac{1}{4}$ (d) $\frac{1}{2}$

7. If $\hat{i}, \hat{j}, \hat{k}$ are unit vectors along three mutually perpendicular directions, then

- (a) $\hat{i} \cdot \hat{j} = 1$ (b) $\hat{i} \times \hat{j} = 1$ (c) $\hat{i} \cdot \hat{k} = 0$ (d) $\hat{i} \times \hat{k} = 0$

8. The graph of the inequality $2x + 3y > 6$ is

- (a) half plane that contains the origin.
(b) half plane that neither contains the origin nor the points of the line $2x + 3y = 6$.
(c) whole XOY -plane excluding the points on the line $2x + 3y = 6$.
(d) entire XOY plane.

9. The lines $\frac{x-2}{1} = \frac{y-3}{1} = \frac{4-z}{k}$ and $\frac{x-1}{k} = \frac{y-4}{2} = \frac{z-5}{-2}$ are mutually perpendicular if the value of k is

- (a) $-\frac{2}{3}$ (b) $\frac{2}{3}$ (c) -2 (d) 2

10. If $y = Ae^{5x} + Be^{-5x}$, then $\frac{d^2y}{dx^2}$ is equal to
(a) $25y$ (b) $5y$ (c) $-25y$ (d) $15y$

11. A relation R on a set A is called _____, if $(a_1, a_2) \in R$ and $(a_2, a_3) \in R$ implies that $(a_1, a_3) \in R$, for $a_1, a_2, a_3 \in A$.

12. The integrating factor of the differential equation $x \frac{dy}{dx} + 2y = x^2$ is _____.

OR

The degree of the differential equation $1 + \left(\frac{dy}{dx}\right)^2 = x$ is _____.

13. The vector equation of a line which passes through the points $(3, 4, -7)$ and $(1, -1, 6)$ is _____.

OR

The line of shortest distance between two skew lines is _____ to both the lines.

14. If $A + B = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ and $A - 2B = \begin{bmatrix} -1 & 1 \\ 0 & -1 \end{bmatrix}$, then matrix $A =$ _____.

15. The least value of the function $f(x) = ax + \frac{b}{x}$ ($a > 0, b > 0, x > 0$) is _____.

16. Evaluate: $\sin\left[\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right]$.

17. Using differential, find the approximate value of $\sqrt{36.6}$ upto 2 decimal places.

OR

Find the slope of tangent to the curve $y = 2\cos^2(3x)$ at $x = \frac{\pi}{6}$.

18. Find the value of $\int_1^4 |x - 5| dx$.

19. If the function f defined as

$$f(x) = \begin{cases} \frac{x^2 - 9}{x - 3}, & x \neq 3 \\ k, & x = 3 \end{cases}$$

is continuous at $x = 3$, find the value of k .

20. For $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$, write A^{-1} .

Section B

21. Find $\int \frac{x+1}{x(1-2x)} dx$.

22. Evaluate $\int \frac{x \sin^{-1}(x^2)}{\sqrt{1-x^4}} dx$.

23. Find the value of $\int_0^1 x(1-x)^n dx$.

24. If $x = a \cos \theta$; $y = b \sin \theta$, then find $\frac{d^2y}{dx^2}$.

OR

Find the differential of $\sin^2 x$ w.r.t. $e^{\cos x}$.

25. Given two independent events A and B such that $P(A) = 0.3$ and $P(B) = 0.6$, find $P(A' \cap B')$.

26. If $f(x) = \frac{4x+3}{6x-4}$, $x \neq \frac{2}{3}$, then show that $(f \circ f)(x) = x$, for all $x \neq \frac{2}{3}$. Also, write inverse of f .

OR

Check if the relation R in the set $\mathbb{R}$ of real numbers defined as $R = \{(a, b) : |a| < b\}$ is (i) symmetric, (ii) transitive.

Section C

27. Prove that $\tan \left[2 \tan^{-1} \left(\frac{1}{2} \right) - \cot^{-1} 3 \right] = \frac{9}{13}$.

28. If $y = (\cos x)^x + \tan^{-1} \sqrt{x}$, find $\frac{dy}{dx}$.

29. If $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$ and $\vec{b} = 2\hat{i} + 4\hat{j} - 5\hat{k}$ represent two adjacent sides of a parallelogram, find unit vectors parallel to the diagonals of the parallelogram.

OR

Using vectors, find the area of the triangle ABC with vertices $A(1, 2, 3)$, $B(2, -1, 4)$ and $C(4, 5, -1)$.

30. A company manufactures two types of novelty souvenirs made of plywood. Souvenirs of type A requires 5 minutes each for cutting and 10 minutes each for assembling. Souvenirs of type B require 8 minutes each for cutting and 8 minutes each for assembling. Given that total time for cutting is 3 hours 20 minutes and for assembling 4 hours. The profit for type A souvenir is ₹ 100 each and for type B souvenir, profit is ₹ 120 each. How many souvenirs of each type should the company manufacture in order to maximize the profit? Formulate the problem as an LPP and solve it graphically.

31. Three rotten apples are mixed with seven fresh apples. Find the probability distribution of the number of rotten apples, if three apples are drawn one by one with replacement. Find the mean of the number of rotten apples.

OR

In a shop X , 30 tins of ghee of type A and 40 tins of ghee of type B which look alike, are kept for sale. While in shop Y , similar 50 tins of ghee of type A and 60 tins of ghee of type B are there. One tin of ghee is purchased from one of the randomly selected shop and is found to be of type B . Find the probability that it is purchased from shop Y .

32. Solve the differential equation:

$$x \sin\left(\frac{y}{x}\right) \frac{dy}{dx} + x - y \sin\left(\frac{y}{x}\right) = 0.$$

Given that $x = 1$ when $y = \frac{\pi}{2}$.

Section D

33. If a, b, c are $p^{\text{th}}, q^{\text{th}}$ and r^{th} terms respectively of a G.P., then prove that

$$\begin{vmatrix} \log a & p & 1 \\ \log b & q & 1 \\ \log c & r & 1 \end{vmatrix} = 0.$$

OR

If $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, then find A^{-1} . Using A^{-1} ,

solve the following system of equations:

$$2x - 3y + 5z = 11$$

$$3x + 2y - 4z = -5$$

$$x + y - 2z = -3$$

34. Find the vector and cartesian equations of the line which is perpendicular to the lines with equations

$$\frac{x+2}{z-3} = \frac{y-3}{2} = \frac{z+1}{4} \text{ and } \frac{x-1}{2} = \frac{y-2}{3} = \frac{1}{4}$$

and passes through the point $(1, 1, 1)$. Also find the angle between the given lines.

35. Using integration find the area of the region bounded between the two circles $x^2 + y^2 = 9$ and $(x-3)^2 + y^2 = 9$.

OR

Evaluate the following integral as the limit of

$$\text{sums } \int_1^4 (x^2 - x) dx.$$

36. Find the point on the curve $y^2 = 4x$ which is nearest to the point $(2, 1)$.

- End of Question Paper -