

Question Paper

Conducted by the Central Board of Secondary Education (CBSE)



Time Allowed: 3 Hours

Maximum Marks: 80

General Instructions

- (i) This question paper contains **36 questions**. All questions are compulsory.
- (ii) Section A: Questions 1 to 20 carry 1 mark each.
- (iii) Section B: Questions 21 to 26 carry 2 marks each.
- (iv) Section C: Questions 27 to 32 carry 4 marks each.
- (v) Section D: Questions 33 to 36 carry 6 marks each.
- (vi) Attempt one alternative where an internal choice is provided. Follow the instructions within each question.
- (vii) Use of calculators is not allowed.

Section A

1. If A is a skew symmetric matrix of order 3, then the value of $|A|$ is

- (a) 3 (b) 0 (c) 9 (d) 27

2. If $\hat{i}, \hat{j}, \hat{k}$ are unit vectors along three mutually perpendicular directions, then

- (a) $\hat{i} \cdot \hat{j} = 1$ (b) $\hat{i} \times \hat{j} = 1$ (c) $\hat{i} \cdot \hat{k} = 0$ (d) $\hat{i} \times \hat{k} = 0$

3. A card is picked at random from a pack of 52 playing cards. Given that the picked card is a queen, the probability of this card to be a card of spade is

- (a) $\frac{1}{3}$ (b) $\frac{4}{13}$ (c) $\frac{1}{4}$ (d) $\frac{1}{2}$

4. If A is a 3×3 matrix such that $|A| = 8$, then $|3A|$ equals

- (a) 8 (b) 24 (c) 72 (d) 216

5. $\int x^2 e^{x^3} dx$ equals

- (a) $\frac{1}{3}e^{x^3} + C$ (b) $\frac{1}{3}e^{x^2} + C$ (c) $\frac{1}{2}e^{x^3} + C$
(d) $\frac{1}{2}e^{x^2} + C$

6. If $y = \log_e \left(\frac{x^2}{e^2} \right)$, then $\frac{d^2y}{dx^2}$ equals

- (a) $-\frac{1}{x}$ (b) $-\frac{1}{x^2}$ (c) $\frac{2}{x^2}$ (d) $-\frac{2}{x^2}$

7. A die is thrown once. Let A be the event that the number obtained is greater than 3. Let B be the event that the number obtained is less than 5. Then $P(A \cup B)$ is

- (a) $\frac{2}{5}$ (b) $\frac{3}{5}$ (c) 0 (d) 1

8. $ABCD$ is a rhombus whose diagonals intersect at E . Then $\vec{EA} + \vec{EB} + \vec{EC} + \vec{ED}$ equals

- (a) $\vec{0}$ (b) $\vec{AD}$ (c) $2\vec{BC}$ (d) $2\vec{AD}$

9. The distance of the origin $(0, 0, 0)$ from the plane $-2x + 6y - 3z = -7$ is

- (a) 1 unit (b) $\sqrt{2}$ units (c) $2\sqrt{2}$ units (d) 3 units

10. The graph of the inequality $2x + 3y > 6$ is

- (a) half plane that contains the origin.
(b) half plane that neither contains the origin nor the points of the line $2x + 3y = 6$.
(c) whole XOY -plane excluding the points on the line $2x + 3y = 6$.
(d) entire XOY plane.

11. If A and B are square matrices each of order 3 and $|A| = 5$, $|B| = 3$, then the value of $|3AB|$ is _____.

12. The least value of the function $f(x) = ax + \frac{b}{x}$ ($a > 0$, $b > 0$, $x > 0$) is _____.

13. The vector equation of a line which passes through the points $(3, 4, -7)$ and $(1, -1, 6)$ is _____.

OR

The line of shortest distance between two skew lines is _____ to both the lines.

14. The integrating factor of the differential equation $x \frac{dy}{dx} + 2y = x^2$ is _____.

OR

The degree of the differential equation $1 + \left(\frac{dy}{dx}\right)^2 = x$ is _____.

15. A relation in a set A is called _____ relation, if each element of A is related to itself.

16. Find the cofactors of all the elements of $\begin{bmatrix} 1 & -2 \\ 4 & 3 \end{bmatrix}$.

17. Let $f(x) = x|x|$, for all $x \in \mathbb{R}$; check its differentiability at $x = 0$.

18. Find the value of $\sin^{-1}\left[\sin\left(-\frac{17\pi}{8}\right)\right]$.

19. Find the value of $\int_1^4 |x - 5| dx$.

20. If $f(x) = x^4 - 10$, then find the approximate value of $f(2.1)$.

OR

Find the slope of the tangent to the curve $y = 2\sin^2(3x)$ at $x = \frac{\pi}{6}$.

Section B

21. Find $\int \frac{x+1}{(x+2)(x+3)} dx$.

22. If $f(x) = \frac{4x+3}{6x-4}$, $x \neq \frac{2}{3}$, then show that $(f \circ f)(x) = x$, for all $x \neq \frac{2}{3}$. Also, write inverse of f .

OR

Check if the relation R in the set $\mathbb{R}$ of real numbers defined as $R = \{(a, b) : a < b\}$ is (i) symmetric, (ii) transitive.

23. Given two independent events A and B such that $P(A) = 0.3$ and $P(B) = 0.6$, find $P(A' \cap B')$.

24. Evaluate $\int_1^2 \left[\frac{1}{x} - \frac{1}{2x^2} \right] e^{2x} dx$.

25. If $x = a \cos \theta$; $y = b \sin \theta$, then find $\frac{d^2 y}{dx^2}$.

OR

Find the differential of $\sin^2 x$ w.r.t. $e^{\cos x}$.

26. Evaluate: $\int_0^1 \tan^{-1} \left(\frac{1-2x}{1+x-x^2} \right) dx$.

Section C

27. Solve the equation for x : $\sin^{-1}\left(\frac{5}{x}\right) + \sin^{-1}\left(\frac{12}{x}\right) = \frac{\pi}{2}$ ($x \neq 0$).

28. Find the general solution of the differential equation

$$y e^{y/x} dx = (x e^{y/x} + y^2) dy, \quad y \neq 0.$$

29. If $y = (\log x)^x + x^{\log x}$, then find $\frac{dy}{dx}$.

30. Three rotten apples are mixed with seven fresh apples. Find the probability distribution of the number of rotten apples, if three apples are drawn one by one with replacement. Find the mean of the number of rotten apples.

OR

In a shop X , 30 tins of ghee of type A and 40 tins of ghee of type B which look alike, are kept for sale. While in shop Y , similar 50 tins of ghee of type A and 60 tins of ghee of type B are there. One tin of ghee is purchased from one of the randomly selected shop and is found to be of type B . Find the probability that it is purchased from shop Y .

31. A company manufactures two types of novelty souvenirs made of plywood. Souvenirs of type A requires 5 minutes each for cutting and 10 minutes each for assembling. Souvenirs of type B require 8 minutes each for cutting and 8 minutes each for assembling. Given that total time for cutting is 3 hours 20 minutes and for assembling 4 hours. The profit for type A souvenir is ₹ 100 each and for type B souvenir, profit is ₹ 120 each. How many souvenirs of each type should the company manufacture in order to maximize the profit? Formulate the problem as an LPP and solve it graphically.

32. If $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$ and $\vec{b} = 2\hat{i} + 4\hat{j} - 5\hat{k}$ represent two adjacent sides of a parallelogram, find unit vectors parallel to the diagonals of the parallelogram.

OR

Using vectors, find the area of the triangle ABC with vertices $A(1, 2, 3)$, $B(2, -1, 4)$ and $C(4, 5, -1)$.

Section D

33. Find the distance of the point $P(3, 4, 4)$ from the point, where the line joining the points $A(3, -4, -5)$ and $B(2, -3, 1)$ intersects the plane $2x + y + z = 7$.

34. Find the minimum value of $(ax + by)$, where $xy = c^2$.

35. If a, b, c are $p^{\text{th}}, q^{\text{th}}$ and r^{th} terms respectively of a G.P., then prove that

$$\begin{vmatrix} \log a & p & 1 \\ \log b & q & 1 \\ \log c & r & 1 \end{vmatrix} = 0.$$

OR

If $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, then find A^{-1} . Using A^{-1} ,

solve the following system of equations:

$$2x - 3y + 5z = 11$$

$$3x + 2y - 4z = -5$$

$$x + y - 2z = -3$$

36. Using integration find the area of the region bounded between the two circles $x^2 + y^2 = 9$ and $(x - 3)^2 + y^2 = 9$.

OR

Evaluate the following integral as the limit of sums $\int_1^4 (x^2 - x) dx$.

- End of Question Paper -