

CBSE Class 12 Physics Question Paper 2020 with Solution PDF-Set 55/1/1



Question Paper with Solutions

Conducted by the Central Board of Secondary Education (CBSE)

Time Allowed: 3 hours

Maximum Marks: 70

General Instructions

- (i) This question paper comprises **four** Sections – **A, B, C** and **D**.
- (ii) There are **37** questions in the question paper. **All** questions are compulsory.
- (iii) **Section A** – Questions no. **1 to 20** are very short answer type questions, carrying **one** mark each.
- (iv) **Section B** – Questions no. **21 to 27** are short answer type questions, carrying **two** marks each.
- (v) **Section C** – Questions no. **28 to 34** are long answer type questions, carrying **three** marks each.
- (vi) **Section D** – Questions no. **35 to 37** are also long answer type questions, carrying **five** marks each.
- (vii) There is no overall choice in the question paper. However, an internal choice has been provided in 2 questions of 1 mark, 2 questions of 2 marks, 1 question of three marks and all the 3 questions of five marks. You have to attempt only one of the choices in such questions.
- (viii) In addition to this, separate instructions are given with each section and question, wherever necessary.
- (ix) Use of calculators and log tables is not permitted.

You may use the following values of physical constants wherever necessary.

$$c = 3 \times 10^8 \text{ m/s}$$

$$h = 6.63 \times 10^{-34} \text{ Js}$$

$$e = 1.6 \times 10^{-19} \text{ C}$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ T m A}^{-1}$$

$$\epsilon_0 = 8.854 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$$

$$\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$$

$$\text{Mass of electron } (m_e) = 9.1 \times 10^{-31} \text{ kg}$$

$$\text{Mass of neutron} = 1.675 \times 10^{-27} \text{ kg}$$

$$\text{Mass of proton} = 1.673 \times 10^{-27} \text{ kg}$$

$$\text{Avogadro's number} = 6.023 \times 10^{23} \text{ per gram mole}$$

$$\text{Boltzmann constant} = 1.38 \times 10^{-23} \text{ JK}^{-1}$$

Section A (20 marks)

Note : Select the most appropriate option from those given below each question :

1. If the net electric flux through a closed surface is zero, then we can infer

[1 mark]

- (A) no net charge is enclosed by the surface.
- (B) uniform electric field exists within the surface.
- (C) electric potential varies from point to point inside the surface.
- (D) charge is present inside the surface.

Correct Answer: (A) no net charge is enclosed by the surface.

Solution:

Concept:

- Gauss's law states that $\Phi_E = \oint \vec{E} \cdot d\vec{A} = Q_{\text{enclosed}}/\epsilon_0$, where Q_{enclosed} is the algebraic sum of enclosed charges.
- Zero net flux constrains net enclosed charge, not the field at individual points.

Step 1: Apply the integral law The given flux is zero. Therefore $Q_{\text{enclosed}} = \epsilon_0 \Phi_E = 0$. Positive and negative enclosed charges may cancel, so the statement is about net charge, not the absence of every charge.

Step 2: Check the other inferences An external charge can produce a nonuniform field inside the surface while the net flux remains zero. A zero field is also possible, so potential need not vary. Charge may be present as cancelling pairs, but its presence is not compulsory. Thus (B), (C) and (D) are not necessary consequences.

Step 3: Conclusion No net charge is enclosed by the surface. Hence, the correct option is (A).

Quick Tip to Solve: For a closed surface, translate net flux directly into algebraic enclosed charge; do not infer the local field.

2. An electric dipole consisting of charges $+q$ and $-q$ separated by a distance L is in stable equilibrium in a uniform electric field $\vec{E}$. The electrostatic potential energy of the dipole is [1 mark]

- (A) qLE
- (B) zero
- (C) $-qLE$
- (D) $-2qLE$

Correct Answer: (C) $-qLE$

Solution:

Concept:

- The electric dipole moment has magnitude $p = qL$, where L is the full separation of the charges.
- In a uniform field the potential energy is $U = -pE \cos \theta$, with θ the angle between $\vec{p}$ and $\vec{E}$.

Step 1: Identify stable equilibrium Stable equilibrium is the minimum of potential energy, reached when the dipole moment is parallel to the field: $\theta = 0$. Antiparallel alignment has zero torque too, but is unstable because a small rotation lowers its energy.

Step 2: Evaluate energy and compare options Substituting the full charge separation gives $U = -qLE \cos 0 = -qLE$. The positive value in (A) belongs to unstable equilibrium; zero in (B) corresponds to perpendicular orientation. The factor two in (D) incorrectly doubles the specified separation.

Step 3: Conclusion The electrostatic potential energy is $-qLE$. Hence, the correct option is (C).

Quick Tip to Solve: Use the stated full separation in $p=qL$, then choose the minimum energy for stable alignment.

3. A potentiometer can measure emf of a cell because [1 mark]

- (A) the sensitivity of potentiometer is large.
- (B) no current is drawn from the cell at balance.
- (C) no current flows in the wire of potentiometer at balance.
- (D) internal resistance of cell is neglected.

Correct Answer: (B) no current is drawn from the cell at balance.

Solution:

Concept:

- A cell of emf $\mathcal{E}$, internal resistance r , and delivered current I has terminal voltage $V = \mathcal{E} - Ir$.
- A potentiometer balances the cell voltage against the potential drop along a calibrated wire.

Step 1: Use the null condition At balance the galvanometer current is zero, so no current is drawn from the cell being measured. Consequently its internal voltage drop Ir vanishes and its terminal voltage equals its emf. The balance length therefore gives $\mathcal{E} = k\ell$, where k is the potential gradient.

Step 2: Distinguish the two circuits The primary circuit must continue to carry current through the potentiometer wire to establish its potential gradient, making (C) false. High sensitivity in (A) improves precision but is not the reason emf is measured. The internal resistance in (D) need not be neglected: its voltage drop is zero at null balance.

Step 3: Conclusion No current is drawn from the cell at balance. Hence, the correct option is (B).

Quick Tip to Solve: At a potentiometer null, distinguish zero current in the test cell from the continuing current in the wire.

4. Two resistors R_1 and R_2 of 4Ω and 6Ω are connected in parallel across a battery. The ratio of power dissipated in them, $P_1 : P_2$ will be [1 mark]

- (A) 4 : 9
- (B) 3 : 2
- (C) 9 : 4
- (D) 2 : 3

Correct Answer: (B) 3 : 2

Solution:

Concept:

- Parallel resistors have the same voltage V across their terminals.
- Power in a resistor is $P = VI = V^2/R$; thus at fixed voltage it varies inversely with resistance.

Step 1: Form the power ratio For the common battery voltage, $P_1 = V^2/R_1$ and $P_2 = V^2/R_2$. Dividing cancels the unknown voltage and gives $P_1/P_2 = R_2/R_1$. No numerical battery voltage is required.

Step 2: Substitute and check direction With $R_1 = 4\Omega$ and $R_2 = 6\Omega$, $P_1/P_2 = 6/4 = 3/2$. The smaller resistance draws more current and dissipates more power, confirming the ratio exceeds one. Option (D) reverses the ratio; (A) and (C) incorrectly involve squared resistance ratios.

Step 3: Conclusion $P_1 : P_2 = 3 : 2$. Hence, the correct option is (B).

Quick Tip to Solve: For parallel branches use inverse resistance for power; for series branches use direct resistance.

5. The magnetic dipole moment of a current carrying coil does not depend upon

[1 mark]

- (A) number of turns of the coil.
- (B) cross-sectional area of the coil.
- (C) current flowing in the coil.
- (D) material of the turns of the coil.

Correct Answer: (D) material of the turns of the coil.

Solution:

Concept:

- The magnetic dipole moment of a plane coil is $\vec{m} = NIA\vec{A}$, with N turns, current I , and area vector $\vec{A}$.
- The area vector is perpendicular to the plane of the coil, in the right-hand-rule direction.

Step 1: Identify explicit dependences The magnitude $m = NIA$ is proportional to the number of turns, the current, and the area enclosed by each turn. Increasing any of these while holding the others fixed increases the dipole moment. Thus (A), (B) and (C) all describe quantities on which it depends.

Step 2: Check the material choice The material of the conducting turns does not appear in $m = NIA$. Different wire materials may alter resistance and hence the current for a fixed applied voltage, but with the current specified their magnetic dipole moments are identical for the same geometry and number of turns.

Step 3: Conclusion The moment does not depend on the material of the turns for a given current. Hence, the correct option is (D).

Quick Tip to Solve: Write $m = NIA$ first and compare each option with its three factors, holding current fixed.

6. Larger aperture of objective lens in an astronomical telescope

[1 mark]

- (A) increases the resolving power of telescope.
- (B) decreases the brightness of the image.
- (C) increases the size of the image.
- (D) decreases the length of the telescope.

Correct Answer: (A) increases the resolving power of telescope.

Solution:

Concept:

- For a circular objective of diameter D , the diffraction limit is approximately $\theta_{\min} = 1.22\lambda/D$.
- Resolving power is inversely proportional to the smallest resolvable angular separation.

Step 1: Relate aperture to resolution A larger aperture means a larger objective diameter D . At a given wavelength this reduces $\theta_{\min}$, allowing the telescope to distinguish stars with a smaller angular separation. Its resolving power therefore increases.

Step 2: Check brightness and image scale The objective also collects more light because its collecting area increases, so (B) has the wrong direction. Image scale depends on focal length, and angular magnification on the ratio of focal lengths; aperture alone does not increase image size. Telescope length in normal adjustment is $f_o + f_e$, so (D) is also unsupported.

Step 3: Conclusion A larger objective aperture increases resolving power. Hence, the correct option is (A).

Quick Tip to Solve: Associate objective diameter with diffraction and light collection; associate focal length with image scale.

7. A biconvex lens of glass having refractive index 1.47 is immersed in a liquid. It becomes invisible and behaves as a plane glass plate. The refractive index of the liquid is [1 mark]

- (A) 1.47
- (B) 1.62
- (C) 1.33
- (D) 1.51

Correct Answer: (A) 1.47

Solution:

Concept:

- A lens immersed in a medium has power $P = (n_g/n_l - 1)(1/R_1 - 1/R_2)$, where n_g and n_l are glass and liquid indices.
- Index matching removes refraction at the glass-liquid interfaces and therefore removes lens power.

Step 1: Use the absence of lens action The biconvex lens has nonzero curvature factor, but behaves without converging power in the liquid. Thus the relative refractive-index factor must vanish: $n_g/n_l - 1 = 0$. Its invisibility likewise indicates matching refractive indices at the boundary.

Step 2: Determine the liquid index Therefore $n_l = n_g = 1.47$. Each other listed index differs from the glass index and gives nonzero optical power; a liquid with a larger index would even make the biconvex lens diverge. Only (A) satisfies both zero lens power and the stated invisibility.

Step 3: Conclusion The liquid has refractive index 1.47. Hence, the correct option is (A).

Quick Tip to Solve: An invisible immersed lens indicates index matching: set the relative refractive index equal to one.

8. For a glass prism, the angle of minimum deviation will be smallest for the light of [1 mark]

- (A) red colour.
- (B) blue colour.
- (C) yellow colour.
- (D) green colour.

Correct Answer: (A) red colour.

Solution:

Concept:

- For prism angle A , $n = \sin[(A + \delta_{\min})/2] / \sin(A/2)$, where $\delta_{\min}$ is minimum deviation.
- In ordinary glass under normal dispersion, refractive index decreases as visible wavelength increases.

Step 1: Compare refractive indices Red light has the longest wavelength among red, yellow, green and blue. The refractive index of glass is therefore smallest for red light. Yellow and green have larger indices, while blue has the largest index of the listed colours.

Step 2: Relate index to minimum deviation For the same prism angle the minimum-deviation formula increases monotonically with refractive index in the physical range. Consequently the smallest index gives the smallest minimum deviation. The other three options all have shorter wavelengths and therefore greater minimum deviations than red.

Step 3: Conclusion The minimum deviation is smallest for red light. Hence, the correct option is (A).

Quick Tip to Solve: For ordinary glass remember red has the smallest refractive index and hence the least prism deviation.

9. Which of the following statements is not correct according to Rutherford model?

[1 mark]

- (A) Most of the space inside an atom is empty.
- (B) The electrons revolve around the nucleus under the influence of coulomb force acting on them.
- (C) Most part of the mass of the atom and its positive charge are concentrated at its centre.
- (D) The stability of atom was established by the model.

Correct Answer: (D) The stability of atom was established by the model.

Solution:

Concept:

- Rutherford scattering established that the atom contains a tiny, massive, positively charged nucleus and is mostly empty space.
- An orbiting electron is an accelerating charge; classical electrodynamics predicts that it radiates energy.

Step 1: Check the experimentally supported statements Most alpha particles passed through the foil, supporting (A). The nuclear positive charge attracts electrons by the Coulomb force, as in (B). Rare large deflections indicate that most mass and positive charge lie at the centre, supporting (C).

Step 2: Examine atomic stability A revolving electron continually changes the direction of its velocity and is therefore accelerated. Classically it should radiate energy, lose orbital energy and spiral into the nucleus. Rutherford's model provided no mechanism to prevent this collapse, so it did not establish atomic stability.

Step 3: Conclusion The incorrect statement is (D): the stability of the atom was established by the model.

Quick Tip to Solve: For a question asking what Rutherford could not explain, test stability against radiation from an accelerating electron.

10. Photons of energies 1 eV and 2 eV are successively incident on a metallic surface of work function 0.5 eV. The ratio of kinetic energy of most energetic photoelectrons in the two cases will be [1 mark]

- (A) 1 : 2
- (B) 1 : 1
- (C) 1 : 3
- (D) 1 : 4

Correct Answer: (C) 1 : 3

Solution:

Concept:

- The maximum photoelectron kinetic energy is $K_{\max} = E_{\gamma} - \phi$, where ϕ is the work function.
- Emission requires $E_{\gamma} \geq \phi$; both incident energies here exceed the threshold.

Step 1: Calculate the two maximum energies For the first photon, $K_1 = (1 - 0.5) \text{ eV} = 0.5 \text{ eV}$. For the second photon, $K_2 = (2 - 0.5) \text{ eV} = 1.5 \text{ eV}$. The work function must be subtracted separately in each case.

Step 2: Form the ratio and test the options Thus $K_1 : K_2 = 0.5 : 1.5 = 1 : 3$. Option (A) merely compares incident photon energies and ignores the energy needed to liberate electrons. Option (B) would require equal excess energies, while (D) gives an incorrect second excess energy. Only (C) agrees with Einstein's equation.

Step 3: Conclusion The ratio is $1 : 3$, corresponding to option (C).

Quick Tip to Solve: Subtract the same work function from each photon energy before taking a photoelectron kinetic-energy ratio.

11. The magnetic field and angle of dip at a place on the earth are 0.3 G and 30° , respectively. The value of vertical component of the earth's magnetic field at the place is _____. [1 mark]

Solution:

Concept:

- The dip angle δ is measured from the horizontal, so the vertical component is $B_V = B \sin \delta$.
- The horizontal component is $B_H = B \cos \delta$; it is distinct from the requested vertical component.

Step 1: Identify the given field and angle The magnitude of the earth's magnetic field is $B = 0.3 \text{ G}$, and its inclination to the horizontal is $\delta = 30^\circ$. Since the vertical direction is opposite this angle in the component triangle, the vertical projection uses the sine.

Step 2: Resolve the field vertically Substitution gives

$$B_V = (0.3 \text{ G}) \sin 30^\circ = (0.3 \text{ G}) \frac{1}{2} = 0.15 \text{ G}.$$

Equivalently, using $1 \text{ G} = 10^{-4} \text{ T}$, the component is $1.5 \times 10^{-5} \text{ T}$.

Step 3: Conclusion The blank is 0.15 G .

Quick Tip to Solve: Dip is measured from the horizontal: use sine for the vertical component and cosine for the horizontal component.

12. Laminated iron sheets are used to minimize _____ currents in the core of a transformer. [1 mark]

Solution:

Concept:

- A changing magnetic flux induces circulating currents within a conducting iron core; these are called eddy currents.
- Insulated laminations interrupt large conducting loops and increase their electrical resistance, reducing Joule heating.

Step 1: Identify the induced currents The alternating current in the transformer windings produces a changing magnetic flux through the iron core. Faraday's law therefore induces electromotive forces within the core itself. In a solid conducting core, these drive closed circulating currents through the bulk metal.

Step 2: Explain why laminations suppress them The core is divided into thin iron sheets separated by electrical insulation. The insulation prevents current from flowing freely across adjacent sheets, breaking up the large circulating paths. The resulting smaller loops and greater effective resistance reduce induced current and its heating loss; the sheets still provide a magnetic path for transformer flux.

Step 3: Conclusion Laminated iron sheets minimize eddy currents.

Quick Tip to Solve: Associate insulated core sheets with interrupted current loops; hysteresis is reduced by choosing suitable soft magnetic material.

13. The number of turns of a solenoid are doubled without changing its length and area of cross-section. The self-inductance of the solenoid will become _____ times.

[1 mark]

Solution:

Concept:

- For a long solenoid, $L = \mu N^2 A / \ell$, where N is its number of turns, A its cross-sectional area and ℓ its length.
- For unchanged core material and geometry, inductance is proportional to the square of the number of turns.

Step 1: Write the original and changed inductances Initially, $L = \mu N^2 A / \ell$. Only the turn count changes: N is replaced by $2N$, while the length, area and permeability remain the same. Thus the new inductance is $L' = \mu (2N)^2 A / \ell$.

Step 2: Compare the two values Taking the ratio cancels the unchanged factors:

$$\frac{L'}{L} = \frac{(2N)^2}{N^2} = 4.$$

One factor of N arises because each turn contributes magnetic field; the other arises because flux links every turn. Hence doubling the turns does not merely double the inductance.

Step 3: Conclusion The self-inductance becomes 4 times its original value.

Quick Tip to Solve: When solenoid geometry stays fixed, square the turn-count multiplication factor to obtain the inductance factor.

14. According to Bohr's atomic model, the circumference of the electron orbit is always an _____ multiple of de Broglie wavelength. [1 mark]

Solution:

Concept:

- Bohr's angular-momentum condition is $mvr = n\hbar = nh/(2\pi)$, with positive integer n .
- The electron's de Broglie wavelength is $\lambda = h/(mv)$; an allowed orbit contains a whole number of wavelengths.

Step 1: Connect quantisation to wavelength Multiply the angular-momentum condition by $2\pi/(mv)$:

$$2\pi r = \frac{nh}{mv} = n\lambda.$$

Here $2\pi r$ is the circumference, and $n = 1, 2, 3, \dots$ is the principal quantum number.

Step 2: Interpret the allowed circumference After a complete circuit, the electron matter wave must join itself in phase to form a standing wave. This requires a whole number of wavelengths around the orbit. A nonintegral circumference-to-wavelength ratio would leave a phase mismatch and would not represent an allowed Bohr orbit.

Step 3: Conclusion The blank is integral (a positive integer) multiple.

Quick Tip to Solve: Remember the standing-wave condition directly as circumference equals a whole number of de Broglie wavelengths.

OR

14. In β -decay, the parent and daughter nuclei have the same number of _____. [1 mark]

Solution:

Concept:

- In beta decay, a nucleon changes type but the total nucleon number $A = Z + N$ is conserved.
- In β^- decay a neutron becomes a proton; in β^+ decay a proton becomes a neutron.

Step 1: Track the nuclear transformation For beta-minus decay, the process is $n \rightarrow p + e^- + \bar{\nu}_e$. One neutron is replaced by one proton, so Z increases by one and the neutron number decreases by one.

There is no loss of a nucleon from the nucleus.

Step 2: Identify the conserved count The daughter therefore has mass number A equal to that of the parent, although its atomic number changes. Beta-plus decay reverses the proton-neutron conversion and also preserves A . Neither proton number nor neutron number separately stays the same; their sum, the number of nucleons, does.

Step 3: Conclusion The blank is nucleons.

Quick Tip to Solve: In beta decay the nucleon changes identity; in alpha decay four nucleons leave, so the mass number changes.

15. A ray of light on passing through an equilateral glass prism, suffers a minimum deviation equal to the angle of the prism. The value of refractive index of the material of the prism is _____. [1 mark]

Solution:

Concept:

- At minimum deviation, the refractive index relative to air is $\mu = \sin[(A + \delta_m)/2] / \sin(A/2)$.
- An equilateral prism has apex angle $A = 60^\circ$; at minimum deviation its path is symmetric.

Step 1: Apply the given equality The minimum deviation equals the prism angle, so $\delta_m = A = 60^\circ$. At the symmetric position, the angle of incidence is $i = (A + \delta_m)/2 = 60^\circ$, and the refraction angle in the prism is $r = A/2 = 30^\circ$.

Step 2: Calculate the refractive index Using Snell's law from air into glass,

$$\mu = \frac{\sin 60^\circ}{\sin 30^\circ} = \frac{\sqrt{3}/2}{1/2} = \sqrt{3}.$$

The ratio is dimensionless. Using the prism angle itself as the internal refraction angle would ignore that the two equal refraction angles add to A .

Step 3: Conclusion The refractive index is $\sqrt{3}$, approximately 1.73.

Quick Tip to Solve: For an equilateral prism with minimum deviation equal to its apex angle, use sine 60 degrees divided by sine 30 degrees.

16. Write the mathematical form of Ampere-Maxwell circuital law. [1 mark]

Solution:

Concept:

- A conduction current I_c and a changing electric flux both produce a magnetic field.
- The displacement current is $I_d = \epsilon_0 d\Phi_E/dt$, where $\Phi_E = \int \vec{E} \cdot d\vec{A}$ is electric flux.

Step 1: Identify the currents For a surface bounded by a closed loop, I_c denotes the conduction current crossing the surface. Maxwell included the displacement current associated with the time variation of electric flux through that same surface.

Step 2: Write the circuital relation The circulation of the magnetic field around the bounding loop is

$$\oint \vec{B} \cdot d\vec{l} = \mu_0(I_c + I_d) = \mu_0 I_c + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}.$$

Here μ_0 and ε_0 are the permeability and permittivity of free space.

Step 3: Conclusion The Ampere-Maxwell law is $\oint \vec{B} \cdot d\vec{l} = \mu_0 \left(I_c + \varepsilon_0 \frac{d\Phi_E}{dt} \right).$

Quick Tip to Solve: Include both conduction current and displacement current in the circuital law.

17. How does an increase in doping concentration affect the width of depletion layer of a p-n junction diode? [1 mark]

Solution:

Concept:

- The depletion layer contains fixed ionised donors and acceptors left behind after carrier diffusion.
- Higher doping gives a greater density of these fixed charges.

Step 1: Relate doping to space charge Increasing the doping concentration increases the number of ionised impurity atoms per unit volume near the junction. Consequently, a given amount of space charge can be supplied by a thinner region.

Step 2: Explain the width change The charge in the depletion region produces the electric field that opposes further diffusion. With more charge per unit volume, this opposing field is established over a smaller distance, so the depletion layer becomes narrower. The depletion region extends farther into the more lightly doped side.

Step 3: Conclusion The width of the depletion layer decreases when the doping concentration increases.

Quick Tip to Solve: More fixed charge per unit volume means a thinner depletion region is sufficient.

18. The nuclear radius of ${}_{13}^{27}\text{Al}$ is 3.6 fermi. Find the nuclear radius of ${}_{29}^{64}\text{Cu}$. [1 mark]

Solution:

Concept:

- The nuclear radius follows $R = R_0 A^{1/3}$, where A is the mass number.
- The same radius constant R_0 is used when comparing the two nuclei.

Step 1: Form the radius ratio The relevant mass numbers are $A_{\text{Al}} = 27$ and $A_{\text{Cu}} = 64$. The atomic numbers do not enter the radius relation. Dividing the two radius equations eliminates R_0 :

$$\frac{R_{\text{Cu}}}{R_{\text{Al}}} = \left(\frac{64}{27} \right)^{1/3} = \frac{4}{3}.$$

Step 2: Substitute the given radius Using $R_{\text{Al}} = 3.6 \text{ fm}$,

$$R_{\text{Cu}} = 3.6 \times \frac{4}{3} \text{ fm} = 4.8 \text{ fm}.$$

The larger mass number therefore corresponds to the larger nuclear radius, as expected.

Step 3: Conclusion The nuclear radius of ${}^{64}_{29}\text{Cu}$ is $\boxed{4.8 \text{ fm}}$.

Quick Tip to Solve: Use cube roots of the mass numbers: 64 gives 4 and 27 gives 3.

OR

18. A proton and an electron have equal speeds. Find the ratio of de Broglie wavelengths associated with them. [1 mark]

Solution:

Concept:

- The de Broglie wavelength is $\lambda = h/p$, where h is Planck's constant and p is momentum.
- For equal nonrelativistic speeds, $p = mv$, so wavelength is inversely proportional to mass.

Step 1: Write each wavelength For a proton of mass m_p and an electron of mass m_e , both moving with speed v ,

$$\lambda_p = \frac{h}{m_p v}, \quad \lambda_e = \frac{h}{m_e v}.$$

Equal speed does not mean equal momentum, because the masses differ.

Step 2: Take the ordered ratio Dividing the proton wavelength by the electron wavelength cancels both h and v :

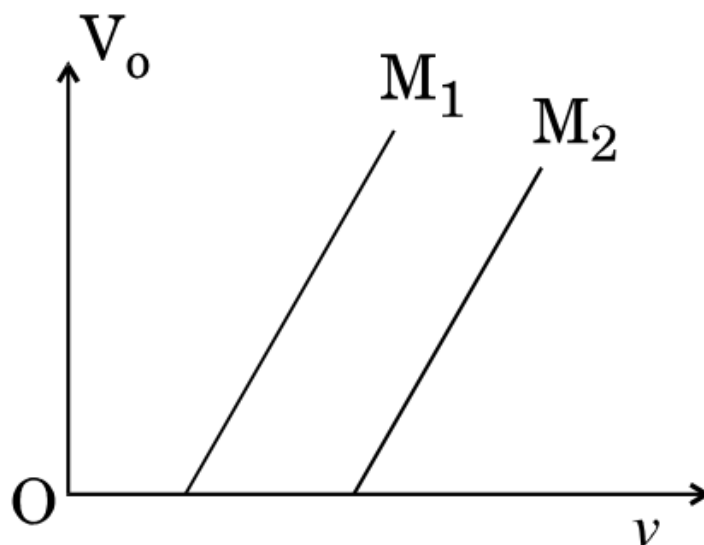
$$\frac{\lambda_p}{\lambda_e} = \frac{m_e}{m_p} \simeq \frac{1}{1836}.$$

The heavier proton has the smaller wavelength; the electron wavelength is about 1836 times larger.

Step 3: Conclusion In proton-to-electron order, $\boxed{\lambda_p : \lambda_e = 1 : 1836}$ approximately.

Quick Tip to Solve: State the order of the ratio explicitly: proton wavelength is the smaller one.

19. The variation of the stopping potential (V_0) with the frequency (ν) of the light incident on two different photosensitive surfaces M_1 and M_2 is shown in the figure. Identify the surface which has greater value of the work function. [1 mark]



Solution:

Concept:

- Einstein's photoelectric equation gives $eV_0 = h\nu - \phi$, with work function ϕ .
- At the threshold frequency ν_0 , the stopping potential is zero and $\phi = h\nu_0$.

Step 1: Read the frequency intercepts The threshold frequency is the point where each straight line meets the frequency axis, because $V_0 = 0$ there. In the printed graph, the line for M_2 meets this axis farther to the right than the line for M_1 . Thus $\nu_{0,2} > \nu_{0,1}$.

Step 2: Compare the work functions Since $\phi = h\nu_0$ and h is the same positive constant for both surfaces, the greater threshold frequency implies a greater work function. The slope h/e is common to the surfaces and does not determine their work-function difference.

Step 3: Conclusion Surface M_2 has the greater work function.

Quick Tip to Solve: Compare frequency-axis intercepts; the farther-right intercept means higher work function.

20. Why cannot we use Si and Ge in fabrication of visible LEDs? [1 mark]

Solution:

Concept:

- An LED emits light when electrons recombine with holes across the semiconductor band gap.
- Visible emission needs suitable photon energy and efficient radiative recombination.

Step 1: Compare the band gaps with visible light Silicon and germanium have small band gaps, approximately 1.1 eV and 0.7 eV, respectively. The corresponding photon wavelengths from $\lambda = hc/E_g$ lie in the infrared, beyond the visible range.

Step 2: Explain the low light efficiency Both materials also have indirect band gaps. Electron-hole recombination requires a change in crystal momentum, and energy is predominantly transferred through lattice vibrations rather than efficient photon emission. They are therefore unsuitable for

making visible LEDs.

Step 3: Conclusion Si and Ge cannot be used for visible LEDs because their small, indirect band gaps give inefficient emission in the infrared rather than visible light.

Quick Tip to Solve: Check both band-gap energy (colour) and direct or indirect gap (emission efficiency).

Section B (14 marks)

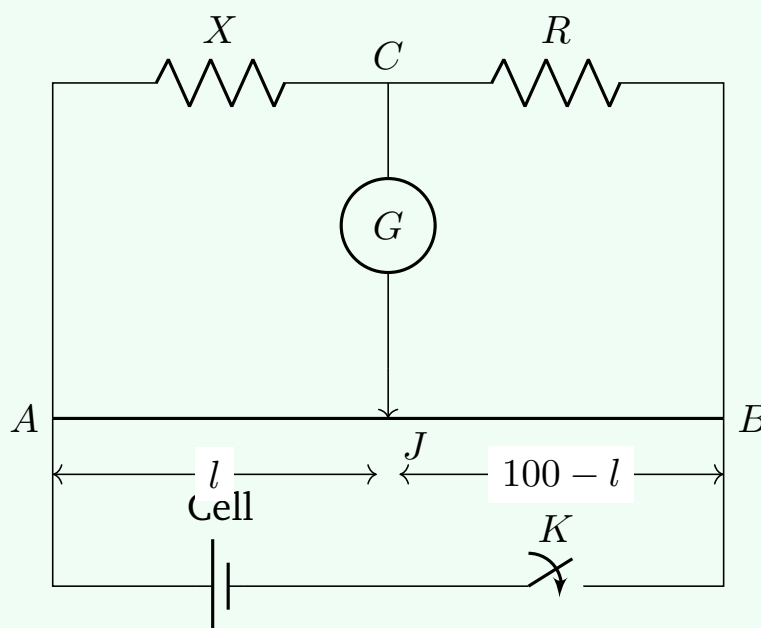
21. Explain the principle of working of a meter bridge. Draw the circuit diagram for determination of an unknown resistance using it. [2 marks]

Solution:

Concept:

- A meter bridge works on the balanced Wheatstone-bridge principle: when the galvanometer current is zero, the ratios of resistances in the two arms are equal.
- For a uniform wire, resistance is proportional to length: $R = \rho L/A$, where ρ is resistivity and A is its cross-sectional area.

Step 1: Arrange the bridge Connect the unknown resistance X in the left gap and a known resistance R in the right gap. Connect a cell and key across the ends A and B of the uniform one-metre wire. Connect the galvanometer between the central junction C and the sliding jockey J .



Meter bridge for measuring the unknown resistance X .

Step 2: Obtain the balance condition Move the jockey until the galvanometer shows zero deflection. If $AJ = l$ centimetres, then $JB = (100 - l)$ centimetres. At balance the potentials of C and J are equal, and the Wheatstone condition gives

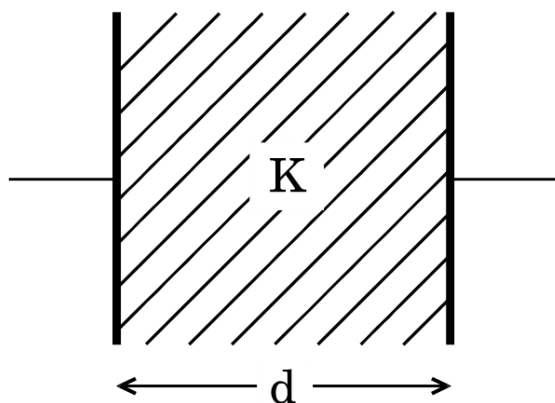
$$\frac{X}{R} = \frac{R_{AJ}}{R_{JB}} = \frac{l}{100 - l}.$$

The wire must be uniform so that its resistance ratio equals its length ratio.

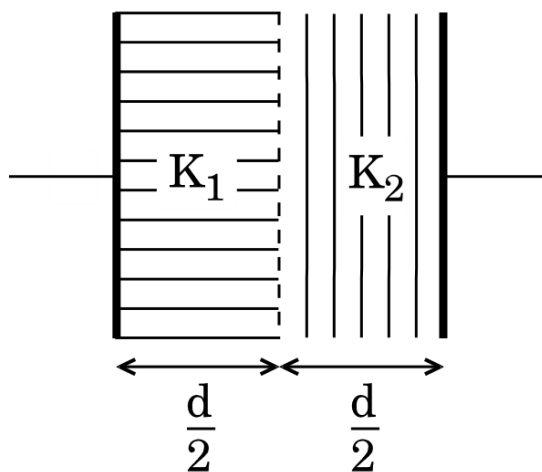
Step 3: Conclusion The unknown resistance is $X = Rl/(100 - l)$. The null galvanometer reading identifies the balanced bridge.

Quick Tip to Solve: Place the unknown in the left gap and use the left balancing length in the numerator.

22. The space between the plates of a parallel plate capacitor is completely filled in two ways. In the first case, it is filled with a slab of dielectric constant K . In the second case, it is filled with two slabs of equal thickness and dielectric constants K_1 and K_2 respectively as shown in the figure. The capacitance of the capacitor is same in the two cases. Obtain the relationship between K , K_1 and K_2 . [2 marks]



(Case 1)



(Case 2)

Solution:

Concept:

- For plate area A , separation d and dielectric constant K , the capacitance is $C = K\epsilon_0 A/d$.
- Dielectric layers placed successively along the electric field act as capacitors in series: their potential differences add while the charge on each is the same.

Step 1: Calculate the first capacitance In case 1, the single slab occupies the entire separation d and the full plate area A . Hence

$$C = \frac{K\epsilon_0 A}{d}.$$

This is the reference capacitance to which the layered arrangement is equated.

Step 2: Combine the two dielectric layers In case 2 each layer has thickness $d/2$, with the same cross-sectional area A . Their individual capacitances are $C_1 = 2K_1\epsilon_0 A/d$ and $C_2 = 2K_2\epsilon_0 A/d$. Since the layers lie along the plate separation,

$$\frac{1}{C_{\text{eq}}} = \frac{d}{2\epsilon_0 A} \left(\frac{1}{K_1} + \frac{1}{K_2} \right), \quad C_{\text{eq}} = \frac{2\epsilon_0 A K_1 K_2}{d(K_1 + K_2)}.$$

Equating C_{eq} to the first capacitance cancels $\epsilon_0 A/d$.

Step 3: Conclusion The required relation is $K = \frac{2K_1 K_2}{K_1 + K_2}$, or $\frac{2}{K} = \frac{1}{K_1} + \frac{1}{K_2}$.

Quick Tip to Solve: Layers along the separation are in series; use the harmonic mean for equal thicknesses.

23. Define the term ‘Half-life’ of a radioactive substance. Two different radioactive substances have half-lives T_1 and T_2 and number of undecayed atoms at an instant, N_1 and N_2 , respectively. Find the ratio of their activities at that instant. [2 marks]

Solution:

Concept:

- Half-life T is the time required for the number of undecayed radioactive nuclei in a sample to become half its initial value.
- Activity is the decay rate $\mathcal{A} = -dN/dt = \lambda N$, where λ is the decay constant; $\lambda = \ln 2/T$.

Step 1: Relate decay constant to half-life The radioactive decay law is $N(t) = N_0 e^{-\lambda t}$. At $t = T$, the defining condition is $N(T) = N_0/2$. Therefore $e^{-\lambda T} = 1/2$, so $\lambda T = \ln 2$. Apply this separately to each substance: $\lambda_1 = \ln 2/T_1$ and $\lambda_2 = \ln 2/T_2$.

Step 2: Form the instantaneous activity ratio At the stated instant, use the given undecayed populations, not their original populations. Thus

$$\mathcal{A}_1 = \frac{\ln 2}{T_1} N_1, \quad \mathcal{A}_2 = \frac{\ln 2}{T_2} N_2.$$

Taking their ratio cancels the common factor $\ln 2$. For equal populations, the substance with the shorter half-life has the larger activity, as expected physically.

Step 3: Conclusion $\frac{\mathcal{A}_1}{\mathcal{A}_2} = \frac{N_1 T_2}{N_2 T_1}$. Activity has units s^{-1} , or becquerel, when the half-lives are in seconds.

Quick Tip to Solve: Write activity as population divided by half-life times $\ln 2$ before taking the ratio.

24. Define wavefront of a travelling wave. Using Huygens principle, obtain the law of refraction at a plane interface when light passes from a denser to rarer medium. [2 marks]

Solution:

Concept:

- A wavefront is the locus of points of a travelling wave that are in the same phase at a given instant.
- Huygens principle: every point on a wavefront emits secondary wavelets; the forward envelope of those wavelets is the new wavefront. Light rays are normal to wavefronts.

Step 1: Construct the refracted wavefront Let AB be an incident plane wavefront in the denser medium, with A reaching the interface first. Let B reach the interface at C after time t . If the speeds in the denser and rarer media are v_1 and v_2 , then $BC = v_1 t$. In the same time the secondary wavelet from A has radius $AD = v_2 t$ in the rarer medium. The tangent from C to this wavelet is the refracted wavefront CD .

Step 2: Derive Snell's law The angles between the wavefronts and the interface equal the ray angles with the normal. Thus $\angle BAC = i$ and $\angle ACD = r$. The right triangles ABC and ADC give

$$\sin i = \frac{BC}{AC} = \frac{v_1 t}{AC}, \quad \sin r = \frac{AD}{AC} = \frac{v_2 t}{AC}.$$

Therefore $\sin i / \sin r = v_1 / v_2$. Using refractive indices $n_1 = c/v_1$ and $n_2 = c/v_2$, this becomes $\sin i / \sin r = n_2 / n_1$.

Step 3: Conclusion The law of refraction is $n_1 \sin i = n_2 \sin r$. Since $n_1 > n_2$, the transmitted ray bends away from the normal ($r > i$ for non-normal incidence), provided refraction is possible.

Quick Tip to Solve: Use the common hypotenuse AC in both Huygens triangles; their sine ratio is the speed ratio.

OR

24. Using lens maker's formula, derive the thin lens formula $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$ for a biconvex lens. [2 marks]

Solution:

Concept:

- For refraction at a spherical surface, $\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$, using Cartesian sign convention.
- For a thin lens of refractive index μ in air, the lens maker's formula is $\frac{1}{f} = (\mu - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$.

Step 1: Refraction at the first surface Let u be the object distance, v_1 the image distance due to the first surface, and R_1 its radius. Light travels from left to right, with all distances measured algebraically along that direction. Refraction from air into glass gives

$$\frac{\mu}{v_1} - \frac{1}{u} = \frac{\mu - 1}{R_1}.$$

Step 2: Refraction at the second surface and addition The first image acts as the object for the second

surface. For negligible lens thickness its object distance is v_1 . Refraction from glass into air gives

$$\frac{1}{v} - \frac{\mu}{v_1} = \frac{1 - \mu}{R_2}.$$

Adding eliminates the intermediate image:

$$\frac{1}{v} - \frac{1}{u} = (\mu - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right).$$

For a biconvex lens, $R_1 > 0$ and $R_2 < 0$. The right side is $1/f$ by the lens maker's formula.

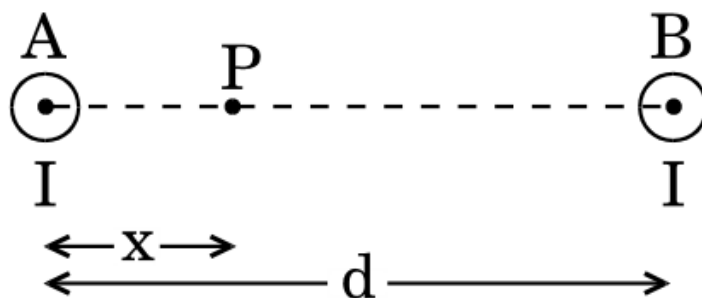
Step 3: Conclusion Thus the thin lens formula is $\boxed{\frac{1}{f} = \frac{1}{v} - \frac{1}{u}}$, with Cartesian signs and a thin lens in air.

Quick Tip to Solve: Add the two spherical-surface refraction equations first; the intermediate-image term cancels automatically.

25. Two long straight parallel wires A and B separated by a distance d , carry equal current I flowing in same direction as shown in the figure.

(a) Find the magnetic field at a point P situated between them at a distance x from one wire.

(b) Show graphically the variation of the magnetic field with distance x for $0 < x < d$. [2 marks]



Solution:

Concept:

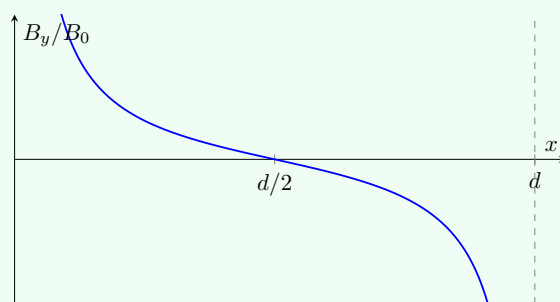
- A long straight wire produces $B = \mu_0 I / (2\pi r)$, where r is the perpendicular distance from the wire.
- Use the right-hand rule and vector superposition. Take an upward field in the plane of the figure as positive.

Step 1: Find the signed field between the wires Both currents point out of the page. At P , to the right of wire A , its field is upward; the field from wire B , to the right of P , is downward. The distances are x and $d - x$. Hence

$$B_y(x) = \frac{\mu_0 I}{2\pi} \left(\frac{1}{x} - \frac{1}{d - x} \right) = \frac{\mu_0 I (d - 2x)}{2\pi x (d - x)}.$$

Its magnitude is the absolute value of this expression.

Step 2: Describe and plot the variation The signed field decreases continuously as the distance from wire A increases. It approaches $+\infty$ as $x \rightarrow 0^+$, is zero at $x = d/2$, and approaches $-\infty$ as $x \rightarrow d^-$. The graph below shows the signed component; a magnitude graph would reflect the negative branch above the horizontal axis. The ideal-wire singularities describe the long, thin-wire approximation.



Signed magnetic field between the wires; $B_0 = \mu_0 I / (2\pi d)$.

Step 3: Conclusion $B_y = \frac{\mu_0 I}{2\pi} \left(\frac{1}{x} - \frac{1}{d-x} \right)$. The field is upward for $x < d/2$, zero at the midpoint, and downward for $x > d/2$.

Quick Tip to Solve: Between equal currents in the same direction, subtract the fields; the nearer wire determines the net direction.

26. Using Bohr's atomic model, derive the expression for the radius of n^{th} orbit of the revolving electron in a hydrogen atom. [2 marks]

Solution:

Concept:

- The Coulomb attraction supplies the centripetal force: $m_e v_n^2 / r_n = e^2 / (4\pi\epsilon_0 r_n^2)$.
- Bohr quantisation gives $m_e v_n r_n = n\hbar = nh / (2\pi)$, where $n = 1, 2, 3, \dots$

Step 1: Apply the force balance For an electron of mass m_e and charge magnitude e revolving around the proton, the electrostatic force acts towards the nucleus. With a stationary nucleus approximation,

$$\frac{m_e v_n^2}{r_n} = \frac{e^2}{4\pi\epsilon_0 r_n^2}, \quad m_e v_n^2 r_n = \frac{e^2}{4\pi\epsilon_0}.$$

Here ϵ_0 is the permittivity of free space.

Step 2: Eliminate the speed using quantised angular momentum The allowed angular momentum is $m_e v_n r_n = n\hbar / (2\pi)$, so $v_n = nh / (2\pi m_e r_n)$. Substitution in the force equation gives

$$\frac{n^2 \hbar^2}{4\pi^2 m_e r_n} = \frac{e^2}{4\pi\epsilon_0}.$$

Rearranging yields $r_n = \epsilon_0 n^2 \hbar^2 / (\pi m_e e^2)$. Thus the permitted orbital radii grow as the square of the orbit number, rather than being arbitrary continuous values.

Step 3: Conclusion $r_n = \frac{\epsilon_0 \hbar^2}{\pi m_e e^2} n^2 = a_0 n^2$, where $a_0 \simeq 5.29 \times 10^{-11} \text{ m}$ is the Bohr radius for hydrogen.

Quick Tip to Solve: Solve angular-momentum quantisation for speed and substitute it into Coulomb force balance; retain the factor 2π .

26. (a) Write two main observations of photoelectric effect experiment which could only be explained by Einstein's photoelectric equation.

(b) Draw a graph showing variation of photocurrent with the anode potential of a photocell. [2 marks]

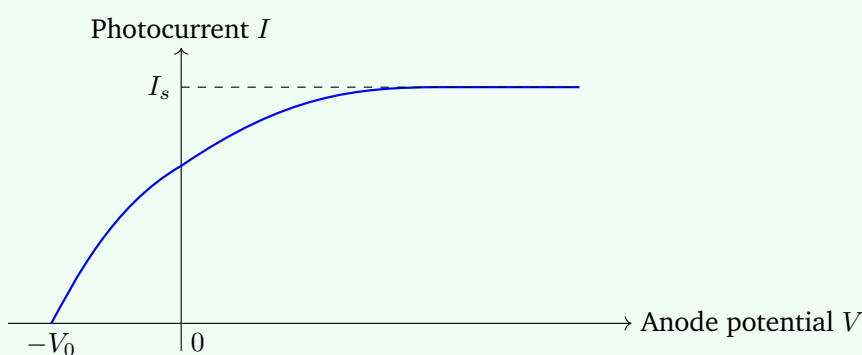
Solution:

Concept:

- Einstein's equation is $K_{\max} = h\nu - \phi = h(\nu - \nu_0)$, where ϕ is the work function and $\nu_0 = \phi/h$ is the threshold frequency.
- At fixed incident frequency above threshold and fixed intensity, the photocurrent rises with collecting potential and approaches saturation.

Step 1: State the two observations and explain them First, photoelectric emission occurs only above a definite threshold frequency, regardless of the incident intensity: when $h\nu < \phi$, a photon cannot supply the energy needed to release an electron. Second, the maximum kinetic energy increases linearly with incident frequency and is independent of intensity, because each emitted electron receives energy from one photon: $K_{\max} = h\nu - \phi$. Raising intensity at fixed frequency increases the photon arrival rate and hence the number of photoelectrons, without raising their maximum kinetic energy.

Step 2: Draw and explain the photocurrent graph For illumination of fixed frequency and intensity, a negative anode potential retards the electrons. At the stopping potential $-V_0$, even the fastest electrons are stopped, so the photocurrent vanishes and $eV_0 = K_{\max}$. A nonzero current flows at zero anode potential because some electrons already have sufficient kinetic energy. As the potential becomes positive, more emitted electrons reach the anode. Eventually all are collected and the current becomes the saturation current I_s .



Photocurrent against anode potential at fixed frequency and intensity.

Step 3: Conclusion The observations are a threshold frequency and a frequency-dependent, intensity-independent maximum kinetic energy. The photocurrent curve starts at $-V_0$ and approaches I_s at positive anode potentials.

Quick Tip to Solve: Keep stopping potential on the negative side of the voltage axis, with a positive photocurrent at zero voltage.

27. Explain the terms 'depletion layer' and 'potential barrier' in a p-n junction diode. How are the (a) width of depletion layer, and (b) value of potential barrier affected when the p-n junction is forward biased? [2 marks]

Solution:

Concept:

- A depletion layer is the region near a p-n junction depleted of mobile majority charge carriers; it contains immobile ionised donor and acceptor atoms.
- The potential barrier is the built-in potential difference across this space-charge region, opposing further diffusion of majority carriers.

Step 1: Explain formation of the depletion layer After the junction is formed, electrons diffuse from the n-side into the p-side and holes diffuse in the opposite direction. They recombine near the junction. The n-side is left with fixed positively charged donor ions and the p-side with fixed negatively charged acceptor ions. This region has very few mobile carriers and is called the depletion layer.

Step 2: Explain the barrier and forward-bias effects The uncovered ions produce an electric field directed from the n-side to the p-side. The associated potential difference opposes majority-carrier diffusion and is called the potential barrier. In forward bias, the p-side is connected to the positive terminal and the n-side to the negative terminal of the supply. The applied field opposes the junction field, pushing majority carriers towards the junction. Thus (a) the depletion-layer width decreases, and (b) the potential barrier decreases, allowing majority carriers to cross the junction more easily.

Step 3: Conclusion Forward bias reduces both the depletion-layer width and the potential barrier.

Quick Tip to Solve: Remember forward bias as p to positive: its applied field opposes the built-in junction field.

Section C (21 marks)

28. (a) Two cells of emf E_1 and E_2 have their internal resistances r_1 and r_2 , respectively. Deduce an expression for the equivalent emf and internal resistance of their parallel combination when connected across an external resistance R . Assume that the two cells are supporting each other.

(b) In case the two cells are identical, each of emf $E = 5\text{ V}$ and internal resistance $r = 2\ \Omega$, calculate the voltage across the external resistance $R = 10\ \Omega$. [3 marks]

Solution:

Concept:

- For supporting cells in parallel, like-polarity terminals are joined, so both cells have the same terminal voltage V .
- For a cell delivering signed current I_j , $V = E_j - I_j r_j$; Kirchhoff's current law gives $I = I_1 + I_2$.
- The equivalent source satisfies $V = E_{\text{eq}} - I r_{\text{eq}}$, and the load obeys $V = IR$.

Step 1: (a) Relate the currents to the common terminal voltage Let I_1 and I_2 be the currents delivered by the two branches and I the external current. Since the cells support each other, their positive terminals are joined and their negative terminals are joined. Applying the loop law separately gives

$$I_1 = \frac{E_1 - V}{r_1}, \quad I_2 = \frac{E_2 - V}{r_2}.$$

These equations remain valid with signed branch currents if unequal emfs cause one branch to receive current. At the junction,

$$I = \frac{E_1}{r_1} + \frac{E_2}{r_2} - V \left(\frac{1}{r_1} + \frac{1}{r_2} \right).$$

Step 2: Identify the equivalent emf and internal resistance Rearranging the current equation into the terminal-voltage form gives

$$V = \frac{E_1 r_2 + E_2 r_1}{r_1 + r_2} - I \frac{r_1 r_2}{r_1 + r_2}.$$

Comparison with $V = E_{\text{eq}} - I r_{\text{eq}}$ yields

$$E_{\text{eq}} = \frac{E_1 r_2 + E_2 r_1}{r_1 + r_2}, \quad r_{\text{eq}} = \frac{r_1 r_2}{r_1 + r_2}.$$

Consequently, $I = E_{\text{eq}}/(R + r_{\text{eq}})$. The equivalent emf is an internal-resistance-weighted average, whereas the internal resistances combine in parallel.

Step 3: (b) Calculate the load voltage for identical cells For $E_1 = E_2 = 5 \text{ V}$ and $r_1 = r_2 = 2 \Omega$,

$$E_{\text{eq}} = 5 \text{ V}, \quad r_{\text{eq}} = 1 \Omega.$$

Thus $I = 5/11 \text{ A}$, and the voltage across the external resistance is

$$V = IR = \frac{5}{11} \times 10 = \frac{50}{11} \text{ V} \approx 4.55 \text{ V}.$$

The terminal voltage is below the emf because there is a voltage drop across the equivalent internal resistance.

Step 4: Conclusion $E_{\text{eq}} = (E_1 r_2 + E_2 r_1)/(r_1 + r_2)$, $r_{\text{eq}} = r_1 r_2/(r_1 + r_2)$; the external voltage is 4.55 V.

Quick Tip to Solve: Identical cells in parallel retain the emf of one cell but halve its internal resistance.

29. (a) Write an expression of magnetic moment associated with a current (I) carrying circular coil of radius r having N turns.

(b) Consider the above mentioned coil placed in YZ plane with its centre at the origin. Derive expression for the value of magnetic field due to it at point $(x, 0, 0)$. [3 marks]

Solution:

Concept:

- The magnetic dipole moment of a plane coil is $\mathbf{m} = NIA \hat{n}$, where $\hat{n}$ is the right-hand-rule normal.
- Biot-Savart law: $d\mathbf{B} = \mu_0 I (d\boldsymbol{\ell} \times \hat{s}) / (4\pi s^2)$, where s runs from the current element to the observation point.
- On the axis of a circular loop, transverse field components cancel pairwise while axial components add.

Step 1: (a) State the magnetic moment The area of each circular turn is $A = \pi r^2$. Hence the magnitude of the magnetic moment of the N -turn coil is $m = NI\pi r^2$. Its direction is normal to the YZ plane, along the positive or negative x -axis as determined by curling the fingers of the right hand in the direction of current.

Step 2: (b) Find the field component of one current element Let $P = (x, 0, 0)$. Every element of the coil is at the same distance $s = \sqrt{r^2 + x^2}$ from P . The tangent element $d\ell$ is perpendicular to the displacement to P , because that displacement has only radial and axial components. Thus Biot-Savart law gives $dB = \mu_0 I d\ell / (4\pi s^2)$. The axial projection of this field has factor r/s , so

$$dB_x = \frac{\mu_0 I r d\ell}{4\pi (r^2 + x^2)^{3/2}}.$$

Here the positive x-direction is chosen as the coil normal for the specified current sense.

Step 3: Integrate over the complete coil For diametrically opposite elements, components perpendicular to the axis are equal and opposite and therefore cancel. The axial components point in the same direction. Integrating $d\ell$ over one turn gives the circumference $2\pi r$, and multiplying by N gives

$$B = \frac{\mu_0 N I r}{4\pi (r^2 + x^2)^{3/2}} (2\pi r) = \frac{\mu_0 N I r^2}{2(r^2 + x^2)^{3/2}}.$$

The field points along the magnetic moment. At the centre, $x = 0$, this reduces to $\mu_0 N I / (2r)$, providing a useful check on the result.

Step 4: Conclusion $m = N I \pi r^2$, and $B(x) = \frac{\mu_0 N I r^2}{2(r^2 + x^2)^{3/2}} \hat{n}$, with direction fixed by the current sense.

Quick Tip to Solve: On a circular coil axis, cancel transverse components first and integrate only the axial projection.

OR

29. (a) Define current sensitivity of a galvanometer. Write its expression.

(b) A galvanometer has resistance G and shows full scale deflection for current I_g .

(i) How can it be converted into an ammeter to measure current up to I_0 ($I_0 > I_g$)?

(ii) What is the effective resistance of this ammeter?

[3 marks]

Solution:

Concept:

- Current sensitivity is angular deflection per unit current: $S_I = \theta / I$.
- For a moving-coil galvanometer in a radial field, magnetic torque $NAB I$ balances restoring torque $k\theta$; N is the number of turns, A coil area, B field and k torsional constant.
- A low-resistance shunt in parallel with the galvanometer diverts the excess current and has the same potential difference as the galvanometer.

Step 1: (a) Define and obtain current sensitivity Current sensitivity measures how much the coil deflects for unit current through it. At equilibrium in the radial magnetic field,

$$NAB I = k\theta, \quad S_I = \frac{\theta}{I} = \frac{NAB}{k}.$$

Its SI unit is rad A^{-1} . Increasing the coil area, number of turns or magnetic field, or decreasing the torsional constant, increases this sensitivity.

Step 2: (b)(i) Determine the shunt for the desired range Connect a suitable low resistance S in

parallel with the galvanometer of resistance G . At the ammeter full-scale current I_0 , the galvanometer must carry only its full-scale current I_g ; therefore the shunt carries $I_0 - I_g$. Equating the potential drops across the parallel branches,

$$I_g G = (I_0 - I_g) S, \quad S = \frac{I_g G}{I_0 - I_g}.$$

The combination is inserted in series with the circuit whose current is measured. The shunt allows the excess current to bypass the sensitive coil.

Step 3: (b)(ii) Obtain the effective ammeter resistance The effective resistance is the parallel combination of G and S :

$$R_A = \frac{GS}{G + S}.$$

Substituting the value of S and cancelling common factors gives

$$R_A = \frac{GI_g}{I_0}.$$

Alternatively, the common full-scale voltage is $V = I_g G$ while the total current is I_0 , so $R_A = V/I_0$ gives the same expression. Since $I_0 > I_g$, this resistance is smaller than G , as expected for an ammeter designed to introduce a small voltage drop.

Step 4: Conclusion $S_I = NAB/k$. Connect shunt $S = I_g G/(I_0 - I_g)$ in parallel; the resulting ammeter resistance is $R_A = GI_g/I_0$.

Quick Tip to Solve: At full scale, write the shunt current as total current minus galvanometer current before equating voltages.

30. A resistance R and a capacitor C are connected in series to a source $V = V_0 \sin \omega t$.

Find:

- (a) The peak value of the voltage across the (i) resistance and (ii) capacitor.
 (b) The phase difference between the applied voltage and current. Which of them is ahead? [3 marks]

Solution:

Concept:

- For a series RC circuit, capacitive reactance is $X_C = 1/(\omega C)$, where ω is the angular frequency and C is the capacitance.
- The impedance magnitude is $Z = \sqrt{R^2 + X_C^2}$, so the peak current is $I_0 = V_0/Z$. The resistor voltage is in phase with current; the capacitor voltage lags current by $\pi/2$.

Step 1: (a)(i) Peak voltage across the resistor The same current flows through both series elements. Ohm's law for the resistor gives $V_R(t) = RI(t)$; hence its amplitude is

$$V_{R0} = I_0 R = \frac{V_0 R}{\sqrt{R^2 + (1/\omega C)^2}}.$$

This is a peak value, so no factor of $\sqrt{2}$ is introduced.

Step 2: (a)(ii) Peak voltage across the capacitor The capacitor voltage amplitude equals the current

amplitude times its reactance:

$$V_{C0} = I_0 X_C = \frac{V_0/(\omega C)}{\sqrt{R^2 + (1/\omega C)^2}} = \frac{V_0}{\sqrt{1 + (\omega CR)^2}}.$$

Because the resistor and capacitor voltages are perpendicular in phase, their peak values add in quadrature, giving $V_0^2 = V_{R0}^2 + V_{C0}^2$, rather than by ordinary scalar addition.

Step 3: (b) Compare the phases of current and applied voltage Taking current as the reference, the applied-voltage phasor has a horizontal component $I_0 R$ and a downward component $I_0 X_C$. Thus applied voltage lags current by an angle ϕ , with

$$\tan \phi = \frac{X_C}{R} = \frac{1}{\omega CR}, \quad \phi = \tan^{-1} \left(\frac{1}{\omega CR} \right).$$

Equivalently, for the given source, $I(t) = I_0 \sin(\omega t + \phi)$, which establishes which quantity is ahead.

Step 4: Conclusion $V_{R0} = V_0 R / \sqrt{R^2 + (1/\omega C)^2}$, $V_{C0} = V_0 / \sqrt{1 + (\omega CR)^2}$. Current leads the applied voltage by $\tan^{-1}[1/(\omega CR)]$.

Quick Tip to Solve: Use the same peak current for both components, and add their voltages in quadrature because they differ in phase by 90° .

31. What is the effect on the interference fringes in Young's double slit experiment due to each of the following operations? Justify your answers.

- (a) The screen is moved away from the plane of the slits.
- (b) The separation between slits is increased.
- (c) The source slit is moved closer to the plane of double slit.

[3 marks]

Solution:

Concept:

- In Young's double slit experiment, fringe width is $\beta = \lambda D/d$, where λ is wavelength, D is slit-to-screen distance and d is slit separation.
- Fringe visibility depends on coherence of illumination. A source slit of finite width subtends a larger angle at the double slit when it is brought closer.

Step 1: (a) Move the screen farther away Increasing D , while keeping λ and d fixed, increases β in direct proportion. For example, doubling the screen distance doubles the distance between adjacent bright fringes (and between adjacent dark fringes). The fringe positions obey $y_n = n\lambda D/d$; consequently the pattern spreads out about the central bright fringe. The fringes become more widely separated.

Step 2: (b) Increase the separation of the slits Increasing d , with wavelength and screen distance unchanged, decreases β inversely. The path difference at a given observation angle is $d \sin \theta$, so the angle required for one additional wavelength of path difference becomes smaller. The bright and dark

fringes therefore come closer together; for instance, doubling the slit separation halves the fringe width.

Step 3: (c) Bring the source slit nearer the double slit For the usual source slit of finite width, bringing it nearer increases its angular extent as seen by the two slits. Different points of this source slit produce mutually incoherent interference patterns with different central-fringe positions. Their superposition reduces the contrast: bright fringes are less distinct and dark fringes are filled in. If the slit is brought sufficiently close, the fringes may disappear. The spacing of the underlying patterns stays $\lambda D/d$, since neither D , d nor λ changes. For an ideal point source on the symmetry axis this loss of visibility does not occur; the stated visibility effect concerns the physical finite source slit.

Step 4: Conclusion (a) Fringe width increases. (b) Fringe width decreases. (c) Fringe visibility decreases for a finite source slit, and fringes may disappear; fringe width is unchanged.

Quick Tip to Solve: Keep D distinct from the source-to-slit distance: only the slit-to-screen distance appears in $\beta = \lambda D/d$.

32. Write the expression for the speed of light in a material medium of relative permittivity ϵ_r and relative magnetic permeability μ_r .

Write the wavelength range and name of the electromagnetic waves which are used in (i) radar systems for aircraft navigation, and (ii) Earth satellites to observe the growth of the crops. [3 marks]

Solution:

Concept:

- The speed of an electromagnetic wave in a homogeneous material is $v = 1/\sqrt{\mu\epsilon}$, where μ is permeability and ϵ is permittivity.
- Relative quantities satisfy $\mu = \mu_r\mu_0$ and $\epsilon = \epsilon_r\epsilon_0$; in vacuum, $c = 1/\sqrt{\mu_0\epsilon_0}$. Electromagnetic band boundaries are conventional and approximate.

Step 1: (a) Obtain the speed in the medium Substitute the relative permeability and permittivity into the wave-speed expression:

$$v = \frac{1}{\sqrt{\mu_r\mu_0\epsilon_r\epsilon_0}} = \frac{c}{\sqrt{\epsilon_r\mu_r}}.$$

Both relative quantities are dimensionless. Thus the refractive index is $n = c/v = \sqrt{\epsilon_r\mu_r}$ for the medium under consideration.

Step 2: (b)(i) Waves for radar navigation Radar systems for aircraft navigation use microwaves, whose approximate wavelength range is

$$10^{-3} \text{ m to } 10^{-1} \text{ m} \quad (1 \text{ mm to } 10 \text{ cm}).$$

Their comparatively short wavelengths permit directional beams and detection of reflected signals from aircraft or other objects. A radar determines distance from the round-trip travel time of the microwave pulse.

Step 3: (b)(ii) Waves for monitoring crop growth Earth satellites use infrared radiation to observe vegetation and the growth of crops. Its approximate wavelength range is

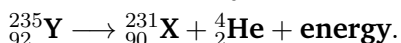
$$7 \times 10^{-7} \text{ m to } 10^{-3} \text{ m} \quad (700 \text{ nm to } 1 \text{ mm}).$$

Vegetation has characteristic infrared reflectance, especially in the near-infrared region, so satellite images can distinguish vegetation and changes in its condition. The full infrared band is quoted here; crop monitoring uses selected bands within it.

Step 4: Conclusion $v = c/\sqrt{\epsilon_r \mu_r}$. Radar uses microwaves (about 1 mm to 10 cm); crop monitoring uses infrared waves (about 700 nm to 1 mm).

Quick Tip to Solve: Associate radar with microwaves and vegetation monitoring with infrared; quote the approximate full band when its wavelength range is asked.

33. The nucleus ${}_{92}^{235}\text{Y}$, initially at rest, decays into ${}_{90}^{231}\text{X}$ by emitting an α -particle.



The binding energies per nucleon of the parent nucleus, the daughter nucleus and α -particle are 7.8 MeV, 7.835 MeV and 7.07 MeV, respectively. Assuming the daughter nucleus to be formed in the unexcited state and neglecting its share in the energy of the reaction, find the speed of the emitted α -particle. (Mass of α -particle = $6.68 \times 10^{-27} \text{ kg}$) [3 marks]

Solution:

Concept:

- Total nuclear binding energy is the number of nucleons times the binding energy per nucleon: $B = A\bar{B}$. A decay releases energy when the total product binding energy exceeds the parent binding energy.
- The release is $Q = B_X + B_\alpha - B_Y$. Under the stated neglect of daughter recoil energy, the alpha particle receives kinetic energy $K_\alpha \simeq Q$, with $K_\alpha = \frac{1}{2}m_\alpha v^2$.

Step 1: Calculate the total binding energies The mass numbers are 235 for the parent, 231 for the daughter and 4 for the alpha particle. Therefore

$$B_Y = 235(7.8) = 1833.0 \text{ MeV},$$

$$B_X = 231(7.835) = 1809.885 \text{ MeV},$$

$$B_\alpha = 4(7.07) = 28.28 \text{ MeV}.$$

The per-nucleon figures must first be multiplied by their respective mass numbers; subtracting the three per-nucleon values directly would not give the energy of the reaction.

Step 2: Find the energy available to the alpha particle The products are more tightly bound in total, so the decrease in rest energy is

$$Q = (1809.885 + 28.28) - 1833.0 = 5.165 \text{ MeV}.$$

The daughter is unexcited, so no energy is retained as nuclear excitation. Neglecting its share of kinetic energy as instructed gives

$$K_{\alpha} = 5.165 \times 10^6 \times 1.602 \times 10^{-19} = 8.27433 \times 10^{-13} \text{ J}.$$

Step 3: Convert kinetic energy to speed Using the supplied alpha-particle mass,

$$v = \sqrt{\frac{2K_{\alpha}}{m_{\alpha}}} = \sqrt{\frac{2(8.27433 \times 10^{-13})}{6.68 \times 10^{-27}}} \simeq 1.57 \times 10^7 \text{ m s}^{-1}.$$

This is about $0.052c$, so the nonrelativistic kinetic-energy formula is appropriate. Momentum conservation gives the daughter a small recoil in an exact treatment, but the question explicitly asks us to neglect its energy share.

Step 4: Conclusion The energy released is 5.165 MeV, and the emitted alpha particle has speed $1.57 \times 10^7 \text{ m s}^{-1}$ under the stated recoil approximation.

Quick Tip to Solve: Multiply each binding energy per nucleon by its own mass number before subtracting; convert MeV to joules before using the mass in kilograms.

34. Draw circuit diagram and explain the working of a zener diode as a dc voltage regulator with the help of its I-V characteristic.

What is the purpose of heavy doping of p- and n-sides of a zener diode?

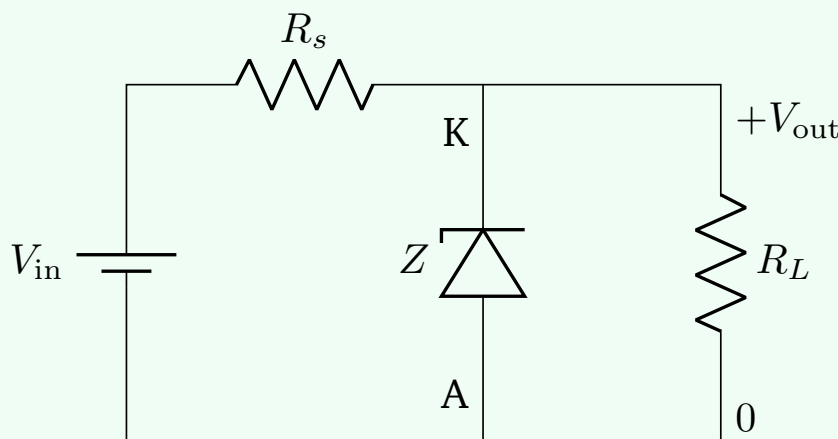
[3 marks]

Solution:

Concept:

- A reverse-biased Zener diode has an almost constant breakdown voltage V_Z , even when its reverse current changes appreciably.
- A series resistor R_s limits current; the load R_L is connected across the Zener, giving $V_{\text{out}} \simeq V_Z$.

Step 1: (a) Connect the regulator circuit Connect the positive supply terminal through R_s to the output node. The Zener cathode K connects to this positive node and its anode A to the negative supply terminal. Connect the load in parallel with the diode. The resistor prevents excessive current, while the load receives the voltage across the Zener.

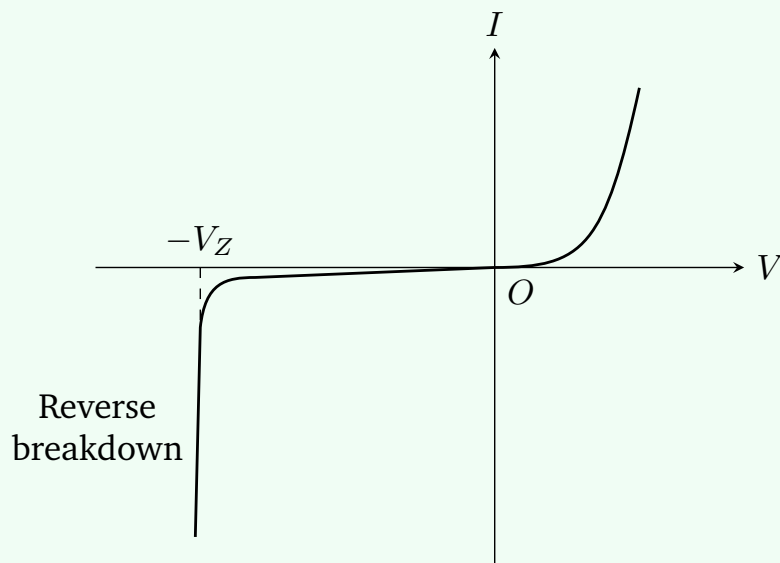


Zener regulator with the load across the reverse-biased diode.

Step 2: Explain regulation from the characteristic At $V = -V_Z$, the reverse characteristic has a sharp knee; beyond it, the reverse current increases steeply with little change in voltage. Using positive current magnitudes,

$$I_s = \frac{V_{in} - V_Z}{R_s}, \quad I_s = I_L + I_Z, \quad I_L = \frac{V_Z}{R_L}.$$

A rise in input voltage increases I_s ; most of the additional current flows through the Zener and the extra voltage drops across R_s . A fall in input decreases the Zener current. Likewise, a rise in load current is accompanied by a fall in Zener current, and a decrease in load current increases Zener current. The output therefore stays approximately constant, provided the diode remains in breakdown and its current and power ratings are respected.



Signed I-V characteristic showing the reverse breakdown branch.

Step 3: (b) Explain heavy doping Heavy doping of both p- and n-sides makes the depletion region very thin. A strong junction electric field can then develop at a relatively small reverse voltage. This produces sharp breakdown at a well-defined designed voltage, suitable for voltage regulation.

Step 4: Conclusion The reverse-biased Zener keeps the load voltage approximately equal to V_Z . Heavy doping provides a thin depletion layer and a strong junction field at low reverse voltage.

Quick Tip to Solve: Place the cathode at the positive output node and the load across the Zener; the resistor must limit the total supply current.

Section D (15 marks)

35. Using Gauss law, derive expression for electric field due to a spherical shell of uniform charge distribution and radius R at a point lying at a distance x from the centre of shell, such that

(i) $0 < x < R$, and

(ii) $x > R$.

An electric field is uniform and acts along $+x$ direction in the region of positive x . It is also uniform with the same magnitude but acts in $-x$ direction in the region of negative x . The value of the field is $E = 200 \text{ N/C}$ for $x > 0$ and $E = -200 \text{ N/C}$ for $x < 0$. A right circular cylinder of length 20 cm and radius 5 cm has its centre at the origin and its axis along the x -axis so that one flat face is at $x = +10 \text{ cm}$ and the other is at $x = -10 \text{ cm}$.

Find:

(i) The net outward flux through the cylinder.

(ii) The net charge present inside the cylinder.

[5 marks]

Solution:

Concept:

- Gauss's law is $\oint \vec{E} \cdot d\vec{A} = Q_{\text{enc}}/\epsilon_0$, where $d\vec{A}$ points outward and ϵ_0 is the permittivity of free space.
- Spherical symmetry makes the shell's field radial and constant in magnitude on a concentric spherical Gaussian surface. Writing its uniform surface charge density as σ , its total charge is $Q = 4\pi R^2\sigma$.
- Electric flux is a signed scalar. Contributions add when the field points along the outward normal on each face, even if the field directions on opposite faces differ.

Step 1: (a)(i) Derive the field inside the shell Choose a concentric spherical Gaussian surface of radius x , with $0 < x < R$. All charge lies on the shell at radius R , so this surface encloses zero charge. By spherical symmetry, $\vec{E} = E(x)\hat{r}$ and its magnitude is the same everywhere on the Gaussian sphere. Hence

$$\oint \vec{E} \cdot d\vec{A} = E(x)4\pi x^2 = \frac{0}{\epsilon_0}.$$

Since $x \neq 0$, this establishes that the electric field vanishes throughout the interior. This inference uses symmetry as well as Gauss's law: zero enclosed charge alone would not imply zero field for an arbitrary charge distribution.

Step 2: (a)(ii) Derive the field outside the shell For a concentric Gaussian sphere with $x > R$, all shell charge is enclosed. The shell area is $4\pi R^2$, so $Q_{\text{enc}} = 4\pi R^2\sigma$. Again the field is radial and constant on the sphere. Therefore

$$E(x)4\pi x^2 = \frac{4\pi R^2\sigma}{\epsilon_0}, \quad \vec{E}(x) = \frac{\sigma R^2}{\epsilon_0 x^2} \hat{r} = \frac{Q}{4\pi\epsilon_0 x^2} \hat{r}.$$

Here $\hat{r}$ points outward; a positive σ gives an outward field and a negative σ an inward field. Outside, the shell thus acts like its entire charge concentrated at its centre.

Step 3: (b)(i) Calculate the outward flux through the cylinder Convert the radius to SI units: $a = 5 \text{ cm} = 0.05 \text{ m}$. Each flat face has area $A = \pi a^2 = 0.0025\pi \text{ m}^2$. At the right face the field and outward normal both point along $+\hat{x}$; at the left face they both point along $-\hat{x}$. Thus each face contributes $+200A$, not opposite fluxes. On the curved surface the normal is perpendicular to the axis,

so $\vec{E} \cdot d\vec{A} = 0$. Consequently

$$\Phi = 200A + 200A + 0 = 400\pi(0.05)^2 = \pi \text{ N m}^2/\text{C} \simeq 3.14 \text{ N m}^2/\text{C}.$$

The cylinder length determines that its two faces are in the stated positive and negative regions; it does not multiply the end-face flux.

Step 4: (b)(ii) Find the enclosed charge Applying Gauss's law to the entire closed cylinder gives

$$Q_{\text{enc}} = \varepsilon_0 \Phi = (8.854 \times 10^{-12})\pi \text{ C} \simeq 2.78 \times 10^{-11} \text{ C}.$$

Its sign is positive because the net flux is outward. No assumption about the detailed distribution of this charge is needed; Gauss's law determines the net amount inside the cylinder.

Step 5: Conclusion For the shell, $\vec{E} = 0$ for $0 < x < R$, and $\vec{E} = \sigma R^2 \hat{r} / (\varepsilon_0 x^2)$ for $x > R$. The cylinder has outward flux $\pi \text{ N m}^2/\text{C}$ and enclosed charge $+2.78 \times 10^{-11} \text{ C}$.

Quick Tip to Solve: Check each outward normal before adding flux: the left field and left normal are both negative, so their dot product is positive.

OR

35. Find the expression for the potential energy of a system of two point charges q_1 and q_2 located at $\vec{r}_1$ and $\vec{r}_2$ respectively in an external electric field $\vec{E}$.

Draw equipotential surfaces due to an isolated point charge $(-q)$ and depict the electric field lines.

Three point charges $+1 \mu\text{C}$, $-1 \mu\text{C}$ and $+2 \mu\text{C}$ are initially infinite distance apart. Calculate the work done in assembling these charges at the vertices of an equilateral triangle of side 10 cm. [5 marks]

Solution:

Concept:

- Potential energy is the work done by an external agent in assembling charges quasistatically. In a prescribed external potential V_{ext} , a charge q has energy qV_{ext} .
- The mutual interaction energy of two point charges separated by r is kq_1q_2/r , where $k = 1/(4\pi\varepsilon_0)$. Count each distinct pair once.
- Equipotential surfaces have constant potential. Electric field lines intersect them normally and point in the direction of decreasing potential.

Step 1: (a) Bring the first charge into the external field Let $V_{\text{ext}}(\vec{r})$ denote the potential produced by the external sources, with a specified reference level. Initially neither of the two charges is present. Bringing q_1 to $\vec{r}_1$ requires external work

$$W_1 = q_1 V_{\text{ext}}(\vec{r}_1).$$

Here the reference configuration assigns zero energy to each charge at the reference potential; when the external sources are localized, this can be taken at infinity. The energy of the external sources themselves is held fixed and is not included in the requested energy.

Step 2: (a) Bring the second charge and add the energies Write $r_{12} = |\vec{r}_2 - \vec{r}_1|$. The potential acting on q_2 consists of the prescribed external potential and the potential kq_1/r_{12} due to the first charge.

Therefore,

$$W_2 = q_2 \left[V_{\text{ext}}(\vec{r}_2) + \frac{kq_1}{r_{12}} \right],$$

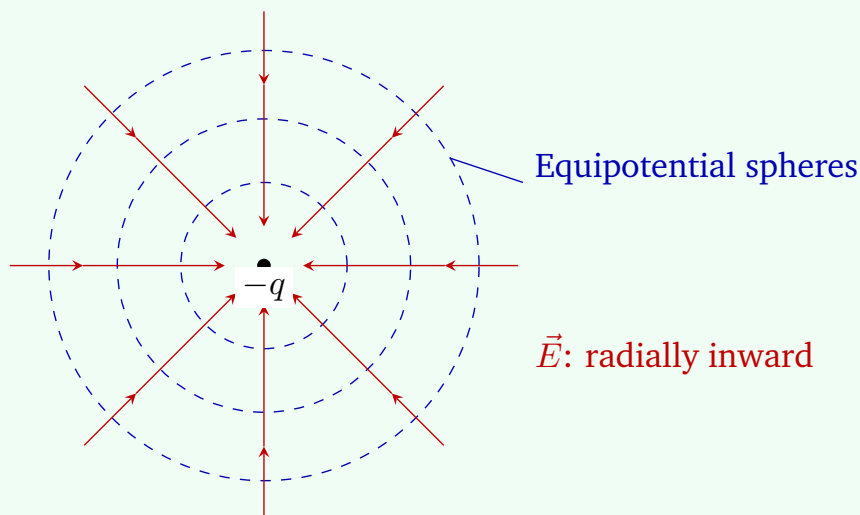
$$U = W_1 + W_2 = q_1 V_{\text{ext}}(\vec{r}_1) + q_2 V_{\text{ext}}(\vec{r}_2) + \frac{q_1 q_2}{4\pi\epsilon_0 r_{12}}.$$

The mutual energy occurs once because it was paid when the second charge was brought in. No self-energy of a point charge is included. The external field determines potential differences by $V_{\text{ext}}(\vec{r}) - V_{\text{ext}}(\vec{r}_0) = -\int_{\vec{r}_0}^{\vec{r}} \vec{E} \cdot d\vec{l}$. If the external field is uniform, this becomes $-\vec{E} \cdot (\vec{r} - \vec{r}_0)$; the general expression above does not assume uniformity.

Step 3: (b) Draw the equipotential surfaces and field lines For an isolated negative charge, with zero potential at infinity,

$$V(r) = -\frac{kq}{r}, \quad \vec{E}(r) = -\frac{kq}{r^2} \hat{r},$$

where $q > 0$ is the magnitude of the charge. Each fixed radius defines a concentric spherical equipotential surface. The diagram is a planar cross-section of these spheres. Field lines are radial and directed inward toward $-q$, meeting every surface at right angles. The potential becomes more negative nearer the charge. There is no work in moving a test charge along a single equipotential surface.



Cross-section of spherical equipotentials around $-q$; electric field lines point inward.

Step 4: (c) Add the three pair energies Initially all pair separations are infinite, so the initial interaction energy is zero. Let the charges be $q_A = 10^{-6} \text{ C}$, $q_B = -10^{-6} \text{ C}$ and $q_C = 2 \times 10^{-6} \text{ C}$. Every separation in the equilateral triangle is $a = 10 \text{ cm} = 0.10 \text{ m}$. The quasistatic external work is the final energy:

$$W_{\text{ext}} = \frac{k}{a} (q_A q_B + q_B q_C + q_C q_A),$$

$$q_A q_B = -10^{-12} \text{ C}^2,$$

$$q_B q_C = -2 \times 10^{-12} \text{ C}^2,$$

$$q_C q_A = +2 \times 10^{-12} \text{ C}^2,$$

$$W_{\text{ext}} = \frac{9 \times 10^9}{0.10} (-10^{-12}) \text{ J}.$$

The two interactions involving $+2 \mu\text{C}$ cancel because its distances from equal and opposite charges are equal. Negative external work means energy is released during assembly; the electric field does positive work of the same magnitude.

Step 5: Conclusion (a) $U = q_1 V_{\text{ext}}(\vec{r}_1) + q_2 V_{\text{ext}}(\vec{r}_2) + kq_1 q_2 / r_{12}$. (b) The equipotentials are concentric spheres, with normally intersecting inward radial field lines. (c) The work done in assembling the charges is -0.090 J .

Quick Tip to Solve: For an equilateral triangle, factor out k/a before adding signed charge products; here the two products involving $+2 \mu\text{C}$ cancel immediately.

36. Derive the expression for the torque acting on the rectangular current carrying coil of a galvanometer. Why is the magnetic field made radial?

An α -particle is accelerated through a potential difference of 10 kV and moves along x -axis. It enters in a region of uniform magnetic field $B = 2 \times 10^{-3} \text{ T}$ acting along y -axis. Find the radius of its path. (Take mass of α -particle $= 6.4 \times 10^{-27} \text{ kg}$) [5 marks]

Solution:

Concept:

- A straight conductor of length l , carrying current I perpendicular to a magnetic field B , experiences a force $F = BIl$. Equal and opposite forces with different lines of action form a couple.
- The magnetic moment of a coil of N turns and area A is $\mu = NIA\hat{n}$, where $\hat{n}$ is its unit normal. Its torque is $\tau = \mu \times B$.
- Acceleration through a potential difference V gives a particle kinetic energy $qV = \frac{1}{2}mv^2$. For motion perpendicular to B , the magnetic force supplies the centripetal force: $qvB = mv^2/r$.

Step 1: (a) Derive the torque on the rectangular coil Consider a rectangular coil of N turns, length l , breadth b , and area $A = lb$, carrying current I in a uniform magnetic field B . Let θ be the angle between the normal to the coil and the field. Choose the two sides of length l perpendicular to the field. Each experiences a force of magnitude $NBIl$; the directions are opposite because the currents in the two sides are opposite. These forces have separate lines of action and form a couple.

The perpendicular distance between their lines of action is $b \sin \theta$. The forces on the other pair of sides do not contribute to the turning couple about the coil's rotation axis. Thus the torque magnitude is

$$\begin{aligned}\tau &= \text{force} \times \text{perpendicular separation} \\ &= NBIl (b \sin \theta) \\ &= NIAB \sin \theta.\end{aligned}$$

Its direction tends to align the magnetic moment of the coil with the field, as expressed by $\tau = \mu \times B$. Here θ refers to the normal, not to the plane of the coil.

Step 2: Explain the purpose of the radial magnetic field In a moving-coil galvanometer, concave pole pieces and a cylindrical soft-iron core produce a radial field. As the coil turns, the magnetic field stays parallel to its plane, so the normal remains perpendicular to the field: $\theta = 90^\circ$. The magnetic torque consequently remains $\tau = NIAB$, independent of the angular position of the coil.

If the suspension has torsional constant k and the coil deflects through an angle ϕ , its restoring torque is $k\phi$. At equilibrium,

$$NIAB = k\phi, \quad \phi = \frac{NAB}{k}I.$$

Hence deflection is directly proportional to current, giving a uniform scale and a constant current sensitivity NAB/k . Without the radial field, an angle-dependent sine factor would make the relation

nonlinear.

Step 3: (b) Calculate the momentum of the accelerated alpha particle An alpha particle has positive charge $q = 2e = 3.2 \times 10^{-19}$ C. Convert the accelerating potential to SI units: $V = 10 \text{ kV} = 1.0 \times 10^4$ V. Starting from negligible kinetic energy, it acquires

$$K = qV = (3.2 \times 10^{-19})(1.0 \times 10^4) = 3.2 \times 10^{-15} \text{ J}.$$

With $m = 6.4 \times 10^{-27}$ kg, its momentum is

$$p = mv = \sqrt{2mK} = \sqrt{2(6.4 \times 10^{-27})(3.2 \times 10^{-15})} = 6.4 \times 10^{-21} \text{ kg m s}^{-1}.$$

Equivalently, its speed is $v = p/m = 1.0 \times 10^6 \text{ m s}^{-1}$. The charge must be $2e$, since this particle is a helium nucleus.

Step 4: Use the magnetic force to determine the radius The velocity is along the x -axis and the magnetic field is along the y -axis, so their angle is 90° . The full magnetic force qvB is perpendicular to the velocity and changes its direction without changing its speed. Since there is no velocity component parallel to the field, the trajectory is circular in the xz -plane, rather than helical.

$$\begin{aligned} qvB &= \frac{mv^2}{r}, \\ r &= \frac{mv}{qB} = \frac{p}{qB} \\ &= \frac{6.4 \times 10^{-21}}{(3.2 \times 10^{-19})(2 \times 10^{-3})} \\ &= 10 \text{ m}. \end{aligned}$$

For motion along positive x and field along positive y , the initial force is along positive z , consistent with $q(\mathbf{v} \times \mathbf{B})$ for a positive particle.

Step 5: Conclusion The rectangular coil experiences $\tau = NIAB \sin \theta$. A radial field keeps $\theta = 90^\circ$, giving $\tau = NIAB$ and a deflection proportional to current. The alpha particle follows a circular path of radius $r = 10 \text{ m}$.

Quick Tip to Solve: Use the angle between the coil normal and the field in the torque formula. For an accelerated alpha particle, substitute $q = 2e$ in both qV and $r = \sqrt{2mqV}/(qB)$.

36. With the help of a labelled diagram, explain the working of a step-up transformer. Give reasons to explain the following:

(i) The core of the transformer is laminated.

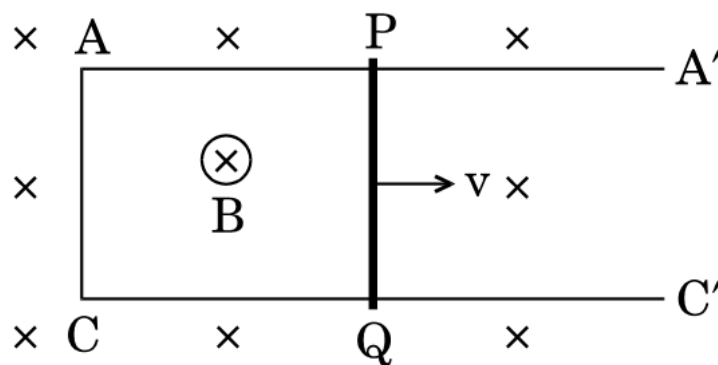
(ii) Thick copper wire is used in windings.

A conducting rod PQ of length 20 cm and resistance $0.1\ \Omega$ rests on two smooth parallel rails of negligible resistance AA' and CC' . It can slide on the rails and the arrangement is positioned between the poles of a permanent magnet producing uniform magnetic field $B = 0.4\ \text{T}$. The rails, the rod and the magnetic field are in three mutually perpendicular directions as shown in the figure. If the ends A and C of the rails are short circuited, find the

(i) external force required to move the rod with uniform velocity $v = 10\ \text{cm/s}$, and

(ii) power required to do so.

[5 marks]



Solution:

Concept:

- A transformer operates by mutual induction: an alternating current in the primary produces a changing magnetic flux Φ linking both windings on a common laminated iron core.
- For an ideal transformer, $V_s/V_p = N_s/N_p$, where N_p, N_s are the primary and secondary turns. A step-up transformer has $N_s > N_p$.
- A rod of length l , moving with speed v perpendicular to a field B , develops motional emf $\varepsilon = Blv$. In a closed circuit of resistance R , $I = \varepsilon/R$ and the magnetic drag is $F_B = BIl$.

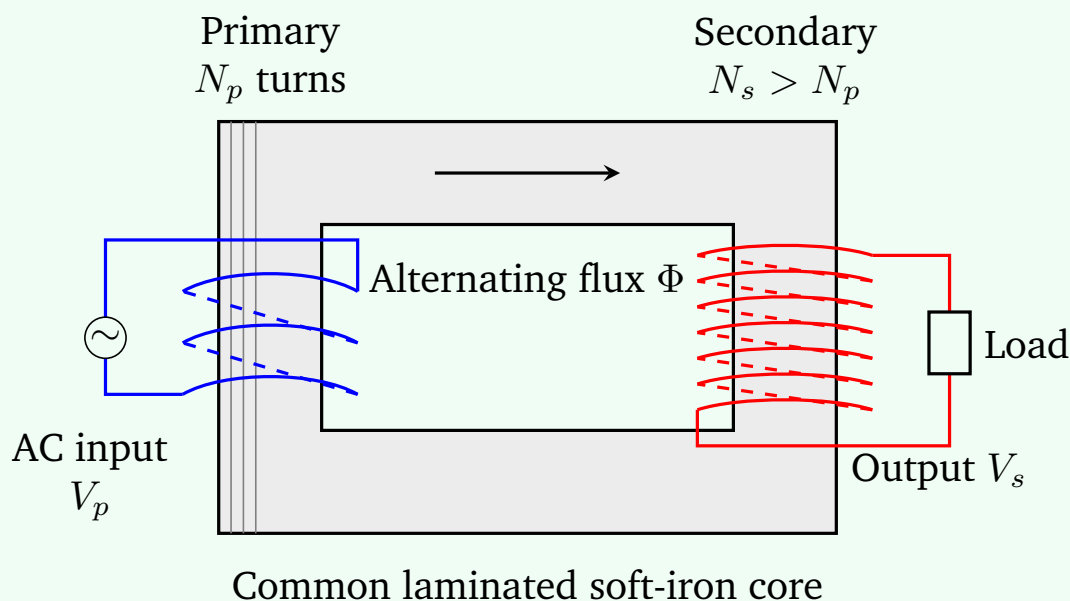
Step 1: (a) Construction and working of a step-up transformer Two electrically insulated windings, the primary and secondary, are wound on a common laminated soft-iron core. Connect the primary to an alternating voltage supply and the secondary to the load. The secondary has more turns than the primary, as indicated in the labelled diagram. The primary alternating current produces an alternating flux in the core. This flux links the secondary and induces an emf there by Faraday's law. Neglecting flux leakage and winding resistance, the same flux links each turn, so

$$e_p = -N_p \frac{d\Phi}{dt}, \quad e_s = -N_s \frac{d\Phi}{dt},$$

$$\frac{V_s}{V_p} = \frac{N_s}{N_p} > 1.$$

Thus the output alternating voltage is higher than the input. Energy passes through the magnetic field, without electrical contact between the windings. Ideally $V_p I_p = V_s I_s$, so the increased voltage is accompanied by reduced secondary current. A steady direct current cannot maintain changing flux

and hence cannot produce continuous transformer action.



Step-up transformer: fewer primary turns and more secondary turns on a common laminated core.

Step 2: (a)(i), (ii) Explain the construction features The core is made of thin iron laminations insulated from one another. These interrupt large eddy-current loops and increase their resistance, reducing eddy-current heating and energy loss while permitting the alternating magnetic flux to pass through the core. Thick copper wire gives the windings a small resistance: $R = \rho L/A$, where ρ is resistivity, L is wire length and A is cross-sectional area. Copper has low resistivity and a larger wire area lowers resistance further. Consequently the copper loss $I^2 R$ and unwanted voltage drop are reduced.

Step 3: (b) Calculate motional emf and induced current Convert both lengths and speeds to SI units:

$$l = 0.20 \text{ m}, \quad v = 0.10 \text{ m s}^{-1}, \quad B = 0.4 \text{ T}, \quad R = 0.1 \Omega.$$

Since the rails have negligible resistance, the rod provides the circuit resistance. Hence

$$\begin{aligned} \varepsilon &= Blv = (0.4)(0.20)(0.10) = 0.008 \text{ V}, \\ I &= \frac{\varepsilon}{R} = \frac{0.008}{0.1} = 0.08 \text{ A}. \end{aligned}$$

The field points into the page and the rod moves right. The force $q\mathbf{v} \times \mathbf{B}$ on positive charges is upward, so P is positive relative to Q . Conventional current flows upward through the rod and returns through the shorted rails. Its magnetic force is leftward, opposing the motion, as required by Lenz's law.

Step 4: (b)(i) Balance the magnetic drag For uniform velocity on smooth rails, the net horizontal force is zero. The external force therefore acts rightward and equals the magnetic drag:

$$F_{\text{ext}} = BIl = \frac{B^2 l^2 v}{R} = (0.4)(0.08)(0.20) = 0.0064 \text{ N}.$$

No force is needed to accelerate the rod because its velocity is constant; the required force supplies energy against electromagnetic resistance.

Step 5: (b)(ii) Find the mechanical power The external force is parallel to the velocity. The mechanical

input power is

$$P_{\text{mech}} = F_{\text{ext}}v = (0.0064)(0.10) = 0.00064 \text{ W}.$$

As a check, the heating rate is $I^2R = (0.08)^2(0.1) = 0.00064 \text{ W}$. Thus all mechanical input becomes Joule heat in the rod under the stated ideal assumptions.

Step 6: Conclusion The step-up transformer uses mutual induction with $N_s > N_p$; laminations reduce eddy-current loss and thick copper wire reduces winding resistance. The external force is $6.4 \times 10^{-3} \text{ N}$ in the direction of motion, and the required power is $6.4 \times 10^{-4} \text{ W}$.

Quick Tip to Solve: Convert cm and cm/s first, then use $F = B^2 l^2 v / R$ and $P = Fv$; verify the power independently with $I^2 R$.

37. Draw the ray diagram of an astronomical telescope when the final image is formed at infinity. Write the expression for the resolving power of the telescope.

An astronomical telescope has an objective lens of focal length 20 m and eyepiece of focal length 1 cm.

(i) Find the angular magnification of the telescope.

(ii) If this telescope is used to view the Moon, find the diameter of the image formed by the objective lens. Given the diameter of the Moon is $3.5 \times 10^6 \text{ m}$ and radius of lunar orbit is $3.8 \times 10^8 \text{ m}$. [5 marks]

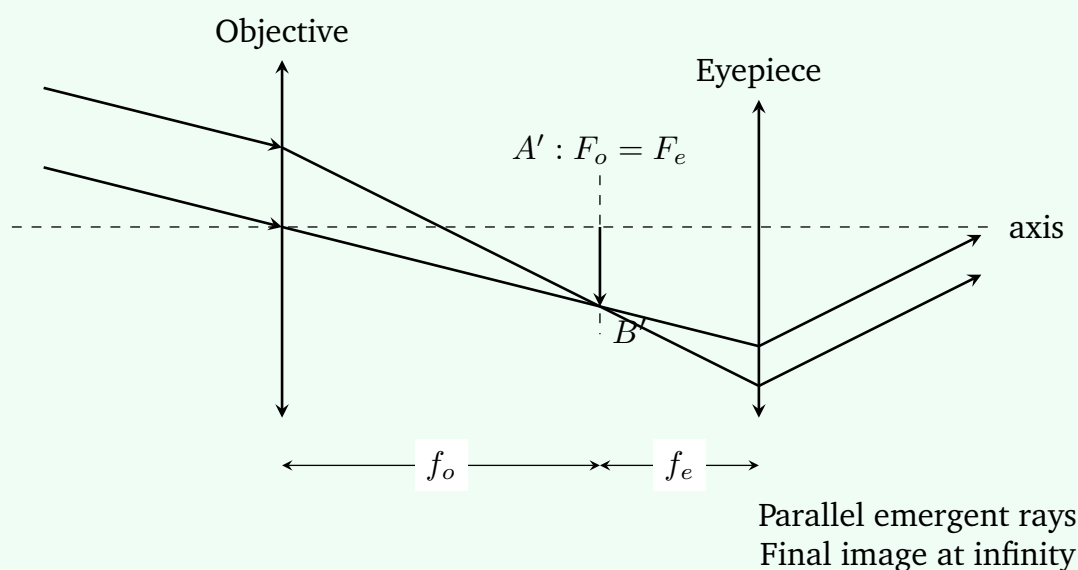
Solution:

Concept:

- In normal adjustment, the real image produced by the objective lies in the first focal plane of the eyepiece. The eyepiece sends out parallel rays, so the final image is at infinity.
- The signed angular magnification is $M = -f_o/f_e$, where f_o and f_e are the focal lengths of the objective and eyepiece. The negative sign indicates inversion.
- For a circular objective of aperture diameter D_o , Rayleigh's criterion gives $\theta_{\min} = 1.22\lambda/D_o$. Resolving power is its reciprocal, where λ is the wavelength of light.

Step 1: (a) Telescope in normal adjustment Parallel rays from a distant object enter the objective lens and converge to a real, inverted intermediate image $A'B'$ in its second focal plane. Place the eyepiece so that this same plane is its first focal plane: $F_o = F_e$. The eyepiece acts as a magnifier and the emerging rays are parallel, giving the final virtual image at infinity. The lens separation is therefore $f_o + f_e$. In the sketch, the incident rays represent one point of the distant object; their intersection is B' . The two focal lengths are drawn schematically rather than to scale.

Parallel incident rays



Astronomical telescope in normal adjustment; the intermediate image lies in the common focal plane and the emergent rays are parallel.

Step 2: (a) Expression for resolving power Two distant point objects are just resolved when the central diffraction maximum of one image coincides with the first minimum of the other. For the circular objective,

$$\theta_{\min} = \frac{1.22\lambda}{D_o}, \quad \text{RP} = \frac{1}{\theta_{\min}} = \frac{D_o}{1.22\lambda}.$$

Here $\theta_{\min}$ is the limiting angular separation in radians. A larger objective aperture or a shorter wavelength improves the ability to distinguish closely spaced objects. Resolving power depends on aperture, whereas angular magnification depends on focal lengths.

Step 3: (b)(i) Calculate angular magnification Convert the eyepiece focal length to the same unit as the objective: $f_e = 1 \text{ cm} = 0.01 \text{ m}$, while $f_o = 20 \text{ m}$. Thus, in normal adjustment,

$$M = -\frac{f_o}{f_e} = -\frac{20}{0.01} = -2000.$$

The telescope increases the angular size by a factor of 2000, and its final image is inverted relative to the object. Angular magnification is dimensionless.

Step 4: (b)(ii) Diameter of the image formed by the objective Let $d_M = 3.5 \times 10^6 \text{ m}$ be the Moon's diameter and $R = 3.8 \times 10^8 \text{ m}$ its distance, approximated by the lunar-orbit radius. Its small angular diameter is $\theta \simeq d_M/R$. Since the Moon is very far away compared with f_o , its objective image lies effectively in the focal plane. If its image diameter is d_i , then $d_i/f_o \simeq \theta$. Consequently,

$$\begin{aligned} d_i &= f_o \frac{d_M}{R} \\ &= \frac{20 \times 3.5 \times 10^6}{3.8 \times 10^8} \text{ m} \\ &\simeq 0.184 \text{ m} = 18.4 \text{ cm}. \end{aligned}$$

This is the diameter of the real intermediate image produced by the objective, so the eyepiece magnification is not applied to it.

Step 5: Conclusion The normal-adjustment diagram has a common focal plane $F_o = F_e$ and parallel emergent rays. The resolving power is $D_o/(1.22\lambda)$; the angular magnification is $M = -2000$ (magnitude 2000); and the objective forms a lunar image of diameter approximately 0.184 m, or 18.4 cm.

Quick Tip to Solve: Convert both focal lengths to metres before taking their ratio; use only the objective focal length when calculating the Moon's intermediate image diameter.

OR

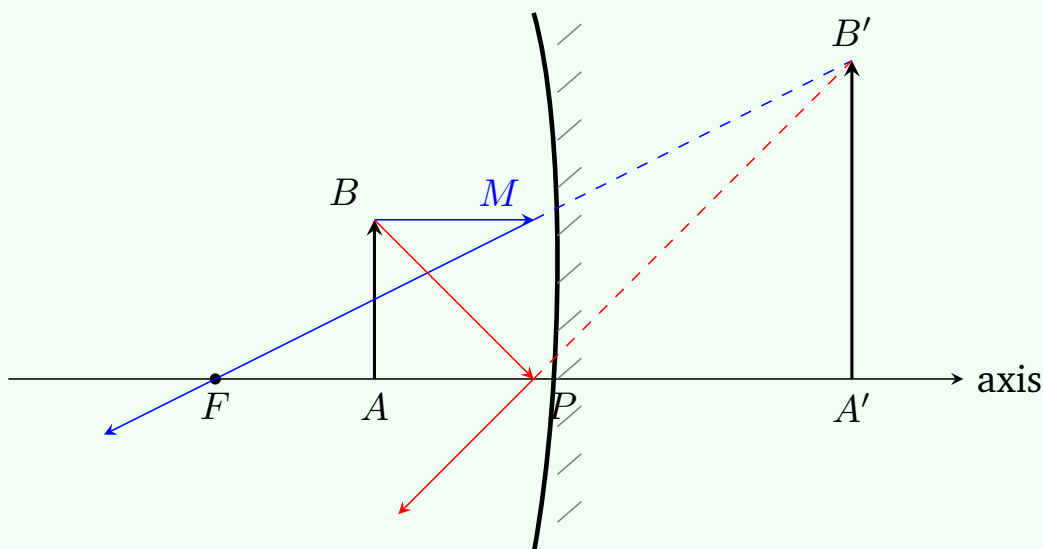
37. An object is placed in front of a concave mirror. It is observed that a virtual image is formed. Draw the ray diagram to show the image formation and hence derive the mirror equation $\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$. An object is placed 30 cm in front of a plano-convex lens with its spherical surface of radius of curvature 20 cm. If the refractive index of the material of the lens is 1.5, find the position and nature of the image formed. [5 marks]

Solution:

Concept:

- A concave mirror forms a virtual, erect, magnified image when the object lies between its pole P and principal focus F . The reflected rays diverge; their backward extensions meet behind the mirror.
- Use the Cartesian convention with the pole as origin and incident light travelling left to right: $u < 0$, $f < 0$, and $v > 0$ for this virtual image. Paraxial rays and a small aperture are assumed in the mirror derivation.
- For a thin lens in air, $1/f_L = (n - 1)(1/R_1 - 1/R_2)$, and $1/f_L = 1/v_L - 1/u_L$. Here n is the refractive index, and the radii carry Cartesian signs.

Step 1: (a) Construct the virtual image Put the upright object AB between F and P . A ray from B parallel to the principal axis reflects towards F . A second ray from B to the pole reflects with equal angles to the principal axis. These reflected rays diverge in front of the mirror, but their dotted backward extensions intersect at B' behind it. The foot A' lies on the axis. Thus $A'B'$ is erect, enlarged and virtual; dotted extensions, rather than the reflected rays themselves, locate it.



Concave mirror with object between focus and pole; dashed backward extensions meet at the virtual image.

Step 2: (a) Derive the mirror equation from the ray geometry Let the object height be h , the image height be h' , and the signed axial coordinates of A, A', F be u, v, f . The pole ray makes equal angles with the axis, so the similar object and image triangles give

$$\frac{h'}{h} = \frac{v}{-u} = -\frac{v}{u}.$$

For the parallel ray, its point of incidence M is approximately in the pole's transverse plane for a paraxial mirror. Thus $PM = h$. The reflected ray through F , extended behind the mirror, has slope $h/(-f)$. At the image plane $x = v$, its height is consequently

$$h' = h + \frac{h}{-f}v = h \left(1 - \frac{v}{f} \right).$$

Equating the two geometric expressions yields

$$\begin{aligned} -\frac{v}{u} &= 1 - \frac{v}{f}, \\ \frac{v}{f} &= 1 + \frac{v}{u}, \\ \frac{1}{f} &= \frac{1}{v} + \frac{1}{u}. \end{aligned}$$

Keeping the signed distances throughout avoids treating the virtual image distance as negative. The derivation is valid in the usual paraxial approximation.

Step 3: (b) Find the focal length of the plano-convex lens Take the curved face first, so $R_1 = +20$ cm and the plane face has $R_2 = \infty$. Reversing the thin lens gives the same positive focal length. With $n = 1.5$, the lens-maker formula gives

$$\frac{1}{f_L} = (1.5 - 1) \left(\frac{1}{20} - 0 \right) = \frac{1}{40} \text{ cm}^{-1}, \quad f_L = +40 \text{ cm}.$$

The object is only 30 cm from this converging lens, so it lies inside the focal length and must produce a virtual image.

Step 4: (b) Calculate the image distance and nature Using $u_L = -30$ cm in the thin-lens equation,

$$\begin{aligned}\frac{1}{40} &= \frac{1}{v_L} - \frac{1}{-30}, \\ \frac{1}{v_L} &= \frac{1}{40} - \frac{1}{30} = -\frac{1}{120}, \\ v_L &= -120 \text{ cm}.\end{aligned}$$

The negative image distance places the image on the object's side of the lens. Also $m = v_L/u_L = (-120)/(-30) = +4$, confirming that the image is erect and four times the object's size.

Step 5: Conclusion (a) The concave mirror produces an erect, magnified virtual image behind it, and the signed-distance derivation gives $1/f = 1/v + 1/u$. (b) The lens image is **120 cm** from the lens on the same side as the object: virtual, erect and magnified four times.

Quick Tip to Solve: Calculate the plano-convex lens focal length first: if the object distance is smaller, expect a negative image distance and positive magnification.