

CBSE Class 12 Physics Question Paper 2020 with Solution PDF-Set 55/1/2



Question Paper with Solutions

Conducted by the Central Board of Secondary Education (CBSE)

Time Allowed: 3 hours

Maximum Marks: 70

General Instructions

- (i) This question paper comprises **four** Sections – **A, B, C** and **D**.
- (ii) There are **37** questions in the question paper. **All** questions are compulsory.
- (iii) **Section A** – Questions no. **1** to **20** are very short answer type questions, carrying **one** mark each.
- (iv) **Section B** – Questions no. **21** to **27** are short answer type questions, carrying **two** marks each.
- (v) **Section C** – Questions no. **28** to **34** are long answer type questions, carrying **three** marks each.
- (vi) **Section D** – Questions no. **35** to **37** are also long answer type questions, carrying **five** marks each.
- (vii) There is no overall choice in the question paper. However, an internal choice has been provided in 2 questions of 1 mark, 2 questions of 2 marks, 1 question of three marks and all the 3 questions of five marks. You have to attempt only one of the choices in such questions.
- (viii) In addition to this, separate instructions are given with each section and question, wherever necessary.
- (ix) Use of calculators and log tables is not permitted.

You may use the following values of physical constants wherever necessary.

$$c = 3 \times 10^8 \text{ m/s}$$

$$h = 6.63 \times 10^{-34} \text{ Js}$$

$$e = 1.6 \times 10^{-19} \text{ C}$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ T m A}^{-1}$$

$$\epsilon_0 = 8.854 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$$

$$\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$$

$$\text{Mass of electron } (m_e) = 9.1 \times 10^{-31} \text{ kg}$$

$$\text{Mass of neutron} = 1.675 \times 10^{-27} \text{ kg}$$

$$\text{Mass of proton} = 1.673 \times 10^{-27} \text{ kg}$$

$$\text{Avogadro's number} = 6.023 \times 10^{23} \text{ per gram mole}$$

$$\text{Boltzmann constant} = 1.38 \times 10^{-23} \text{ JK}^{-1}$$

Section A (20 marks)

Note : Select the most appropriate option from those given below each question :

1. Photons of energies 1 eV and 2 eV are successively incident on a metallic surface of work function 0.5 eV. The ratio of kinetic energy of most energetic photoelectrons in the two cases will be [1 mark]

- (A) 1 : 2
- (B) 1 : 1
- (C) 1 : 3
- (D) 1 : 4

Correct Answer: (C) 1 : 3

Solution:

Concept:

- Einstein's photoelectric equation is $K_{\max} = E - \phi$, where E is the incident photon energy and ϕ is the metal's work function.
- Emission occurs for $E \geq \phi$; only the energy left after overcoming the work function becomes the maximum kinetic energy.

Step 1: Calculate both maximum kinetic energies Both photon energies exceed $\phi = 0.5$ eV. For the same metallic surface,

$$K_1 = (1 - 0.5) \text{ eV} = 0.5 \text{ eV}, \quad K_2 = (2 - 0.5) \text{ eV} = 1.5 \text{ eV}.$$

Step 2: Compare the energies in the given order The required ratio is $K_1 : K_2 = 0.5 : 1.5$. The ratio 1 : 2 compares the incident photon energies and neglects the work function. Equal kinetic energies are impossible for these unequal photon energies, and 1 : 4 does not follow from Einstein's equation.

Step 3: Conclusion The ratio of maximum kinetic energies is $\boxed{1 : 3}$. Hence, the correct option is (C).

Quick Tip to Solve: Subtract the same work function from each photon energy before taking their ratio; photon-energy ratios are not kinetic-energy ratios.

2. Which of the following statements is not correct according to Rutherford model? [1 mark]

- (A) Most of the space inside an atom is empty.
- (B) The electrons revolve around the nucleus under the influence of coulomb force acting on them.
- (C) Most part of the mass of the atom and its positive charge are concentrated at its centre.
- (D) The stability of atom was established by the model.

Correct Answer: (D) The stability of atom was established by the model.

Solution:

Concept:

- Rutherford's nuclear model places almost all the positive charge and atomic mass in a small central nucleus; most atomic volume is empty space.
- An orbiting electron is accelerated. Classical electrodynamics predicts that an accelerating charge radiates electromagnetic energy.

Step 1: Identify the statements consistent with the model Most alpha particles passed through the foil, supporting the empty-space picture in (A). Large deflections supported the concentrated positive nucleus in (C). In the model, the electrostatic Coulomb attraction supplies the centripetal force for revolving electrons, as stated in (B).

Step 2: Examine the stability claim A revolving electron should radiate energy continuously, lose orbital energy and spiral towards the nucleus. The model therefore cannot explain why atoms remain stable. Statement (D) attributes a success to Rutherford's model that it did not achieve; it is the incorrect statement requested here.

Step 3: Conclusion Atomic stability was not established by Rutherford's model. Hence, the correct option is (D).

Quick Tip to Solve: In a Rutherford-model question, separate its successful nuclear picture from its failure to explain stable electron orbits.

3. The resolving power of a telescope can be increased by increasing

[1 mark]

- (A) wavelength of light.
- (B) diameter of objective.
- (C) length of the tube.
- (D) focal length of eyepiece.

Correct Answer: (B) diameter of objective.

Solution:

Concept:

- A telescope resolves two distant point objects when their angular separation exceeds the diffraction limit of its objective.
- For a circular objective, $\theta_{\min} = 1.22\lambda/D$, where D is its diameter and λ is the wavelength. Resolving power is $1/\theta_{\min}$.

Step 1: Express the resolving power Applying the Rayleigh criterion gives

$$\text{Resolving power} = \frac{D}{1.22\lambda}.$$

At a fixed wavelength, a larger objective diameter produces a narrower diffraction pattern and permits smaller angular separations to be distinguished.

Step 2: Check the proposed changes Increasing wavelength increases the diffraction limit and reduces resolving power, so (A) fails. Increasing the tube length or the eyepiece focal length does not increase the objective aperture or reduce its diffraction limit. These changes affect the optical arrangement or magnification, rather than the resolving power described here.

Step 3: Conclusion Increasing the diameter of the objective increases the resolving power. Hence, the correct option is (B).

Quick Tip to Solve: For telescope resolution, use D/λ ; distinguish resolving two points from merely magnifying their images.

4. The magnetic dipole moment of a current carrying coil does not depend upon

[1 mark]

- (A) number of turns of the coil.
- (B) cross-sectional area of the coil.
- (C) current flowing in the coil.
- (D) material of the turns of the coil.

Correct Answer: (D) material of the turns of the coil.

Solution:

Concept:

- A plane coil of N turns, carrying current I , has magnetic dipole moment $\vec{m} = NI\vec{A}$, where $\vec{A}$ is the area vector of each turn.
- The magnitude $m = NIA$ depends on the number of turns, the current and the area enclosed by each turn. Its SI unit is A m^2 .

Step 1: Match the stated quantities to the formula The number of turns in (A), the coil's enclosed area in (B), and the current in (C) all occur explicitly in $m = NIA$. Increasing any one of these quantities, while keeping the others fixed, increases the magnetic dipole moment in the same proportion.

Step 2: Assess the material dependence The material of the wire does not appear in this expression. Changing material can change resistance and thereby the current under a fixed applied voltage, but when the actual current I is specified there is no separate dependence on the wire material. The question concerns the moment of the current loop itself.

Step 3: Conclusion The magnetic dipole moment does not independently depend on the material of the turns. Hence, the correct option is (D).

Quick Tip to Solve: Write $m = NIA$ first and select the quantity absent from it; the area means the area enclosed by a turn, not the wire's cross-section.

5. For a glass prism, the angle of minimum deviation will be smallest for the light of

[1 mark]

- (A) red colour.
- (B) blue colour.
- (C) yellow colour.
- (D) green colour.

Correct Answer: (A) red colour.

Solution:

Concept:

- For a prism of angle A , the minimum deviation δ_m satisfies $\mu = \sin[(A + \delta_m)/2] / \sin(A/2)$, where μ is the glass refractive index relative to air.
- In ordinary dispersion of visible light in glass, the refractive index decreases as wavelength increases. Red light has the longest wavelength among the colours listed.

Step 1: Relate minimum deviation to refractive index For a fixed prism angle,

$$\delta_m = 2 \sin^{-1} \left[\mu \sin \left(\frac{A}{2} \right) \right] - A.$$

Over the physical range for transmission through the prism, this expression increases with μ . Thus the colour with the smallest refractive index has the smallest minimum deviation.

Step 2: Compare the given colours The wavelength decreases in the order red, yellow, green, blue. For glass under normal dispersion, the corresponding refractive indices increase in that order. Blue, yellow and green therefore each undergo a larger minimum deviation than red in the same prism.

Step 3: Conclusion Red light has the smallest angle of minimum deviation. Hence, the correct option is (A).

Quick Tip to Solve: For a glass prism in visible light, remember: longer wavelength means smaller refractive index and smaller deviation.

6. A biconvex lens of glass having refractive index 1.47 is immersed in a liquid. It becomes invisible and behaves as a plane glass plate. The refractive index of the liquid is [1 mark]

- (A) 1.47
- (B) 1.62
- (C) 1.33
- (D) 1.51

Correct Answer: (A) 1.47

Solution:

Concept:

- Refraction at an interface depends on the refractive index of one medium relative to the other, $\mu_{\text{rel}} = \mu_g / \mu_l$, where the subscripts denote glass and liquid.
- The lens-maker relation in a surrounding medium is $1/f = (\mu_g / \mu_l - 1)(1/R_1 - 1/R_2)$, with signed surface radii R_1, R_2 .

Step 1: Apply the condition for vanishing lens power A biconvex lens has a nonzero curvature factor. For it to stop converging or diverging light, the refractive-index factor must vanish:

$$\frac{\mu_g}{\mu_l} - 1 = 0 \quad \Rightarrow \quad \mu_l = \mu_g.$$

With equal indices, rays do not bend at either curved boundary, so the lens loses its optical contrast with the liquid.

Step 2: Substitute and test the alternatives The glass index is 1.47, so the matching liquid must have the same index. Each of 1.62, 1.33 and 1.51 produces a relative index different from unity; the curved interfaces would still refract light and the lens would have nonzero power.

Step 3: Conclusion The liquid has refractive index 1.47. Hence, the correct option is (A).

Quick Tip to Solve: When a transparent lens disappears optically in a liquid, equate their refractive indices; use the relative index, not the glass index alone.

7. The resistance of a metal wire increases with increasing temperature on account of

[1 mark]

- (A) decrease in free electron density.
- (B) decrease in relaxation time.
- (C) increase in mean free path.
- (D) increase in the mass of electron.

Correct Answer: (B) decrease in relaxation time.

Solution:

Concept:

- In the free-electron model, $\rho = m/(ne^2\tau)$, where n is free-electron number density, m is electron mass, e is the charge magnitude and τ is relaxation time.
- Resistance is $R = \rho L/A$. For a metal, the main temperature dependence comes from electron scattering and hence from τ , rather than from changes in carrier density.

Step 1: Connect heating to electron scattering As the wire becomes hotter, lattice ions vibrate more strongly. Conduction electrons are scattered more frequently, reducing the average time between successive scattering events. Thus the relaxation time τ decreases.

Step 2: Apply the inverse dependence on relaxation time Because $\rho \propto 1/\tau$, a decrease in relaxation time raises resistivity and therefore resistance. In a metal, free-electron density is approximately constant over the usual temperature range, so (A) is not the explanation. Increased scattering does not imply the increase in mean free path proposed in (C). The electron's mass does not increase on heating the wire, excluding (D).

Step 3: Conclusion The resistance increases because the relaxation time decreases. Hence, the correct option is (B).

Quick Tip to Solve: For a heated metal, follow the chain: stronger lattice vibrations, more frequent scattering, smaller τ , larger resistance.

8. An electric dipole placed in a non-uniform electric field can experience

[1 mark]

- (A) a force but not a torque.
- (B) a torque but not a force.
- (C) always a force and a torque.
- (D) neither a force nor a torque.

Correct Answer: Ambiguous as printed: (A) is the usual intended answer, but (B) is also possible for a suitable non-uniform field; no geometry is specified.

Solution:

Concept:

- A dipole consists of charges $+q$ and $-q$, with dipole moment directed from negative to positive. Its net force is $\vec{F} = q[\vec{E}(\vec{r}_+) - \vec{E}(\vec{r}_-)]$.
- For a small dipole, torque about its centre is $\vec{\tau} = \vec{p} \times \vec{E}$. Force depends on the field difference at the charges, whereas torque also depends on orientation.

Step 1: Demonstrate the usual intended possibility Consider the non-uniform electrostatic field $\vec{E} = (E_0 + kx)\hat{x}$, with E_0 and k nonzero constants. Put a dipole along the x -axis, centred at the origin,

with charges separated by d . Its unequal collinear forces give $\vec{F} = qkd\hat{x}$, but zero torque. Thus (A) is possible, and the word “always” makes (C) false.

Step 2: Test whether the answer is unique In the same field, put the dipole along the y -axis, centred at the origin. Both charges now lie at $x = 0$, where the field is $E_0\hat{x}$. Their forces cancel, but form a couple with torque magnitude qdE_0 . Thus (B) is also possible despite the field being non-uniform in space. Even (D) can occur in a specially arranged field if the field vectors at the two charges are equal and directed along the dipole axis; spatial non-uniformity elsewhere does not prohibit that configuration.

Step 3: Conclusion The usual intended choice is (A), but the printed words “can experience” and the absence of a specified field geometry do not identify a unique option. Both (A) and (B) have explicit counterexamples to uniqueness; (C) is not universally true.

Quick Tip to Solve: Check the field at both charges and the dipole orientation separately; a field varying in space need not differ at those two particular positions.

9. If the net electric flux through a closed surface is zero, then we can infer

[1 mark]

- (A) no net charge is enclosed by the surface.
- (B) uniform electric field exists within the surface.
- (C) electric potential varies from point to point inside the surface.
- (D) charge is present inside the surface.

Correct Answer: (A) no net charge is enclosed by the surface.

Solution:

Concept:

- Gauss's law relates the net outward electric flux through a closed surface to its enclosed charge:
 $\Phi_E = \oint \vec{E} \cdot d\vec{A} = Q_{\text{enc}}/\epsilon_0$.
- The enclosed charge is an algebraic sum; positive and negative charges may cancel even when individual charges are present.

Step 1: Apply the zero-flux condition Given $\Phi_E = 0$, Gauss's law gives $Q_{\text{enc}} = \epsilon_0\Phi_E = 0$. Thus the surface encloses zero net charge. This condition concerns the total charge, not the local field at every point.

Step 2: Check the other inferences Option (B) is not necessary: external charges can produce a nonuniform field with zero net enclosed charge. Option (C) is not necessary: the potential could be constant, for example when the field vanishes. Option (D) is not necessary either: there may be no charge inside, or equal positive and negative charges. Only the zero algebraic sum follows in every case.

Step 3: Conclusion No net charge is enclosed by the surface. Hence, the correct option is (A).

Quick Tip to Solve: Zero flux fixes the algebraic enclosed charge; it does not by itself say that the field vanishes or that every enclosed charge is absent.

10. Kirchhoff's first rule at a junction in an electrical network, deals with conservation of [1 mark]

- (A) energy
- (B) charge
- (C) momentum
- (D) both energy and charge

Correct Answer: (B) charge

Solution:

Concept:

- Kirchhoff's junction rule states that the total current entering a junction equals the total current leaving it.
- Current is charge per unit time, $I = \Delta Q / \Delta t$. In a steady circuit, charge does not continuously accumulate at a junction.

Step 1: Apply charge balance at the junction During a time interval Δt , the entering charge is $\Delta t \sum I_{\text{in}}$ and the leaving charge is $\Delta t \sum I_{\text{out}}$. Conservation of charge and no steady accumulation require these quantities to be equal, giving $\sum I_{\text{in}} = \sum I_{\text{out}}$.

Step 2: Distinguish the conservation laws Energy conservation leads to Kirchhoff's second, or loop, rule, not the junction rule. Thus (A) and (D) incorrectly attach energy conservation to the first rule. Momentum conservation is not the basis of the current balance, so (C) is also excluded.

Step 3: Conclusion Kirchhoff's first rule expresses conservation of charge. Hence, the correct option is (B).

Quick Tip to Solve: Associate a junction with charge balance and a closed loop with energy balance before selecting the conservation law.

11. A ray of light on passing through an equilateral glass prism, suffers a minimum deviation equal to the angle of the prism. The value of refractive index of the material of the prism is _____. [1 mark]

Solution:

Concept:

- For a prism in air at minimum deviation, $\mu = \sin[(A + \delta_m)/2] / \sin(A/2)$, where A is the prism angle and δ_m is its minimum deviation.
- An equilateral prism has $A = 60^\circ$. At minimum deviation the ray path is symmetric, so each internal refraction angle equals $A/2$.

Step 1: Insert the specified angles The minimum deviation equals the prism angle, hence $\delta_m = A = 60^\circ$. The numerator angle is therefore $(A + \delta_m)/2 = 60^\circ$, while the denominator angle is $A/2 = 30^\circ$.

Step 2: Evaluate the refractive index Using the standard sine values,

$$\mu = \frac{\sin 60^\circ}{\sin 30^\circ} = \frac{\sqrt{3}/2}{1/2} = \sqrt{3}.$$

The refractive index is dimensionless because it is a ratio of speeds, equivalently a ratio of sines in Snell's law.

Step 3: Conclusion The blank is $\sqrt{3}$ (approximately 1.73).

Quick Tip to Solve: When minimum deviation equals the prism angle, simplify to $\mu = 2 \cos(A/2)$, then use $A = 60^\circ$.

12. According to Bohr's atomic model, the circumference of the electron orbit is always an _____ multiple of de Broglie wavelength. [1 mark]

Solution:

Concept:

- Bohr's angular-momentum condition is $mvr = n\hbar = nh/(2\pi)$, where $n = 1, 2, 3, \dots$
- The de Broglie wavelength is $\lambda = h/(mv)$; an allowed circular orbit fits a whole number of wavelengths.

Step 1: Rewrite the quantisation condition Multiply the angular-momentum condition by $2\pi/(mv)$. This gives

$$2\pi r = n \frac{h}{mv}.$$

Here $2\pi r$ is the circumference of the circular electron orbit.

Step 2: Identify the wavelength multiple Substituting $\lambda = h/(mv)$ gives $2\pi r = n\lambda$. Since the principal quantum number n takes positive integer values, the circumference contains a whole number of wavelengths. The electron wave then joins smoothly after one complete revolution, producing a standing-wave condition.

Step 3: Conclusion The blank is integral : the circumference is an integral multiple of the de Broglie wavelength.

Quick Tip to Solve: Remember the standing-wave condition $2\pi r = n\lambda$: it is the circumference, not the radius, that contains an integer number of wavelengths.

OR

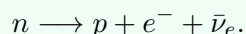
12. In β -decay, the parent and daughter nuclei have the same number of _____. [1 mark]

Solution:

Concept:

- The mass number $A = Z + N$ is the total number of nucleons: protons plus neutrons.
- In beta decay, a neutron changes into a proton or a proton changes into a neutron; the total number of nucleons is unchanged.

Step 1: Track a beta-decay transformation For beta-minus decay,



The daughter gains one proton and loses one neutron, so Z increases by one and N decreases by one. The emitted electron and antineutrino are not nucleons.

Step 2: Compare parent and daughter Adding the daughter proton and neutron counts gives $(Z + 1) + (N - 1) = Z + N = A$. In beta-plus decay the changes are reversed, and the sum remains A again. Thus proton number and neutron number individually change, but their total does not.

Step 3: Conclusion The blank is nucleons. Parent and daughter have the same mass number.

Quick Tip to Solve: For any beta decay keep A unchanged and change Z by one; do not confuse equal nucleon number with equal neutron number.

13. The number of turns of a solenoid are doubled without changing its length and area of cross-section. The self-inductance of the solenoid will become _____ times.

[1 mark]

Solution:

Concept:

- The self-inductance of a long solenoid is $L = \mu N^2 A / \ell$, where N is its number of turns, A its cross-sectional area and ℓ its length.
- When the core material, length and area stay unchanged, self-inductance is proportional to the square of the number of turns: $L \propto N^2$.

Step 1: Apply the changed turn count The number of turns becomes $N' = 2N$, while A , ℓ and permeability μ remain unchanged. Substituting this turn count gives $L' = \mu(2N)^2 A / \ell$.

Step 2: Compare the two inductances Divide the new expression by the original expression:

$$\frac{L'}{L} = \frac{(2N)^2}{N^2} = 4.$$

The square occurs because more turns both increase the magnetic field per unit current and increase the number of turns linking the flux.

Step 3: Conclusion The self-inductance becomes 4 times its original value.

Quick Tip to Solve: With a fixed core and fixed solenoid dimensions, square the factor by which the turn count changes to obtain the inductance factor.

14. Laminated iron sheets are used to minimize _____ currents in the core of a transformer. [1 mark]

Solution:

Concept:

- A changing magnetic flux induces circulating currents within a solid conducting transformer core; these are called eddy currents.
- Eddy currents dissipate electrical energy as heat. Thin iron laminations are insulated from one

another to interrupt large circulating current paths.

Step 1: Identify the induced core currents The alternating current in the primary coil creates a time-varying magnetic flux in the iron core. By Faraday's law, this varying flux induces an emf within the conducting iron and drives closed loops of current.

Step 2: Explain why laminations reduce them Dividing the core into thin insulated sheets breaks up the large conducting loops and increases their effective electrical resistance. The induced circulating currents are consequently reduced, lowering the associated heating loss. This lamination method addresses eddy-current loss; choosing a material with a narrow hysteresis loop addresses hysteresis loss.

Step 3: Conclusion The blank is eddy: laminated iron sheets minimise eddy currents.

Quick Tip to Solve: Link insulated laminations to reduced eddy-current paths; link a narrow magnetic hysteresis loop to reduced hysteresis loss.

15. The magnetic field lines are _____ by a diamagnetic substance.

[1 mark]

Solution:

Concept:

- A diamagnetic substance has negative magnetic susceptibility, $\chi_m < 0$, so its induced magnetisation opposes the applied magnetic field.
- Its relative permeability $\mu_r = 1 + \chi_m$ is slightly less than one; magnetic field lines are less dense inside it than in the surrounding vacuum.

Step 1: Determine the induced magnetic response On placing a diamagnetic material in an external magnetic field, the induced magnetic moments point opposite to the applied field. Their contribution reduces the magnetic field within the substance.

Step 2: Interpret the field-line pattern The field lines bend away from the diamagnetic specimen and are relatively sparse within it. This is described as the magnetic field lines being repelled by the substance. The effect is weak for ordinary diamagnetic materials; it does not mean that every field line is completely excluded from the interior.

Step 3: Conclusion The blank is repelled.

Quick Tip to Solve: Use $\chi_m < 0$ and $\mu_r < 1$ to remember that diamagnetic materials weaken the internal field and repel its field lines.

16. Why cannot we use Si and Ge in fabrication of visible LEDs?

[1 mark]

Solution:

Concept:

- In an LED, electron-hole recombination releases energy approximately equal to the semiconductor band gap, E_g .
- For visible emission, the photon energy $h\nu = hc/\lambda$ must lie in the visible range, approximately 1.8 to 3.1 eV.

Step 1: Compare the band gaps with visible photon energies Silicon and germanium have band gaps of approximately 1.1 eV and 0.7 eV, respectively. These energies are below the visible range; photons corresponding to them have infrared wavelengths.

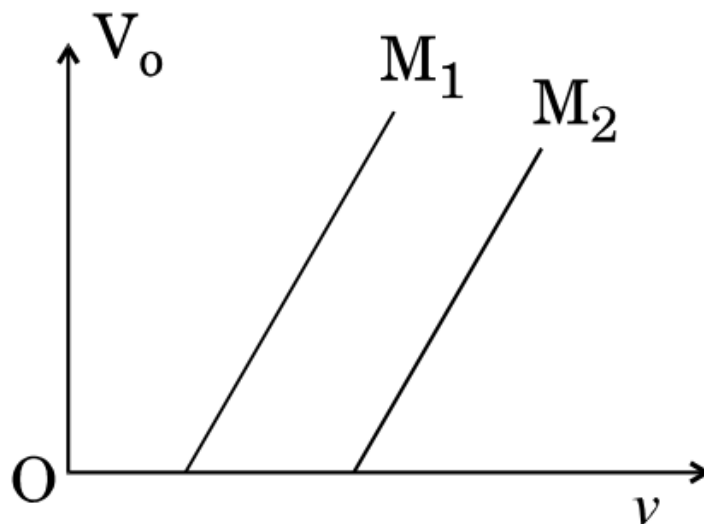
Step 2: Consider the efficiency of light emission Both Si and Ge are indirect-band-gap materials. Recombination therefore releases energy mainly through lattice vibrations rather than efficient photon emission, making them unsuitable for ordinary visible LEDs.

Step 3: Conclusion Si and Ge cannot be used to fabricate ordinary visible LEDs because their band gaps are too small for visible photons and their radiative recombination is inefficient.

Quick Tip to Solve: Compare the semiconductor band gap with about 1.8 eV, the low-energy end of visible light; Si and Ge both fall below it.

17. The variation of the stopping potential (V_0) with the frequency (ν) of the light incident on two different photosensitive surfaces M_1 and M_2 is shown in the figure. Identify the surface which has greater value of the work function.

[1 mark]



Solution:

Concept:

- Einstein's photoelectric equation is $eV_0 = h\nu - \phi$, where V_0 is stopping potential, e is the elementary charge and ϕ is work function.
- At the threshold frequency ν_0 , the maximum kinetic energy is zero, so $\phi = h\nu_0$.

Step 1: Read the frequency-axis intercepts The frequency-axis intercept of each line occurs at $V_0 = 0$, and hence gives its threshold frequency. In the printed graph, the line for M_2 cuts the frequency axis

to the right of the line for M_1 . Thus $\nu_{02} > \nu_{01}$.

Step 2: Relate threshold frequency to work function Multiply the threshold frequencies by the same positive Planck constant h . Since $\phi_2 = h\nu_{02}$ and $\phi_1 = h\nu_{01}$, their ordering follows directly. The common slope h/e does not distinguish the work functions.

Step 3: Conclusion Surface M_2 has the greater work function: $\phi_2 > \phi_1$.

Quick Tip to Solve: On a stopping-potential versus frequency graph, compare the frequency-axis intercepts: the farther-right intercept means a larger work function.

18. How does an increase in doping concentration affect the width of depletion layer of a p-n junction diode? [1 mark]

Solution:

Concept:

- The depletion layer contains immobile ionised donors and acceptors left behind when mobile carriers diffuse across a p-n junction and recombine.
- Its electric field opposes further diffusion. For a given junction potential, a higher density of fixed ions requires a smaller depletion width to establish the barrier.

Step 1: Identify the effect of heavier doping Increasing doping increases the concentration of donor or acceptor atoms. Hence a depleted region of a given width contains a greater density of uncovered fixed charge than in a lightly doped junction.

Step 2: Relate fixed charge density to the layer width The opposing junction field can then develop over a shorter distance. Thus, under otherwise comparable conditions, depletion extends less far into a more heavily doped region; a heavily doped junction has a thinner depletion layer.

Step 3: Conclusion Increasing the doping concentration decreases the width of the depletion layer of the p-n junction diode.

Quick Tip to Solve: Remember the inverse trend: heavier doping gives more fixed charge per unit volume, so a thinner depletion region establishes the junction barrier.

19. The nuclear radius of ${}^{27}_{13}\text{Al}$ is 3.6 fermi. Find the nuclear radius of ${}^{64}_{29}\text{Cu}$. [1 mark]

Solution:

Concept:

- The nuclear radius follows $R = R_0 A^{1/3}$, where A is the mass number and R_0 is approximately the same for different nuclei.
- Taking the ratio of two radii eliminates R_0 . One fermi equals 10^{-15} m.

Step 1: Form the ratio using the mass numbers The superscripts give $A_{\text{Al}} = 27$ and $A_{\text{Cu}} = 64$. The atomic numbers 13 and 29 are not used in the nuclear-radius law. Therefore,

$$\frac{R_{\text{Cu}}}{R_{\text{Al}}} = \left(\frac{64}{27}\right)^{1/3} = \frac{4}{3}.$$

Step 2: Substitute the given aluminium radius With $R_{Al} = 3.6$ fermi, the copper radius is obtained by multiplying by this dimensionless ratio:

$$R_{Cu} = 3.6 \times \frac{4}{3} \text{ fermi} = 4.8 \text{ fermi}.$$

Step 3: Conclusion The nuclear radius of ${}^{64}_{29}\text{Cu}$ is $4.8 \text{ fermi} = 4.8 \times 10^{-15} \text{ m}$.

Quick Tip to Solve: Use the cube roots of the mass numbers: $64 = 4^3$ and $27 = 3^3$, so the radius ratio is immediately $4 : 3$.

OR

19. A proton and an electron have equal speeds. Find the ratio of de Broglie wavelengths associated with them. [1 mark]

Solution:

Concept:

- The de Broglie wavelength of a particle of momentum p is $\lambda = h/p$, where h is Planck's constant.
- For non-relativistic motion, $p = mv$. At equal speeds, wavelength is inversely proportional to the particle's mass.

Step 1: Write the wavelengths at the common speed Let the common speed be v , and let m_p and m_e denote the proton and electron masses. Their wavelengths are

$$\lambda_p = \frac{h}{m_p v}, \quad \lambda_e = \frac{h}{m_e v}.$$

The particles have equal speeds, but their momenta differ because their masses differ.

Step 2: Divide in proton-to-electron order Dividing cancels the common quantities h and v . Using $m_p \simeq 1836m_e$,

$$\frac{\lambda_p}{\lambda_e} = \frac{m_e}{m_p} \simeq \frac{1}{1836}.$$

The heavier proton consequently has the shorter wavelength.

Step 3: Conclusion The ratio of proton to electron de Broglie wavelengths is

$$\lambda_p : \lambda_e = m_e : m_p \simeq 1 : 1836.$$

Quick Tip to Solve: Write the ratio's order first: at equal speeds the wavelength ratio reverses the mass ratio, so the proton wavelength must be smaller.

20. How is displacement current produced between the plates of a parallel plate capacitor during charging? [1 mark]

Solution:

Concept:

- A time-varying electric field produces displacement current. For an air or vacuum gap, $I_d = \epsilon_0 d\Phi_E/dt$, where Φ_E is electric flux.
- Between ideal parallel plates of area A , the field is approximately uniform, so $\Phi_E = EA$. Charge need not cross the insulating gap for displacement current to exist.

Step 1: Identify what changes during charging Charge supplied through the connecting wires accumulates on the plates. As the plate charge Q increases, the electric field E , and therefore the electric flux between the plates, also increases with time.

Step 2: Apply Maxwell's displacement-current expression Neglecting edge effects, $E = Q/(\epsilon_0 A)$. Thus the changing field produces

$$I_d = \epsilon_0 \frac{d(EA)}{dt} = \frac{dQ}{dt}.$$

This equals the conduction current in the charging wires, although no conduction charge flows through the ideal insulating space between the plates.

Step 3: Conclusion Displacement current is produced by the time-varying electric flux between the capacitor plates during charging: $I_d = \epsilon_0 d\Phi_E/dt$.

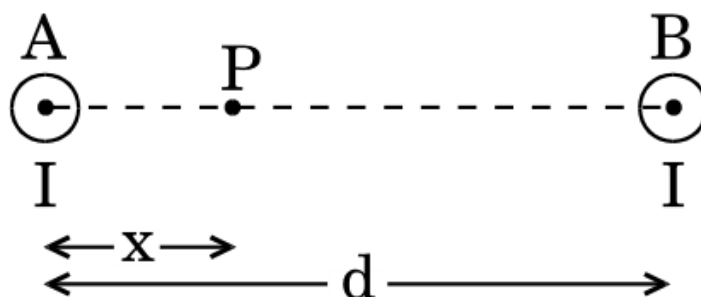
Quick Tip to Solve: Trace the sequence: increasing plate charge changes the electric field, which changes electric flux and therefore produces displacement current.

Section B (14 marks)

21. Two long straight parallel wires A and B separated by a distance d , carry equal current I flowing in same direction as shown in the figure.

(a) Find the magnetic field at a point P situated between them at a distance x from one wire.

(b) Show graphically the variation of the magnetic field with distance x for $0 < x < d$. [2 marks]



Solution:

Concept:

- An infinitely long straight wire carrying current I produces magnetic field of magnitude $B = \mu_0 I/(2\pi r)$ at perpendicular distance r , where μ_0 is the permeability of free space.
- The right-hand thumb rule gives the field direction. Magnetic fields add vectorially, so oppositely directed fields between equal parallel currents subtract.

Step 1: (a) Resolve the fields at the point between the wires Take x from wire A , so the distance from B is $d - x$. The dots in the printed figure represent currents out of the page. At P , the field of A is upward in the page, while that of B is downward. With upward taken positive,

$$B_A = \frac{\mu_0 I}{2\pi x}, \quad B_B = -\frac{\mu_0 I}{2\pi(d-x)}.$$

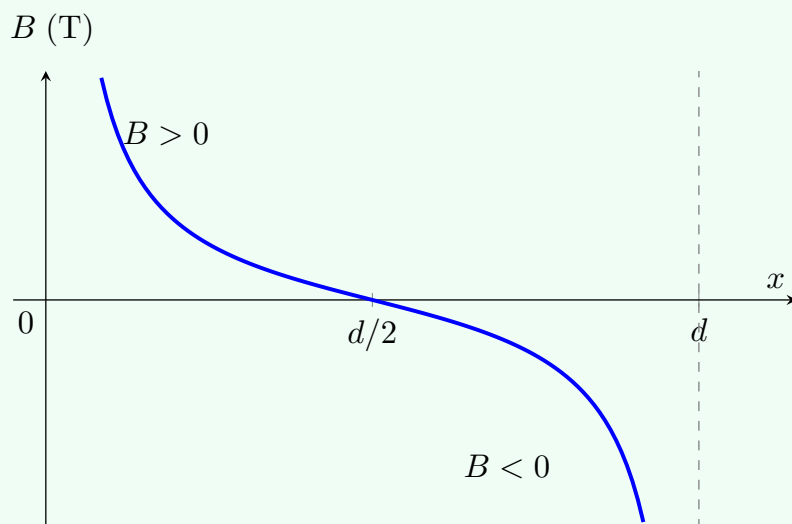
Superposition therefore gives the signed resultant component

$$B(x) = \frac{\mu_0 I}{2\pi} \left(\frac{1}{x} - \frac{1}{d-x} \right) = \frac{\mu_0 I(d-2x)}{2\pi x(d-x)}.$$

Step 2: (b) Sketch the signed magnetic field The field tends to $+\infty$ as $x \rightarrow 0^+$, vanishes at $x = d/2$, and tends to $-\infty$ as $x \rightarrow d^-$, in the ideal thin-wire model. The graph decreases throughout the interval because

$$\frac{dB}{dx} = -\frac{\mu_0 I}{2\pi} \left(\frac{1}{x^2} + \frac{1}{(d-x)^2} \right) < 0.$$

Its positive and negative signs specify upward and downward directions respectively. If only the magnitude is required, use $|B(x)|$, reflecting the negative part of this signed graph above the horizontal axis.



Signed field between the wires (upward positive); the dashed line marks the second ideal-wire limit.

Step 3: Conclusion $B = \frac{\mu_0 I}{2\pi} \left(\frac{1}{x} - \frac{1}{d-x} \right)$, taking upward as positive. It is upward for $0 < x < d/2$, zero at the midpoint and downward for $d/2 < x < d$.

Quick Tip to Solve: Between currents in the same direction, subtract the two fields; use the midpoint cancellation to check the graph's zero.

22. Using Bohr's atomic model, derive the expression for the radius of n^{th} orbit of the revolving electron in a hydrogen atom. [2 marks]

Solution:

Concept:

- In Bohr's model, electrostatic attraction between the proton and electron supplies the centripetal force. Treat the proton as stationary and let m_e , e , v_n and r_n denote the electron mass, charge magnitude, speed and orbital radius.
- Allowed circular orbits satisfy angular momentum quantisation: $m_e v_n r_n = n\hbar = nh/(2\pi)$, where $n = 1, 2, 3, \dots$ and h is Planck's constant.

Step 1: Equate the electrostatic and centripetal forces For hydrogen, the nucleus has charge $+e$, so Coulomb's law gives

$$\frac{e^2}{4\pi\epsilon_0 r_n^2} = \frac{m_e v_n^2}{r_n}.$$

Thus $m_e v_n^2 r_n = e^2/(4\pi\epsilon_0)$, with ϵ_0 the permittivity of free space. This force equation alone permits a continuous set of radii; the quantum condition selects the allowed ones.

Step 2: Eliminate the speed using angular momentum quantisation Bohr's condition yields $v_n = nh/(2\pi m_e r_n)$. Substitute this expression into the force equation:

$$\begin{aligned} \frac{e^2}{4\pi\epsilon_0 r_n^2} &= \frac{m_e}{r_n} \frac{n^2 h^2}{4\pi^2 m_e^2 r_n^2}, \\ \frac{e^2 r_n}{4\pi\epsilon_0} &= \frac{n^2 h^2}{4\pi^2 m_e}. \end{aligned}$$

Rearranging isolates the radius and shows its dependence on the square of the principal quantum number.

Step 3: Conclusion The radius of the n^{th} hydrogen orbit is

$$r_n = \frac{\epsilon_0 h^2}{\pi m_e e^2} n^2 = a_0 n^2, \quad a_0 \simeq 5.29 \times 10^{-11} \text{ m}.$$

Quick Tip to Solve: Use the force equation together with $mvr = n\hbar$; eliminate v and check that the hydrogen radius scales as n^2 .

OR

22. (a) Write two main observations of photoelectric effect experiment which could only be explained by Einstein's photoelectric equation.

(b) Draw a graph showing variation of photocurrent with the anode potential of a photocell. [2 marks]

Solution:

Concept:

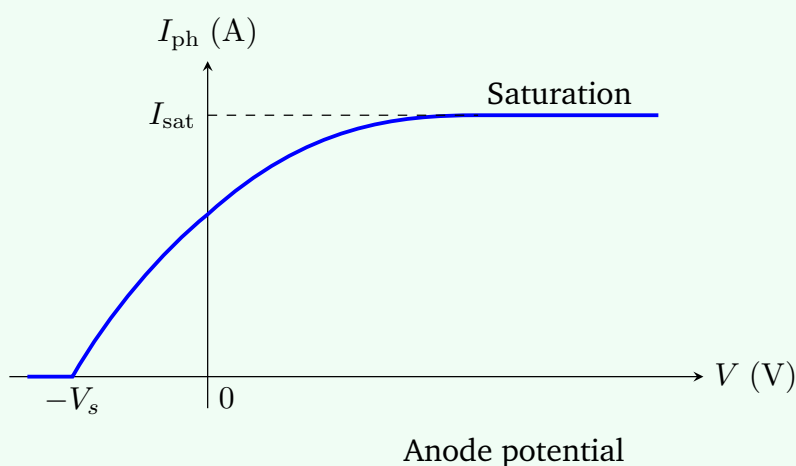
- Einstein's equation is $K_{\text{max}} = h\nu - \phi = eV_s$, where ν is the incident frequency, ϕ the work function, K_{max} the maximum electron kinetic energy and V_s the positive magnitude of stopping potential.
- A photon transfers energy $h\nu$ to one electron. For fixed frequency above threshold, raising intensity

increases the number of emitted electrons, while anode potential controls their collection.

Step 1: (a) State two observations explained by the photon equation **Threshold frequency:** No photoelectrons are emitted for $\nu < \nu_0$, however intense the light, where $\nu_0 = \phi/h$. Each photon must supply at least the work function; increasing the number of sub-threshold photons does not increase their individual energies.

Frequency dependence of maximum energy: Above threshold, $K_{\max}$ increases linearly with frequency and is independent of intensity at a fixed frequency. Einstein's equation gives slope h for the $K_{\max}$ -versus- ν graph. Greater intensity supplies more photons, hence more electrons, without changing the maximum energy per electron.

Step 2: (b) Draw and interpret the current versus anode potential curve Keep the incident frequency and intensity fixed, with $\nu > \nu_0$. A positive anode potential attracts emitted electrons; the photocurrent rises and reaches saturation when all emitted electrons are collected. A finite current exists at zero potential because electrons emerge with kinetic energy. Making the anode negative retards them, and the current falls to zero at $V = -V_s$, where even the fastest electrons are stopped. It stays zero for more negative potentials.



Photocurrent at fixed frequency and intensity; the negative intercept is the stopping potential.

Step 3: Conclusion The threshold frequency and the intensity-independent, linear frequency dependence of $K_{\max}$ follow from $K_{\max} = h\nu - \phi$. The photocurrent curve reaches zero at $-V_s$ and saturates at sufficiently positive anode potential.

Quick Tip to Solve: Put the stopping-potential intercept on the negative side of the voltage axis, and show nonzero photocurrent at zero anode potential.

23. Define wavefront of a travelling wave. Using Huygens principle, obtain the law of refraction at a plane interface when light passes from a rarer to a denser medium. [2 marks]

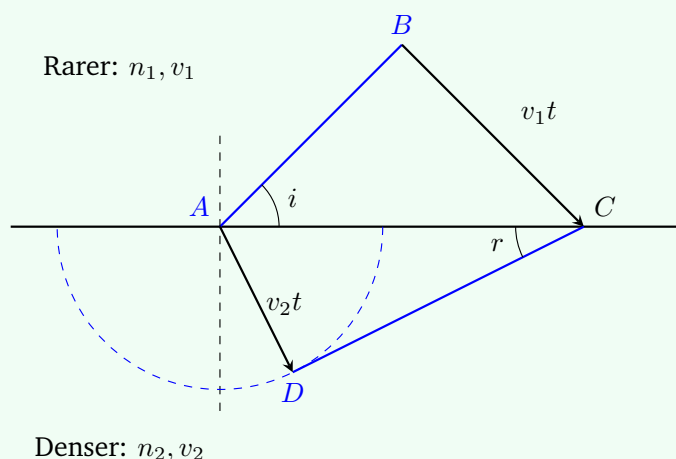
Solution:

Concept:

- A wavefront is the locus of points of a travelling wave that are in the same phase at a given instant. In an isotropic medium, rays are normal to the wavefront.
- Huygens' principle states that every point of a wavefront acts as a source of secondary wavelets. Their forward common envelope gives the later wavefront; the wavelets travel at the speed of light

in that medium.

Step 1: Construct the refracted wavefront Let light travel from the rarer medium of index n_1 , speed v_1 , into the denser medium of index $n_2 > n_1$, speed $v_2 < v_1$. The incident plane wavefront AB first touches the interface at A . In time t , its point B reaches the interface at C , so $BC = v_1 t$. During the same time the secondary wavelet from A travels a distance $AD = v_2 t$ in the denser medium. The tangent CD to this wavelet is the refracted wavefront, with AD perpendicular to CD .



Huygens' construction for light passing from a rarer to a denser medium; rays are normal to their wavefronts.

Step 2: Use the two right triangles to obtain Snell's law The angle between incident wavefront AB and the interface AC equals the incidence angle i , measured between the incident ray and the normal. Similarly, the angle between CD and AC equals the refraction angle r . Therefore,

$$\sin i = \frac{BC}{AC} = \frac{v_1 t}{AC}, \quad \sin r = \frac{AD}{AC} = \frac{v_2 t}{AC}.$$

Dividing and using $v = c/n$,

$$\frac{\sin i}{\sin r} = \frac{v_1}{v_2} = \frac{n_2}{n_1}.$$

The incident ray, refracted ray and interface normal lie in the same plane of the construction, giving the other law of refraction.

Step 3: Conclusion A wavefront joins points of equal phase. Huygens' construction gives $n_1 \sin i = n_2 \sin r$. Since $n_2 > n_1$, $r < i$ for oblique incidence: the ray bends towards the normal on entering the denser medium.

Quick Tip to Solve: Compare the distances travelled in the same time: the slower secondary wavelet in the denser medium gives the smaller refraction angle.

23. Using lens maker's formula, derive the thin lens formula $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$ for a biconvex lens. [2 marks]

Solution:

Concept:

- For paraxial refraction at a spherical surface, $n_2/v - n_1/u = (n_2 - n_1)/R$, with all distances measured using the Cartesian sign convention.
- For a thin lens of refractive index μ in air, the lens maker's formula is $1/f = (\mu - 1)(1/R_1 - 1/R_2)$, where R_1 and R_2 are the signed radii of its surfaces.

Step 1: Apply refraction at the two surfaces Let light travel from left to right through a thin biconvex lens. Distances to the right are positive, so $R_1 > 0$ and $R_2 < 0$. Let u be the object distance and v_1 the image distance due to its first surface. Refraction from air into glass gives

$$\frac{\mu}{v_1} - \frac{1}{u} = \frac{\mu - 1}{R_1}.$$

This intermediate image acts as the object for the second surface. Since the lens is thin, its distance from the second surface is approximately v_1 . If the final image is at v , refraction from glass into air gives

$$\frac{1}{v} - \frac{\mu}{v_1} = \frac{1 - \mu}{R_2}.$$

Step 2: Add the equations and use the lens maker's formula Adding cancels the intermediate-image term. Thus the optical effect of both surfaces is

$$\frac{1}{v} - \frac{1}{u} = (\mu - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right).$$

The expression on the right equals $1/f$ by the lens maker's formula. The derivation uses the thin-lens and paraxial approximations; the signed distances therefore refer to the same optical centre.

Step 3: Conclusion The required thin lens formula for the biconvex lens is

$$\boxed{\frac{1}{f} = \frac{1}{v} - \frac{1}{u}}.$$

Quick Tip to Solve: Add the two surface-refraction equations first: the intermediate image cancels, leaving exactly the curvature combination in the lens maker's formula.

24. Explain the principle of working of a meter bridge. Draw the circuit diagram for determination of an unknown resistance using it. [2 marks]

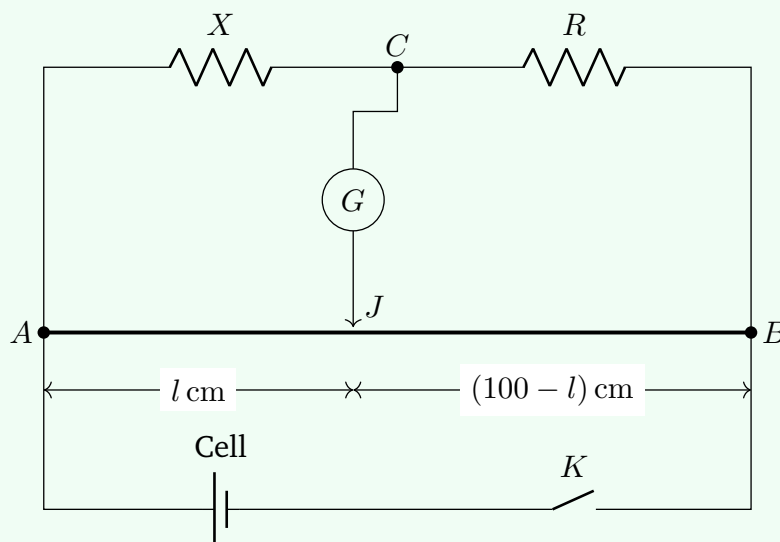
Solution:

Concept:

- A metre bridge works on the Wheatstone bridge principle: at balance, no current flows through the galvanometer and the ratios of resistances in the two arms are equal.
- For a uniform wire of cross-sectional area A and resistivity ρ , its resistance is $r = \rho L/A$; hence the

resistances of its two segments are proportional to their lengths.

Step 1: Connect the unknown resistance and locate the null point Place the unknown resistance X in the left gap and a known resistance R in the right gap. Connect a cell and key K across the ends A and B of the uniform one-metre wire. Connect the galvanometer G between the central junction C and a sliding jockey J . Close the key and move the jockey until the galvanometer shows zero deflection. Then C and J have equal potentials.



Metre bridge: X is the unknown resistance, R the known resistance, and J the jockey at the null point.

Step 2: Use the balance condition to determine the unknown Measure the balance length $AJ = l$ in centimetres, so $JB = (100 - l)$ centimetres. Since both wire segments have the same material and cross-sectional area, the Wheatstone balance condition gives

$$\frac{X}{R} = \frac{r_{AJ}}{r_{JB}} = \frac{l}{100 - l}.$$

The known resistance and measured balance length therefore determine X . Choose R so that the null point lies near the middle of the wire for better sensitivity.

Step 3: Conclusion A metre bridge uses the null condition of a Wheatstone bridge. For the connections shown, the unknown resistance is

$$X = R \frac{l}{100 - l}.$$

Quick Tip to Solve: Write the resistance ratio in the same left-to-right order as the wire lengths: unknown in the left gap means $X/R = l/(100 - l)$.

25. Explain the terms ‘depletion layer’ and ‘potential barrier’ in a p-n junction diode. How are the (a) width of depletion layer, and (b) value of potential barrier affected when the p-n junction is forward biased? [2 marks]

Solution:

Concept:

- Carrier diffusion across a newly formed p-n junction leaves fixed ionised donors on the n-side and fixed ionised acceptors on the p-side near the junction.
- These fixed charges establish an internal electric field and a built-in potential difference that opposes further diffusion of majority carriers.

Step 1: Define the depletion layer and potential barrier Electrons diffuse from the n-region into the p-region, while holes diffuse in the opposite direction, and they recombine near the junction. The resulting narrow region on either side of the junction is depleted of mobile charge carriers and is called the depletion layer. It contains immobile positive donor ions on the n-side and immobile negative acceptor ions on the p-side; it is therefore not a region empty of all charge. The potential difference developed across this layer is the potential barrier. Its electric field is directed from the n-side to the p-side and opposes further majority-carrier diffusion across the junction.

Step 2: Explain the effect of forward bias Forward bias connects the p-side to the positive terminal and the n-side to the negative terminal of an external supply. The applied field opposes the junction’s internal field, making it easier for majority carriers to cross. Consequently, (a) the width of the depletion layer decreases and (b) the effective potential barrier decreases. In the elementary junction model, an applied forward voltage V_F reduces the barrier from V_b to approximately $V_b - V_F$.

Step 3: Conclusion The depletion layer is the junction region deficient in mobile carriers; the potential barrier is the built-in potential difference opposing majority-carrier diffusion. Forward bias reduces both the depletion-layer width and the effective potential barrier.

Quick Tip to Solve: Remember the forward connection as p to positive and n to negative: it opposes the built-in field, so both junction width and barrier decrease.

26. N small conducting liquid droplets, each of radius r , are charged to a potential V each. These droplets coalesce to form a single large drop without any charge leakage. Find the potential of the large drop. [2 marks]

Solution:

Concept:

- The potential of an isolated conducting sphere of radius a and charge Q , relative to infinity, is $V_s = Q/(4\pi\epsilon_0 a)$.
- Coalescence conserves the total liquid volume and, in the absence of leakage, the total charge. The electric potential itself is not an additive conserved quantity.

Step 1: Determine the radius of the large drop Let R be the radius of the final spherical drop. Since all N small drops have radius r , volume conservation gives

$$\frac{4}{3}\pi R^3 = N\frac{4}{3}\pi r^3, \quad R = N^{1/3}r.$$

The radius therefore grows as the cube root of the number of drops, because volume is proportional

to the cube of radius.

Step 2: Conserve charge and evaluate the new potential If each small drop has charge q and potential V , the spherical-conductor formula gives

$$q = 4\pi\epsilon_0 rV.$$

There is no charge leakage, so the final charge is $Q = Nq$. Its potential V_{large} , using the same surrounding medium and reference at infinity, is

$$\begin{aligned} V_{\text{large}} &= \frac{Q}{4\pi\epsilon_0 R} = \frac{N(4\pi\epsilon_0 rV)}{4\pi\epsilon_0 N^{1/3}r} \\ &= N^{2/3}V. \end{aligned}$$

Charge rises by a factor N , whereas radius rises by only $N^{1/3}$; their ratio determines the potential increase.

Step 3: Conclusion The potential of the large drop is

$$V_{\text{large}} = N^{2/3}V.$$

It retains the sign of the initial potential; for $N = 1$, the expression correctly returns V .

Quick Tip to Solve: First conserve volume to obtain $R = N^{1/3}r$, then use potential proportional to charge divided by radius; do not add the initial potentials.

27. Define activity of a sample of a radioactive substance. The value of the disintegration constant of a radioactive substance is 0.0693 h^{-1} . Find the time after which the activity of a sample of this substance reduces to one-half that of its present value. [2 marks]

Solution:

Concept:

- Activity is the number of radioactive disintegrations occurring per unit time in a sample: $A = -dN/dt = \lambda N$, where N is the number of undecayed nuclei and λ is the decay constant.
- Radioactive decay gives $N = N_0 e^{-\lambda t}$. Consequently, the activity follows $A = A_0 e^{-\lambda t}$, and the time for it to halve is the half-life.

Step 1: Define the activity and its unit Activity measures the rate of spontaneous nuclear disintegration of the sample. Since the number of undecayed nuclei decreases, dN/dt is negative; the minus sign makes the activity positive. Its SI unit is the becquerel: $1 \text{ Bq} = 1 \text{ disintegration per second}$.

Step 2: Apply the half-activity condition Take the present activity as A_0 . At the required time, $A = A_0/2$. Dividing the decay law by A_0 and taking logarithms gives

$$\frac{1}{2} = e^{-\lambda t}, \quad \lambda t = \ln 2, \quad t = \frac{0.693}{0.0693 \text{ h}^{-1}}.$$

The activity and the number of undecayed nuclei fall by the same fraction because the decay constant remains fixed. The time obtained is in hours, since the supplied constant is in inverse hours.

Step 3: Conclusion Activity is the positive disintegration rate $A = -dN/dt$. The activity becomes

one-half of its present value after 10 h.

Quick Tip to Solve: For any halving of activity, use $t = 0.693/\lambda$; the original activity is unnecessary and the time unit comes from λ .

Section C (21 marks)

28. In a single slit diffraction experiment, light of wavelength λ illuminates the slit of width a and the diffraction pattern is observed on a screen.

(a) Show the intensity distribution in the pattern with the angular position θ .

(b) How are the intensity and angular width of central maxima affected when

(i) width of slit is increased, and

(ii) separation between slit and screen is decreased?

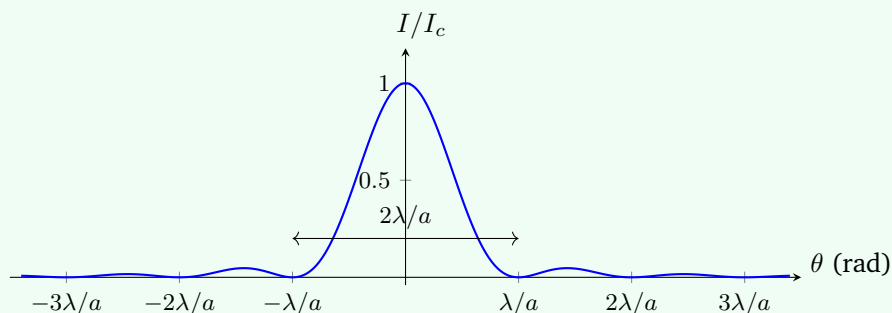
[3 marks]

Solution:

Concept:

- The coherent secondary wavelets across a slit interfere to give the single-slit distribution $I(\theta) = I_c(\sin \beta / \beta)^2$, where $\beta = \pi a \sin \theta / \lambda$ and I_c is the central intensity.
- Minima occur at $a \sin \theta = n\lambda$, for nonzero integers n . For small angles, the central maximum's full angular width is $\Delta\theta = 2\lambda/a$.

Step 1: Sketch the angular intensity distribution At $\theta = 0$, all parts of the slit contribute in phase and the intensity is greatest. The first minima lie at $\theta \simeq \pm\lambda/a$; weaker secondary maxima lie between successive minima on either side. The central maximum is twice as wide as the secondary maxima, approximately, and the pattern is symmetric about the forward direction. The graph shows relative intensity in the small-angle approximation.



Single-slit intensity distribution; the central peak lies between the first two minima.

Step 2: Increase the slit width For the same incident illumination and screen distance, increasing a admits more light. At the centre, the amplitudes add in phase, so the central amplitude is proportional to a and $I_c \propto a^2$. Thus the central maximum becomes brighter. Meanwhile $\Delta\theta = 2\lambda/a$ decreases: its bounding minima move towards the axis and the maximum becomes angularly narrower.

Step 3: Decrease the slit-to-screen separation Let the screen distance be D , and keep a , λ and the

incident illumination fixed. Within the far-field approximation, central screen intensity varies as $I_c \propto a^2/D^2$, so it increases when D decreases. The angular positions of the minima depend on a and λ , not on D ; hence the angular width remains unchanged. Only the linear width on the screen, $W = D\Delta\theta = 2D\lambda/a$, decreases. These statements assume the screen remains in the far-field diffraction regime.

Step 4: Conclusion Increasing slit width increases central intensity and decreases angular width. Decreasing screen distance increases central screen intensity while leaving angular width unchanged; the linear width decreases.

Quick Tip to Solve: Distinguish angular width $2\lambda/a$ from screen width $2D\lambda/a$; only the latter changes with screen distance.

29. With the help of a simple diagram, explain the working of a silicon solar cell, giving all three basic processes involved. Draw its I-V characteristic. [3 marks]

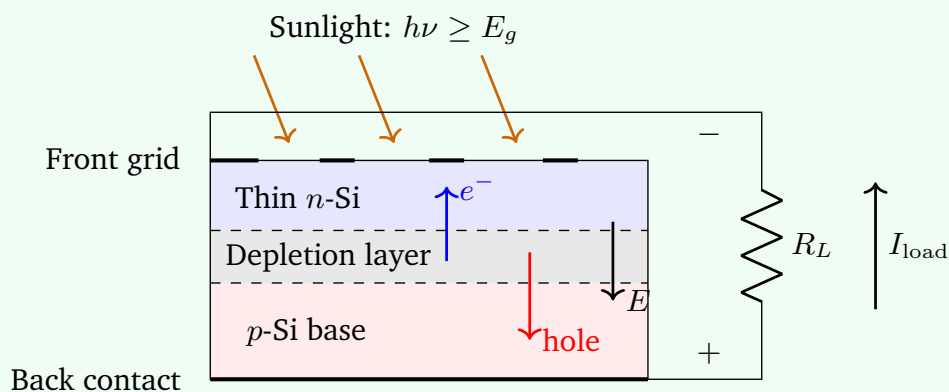
Solution:

Concept:

- A solar cell is a large-area semiconductor p - n junction that converts incident light into electrical energy by the photovoltaic effect, without an external bias supply.
- Photons with energy $h\nu \geq E_g$ generate electron-hole pairs. For silicon, the band gap E_g is approximately 1.1 eV.
- The depletion-region electric field separates the photogenerated charges. Metal contacts collect them and supply current to an external load.

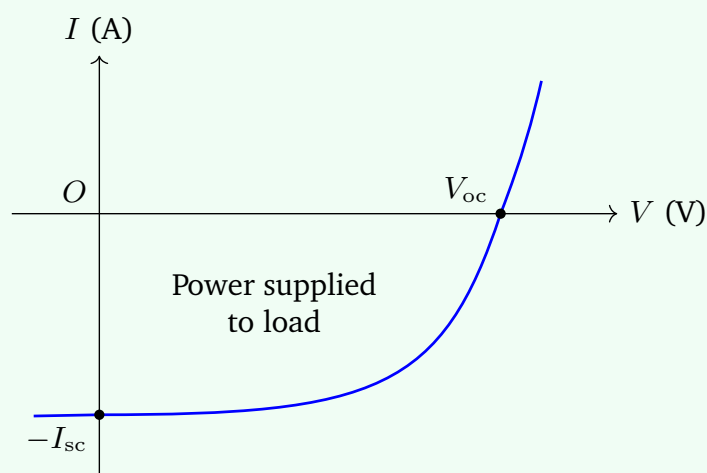
Step 1: Identify the construction and generate charge carriers A thin n -type silicon layer is formed over a thicker p -type base. A front metal grid admits sunlight, and a back metal contact completes the connection. The junction is kept close to the illuminated surface so that photogenerated carriers can reach its depletion region. The first process is **generation**: absorbed photons of sufficient energy excite electrons into the conduction band, leaving holes in the valence band and creating electron-hole pairs in or near the junction.

Step 2: Separate and collect the carriers The second process is **separation**: the built-in electric field points from the n side to the p side. It drives electrons towards the n side and holes towards the p side, preventing their immediate recombination. The third process is **collection**: the contacts collect electrons on the n side and holes on the p side. Thus the p terminal becomes positive and the n terminal negative. With a load connected, conventional current flows through it from the p contact to the n contact; electrons flow oppositely. Continuous illumination maintains the electrical output.



Illuminated silicon solar cell: the junction field separates carriers, and contacts collect them.

Step 3: Draw and interpret the illuminated current-voltage curve Use $V = V_p - V_n$, and define positive I as forward diode current entering the p terminal. Illumination shifts the diode characteristic downwards. At zero voltage, $I = -I_{sc}$, where I_{sc} is the short-circuit current magnitude; at zero current, $V = V_{oc}$, the open-circuit voltage. Between these intercepts, $V > 0$ and $I < 0$: the cell delivers power to a load. Thus its power-generating part lies in the fourth quadrant in this convention. The diagram shows this characteristic schematically for fixed illumination.



Illuminated I - V characteristic with current positive into the p terminal.

Step 4: Conclusion The three processes are generation, separation and collection of electron-hole pairs. Their action produces a photovoltage and supplies load current, with electrical power generated in the fourth-quadrant region of the stated I - V convention.

Quick Tip to Solve: Label the p terminal positive, the n terminal negative, and specify the current convention before drawing the solar-cell characteristic.

30. A resistor R and an inductor L are connected in series to a source $V = V_0 \sin \omega t$.

Find the

(a) peak value of the voltage drops across R and across L ,

(b) phase difference between the applied voltage and current. Which of them is ahead?

[3 marks]

Solution:

Concept:

- The inductive reactance of an ideal inductor is $X_L = \omega L$; the impedance of a series resistor-inductor circuit is $Z = \sqrt{R^2 + X_L^2}$.
- The resistor voltage is in phase with the current, while the inductor voltage leads the current by $\pi/2$. Their amplitudes therefore add as perpendicular phasors.

Step 1: Find the peak current The same alternating current flows through both series elements. Since the applied voltage has peak value V_0 , its steady-state peak current is

$$I_0 = \frac{V_0}{Z} = \frac{V_0}{\sqrt{R^2 + (\omega L)^2}}.$$

The phase shift of the current must be retained; in general it is not $I_0 \sin \omega t$.

Step 2: Obtain the two peak voltage drops (a) For the resistor, the voltage amplitude equals $I_0 R$, whereas for the inductor it equals $I_0 X_L$. Thus

$$V_{R0} = \frac{V_0 R}{\sqrt{R^2 + (\omega L)^2}},$$
$$V_{L0} = \frac{V_0 \omega L}{\sqrt{R^2 + (\omega L)^2}}.$$

These peaks occur at different times. Consequently $V_0^2 = V_{R0}^2 + V_{L0}^2$, rather than an arithmetic addition of the amplitudes.

Step 3: Determine which quantity leads (b) Taking current as the reference, the source-voltage phasor has horizontal component $I_0 R$ and positive perpendicular component $I_0 \omega L$. Its lead angle ϕ satisfies

$$\tan \phi = \frac{V_{L0}}{V_{R0}} = \frac{\omega L}{R}, \quad I(t) = I_0 \sin(\omega t - \phi).$$

For positive R, L , the angle lies between zero and $\pi/2$, consistent with the inductive circuit opposing changes in current.

Step 4: Conclusion The peak drops are $V_{R0} = V_0 R / \sqrt{R^2 + (\omega L)^2}$ and $V_{L0} = V_0 \omega L / \sqrt{R^2 + (\omega L)^2}$. The applied voltage leads the current by $\phi = \tan^{-1}(\omega L / R)$.

Quick Tip to Solve: Find I_0 once, then multiply by R and ωL ; use their ratio for the phase angle because the two voltage phasors are perpendicular.

31. (a) Write the expression for the speed of light in a material medium of relative permittivity ϵ_r and relative magnetic permeability μ_r .

(b) Write the wavelength range and name of the electromagnetic waves which are used in (i) radar systems for aircraft navigation, and (ii) Earth satellites to observe the growth of the crops. [3 marks]

Solution:

Concept:

- The propagation speed of an electromagnetic wave in a medium is $v = 1/\sqrt{\mu\epsilon}$, where μ is its magnetic permeability and ϵ its electric permittivity.
- Relative quantities satisfy $\mu = \mu_r\mu_0$ and $\epsilon = \epsilon_r\epsilon_0$. In vacuum, $c = 1/\sqrt{\mu_0\epsilon_0}$.
- Electromagnetic bands are identified by their approximate wavelength ranges. Microwaves are useful for radar; infrared radiation is useful in satellite observations of vegetation.

Step 1: Express the speed using relative properties (a) Substitute the definitions of relative permeability and relative permittivity in the wave-speed relation:

$$v = \frac{1}{\sqrt{\mu_r\mu_0\epsilon_r\epsilon_0}} = \frac{c}{\sqrt{\mu_r\epsilon_r}}.$$

Thus both electrical and magnetic properties of the material enter the expression. The commonly used form $c/\sqrt{\epsilon_r}$ applies only when μ_r is approximately unity.

Step 2: Identify the radiation used in radar (b)(i) Aircraft-navigation radar uses **microwaves**, whose usual school-level wavelength range is approximately

$$1 \text{ mm to } 0.1 \text{ m} \quad (10^{-3} \text{ m to } 10^{-1} \text{ m}).$$

Their short wavelengths allow directional beams. A radar transmits such waves and receives reflections from objects; the travel time of the echo gives distance.

Step 3: Identify the radiation used to monitor crops (b)(ii) Earth satellites use **infrared waves** to observe crop growth. The infrared band extends approximately from

$$700 \text{ nm to } 1 \text{ mm} \quad (7 \times 10^{-7} \text{ m to } 10^{-3} \text{ m}).$$

Vegetation has a characteristic response in the infrared region. Changes in the infrared radiation detected from it help distinguish vegetation and assess crop condition; the range quoted is that of the band, not a single satellite channel.

Step 4: Conclusion $v = c/\sqrt{\epsilon_r\mu_r}$. Radar uses microwaves (approximately 1 mm to 0.1 m); crop observations use infrared waves (approximately 700 nm to 1 mm).

Quick Tip to Solve: Link radar with directional microwave beams and crop monitoring with infrared; write both band names and wavelength limits when the question asks for ranges.

32. (a) Two cells of emf E_1 and E_2 have their internal resistances r_1 and r_2 , respectively. Deduce an expression for the equivalent emf and internal resistance of their parallel combination when connected across an external resistance R . Assume that the two cells are supporting each other.

(b) In case the two cells are identical, each of emf $E = 5\text{ V}$ and internal resistance $r = 2\ \Omega$, calculate the voltage across the external resistance $R = 10\ \Omega$. [3 marks]

Solution:

Concept:

- Supporting cells in parallel have like terminals joined and the same terminal voltage V . For branch current I_j delivered by a cell, $V = E_j - I_j r_j$.
- Kirchhoff's junction rule gives the load current $I = I_1 + I_2$. An equivalent source obeys $V = E_{\text{eq}} - I r_{\text{eq}}$.

Step 1: Express the branch currents (a) Let $V = IR$ be the potential difference across the external resistance. Join both positive terminals together and both negative terminals together, as required for supporting cells. The two branch equations give

$$I_1 = \frac{E_1 - V}{r_1}, \quad I_2 = \frac{E_2 - V}{r_2}.$$

Step 2: Compare with a single equivalent cell Adding the branch currents and rearranging isolates the common terminal voltage:

$$I = \frac{E_1}{r_1} + \frac{E_2}{r_2} - V \left(\frac{1}{r_1} + \frac{1}{r_2} \right),$$

$$V = \frac{E_1 r_2 + E_2 r_1}{r_1 + r_2} - I \frac{r_1 r_2}{r_1 + r_2}.$$

Comparison with $V = E_{\text{eq}} - I r_{\text{eq}}$ identifies

$$E_{\text{eq}} = \frac{E_1 r_2 + E_2 r_1}{r_1 + r_2}, \quad r_{\text{eq}} = \frac{r_1 r_2}{r_1 + r_2}.$$

The load current is $I = E_{\text{eq}} / (R + r_{\text{eq}})$.

Step 3: Calculate the voltage for identical cells (b) Put $E_1 = E_2 = 5\text{ V}$ and $r_1 = r_2 = 2\ \Omega$. The equivalent emf stays equal to one cell's emf:

$$E_{\text{eq}} = 5\text{ V}, \quad r_{\text{eq}} = 1\ \Omega.$$

With the external resistance $R = 10\ \Omega$,

$$I = \frac{5}{10 + 1} = \frac{5}{11}\text{ A}, \quad V = IR = \frac{50}{11}\text{ V} \approx 4.55\text{ V}.$$

Step 4: Conclusion $E_{\text{eq}} = (E_1 r_2 + E_2 r_1) / (r_1 + r_2)$, $r_{\text{eq}} = r_1 r_2 / (r_1 + r_2)$, and the voltage across the $10\ \Omega$ resistance is $50/11\text{ V} \approx 4.55\text{ V}$.

Quick Tip to Solve: For two identical cells in parallel, retain the emf and halve the internal resistance; then use $V = ER / (R + r/2)$ for the load voltage.

33. (a) Write an expression of magnetic moment associated with a current (I) carrying circular coil of radius r having N turns.

(b) Consider the above mentioned coil placed in YZ plane with its centre at the origin. Derive expression for the value of magnetic field due to it at point $(x, 0, 0)$. [3 marks]

Solution:

Concept:

- The magnetic dipole moment of a plane coil is $\vec{m} = NIA\hat{n}$, where N is the number of turns, I the current, A the area per turn, and $\hat{n}$ the unit normal given by the right-hand rule.
- Biot-Savart's law gives $d\vec{B} = (\mu_0 I / 4\pi) d\vec{\ell} \times \vec{s} / s^2$, where $\vec{s}$ runs from the current element to the observation point.

Step 1: Write the magnetic moment (a) Each circular turn has area $A = \pi r^2$. Therefore the magnitude of the coil's magnetic moment is

$$m = NI\pi r^2.$$

Its direction is perpendicular to the YZ plane, along the positive or negative x -axis according to the current direction. Take the normal as $+\hat{x}$ for the following derivation; reversing the current reverses the field.

Step 2: Resolve the field of a current element (b) At $P = (x, 0, 0)$, every element of the ring is at the same distance $s = \sqrt{r^2 + x^2}$. The tangent element $d\vec{\ell}$ is perpendicular to $\vec{s}$, so for one turn

$$dB = \frac{\mu_0 I d\ell}{4\pi(r^2 + x^2)}.$$

Diametrically opposite elements give equal and opposite components perpendicular to the axis; these cancel. Their axial components point in the same direction and add. Since the axial projection factor is r/s ,

$$dB_x = dB \frac{r}{s} = \frac{\mu_0 I r d\ell}{4\pi(r^2 + x^2)^{3/2}}.$$

Step 3: Integrate around all turns The factors r and x are constant during integration. For a circular turn $\oint d\ell = 2\pi r$, and the N coincident turns contribute equally. Hence

$$\begin{aligned} B_x &= \frac{\mu_0 N I r}{4\pi(r^2 + x^2)^{3/2}} \oint d\ell \\ &= \frac{\mu_0 N I r^2}{2(r^2 + x^2)^{3/2}}. \end{aligned}$$

At the centre this reduces to $\mu_0 N I / (2r)$, the familiar centre-field expression. This check confirms both the radius dependence and the factor of two.

Step 4: Conclusion The magnetic moment is $\vec{m} = NI\pi r^2 \hat{n}$. The axial field magnitude is $B = \mu_0 N I r^2 / [2(r^2 + x^2)^{3/2}]$; its direction is along the coil's magnetic moment, either $+\hat{x}$ or $-\hat{x}$ as fixed by the right-hand rule.

Quick Tip to Solve: On a loop's axis, cancel the transverse components first; multiply the element field by $r/\sqrt{r^2 + x^2}$ before integrating around the ring.

33. (a) Define current sensitivity of a galvanometer. Write its expression.

(b) A galvanometer has resistance G and shows full scale deflection for current I_g .

(i) How can it be converted into an ammeter to measure current up to I_0 ($I_0 > I_g$)?

(ii) What is the effective resistance of this ammeter?

[3 marks]

Solution:

Concept:

- Current sensitivity is angular deflection per unit current: $S_I = \theta/I$. For a moving-coil galvanometer in a radial field, magnetic torque is $NAB I$.
- At equilibrium, the restoring torque $k\theta$ equals the magnetic torque. Here N is the number of turns, A the coil area, B the field and k the torsional constant.
- Parallel branches have equal potential differences, while their currents add. A small shunt resistance bypasses the excess current around the galvanometer.

Step 1: (a) Obtain the current sensitivity Current sensitivity measures how much angular deflection is produced by unit current in the coil. Balancing the magnetic and restoring torques gives

$$NAB I = k\theta \Rightarrow S_I = \frac{\theta}{I} = \frac{NAB}{k}.$$

Its SI unit is rad A^{-1} . A larger coil area, more turns or a stronger magnetic field increases sensitivity, whereas a stiffer suspension reduces it.

Step 2: (b)(i) Choose the parallel shunt Connect a low resistance S in parallel with the galvanometer. When the total current is I_0 , the galvanometer must carry only its full-scale current I_g , so the shunt carries $I_0 - I_g$. Equal potential differences across the two branches require

$$I_g G = (I_0 - I_g) S \Rightarrow S = \frac{I_g G}{I_0 - I_g}.$$

The instrument is then calibrated to read total current and connected in series with the external circuit. A series resistance would not provide the required bypass.

Step 3: (b)(ii) Calculate the effective resistance The resistance of the converted ammeter is the parallel combination of G and S . Substituting the shunt value,

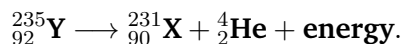
$$R_A = \frac{GS}{G+S} = \frac{GI_g}{I_0}.$$

Equivalently, the terminal voltage at full scale is $I_g G$, so $R_A = V/I_0$. Since $I_0 > I_g$, this resistance is less than G , as required for an ammeter that disturbs the circuit only slightly.

Step 4: Conclusion The current sensitivity is $S_I = NAB/k$. Connect $S = I_g G/(I_0 - I_g)$ in parallel; the resulting ammeter has effective resistance $R_A = GI_g/I_0$.

Quick Tip to Solve: At full scale write the two branch currents first: I_g through the coil and $I_0 - I_g$ through the shunt. Their voltage drops must be equal.

34. The nucleus ${}_{92}^{235}\text{Y}$, initially at rest, decays into ${}_{90}^{231}\text{X}$ by emitting an α -particle



The binding energies per nucleon of the parent nucleus, the daughter nucleus and α -particle are 7.8 MeV, 7.835 MeV and 7.07 MeV, respectively. Assuming the daughter nucleus to be formed in the unexcited state and neglecting its share in the energy of the reaction, find the speed of the emitted α -particle. (Mass of α -particle = 6.68×10^{-27} kg) [3 marks]

Solution:

Concept:

- The total binding energy of a nucleus of mass number A is A times its binding energy per nucleon. The energy released in a decay is the total binding energy of products minus that of the parent.
- With no nuclear excitation and the daughter's kinetic energy neglected as instructed, the released energy becomes the alpha particle's kinetic energy: $Q = \frac{1}{2}m_{\alpha}v^2$.
- For conversion to SI units, $1 \text{ MeV} = 1.6 \times 10^{-13} \text{ J}$.

Step 1: Calculate the total binding energies Multiply each given binding energy per nucleon by its corresponding mass number:

$$\begin{aligned}E_{b,Y} &= 235(7.8) = 1833.0 \text{ MeV}, \\E_{b,X} &= 231(7.835) = 1809.885 \text{ MeV}, \\E_{b,\alpha} &= 4(7.07) = 28.28 \text{ MeV}.\end{aligned}$$

Nucleon number and charge are conserved, so free-nucleon rest energies cancel in the energy difference.

Step 2: Find the energy released The products are more tightly bound in total, so their combined rest energy is lower. The resulting positive energy release is

$$\begin{aligned}Q &= E_{b,X} + E_{b,\alpha} - E_{b,Y} \\&= 1809.885 + 28.28 - 1833.0 \\&= 5.165 \text{ MeV} = 8.264 \times 10^{-13} \text{ J}.\end{aligned}$$

Step 3: Convert kinetic energy into speed Under the specified approximation, $K_{\alpha} = Q$. Using the alpha-particle mass printed in this question,

$$\begin{aligned}v &= \sqrt{\frac{2Q}{m_{\alpha}}} \\&= \sqrt{\frac{2(8.264 \times 10^{-13})}{6.68 \times 10^{-27}}} \\&= \sqrt{2.474 \times 10^{14}} \text{ m s}^{-1}.\end{aligned}$$

Step 4: Conclusion The released energy is 5.165 MeV, giving the emitted alpha particle a speed of $v \simeq 1.57 \times 10^7 \text{ m s}^{-1}$ under the stated approximation.

Quick Tip to Solve: Multiply each binding energy per nucleon by its own mass number first; then subtract parent binding energy from the total for both products.

Section D (15 marks)

35. (a) Derive the expression for the torque acting on the rectangular current carrying coil of a galvanometer. Why is the magnetic field made radial?

(b) An α -particle is accelerated through a potential difference of 10 kV and moves along x -axis. It enters in a region of uniform magnetic field $B = 2 \times 10^{-3}$ T acting along y -axis. Find the radius of its path. (Take mass of α -particle $= 6.4 \times 10^{-27}$ kg) [5 marks]

Solution:

Concept:

- A straight conductor of length l carrying current I perpendicular to a magnetic field B experiences force $F = BIl$. Equal, opposite forces with separated lines of action form a couple.
- The magnetic moment of an N -turn coil of area A is $\mu = NIA\hat{n}$. Its torque is $\tau = \mu \times B$.
- A charge q accelerated from rest through potential difference V gains kinetic energy qV . In a perpendicular magnetic field, $qvB = mv^2/r$.

Step 1: (a) Identify the forces on the rectangular coil Let the coil have N turns, length l , breadth b and area $A = lb$. Let its normal make angle θ with the field. Take the rotation axis along the length, perpendicular to the field. The two lengthwise sides each experience a force of magnitude $NBIl$, in opposite directions. Their resultant force is zero, but they produce a turning couple. Forces on the other pair of sides do not produce torque about this rotation axis.

Step 2: Derive the torque and explain the radial field The perpendicular distance between the lines of action of the two forces is $b \sin \theta$. Thus the magnitude of the couple is

$$\tau = (NBIl)(b \sin \theta) = NIAB \sin \theta.$$

In a galvanometer, curved pole pieces and a soft-iron cylindrical core provide a radial magnetic field. The plane of the coil remains parallel to the local field at every angular position, so the angle between its normal and the field is always 90° . Hence

$$\tau = NIAB.$$

The torque is maximum and independent of the coil's angular position for a fixed current. If k is the spring's torsional constant and ϕ the deflection, equilibrium requires

$$k\phi = NIAB \quad \Rightarrow \quad \phi = \frac{NAB}{k}I.$$

Consequently deflection is directly proportional to current, producing a uniform scale.

Step 3: (b) Calculate the alpha particle's speed An alpha particle has charge $q = 2e = 3.2 \times 10^{-19}$ C. The accelerating voltage is $V = 10$ kV $= 10^4$ V. Using the mass specified in this question,

$$\begin{aligned} \frac{1}{2}mv^2 &= qV, \\ v &= \sqrt{\frac{2(3.2 \times 10^{-19})(10^4)}{6.4 \times 10^{-27}}} = 10^6 \text{ m s}^{-1}. \end{aligned}$$

Step 4: Use magnetic force as the centripetal force Velocity is along the x -axis and the field along the y -axis, so they are perpendicular. The magnetic force changes direction of motion without changing

speed, giving a circular path in the xz -plane:

$$qvB = \frac{mv^2}{r} \Rightarrow r = \frac{mv}{qB},$$

$$r = \frac{(6.4 \times 10^{-27})(10^6)}{(3.2 \times 10^{-19})(2 \times 10^{-3})} = 10 \text{ m}.$$

For motion initially along positive x and field along positive y , the positive alpha particle initially experiences force along positive z .

Step 5: Conclusion The general coil torque is $\tau = NIAB \sin \theta$; a radial field gives $\tau = NIAB$ and a uniform current scale. The alpha particle follows a circle of radius $r = 10 \text{ m}$.

Quick Tip to Solve: For an alpha particle use $q = 2e$, not e . Combining the two laws directly gives $r = \sqrt{2mV/q}/B$ when the velocity is perpendicular to the field.

OR

35. (a) With the help of a labelled diagram, explain the working of a step-up transformer. Give reasons to explain the following:

(i) The core of the transformer is laminated.

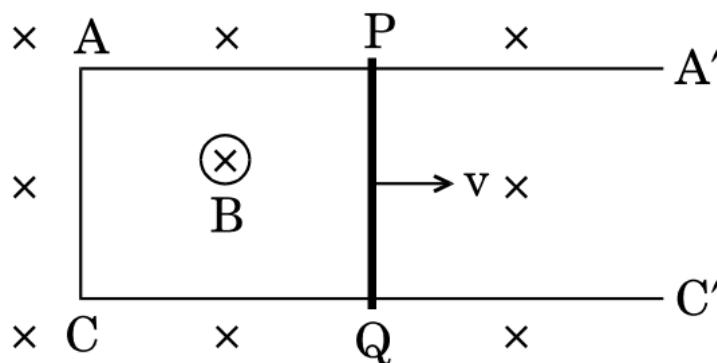
(ii) Thick copper wire is used in windings.

(b) A conducting rod PQ of length 20 cm and resistance 0.1Ω rests on two smooth parallel rails of negligible resistance AA' and CC' . It can slide on the rails and the arrangement is positioned between the poles of a permanent magnet producing uniform magnetic field $B = 0.4 \text{ T}$. The rails, the rod and the magnetic field are in three mutually perpendicular directions as shown in the figure. If the ends A and C of the rails are short circuited, find the

(i) external force required to move the rod with uniform velocity $v = 10 \text{ cm/s}$, and

(ii) power required to do so.

[5 marks]



Solution:

Concept:

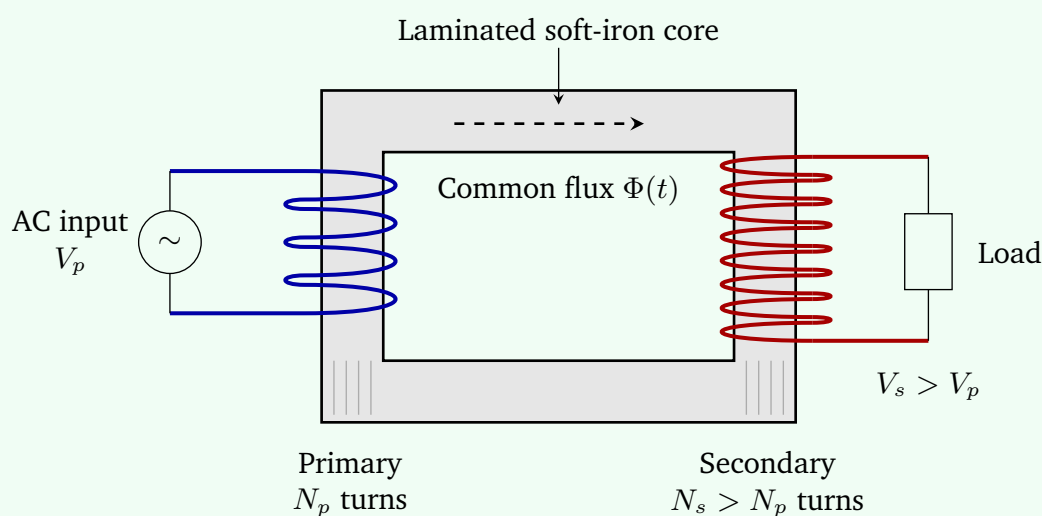
- A transformer works by mutual induction: the alternating flux Φ in a common core induces emfs $e_p = -N_p d\Phi/dt$ and $e_s = -N_s d\Phi/dt$. Here N_p, N_s denote the primary and secondary turns.
- For an ideal transformer, the rms voltages satisfy $V_s/V_p = N_s/N_p$, and conservation of power gives $V_p I_p = V_s I_s$.
- A rod of length l moving with speed v , perpendicular to a magnetic field B , develops motional

emf $\varepsilon = Blv$. In a closed circuit of resistance R , $I = \varepsilon/R$, the magnetic force is $F_B = BIl$, and mechanical power is $P = Fv$.

Step 1: (a) Explain the step-up transformer Wind two insulated copper coils on a common laminated soft-iron core. Connect the primary, of N_p turns, to an alternating-voltage supply; connect the secondary, with $N_s > N_p$ turns, to the load. The alternating primary current establishes a changing magnetic flux in the core, which links both coils. By Faraday's law this changing flux induces an alternating emf in the secondary even though the two windings have no conducting connection.

$$\frac{|e_s|}{|e_p|} = \frac{N_s}{N_p} \implies \frac{V_s}{V_p} = \frac{N_s}{N_p} > 1.$$

Thus the secondary voltage is raised. For an ideal transformer the current is correspondingly reduced, $I_s/I_p = N_p/N_s$, so there is no gain in power. A steady direct-current supply cannot maintain the changing flux needed for continuous transformer action.



Step-up transformer: a common laminated core links the primary and the secondary, with $N_s > N_p$.

Step 2: (a)(i) Explain why the core is laminated The changing magnetic flux also induces circulating eddy currents in the iron core. A solid core would allow large loops of such currents, causing appreciable heating and wasted power. Thin laminations, electrically insulated from one another, interrupt these conducting paths and increase their resistance. They therefore reduce eddy-current loss while still providing a path for magnetic flux.

Step 3: (a)(ii) Explain the use of thick copper wire Copper has low resistivity, and a thick wire has a large cross-sectional area A_w . Since winding resistance is $R_w = \rho L_w/A_w$, thick copper conductors reduce resistance and hence the copper heating loss $I^2 R_w$. This is especially useful in the winding carrying the larger current, which is the primary of a step-up transformer.

Step 4: (b) Calculate the emf and the induced current Convert the given length and speed to SI units:

$$l = 0.20 \text{ m}, \quad v = 0.10 \text{ m s}^{-1}, \quad B = 0.4 \text{ T}, \quad R = 0.1 \Omega.$$

The rails and short-circuit connection have negligible resistance, so the rod provides the circuit resistance. Because the velocity, rod and field are mutually perpendicular,

$$\varepsilon = Blv = (0.4)(0.20)(0.10) = 0.008 \text{ V}, \quad I = \frac{0.008}{0.1} = 0.08 \text{ A}.$$

In the printed figure the field points into the page and the rod moves right. The force $qv \times \mathbf{B}$ on positive charges is upwards, so the induced conventional current travels from Q to P through the rod and returns along $P \rightarrow A \rightarrow C \rightarrow Q$.

Step 5: (b)(i) Balance the magnetic retarding force The current-carrying rod experiences a force to the left, opposing its motion in accordance with Lenz's law. The rails are smooth, so uniform velocity requires an equal applied force to the right:

$$F_{\text{ext}} = BIl = \frac{B^2 l^2 v}{R} = (0.4)(0.08)(0.20) = 6.4 \times 10^{-3} \text{ N}.$$

Step 6: (b)(ii) Calculate the mechanical power The applied force and velocity point in the same direction. Hence

$$P = F_{\text{ext}}v = (6.4 \times 10^{-3})(0.10) = 6.4 \times 10^{-4} \text{ W}.$$

As an energy check, the electrical heating rate in the rod is $I^2 R = (0.08)^2(0.1) = 6.4 \times 10^{-4} \text{ W}$, equal to the mechanical power supplied.

Step 7: Conclusion The transformer raises voltage by using $N_s > N_p$; laminations reduce eddy-current loss and thick copper windings reduce resistive heating. For the rod, the required external force is

$$6.4 \times 10^{-3} \text{ N} \text{ to the right, and the required power is } 6.4 \times 10^{-4} \text{ W}.$$

Quick Tip to Solve: Convert both centimetre quantities to metres before using Blv . For this short-circuited sliding rod, use $F = B^2 l^2 v / R$ and verify $Fv = I^2 R$.

36. (a) Draw the ray diagram of an astronomical telescope when the final image is formed at infinity. Write the expression for the resolving power of the telescope.

(b) An astronomical telescope has an objective lens of focal length 20 m and eyepiece of focal length 1 cm.

(i) Find the angular magnification of the telescope.

(ii) If this telescope is used to view the Moon, find the diameter of the image formed by the objective lens. Given the diameter of the Moon is $3.5 \times 10^6 \text{ m}$ and radius of lunar orbit is $3.8 \times 10^8 \text{ m}$. [5 marks]

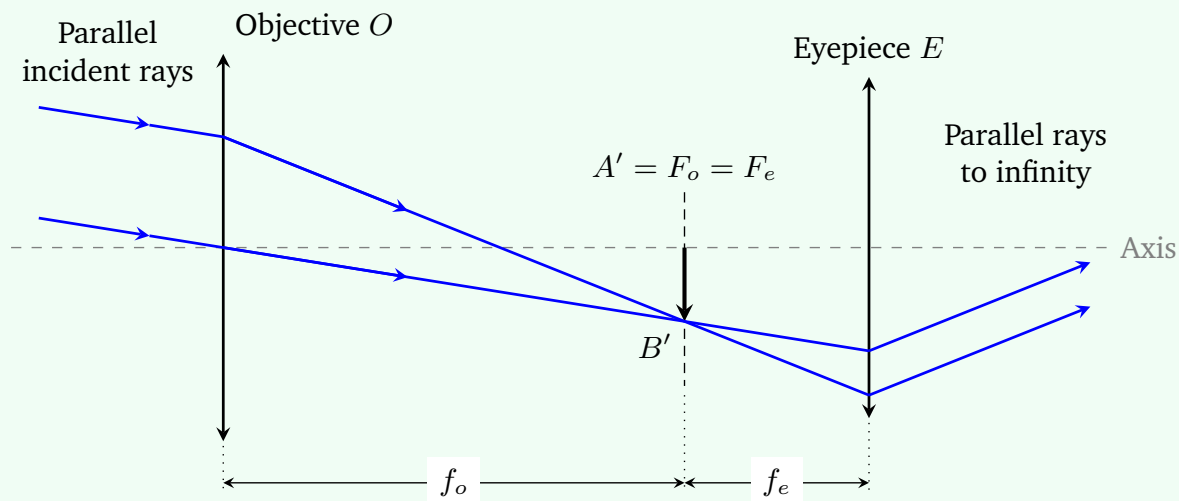
Solution:

Concept:

- In normal adjustment, the objective forms a real image in its focal plane, which coincides with the first focal plane of the eyepiece. The final image is at infinity and the lens separation is $f_o + f_e$.
- Angular magnification is $M = \beta/\alpha = -f_o/f_e$, where α and β are the small angular sizes of the object and final image. The minus sign indicates inversion.
- For a circular objective of aperture diameter D , Rayleigh's criterion gives $\theta_{\min} = 1.22\lambda/D$. Resolving power is $1/\theta_{\min}$, with wavelength λ and angles in radians.
- A distant object of diameter d at distance R subtends $\theta \simeq d/R$, so the objective's image diameter is $d' = f_o\theta$.

Step 1: (a) Construct the telescope in normal adjustment Parallel rays from a distant off-axis point fall on the large objective O . They converge at B' , forming a real, inverted intermediate image $A'B'$. Place the eyepiece E so that this image lies in its first focal plane, with $F_o = F_e = A'$. After refraction through the eyepiece, the rays from B' emerge parallel, so the final magnified image is at infinity. The

drawing is schematic: the actual focal-length ratio is much larger than shown.



Astronomical telescope in normal adjustment: rays from one distant off-axis point emerge parallel. The intermediate image lies in the common focal plane.

Step 2: (a) State the resolving power Two distant point sources are just resolved when the central diffraction maximum of one falls on the first minimum of the other. For an objective with a circular aperture,

$$\theta_{\min} = \frac{1.22\lambda}{D}, \quad \text{Resolving power} = \frac{D}{1.22\lambda}.$$

Here D is the objective diameter, not the Moon's diameter. Increasing the aperture or using a smaller wavelength improves resolution. Magnification alone does not determine resolving power; no numerical value can be calculated without the aperture and wavelength.

Step 3: (b)(i) Calculate the angular magnification For an intermediate image of height h , the small-angle relations are $|\alpha| \simeq h/f_o$ and $|\beta| \simeq h/f_e$. Their ratio gives $|M| = f_o/f_e$. Convert the eyepiece focal length into the same unit as the objective focal length before substitution:

$$f_o = 20 \text{ m}, \quad f_e = 1 \text{ cm} = 0.01 \text{ m},$$

$$M = -\frac{20}{0.01} = -2000.$$

Thus the angular magnification has magnitude 2000; the negative sign describes the inverted view. Magnification is dimensionless.

Step 4: (b)(ii) Find the image diameter at the objective focus The Moon is sufficiently distant that the objective forms its image essentially in the focal plane. Using its orbital radius as the given distance from the observer, the Moon's angular diameter is

$$\theta \simeq \frac{d}{R} = \frac{3.5 \times 10^6}{3.8 \times 10^8} = 9.21 \times 10^{-3} \text{ rad}.$$

The full diameter of the image, rather than its radius, is therefore

$$d' = f_o \theta = 20 \times 9.21 \times 10^{-3} = 0.1842 \text{ m} \simeq 18.4 \text{ cm}.$$

This is the real intermediate image made by the objective. The eyepiece changes its apparent angular size but is not used to calculate this image diameter.

Step 5: Conclusion In normal adjustment, $F_o = F_e$ and the final image is at infinity. The resolving power is $D/(1.22\lambda)$. The angular magnification is $M = -2000$ (magnitude 2000, inverted image), and the Moon's image formed by the objective has diameter approximately 0.184 m, or 18.4 cm.

Quick Tip to Solve: Convert both focal lengths to metres for magnification; for the Moon's objective image use $f_o d/R$, with the full lunar diameter and no eyepiece factor.

OR

36. (a) An object is placed in front of a concave mirror. It is observed that a virtual image is formed. Draw the ray diagram to show the image formation and hence derive the mirror equation $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$.

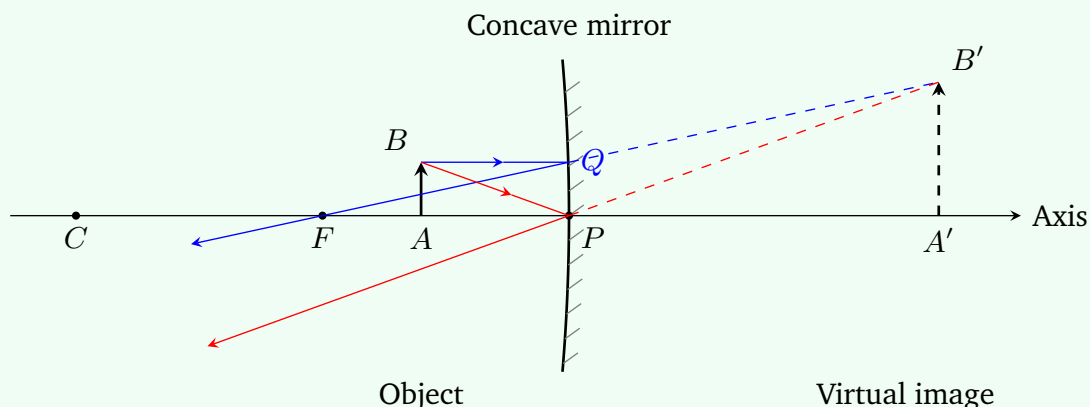
(b) An object is placed 30 cm in front of a plano-convex lens with its spherical surface of radius of curvature 20 cm. If the refractive index of the material of the lens is 1.5, find the position and nature of the image formed. [5 marks]

Solution:

Concept:

- A concave mirror forms an erect, enlarged virtual image when the object lies between its pole P and focus F . Reflected rays diverge, and their backward extensions meet behind the mirror.
- Use the paraxial approximation for a spherical mirror. With the Cartesian convention and light incident from the left, $u < 0$, $f < 0$, and a virtual image has $v > 0$.
- For a thin lens in air, the lens-maker relation is $1/f_L = (n - 1)(1/R_1 - 1/R_2)$, and the lens equation is $1/v_L - 1/u_L = 1/f_L$. Here n is refractive index and R_1, R_2 are signed surface radii.

Step 1: (a) Construct the virtual image Place the object AB between P and F . From its tip B , draw one incident ray parallel to the principal axis; after reflection at Q , it travels through F . Draw a second ray from B to P ; it reflects at the same angle to the principal axis. Extend both reflected rays backwards with dashed lines. Their intersection is B' , and the foot of the image is A' . The solid arrows show actual light travelling back to the left; the dashed portions behind the mirror are only extensions.



Concave mirror: object between focus and pole; dashed extensions meet at the enlarged virtual image. Paraxial construction, not to scale.

Step 2: Relate object and image heights by similar triangles Let the positive distance magnitudes be $PA = a$, $PA' = b$, and $PF = F_0$. Write the object and image heights as $AB = h$ and $A'B' = h'$. The

pole ray makes equal angles with the axis, so the right triangles ABP and $A'B'P$ are similar:

$$\frac{h'}{h} = \frac{b}{a}.$$

For a small aperture, the reflection point Q can be taken in the plane through P , so $PQ \simeq h$. The reflected parallel ray and its backward extension form similar triangles FQP and $FB'A'$. Since the image is behind the mirror, $FA' = F_0 + b$, giving

$$\frac{h'}{h} = \frac{F_0 + b}{F_0}.$$

Step 3: Derive the signed mirror equation Equate the two height ratios and rearrange:

$$\begin{aligned}\frac{b}{a} &= \frac{F_0 + b}{F_0}, \\ bF_0 &= aF_0 + ab, \\ \frac{1}{a} &= \frac{1}{b} + \frac{1}{F_0}, \\ -\frac{1}{F_0} &= -\frac{1}{a} + \frac{1}{b}.\end{aligned}$$

Now substitute the Cartesian distances $u = -a$, $v = b$, and $f = -F_0$. This gives the required relation in signed form. The derivation uses paraxial rays, for which spherical aberration is neglected.

Step 4: (b) Find the focal length of the plano-convex lens Take light to enter the convex face first. Then $R_1 = +20$ cm, while the plane second face has $R_2 = \infty$. With $n = 1.5$,

$$\frac{1}{f_L} = (1.5 - 1) \left(\frac{1}{20} - 0 \right) = \frac{1}{40} \text{ cm}^{-1}, \quad f_L = +40 \text{ cm}.$$

Reversing the lens gives the same positive focal length: the plane first surface contributes zero and the second radius becomes negative. The object distance is $u_L = -30$ cm, so the object is inside the focal length.

Step 5: Determine the lens image distance and magnification Apply the thin-lens equation with signed distances:

$$\begin{aligned}\frac{1}{v_L} - \frac{1}{-30} &= \frac{1}{40}, \\ \frac{1}{v_L} &= \frac{1}{40} - \frac{1}{30} = -\frac{1}{120} \text{ cm}^{-1}, \\ v_L &= -120 \text{ cm}, \\ m_L &= \frac{v_L}{u_L} = \frac{-120}{-30} = +4.\end{aligned}$$

The negative image distance places the image on the object's side of the lens. The positive magnification establishes that it is erect; its magnitude establishes the enlargement.

Step 6: Conclusion (a) The ray construction gives a virtual, erect and enlarged mirror image, with $1/f = 1/v + 1/u$. (b) The lens has focal length 40 cm and forms its image 120 cm in front of the lens, on the same side as the object. The image is virtual, erect and four times the object's size.

Quick Tip to Solve: For the mirror proof, use positive lengths in similar triangles first and then insert Cartesian signs. For the lens, compare 30 cm with $f_L = 40$ cm: an object inside the focus must give a virtual image.

37. (a) Using Gauss law, derive expression for electric field due to a spherical shell of uniform charge distribution σ and radius R at a point lying at a distance x from the centre of shell, such that

(i) $0 < x < R$, and

(ii) $x > R$.

(b) An electric field is uniform and acts along $+x$ direction in the region of positive x . It is also uniform with the same magnitude but acts in $-x$ direction in the region of negative x . The value of the field is $E = 200 \text{ N/C}$ for $x > 0$ and $E = -200 \text{ N/C}$ for $x < 0$. A right circular cylinder of length 20 cm and radius 5 cm has its centre at the origin and its axis along the x -axis so that one flat face is at $x = +10 \text{ cm}$ and the other is at $x = -10 \text{ cm}$.

Find:

(i) The net outward flux through the cylinder.

(ii) The net charge present inside the cylinder.

[5 marks]

Solution:

Concept:

- Gauss's law relates the outward electric flux through a closed surface to the enclosed charge: $\oint \vec{E} \cdot d\vec{A} = Q_{\text{enc}}/\epsilon_0$, where $d\vec{A}$ points outward.
- A uniformly charged spherical shell has total charge $Q = 4\pi R^2\sigma$. Spherical symmetry makes its electric field radial and its radial component constant on any concentric spherical Gaussian surface.
- For a uniform field on a flat surface, $\Phi = EA \cos \theta$, where θ is the angle between the field and the outward area vector. Fluxes through the separate parts of a closed surface add algebraically.

Step 1: (a) Choose a concentric Gaussian sphere Let the Gaussian sphere have radius x , centred on the charged shell. By spherical symmetry, there is no preferred tangential direction, so the electric field is radial. Its radial component E_r has the same value at every point of this Gaussian sphere. Hence

$$\oint \vec{E} \cdot d\vec{A} = E_r \oint dA = E_r 4\pi x^2.$$

The crucial quantity on the right side of Gauss's law is the charge enclosed by the Gaussian sphere, not the charge on all surrounding objects.

Step 2: (a)(i) Apply Gauss's law inside the shell For $0 < x < R$, the Gaussian sphere lies entirely inside the hollow shell. All the charge is on the shell at radius R , so $Q_{\text{enc}} = 0$. Thus

$$E_r 4\pi x^2 = 0 \quad \Rightarrow \quad E_r = 0.$$

Zero enclosed charge implies zero net flux in general; here spherical symmetry additionally makes the field magnitude constant and radial, allowing us to infer that the field itself vanishes everywhere inside.

Step 3: (a)(ii) Apply Gauss's law outside the shell For $x > R$, the Gaussian sphere encloses the entire shell charge $4\pi R^2\sigma$. Therefore

$$E_r 4\pi x^2 = \frac{4\pi R^2\sigma}{\epsilon_0},$$

$$\vec{E}(x) = \frac{\sigma R^2}{\epsilon_0 x^2} \hat{r} = \frac{Q}{4\pi\epsilon_0 x^2} \hat{r}.$$

The exterior field equals that of a point charge Q at the centre. It points outward for positive σ and

inward for negative σ ; its magnitude uses $|\sigma|$.

Step 4: (b)(i) Evaluate the flux through each cylinder surface The radius is $a = 5 \text{ cm} = 0.05 \text{ m}$, so either flat face has area $A = \pi a^2 = \pi(0.05)^2 \text{ m}^2$. At the right face both $\vec{E}$ and its outward normal point along $+\hat{x}$. At the left face both point along $-\hat{x}$. Their dot products are therefore positive, even though the field components on the two faces have opposite signs:

$$\Phi_{\text{right}} = 200A, \quad \Phi_{\text{left}} = (-200)(-A) = 200A.$$

On the curved surface the outward normal is perpendicular to the cylinder axis, whereas the field is axial, so the curved-surface flux is zero. Adding all three contributions gives

$$\Phi_{\text{net}} = 2(200)\pi(0.05)^2 = \pi \text{ N m}^2/\text{C} \simeq 3.14 \text{ N m}^2/\text{C}.$$

The cylinder length locates its faces in the two specified regions; it does not enter their area or the flux.

Step 5: (b)(ii) Determine the enclosed charge Apply Gauss's law to the whole closed cylinder. Using $\epsilon_0 = 8.854 \times 10^{-12} \text{ C}^2/(\text{N m}^2)$,

$$Q_{\text{enc}} = \epsilon_0 \Phi_{\text{net}} = (8.854 \times 10^{-12})\pi \text{ C} \simeq 2.78 \times 10^{-11} \text{ C}.$$

The sign is positive because field lines leave the cylinder through both ends. The negative field component on the left face therefore does not represent negative outward flux.

Step 6: Conclusion (a) The shell field is $\vec{E} = 0$ for $0 < x < R$, and $\vec{E} = \sigma R^2 \hat{r}/(\epsilon_0 x^2)$ for $x > R$. (b) The net outward flux is $+3.14 \text{ N m}^2/\text{C}$, and the enclosed charge is $+2.78 \times 10^{-11} \text{ C}$.

Quick Tip to Solve: For each cylinder end, compare the field with that end's outward normal: both reverse at the left end, so the two positive fluxes add rather than cancel.

OR

37. (a) Find the expression for the potential energy of a system of two point charges q_1 and q_2 located at $\vec{r}_1$ and $\vec{r}_2$ respectively in an external electric field $\vec{E}$.

(b) Draw equipotential surfaces due to an isolated point charge $(-q)$ and depict the electric field lines.

(c) Three point charges $+1 \mu\text{C}$, $-1 \mu\text{C}$ and $+2 \mu\text{C}$ are initially infinite distance apart. Calculate the work done in assembling these charges at the vertices of an equilateral triangle of side 10 cm. [5 marks]

Solution:

Concept:

- Electrostatic potential energy equals the work done by an external agent in assembling charges slowly, with no change in their kinetic energy. A charge q at external potential V_{ext} contributes qV_{ext} .
- The mutual energy of two point charges is $U_{12} = kq_1q_2/r_{12}$, where $k = 1/(4\pi\epsilon_0)$ and $r_{12} = |\vec{r}_1 - \vec{r}_2|$. Count each distinct pair once.
- For a point charge $-q$, with $q > 0$, $V = -kq/r$. An equipotential surface has constant V , and $\vec{E} = -\nabla V$ is perpendicular to that surface.

Step 1: (a) Place the first charge in the external field Let $V_{\text{ext}}(\vec{r})$ be the potential due only to the

sources of the given external field. These sources are assumed fixed while the two charges are assembled. Choose a common zero of potential and exclude the unchanged energy of the external sources and the self-energies of the point charges. The work required to place q_1 at $\vec{r}_1$, relative to this reference, is

$$W_1 = q_1 V_{\text{ext}}(\vec{r}_1).$$

The external field and its potential are related by $\vec{E} = -\nabla V_{\text{ext}}$. Thus the external-field contribution can be expressed in terms of the potential even when the field is not uniform.

Step 2: (a) Add the second charge and count the interaction once After q_1 is in place, the potential at $\vec{r}_2$ is the sum of the external potential and the potential due to q_1 . Therefore the work to place q_2 is

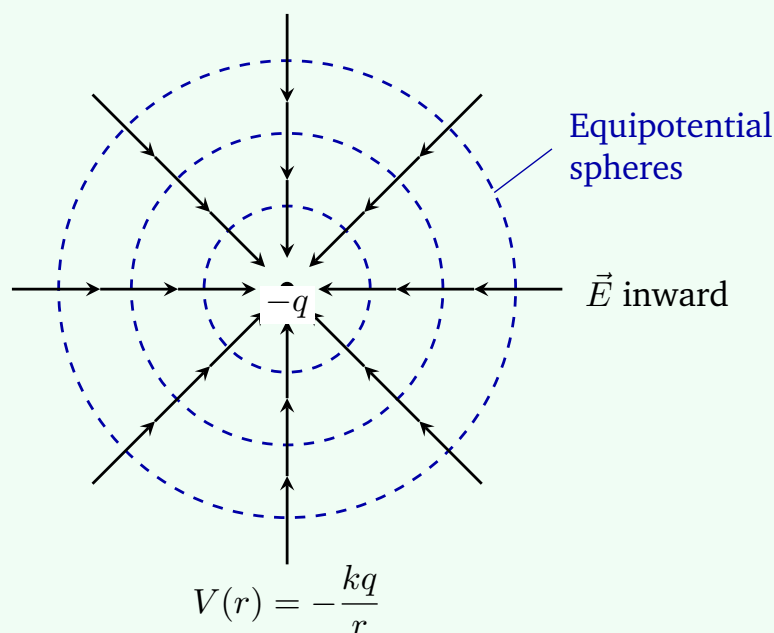
$$W_2 = q_2 \left[V_{\text{ext}}(\vec{r}_2) + \frac{kq_1}{r_{12}} \right].$$

Adding the two assembly contributions gives

$$U = q_1 V_{\text{ext}}(\vec{r}_1) + q_2 V_{\text{ext}}(\vec{r}_2) + \frac{q_1 q_2}{4\pi\epsilon_0 r_{12}}.$$

The first two terms describe interaction with the external sources; the last describes the mutual interaction of the two added charges. Including the mutual term again for q_1 would double-count the same pair. For a uniform external field, one may use $V_{\text{ext}}(\vec{r}) = -\vec{E} \cdot \vec{r} + C$, where C fixes the reference potential.

Step 3: (b) Draw the equipotentials and field lines Since $V = -kq/r$ depends only on the distance r , each equipotential surface is a sphere centred on the isolated negative charge. The dashed concentric circles in the figure are planar sections of these spheres. The electric field lines are radial, point inward towards $-q$, and meet every sphere normally. Potential becomes more negative as the charge is approached, so the inward field direction is consistent with $\vec{E} = -\nabla V$. No work is done in moving a test charge along any one sphere.



Planar sections of spherical equipotentials around $-q$; arrows show the inward electric field.

Step 4: (c) Write the energy of the three distinct pairs Denote the three charges by $q_1 = +1 \times 10^{-6} \text{ C}$, $q_2 = -1 \times 10^{-6} \text{ C}$, and $q_3 = +2 \times 10^{-6} \text{ C}$. Each separation in the equilateral triangle is $a = 10 \text{ cm} =$

0.10 m. Initially, all separations are infinite, so their mutual potential energy is zero. The final energy is

$$U_f = \frac{k}{a} (q_1 q_2 + q_2 q_3 + q_3 q_1).$$

Retain the signs of the charges when forming the products:

$$q_1 q_2 = -1 \times 10^{-12} \text{ C}^2,$$

$$q_2 q_3 = -2 \times 10^{-12} \text{ C}^2,$$

$$q_3 q_1 = +2 \times 10^{-12} \text{ C}^2.$$

The attractive and repulsive interactions involving q_3 cancel because its two neighbours are equal and opposite and are equally distant from it.

Step 5: (c) Calculate the assembly work and interpret its sign Using $k = 9 \times 10^9 \text{ N m}^2 \text{C}^{-2}$, the external work for slow assembly is

$$\begin{aligned} W_{\text{ext}} &= U_f - U_i \\ &= \frac{9 \times 10^9}{0.10} (-1 - 2 + 2) \times 10^{-12} \text{ J} \\ &= -9.0 \times 10^{-2} \text{ J}. \end{aligned}$$

A negative external work means that the system releases energy during assembly. The electric field does positive work of the same magnitude, while an external agent removes that energy to keep the assembly quasistatic.

Step 6: Conclusion (a) $U = q_1 V_{\text{ext}}(\vec{r}_1) + q_2 V_{\text{ext}}(\vec{r}_2) + q_1 q_2 / (4\pi\epsilon_0 r_{12})$. (b) Equipotentials are concentric spheres with inward radial electric field lines. (c) The work done in assembling the triangle is -0.090 J ; the electric field does $+0.090 \text{ J}$.

Quick Tip to Solve: For equal separations, add the signed pair products before multiplying by k/a . Here the two products involving $+2 \mu\text{C}$ cancel immediately.