

CBSE Class 12 Physics Question Paper 2020 with Solution PDF-Set 55/2/1



Question Paper with Solutions

Conducted by the Central Board of Secondary Education (CBSE)

Time Allowed: 3 hours

Maximum Marks: 70

General Instructions

- (i) This question paper comprises **four** Sections – **A, B, C and D**.
- (ii) There are **37** questions in the question paper. **All** questions are compulsory.
- (iii) **Section A** – Questions no. **1 to 20** are very short answer type questions, carrying **1** mark each.
- (iv) **Section B** – Questions no. **21 to 27** are short answer type questions, carrying **2** marks each.
- (v) **Section C** – Questions no. **28 to 34** are long answer type questions, carrying **3** marks each.
- (vi) **Section D** – Questions no. **35 to 37** are also long answer type questions, carrying **5** marks each.
- (vii) There is no overall choice in the question paper. However, an internal choice has been provided in **2** questions of **1** mark, **2** questions of **2** marks, **1** question of three marks and all the **3** questions of five marks. You have to attempt only one of the choices in such questions.
- (viii) In addition to this, separate instructions are given with each section and question, wherever necessary.
- (ix) Use of calculators and **log tables is not** permitted.

You may use the following values of physical constants wherever necessary :

$$c = 3 \times 10^8 \text{ m/s}$$

$$h = 6.63 \times 10^{-34} \text{ Js}$$

$$e = 1.6 \times 10^{-19} \text{ C}$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ T m A}^{-1}$$

$$\epsilon_0 = 8.854 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$$

$$\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$$

$$\text{Mass of electron } (m_e) = 9.1 \times 10^{-31} \text{ kg}$$

$$\text{Mass of neutron} = 1.675 \times 10^{-27} \text{ kg}$$

$$\text{Mass of proton} = 1.673 \times 10^{-27} \text{ kg}$$

$$\text{Avogadro's number} = 6.023 \times 10^{23} \text{ per gram mole}$$

$$\text{Boltzmann constant} = 1.38 \times 10^{-23} \text{ JK}^{-1}$$

Section A (20 marks)

Note : Select the most appropriate option from those given below each question :

1. A cell of internal resistance r connected across an external resistance R can supply maximum current when [1 mark]

- (A) $R = r$
- (B) $R > r$
- (C) $R = \frac{r}{2}$
- (D) $R = 0$

Correct Answer: (D) $R = 0$

Solution:

Concept:

- A cell of emf $\mathcal{E}$ and internal resistance r supplies current $I = \mathcal{E}/(R + r)$, where R is the external resistance.
- For a fixed cell, maximum current requires the smallest possible total resistance.

Step 1: Apply the circuit equation The external and internal resistances are in series, so $\mathcal{E} = I(R + r)$. For a given positive emf and internal resistance, increasing R increases the denominator and reduces the current.

Step 2: Compare the printed options At $R = r$, the current is $\mathcal{E}/(2r)$. For $R > r$, it is smaller still. At $R = r/2$, it is $2\mathcal{E}/(3r)$. The smallest permitted external resistance is $R = 0$, giving the largest current $\mathcal{E}/r$. The condition $R = r$ maximises power delivered to the load, which is a different quantity.

Step 3: Conclusion Maximum current is supplied when $R = 0$. Hence, the correct option is (D).

Quick Tip to Solve: For maximum current minimise total resistance; do not use the maximum-power condition $R = r$.

2. In a current carrying conductor, the ratio of the electric field and the current density at a point is called [1 mark]

- (A) Resistivity
- (B) Conductivity
- (C) Resistance
- (D) Mobility

Correct Answer: (A) Resistivity

Solution:

Concept:

- Local Ohm's law is $\mathbf{E} = \rho \mathbf{J}$, where ρ is resistivity and $\mathbf{J}$ is current density.
- Conductivity σ is the reciprocal of resistivity: $\sigma = 1/\rho$.

Step 1: Identify the requested ratio In an ordinary ohmic conductor, the electric field and current density are parallel. Taking their magnitudes in $\mathbf{E} = \rho \mathbf{J}$ gives $E/J = \rho$. This is a property of the material at the specified conditions.

Step 2: Distinguish the other quantities Conductivity is the reverse ratio J/E , so option (B) is incorrect. Resistance is the whole conductor's ratio V/I and depends on its dimensions through $R = \rho L/A$. Mobility is drift-speed magnitude per unit electric field, $\mu = v_d/E$, rather than field divided by current density. Thus neither (C) nor (D) names the requested local ratio.

Step 3: Conclusion The ratio E/J is resistivity. Hence, the correct option is (A).

Quick Tip to Solve: Remember the local relation $E = \rho J$: field divided by current density gives resistivity.

3. An electron is released from rest in a region of uniform electric and magnetic fields acting parallel to each other. The electron will [1 mark]

- (A) move in a straight line.
- (B) move in a circle.
- (C) remain stationary.
- (D) move in a helical path.

Correct Answer: (A) move in a straight line.

Solution:

Concept:

- The Lorentz force is $\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$.
- An electron has charge $-e$, so its electric force is opposite to $\mathbf{E}$.

Step 1: Determine the initial motion The electron is released from rest, so initially $\mathbf{v} = 0$ and the magnetic force is zero. The nonzero electric force $-e\mathbf{E}$ accelerates it opposite to the electric field. It therefore cannot remain stationary.

Step 2: Check whether the path can curve The electric and magnetic fields are parallel, so the acquired velocity is antiparallel to $\mathbf{B}$. Thus $\mathbf{v} \times \mathbf{B} = 0$ throughout the motion. There is no transverse force to bend the path into a circle. A helix would require a nonzero initial velocity component perpendicular to the magnetic field; none is present here.

Step 3: Conclusion The electron moves in a straight line opposite to the electric field. Hence, the correct option is (A).

Quick Tip to Solve: For release from rest with parallel fields, the electric force creates no velocity perpendicular to $\mathbf{B}$.

4. Above Curie temperature, a

[1 mark]

- (A) ferromagnetic material becomes diamagnetic.
- (B) ferromagnetic material becomes paramagnetic.
- (C) paramagnetic material becomes ferromagnetic.
- (D) paramagnetic material becomes diamagnetic.

Correct Answer: (B) ferromagnetic material becomes paramagnetic.

Solution:

Concept:

- Ferromagnetic materials show spontaneous alignment of magnetic moments below their Curie temperature T_C .
- Above T_C , thermal agitation destroys spontaneous magnetic ordering.

Step 1: Apply the Curie-temperature condition When a ferromagnetic material is heated above its Curie temperature, thermal disorder prevents its magnetic moments from maintaining spontaneous collective alignment. The ferromagnetic state and its spontaneous magnetisation disappear.

Step 2: Identify the resulting response The magnetic moments still exist and can weakly align with an applied magnetic field, giving a paramagnetic response. Hence option (B) states the transition correctly. Option (A) incorrectly assigns diamagnetism. Options (C) and (D) concern a paramagnetic starting material and do not describe loss of ferromagnetic ordering on heating above the Curie temperature.

Step 3: Conclusion Above the Curie temperature, a ferromagnetic material becomes paramagnetic. Hence, the correct option is (B).

Quick Tip to Solve: Heating above T_C destroys ordered alignment, but the magnetic moments still respond paramagnetically.

5. Displacement current exists only when

[1 mark]

- (A) electric field is changing.
- (B) magnetic field is changing.
- (C) electric field is not changing.
- (D) magnetic field is not changing.

Correct Answer: (A) electric field is changing.

Solution:

Concept:

- Maxwell's displacement current is $I_d = \epsilon_0 d\Phi_E/dt$, where Φ_E is electric flux.
- For a fixed surface in a uniform field, $\Phi_E = EA \cos \theta$; a changing electric field produces displacement current.

Step 1: Use the displacement-current expression The displacement-current density in vacuum is $\mathbf{J}_d = \epsilon_0 \partial \mathbf{E} / \partial t$. Therefore a nonzero displacement current requires a time-varying electric field. A steady electric field has zero time derivative and produces no displacement current.

Step 2: Check the alternatives Option (A) gives the required changing electric field. Option (C), a field that is not changing, gives zero displacement current. A changing magnetic field is associated with an

induced electric field by Faraday's law, but does not itself define displacement current. Whether the magnetic field is changing or constant, as in (B) and (D), is not the required condition.

Step 3: Conclusion Displacement current exists when the electric field is changing. Hence, the correct option is (A).

Quick Tip to Solve: Use $I_d = \epsilon_0 d\Phi_E/dt$; displacement current is tied to a changing electric flux.

6. Electromagnetic waves used as a diagnostic tool in medicine are

[1 mark]

- (A) X-rays.
- (B) ultraviolet rays.
- (C) infrared radiation.
- (D) ultrasonic waves.

Correct Answer: (A) X-rays.

Solution:

Concept:

- X-rays are electromagnetic waves that penetrate body tissue and are absorbed by different tissues to different extents.
- Ultrasonic waves are mechanical sound waves, even though ultrasound is also used for medical diagnosis.

Step 1: Identify the diagnostic electromagnetic radiation X-rays form images because different body structures attenuate them differently. Bones absorb X-rays more strongly than many soft tissues, creating contrast on the detector. Their penetration and differential absorption make X-rays a standard diagnostic tool in medicine.

Step 2: Check each remaining choice Ultraviolet radiation is mainly associated here with surface effects and sterilisation rather than penetrating diagnostic radiography. Infrared radiation is associated with thermal effects and does not provide the standard X-ray diagnostic image. Ultrasound does have diagnostic uses, but ultrasonic waves are mechanical and require a medium; they are not electromagnetic waves. Thus (D) fails the explicit electromagnetic requirement.

Step 3: Conclusion The required electromagnetic waves are X-rays. Hence, the correct option is (A).

Quick Tip to Solve: Check both words in the question: ultrasound is diagnostic, but X-rays are diagnostic electromagnetic waves.

7. At equilibrium, in a p-n junction diode the net current is

[1 mark]

- (A) due to diffusion of majority charge carriers.
- (B) due to drift of minority charge carriers.
- (C) zero as diffusion and drift currents are equal and opposite.
- (D) zero as no charge carriers cross the junction.

Correct Answer: (C) zero as diffusion and drift currents are equal and opposite.

Solution:

Concept:

- Diffusion current results from majority carriers moving across the junction down their concentration gradients.
- The built-in electric field drives a drift current opposite to the diffusion current at equilibrium.

Step 1: Identify the two opposing currents Electrons diffuse from the n-side towards the p-side, while holes diffuse from the p-side towards the n-side. The depletion-region electric field produces the oppositely directed drift current, associated with minority carriers. Both transport processes occur at equilibrium.

Step 2: Find the net equilibrium current At thermal equilibrium with no applied bias, the drift and diffusion currents have equal magnitudes and opposite directions. Consequently $I_{\text{net}} = I_{\text{diffusion}} + I_{\text{drift}} = 0$ when signed currents are used. Options (A) and (B) each describe only one contribution. Option (D) is incorrect because zero net current does not imply that charge carriers stop crossing the junction.

Step 3: Conclusion The net current is zero because diffusion and drift currents are equal and opposite. Hence, the correct option is (C).

Quick Tip to Solve: Equilibrium means equal opposing currents; it does not mean that carrier motion across the junction ceases.

8. In an n-type semiconductor, the donor energy level lies

[1 mark]

- (A) at the centre of the energy gap.
- (B) just below the conduction band.
- (C) just above the valance band.
- (D) in the conduction band.

Correct Answer: (B) just below the conduction band.

Solution:

Concept:

- A pentavalent impurity supplies an extra electron to an n-type semiconductor, producing a donor energy level.
- A small energy difference $E_C - E_D$ separates the donor level E_D from the conduction-band edge E_C .

Step 1: Locate the donor level The fifth valence electron of the donor is weakly bound. Ordinary thermal energy can promote it into the conduction band, so its initial level must be in the forbidden gap very close to the conduction-band edge.

Step 2: Compare the alternatives A level at the centre of the gap would require much more excitation energy. A level just above the valence band describes an acceptor impurity in a p-type semiconductor. The donor level is a bound impurity level and does not lie within the conduction band itself.

Step 3: Conclusion The donor energy level lies just below the conduction band. Hence, the correct option is (B).

Quick Tip to Solve: Associate donors with the conduction band and acceptors with the valence band; their levels lie inside the gap near those edges.

9. When two nuclei ($A \leq 10$) fuse together to form a heavier nucleus, the

[1 mark]

- (A) binding energy per nucleon increases.
- (B) binding energy per nucleon decreases.
- (C) binding energy per nucleon does not change.
- (D) total binding energy decreases.

Correct Answer: (A) binding energy per nucleon increases.

Solution:

Concept:

- Binding energy per nucleon B/A measures how tightly the nucleons are bound on average; B is total binding energy and A is mass number.
- For light nuclei, the overall binding-energy-per-nucleon curve rises toward the intermediate-mass region.

Step 1: Identify the region of the binding-energy curve The printed condition $A \leq 10$ refers to very light nuclei. When these nuclei fuse to form a heavier nucleus, the product lies farther along the rising light-nucleus region of the binding-energy curve.

Step 2: Relate increased binding to released energy The product is more tightly bound per nucleon. The increase in total binding energy corresponds to a reduction in total rest mass and is released as energy. A decrease or no change in average binding would not describe the usual energy-releasing fusion of light nuclei; the total binding energy increases rather than decreases.

Step 3: Conclusion The binding energy per nucleon increases. Hence, the correct option is (A).

Quick Tip to Solve: On the binding-energy-per-nucleon curve, light nuclei release energy by moving upward through fusion toward more tightly bound nuclei.

10. In β^- decay, a

[1 mark]

- (A) neutron converts into a proton emitting antineutrino.
- (B) neutron converts into a proton emitting neutrino.
- (C) proton converts into a neutron emitting antineutrino.
- (D) proton converts into a neutron emitting neutrino.

Correct Answer: (A) neutron converts into a proton emitting antineutrino.

Solution:

Concept:

- In β^- decay a neutron becomes a proton, emitting an electron and an electron antineutrino.
- Electric charge and lepton number are conserved in $n \rightarrow p + e^- + \bar{\nu}_e$.

Step 1: Write the neutron conversion The microscopic process is

$$n \rightarrow p + e^- + \bar{\nu}_e.$$

The emitted electron is the beta-minus particle. The nucleus therefore gains one proton while its total number of nucleons remains unchanged.

Step 2: Check the particle named in each option The electron has lepton number +1, so the accompanying particle must be an antineutrino with lepton number -1. A neutrino would not conserve lepton number in this process. Proton-to-neutron conversion instead belongs to beta-plus decay or electron capture, so both proton-conversion alternatives fail.

Step 3: Conclusion A neutron converts into a proton with emission of an electron and an antineutrino. Hence, the correct option is (A).

Quick Tip to Solve: Remember the minus sign of the emitted electron: beta-minus decay increases nuclear atomic number by one and emits an antineutrino.

11. If the electric flux entering and leaving a closed surface in air are ϕ_1 and ϕ_2 respectively, the net electric charge enclosed within the surface is _____. [1 mark]

Solution:

Concept:

- Gauss's law gives $\Phi_{\text{net}} = Q_{\text{enc}}/\epsilon_0$, with Q_{enc} the enclosed charge and ϵ_0 the permittivity of free space.
- Closed-surface flux uses the outward normal: leaving flux is positive and entering flux is negative.

Step 1: Assign the signs of the two fluxes The stated ϕ_1 and ϕ_2 are the magnitudes of entering and leaving flux respectively. Entering electric field points opposite to the outward area vector, giving $-\phi_1$, whereas leaving field gives $+\phi_2$. Thus

$$\Phi_{\text{net}} = \phi_2 - \phi_1.$$

Step 2: Apply Gauss's law Treat air as having permittivity approximately ϵ_0 . Multiplying net flux by this permittivity gives

$$Q_{\text{enc}} = \epsilon_0 \Phi_{\text{net}}.$$

More leaving than entering flux corresponds to positive enclosed charge, as required by the outward-normal convention.

Step 3: Conclusion The blank is $\boxed{\varepsilon_0(\phi_2 - \phi_1)}$.

Quick Tip to Solve: For a closed surface, subtract entering flux from leaving flux before multiplying by ε_0 .

12. In Young's double slit experiment, the path difference between two interfering waves at a point on the screen is $\frac{5\lambda}{2}$, λ being wavelength of the light used. The _____ dark fringe will lie at this point. [1 mark]

Solution:

Concept:

- Destructive interference occurs for a path difference $\Delta = (2m - 1)\lambda/2$, where $m = 1, 2, 3, \dots$ numbers the dark fringes from the central maximum.
- Successive dark fringes therefore have path differences $\lambda/2, 3\lambda/2, 5\lambda/2$, and so on.

Step 1: Compare with the dark-fringe condition The official paper gives $\Delta = 5\lambda/2$. Equating this to the expression for the m -th dark fringe gives

$$\frac{(2m - 1)\lambda}{2} = \frac{5\lambda}{2}.$$

Both sides use the same wavelength, so the common factor $\lambda/2$ cancels.

Step 2: Determine the fringe number The remaining equation is $2m - 1 = 5$, hence $2m = 6$ and $m = 3$. Counting explicitly also gives the first dark fringe at $\lambda/2$, the second at $3\lambda/2$, and the next at the stated difference.

Step 3: Conclusion The third dark fringe lies at this point; fill the blank with $\boxed{\text{third}}$.

Quick Tip to Solve: For dark fringes, an odd multiplier k in $k\lambda/2$ corresponds to fringe number $(k + 1)/2$.

OR

12. If one of the slits in Young's double slit experiment is fully closed, the new pattern has _____ central maximum in angular size. [1 mark]

Solution:

Concept:

- With both slits open, the angular width of a Young interference bright fringe is approximately λ/d , where d is the slit separation.
- With only one finite-width slit open, single-slit diffraction has a central maximum of angular width approximately $2\lambda/a$, where a is the slit width.

Step 1: Identify the pattern after closing one slit Closing one slit removes the pair of coherent beams required for Young's interference fringes. Light through the remaining finite-width slit still diffracts, giving a single-slit pattern with a central maximum between its first two minima.

Step 2: Compare angular sizes The first diffraction minima satisfy $a \sin \theta = \pm\lambda$. At small angles their

separation is $2\lambda/a$. Since slit width is smaller than slit separation in the usual Young arrangement, this central maximum is wider than the original interference bright fringe of width λ/d .

Step 3: Conclusion The new pattern has a broader central maximum in angular size. Fill the blank with a broader.

Quick Tip to Solve: Closing one slit replaces the fine interference fringes by the much broader central maximum of the remaining slit's diffraction pattern.

13. For a higher resolving power of a compound microscope, the wavelength of light used should be _____. [1 mark]

Solution:

Concept:

- Resolving power is the reciprocal of the minimum separation of two object points that can be distinguished.
- For a microscope, $d_{\min} = 1.22\lambda/(2n \sin \alpha)$, where n is the medium's refractive index and α is the objective's semi-angle of collection.

Step 1: Express the wavelength dependence For the same microscope and immersion medium, the numerical aperture $n \sin \alpha$ remains fixed. The minimum resolvable separation is then directly proportional to the wavelength λ . Shorter wavelengths therefore allow two closer points to remain distinguishable.

Step 2: Relate separation to resolving power Taking the reciprocal gives

$$\text{RP} = \frac{1}{d_{\min}} = \frac{2n \sin \alpha}{1.22\lambda}.$$

Thus resolving power increases as wavelength decreases. Increasing wavelength would broaden the diffraction images and make nearby object details harder to distinguish.

Step 3: Conclusion The wavelength should be smaller (shorter); fill the blank with smaller.

Quick Tip to Solve: At fixed numerical aperture, microscope resolving power varies as $1/\lambda$; choose shorter-wavelength light to resolve finer details.

14. Unpolarised light passes from a rarer into a denser medium. If the reflected and the refracted rays are mutually perpendicular, the reflected light is linearly polarised _____ to the plane of incidence. [1 mark]

Solution:

Concept:

- At Brewster's angle i_B , the reflected and refracted rays are perpendicular: $i_B + r = 90^\circ$.
- The plane of incidence contains the incident ray and the normal; polarisation describes the direction of the electric field.

Step 1: Identify the condition The given perpendicularity of the two outgoing rays identifies incidence at the polarising (Brewster) angle. For this condition, Brewster's law is $\tan i_B = n_2/n_1$, where n_1 and

n_2 are the refractive indices of the incident and transmitted media.

Step 2: Determine the electric-field direction At this angle the reflected component with its electric field parallel to the plane of incidence vanishes. Only the component with its electric field normal to that plane remains, so the reflected beam is completely linearly polarised. The direction refers to electric-field vibration, not to the direction in which the ray travels.

Step 3: Conclusion The blank is perpendicular to the plane of incidence.

Quick Tip to Solve: When reflected and refracted rays are perpendicular, use Brewster polarisation: the reflected electric field is perpendicular to the incidence plane.

15. Out of red, blue and yellow lights, the scattering of _____ light is maximum.

[1 mark]

Solution:

Concept:

- For Rayleigh scattering, the scattered intensity satisfies $I_s \propto 1/\lambda^4$, where λ is the wavelength.
- Among the given colours, $\lambda_{\text{blue}} < \lambda_{\text{yellow}} < \lambda_{\text{red}}$.

Step 1: Compare the wavelengths Blue light has a shorter wavelength than yellow light and red light. We compare only the three colours specified in the question; no numerical wavelength is required.

Step 2: Apply the scattering law The fourth power of wavelength occurs in the denominator of the scattering law. Thus decreasing wavelength strongly increases scattering. Yellow light is scattered more than red light, but less than blue light. Red, having the largest wavelength of the three, has the least scattering under the same conditions.

Step 3: Conclusion The scattering of blue light is maximum.

Quick Tip to Solve: Order the given colours by wavelength, then choose the shortest wavelength because Rayleigh scattering varies as its inverse fourth power.

16. What is the impedance of a capacitor of capacitance C in an ac circuit using source of frequency n Hz?

[1 mark]

Solution:

Concept:

- The angular frequency is $\omega = 2\pi n$, where n is the source frequency in hertz.
- For an ideal capacitor, the capacitive reactance is $X_C = 1/(\omega C)$; this equals the impedance magnitude.

Step 1: Relate frequency to angular frequency The source completes n cycles each second, so its angular frequency is $2\pi n$ radians per second. The factor 2π must be included when frequency is specified in hertz.

Step 2: Write the capacitive impedance For sinusoidal ac, the ratio of voltage amplitude to current amplitude is $V_0/I_0 = 1/(\omega C)$. Substitution gives $X_C = 1/(2\pi n C)$, measured in ohms. The ideal capacitor has no resistive part; its current leads its voltage by 90° . In complex notation, the impedance is $Z_C = -j/(2\pi n C)$, with $j^2 = -1$.

Step 3: Conclusion The impedance magnitude is $|Z_C| = \frac{1}{2\pi nC} \Omega$, for C in farads and n in hertz.

Quick Tip to Solve: Convert hertz to angular frequency first: replace ω by $2\pi n$ in $X_C = 1/(\omega C)$.

OR

16. What is the value of impedance of a resonant series LCR circuit?

[1 mark]

Solution:

Concept:

- For a series LCR circuit, $Z = \sqrt{R^2 + (X_L - X_C)^2}$, where R is resistance.
- Resonance occurs when $X_L = \omega L = X_C = 1/(\omega C)$.

Step 1: Use the resonance condition At the resonant angular frequency $\omega_0 = 1/\sqrt{LC}$, the inductive and capacitive reactances are equal. Their voltages have opposite phases, so their contributions to the net series reactance cancel.

Step 2: Simplify the impedance Substituting $X_L - X_C = 0$ in the series impedance formula gives $Z = \sqrt{R^2 + 0} = R$. Therefore the impedance is purely resistive and is minimum at resonance. The applied voltage and current are in phase, and the current amplitude is V_0/R . Cancellation of reactance does not cancel the resistance.

Step 3: Conclusion At resonance, the series LCR circuit has impedance $Z = R$, its resistance.

Quick Tip to Solve: In a series circuit at resonance cancel X_L against X_C , then retain the resistance R .

17. A conducting rod of length l is kept parallel to a uniform magnetic field $\vec{B}$. It is moved along the magnetic field with a velocity $\vec{v}$. What is the value of emf induced in the conductor?

[1 mark]

Solution:

Concept:

- The magnetic force on a charge moving with the rod is $\vec{F}_B = q(\vec{v} \times \vec{B})$.
- Motional emf is $\varepsilon = \int (\vec{v} \times \vec{B}) \cdot d\vec{l}$, integrated along the conductor.

Step 1: Examine the direction of motion The rod moves along the magnetic field, so $\vec{v}$ is parallel to $\vec{B}$. The angle between these vectors is zero and their cross product vanishes: $\vec{v} \times \vec{B} = \vec{0}$.

Step 2: Find the induced potential difference Since the magnetic force on the moving free charges is zero, there is no magnetic separation of charge between the ends of the rod. The integrand in the motional-emf expression is zero everywhere along the conductor. Consequently its integral is zero, irrespective of the rod length or the magnitudes of velocity and field. The usual formula Blv requires mutually perpendicular directions and cannot be applied here.

Step 3: Conclusion The induced emf is $\varepsilon = 0 \text{ V}$.

Quick Tip to Solve: Check $\vec{v} \times \vec{B}$ before using Blv : motion parallel to the field gives zero motional emf.

18. Draw the graph showing variation of the value of the induced emf as a function of rate of change of current flowing through an ideal inductor. [1 mark]

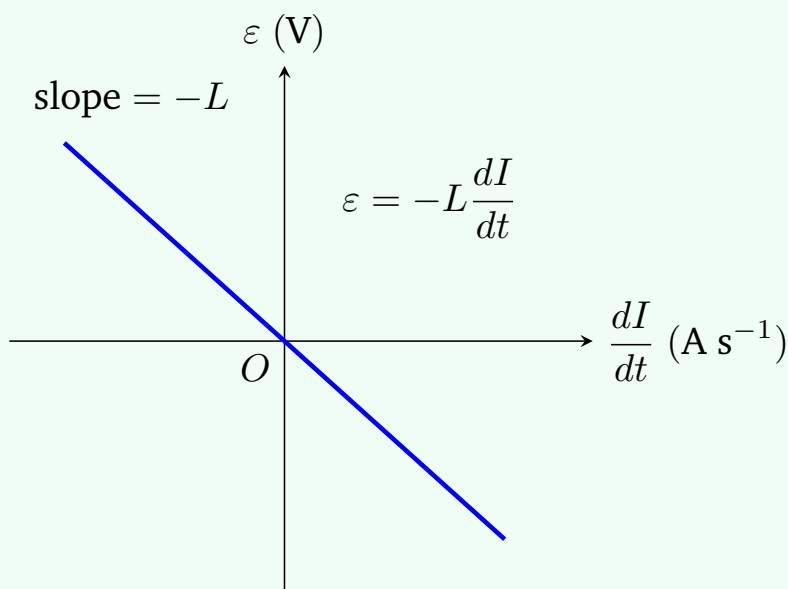
Solution:

Concept:

- The self-induced emf of an ideal inductor is $\varepsilon = -L \, dI/dt$, with constant self-inductance $L > 0$.
- The negative sign expresses Lenz's law: induced emf opposes the change in current.

Step 1: Choose the axes Take the rate of change of current dI/dt , in amperes per second, on the horizontal axis and signed induced emf ε , in volts, on the vertical axis. The sign of emf is measured relative to the chosen positive direction of current.

Step 2: Determine the line and its slope Writing $x = dI/dt$ gives $\varepsilon = -Lx$. Thus the plot is a straight line through the origin with slope $-L$. Positive rates of current increase give negative emf, and negative rates give positive emf. At zero rate of change the induced emf is zero. If only the magnitude is plotted for a positive rate, $|\varepsilon| = L \, dI/dt$ has positive slope L ; the drawn graph shows the signed emf including Lenz's-law sign.



Signed induced emf versus rate of change of current; slope $-L$.

Step 3: Conclusion The required signed-emf graph is a straight line through the origin with slope $= -L$.

Quick Tip to Solve: Put dI/dt , not current or time, on the horizontal axis; the signed-emf slope is $-L$.

19. What is the wavelength of a photon of energy $3.3 \times 10^{-19} \text{ J}$? [1 mark]

Solution:

Concept:

- A photon of frequency ν has energy $E = h\nu$, where h is Planck's constant.
- In vacuum, $c = \nu\lambda$; hence $E = hc/\lambda$, with λ the wavelength and c the speed of light.

Step 1: Express wavelength in terms of photon energy Combining the two photon relations gives

$\lambda = hc/E$. The energy is supplied directly in joules, so no conversion to electron volts is needed.

Step 2: Substitute the given energy Using $h = 6.63 \times 10^{-34} \text{ J s}$, $c = 3.0 \times 10^8 \text{ m s}^{-1}$ and $E = 3.3 \times 10^{-19} \text{ J}$,

$$\lambda = \frac{(6.63 \times 10^{-34})(3.0 \times 10^8)}{3.3 \times 10^{-19}} \text{ m} = 6.03 \times 10^{-7} \text{ m}.$$

The joule cancels and the remaining unit is metre. The result lies in the visible wavelength range, consistent with the given photon energy.

Step 3: Conclusion The wavelength is $\lambda \simeq 6.0 \times 10^{-7} \text{ m} = 600 \text{ nm}$ to the precision of the supplied energy.

Quick Tip to Solve: When photon energy is in joules, use hc/E directly and combine the powers of ten before converting metres to nanometres.

20. Define the term ‘threshold frequency’ in photoelectric emission.

[1 mark]

Solution:

Concept:

- Photoelectric emission occurs when an incident photon transfers enough energy to an electron to overcome the metal’s work function ϕ .
- The photon energy is $h\nu$; the threshold condition is $h\nu_0 = \phi$, where ν_0 denotes threshold frequency.

Step 1: State the limiting frequency Threshold frequency is the minimum frequency of incident electromagnetic radiation needed to just cause photoelectric emission from a given metal surface. It is a property of the material, since its work function fixes the energy required to release an electron.

Step 2: Explain the threshold condition At $\nu = \nu_0$, the most energetic emitted electrons have zero kinetic energy because the photon energy just equals the work function. For $\nu < \nu_0$, no photoelectric emission occurs, however large the intensity of the incident radiation. Increasing intensity increases the number of incident photons; it does not increase each photon’s energy.

Step 3: Conclusion $\nu_0 = \phi/h$ is the minimum incident frequency required for photoelectric emission from the given surface.

Quick Tip to Solve: Define the threshold using the minimum frequency for a specified metal; intensity cannot compensate for a frequency below it.

Section B (14 marks)

21. Define the term ‘mobility’ of charge carriers in a current carrying conductor. Obtain the relation for mobility in terms of relaxation time.

[2 marks]

Solution:

Concept:

- Mobility μ is the magnitude of the drift velocity acquired by a charge carrier per unit electric field:
 $\mu = v_d/E$.
- Relaxation time τ is the mean time between successive collisions of a conduction electron with the lattice; electron mass is m and its charge is $-e$, with $e > 0$.

Step 1: Define mobility If an applied electric field of magnitude E produces a steady drift speed v_d in the conductor, the mobility is $\mu = v_d/E$. It measures how readily charge carriers move under the applied field. Its SI unit is

$$[\mu] = \frac{\text{m s}^{-1}}{\text{V m}^{-1}} = \text{m}^2 \text{V}^{-1} \text{s}^{-1}.$$

Mobility is conventionally positive; an electron's drift velocity is opposite to the field.

Step 2: Obtain the drift velocity using relaxation time An electron in the electric field experiences $\vec{F} = -e\vec{E}$. Therefore its acceleration between collisions is $\vec{a} = -e\vec{E}/m$. The random thermal velocities average to zero. Over the mean free time τ , the field contributes the average drift velocity

$$\vec{v}_d = \vec{a}\tau = -\frac{e\tau}{m}\vec{E}, \quad v_d = \frac{eE\tau}{m}.$$

Dividing the drift speed by the field magnitude yields the required dependence on relaxation time.

Step 3: Conclusion For electrons, $\mu = \frac{v_d}{E} = \frac{e\tau}{m}$. A longer relaxation time gives a greater mobility because carriers accelerate for longer between collisions.

Quick Tip to Solve: Use drift speed in the definition of mobility; keep the minus sign only in the electron's vector drift-velocity relation.

OR

21. Define the term 'drift velocity' of electrons in a current carrying conductor. Obtain the relationship between the current density and the drift velocity of electrons. [2 marks]

Solution:

Concept:

- Drift velocity is the average velocity acquired by free electrons in a conductor under an applied electric field, superposed on their random thermal motion.
- Current density is current per unit cross-sectional area: $J = I/A$. Let n be the number of free electrons per unit volume and e the positive magnitude of the electronic charge.

Step 1: Define electron drift velocity In the absence of an electric field, electrons move randomly and their vector average velocity is zero. Applying a field produces a small directed average velocity, called the drift velocity $\vec{v}_d$. Since electrons carry negative charge, this velocity is opposite to the applied field and to the conventional current direction. The drift speed v_d is its magnitude.

Step 2: Calculate the charge crossing a section Consider a conductor of cross-sectional area A . In time Δt , electrons with drift speed v_d cross the section from a length $v_d\Delta t$. The corresponding volume is $Av_d\Delta t$, containing $nAv_d\Delta t$ electrons. The magnitude of the transported charge is therefore

$$\Delta Q = neAv_d\Delta t, \quad I = \frac{\Delta Q}{\Delta t} = neAv_d.$$

Dividing by A eliminates the cross-sectional area. For the vector relation, the negative electron charge reverses the direction of current relative to electron drift.

Step 3: Conclusion $J = nev_d$ in magnitude, or $\vec{J} = -ne\vec{v}_d$ for electrons. Conventional current density points opposite to their drift velocity.

Quick Tip to Solve: Count electrons in the swept volume $Av_d\Delta t$; remember that conventional current is opposite to electron motion.

22. An ammeter of resistance $0.8\ \Omega$ can measure a current up to $1.0\ \text{A}$. Find the value of shunt resistance required to convert this ammeter to measure a current up to $5.0\ \text{A}$. [2 marks]

Solution:

Concept:

- A low resistance shunt is connected in parallel with an ammeter to divert the excess current while the instrument carries no more than its full-scale current.
- Parallel branches have equal potential difference: $I_A R_A = I_s S$, where R_A is ammeter resistance, I_A its current, and S the shunt resistance.

Step 1: Determine the current through the shunt At the new full-scale current $I = 5.0\ \text{A}$, the original ammeter must still carry only its rated current $I_A = 1.0\ \text{A}$. By the junction rule, the remaining current passes through the parallel shunt:

$$I_s = I - I_A = 5.0 - 1.0 = 4.0\ \text{A}.$$

The shunt thus carries four times the current in the ammeter. A series resistor would not provide this alternate path and cannot extend the current range in this way.

Step 2: Equate the potential differences The voltage across the ammeter at full scale is

$$V_A = I_A R_A = (1.0)(0.8) = 0.8\ \text{V}.$$

Since the shunt is connected in parallel, its voltage must also be $0.8\ \text{V}$. Therefore,

$$S = \frac{I_A R_A}{I_s} = \frac{(1.0)(0.8)}{4.0}\ \Omega = 0.20\ \Omega.$$

This resistance is one-fourth of the ammeter resistance, so the current division is consistent with the required ratio.

Step 3: Conclusion Connect a shunt of resistance $S = 0.20\ \Omega$ in parallel with the ammeter to increase its range to $5.0\ \text{A}$.

Quick Tip to Solve: Use the excess current $5.0 - 1.0$ in the shunt denominator, while the ammeter branch retains its original full-scale current.

23. (a) Explain the term 'sharpness of resonance' in ac circuit.

(b) In a series LCR circuit, $V_L = V_C \neq V_R$. What is the value of power factor for this circuit? [2 marks]

Solution:

Concept:

- Resonance in a series LCR circuit occurs when $X_L = X_C$, at $\omega_0 = 1/\sqrt{LC}$, where L and C are inductance and capacitance.
- Impedance is $Z = \sqrt{R^2 + (X_L - X_C)^2}$; power factor is $\cos \phi = R/Z$, where ϕ is the source voltage-current phase difference.

Step 1: (a) Explain sharpness of resonance Sharpness of resonance describes how rapidly current falls as the driving frequency is moved away from the resonant frequency. A narrow, high current peak means sharp resonance: appreciable current occurs only in a small frequency interval around resonance. The quality factor is $Q = \omega_0/\Delta\omega = \omega_0 L/R$, where $\Delta\omega$ is the bandwidth between the half-power frequencies. A larger Q , or smaller resistance for fixed L, C , gives a sharper peak.

Step 2: (b) Cancel the reactive voltages The same current flows through all series components. Since $V_L = IX_L$ and $V_C = IX_C$, the equality $V_L = V_C$ implies $X_L = X_C$. The inductor and capacitor voltages oppose each other in phase and cancel in the resultant source voltage. Consequently $Z = R$, the source voltage is in phase with current, and $\phi = 0$. The inequality $V_L \neq V_R$ does not alter the cancellation; it merely compares individual voltage magnitudes.

Step 3: Conclusion Sharpness is the narrowness of the resonance current peak, measured by Q . For the given circuit, $\cos \phi = 1$.

Quick Tip to Solve: Equal opposing reactive voltages imply unity power factor, regardless of their size relative to V_R .

OR

23. An ac source of emf $V = V_0 \sin \omega t$ is connected to a capacitor of capacitance C . Deduce the expression for the current (I) flowing in it. Plot the graph of (i) V vs. ωt , and (ii) I vs. ωt . [2 marks]

Solution:

Concept:

- The charge on a capacitor is $q = CV$, where C is its capacitance and V its instantaneous potential difference. Current is the rate of change of charge, $I = dq/dt$.
- Capacitive reactance is $X_C = 1/(\omega C)$, where ω is angular frequency. In a purely capacitive circuit, current leads voltage by $\pi/2$.

Step 1: Obtain charge and differentiate With $V = V_0 \sin \omega t$, the charge supplied to the capacitor is

$$q(t) = CV_0 \sin \omega t.$$

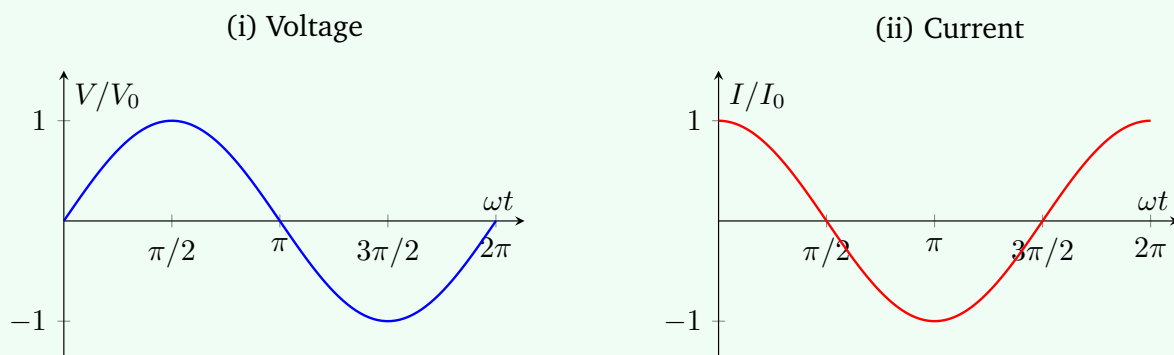
Taking current positive in the direction which increases this charge,

$$I(t) = \frac{dq}{dt} = \omega CV_0 \cos \omega t.$$

Therefore the current amplitude is $I_0 = \omega CV_0 = V_0/X_C$. Current is largest when the voltage changes

most rapidly, rather than when the voltage itself is largest.

Step 2: Express the phase relation and plot both waves Using $\cos \omega t = \sin(\omega t + \pi/2)$, the current can also be written as $I = I_0 \sin(\omega t + \pi/2)$. At $\omega t = 0$, voltage is zero and rising, whereas current is at its positive maximum. At $\omega t = \pi/2$, voltage reaches V_0 and current is zero. The plotted curves show one complete cycle; the current reaches corresponding points a quarter-cycle before the voltage.



Voltage and current against ωt ; their vertical scales are V_0 and I_0 , respectively.

Step 3: Conclusion $I = \omega C V_0 \cos \omega t = I_0 \sin(\omega t + \pi/2)$, with $I_0 = \omega C V_0$. Current leads voltage by $\pi/2$.

Quick Tip to Solve: Differentiate the sine voltage to obtain a cosine current: a capacitor's current leads its voltage by a quarter-cycle.

24. Which of the following electromagnetic waves has (a) minimum wavelength, and (b) minimum frequency? Write one use of each of these two waves.

Infrared waves, Microwaves, γ -rays and X-rays

[2 marks]

Solution:

Concept:

- Electromagnetic waves in vacuum satisfy $c = \nu \lambda$, where c is the speed of light, ν the frequency and λ the wavelength. Frequency and wavelength therefore vary inversely.
- Photon energy is $E = h\nu$, where h is Planck's constant. High-frequency radiation has higher photon energy and greater penetrating ability in the usual comparison of these spectral regions.

Step 1: (a) Identify the shortest wavelength and its use Among the listed waves, the order of increasing wavelength is gamma rays, X-rays, infrared waves, and microwaves. Hence gamma rays have the minimum wavelength. One use is cancer treatment (radiotherapy): carefully directed gamma radiation damages cancerous cells. This use is associated with their high photon energy. X-rays also have short wavelengths, but in the standard electromagnetic spectrum they lie below gamma rays in frequency.

Step 2: (b) Identify the lowest frequency and its use Since $\nu = c/\lambda$, the largest wavelength corresponds to the smallest frequency. Microwaves have the largest wavelength among these four regions, so they have the minimum frequency. One use is in radar, where transmitted microwaves reflect from an object and the returned signal helps determine its location or distance. Infrared radiation has a higher frequency than microwaves, despite having a lower frequency than X-rays and gamma rays.

Step 3: Conclusion (a) Minimum wavelength: gamma rays, used in cancer radiotherapy. (b) Minimum frequency: microwaves, used in radar.

Quick Tip to Solve: Order the four bands by wavelength once; the shortest wavelength gives the highest frequency, and the longest gives the lowest.

25. An object is kept 20 cm in front of a concave mirror of radius of curvature 60 cm. Find the nature and position of the image formed. [2 marks]

Solution:

Concept:

- For a spherical mirror, focal length is half the radius of curvature: $f = R/2$. In the Cartesian convention, distances opposite the incident-light direction are negative.
- The mirror formula is $1/f = 1/v + 1/u$, and transverse magnification is $m = -v/u$, where u and v are object and image distances from the pole.

Step 1: Assign signed distances The object is in front of the mirror, so $u = -20$ cm. A concave mirror's centre of curvature and focus are also in front of it. Thus $R = -60$ cm and

$$f = \frac{R}{2} = -30 \text{ cm.}$$

The object is closer to the pole than the focus, since $|u| < |f|$. This already suggests a virtual image behind the mirror.

Step 2: Calculate image distance and magnification Substituting in the mirror formula,

$$\begin{aligned}\frac{1}{v} &= \frac{1}{f} - \frac{1}{u} = -\frac{1}{30} + \frac{1}{20} = \frac{1}{60}, \\ v &= +60 \text{ cm.}\end{aligned}$$

The positive image distance places the image behind the mirror, where reflected rays do not actually meet; their backward extensions meet there. Hence it is virtual. Its magnification is

$$m = -\frac{v}{u} = -\frac{60}{-20} = +3.$$

Positive magnification means an erect image, and its magnitude shows that the image is three times the object's size.

Step 3: Conclusion The image forms 60 cm behind the mirror. It is virtual, erect and magnified three times.

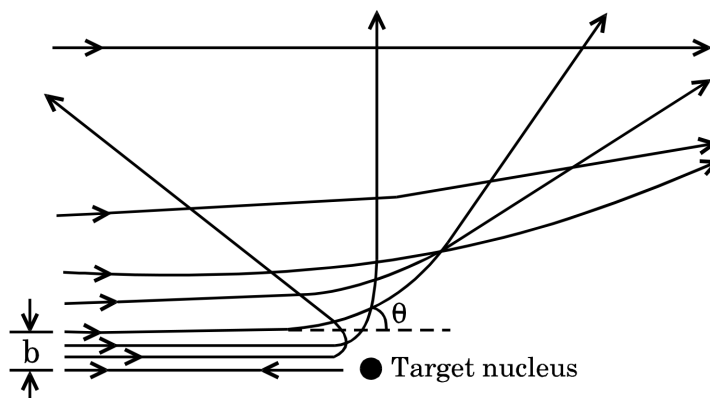
Quick Tip to Solve: An object inside a concave mirror's focal length produces a virtual, erect image; use this to check the sign of your calculated image distance.

26. In Geiger-Marsden scattering experiment, the trajectory of α -particles in Coulomb's field of a heavy nucleus is shown in the figure.

(a) What do ' b ' and ' θ ' represent in the figure?

(b) What will be the value of ' b ' for (i) $\theta = 0^\circ$, and (ii) $\theta = 180^\circ$?

[2 marks]



Solution:

Concept:

- Rutherford scattering is the deflection of a positively charged α -particle by the repulsive Coulomb field of a positive nucleus.
- At fixed incident kinetic energy K , $b = \frac{1}{4\pi\epsilon_0} \frac{Ze^2}{K} \cot(\theta/2)$, where the nuclear charge is Ze and the alpha-particle charge is $2e$.

Step 1: (a) Identify the geometrical quantities The impact parameter b is the perpendicular distance from the initial, undeflected line of motion of the incoming α -particle to the centre of the target nucleus. The scattering angle θ is the angle between the initial direction of motion and the final direction after scattering. Thus b specifies how far off-centre the incident trajectory is, whereas θ measures its deflection.

Step 2: (b) Examine the two limiting trajectories For $\theta = 0^\circ$, the particle undergoes no deflection. In the ideal Coulomb scattering model, $\cot(\theta/2)$ tends to infinity as θ tends to zero, so the impact parameter tends to infinity. A sufficiently large finite b gives an approximately straight trajectory. For $\theta = 180^\circ$, $\cot 90^\circ = 0$: the particle approaches the nucleus head-on, stops momentarily and returns along its incident line.

Step 3: Conclusion b is the impact parameter and θ the scattering angle. For (i) $\theta = 0^\circ$, $b \rightarrow \infty$; for (ii) $\theta = 180^\circ$, $b = 0$.

Quick Tip to Solve: A head-on alpha-particle has zero impact parameter and maximum deflection; larger impact parameters give smaller deflections.

27. Draw V-I characteristics of a p-n junction diode. Explain, why the current under reverse bias is almost independent of the applied voltage up to the critical voltage.

[2 marks]

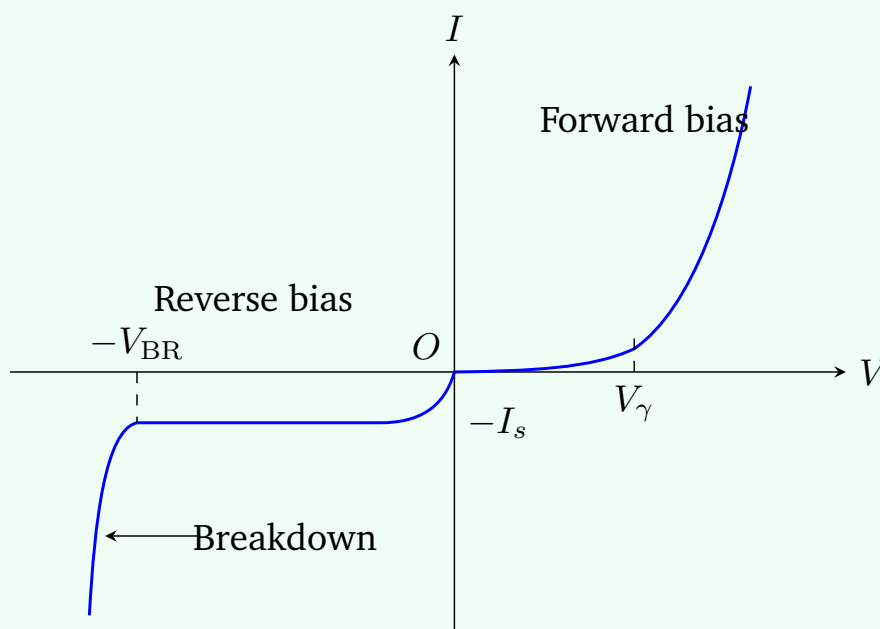
Solution:

Concept:

- Forward bias reduces the junction barrier and allows a large majority-carrier current after the knee voltage V_γ .

- Reverse bias widens the depletion region. The small reverse saturation current I_s is mainly due to thermally generated minority carriers; its value depends strongly on temperature.

Step 1: Draw the forward and reverse characteristics Plot diode current I against applied diode voltage V . In forward bias, current is initially small and rises rapidly beyond the knee voltage V_γ . In reverse bias, the current quickly reaches a small, nearly constant negative value $-I_s$. At the critical reverse voltage, the breakdown voltage V_{BR} , its magnitude rises sharply. The sketch uses different scales for forward and reverse current so that the small reverse current is visible.



Diode V-I characteristic (schematic; unequal forward and reverse current scales)

Step 2: Explain the nearly constant reverse current The reverse electric field sweeps minority carriers across the depletion layer. At a fixed temperature, their supply is governed mainly by thermal generation, rather than by the applied reverse voltage. Once these carriers are collected, increasing the voltage changes their drift speed and the depletion width but adds very few carriers to the supply; consequently the reverse current remains almost constant. At breakdown, carrier multiplication or tunnelling causes a rapid increase, so the nearly constant-current description ceases to apply.

Step 3: Conclusion The reverse-bias current is nearly voltage-independent below breakdown because it is limited mainly by the supply of thermally generated minority carriers at a fixed temperature.

Quick Tip to Solve: Label the knee, reverse saturation current and breakdown voltage; make clear that forward and reverse current scales differ.

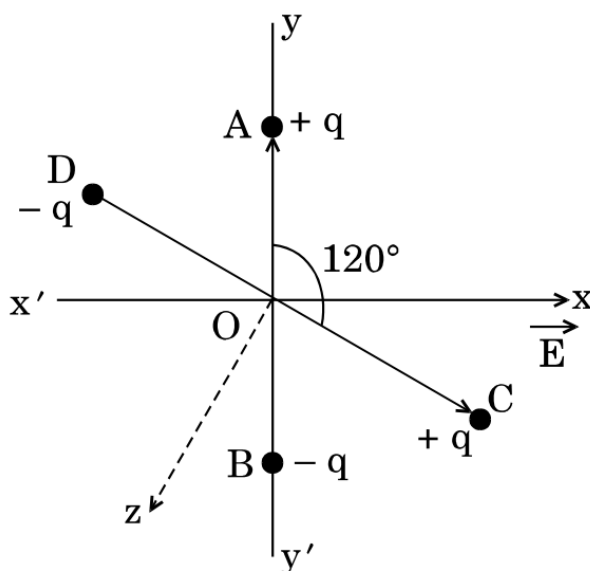
Section C (21 marks)

28. Two small identical electric dipoles AB and CD, each of dipole moment $\vec{p}$ are kept at an angle of 120° to each other in an external electric field $\vec{E}$ pointing along the x -axis as shown in the figure. Find the

(a) dipole moment of the arrangement, and

(b) magnitude and direction of the net torque acting on it.

[3 marks]



Solution:

Concept:

- An electric dipole moment points from the negative charge to the positive charge. For charges $\pm q$ separated by distance d , its magnitude is $p = qd$.
- Dipole moments add vectorially: $\vec{p}_{\text{net}} = \vec{p}_{AB} + \vec{p}_{CD}$.
- In a uniform external field, $\vec{\tau} = \vec{p} \times \vec{E}$. Here $\vec{E} = E\hat{i}$, and $\hat{k}$ denotes the positive z -direction in a right-handed coordinate system.

Step 1: Read the dipole directions from the charge signs Dipole AB points from B ($-q$) to A ($+q$), along the positive y -axis. Dipole CD points from D ($-q$) to C ($+q$), at 30° below the positive x -axis. These directions make the printed angle 120° . Thus

$$\vec{p}_{AB} = p\hat{j}, \quad \vec{p}_{CD} = \frac{\sqrt{3}p}{2}\hat{i} - \frac{p}{2}\hat{j}.$$

Step 2: (a) Add the moments and find the resultant direction Adding corresponding components gives

$$\vec{p}_{\text{net}} = \frac{\sqrt{3}p}{2}\hat{i} + \frac{p}{2}\hat{j}.$$

Its magnitude and angle ϕ above the field direction are

$$|\vec{p}_{\text{net}}| = \sqrt{\frac{3p^2}{4} + \frac{p^2}{4}} = p, \quad \tan \phi = \frac{p/2}{\sqrt{3}p/2} = \frac{1}{\sqrt{3}}.$$

Therefore $\phi = 30^\circ$. The resultant lies on the bisector of the angle between the two equal dipole moments.

Step 3: (b) Calculate the net torque The torques add, so their sum equals the torque of the resultant moment:

$$\begin{aligned}\vec{\tau}_{\text{net}} &= \vec{p}_{\text{net}} \times E\hat{i} \\ &= \frac{pE}{2}(\hat{j} \times \hat{i}) = -\frac{pE}{2}\hat{k}.\end{aligned}$$

AB produces torque $-pE\hat{k}$, whereas CD produces $+pE\hat{k}/2$; this independently verifies the sign and magnitude. The net torque is directed into the plane of the paper, along negative z , and tends to rotate the arrangement clockwise towards alignment with $\vec{E}$.

Step 4: Conclusion (a) The resultant dipole moment has magnitude p and points 30° above the positive x -axis: $\vec{p}_{\text{net}} = (\sqrt{3}p/2)\hat{i} + (p/2)\hat{j}$. (b) The net torque has magnitude $pE/2$, directed along negative z , into the paper (clockwise).

Quick Tip to Solve: Read each dipole from minus to plus first; the two torques oppose each other, so subtract their magnitudes.

OR

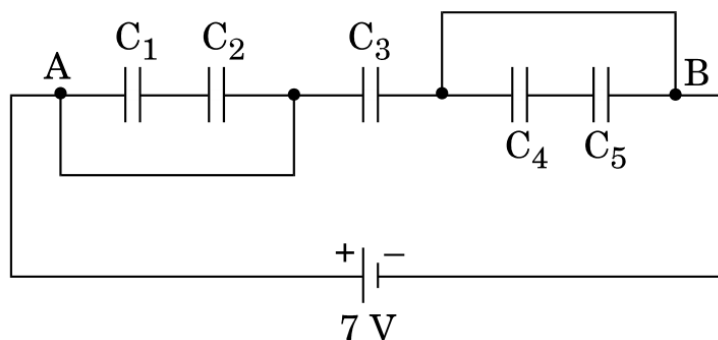
28. In the figure given below, find the

(a) equivalent capacitance of the network between points A and B. Given: $C_1 = C_5 = 8 \mu\text{F}$, $C_2 = C_3 = C_4 = 4 \mu\text{F}$.

(b) maximum charge supplied by the battery, and

(c) total energy stored in the network.

[3 marks]



Solution:

Concept:

- Points joined directly by an ideal conducting wire have the same potential. Before applying series or parallel capacitor formulae, identify these equipotential nodes.
- For a capacitor across a battery, its final charge is $Q = CV$ and its stored electrostatic energy is $U = \frac{1}{2}CV^2$, where C is capacitance and V is the potential difference.

Step 1: (a) Identify the wires that bypass the outer capacitor pairs Let X be the junction between C_2 and C_3 , and Y the junction between C_3 and C_4 . The lower connecting wire joins A directly to X , so $V_X = V_A$. The upper connecting wire joins Y directly to B , so $V_Y = V_B$. Thus the series pair C_1, C_2 has zero voltage across its ends, as does the pair C_4, C_5 . For initially uncharged capacitors, these bypassed pairs acquire no charge and store no energy. Only C_3 lies between the two distinct battery terminals. Consequently,

$$C_{\text{eq}} = C_3 = 4 \mu\text{F}.$$

Adding all five capacitors in series would ignore the two direct wire connections shown in the printed circuit.

Step 2: (b) Calculate the maximum charge supplied by the battery The figure gives the battery voltage as $V = 7\text{ V}$. After charging is complete, the charge delivered to one terminal of the equivalent capacitor is

$$Q_{\max} = C_{\text{eq}}V = (4 \times 10^{-6})(7)\text{ C} = 28\text{ }\mu\text{C}.$$

This is the battery's transferred charge; equal and opposite charges on the two capacitor plates are not added together.

Step 3: (c) Calculate the total stored energy Since the bypassed pairs remain uncharged, the total energy equals the energy in C_3 :

$$U = \frac{1}{2}C_{\text{eq}}V^2 = \frac{1}{2}(4 \times 10^{-6})(7)^2\text{ J} = 98 \times 10^{-6}\text{ J}.$$

The square of the voltage is essential; the energy is not CV , which has the dimensions of charge.

Step 4: Conclusion (a) $C_{\text{eq}} = 4\text{ }\mu\text{F}$, (b) $Q_{\max} = 28\text{ }\mu\text{C}$, and (c) $U = 98\text{ }\mu\text{J}$.

Quick Tip to Solve: Trace every unbroken wire first: the lower and upper wires bypass the outer pairs, leaving only the central capacitor across the battery.

29. (a) Derive the condition of balance for Wheatstone bridge.

(b) Draw the circuit diagram of a meter bridge to explain how it is based on Wheatstone bridge.

[3 marks]

Solution:

Concept:

- A Wheatstone bridge has four resistances in two parallel branches, with a galvanometer connecting their middle junctions. At balance its current is zero, so the two junctions have equal potential.
- For a uniform wire, resistance is $R = \rho L/A$, where ρ is resistivity, L is length and A is cross-sectional area. Its resistance ratio therefore equals its length ratio.

Step 1: (a) Derive the balance condition of the Wheatstone bridge Connect resistances P, Q in the branch $A:D:B$ and resistances R, S in the branch $A:J:B$. Connect the cell between A and B , and the galvanometer between D and J . Let branch currents be I_1 through P, Q and I_2 through R, S . At balance the galvanometer shows zero deflection, so no current passes between D and J , and $V_D = V_J$. The potential drops from A to these junctions, and from these junctions to B , give

$$I_1P = I_2R,$$

$$I_1Q = I_2S.$$

Dividing the first equation by the second cancels the nonzero branch currents:

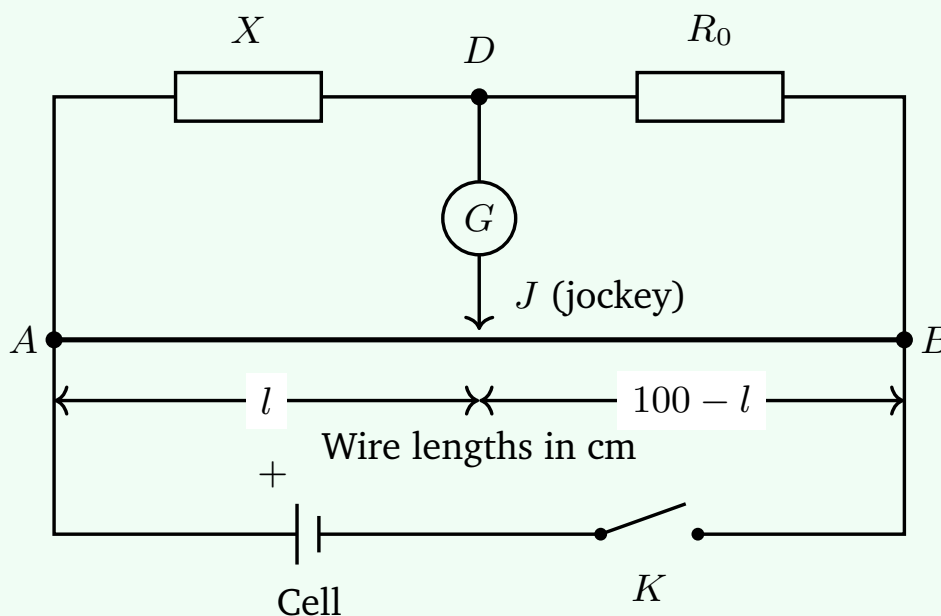
$$\frac{P}{Q} = \frac{R}{S}, \quad PS = QR.$$

This derivation uses the null-current condition, rather than assuming equal currents in the two separate branches.

Step 2: (b) Explain the meter bridge using its circuit In the diagram, X is the unknown resistance and R_0 the known resistance in the two gaps. The uniform resistance wire AB is 100 cm long. A galvanometer connects their junction D to a sliding jockey J on the wire. The cell and key K are connected across A, B . Move the jockey until the galvanometer reads zero. If $AJ = l$ cm, then $JB = (100 - l)$ cm. The four bridge arms are X, R_0, R_{AJ}, R_{JB} ; the uniform wire provides the latter two. Applying the balance condition gives

$$\frac{X}{R_0} = \frac{R_{AJ}}{R_{JB}} = \frac{l}{100 - l}, \quad X = R_0 \frac{l}{100 - l}.$$

The meter bridge thus measures an unknown resistance by realizing a Wheatstone bridge with two adjustable wire lengths. The same wire material and cross-section make ρ/A cancel in the ratio.



Meter bridge: the galvanometer joins the gap junction to the jockey on a uniform metre-long wire.

Step 3: Conclusion The Wheatstone bridge balances when $P/Q = R/S$. For the meter bridge shown, its balance condition is $X/R_0 = l/(100 - l)$.

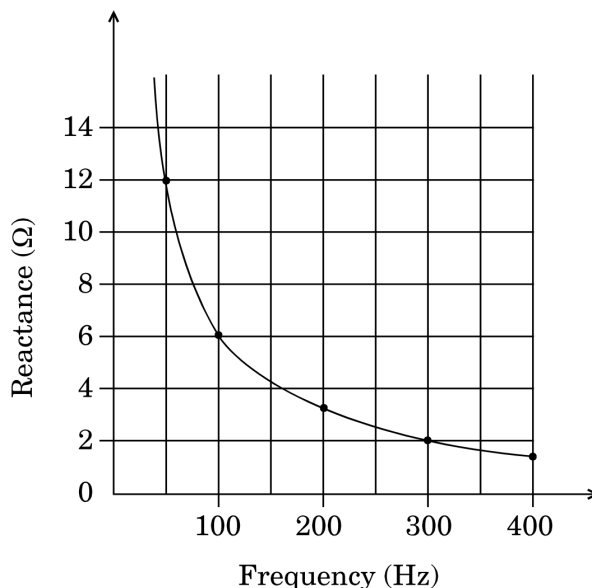
Quick Tip to Solve: Match the unknown in the left gap to the left wire length: its ratio to the right gap resistance is the left-to-right length ratio.

30. The figure shows the graphical variation of the reactance of a capacitor with frequency of ac source.

(a) Find the capacitance of the capacitor.

(b) An ideal inductor has the same reactance at 100 Hz frequency as the capacitor has at the same frequency. Find the value of inductance of the inductor.

(c) Draw the graph showing the variation of the reactance of this inductor with frequency. [3 marks]



Solution:

Concept:

- Capacitive reactance is $X_C = 1/(2\pi fC)$, where f is the frequency in hertz and C the capacitance in farads. Thus fX_C remains constant for a fixed capacitor.
- Inductive reactance is $X_L = 2\pi fL$, where L is the inductance in henries. For an ideal inductor it is directly proportional to frequency and vanishes at zero frequency.

Step 1: (a) Read the capacitor graph and find its capacitance The marked point at $f = 100$ Hz has capacitive reactance $X_C = 6\Omega$. Use this pair of coordinates in the capacitor formula:

$$C = \frac{1}{2\pi fX_C} = \frac{1}{2\pi(100)(6)} \text{ F} = \frac{1}{1200\pi} \text{ F}.$$

Numerically this is 2.65×10^{-4} F, or approximately $265 \mu\text{F}$. As a check, the graph also gives $X_C = 3\Omega$ at 200 Hz; both coordinates give $fX_C = 600 \text{ Hz}\Omega$, consistent with the same capacitance.

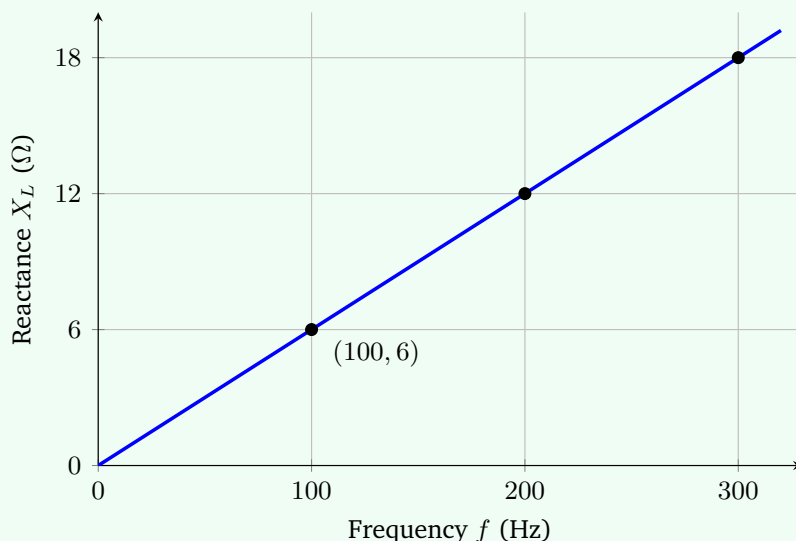
Step 2: (b) Apply the equality of reactances at the stated frequency The equality applies specifically at 100 Hz, where the capacitor has reactance 6Ω . Therefore the inductor must also have $X_L = 6\Omega$ at that frequency. Rearranging its reactance formula gives

$$L = \frac{X_L}{2\pi f} = \frac{6}{2\pi(100)} \text{ H} = \frac{3}{100\pi} \text{ H} \simeq 9.55 \times 10^{-3} \text{ H}.$$

The two devices have equal reactance magnitudes at this one frequency, although the capacitor's reactance decreases and the inductor's reactance increases as frequency rises.

Step 3: (c) Plot inductive reactance against frequency Substituting the value of L into $X_L = 2\pi fL$ gives $X_L = 0.060f$, with X_L in ohms when f is in hertz. The required graph is consequently a straight line through the origin with slope $0.060 \Omega \text{ Hz}^{-1}$. It passes through (100 Hz, 6Ω), (200 Hz, 12Ω) and

(300 Hz, $18\ \Omega$); these points fix its scale and direction.



Inductive reactance increases linearly with frequency.

Step 4: Conclusion $C \simeq 265\ \mu\text{F}$ and $L \simeq 9.55\ \text{mH}$. The inductor's reactance-frequency graph is the increasing straight line $X_L = 0.060f$ through the origin.

Quick Tip to Solve: Read one labelled point first: here (100, 6) determines both devices. Check the capacitor using constant fX_C , then draw the inductor using constant X_L/f .

31. What is the difference in the construction of an astronomical telescope and a compound microscope? The focal lengths of the objective and eyepiece of a compound microscope are 1.25 cm and 5.0 cm, respectively. Find the position of the object relative to the objective in order to obtain an angular magnification of 30 when the final image is formed at the near point. [3 marks]

Solution:

Concept:

- A telescope observes distant objects and uses an objective of long focal length and large aperture. A compound microscope observes small nearby objects and uses an objective of very short focal length and small aperture; both use a short-focal-length eyepiece.
- For a microscope with its final image at the near point, the magnitude of angular magnification is $M = |m_o|(1 + D/f_e)$. Here $m_o = v_o/u_o$ is the objective's signed linear magnification and $D = 25\ \text{cm}$ is the least distance of distinct vision.
- The objective obeys the thin-lens formula $1/f_o = 1/v_o - 1/u_o$. With Cartesian signs, its real object has $u_o < 0$, its real intermediate image has $v_o > 0$, and its convex-lens focal length has $f_o > 0$.

Step 1: Compare the construction of the two instruments The astronomical telescope has a large-aperture objective of long focal length, which collects light from distant faint objects and forms a real image. Its eyepiece has a much shorter focal length. In contrast, the compound microscope has a small-aperture objective of very short focal length, placed close to the small object to produce an enlarged real intermediate image. Its eyepiece also has a short focal length, but this is longer than the objective's focal length. Thus the ordering of the focal lengths differs: $f_o > f_e$ for the telescope, while $f_o < f_e$ for the microscope.

Step 2: Determine the magnification required from the objective Since the final image is formed at the near point, use the eyepiece factor $1 + D/f_e$, rather than the factor for an image at infinity. With $f_e = 5.0 \text{ cm}$,

$$m_e = 1 + \frac{25}{5.0} = 6, \quad |m_o| = \frac{M}{m_e} = \frac{30}{6} = 5.$$

The objective forms an inverted real intermediate image, so its signed magnification is $m_o = -5$. Therefore $v_o/u_o = -5$, giving $v_o = -5u_o$.

Step 3: Substitute the magnification relation into the lens equation With $f_o = 1.25 \text{ cm}$, substitute $v_o = -5u_o$ into the Cartesian lens formula:

$$\begin{aligned} \frac{1}{1.25} &= \frac{1}{-5u_o} - \frac{1}{u_o} = -\frac{6}{5u_o}, \\ u_o &= -\frac{6}{5}(1.25) \text{ cm} = -1.50 \text{ cm}, \\ v_o &= -5u_o = 7.50 \text{ cm}. \end{aligned}$$

The negative object distance denotes the incident-light side of the objective. Its magnitude is just greater than the objective's focal length, as required for an enlarged real image. The positive image distance verifies that the intermediate image is real.

Step 4: Conclusion The telescope uses a long-focal-length, large-aperture objective, whereas the microscope uses a very short-focal-length, small-aperture objective. The microscope's object must be placed 1.50 cm in front of its objective, i.e. $u_o = -1.50 \text{ cm}$.

Quick Tip to Solve: Divide the total magnification by $1 + D/f_e$ first, then use the resulting objective magnification in the lens formula. Keep the real object's distance negative.

32. The maximum kinetic energy of the photoelectrons emitted is doubled when the wavelength of light incident on the photosensitive surface changes from λ_1 to λ_2 . Deduce expressions for the threshold wavelength and work function for the metal surface in terms of λ_1 and λ_2 . [3 marks]

Solution:

Concept:

- Einstein's photoelectric equation is $K_{\max} = hc/\lambda - \phi$, where h is Planck's constant, c the speed of light, λ the incident wavelength and ϕ the metal's work function.
- At the threshold wavelength λ_0 , electrons just escape with zero maximum kinetic energy. Hence $\phi = hc/\lambda_0$. A shorter incident wavelength supplies more energy per photon.

Step 1: Write the energy equation at each incident wavelength Let the initial maximum kinetic energy be K_1 . The metal surface is unchanged, so the same work function enters both equations. Since the kinetic energy doubles when the wavelength changes from λ_1 to λ_2 ,

$$\begin{aligned} K_1 &= \frac{hc}{\lambda_1} - \phi, \\ 2K_1 &= \frac{hc}{\lambda_2} - \phi. \end{aligned}$$

Only the excess photon energy above the fixed work function doubles.

Step 2: Eliminate kinetic energy to obtain the work function Substitute the first equation into the

second, and collect the work-function terms:

$$\begin{aligned}2\left(\frac{hc}{\lambda_1} - \phi\right) &= \frac{hc}{\lambda_2} - \phi, \\ \phi &= hc\left(\frac{2}{\lambda_1} - \frac{1}{\lambda_2}\right) \\ &= \frac{hc(2\lambda_2 - \lambda_1)}{\lambda_1\lambda_2}.\end{aligned}$$

With wavelengths in metres, this expression gives the work function in joules.

Step 3: Use the threshold condition and check the wavelength ordering At threshold, $K_{\max} = 0$, so substitute $\phi = hc/\lambda_0$ into the expression just derived:

$$\frac{1}{\lambda_0} = \frac{2}{\lambda_1} - \frac{1}{\lambda_2}, \quad \lambda_0 = \frac{\lambda_1\lambda_2}{2\lambda_2 - \lambda_1}.$$

The stated increase in kinetic energy requires $\lambda_2 < \lambda_1$. A positive work function also requires $2\lambda_2 > \lambda_1$. Thus the physical situation has $\lambda_1/2 < \lambda_2 < \lambda_1$, and the derived threshold wavelength is greater than λ_1 , as expected for emission at both incident wavelengths.

Step 4: Conclusion The required threshold wavelength and work function are

$$\boxed{\lambda_0 = \frac{\lambda_1\lambda_2}{2\lambda_2 - \lambda_1}}, \quad \boxed{\phi = hc\left(\frac{2}{\lambda_1} - \frac{1}{\lambda_2}\right)}.$$

Quick Tip to Solve: Double the whole expression $hc/\lambda_1 - \phi$, keeping the work function fixed in the second equation. A positive denominator checks the resulting threshold wavelength.

33. (a) Differentiate between half-life and average life of a radioactive substance.

(b) A radioactive substance decays for an interval of time equal to its mean life. Find the fraction of the amount of the substance which is left undecayed after this time interval. [3 marks]

Solution:

Concept:

- Radioactive decay is a random process with a constant decay probability per unit time, the decay constant λ . The number of undecayed nuclei is $N(t) = N_0 e^{-\lambda t}$, where N_0 is the initial number.
- Half-life measures the time for an ensemble to decrease to half its initial size, whereas mean life is the average survival time of its nuclei. Both depend on λ , but they are different quantities.

Step 1: (a) Define and obtain the half-life The half-life $T_{1/2}$ of a radioactive substance is the time in which half the original radioactive nuclei decay, so that half remain undecayed. Applying the decay law gives

$$\frac{N_0}{2} = N_0 e^{-\lambda T_{1/2}} \implies T_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda}.$$

It is independent of the initial amount: each successive half-life reduces the remaining amount by a further factor of two.

Step 2: (a) Define and obtain the average life The average life, or mean life, τ is the arithmetic average of the lifetimes of all the radioactive nuclei in a large initial sample. The fraction decaying

between t and $t + dt$ is $\lambda e^{-\lambda t} dt$; hence

$$\tau = \int_0^{\infty} t \lambda e^{-\lambda t} dt = \frac{1}{\lambda}.$$

Thus $T_{1/2} = (\ln 2)\tau$, or $\tau \simeq 1.443T_{1/2}$. Mean life is longer than half-life; it is not the time by which all nuclei must have decayed.

Step 3: (b) Evaluate the undecayed fraction after one mean life Set the elapsed time equal to $\tau = 1/\lambda$ in the exponential decay law. Then

$$\frac{N(\tau)}{N_0} = e^{-\lambda\tau} = e^{-1}.$$

Since the amount of the original radioactive substance is proportional to the number of its undecayed nuclei, this is also the required fraction of the original amount. It is the surviving fraction; the decayed fraction would instead be $1 - e^{-1}$.

Step 4: Conclusion Half-life is $T_{1/2} = 0.693/\lambda$, while average life is $\tau = 1/\lambda$. After one mean life, the fraction left undecayed is $1/e \simeq 0.368$, approximately 36.8%.

Quick Tip to Solve: At one half-life the surviving fraction is $1/2$; at one mean life it is $1/e$. Substitute the correct time before evaluating the exponential.

34. What is the function of a solar cell? Briefly explain its working and draw its I-V characteristic curve. [3 marks]

Solution:

Concept:

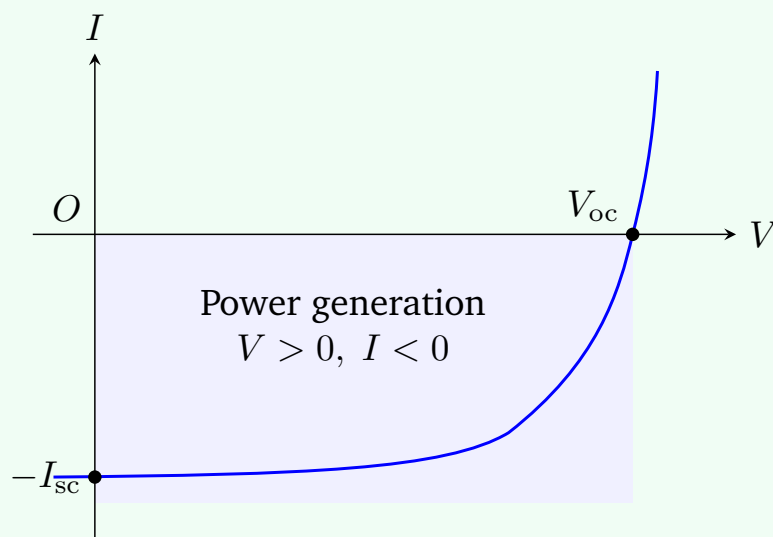
- A solar cell is a semiconductor p - n junction device that converts incident solar energy directly into electrical energy by the photovoltaic effect, without an externally applied bias.
- Photons with energy $h\nu \geq E_g$, where E_g is the semiconductor band gap, generate electron-hole pairs. The built-in electric field of the depletion region separates these charge carriers.
- With diode forward current taken as positive, an illuminated cell has approximately $I = I_0(e^{qV/(nkT)} - 1) - I_{ph}$. Here I_0 is the dark saturation current, I_{ph} the photocurrent, and n the diode ideality factor.

Step 1: State the function and explain carrier generation The function of a solar cell is to supply electrical power from sunlight. A large-area junction with a thin illuminated surface layer allows light to reach the junction region. Photons of sufficient energy are absorbed and create electron-hole pairs in or near the depletion region. A metal contact grid collects charge while allowing light to enter; the opposite surface has a collecting contact.

Step 2: Explain separation and delivery of power The depletion-region field points from the positively charged donor ions on the n side towards the negatively charged acceptor ions on the p side. It drives electrons towards the n side and holes towards the p side. The charge separation makes the p contact positive relative to the n contact and produces a photovoltage. When a load connects the contacts, conventional current flows through the load from the p contact to the n contact, delivering electrical energy. Continuous illumination maintains the generation and separation of carriers.

Step 3: Describe the illuminated I-V characteristic In the graph, V is the p -contact potential relative to the n contact, and positive I follows the forward-diode convention. At zero voltage the current

is $-I_{sc}$, where I_{sc} is the magnitude of the short-circuit current. At zero current the voltage is V_{oc} , the open-circuit voltage. Between these intercepts, $V > 0$ and $I < 0$: the operating region lies in the fourth quadrant and the cell delivers power. The curve is nearly horizontal initially, then bends upwards towards V_{oc} . The shaded region identifies this useful operating quadrant, rather than a region requiring external reverse bias.



Illuminated solar-cell characteristic, using positive forward-diode current.

Step 4: Conclusion A solar cell converts sunlight into electrical power by generating and separating electron-hole pairs at an illuminated p - n junction. Its illuminated I - V curve passes through $(0, -I_{sc})$ and $(V_{oc}, 0)$, with power generation between them in the fourth quadrant under the diode-current convention.

Quick Tip to Solve: Label both intercepts and state the current convention: a solar cell delivers power at positive terminal voltage and negative diode current.

Section D (15 marks)

35. (a) Use Gauss's law to show that due to a uniformly charged spherical shell of radius R , the electric field at any point situated outside the shell at a distance r from its centre is equal to the electric field at the same point, when the entire charge on the shell were concentrated at its centre. Also plot the graph showing the variation of electric field with r , for $r \leq R$ and $r \geq R$.

(b) Two point charges of $+1 \mu\text{C}$ and $+4 \mu\text{C}$ are kept 30 cm apart. How far from the $+1 \mu\text{C}$ charge on the line joining the two charges, will the net electric field be zero? [5 marks]

Solution:

Concept:

- Gauss's law states $\oint_S \vec{E} \cdot d\vec{A} = Q_{enc}/\epsilon_0$, where Q_{enc} is the charge enclosed by the closed Gaussian surface and ϵ_0 is the permittivity of free space.
- For a uniformly charged spherical shell, spherical symmetry makes the electric field radial and its magnitude constant on a concentric Gaussian sphere. Outside the shell the enclosed charge is its total charge Q ; inside it the enclosed charge is zero.
- Fields obey superposition. A positive point charge produces a field directed away from itself, of

magnitude kq/d^2 , where $k = 1/(4\pi\epsilon_0)$ and d is the distance from the charge.

Step 1: (a) Choose a Gaussian sphere outside the shell Let the total uniformly distributed charge on the shell be Q . Choose a concentric spherical Gaussian surface of radius $r > R$. By symmetry, $\vec{E}$ is radial everywhere on this surface and has the same magnitude $E(r)$ at each point. For positive Q , the field is parallel to the outward area vector, so its total outward flux is

$$\oint_S \vec{E} \cdot d\vec{A} = E(r) \oint_S dA = E(r) 4\pi r^2.$$

The entire shell lies inside this Gaussian surface, hence $Q_{\text{enc}} = Q$. Applying Gauss's law gives

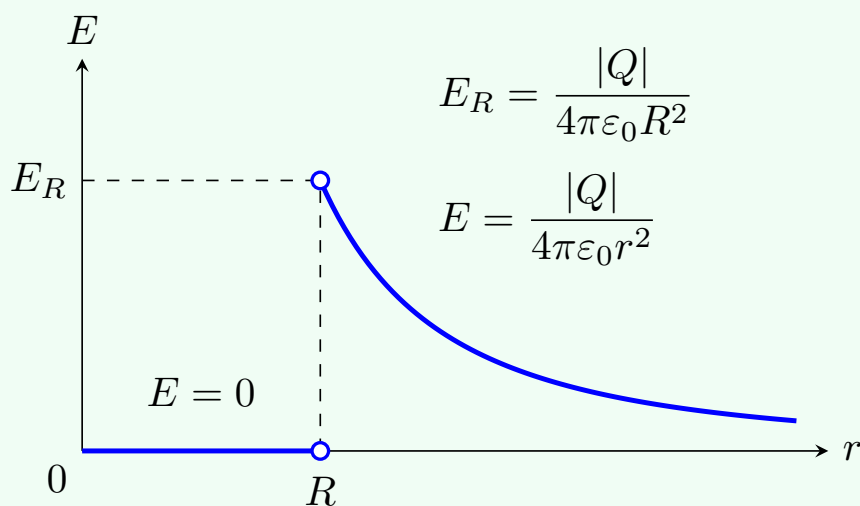
$$E(r) 4\pi r^2 = \frac{Q}{\epsilon_0}, \quad \vec{E}(r) = \frac{Q}{4\pi\epsilon_0 r^2} \hat{r}.$$

This is exactly the field of a point charge Q at the centre: the distance from that point charge to the observation point is also r . A negative shell charge reverses the radial direction; its field magnitude is obtained with $|Q|$.

Step 2: (a) Determine the interior field and draw its variation For a concentric Gaussian sphere with $r < R$, no shell charge is enclosed. Spherical symmetry still allows the constant field magnitude to be taken outside the flux integral. Thus

$$E(r) 4\pi r^2 = 0 \implies E(r) = 0.$$

The field at the centre is also zero by symmetry. Define $E_R = |Q|/(4\pi\epsilon_0 R^2)$. The magnitude is zero throughout the interior, while outside it obeys $E(r) = E_R(R/r)^2$ and tends to zero as r increases without bound. The graph therefore has a jump at the charged surface. For the ideal infinitesimally thin shell, the values at the surface are understood as the two limiting values: $E(R^-) = 0$ and $E(R^+) = E_R$. The printed overlapping intervals at R do not imply that the field is continuous there; the graph explicitly displays the interior and exterior limits.



Electric-field magnitude of a uniformly charged thin spherical shell; circles indicate the limits at its surface.

Step 3: (b) Locate the region where cancellation is possible Place the $+1 \mu\text{C}$ charge at A and the $+4 \mu\text{C}$ charge at B , with $AB = 30 \text{ cm}$. At points outside the segment AB , both fields point in the same direction along the joining line, so cancellation is impossible. Between the charges, the field due to A points towards B , whereas that due to B points towards A . Only in this region can the two equal

and opposite fields give zero resultant. Let the null point be x cm from A , so its distance from B is $(30 - x)$ cm, with $0 < x < 30$.

Step 4: (b) Equate the field magnitudes At the null point, superposition requires equal magnitudes in opposite directions:

$$\frac{k(1 \times 10^{-6})}{x^2} = \frac{k(4 \times 10^{-6})}{(30 - x)^2}.$$

The distances may both be expressed in centimetres here because the common conversion factor cancels in this equality. Cancelling k and the common charge factor gives

$$(30 - x)^2 = 4x^2.$$

Both distances are positive between the charges, so taking the positive square root gives $30 - x = 2x$. Therefore $3x = 30$, giving $x = 10$ cm. The distance from the larger charge is 20 cm, which is twice as large; its fourfold charge is therefore exactly balanced by the fourfold distance-squared denominator.

Step 5: Conclusion (a) The shell's exterior field is $\vec{E} = Q\hat{r}/(4\pi\epsilon_0 r^2)$, identical to that of its entire charge placed at the centre; its interior field is zero. The exterior magnitude falls as $1/r^2$, with a jump from zero to $|Q|/(4\pi\epsilon_0 R^2)$ across the surface. (b) The net electric field vanishes 10 cm from the $+1 \mu\text{C}$ charge, between the charges towards the $+4 \mu\text{C}$ charge.

Quick Tip to Solve: Between two like charges, the null point divides their separation in the ratio of the square roots of their charges: here the distances are 1 : 2, so the smaller charge is one-third of the separation away.

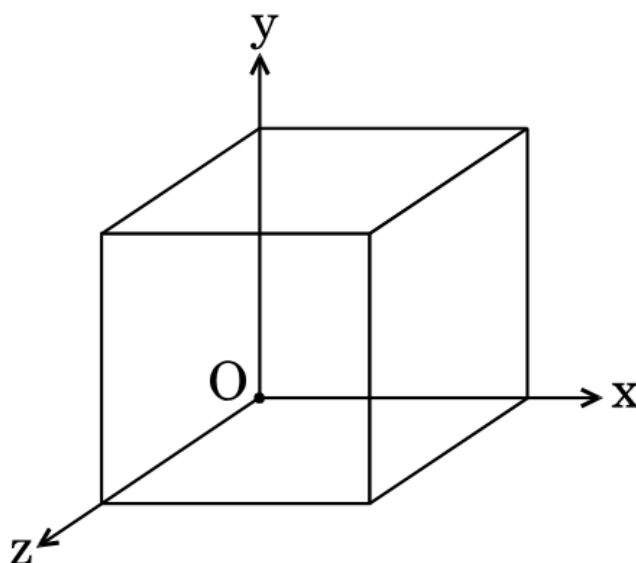
OR

35. (a) Two point charges q_1 and q_2 are kept r distance apart in a uniform external electric field $\vec{E}$. Find the amount of work done in assembling this system of charges.

(b) A cube of side 20 cm is kept in a region as shown in the figure. An electric field $\vec{E}$ exists in the region such that the potential at a point is given by $V = 10x + 5$, where V is in volt and x is in m.

Find the (i) electric field $\vec{E}$, and (ii) total electric flux through the cube.

[5 marks]



Solution:

Concept:

- The work done by an external agent in assembling charges quasistatically equals the increase in their electrostatic potential energy. If $V_{\text{ext}}(\mathbf{r})$ is the external potential, a charge q contributes $qV_{\text{ext}}(\mathbf{r})$.
- Two point charges separated by r have mutual interaction energy $q_1q_2/(4\pi\epsilon_0r)$. Count this interaction once, while including both external-field contributions.
- Electric field is the negative potential gradient, $\mathbf{E} = -\nabla V$; electric flux through a closed surface is $\Phi = \oint \mathbf{E} \cdot d\mathbf{A}$, with $d\mathbf{A}$ pointing outward.

Step 1: (a) Assemble the first charge Let the final positions be $\mathbf{r}_1$ and $\mathbf{r}_2$, with $|\mathbf{r}_2 - \mathbf{r}_1| = r$. Choose a common reference for the external potential and denote its values at the positions by V_1 and V_2 . Bringing q_1 from this reference to $\mathbf{r}_1$, while q_2 is absent, requires external work $W_1 = q_1V_1$. The existing sources of the uniform external field are held fixed during assembly.

Step 2: (a) Assemble the second charge and add the work At $\mathbf{r}_2$, the potential consists of the external potential V_2 plus the potential $q_1/(4\pi\epsilon_0r)$ due to the first charge. Hence

$$W_2 = q_2V_2 + \frac{q_1q_2}{4\pi\epsilon_0r}, \quad W = W_1 + W_2 = q_1V_1 + q_2V_2 + \frac{q_1q_2}{4\pi\epsilon_0r}.$$

For a uniform field and a finite reference point $\mathbf{r}_0$ where the external potential is set to zero, $V_i = -\mathbf{E} \cdot (\mathbf{r}_i - \mathbf{r}_0)$. This also gives the external-field terms explicitly. A truly uniform field extending to infinity has no finite zero-potential reference at infinity, so the chosen reference must be stated; the separation alone cannot determine those terms. Negative work means the external agent extracts energy.

Step 3: (b)(i) Differentiate the given potential Here $V(x, y, z) = 10x + 5$, where x is measured in metres. Thus $\partial V/\partial x = 10 \text{ V m}^{-1}$, and both the y and z derivatives vanish. Therefore

$$\mathbf{E} = -\left(\frac{\partial V}{\partial x}\hat{\mathbf{i}} + \frac{\partial V}{\partial y}\hat{\mathbf{j}} + \frac{\partial V}{\partial z}\hat{\mathbf{k}}\right) = -10\hat{\mathbf{i}} \text{ N C}^{-1}.$$

The constant 5 V changes the potential reference but contributes no electric field. The field points along the negative x -axis, toward decreasing potential.

Step 4: (b)(ii) Add the fluxes through all six faces The face area is $A = (0.20)^2 = 0.040 \text{ m}^2$. Through the face whose outward normal is $+\hat{\mathbf{i}}$, the flux is $-10A = -0.40 \text{ N m}^2\text{C}^{-1}$. Through the opposite face, whose outward normal is $-\hat{\mathbf{i}}$, it is $+0.40 \text{ N m}^2\text{C}^{-1}$. The field is parallel to each of the remaining four faces, giving zero flux through them. Thus the incoming and outgoing fluxes cancel. Equivalently, a spatially constant field has zero divergence and encloses no net charge.

Step 5: Conclusion (a) $W = q_1V_1 + q_2V_2 + q_1q_2/(4\pi\epsilon_0r)$, for a specified external-potential reference. (b)(i) $\mathbf{E} = -10\hat{\mathbf{i}} \text{ N C}^{-1}$; (ii) the total outward flux is $\boxed{0 \text{ N m}^2\text{C}^{-1}}$.

Quick Tip to Solve: In an external field, add both external-potential energies and only one mutual interaction term. For a constant field through a closed cube, opposite-face fluxes cancel.

36. (a) A circular loop of radius R carries a current I . Obtain an expression for the magnetic field at a point on its axis at a distance x from its centre.

(b) A conducting rod of length 2 m is placed on a horizontal table in north-south direction. It carries a current of 5 A from south to north. Find the direction and magnitude of the magnetic force acting on the rod. Given that the Earth's magnetic field at the place is 0.6×10^{-4} T and angle of dip is $\frac{\pi}{6}$. [5 marks]

Solution:

Concept:

- Biot–Savart law gives $d\mathbf{B} = (\mu_0 I / 4\pi) d\ell \times \hat{s} / s^2$, where $d\ell$ follows the current and s joins the current element to the observation point.
- By rotational symmetry, transverse field components of opposite elements of a circular loop cancel on its axis, while axial components add. The right-hand grip rule fixes the axial direction.
- A straight current-carrying rod experiences $\mathbf{F} = I\mathbf{L} \times \mathbf{B}$, where $\mathbf{L}$ points along the current. At dip δ , Earth's field has horizontal component $B \cos \delta$ and vertically downward component $B \sin \delta$.

Step 1: (a) Set up the geometry of an element Let O be the loop centre and P lie on its axis at distance x . Every element of the radius- R loop is at the same distance $s = \sqrt{R^2 + x^2}$ from P . The tangent element $d\ell$ is perpendicular to the line joining that element to P , because this line lies in the plane formed by the local radial direction and the loop axis. Thus Biot–Savart law gives the magnitude

$$dB = \frac{\mu_0 I}{4\pi} \frac{d\ell}{R^2 + x^2}.$$

To add the vectors, resolve each contribution into its axial and transverse parts rather than adding their full magnitudes directly.

Step 2: (a) Resolve the field along the axis The projection factor for the axial component of $d\mathbf{B}$ is R/s . For example, take the axis as the z -direction: $\mathbf{s} = x\hat{\mathbf{z}} - R\hat{\mathbf{r}}$ and $d\ell = d\ell\hat{\phi}$, so the axial part of $d\ell \times \mathbf{s}$ has magnitude $R d\ell$. Therefore

$$dB_{\text{axis}} = dB \frac{R}{s} = \frac{\mu_0 I R d\ell}{4\pi (R^2 + x^2)^{3/2}}.$$

Diametrically opposite elements have equal and opposite transverse components. Those components cancel around the whole loop, leaving a resultant entirely along the axis.

Step 3: (a) Integrate around the loop Since R , I and x are constant throughout the integral, and the circumference is $\oint d\ell = 2\pi R$,

$$B_{\text{axis}} = \frac{\mu_0 I R}{4\pi (R^2 + x^2)^{3/2}} \oint d\ell = \frac{\mu_0 I R^2}{2(R^2 + x^2)^{3/2}}.$$

The direction is the loop's magnetic-moment direction: curl the fingers of the right hand along the current and the thumb gives the axial direction. At $x = 0$, the formula reduces to $B = \mu_0 I / (2R)$, providing a check on the derivation.

Step 4: (b) Identify which component of Earth's field acts on the rod The current is northward along the horizontal rod. Earth's horizontal field points north and is parallel to the rod, so it produces no force. Its downward vertical component is perpendicular to the rod and equals

$$B_v = B \sin \delta = (0.6 \times 10^{-4}) \sin \frac{\pi}{6} = 3.0 \times 10^{-5} \text{ T}.$$

This is also why using the total field as though it were perpendicular to the rod would overestimate the force.

Step 5: (b) Calculate the magnitude and direction of force Substitute $I = 5 \text{ A}$, $L = 2 \text{ m}$ and the perpendicular component B_v :

$$F = ILB_v = 5 \times 2 \times 3.0 \times 10^{-5} = 3.0 \times 10^{-4} \text{ N}.$$

To establish the direction explicitly, choose east, north and upward as $\hat{i}, \hat{j}, \hat{k}$. Then $\mathbf{L} = L\hat{j}$ and $\mathbf{B} = B \cos \delta \hat{j} - B \sin \delta \hat{k}$. Consequently $\mathbf{F} = -ILB \sin \delta \hat{i}$, because $\hat{j} \times \hat{k} = \hat{i}$. The negative east direction is west.

Step 6: Conclusion (a) The magnetic field on the loop axis is $B = \mu_0 IR^2 / [2(R^2 + x^2)^{3/2}]$, directed along the axis by the right-hand grip rule. (b) The force on the rod is $3.0 \times 10^{-4} \text{ N}$, horizontally westward.

Quick Tip to Solve: On a loop axis, integrate only the axial component. For a northward horizontal rod in Earth's field, use the downward component $B \sin \delta$ and then apply the cross product for direction.

OR

36. (a) Obtain the expression for the deflecting torque acting on the current carrying rectangular coil of a galvanometer in a uniform magnetic field. Why is a radial magnetic field employed in the moving coil galvanometer?

(b) Particles of mass $1.6 \times 10^{-27} \text{ kg}$ and charge $1.6 \times 10^{-19} \text{ C}$ are accelerated in a cyclotron of dee radius 40 cm. It employs a magnetic field 0.4 T. Find the kinetic energy (in MeV) of the particle beam imparted by the accelerator. [5 marks]

Solution:

Concept:

- The magnetic force on a straight wire of length l , carrying current I , is $\mathbf{F} = I\mathbf{l} \times \mathbf{B}$. A pair of equal and opposite forces with separated lines of action forms a couple.
- A coil of N turns and area A has magnetic moment $\boldsymbol{\mu} = NIA \hat{n}$, where $\hat{n}$ is its normal. Its torque is $\boldsymbol{\tau} = \boldsymbol{\mu} \times \mathbf{B}$.
- In a cyclotron, the magnetic force provides centripetal force: $qvB = mv^2/r$. The magnetic field bends the path; the alternating electric field across the gap between the dees supplies the kinetic energy.

Step 1: (a) Identify the forces on the rectangular coil Let a rectangular coil have N turns, side lengths l and b , and area $A = lb$. Let θ be the angle between the normal to its plane and the uniform magnetic field $\mathbf{B}$. Choose the pair of sides of length l perpendicular to $\mathbf{B}$; the rotation axis is parallel to these sides. The current directions in these opposite sides are opposite, so their magnetic forces have equal magnitude $F = NILB$ and opposite directions. Their resultant force is zero, but they form a couple. The forces on the other pair of sides do not contribute torque about this rotation axis.

Step 2: (a) Obtain the deflecting torque The perpendicular distance between the lines of action of

the two forces is $b \sin \theta$. Hence the magnitude of the torque of the couple is

$$\begin{aligned}\tau_d &= F(b \sin \theta) \\ &= NIlB b \sin \theta \\ &= NIAB \sin \theta.\end{aligned}$$

Thus $\tau_d = \mu \times \mathbf{B}$; its direction tends to align the coil's magnetic moment with the field. The factor N accounts for all turns, while for a single turn the result is $IAB \sin \theta$.

Step 3: (a) Explain the purpose of the radial magnetic field Shaped pole pieces and a soft-iron cylindrical core produce a radial field in the region occupied by the coil. As the coil rotates, the magnetic field remains parallel to the plane of the coil, so its normal remains perpendicular to the field: $\theta = 90^\circ$. Therefore $\sin \theta = 1$ at every deflection and $\tau_d = NIAB$. If the suspension has torsional constant k and angular deflection ϕ , the restoring torque is $k\phi$. At equilibrium, $k\phi = NIAB$, giving $\phi = (NAB/k)I$. Thus deflection is directly proportional to current, giving a uniform current scale and a torque that has no varying angular factor.

Step 4: (b) Find the limiting speed and kinetic energy in the cyclotron At the outer edge of a dee, the orbit radius is $r = 0.40$ m. Using the magnitude of the charge, the circular-motion equation gives

$$v = \frac{qBr}{m} = \frac{(1.6 \times 10^{-19})(0.4)(0.40)}{1.6 \times 10^{-27}} = 1.6 \times 10^7 \text{ m s}^{-1}.$$

The nonrelativistic kinetic energy imparted at this radius is

$$\begin{aligned}K &= \frac{1}{2}mv^2 = \frac{q^2 B^2 r^2}{2m} \\ &= \frac{1}{2}(1.6 \times 10^{-19})(1.6 \times 10^7)^2 \\ &= 2.048 \times 10^{-13} \text{ J}.\end{aligned}$$

Since $1 \text{ MeV} = 1.6 \times 10^{-13} \text{ J}$,

$$K = \frac{2.048 \times 10^{-13}}{1.6 \times 10^{-13}} = 1.28 \text{ MeV}.$$

The speed is much smaller than the speed of light, consistent with the classical cyclotron formula used here.

Step 5: Conclusion (a) The deflecting torque is $\tau_d = NIAB \sin \theta$, where θ is the angle between the coil normal and the field. A radial field keeps $\theta = 90^\circ$, so $\tau_d = NIAB$ and galvanometer deflection is proportional to current. (b) The particle beam has kinetic energy 1.28 MeV .

Quick Tip to Solve: Define the torque angle using the coil normal, and convert the dee radius from centimetres to metres before using $K = q^2 B^2 r^2 / (2m)$.

37. (a) Derive lens maker's formula for a biconvex lens.

(b) A point object is placed at a distance of 12 cm on the principal axis of a convex lens of focal length 10 cm. A convex mirror is placed coaxially on the other side of the lens at a distance of 10 cm. If the final image coincides with the object, sketch the ray diagram and find the focal length of the convex mirror.

[5 marks]

Solution:

Concept:

- For refraction at a spherical surface, $\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$, where the distances are measured from the surface pole using the Cartesian sign convention.
- For a thin lens in air, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$. Its two refracting surfaces act successively; the image formed by the first surface is the object for the second.
- Optical paths are reversible. A ray incident normally on a spherical mirror retraces its path; all such rays are directed towards its centre of curvature, and $f_m = R_m/2$.

Step 1: (a) Refraction at the first surface Consider a thin biconvex lens of refractive index μ in air. Light travels from left to right. Let the signed radii of its first and second surfaces be R_1 and R_2 , respectively. For a biconvex lens, $R_1 > 0$ and $R_2 < 0$. Let u be the object distance and v_1 the intermediate image distance produced by the first surface. Applying spherical refraction from air to glass gives

$$\frac{\mu}{v_1} - \frac{1}{u} = \frac{\mu - 1}{R_1}. \quad (1)$$

This relation assumes paraxial rays, so the small-angle approximation applies.

Step 2: (a) Refraction at the second surface and the focal length The intermediate image is the object for refraction from glass to air at the second surface. Since the lens is thin, the separation of its surfaces is neglected and the intermediate distance is still v_1 , with its algebraic sign retained. If v is the final image distance,

$$\frac{1}{v} - \frac{\mu}{v_1} = \frac{1 - \mu}{R_2}. \quad (2)$$

Adding equations (1) and (2) eliminates v_1 :

$$\frac{1}{v} - \frac{1}{u} = (\mu - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right).$$

For a distant object, $u \rightarrow -\infty$ and $v = f$. Hence the lens maker's formula in air is

$$\boxed{\frac{1}{f} = (\mu - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)}.$$

The opposite signs of the radii make the right-hand side positive for a biconvex glass lens in air. The assumptions are a thin lens, spherical refracting surfaces, a homogeneous lens material and paraxial rays.

Step 3: (b) Find where the lens alone would form the image Take the optical centre of the lens as the origin, with the positive direction along the incident light. The point object is at $u = -12$ cm and the

convex lens has $f = +10$ cm. Thus

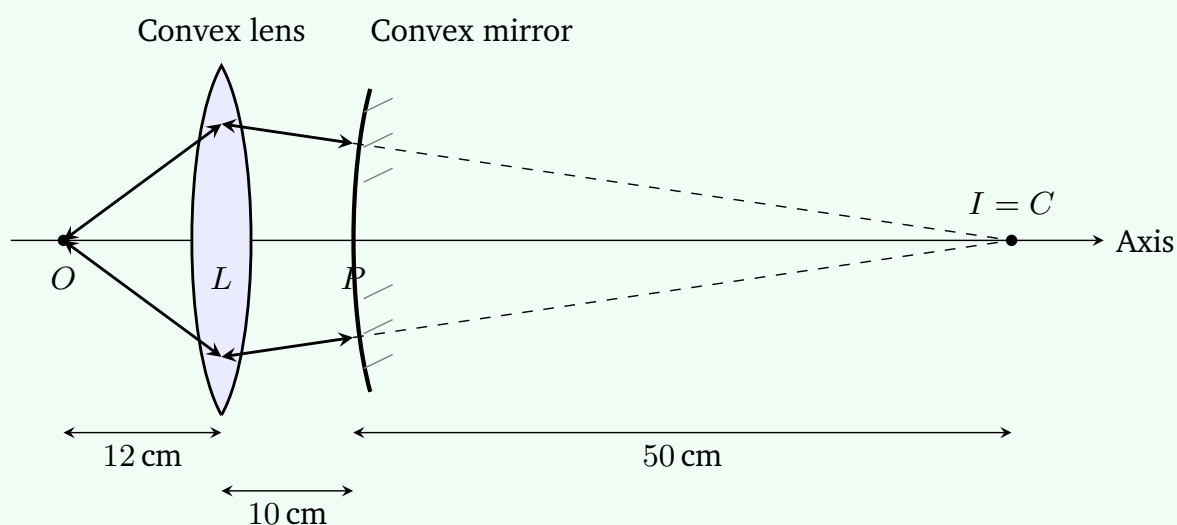
$$\begin{aligned}\frac{1}{v} - \frac{1}{-12} &= \frac{1}{10}, \\ \frac{1}{v} &= \frac{1}{10} - \frac{1}{12} = \frac{1}{60}, \\ v &= +60 \text{ cm}.\end{aligned}$$

The rays emerging from the lens would converge at a point I , 60 cm to its right. The mirror intercepts them at 10 cm from the lens, so I lies 50 cm behind the mirror and acts as its virtual object.

Step 4: (b) Apply the image-coincidence condition For the final image to coincide with the original object O , the reflected rays must return through the lens along the original paths. Reversibility then requires the mirror to send the rays back along the same converging lines. These lines are normal to the mirror when their intersection I is the mirror's centre of curvature C . Therefore,

$$R_m = LC - LP = 60 - 10 = 50 \text{ cm}, \quad f_m = \frac{R_m}{2} = 25 \text{ cm}.$$

Equivalently, using the mirror convention with the incident direction positive, its virtual object and virtual image are both behind the mirror: $u_m = v_m = +50$ cm. The mirror formula gives $1/f_m = 1/50 + 1/50$, confirming $f_m = +25$ cm. The sketch shows the incident and reflected paths as coincident lines with arrows in both directions. Dotted extensions meet at $I = C$; light does not reach that point because the mirror reflects it first.



Lens-mirror arrangement (schematic): rays retrace their paths because their extensions meet at the mirror's centre C .

Step 5: Conclusion (a) For a thin biconvex lens in air, $\boxed{1/f = (\mu - 1)(1/R_1 - 1/R_2)}$, with $R_1 > 0$ and $R_2 < 0$. (b) The convex mirror has focal length $\boxed{f_m = +25 \text{ cm}}$; its centre of curvature is 50 cm behind the mirror, ensuring that the final image coincides with the object.

Quick Tip to Solve: First locate the image that the lens alone would form. For retracing rays, this point is the mirror's centre of curvature, so subtract the lens-mirror spacing before halving the distance.

37. (a) What is a wavefront? How does it propagate? Using Huygens' principle, explain reflection of a plane wavefront from a surface and verify the laws of reflection.

(b) A parallel beam of light of wavelength 500 nm falls on a narrow slit and the resulting diffraction pattern is obtained on a screen 1 m away. If the first minimum is formed at a distance of 2.5 mm from the centre of the screen, find the (i) width of the slit, and (ii) distance of first secondary maximum from the centre of the screen. [5 marks]

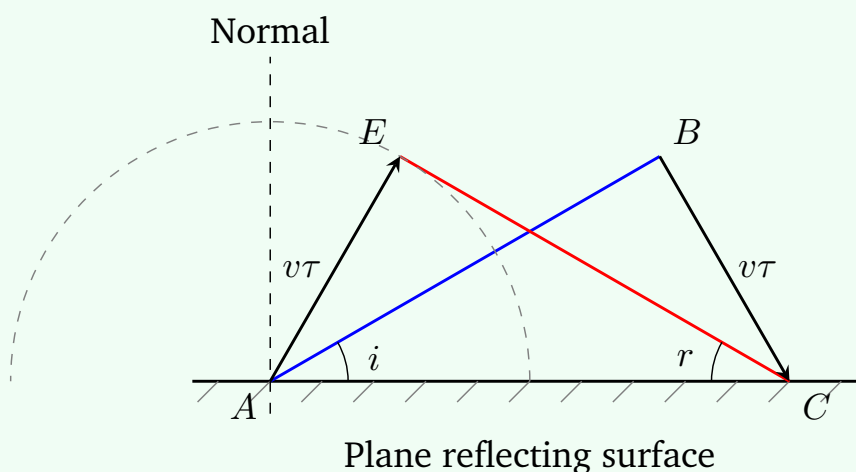
Solution:

Concept:

- A wavefront is a surface joining points of a wave that are in the same phase. Rays in an isotropic medium are normal to the wavefront.
- Huygens' principle: every point of a wavefront acts as a source of secondary wavelets. The forward envelope of these wavelets after a time interval is the new wavefront.
- For a slit of width a , diffraction minima satisfy $a \sin \theta = n\lambda$, where λ is the wavelength and $n = 1, 2, \dots$. For small angles, the screen displacement is $y \simeq D \sin \theta$, with screen distance D .

Step 1: (a) Define the wavefront and describe its propagation A wavefront is the locus of all points oscillating in the same phase at a given instant. A point source produces a spherical wavefront; a sufficiently distant source produces an approximately plane wavefront. In a homogeneous isotropic medium, the wavefront advances along its normal at the speed v of light in that medium. After time τ , each secondary wavelet has radius $v\tau$; the common forward envelope gives the wavefront's new position. The backward envelope is not the physical advancing wavefront.

Step 2: Construct the reflected wavefront using secondary wavelets Let AB be a plane wavefront incident obliquely on the plane reflecting surface AC . Point A reaches the surface first. During time τ , the disturbance at B travels along the incident ray to C , so $BC = v\tau$. Meanwhile, the reflected secondary wavelet from A grows to radius $AE = v\tau$. Draw the tangent CE from C to this wavelet. It is the reflected plane wavefront, since it is also the common envelope of the reflected wavelets from the points between A and C . The incident ray BC is perpendicular to AB , and the reflected ray AE is perpendicular to CE .



Huygens' construction: $BC = AE = v\tau$; AB is the incident wavefront and CE is its reflected wavefront.

Step 3: Verify both laws of reflection Triangles ABC and AEC are right triangles, with common hypotenuse AC and equal sides $BC = AE = v\tau$. They are therefore congruent. Let the incident and

reflected rays make angles i and r with the normal to the surface. By perpendicularity, $\angle BAC = i$ and $\angle ACE = r$. Hence

$$\sin i = \frac{BC}{AC} = \frac{AE}{AC} = \sin r.$$

Since both angles lie between 0 and 90° , $i = r$. This is the first law of reflection. The construction lies in the plane containing the incident ray and the surface normal; the reflected ray, normal to CE in that plane, lies in it as well. Thus the incident ray, reflected ray and normal are coplanar, establishing the second law.

Step 4: (b)(i) Calculate the slit width from the first minimum Convert the given quantities to SI units:

$$\lambda = 500 \times 10^{-9} \text{ m}, \quad D = 1 \text{ m}, \quad y_1 = 2.5 \times 10^{-3} \text{ m}.$$

The small-angle approximation is justified because $y_1/D = 0.0025 \ll 1$. At the first minimum, $a \sin \theta_1 = \lambda$, so

$$a \frac{y_1}{D} = \lambda, \quad a = \frac{\lambda D}{y_1} = \frac{500 \times 10^{-9} \times 1}{2.5 \times 10^{-3}} = 2.0 \times 10^{-4} \text{ m}.$$

Step 5: (b)(ii) Locate the first secondary maximum In the usual school-level approximation, a secondary maximum lies approximately midway between its neighbouring minima. The first secondary maximum is between the first and second minima, giving

$$a \sin \theta_{s1} \simeq \frac{3\lambda}{2}, \quad y_{s1} \simeq \frac{3\lambda D}{2a} = \frac{3}{2} y_1 = \frac{3}{2} (2.5 \text{ mm}) = 3.75 \text{ mm}.$$

This distance is measured from the centre on either side of the central maximum. The midpoint expression is an approximation: the exact intensity function $I \propto (\sin \beta / \beta)^2$, with $\beta = \pi a \sin \theta / \lambda$, gives $\tan \beta = \beta$. Its first nonzero maximum occurs at $\beta \simeq 4.493$, corresponding to approximately 3.58 mm; the standard approximate result requested here is 3.75 mm.

Step 6: Conclusion Huygens' construction gives equal angles $i = r$ and places the incident ray, reflected ray and normal in one plane. The slit width is $\boxed{0.20 \text{ mm}}$, and the first secondary maximum is approximately $\boxed{3.75 \text{ mm}}$ from the centre on either side, using the usual midpoint approximation.

Quick Tip to Solve: Use the first minimum to find $a = \lambda D / y_1$; then the usual approximate first-secondary-maximum distance is $1.5y_1$, without repeating the substitutions.