

CBSE Class 12 Physics Question Paper 2020 with Solution PDF-Set 55/2/2



Question Paper with Solutions

Conducted by the Central Board of Secondary Education (CBSE)

Time Allowed: 3 hours

Maximum Marks: 70

General Instructions

- (i) This question paper comprises **four** Sections – **A, B, C** and **D**.
- (ii) There are **37** questions in the question paper. **All** questions are compulsory.
- (iii) **Section A** – Questions no. **1 to 20** are very short answer type questions, carrying **1** mark each.
- (iv) **Section B** – Questions no. **21 to 27** are short answer type questions, carrying **2** marks each.
- (v) **Section C** – Questions no. **28 to 34** are long answer type questions, carrying **3** marks each.
- (vi) **Section D** – Questions no. **35 to 37** are also long answer type questions, carrying **5** marks each.
- (vii) There is no overall choice in the question paper. However, an internal choice has been provided in **2** questions of **1** mark, **2** questions of **2** marks, **1** question of three marks and all the **3** questions of five marks. You have to attempt only one of the choices in such questions.
- (viii) In addition to this, separate instructions are given with each section and question, wherever necessary.
- (ix) Use of calculators and log tables is **not** permitted.

You may use the following values of physical constants wherever necessary :

$$c = 3 \times 10^8 \text{ m/s}$$

$$h = 6.63 \times 10^{-34} \text{ Js}$$

$$e = 1.6 \times 10^{-19} \text{ C}$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ T m A}^{-1}$$

$$\epsilon_0 = 8.854 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$$

$$\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$$

$$\text{Mass of electron } (m_e) = 9.1 \times 10^{-31} \text{ kg}$$

$$\text{Mass of neutron} = 1.675 \times 10^{-27} \text{ kg}$$

$$\text{Mass of proton} = 1.673 \times 10^{-27} \text{ kg}$$

$$\text{Avogadro's number} = 6.023 \times 10^{23} \text{ per gram mole}$$

$$\text{Boltzmann constant} = 1.38 \times 10^{-23} \text{ JK}^{-1}$$

Section A (20 marks)

Note : Select the most appropriate option from those given below each question :

1. Resistivity of a given conductor depends upon

[1 mark]

- (A) temperature.
- (B) length of conductor.
- (C) area of cross-section.
- (D) shape of the conductor.

Correct Answer: (A) temperature.

Solution:

Concept:

- Resistivity ρ is a property of the material at a specified temperature.
- Resistance is $R = \rho L/A$, where L is length and A is cross-sectional area.

Step 1: Separate material property from geometry For a given conducting material, changing length or area changes resistance. It does not change the intrinsic resistivity: rearranging gives $\rho = RA/L$. Thus options (B) and (C) describe geometrical factors in resistance, not independent causes of a change in resistivity.

Step 2: Identify the relevant physical condition Temperature affects the microscopic scattering of charge carriers and therefore the resistivity of a conductor. Changing its shape likewise alters its geometrical resistance without changing the material property at the same temperature. Hence option (D) also fails.

Step 3: Conclusion Resistivity depends upon temperature. The correct option is (A).

Quick Tip to Solve: Use $R = \rho L/A$ to distinguish changes in resistance from changes in resistivity.

2. The ratio of current density and electric field is called

[1 mark]

- (A) Resistivity
- (B) Conductivity
- (C) Drift velocity
- (D) Mobility

Correct Answer: (B) Conductivity

Solution:

Concept:

- The local form of Ohm's law is $\mathbf{J} = \sigma \mathbf{E}$, with current density $\mathbf{J}$, electric field $\mathbf{E}$, and conductivity σ .
- Resistivity is $\rho = 1/\sigma$, while mobility is $\mu = v_d/E$.

Step 1: Use the local electrical relation For an isotropic ohmic conductor, current density and electric field are parallel. Taking their magnitudes gives $J = \sigma E$, so the ratio asked for is $J/E = \sigma$. Its SI unit is S m^{-1} .

Step 2: Check the other choices Resistivity is the inverse ratio E/J , so (A) is incorrect. Drift velocity is the average directed speed of carriers, not current density divided by field, excluding (C). Mobility

is drift velocity divided by field, excluding (D).

Step 3: Conclusion The ratio is conductivity. The correct option is (B).

Quick Tip to Solve: Remember the order: $J/E = \sigma$, whereas $E/J = \rho$.

3. In β^- decay, a

[1 mark]

- (A) neutron converts into a proton emitting antineutrino.
- (B) neutron converts into a proton emitting neutrino.
- (C) proton converts into a neutron emitting antineutrino.
- (D) proton converts into a neutron emitting neutrino.

Correct Answer: (A) neutron converts into a proton emitting antineutrino.

Solution:

Concept:

- In beta-minus decay, a neutron changes into a proton and emits an electron and an electron antineutrino.
- The nuclear mass number stays unchanged; the atomic number increases by one.

Step 1: Write the elementary transformation The process is

$$n \longrightarrow p + e^- + \bar{\nu}_e.$$

The emitted electron is the beta-minus particle. Charge is conserved because the proton has charge $+e$ and the electron has charge $-e$.

Step 2: Distinguish neutrino and nucleon alternatives The electron is accompanied by an antineutrino, which conserves electron lepton number. Option (B) gives the wrong neutral particle. Options (C) and (D) start with a proton changing into a neutron, so neither describes beta-minus decay. The omission of the electron from the short option does not negate the stated neutron-to-proton transformation.

Step 3: Conclusion A neutron converts into a proton, emitting an antineutrino along with the beta-minus electron. The correct option is (A).

Quick Tip to Solve: For beta-minus emission, remember $n \rightarrow p + e^- + \bar{\nu}_e$: atomic number rises by one.

4. The material which is not suitable for making a permanent magnet is

[1 mark]

- (A) Steel
- (B) Ticonal
- (C) Lead
- (D) Alnico

Correct Answer: (C) Lead

Solution:

Concept:

- A permanent magnet requires high retentivity so that substantial magnetisation remains after the external field is removed.
- High coercivity helps it resist demagnetisation; hard ferromagnetic materials are suitable.

Step 1: Apply the material requirements A permanent magnet must retain its magnetic state for long periods. Steel and the magnetic alloys Ticonal and Alnico have properties suited to this purpose, so options (A), (B) and (D) are usable permanent-magnet materials.

Step 2: Examine lead Lead is diamagnetic. Its weak induced magnetisation opposes an applied field and does not provide the strong remanent magnetisation needed for a permanent magnet. It therefore fails the basic material requirement, unlike the three hard magnetic choices.

Step 3: Conclusion Lead is not suitable for making a permanent magnet. The correct option is (C).

Quick Tip to Solve: For permanent magnets seek high retentivity and coercivity; exclude diamagnetic lead.

5. A welder wears special glasses to protect his eyes mostly from the harmful effect of

[1 mark]

- (A) very intense visible light.
- (B) infrared radiation.
- (C) ultraviolet rays.
- (D) microwaves.

Correct Answer: (C) ultraviolet rays.

Solution:

Concept:

- An electric welding arc emits ultraviolet radiation as well as strong visible light and infrared radiation.
- Ultraviolet radiation can injure the exposed surface of the eye; protective welding filters block it.

Step 1: Identify the principal harmful radiation The radiation associated with the characteristic eye injury from a welding arc is ultraviolet radiation. A welding shield or specially filtered glasses protect the eyes by strongly attenuating this component of the arc emission.

Step 2: Compare all the listed choices Intense visible light produces glare and infrared radiation

has heating effects, so proper protection also reduces these components. However, the question asks mostly for the harmful radiation emphasised in this application: ultraviolet rays. Microwaves are not the relevant damaging emission of a welding arc.

Step 3: Conclusion The intended radiation is ultraviolet rays. The correct option is (C).

Quick Tip to Solve: Associate welding-arc eye protection with ultraviolet radiation, even though the arc also emits visible and infrared light.

6. In an n-type semiconductor, the donor energy level lies

[1 mark]

- (A) at the centre of the energy gap.
- (B) just below the conduction band.
- (C) just above the valence band.
- (D) in the conduction band.

Correct Answer: (B) just below the conduction band.

Solution:

Concept:

- Pentavalent donor impurities supply electrons to an n-type semiconductor.
- The donor level E_D lies slightly below the conduction-band edge E_C , giving a small ionisation energy $E_C - E_D$.

Step 1: Locate the shallow donor state Four of the donor atom's valence electrons form covalent bonds; the extra electron is weakly bound. Its allowed energy is inside the forbidden gap, close to the conduction band. A small amount of thermal energy can lift it into that band.

Step 2: Exclude the other locations A central-gap level would require much greater excitation energy and is not the usual shallow donor level, excluding (A). A level just above the valence band is characteristic of an acceptor, excluding (C). The bound donor state lies below the conduction band, not already within it, excluding (D).

Step 3: Conclusion The donor energy level lies just below the conduction band. The correct option is (B).

Quick Tip to Solve: Donors donate electrons upward into the nearby conduction band; acceptors lie near the valence band.

7. When two nuclei ($A \leq 10$) fuse together to form a heavier nucleus, the

[1 mark]

- (A) binding energy per nucleon increases.
- (B) binding energy per nucleon decreases.
- (C) binding energy per nucleon does not change.
- (D) total binding energy decreases.

Correct Answer: (A) binding energy per nucleon increases.

Solution:

Concept:

- Binding energy per nucleon measures the average energy binding each nucleon and indicates nuclear stability.
- For light nuclei, the binding-energy-per-nucleon curve rises towards the intermediate-mass region.

Step 1: Compare the nuclei on the binding-energy curve The question concerns initially very light nuclei with $A \leq 10$. Fusion forms a heavier nucleus in the rising region of the binding-energy-per-nucleon curve. Its nucleons are more tightly bound on average than in the initial light nuclei.

Step 2: Relate the change to released energy The total number of nucleons is conserved in the fusion reaction. An increase in their average binding energy means the final total binding energy is greater, and the corresponding decrease in mass appears as released energy. Thus (B) and (C) contradict the rising curve, and (D) gives the wrong direction for total binding energy.

Step 3: Conclusion The binding energy per nucleon increases. The correct option is (A).

Quick Tip to Solve: For fusion of very light nuclei, move right along the rising portion of the binding-energy-per-nucleon curve.

8. Electromagnetic waves used as a diagnostic tool in medicine are

[1 mark]

- (A) X-rays.
- (B) ultraviolet rays.
- (C) infrared radiation.
- (D) ultrasonic waves.

Correct Answer: (A) X-rays.

Solution:

Concept:

- X-rays are electromagnetic radiation with high penetrating power.
- Different absorption by bone and soft tissue makes X-rays useful for diagnostic images.

Step 1: Connect penetration with medical imaging X-rays pass through the body to different extents depending on tissue composition and thickness. Bone attenuates them more strongly than many soft tissues, producing contrast on a detector and enabling radiographic diagnosis.

Step 2: Examine the alternatives Ultraviolet radiation is strongly absorbed near the surface and does not provide the usual internal radiographic image. Infrared radiation is associated mainly with thermal information rather than this standard diagnostic application. Ultrasound is indeed used in medicine, but ultrasonic waves are mechanical sound waves and therefore are not electromagnetic waves.

Step 3: Conclusion X-rays are the electromagnetic waves intended here. The correct option is (A).

Quick Tip to Solve: In this list, choose penetrating X-rays; ultrasound is diagnostic but is a mechanical wave.

9. At equilibrium, in a p-n junction diode the net current is

[1 mark]

- (A) due to diffusion of majority charge carriers.
- (B) due to drift of minority charge carriers.
- (C) zero as diffusion and drift currents are equal and opposite.
- (D) zero as no charge carriers cross the junction.

Correct Answer: (C) zero as diffusion and drift currents are equal and opposite.

Solution:

Concept:

- Diffusion carries majority carriers across a junction because of their concentration gradient.
- The depletion-layer electric field produces a drift current opposite to the diffusion current.

Step 1: Identify the two currents Electrons diffuse from the n-side to the p-side and holes diffuse in the reverse direction. The depletion-region field also drives minority carriers, producing drift current. Thus charge carriers continue to cross the junction even at equilibrium.

Step 2: Apply the equilibrium condition At thermal equilibrium the two current magnitudes are equal: $I_{\text{diff}} + I_{\text{drift}} = 0$, with signs denoting opposite directions. Options (A) and (B) describe individual contributions rather than the net current. Option (D) incorrectly denies the continuing carrier motion.

Step 3: Conclusion The net current is zero because diffusion and drift currents are equal and opposite. Hence, the correct option is (C).

Quick Tip to Solve: Equilibrium cancels opposing currents; it does not stop individual carriers crossing the junction.

10. An electron is released from rest in a region of uniform electric and magnetic fields acting parallel to each other. The electron will

[1 mark]

- (A) move in a straight line.
- (B) move in a circle.
- (C) remain stationary.
- (D) move in a helical path.

Correct Answer: (A) move in a straight line.

Solution:

Concept:

- The Lorentz force is $\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$, where $q = -e$ for an electron.
- Magnetic force is zero when velocity is parallel or antiparallel to the magnetic field.

Step 1: Find the initial force The electron starts from rest, so initially $\mathbf{v} \times \mathbf{B} = 0$. Its electric force is $-e\mathbf{E}$, and hence it accelerates opposite to the electric field. Since the two fields are parallel, this direction lies along the magnetic-field axis.

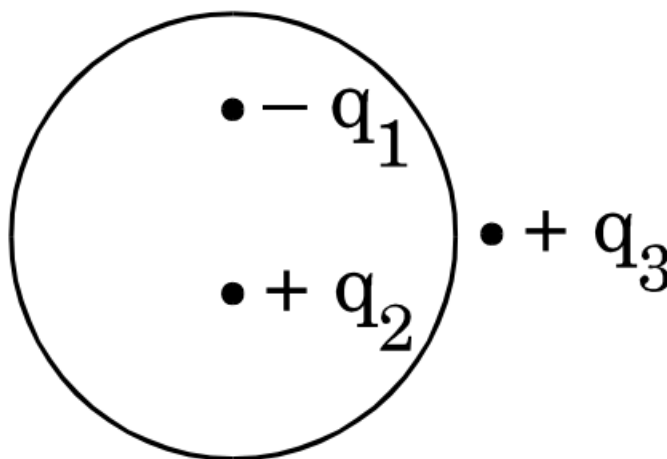
Step 2: Follow the subsequent motion The acquired velocity remains antiparallel to $\mathbf{E}$ and along the common field axis. Therefore the magnetic force stays zero at every instant. A circle or helix would require a velocity component perpendicular to $\mathbf{B}$, which is absent. The electron cannot remain stationary because its electric force is nonzero.

Step 3: Conclusion The electron moves in a straight line opposite to the electric field. Hence, the correct option is (A).

Quick Tip to Solve: Check the initial velocity first: a particle released from rest has no magnetic force.

11. Electric flux through a spherical surface shown in the figure, is ____.

[1 mark]



Solution:

Concept:

- Gauss's law gives net outward flux $\Phi_E = Q_{\text{enclosed}}/\epsilon_0$, where ϵ_0 is vacuum permittivity.
- Only charges inside a closed surface enter its net flux; exterior charges can change the local field.

Step 1: Read the enclosed charges The printed spherical surface encloses the negative charge $-q_1$ and the positive charge $+q_2$. The positive charge $+q_3$ lies outside the surface. Keeping the printed signs, the algebraic enclosed charge is $Q_{\text{enclosed}} = -q_1 + q_2$.

Step 2: Apply Gauss's law For the entire closed surface,

$$\Phi_E = \oint \mathbf{E} \cdot d\mathbf{A} = \frac{-q_1 + q_2}{\epsilon_0}.$$

Although the external charge contributes to the electric field on the sphere, its inward and outward flux contributions cancel. The radius of the sphere does not enter this result.

Step 3: Conclusion The blank is $(q_2 - q_1)/\epsilon_0$, the net outward electric flux.

Quick Tip to Solve: List enclosed charges with their signs before using Gauss's law; exclude the exterior charge.

12. Out of red, blue and yellow lights, the scattering of _____ light is maximum.

[1 mark]

Solution:

Concept:

- For Rayleigh scattering, scattered intensity varies inversely as the fourth power of wavelength:
 $I_s \propto 1/\lambda^4$.
- Among the given visible colours, blue has a shorter wavelength than yellow and red.

Step 1: Compare the wavelengths The order of wavelengths for the three given colours is $\lambda_{\text{blue}} < \lambda_{\text{yellow}} < \lambda_{\text{red}}$. The question asks for the colour most strongly scattered, rather than the colour with the greatest wavelength.

Step 2: Use the wavelength dependence For small scattering particles and comparable incident intensities, $I_s \propto \lambda^{-4}$. Decreasing wavelength therefore increases the scattering strongly. Blue is scattered more than yellow, while red is scattered least among these choices. The fourth-power dependence explains why even a moderate wavelength difference matters.

Step 3: Conclusion The blank is blue.

Quick Tip to Solve: For these colour comparisons, choose the shortest wavelength because scattering follows the inverse fourth power.

13. A compound microscope is used because a realistic simple microscope does not have _____ magnification.

[1 mark]

Solution:

Concept:

- A simple microscope uses a single converging lens to increase the angular size of a nearby object.
- Its magnifying power is D/f for a relaxed eye, or $1 + D/f$ at the near point; D is the least distance of distinct vision.

Step 1: Consider the practical limitation Large magnification with a simple microscope requires a very small focal length f . A realistic single lens cannot provide arbitrarily high useful magnification: short focal length makes viewing difficult and lens aberrations limit the quality of the enlarged image.

Step 2: Explain the compound arrangement A compound microscope first uses its objective to produce an enlarged real intermediate image. The eyepiece then magnifies that image for the eye. The overall magnifying power is the product of these two contributions, allowing useful magnification far greater than a practical simple microscope.

Step 3: Conclusion The blank is high (sufficiently large) magnification.

Quick Tip to Solve: Use two-stage magnification when a practical single lens cannot give the required high magnifying power.

14. An unpolarised light of intensity I_0 is passed through a polaroid. The intensity of a plane polarised light obtained is _____. [1 mark]

Solution:

Concept:

- An ideal polaroid transmits the electric-field component along its transmission axis.
- Unpolarised light has uniformly distributed transverse vibration directions, with $\langle \cos^2 \theta \rangle = 1/2$.

Step 1: Resolve each vibration component If a component of the incident light makes angle θ with the transmission axis, its transmitted intensity follows the projection factor $\cos^2 \theta$. A single fixed angle cannot describe the whole unpolarised incident beam.

Step 2: Average over the incident directions All transverse directions are equally represented in unpolarised light. Thus

$$I = I_0 \langle \cos^2 \theta \rangle = \frac{I_0}{2}.$$

The emerging vibrations lie along the polaroid transmission axis, so the transmitted beam is plane polarised. This is the standard result for an ideal first polariser.

Step 3: Conclusion The blank is $I_0/2$.

Quick Tip to Solve: The first ideal polaroid halves unpolarised intensity; use Malus's law for a later analyser.

15. In Young's double slit experiment, the path difference between two interfering waves at a point on the screen is $\frac{5\lambda}{2}$, λ being wavelength of the light used. The _____ dark fringe will lie at this point. [1 mark]

Solution:

Concept:

- Destructive interference occurs when path difference is an odd multiple of $\lambda/2$.
- The n th dark fringe has $\Delta = (2n - 1)\lambda/2$, counting the first dark fringe as $n = 1$.

Step 1: Use the printed path difference The official paper gives $\Delta = 5\lambda/2$. The consecutive dark-fringe path differences on either side of the central bright fringe are $\lambda/2$, $3\lambda/2$, $5\lambda/2$, and so on. Thus fringe order must be counted from the first minimum.

Step 2: Solve for the fringe order Equating the given value to the dark-fringe condition,

$$\frac{(2n - 1)\lambda}{2} = \frac{5\lambda}{2} \Rightarrow 2n - 1 = 5 \Rightarrow n = 3.$$

The result is not the fifth dark fringe: the coefficient five labels the odd half-wavelength multiple, not the fringe number.

Step 3: Conclusion The blank is $\boxed{\text{third}}$ dark fringe.

Quick Tip to Solve: For an odd half-wavelength coefficient k , the dark-fringe number is $(k+1)/2$.

15. If one of the slits in Young's double slit experiment is fully closed, the new pattern has _____ central maximum in angular size. [1 mark]

Solution:

Concept:

- Two open slits produce interference fringes; one open slit produces a single-slit diffraction pattern.
- For slit width a , the single-slit central maximum has small-angle angular width $2\lambda/a$.

Step 1: Identify the new pattern Closing one slit removes interference between the two slit beams. Light from the remaining finite-width slit still diffracts, giving a central diffraction maximum bounded by its first minima. It is therefore a broad central maximum rather than the narrow central interference fringe.

Step 2: Compare angular sizes The first diffraction minima satisfy $a \sin \theta = \pm \lambda$, so the central diffraction width is approximately $2\lambda/a$. The interference-fringe angular width with both slits open is approximately λ/d , where d is slit separation. In the usual arrangement $d > a$, so the single-slit central maximum is wider in angular size.

Step 3: Conclusion The blank is wider (larger): the new central maximum is wider in angular size.

Quick Tip to Solve: After closing a slit, compare the broad single-slit diffraction maximum with the original interference fringe.

16. Define the term 'threshold frequency' in photoelectric emission. [1 mark]

Solution:

Concept:

- Each incident photon has energy $h\nu$, where h is Planck's constant and ν is the frequency.
- The work function ϕ is the minimum energy needed to release an electron from the surface.

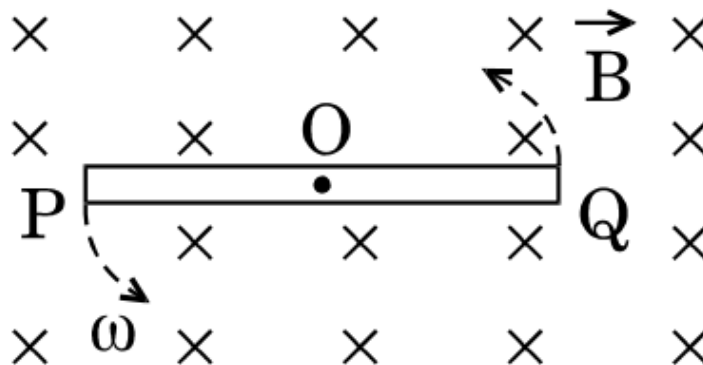
Step 1: Apply the emission condition Photoelectric emission requires the energy of an individual photon to be at least the work function: $h\nu \geq \phi$. At the limiting frequency the emitted electron has zero maximum kinetic energy.

Step 2: Identify the threshold The limiting frequency is $\nu_0 = \phi/h$. Below ν_0 , photons cannot liberate electrons, however large the intensity of the incident light is.

Step 3: Conclusion Threshold frequency is the minimum frequency of incident radiation required to cause photoelectric emission from a given surface: $\nu_0 = \phi/h$.

Quick Tip to Solve: Remember that frequency sets photon energy; increasing intensity below threshold does not cause emission.

17. A metallic rod PQ of length l is rotated with an angular velocity ω about an axis passing through its mid-point (O) and perpendicular to the plane of the paper, in uniform magnetic field $\vec{B}$, as shown in the figure. What is the potential difference developed between the two ends of the rod, P and Q? [1 mark]



Solution:

Concept:

- A rotating rod has local speed $v = \omega r$, where r is the distance from its rotation axis.
- The motional potential difference along a radial half of the rod has magnitude $\int_0^{l/2} B\omega r \, dr$.

Step 1: Compare each end with the midpoint Both halves have the same length $l/2$ and rotate in the same uniform field. Their end-to-midpoint potential differences therefore have equal magnitude $B\omega l^2/8$.

Step 2: Compare the polarities On opposite sides of O, both the position vector and the velocity reverse. Consequently the magnetic force on positive charges has the same radial sense on both halves: the ends P and Q acquire the same potential relative to O. Thus $V_P - V_O = V_Q - V_O$.

Step 3: Conclusion The potential difference between the two ends is $V_P - V_Q = 0 \text{ V}$.

Quick Tip to Solve: For rotation about the midpoint, compare each end with the centre before adding any motional emfs.

18. What is the impedance of a capacitor of capacitance C in an ac circuit using source of frequency n Hz? [1 mark]

Solution:

Concept:

- The magnitude of a capacitor's impedance is its capacitive reactance $X_C = 1/(\omega C)$.
- The angular frequency is $\omega = 2\pi n$, where n is the source frequency in hertz.

Step 1: Write the capacitor relation For a sinusoidal applied voltage, $i = C \, dv/dt$, so the current amplitude is $I_0 = \omega C V_0$. The ratio V_0/I_0 gives the impedance magnitude.

Step 2: Substitute the given frequency Using $\omega = 2\pi n$, the ratio is $V_0/I_0 = 1/(2\pi n C)$. A larger frequency or capacitance therefore gives a smaller opposition to alternating current. In complex notation, $Z_C = -j/(2\pi n C)$.

Step 3: Conclusion The required impedance magnitude is $Z = X_C = \frac{1}{2\pi n C} \, \Omega$, with n in hertz and

C in farads.

Quick Tip to Solve: Convert ordinary frequency to angular frequency first; the factor of two pi belongs in the denominator.

OR

18. What is the value of impedance of a resonant series LCR circuit?

[1 mark]

Solution:

Concept:

- The magnitude of series LCR impedance is $Z = \sqrt{R^2 + (X_L - X_C)^2}$.
- At resonance the inductive and capacitive reactances are equal: $X_L = X_C$.

Step 1: Apply the resonance condition The inductor and capacitor produce equal and opposite reactive voltage contributions at resonance. Their net reactance is therefore zero, although each separate reactance need not be zero.

Step 2: Evaluate the impedance Substituting $X_L - X_C = 0$ into the series impedance gives $Z = \sqrt{R^2} = R$. The remaining opposition to current is entirely resistive; the current and applied voltage are in phase.

Step 3: Conclusion The impedance of a resonant series LCR circuit is $Z = R$, the resistance of the circuit.

Quick Tip to Solve: At series resonance cancel the reactances before calculating impedance; retain the circuit resistance.

19. The threshold wavelength for two photosensitive surfaces A and B are λ_1 and λ_2 , respectively. What is the ratio of the work functions of the two surfaces?

[1 mark]

Solution:

Concept:

- At the threshold, the photon energy just equals the surface work function: $\phi = hc/\lambda_0$.
- Here h is Planck's constant, c is the speed of light and λ_0 is the threshold wavelength.

Step 1: Express the two work functions For surface A, $\phi_A = hc/\lambda_1$. For surface B, $\phi_B = hc/\lambda_2$. At each threshold the maximum kinetic energy of the emitted electrons is zero.

Step 2: Take the ratio Dividing the first expression by the second cancels the common factor hc : $\phi_A/\phi_B = (hc/\lambda_1)/(hc/\lambda_2) = \lambda_2/\lambda_1$. A larger threshold wavelength corresponds to a smaller work function.

Step 3: Conclusion The work-function ratio is $\phi_A : \phi_B = \lambda_2 : \lambda_1$.

Quick Tip to Solve: Work function is inversely proportional to threshold wavelength, so reverse the wavelength ratio.

20. Draw the graph showing variation of the value of the induced emf as a function of rate of change of current flowing through an ideal inductor. [1 mark]

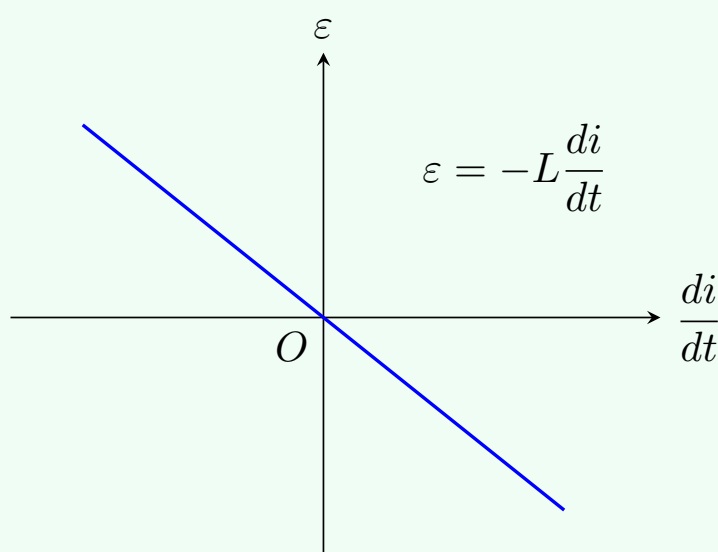
Solution:

Concept:

- For an ideal inductor of constant self-inductance L , the induced emf is $\varepsilon = -L \frac{di}{dt}$.
- The negative sign expresses Lenz's law: the induced emf opposes the change in current.

Step 1: Choose the axes Plot the signed induced emf ε on the vertical axis and the rate of change of current $\frac{di}{dt}$ on the horizontal axis. When $\frac{di}{dt} = 0$, the induced emf is also zero.

Step 2: Determine the slope The relation $\varepsilon = -L(\frac{di}{dt})$ is linear. Hence the graph passes through the origin and has constant slope $-L$. A positive rate of change gives a negative emf; a negative rate gives a positive emf. If only the magnitude is considered for positive rates, $|\varepsilon| = L|\frac{di}{dt}|$ has slope $+L$.



Signed induced emf versus rate of change of current; slope $-L$.

Step 3: Conclusion The signed emf graph is a straight line through the origin with slope $-L$, as shown.

Quick Tip to Solve: Decide whether the vertical axis is signed emf or its magnitude; Lenz's minus sign sets the signed slope.

Section B (14 marks)

21. Which of the following electromagnetic waves has (a) minimum wavelength, and (b) minimum frequency? Write one use of each of these two waves.

Infrared waves, Microwaves, γ -rays and X-rays

[2 marks]

Solution:

Concept:

- In vacuum every electromagnetic wave travels at speed c , so $c = \nu\lambda$, where ν is frequency and λ is wavelength.

- Higher frequency means shorter wavelength; compare the positions of the named waves in the electromagnetic spectrum.

Step 1: Arrange the named waves The order of increasing frequency is microwaves, infrared waves, X-rays and gamma rays. Therefore the order of increasing wavelength is reversed: gamma rays, X-rays, infrared waves and microwaves. This comparison is restricted to the four types given in the question.

Step 2: Identify the minimum wavelength and a use Gamma rays have the highest frequency among the listed waves and hence the minimum wavelength. One use of gamma rays is radiotherapy: carefully directed radiation is used to destroy cancer cells.

Step 3: Identify the minimum frequency and a use Microwaves have the lowest frequency among the listed waves, and hence the longest wavelength in this list. One use is radar, where microwave pulses are transmitted and their reflected signals help determine the position or distance of an object.

Step 4: Conclusion (a) Minimum wavelength: γ -rays; use: cancer treatment by radiotherapy. (b) Minimum frequency: microwaves; use: radar.

Quick Tip to Solve: Write the spectrum order first; use $\nu\lambda = c$ to reverse the order when comparing wavelengths.

22. A galvanometer of resistance $16\ \Omega$ shows full scale deflection for a current of $4\ \text{mA}$. How will you convert it into a voltmeter to measure a voltage up to $3\ \text{V}$? [2 marks]

Solution:

Concept:

- A galvanometer is converted into a voltmeter by connecting a high resistance R in series with its coil of resistance G .
- At the maximum measured voltage V , the coil must carry only its full-scale current I_g : $V = I_g(G + R)$.

Step 1: Convert the full-scale current to amperes The coil resistance is $G = 16\ \Omega$, the full-scale current is $I_g = 4\ \text{mA} = 0.004\ \text{A}$, and the required upper voltage limit is $V = 3\ \text{V}$. The required resistance is a series multiplier, so the same current passes through it and the galvanometer.

Step 2: Calculate the required series resistance Applying Ohm's law to the complete series combination,

$$G + R = \frac{V}{I_g} = \frac{3}{0.004} = 750\ \Omega.$$

Therefore

$$R = 750 - 16 = 734\ \Omega.$$

At full scale the coil drops $I_g G = 0.064\ \text{V}$, and the series resistor drops $2.936\ \text{V}$, giving a total of $3\ \text{V}$.

Step 3: Describe the connection Connect the calculated resistance in series with the galvanometer and calibrate its scale from zero to the required maximum voltage. Connect this complete combination across the two points whose potential difference is to be measured; its high total resistance limits the

current through the coil.

Step 4: Conclusion Connect a $734\ \Omega$ resistance in series with the galvanometer to obtain a 0-3 V voltmeter.

Quick Tip to Solve: For a voltmeter calculate V/I_g first, then subtract the coil resistance; a parallel shunt converts it into an ammeter.

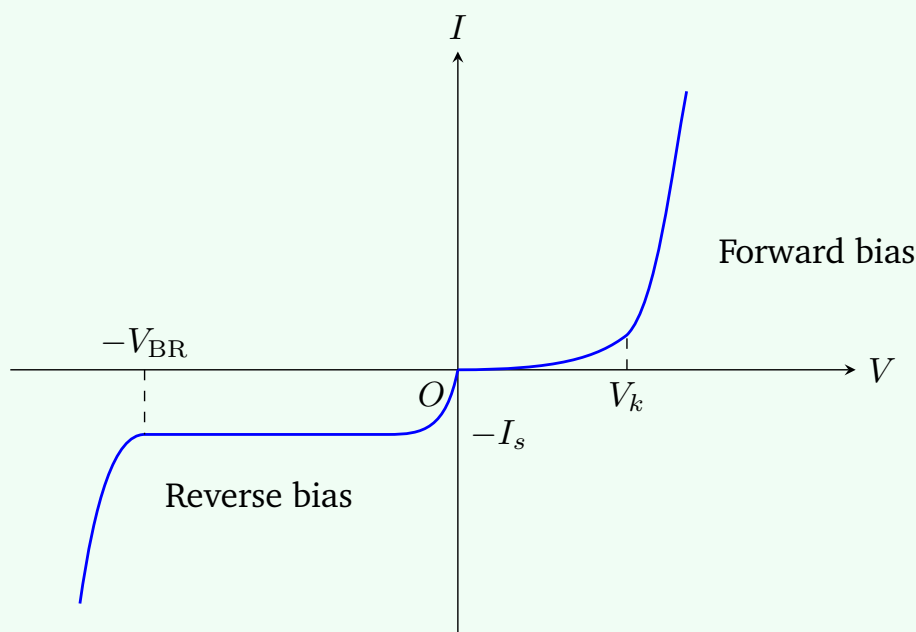
23. Draw V-I characteristics of a p-n junction diode. Explain, why the current under reverse bias is almost independent of the applied voltage up to the critical voltage. [2 marks]

Solution:

Concept:

- Forward bias lowers the junction barrier and produces rapidly increasing current beyond the knee voltage.
- Reverse bias widens the depletion region; the small reverse saturation current is carried by thermally generated minority carriers.

Step 1: Draw and interpret the characteristic The graph plots diode current I vertically against voltage V horizontally. In forward bias the current is initially small and then increases steeply beyond the knee V_k . Under reverse bias, a small negative current $-I_s$ is nearly constant until the reverse voltage reaches the breakdown value $-V_{BR}$. Beyond this critical voltage the magnitude of the reverse current increases sharply. The reverse-current scale is expanded in the schematic so that its small value is visible.



Schematic p-n diode V-I characteristic; reverse-current scale expanded.

Step 2: Explain the almost constant reverse current Reverse current is due mainly to minority electrons

in the p-region and minority holes in the n-region. These carriers are generated thermally. At a fixed temperature, their rate of generation is nearly fixed. The reverse electric field sweeps them across the junction, so increasing the reverse voltage does not appreciably increase their supply. Thus the current remains almost independent of voltage until breakdown, when additional carrier-generation mechanisms cause a rapid increase.

Step 3: Conclusion The reverse current is nearly the constant saturation current I_s up to the critical reverse voltage because it is limited by the thermal generation of minority carriers.

Quick Tip to Solve: Distinguish the source of reverse current (minority carriers) from breakdown; increasing reverse voltage alone does not supply more thermal carriers.

24. Define the term ‘mobility’ of charge carriers in a current carrying conductor. Obtain the relation for mobility in terms of relaxation time. [2 marks]

Solution:

Concept:

- Mobility μ is the magnitude of drift velocity per unit electric field: $\mu = v_d/E$.
- Relaxation time τ is the mean time between successive collisions of a charge carrier; e is the magnitude of electron charge and m is electron mass.

Step 1: Define mobility and its unit When an electric field is applied, charge carriers acquire a small average drift velocity in addition to their random thermal motion. Mobility measures how readily a carrier drifts in that field:

$$\mu = \frac{|\vec{v}_d|}{|\vec{E}|}.$$

Its SI unit is $\text{m}^2 \text{V}^{-1} \text{s}^{-1}$, since drift speed is measured in m s^{-1} and field strength in V m^{-1} .

Step 2: Obtain the drift velocity from relaxation time For an electron, the force due to the field is $\vec{F} = -e\vec{E}$; hence $\vec{a} = -e\vec{E}/m$. The random velocity has zero ensemble average. In the relaxation-time model, the mean field-induced velocity acquired between collisions is therefore

$$\vec{v}_d = -\frac{e\tau}{m}\vec{E}.$$

The negative sign means that electron drift is opposite to the electric field. Taking magnitudes gives $v_d = eE\tau/m$.

Step 3: Express mobility in terms of relaxation time Divide the drift-speed relation by the electric-field magnitude:

$$\mu = \frac{v_d}{E} = \frac{eE\tau}{mE} = \frac{e\tau}{m}.$$

Thus, for a given carrier mass and charge magnitude, a longer relaxation time leads to greater mobility.

Step 4: Conclusion The mobility is the drift speed per unit electric field, and for electrons $\mu = e\tau/m$, with SI unit $\text{m}^2 \text{V}^{-1} \text{s}^{-1}$.

Quick Tip to Solve: Use magnitudes in the definition of mobility; retain the negative sign only when writing the electron drift-velocity vector.

24. Define the term ‘drift velocity’ of electrons in a current carrying conductor. Obtain the relationship between the current density and the drift velocity of electrons. [2 marks]

Solution:

Concept:

- Drift velocity $\vec{v}_d$ is the average velocity acquired by free electrons in a conductor under an applied electric field, superposed on their random thermal motion.
- Current density $\vec{J}$ is conventional current per unit cross-sectional area. Let n be the number of free electrons per unit volume and e the positive magnitude of electronic charge.

Step 1: Define the direction of drift Without an electric field, random electron velocities average to zero. An applied field produces a small average electron velocity opposite to the field because the electron charge is $-e$. This directed average is the drift velocity; it is not the instantaneous speed of an individual electron.

Step 2: Count the charge crossing a section For a conductor of cross-sectional area A , electrons in a cylinder of length $v_d \Delta t$ cross the section in time Δt . Their number is $nAv_d \Delta t$, so the magnitude of charge transported is $\Delta Q = neAv_d \Delta t$. Therefore $I = \Delta Q / \Delta t = neAv_d$, and dividing by A gives $J = nev_d$. Conventional current points opposite to electron drift, giving the vector relation $\vec{J} = -ne\vec{v}_d$.

Step 3: Conclusion The required relationship is $J = nev_d$ in magnitude, or $\vec{J} = -ne\vec{v}_d$ as a vector.

Quick Tip to Solve: Use a cylinder of length drift speed times time to count the electrons crossing a section.

25. The focal length of an equiconcave lens is $\frac{3}{4}$ times of radius of curvature of its surfaces. Find the refractive index of the material of the lens. Under what condition will this lens behave as a converging lens? [2 marks]

Solution:

Concept:

- For a thin lens in air, the lens maker formula is $1/f = (\mu - 1)(1/R_1 - 1/R_2)$, where μ is its refractive index relative to air.
- With light incident from the left, an equiconcave lens has $R_1 = -R$, $R_2 = +R$, and negative focal length in air.

Step 1: Use the signed focal length The stated focal length is the magnitude: $|f| = 3R/4$. Since the lens is concave in air, $f = -3R/4$. Substitution gives

$$-\frac{4}{3R} = (\mu - 1) \left(-\frac{1}{R} - \frac{1}{R} \right) = -\frac{2(\mu - 1)}{R}.$$

Cancelling the common negative factor gives $\mu - 1 = 2/3$, hence $\mu = 5/3$.

Step 2: Find the condition for convergence If the surrounding medium has refractive index μ_m , replace μ in the formula by μ/μ_m . The curvature factor for this lens remains negative. Positive power and positive focal length therefore require $\mu/\mu_m - 1 < 0$, that is, a surrounding medium optically denser than the lens material. Equality would give zero power, not convergence.

Step 3: Conclusion $\mu = 5/3$. The lens converges when immersed in a medium with $\mu_m > 5/3$.

Quick Tip to Solve: Assign a negative focal length to a concave lens before substituting into the lens maker formula.

26. (a) For a radioactive substance, show the variation of the total mass disintegrated as a function of time t graphically.

(b) Initial mass of a radioactive substance is 3.2 mg. It has a half-life of 4 h. Find the mass of the substance left undecayed after 8 h. [2 marks]

Solution:

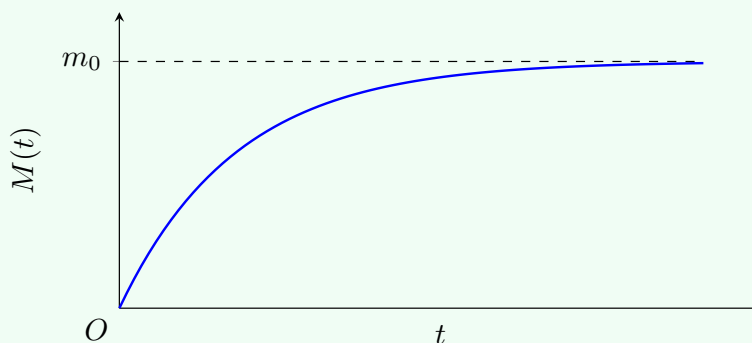
Concept:

- Radioactive decay leaves parent-substance mass $m(t) = m_0 e^{-\lambda t}$, where m_0 is its initial mass and λ is its decay constant.
- The total parent-substance mass disintegrated is $M(t) = m_0 - m(t)$. After each half-life the remaining parent mass is halved.

Step 1: (a) Describe and plot the disintegrated mass The required quantity is the cumulative mass that has undergone decay, so

$$M(t) = m_0 (1 - e^{-\lambda t}).$$

It starts at zero, rises continuously, and approaches m_0 asymptotically. Its slope is $\lambda m_0 e^{-\lambda t}$, which is positive but decreases with time. Thus the graph is concave downward and does not reach the horizontal limiting line at any finite time.



Total parent-substance mass disintegrated versus time

Step 2: (b) Calculate the remaining mass The elapsed time corresponds to $n = t/T_{1/2} = 8/4 = 2$ half-lives. Hence

$$m = m_0 \left(\frac{1}{2}\right)^n = 3.2 \left(\frac{1}{2}\right)^2 \text{ mg} = 0.8 \text{ mg}.$$

Equivalently, the mass falls from 3.2 mg to 1.6 mg after four hours and then to 0.8 mg after eight hours.

Step 3: Conclusion The disintegrated mass rises toward m_0 as shown; the undecayed mass after 8 h is 0.8 mg .

Quick Tip to Solve: Distinguish remaining mass (a falling curve) from mass disintegrated (a rising curve).

27. (a) Explain the term 'sharpness of resonance' in ac circuit.

(b) In a series LCR circuit, $V_L = V_C \neq V_R$. What is the value of power factor for this circuit? [2 marks]

Solution:

Concept:

- In a series LCR circuit, resonance occurs at $\omega_0 = 1/\sqrt{LC}$, where inductive and capacitive reactances cancel.
- The power factor is $\cos \phi = R/Z$, with $Z = \sqrt{R^2 + (X_L - X_C)^2}$, and ϕ the phase difference between applied voltage and current.

Step 1: (a) Explain sharpness of resonance Sharpness of resonance describes how narrowly the large current response is concentrated around the resonant frequency: a sharper resonance has a tall, narrow peak and the current decreases rapidly when the frequency is moved away from resonance. It is quantified by the quality factor $Q = \omega_0/\Delta\omega = \omega_0 L/R$, where $\Delta\omega$ is the bandwidth between half-power frequencies. For fixed L and C , smaller resistance produces greater sharpness.

Step 2: (b) Use the equal reactive voltages The same current flows through every element, so $V_L = IX_L$ and $V_C = IX_C$. Their equality implies $X_L = X_C$. These voltages have opposite phasor directions and cancel; the total applied voltage is consequently in phase with the current and equals the resistor voltage. Thus $Z = R$, $\phi = 0$, and $\cos \phi = R/R = 1$. The fact that the individual reactive voltages differ from V_R does not change their cancellation.

Step 3: Conclusion Sharpness measures the narrowness of the resonance peak; for the stated circuit, power factor = 1.

Quick Tip to Solve: Equal inductor and capacitor voltage magnitudes cancel as phasors in a series circuit.

OR

27. An ac source of emf $V = V_0 \sin \omega t$ is connected to a capacitor of capacitance C . Deduce the expression for the current (I) flowing in it. Plot the graph of (i) V vs. ωt , and (ii) I vs. ωt . [2 marks]

Solution:

Concept:

- For a capacitor, $q = CV$, where q is instantaneous plate charge, C is capacitance and V is the applied voltage.
- Current is $I = dq/dt$. The derivative of a sine is a cosine, so capacitive current leads voltage by a quarter-cycle.

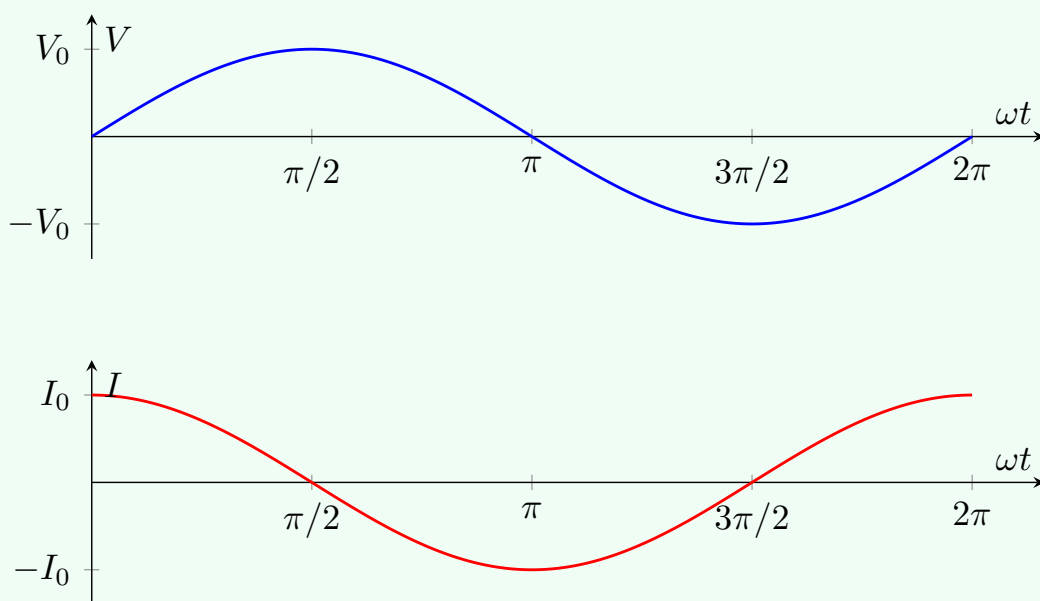
Step 1: Differentiate the capacitor charge The applied voltage gives $q = CV_0 \sin \omega t$. For constant capacitance, differentiation yields

$$I = \frac{dq}{dt} = \omega CV_0 \cos \omega t = I_0 \sin(\omega t + \pi/2), \quad I_0 = \omega CV_0.$$

Equivalently, $I_0 = V_0/X_C$, where $X_C = 1/(\omega C)$. The current has the same frequency as the applied voltage.

Step 2: Plot both quantities against phase Voltage starts at zero and increases to $+V_0$ at $\omega t = \pi/2$; it returns to zero at π . Current starts at $+I_0$, becomes zero at $\pi/2$, and reaches $-I_0$ at π . Both waves

repeat after 2π . Thus, the current maximum precedes the voltage maximum by $\pi/2$. The two graphs use separate peak values on their vertical axes.



Voltage and current against ωt .

Step 3: Conclusion $I = \omega CV_0 \cos \omega t$. The current leads the voltage by $\pi/2$.

Quick Tip to Solve: Differentiate the voltage sine wave; the cosine immediately reveals the current maximum at zero phase.

Section C (21 marks)

28. (a) Two electric field lines cannot cross each other. Also, they cannot form closed loops. Give reasons.

(b) A particle of charge $2 \mu\text{C}$ and mass 1.6 g is moving with a velocity $4\hat{i} \text{ m s}^{-1}$. At $t = 0$ the particle enters in a region having an electric field $\vec{E}$ (in NC^{-1}) $= 80\hat{i} + 60\hat{j}$. Find the velocity of the particle at $t = 5 \text{ s}$. [3 marks]

Solution:

Concept:

- The tangent to an electric field line gives the unique direction of the electric field at a point. Electrostatic fields are conservative: $\oint \vec{E} \cdot d\vec{l} = 0$.
- A charge q of mass m in a uniform field has constant acceleration $\vec{a} = q\vec{E}/m$, so $\vec{v} = \vec{u} + \vec{a}t$.

Step 1: (a) Explain why lines cannot cross If two electric field lines crossed, their two tangents at the intersection would prescribe two different directions for the electric field at the same point. However, the resultant field at any point has one definite direction. Thus, field lines cannot cross.

Step 2: (a) Explain why electrostatic lines cannot close An electrostatic field line is directed from higher to lower potential. Along its direction, potential continuously decreases; it cannot return to its initial potential by forming a closed loop. Equivalently, following a closed field line would give a nonzero circulation, whereas a conservative electrostatic field satisfies $\oint \vec{E} \cdot d\vec{l} = 0$.

Step 3: (b) Calculate acceleration and final velocity Convert the given quantities: $q = 2 \times 10^{-6} \text{ C}$ and

$m = 1.6 \times 10^{-3} \text{ kg}$. Since the charge is positive, acceleration is along the field. Thus

$$\vec{a} = \frac{2 \times 10^{-6}}{1.6 \times 10^{-3}} (80\hat{i} + 60\hat{j}) = (0.10\hat{i} + 0.075\hat{j}) \text{ m s}^{-2}.$$

The initial velocity is $\vec{u} = 4\hat{i} \text{ m s}^{-1}$, with no initial y -component. At $t = 5 \text{ s}$,

$$\vec{v} = 4\hat{i} + 5(0.10\hat{i} + 0.075\hat{j}) = (4.50\hat{i} + 0.375\hat{j}) \text{ m s}^{-1}.$$

Step 4: Conclusion Field lines cannot cross because field direction is unique; electrostatic lines cannot close because potential decreases along them. The velocity at 5 s is $(4.50\hat{i} + 0.375\hat{j}) \text{ m s}^{-1}$.

Quick Tip to Solve: Convert grams to kilograms before dividing by mass, and update the two velocity components separately.

OR

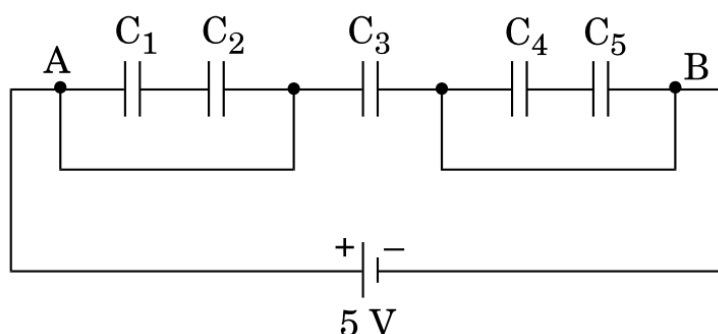
28. In the figure given below, find the

(a) equivalent capacitance of the network between points A and B. Given: $C_1 = C_5 = 4 \mu\text{F}$, $C_2 = C_3 = C_4 = 2 \mu\text{F}$.

(b) maximum charge supplied by the battery, and

(c) total energy stored in the network.

[3 marks]



Solution:

Concept:

- Points joined by an ideal wire are at the same potential. A capacitor branch whose endpoints are shorted has zero net applied voltage.
- Battery charge and total stored energy are $Q = C_{\text{eq}}V$ and $U = \frac{1}{2}C_{\text{eq}}V^2$, where C_{eq} is the equivalent capacitance.

Step 1: (a) Identify the conducting bypasses The lower left wire directly joins A to the junction between C_2 and C_3 . Hence the series branch containing C_1 and C_2 is bypassed. Similarly, the lower right wire joins the junction between C_3 and C_4 directly to B, bypassing the branch containing C_4 and C_5 . Each bypassed branch has zero endpoint potential difference; for initially uncharged capacitors it stores no charge or energy. The given values of those four capacitors therefore do not affect the equivalent capacitance.

Step 2: (a) Find the effective capacitance The left plate of C_3 is at potential A and its right plate is at

potential B. Thus only C_3 remains effectively connected across the battery:

$$C_{\text{eq}} = C_3 = 2 \mu\text{F}.$$

It would be incorrect to combine all five capacitors in series because the conducting wires change the network connections.

Step 3: (b) Calculate the charge supplied The figure shows a 5 V battery. At complete charging, the maximum charge supplied is

$$Q = C_{\text{eq}} V = (2 \times 10^{-6})(5) = 10 \times 10^{-6} \text{ C} = 10 \mu\text{C}.$$

This is the charge magnitude on each plate of the effective capacitor, not the sum of both opposite plate charges.

Step 4: (c) Calculate stored energy Only the effective capacitor contributes to the final stored energy. Therefore

$$U = \frac{1}{2} C_{\text{eq}} V^2 = \frac{1}{2} (2 \times 10^{-6})(5)^2 = 25 \times 10^{-6} \text{ J} = 25 \mu\text{J}.$$

This is energy stored in the capacitors; the battery work during charging is not the same quantity.

Step 5: Conclusion $C_{\text{eq}} = 2 \mu\text{F}$, maximum battery charge $Q = 10 \mu\text{C}$, and total stored energy $U = 25 \mu\text{J}$.

Quick Tip to Solve: First trace ideal wires to merge equal-potential nodes; the two bypass wires leave only the central capacitor active.

29. (a) Differentiate between half-life and average life of a radioactive substance.

(b) A radioactive substance decays for an interval of time equal to its mean life. Find the fraction of the amount of the substance which is left undecayed after this time interval. [3 marks]

Solution:

Concept:

- Radioactive decay follows $N(t) = N_0 e^{-\lambda t}$, where N_0 is the initial number of undecayed nuclei and λ is the decay constant.
- Half-life $T_{1/2}$ is the time in which half the original nuclei decay; mean or average life τ is the average survival time of a nucleus.

Step 1: (a) Define and obtain the half-life At the half-life, the number remaining is $N_0/2$. Substituting this condition in the decay law gives

$$\frac{1}{2} = e^{-\lambda T_{1/2}}, \quad T_{1/2} = \frac{\ln 2}{\lambda}.$$

Thus half-life describes the time required for the undecayed population, or equivalently its activity, to fall to half its initial value.

Step 2: (a) Define and compare the average life The fraction of nuclei decaying between t and $t + dt$ is $\lambda e^{-\lambda t} dt$. Therefore the average of their individual lifetimes is

$$\tau = \int_0^{\infty} t \lambda e^{-\lambda t} dt = \frac{1}{\lambda}.$$

Unlike half-life, this is a statistical average over individual nuclear lifetimes. The two quantities satisfy $T_{1/2} = (\ln 2)\tau$, so $\tau \simeq 1.443T_{1/2}$.

Step 3: (b) Substitute one mean life in the decay law For the given interval $t = \tau = 1/\lambda$, the fraction remaining undecayed is obtained directly from

$$\frac{N(\tau)}{N_0} = e^{-\lambda\tau} = e^{-\lambda(1/\lambda)} = e^{-1}.$$

The same fraction applies to the amount of the original radioactive substance, because that amount is proportional to its number of undecayed nuclei.

Step 4: Conclusion $T_{1/2} = \ln 2/\lambda$, whereas $\tau = 1/\lambda$. After one mean life the fraction left undecayed is $1/e \simeq 0.368$, or approximately 36.8%.

Quick Tip to Solve: At one mean life the exponent is exactly -1 ; do not replace mean life by half-life and answer one-half.

30. What is the difference in the construction of an astronomical telescope and a compound microscope? The focal lengths of the objective and eyepiece of a compound microscope are 1.25 cm and 5.0 cm, respectively. Find the position of the object relative to the objective in order to obtain an angular magnification of 30 when the final image is formed at the near point. [3 marks]

Solution:

Concept:

- A compound microscope uses a short-focal-length objective to produce a real magnified intermediate image, which a short-focal-length eyepiece magnifies.
- For a final image at the near point, the eyepiece angular magnification is $M_e = 1 + D/f_e$, where the least distance of distinct vision is $D = 25$ cm. The total magnitude is $M = |m_o|M_e$.
- For the objective, the Cartesian lens formula is $1/f_o = 1/v_o - 1/u_o$ and its linear magnification is $m_o = v_o/u_o$.

Step 1: Compare the construction of the two instruments An astronomical telescope has an objective of large aperture and long focal length, together with an eyepiece of comparatively short focal length. Its large objective collects light from distant faint objects. In a compound microscope, both lenses have short focal lengths, with the objective focal length smaller than the eyepiece focal length; the objective has a small aperture and views an object placed close to its first focus. Thus the principal construction contrast is a long-focal-length, large-aperture telescope objective versus a very short-focal-length, small-aperture microscope objective.

Step 2: Find the required objective magnification Using $f_e = 5.0$ cm, the near-point eyepiece magnification is

$$M_e = 1 + \frac{25}{5.0} = 6.$$

The specified total angular magnification has magnitude 30, so $|m_o| = 30/6 = 5$. The objective forms a real inverted intermediate image, hence $m_o = -5$ and $v_o = -5u_o$.

Step 3: Apply the objective lens formula With $f_o = 1.25$ cm and $v_o = -5u_o$,

$$\frac{1}{1.25} = \frac{1}{-5u_o} - \frac{1}{u_o} = -\frac{6}{5u_o},$$

$$u_o = -\frac{6}{5}(1.25) = -1.50 \text{ cm}.$$

The corresponding real intermediate image distance is $v_o = 7.50$ cm. The negative object distance signifies an object on the incident-light side of the objective, slightly beyond its focal point.

Step 4: Conclusion The object must be placed 1.50 cm in front of the objective (Cartesian distance $u_o = -1.50$ cm). The telescope uses a long-focal-length, large-aperture objective; the microscope uses a short-focal-length, small-aperture objective.

Quick Tip to Solve: First divide the total magnification by $1 + 25/f_e$, then use the exact objective lens formula; no tube length is needed.

31. The maximum kinetic energy of the photoelectrons emitted is doubled when the wavelength of light incident on the photosensitive surface changes from λ_1 to λ_2 . Deduce expressions for the threshold wavelength and work function for the metal surface in terms of λ_1 and λ_2 . [3 marks]

Solution:

Concept:

- Einstein's photoelectric equation is $K_{\max} = hc/\lambda - \phi$, where ϕ is the work function, h is Planck's constant and c is the speed of light.
- The threshold wavelength λ_0 is the longest wavelength that can cause emission, so $\phi = hc/\lambda_0$.

Step 1: Write the two energy equations Let the initial maximum kinetic energy be K_1 for wavelength λ_1 . The given change makes the energy $K_2 = 2K_1$ at wavelength λ_2 . For the same metal the work function remains unchanged, so

$$K_1 = \frac{hc}{\lambda_1} - \phi, \quad 2K_1 = \frac{hc}{\lambda_2} - \phi.$$

Since photon energy increases as wavelength decreases, the doubled positive kinetic energy requires $\lambda_2 < \lambda_1$.

Step 2: Eliminate the kinetic energy to find the work function Substitute the first equation into the second and collect the work-function terms:

$$2\left(\frac{hc}{\lambda_1} - \phi\right) = \frac{hc}{\lambda_2} - \phi,$$

$$\phi = \frac{2hc}{\lambda_1} - \frac{hc}{\lambda_2} = hc \frac{2\lambda_2 - \lambda_1}{\lambda_1 \lambda_2}.$$

This subtraction retains the same work function on both sides; doubling the kinetic energy does not mean doubling the incident photon energy.

Step 3: Use the threshold condition At threshold the emitted electron has zero maximum kinetic energy, giving $hc/\lambda_0 = \phi$. Therefore

$$\frac{1}{\lambda_0} = \frac{2}{\lambda_1} - \frac{1}{\lambda_2}, \quad \lambda_0 = \frac{\lambda_1 \lambda_2}{2\lambda_2 - \lambda_1}.$$

For a physical metal with positive work function and positive initial kinetic energy, the given wavelengths satisfy $\lambda_1/2 < \lambda_2 < \lambda_1$, and the derived threshold wavelength is longer than λ_1 , as expected.

Step 4: Conclusion The required expressions are $\lambda_0 = \frac{\lambda_1 \lambda_2}{2\lambda_2 - \lambda_1}$ and $\phi = hc \left(\frac{2}{\lambda_1} - \frac{1}{\lambda_2} \right)$.

Quick Tip to Solve: Keep ϕ unchanged in both Einstein equations and eliminate K_1 ; this prevents reversing the denominator sign.

32. (a) Derive the condition of balance for Wheatstone bridge.

(b) Draw the circuit diagram of a meter bridge to explain how it is based on Wheatstone bridge.

[3 marks]

Solution:

Concept:

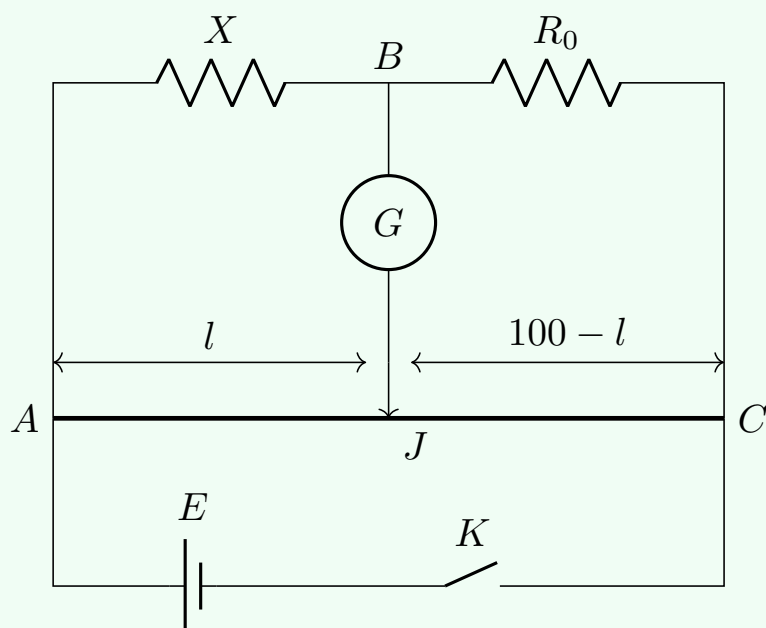
- A Wheatstone bridge consists of four resistances in two series branches, with a galvanometer joining their intermediate junctions. Balance means zero galvanometer current, so the junctions are at equal potential.
- Ohm's law is $V = IR$. For a uniform bridge wire, resistance is proportional to length: $r = \rho l/A$, where ρ is resistivity and A is cross-sectional area.

Step 1: (a) Apply the null condition to the Wheatstone bridge Let the four arms be $AB = P$, $BC = Q$, $AD = R$ and $DC = S$. Connect a cell between A and C , and a galvanometer between B and D . At balance, $I_G = 0$, hence $V_B = V_D$. The current I_1 therefore flows through both P and Q , while I_2 flows through both R and S . Equating the potential drops from A to the equal-potential junctions gives $I_1 P = I_2 R$. Equating the drops from these junctions to C gives $I_1 Q = I_2 S$. Dividing the first equation by the second eliminates both currents and gives the balance relation $P/Q = R/S$, or $PS = QR$. No current passes through the galvanometer only when this ratio condition holds.

Step 2: (b) Identify the four arms of the meter bridge In the circuit below, place unknown resistance X and known resistance R_0 in the two gaps. Connect the cell and key across the ends A and C of the uniform one-metre wire. The galvanometer connects the central junction B to the movable jockey J . Move J until the galvanometer shows zero deflection. The four Wheatstone arms are now X , R_0 , wire segment AJ , and wire segment JC . If $AJ = l$ cm, then $JC = (100 - l)$ cm. Since the same material and cross-section occur in both wire segments, $r_{AJ}/r_{JC} = l/(100 - l)$. Applying the derived balance condition gives

$$\frac{X}{R_0} = \frac{r_{AJ}}{r_{JC}} = \frac{l}{100 - l}.$$

The meter bridge is thus a practical Wheatstone bridge whose two lower arms are lengths of the same uniform wire.



Meter bridge at balance: the central junction B and jockey J have equal potential.

Step 3: Conclusion The Wheatstone balance condition is $\frac{P}{Q} = \frac{R}{S}$. In the meter bridge shown, $X = R_0 l / (100 - l)$, with l measured from the end adjoining X .

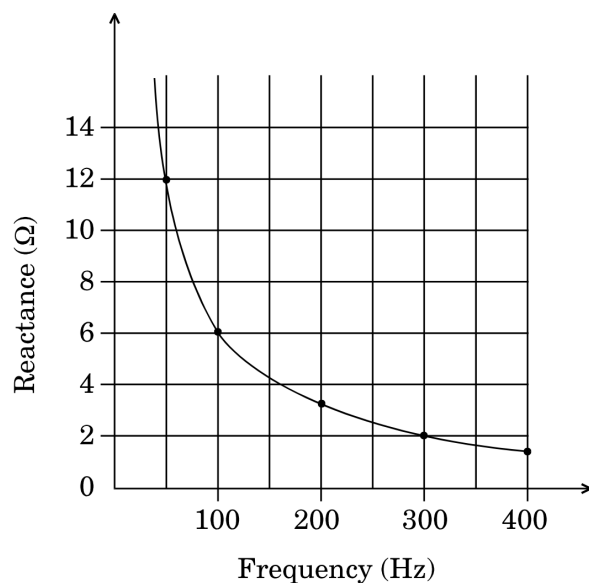
Quick Tip to Solve: Match each gap resistance to the wire length on the same side of the jockey before forming the balance ratio.

33. The figure shows the graphical variation of the reactance of a capacitor with frequency of ac source.

(a) Find the capacitance of the capacitor.

(b) An ideal inductor has the same reactance at 100 Hz frequency as the capacitor has at the same frequency. Find the value of inductance of the inductor.

(c) Draw the graph showing the variation of the reactance of this inductor with frequency. [3 marks]



Solution:

Concept:

- Capacitive reactance is $X_C = 1/(2\pi fC)$, where f is the source frequency and C is the capacitance. Thus fX_C is constant for a fixed capacitor.
- Inductive reactance is $X_L = 2\pi fL$, where L is inductance. For a fixed ideal inductor, reactance grows linearly with frequency and vanishes at zero frequency.

Step 1: (a) Read the capacitor graph and calculate its capacitance The printed graph gives $X_C = 6\ \Omega$ at $f = 100\text{ Hz}$. Use this clearly marked point rather than treating either axis intercept as finite. Rearranging the capacitor formula gives

$$C = \frac{1}{2\pi fX_C} = \frac{1}{2\pi \times 100 \times 6}\text{ F} = 2.65 \times 10^{-4}\text{ F}.$$

This equals approximately $265\ \mu\text{F}$. The reading is consistent with the other clear points: at 50 Hz , the reactance is $12\ \Omega$, and at 300 Hz , it is $2\ \Omega$. In each case, $fX_C = 600\text{ Hz}\ \Omega$, as required by the inverse-frequency relation.

Step 2: (b) Find the inductance using the equality at 100 Hz At the specified frequency, the inductor also has reactance $6\ \Omega$. Substitute this value into the inductive formula, keeping the factor 2π because the given frequency is in hertz rather than radians per second:

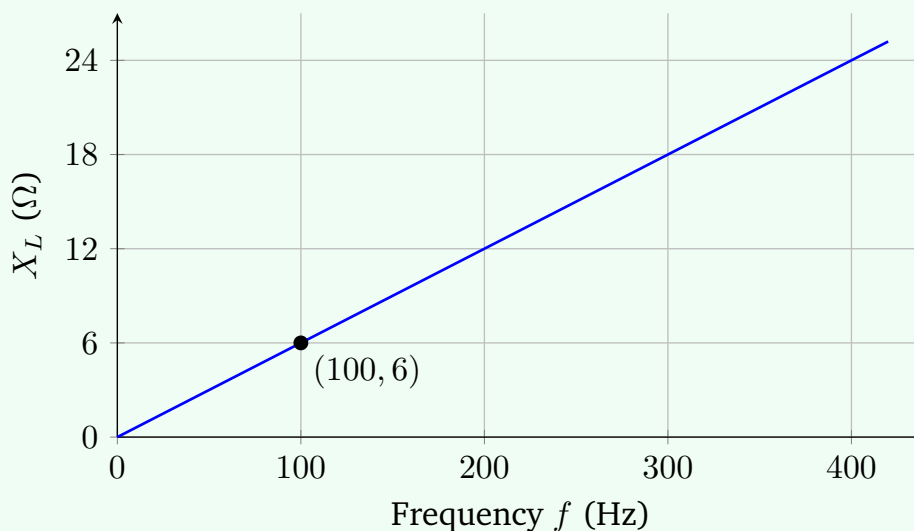
$$L = \frac{X_L}{2\pi f} = \frac{6}{2\pi \times 100}\text{ H} = 9.55 \times 10^{-3}\text{ H} = 9.55\text{ mH}.$$

Equal reactances at one frequency do not imply equal reactances at other frequencies: the capacitor reactance decreases with f , whereas the inductor reactance increases with f .

Step 3: (c) Construct the inductor reactance graph For the calculated inductance,

$$X_L = 2\pi Lf = 0.060f\ \Omega,$$

when the numerical value of f is in hertz. The graph is a straight line through the origin with positive slope $0.060\ \Omega\text{ Hz}^{-1}$. It passes through $(100\text{ Hz}, 6\ \Omega)$, $(200\text{ Hz}, 12\ \Omega)$ and $(400\text{ Hz}, 24\ \Omega)$. Label frequency on the horizontal axis and inductive reactance on the vertical axis, as shown below.



Inductive reactance increases linearly with frequency.

Step 4: Conclusion $C \approx 265 \mu\text{F}$ and $L \approx 9.55 \text{ mH}$. The inductor graph is the straight line $X_L = 0.060f$, through the origin and the point (100 Hz, 6Ω).

Quick Tip to Solve: Read the common reactance at 100 Hz once, then use the reciprocal formula for C and the direct formula for L.

34. (a) Plot V-I characteristics for an illuminated photodiode under reverse bias for three different illumination intensities $I_1 > I_2 > I_3$.

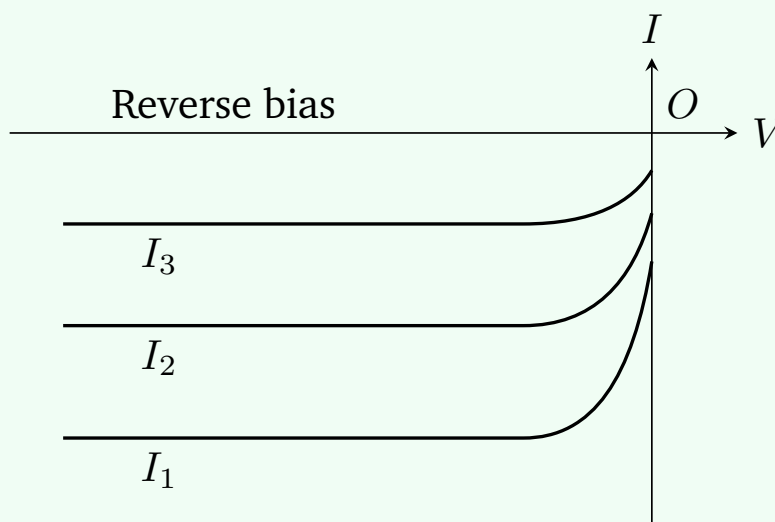
(b) In a zener regulated power supply, a zener diode with $V_Z = 6 \text{ V}$ is used for regulation. The load current should be 4.0 mA and zener current is five times the load current for unregulated input of 10 V . Find the series resistor R_s used in the power supply. [3 marks]

Solution:

Concept:

- A photodiode operates in reverse bias. Incident light creates electron-hole pairs, increasing the magnitude of the reverse photocurrent with illumination intensity.
- In a zener regulator, the load voltage is the breakdown voltage V_Z . The series resistor supplies both the load current I_L and the zener current I_Z .

Step 1: (a) Draw the illuminated photodiode characteristics Take diode voltage V on the horizontal axis and conventional diode current I on the vertical axis. Reverse bias and reverse current occupy the third quadrant. Away from the origin and before breakdown, each reverse photocurrent is nearly independent of reverse voltage. Since the specified illumination intensities obey $I_1 > I_2 > I_3$, the curve for illumination I_1 has the largest magnitude of negative current and lies lowest. The curve for I_3 lies closest to the voltage axis. Here the curve labels refer to illumination intensity, rather than the signed diode current. The sketch shows the reverse-bias operating region only.



Reverse-bias photodiode characteristics for illumination intensities $I_1 > I_2 > I_3$; the current magnitude increases with illumination.

Step 2: (b) Calculate the current through the series resistor The zener is in parallel with the load, so the regulated output is 6 V . The specified load current is $I_L = 4.0 \text{ mA}$, and hence

$$I_Z = 5I_L = 20.0 \text{ mA}, \quad I_s = I_L + I_Z = 24.0 \text{ mA}.$$

The resistor drops the difference between the unregulated input and regulated output:

$$R_s = \frac{10 - 6}{24.0 \times 10^{-3}} = 166.7 \Omega.$$

Using only the load current would omit the current required by the zener and give the wrong resistance.

Step 3: Conclusion The reverse photocurrent magnitude is greatest for illumination I_1 , followed by I_2 and I_3 , as shown. The required series resistance is $R_s \simeq 167 \Omega$.

Quick Tip to Solve: For a shunt zener regulator, add the zener and load currents before dividing the input-output voltage difference by the current.

Section D (15 marks)

35. (a) A circular loop of radius R carries a current I . Obtain an expression for the magnetic field at a point on its axis at a distance x from its centre.

(b) A conducting rod of length 2 m is placed on a horizontal table in north-south direction. It carries a current of 5 A from south to north. Find the direction and magnitude of the magnetic force acting on the rod. Given that the Earth's magnetic field at the place is 0.6×10^{-4} T and angle of dip is $\frac{\pi}{6}$. [5 marks]

Solution:

Concept:

- Biot-Savart law gives $d\mathbf{B} = (\mu_0 I / 4\pi) d\boldsymbol{\ell} \times \hat{\mathbf{r}} / r^2$, where $\mathbf{r}$ points from the current element to the observation point.
- For a circular loop, symmetry cancels transverse field components on its axis and adds the axial components.
- The force on a straight current-carrying rod is $\mathbf{F} = I\mathbf{L} \times \mathbf{B}$. At dip angle δ , the Earth's field has horizontal component $B \cos \delta$ and downward component $B \sin \delta$.

Step 1: (a) Resolve the field of a loop element along the axis Let O be the loop centre and P the axial observation point, with $OP = x$. Every current element is the same distance $r = \sqrt{R^2 + x^2}$ from P . The element $d\boldsymbol{\ell}$ is tangential to the loop and perpendicular to the line joining it to P . Thus Biot-Savart law gives the magnitude

$$dB = \frac{\mu_0 I d\ell}{4\pi(R^2 + x^2)^{3/2}}.$$

If α is the angle between $d\mathbf{B}$ and the axis, geometry gives $\cos \alpha = R/r$. For diametrically opposite elements, components perpendicular to the axis cancel, whereas axial components point in the same direction. Therefore

$$dB_{\text{axis}} = dB \cos \alpha = \frac{\mu_0 I R d\ell}{4\pi(R^2 + x^2)^{3/2}}.$$

Step 2: (a) Integrate around the complete loop All factors except $d\ell$ are constant over the loop and its circumference is $2\pi R$. Hence

$$B(x) = \frac{\mu_0 I R}{4\pi(R^2 + x^2)^{3/2}} \oint d\ell = \frac{\mu_0 I R^2}{2(R^2 + x^2)^{3/2}}.$$

The field is directed along the axis according to the right-hand thumb rule: curl the fingers in the current direction and the thumb gives the field direction. At the centre $x = 0$, the expression reduces to $\mu_0 I / (2R)$, providing a check on the derivation.

Step 3: (b) Identify the field component producing the rod's force The rod and current point horizontally northward. The horizontal component of the Earth's field also points north, so it is parallel to the current and contributes zero force. Only the downward vertical component is perpendicular to the rod:

$$B_v = B \sin \delta = (0.6 \times 10^{-4}) \sin(\pi/6) = 3.0 \times 10^{-5} \text{ T}.$$

Consequently, using $I = 5 \text{ A}$ and $L = 2 \text{ m}$,

$$F = ILB_v = 5 \times 2 \times 3.0 \times 10^{-5} = 3.0 \times 10^{-4} \text{ N}.$$

Step 4: (b) Determine the direction by the vector product Choose east, north and upward as the positive x, y, z directions. Then $\mathbf{L} = L\hat{y}$ and the relevant field is $-B_v\hat{z}$. Therefore $\mathbf{F} = -ILB_v\hat{x}$, which points horizontally westward. The magnetic force direction is independent of any friction or support force exerted by the table.

Step 5: Conclusion The magnetic field on the loop axis is $B(x) = \mu_0 IR^2 / [2(R^2 + x^2)^{3/2}]$, directed by the right-hand rule. The rod experiences a magnetic force of $3.0 \times 10^{-4} \text{ N}$, horizontally towards the west.

Quick Tip to Solve: For a north-south horizontal rod, discard the Earth's parallel horizontal field and use the vertical component $B \sin \delta$ in ILB .

OR

35. (a) Obtain the expression for the deflecting torque acting on the current carrying rectangular coil of a galvanometer in a uniform magnetic field. Why is a radial magnetic field employed in the moving coil galvanometer?

(b) Particles of mass $1.6 \times 10^{-27} \text{ kg}$ and charge $1.6 \times 10^{-19} \text{ C}$ are accelerated in a cyclotron of dee radius 40 cm. It employs a magnetic field 0.4 T. Find the kinetic energy (in MeV) of the particle beam imparted by the accelerator. [5 marks]

Solution:

Concept:

- A straight conductor of length l , carrying current I in magnetic field B , experiences force $F = BIl \sin \alpha$, where α is the angle between the current and the field.
- A coil of N turns and area A has magnetic moment $\vec{m} = NIA\hat{n}$, where $\hat{n}$ is its unit normal. Its torque is $\vec{\tau} = \vec{m} \times \vec{B}$.
- In a cyclotron the magnetic force supplies centripetal force: $qvB = mv^2/r$. The kinetic energy on emerging at the dee radius R is $K = q^2 B^2 R^2 / (2m)$ in the non-relativistic approximation.

Step 1: (a) Forces on the rectangular coil Consider a rectangular coil with N turns, length l , breadth b , and area $A = lb$. Let its normal make an angle θ with the uniform magnetic field $\vec{B}$. Choose the pair of sides of length l perpendicular to the field. The currents in these two sides are opposite, so each experiences a magnetic force of magnitude $F = NBIl$, with the two forces oppositely directed.

Their net force is zero, but their distinct lines of action form a couple. Forces on the other pair of sides do not contribute to torque about the coil's rotation axis.

Step 2: (a) Derive the deflecting torque The perpendicular separation of the lines of action of the two forces is $b \sin \theta$. Therefore the moment of the couple is

$$\begin{aligned}\tau_d &= F(b \sin \theta) \\ &= NBIlb \sin \theta \\ &= NIBA \sin \theta.\end{aligned}$$

Thus the torque tends to turn the coil's magnetic moment towards the field, in agreement with $\vec{\tau}_d = \vec{m} \times \vec{B}$. For a single turn the expression is $IBA \sin \theta$; the factor N accounts for all turns of a galvanometer coil.

Step 3: (a) Explain the radial field Curved pole pieces and a soft iron cylindrical core provide a radial magnetic field. Throughout the coil's permitted rotation the field lies in the plane of the coil, so the normal is perpendicular to the field: $\theta = 90^\circ$. Consequently $\tau_d = NIBA$ is independent of the angular deflection and is directly proportional to current. If the suspension has torsional constant κ , its restoring torque is $\kappa\phi$, where ϕ is the deflection. At equilibrium, $\kappa\phi = NIBA$, giving $\phi = (NBA/\kappa)I$. The radial field therefore produces a uniform current scale and maintains maximum magnetic torque for a given current.

Step 4: (b) Determine the exit momentum and kinetic energy Convert the dee radius to SI units: $R = 40 \text{ cm} = 0.40 \text{ m}$. At the outermost orbit the magnetic force supplies centripetal force, so

$$qvB = \frac{mv^2}{R}, \quad mv = qBR.$$

Substituting this momentum into $K = p^2/(2m)$,

$$\begin{aligned}K &= \frac{q^2 B^2 R^2}{2m} \\ &= \frac{(1.6 \times 10^{-19})^2 (0.4)^2 (0.40)^2}{2(1.6 \times 10^{-27})} \\ &= 2.048 \times 10^{-13} \text{ J}.\end{aligned}$$

The magnetic field bends the path; the electric field across the dee gap supplies the energy. The specified radius and magnetic field determine the maximum momentum reached before extraction.

Step 5: (b) Convert joules to MeV Using $1 \text{ MeV} = 1.6 \times 10^{-13} \text{ J}$,

$$K = \frac{2.048 \times 10^{-13}}{1.6 \times 10^{-13}} \text{ MeV} = 1.28 \text{ MeV}.$$

As a check, $v = qBR/m = 1.6 \times 10^7 \text{ m s}^{-1}$, which is much smaller than the speed of light and supports the non-relativistic expression used here.

Step 6: Conclusion (a) The deflecting torque is $\tau_d = NIBA \sin \theta$. In a radial field it becomes $\tau_d = NIBA$, giving deflection proportional to current and a uniform scale. (b) The beam kinetic energy is 1.28 MeV .

Quick Tip to Solve: Define the torque angle with the coil's normal, and convert the dee radius from centimetres to metres before squaring it in the cyclotron energy formula.

36. (a) Derive lens maker's formula for a biconvex lens.

(b) A point object is placed at a distance of 12 cm on the principal axis of a convex lens of focal length 10 cm. A convex mirror is placed coaxially on the other side of the lens at a distance of 10 cm. If the final image coincides with the object, sketch the ray diagram and find the focal length of the convex mirror. [5 marks]

Solution:

Concept:

- For paraxial refraction at a spherical surface, $n_2/v - n_1/u = (n_2 - n_1)/R$, where distances are measured from the surface pole using the Cartesian sign convention.
- For a thin lens in air, $1/v - 1/u = 1/f$. The lens maker's formula relates its focal length f to its refractive index μ and signed surface radii R_1, R_2 .
- A ray directed towards the centre of curvature of a spherical mirror is incident normally and retraces its path after reflection. The mirror focal length has magnitude $R/2$.

Step 1: (a) Refraction at the first surface Consider a thin biconvex lens of refractive index μ surrounded by air. Light travels from left to right, and all distances towards the right are positive. Let an axial object be at signed distance u . The first surface alone would form an image at signed distance v_1 . Refraction from air into glass gives

$$\frac{\mu}{v_1} - \frac{1}{u} = \frac{\mu - 1}{R_1}.$$

For the biconvex lens, the centre of curvature of this first surface lies to its right, so $R_1 > 0$. The derivation assumes a small aperture and paraxial rays, so the spherical-surface formula applies.

Step 2: (a) Refraction at the second surface and the focal length The image due to the first surface acts as the object for the second surface. Neglecting the thickness of the lens, its signed object distance is v_1 . If the final image distance is v , refraction from glass into air gives

$$\frac{1}{v} - \frac{\mu}{v_1} = \frac{1 - \mu}{R_2}.$$

The second surface has its centre of curvature to the left, so $R_2 < 0$. Adding the two surface equations eliminates the intermediate image distance:

$$\frac{1}{v} - \frac{1}{u} = (\mu - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right).$$

For an object at infinity, $1/u \rightarrow 0$ and $v = f$. Thus the right-hand side gives the reciprocal focal length of the lens. For two convex surfaces the signed-radius difference is positive, consistent with a converging lens.

Step 3: (b) Locate the image produced by the lens alone Take the lens optical centre L as the origin, with rightward distances positive. The point object O is at $u = -12$ cm, and the convex lens has $f_L = +10$ cm. Therefore,

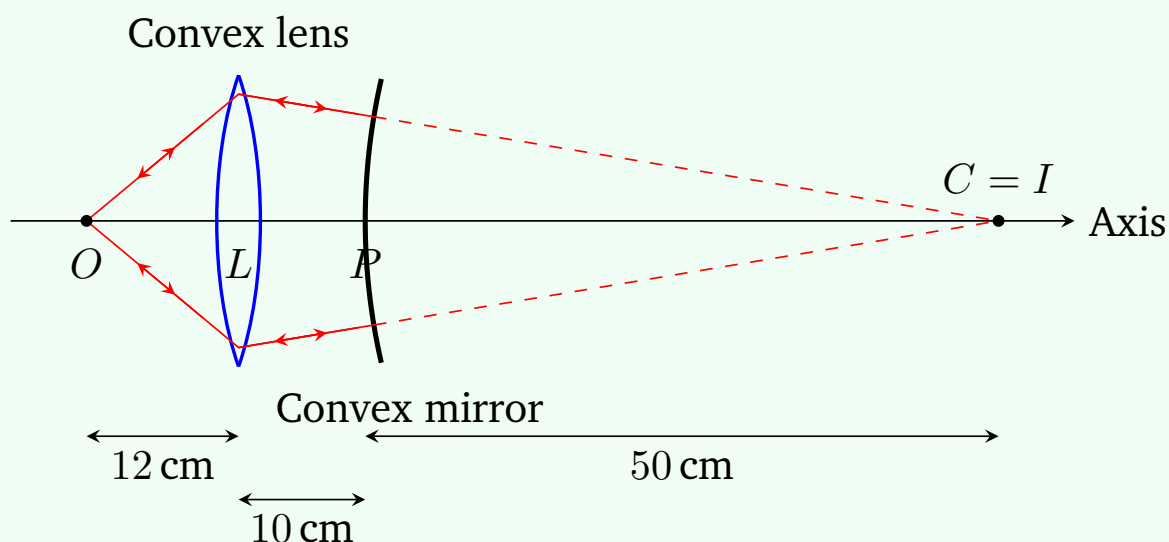
$$\begin{aligned} \frac{1}{v} - \frac{1}{-12} &= \frac{1}{10}, \\ \frac{1}{v} &= \frac{1}{10} - \frac{1}{12} = \frac{1}{60}, \\ v &= +60 \text{ cm.} \end{aligned}$$

In the absence of the mirror, rays leaving the lens would meet at I , located 60 cm to its right. Since the mirror pole P is only 10 cm to the right of the lens, the incident rays at the mirror form a converging beam directed towards a point 50 cm behind it.

Step 4: (b) Apply the condition that the image returns to the object By reversibility of light, rays returning through the lens form an image at O if the reflected beam retraces the rays that originally emerged from the lens. The mirror must therefore reflect the converging rays back along themselves. This happens when their common target I is the mirror's centre of curvature C : each incident ray is along a radius and hence strikes the mirror normally. Thus

$$PC = LC - LP = 60 - 10 = 50 \text{ cm.}$$

Equivalently, using rightward-positive distances at the mirror, its virtual object and virtual image both lie at +50 cm. The mirror formula gives $1/f_M = 1/50 + 1/50$. The figure shows two representative paraxial rays; their dashed continuations meet at C , while the reflected rays return through the lens to O .



Lens-mirror arrangement: incident and reflected rays retrace; the diagram is schematic.

Step 5: Conclusion (a) For a thin biconvex lens in air,

$$\frac{1}{f} = (\mu - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right).$$

(b) The convex mirror has radius of curvature 50 cm and focal length $f_M = +25 \text{ cm}$, with its focus behind the mirror. The final image coincides with the original object because the rays retrace their paths.

Quick Tip to Solve: First find the image distance for the lens alone; subtract the lens-mirror separation to obtain the mirror radius when the rays must retrace their paths.

36. (a) What is a wavefront? How does it propagate? Using Huygens' principle, explain reflection of a plane wavefront from a surface and verify the laws of reflection.

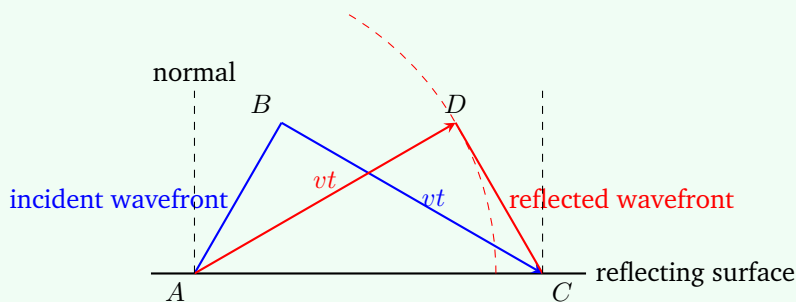
(b) A parallel beam of light of wavelength 500 nm falls on a narrow slit and the resulting diffraction pattern is obtained on a screen 1 m away. If the first minimum is formed at a distance of 2.5 mm from the centre of the screen, find the (i) width of the slit, and (ii) distance of first secondary maximum from the centre of the screen. [5 marks]

Solution:

Concept:

- A wavefront is the locus of points of a wave that are in the same phase. Rays in an isotropic medium are normal to the wavefront.
- Huygens' principle: every point of a wavefront emits secondary wavelets; their forward common envelope is the new wavefront. In time t , each wavelet travels a distance vt , where v is the wave speed.
- For single-slit diffraction, the first minimum satisfies $a \sin \theta = \lambda$. At small angles its screen position is $y_1 = D\lambda/a$, where a is the slit width and D the screen distance.

Step 1: (a) Define the wavefront and construct its reflected position All points on a wavefront have the same phase at a given instant. A plane wave has plane wavefronts. Successive wavefronts propagate through the medium as the common forward envelope of the secondary wavelets, with the rays along their normals. Let AB be a plane incident wavefront and AC the plane reflecting surface. The point A reaches the surface first. During the time t in which point B reaches C , the incident wave travels $BC = vt$. Meanwhile the reflected secondary wavelet centred at A has radius $AD = vt$. Draw the tangent CD from C to that wavelet. The common tangent CD is the reflected plane wavefront; AD is normal to it and gives the direction of a reflected ray.



Huygens construction: AB is incident and CD reflected wavefront; $BC = AD = vt$.

Step 2: (a) Verify the two laws of reflection The incident ray BC is perpendicular to AB , and the reflected ray AD is perpendicular to CD . Thus ABC and ADC are right triangles. They share hypotenuse AC and have $BC = AD = vt$. If i and r are the angles of the incident and reflected rays with the surface normal, the corresponding angles between the wavefronts and the surface are also i and r . Hence

$$\sin i = \frac{BC}{AC} = \frac{vt}{AC}, \quad \sin r = \frac{AD}{AC} = \frac{vt}{AC}.$$

Both angles lie between 0° and 90° , so $i = r$. The construction lies in the plane containing the incident ray and the normal to the reflecting surface. The reflected ray lies in this same plane. These establish both laws of reflection.

Step 3: (b)(i) Calculate the slit width Convert the given quantities to SI units:

$$\lambda = 500 \times 10^{-9} \text{ m}, \quad D = 1 \text{ m}, \quad y_1 = 2.5 \times 10^{-3} \text{ m}.$$

The small-angle approximation is appropriate since $y_1/D = 0.0025$. Applying the condition for the first minimum gives

$$a = \frac{D\lambda}{y_1} = \frac{1 \times 500 \times 10^{-9}}{2.5 \times 10^{-3}} = 2.0 \times 10^{-4} \text{ m} = 0.20 \text{ mm}.$$

Step 4: (b)(ii) Locate the first secondary maximum In the standard school-level approximation, the first secondary maximum lies halfway between the first and second minima on either side of the central maximum. Its position therefore satisfies $a \sin \theta \simeq 3\lambda/2$, giving

$$y_{\text{secondary}} \simeq \frac{3D\lambda}{2a} = \frac{3}{2}y_1 = \frac{3}{2}(2.5 \text{ mm}) = 3.75 \text{ mm}.$$

This is an approximate maximum position: the exact intensity extremum obeys $\tan \beta = \beta$, with $\beta = \pi a \sin \theta / \lambda$. Its first nonzero root is about 4.493, which would give 3.58 mm. The usual CBSE approximation requested here gives 3.75 mm.

Step 5: Conclusion A wavefront is a surface of constant phase and propagates as the envelope of Huygens' secondary wavelets. The construction gives $i = r$, with the incident ray, reflected ray and normal coplanar. The slit width is 0.20 mm; the first secondary maximum is approximately 3.75 mm from the centre on either side.

Quick Tip to Solve: Use the first-minimum distance directly: slit width is $D\lambda/y_1$, and the school-level first secondary maximum is at $1.5y_1$.

37. (a) Use Gauss's law to show that due to a uniformly charged spherical shell of radius R , the electric field at any point situated outside the shell at a distance r from its centre is equal to the electric field at the same point, when the entire charge on the shell were concentrated at its centre. Also plot the graph showing the variation of electric field with r , for $r \leq R$ and $r \geq R$.

(b) Two point charges of $+1 \mu\text{C}$ and $+4 \mu\text{C}$ are kept 30 cm apart. How far from the $+1 \mu\text{C}$ charge on the line joining the two charges, will the net electric field be zero? [5 marks]

Solution:

Concept:

- Gauss's law states $\oint \vec{E} \cdot d\vec{A} = Q_{\text{enc}}/\epsilon_0$, where Q_{enc} is the charge enclosed by a closed surface and ϵ_0 is the permittivity of free space.
- Spherical symmetry makes the electric field radial and its magnitude constant on every sphere concentric with a uniformly charged spherical shell. The area of such a sphere of radius r is $4\pi r^2$.
- The field of a positive point charge q has magnitude kq/d^2 , where d is its distance and $k = 1/(4\pi\epsilon_0)$. Fields combine by vector addition; cancellation requires opposite directions as well as equal magnitudes.

Step 1: (a) Field outside the shell Let the shell carry total charge Q . Choose a spherical Gaussian surface of radius $r > R$, with its centre at the shell's centre. At every point of this surface the field is radial, so its direction is parallel to the outward area vector for $Q > 0$. Its magnitude is the same

everywhere by spherical symmetry. The enclosed charge is the whole charge Q . Therefore,

$$\oint \vec{E} \cdot d\vec{A} = E \oint dA = E(4\pi r^2) = \frac{Q}{\epsilon_0},$$

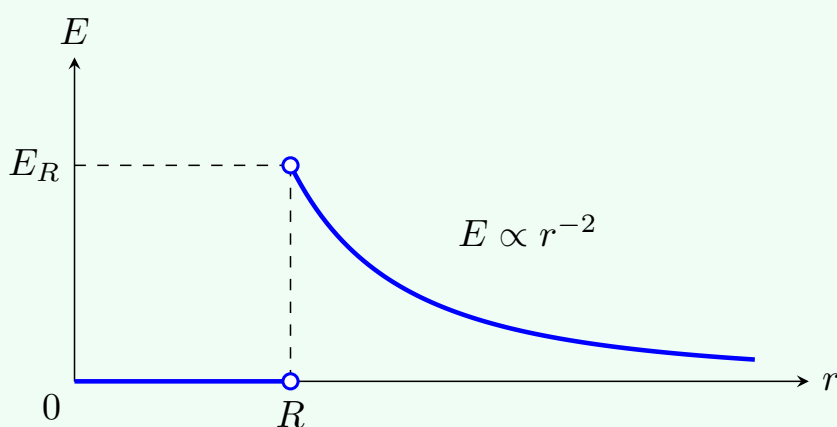
$$\vec{E}(r) = \frac{Q}{4\pi\epsilon_0 r^2} \hat{r} \quad (r > R).$$

A point charge Q placed at the centre produces precisely this field at the same position. Thus the shell and the central point charge have identical external electric fields. For negative Q , the signed radial expression remains valid and the field points inward.

Step 2: (a) Field inside and the graph at the shell For a concentric Gaussian sphere with $r < R$, no charge is enclosed, since all the charge lies on the shell. Spherical symmetry again makes the flux $E 4\pi r^2$, so Gauss's law gives $E = 0$. The field at the centre is also zero by symmetry. For a positive shell, the magnitude therefore varies as

$$E(r) = \begin{cases} 0, & 0 \leq r < R, \\ \frac{Q}{4\pi\epsilon_0 r^2}, & r > R. \end{cases}$$

The ideal infinitely thin charged surface has a discontinuity: the inner limit is $E(R^-) = 0$, whereas the outer limit is $E(R^+) = E_R = Q/(4\pi\epsilon_0 R^2)$. The graph shows these two limits at R ; its exterior branch decreases as $1/r^2$ and approaches zero at large distances. The inclusive inequalities in the question are interpreted using these inner and outer surface limits, since there is no single continuous field value on an ideal charged surface.



Field magnitude for a positive thin spherical shell; open circles show the two one-sided limits at its surface.

Step 3: (b) Locate the region where cancellation is possible Put the $+1 \mu\text{C}$ charge at $x = 0$ and the $+4 \mu\text{C}$ charge at $x = 30 \text{ cm}$. Between them, the first field points towards increasing x , while the second points towards decreasing x . Outside this interval, both fields point in the same direction, so they cannot cancel. Let the zero-field point be a distance x from the smaller charge towards the larger charge. Its distance from the larger charge is $30 - x \text{ cm}$.

Step 4: (b) Equate the two field magnitudes Using the same distance unit on both sides, cancellation

requires

$$\frac{k(1\ \mu\text{C})}{x^2} = \frac{k(4\ \mu\text{C})}{(30-x)^2},$$

$$\frac{30-x}{x} = \sqrt{4} = 2,$$

$$30-x = 2x, \quad x = 10\ \text{cm}.$$

The positive square root is chosen because both distances are positive in the interval considered. As a check, at distances 0.10 m and 0.20 m, respectively, each field has magnitude $9.0 \times 10^5\ \text{N C}^{-1}$, using $k = 9.0 \times 10^9\ \text{N m}^2\text{C}^{-2}$. Their directions are opposite, so their vector sum is zero.

Step 5: Conclusion (a) Outside the shell, $\vec{E} = Q\hat{r}/(4\pi\epsilon_0 r^2)$, exactly as for a charge Q concentrated at its centre. Inside the shell the field is zero, and the external field follows an inverse-square curve after a jump at the surface. (b) The net field vanishes 10 cm from the $+1\ \mu\text{C}$ charge, between the charges, towards the $+4\ \mu\text{C}$ charge.

Quick Tip to Solve: For two like charges, first restrict the null point to the interval between them; the distances from the charges are in the ratio of the square roots of their charges.

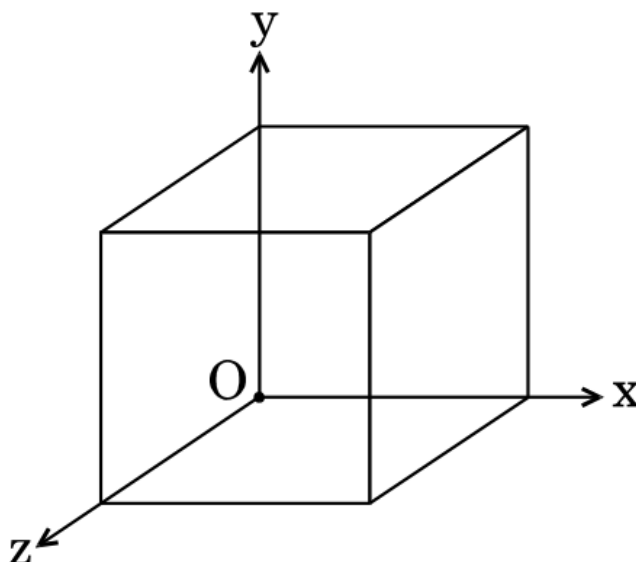
OR

37. (a) Two point charges q_1 and q_2 are kept r distance apart in a uniform external electric field $\vec{E}$. Find the amount of work done in assembling this system of charges.

(b) A cube of side 20 cm is kept in a region as shown in the figure. An electric field $\vec{E}$ exists in the region such that the potential at a point is given by $V = 10x + 5$, where V is in volt and x is in m.

Find the (i) electric field $\vec{E}$, and (ii) total electric flux through the cube.

[5 marks]



Solution:

Concept:

- The work done by an external agent in assembling stationary charges slowly equals the increase in electrostatic potential energy. Both the external-field potential and the mutual interaction of the charges contribute.
- If $V_{\text{ext}}(\vec{r})$ is the potential of the imposed field, the energy of a charge q at $\vec{r}$ is $qV_{\text{ext}}(\vec{r})$. The mutual

energy of two point charges separated by r is $q_1 q_2 / (4\pi\epsilon_0 r)$.

- Electric field is the negative potential gradient: $\vec{E} = -\nabla V$. Electric flux through a closed surface is $\Phi = \oint \vec{E} \cdot d\vec{A}$, where $d\vec{A}$ points outward.

Step 1: (a) Bring the first charge into the external field Denote the final positions of the charges by $\vec{r}_1$ and $\vec{r}_2$, with $|\vec{r}_2 - \vec{r}_1| = r$. Choose a reference for the external potential and write its values at these positions as V_1 and V_2 . Bringing q_1 slowly from the chosen zero-potential reference gives the external work $W_1 = q_1 V_1$. There is no mutual interaction term yet because the second charge has not been placed.

Step 2: (a) Add the second charge and obtain the total work The potential experienced by q_2 at its final position is the sum of the imposed potential V_2 and the potential $q_1/(4\pi\epsilon_0 r)$ due to the first charge. Thus

$$W_2 = q_2 V_2 + \frac{q_1 q_2}{4\pi\epsilon_0 r}, \quad W = W_1 + W_2 = q_1 V_1 + q_2 V_2 + \frac{q_1 q_2}{4\pi\epsilon_0 r}.$$

The interaction term is counted once, since it is introduced when the second charge is added. For a uniform external field, $V_{\text{ext}}(\vec{r}) = V_0 - \vec{E} \cdot \vec{r}$, so an equivalent expression is

$$W = (q_1 + q_2)V_0 - \vec{E} \cdot (q_1 \vec{r}_1 + q_2 \vec{r}_2) + \frac{q_1 q_2}{4\pi\epsilon_0 r}.$$

A numerical work cannot follow from separation alone: the external potential reference and final positions must also be specified. For an ideal field uniform throughout space, infinity cannot be used as a finite zero-potential reference; the expression uses the chosen reference for assembly.

Step 3: (b)(i) Differentiate the given potential The potential $V = 10x + 5$ depends only on x . Its derivatives are $\partial V/\partial x = 10 \text{ V m}^{-1}$, $\partial V/\partial y = 0$ and $\partial V/\partial z = 0$. Hence

$$\vec{E} = -\left(\frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j} + \frac{\partial V}{\partial z}\hat{k}\right) = -10\hat{i} \text{ V m}^{-1}.$$

The field points along the negative x -axis, towards decreasing potential. The constant 5 V makes no contribution to the gradient.

Step 4: (b)(ii) Sum the outward fluxes through the cube The cube side is $a = 20 \text{ cm} = 0.20 \text{ m}$, so each face has area $A = a^2 = 0.040 \text{ m}^2$. On the face with outward normal $+\hat{i}$, the flux is $-10A = -0.40 \text{ N m}^2 \text{ C}^{-1}$. On the opposite face, whose outward normal is $-\hat{i}$, it is $+10A = +0.40 \text{ N m}^2 \text{ C}^{-1}$. The field is parallel to the other four faces, giving zero flux through each. Therefore

$$\Phi_{\text{cube}} = -0.40 + 0.40 + 0 = 0 \text{ N m}^2 \text{ C}^{-1}.$$

Equal flux enters and leaves the cube because the field is spatially uniform; this also agrees with Gauss's law for zero net enclosed charge.

Step 5: Conclusion (a) The assembly work is $W = q_1 V_{\text{ext}}(\vec{r}_1) + q_2 V_{\text{ext}}(\vec{r}_2) + q_1 q_2 / (4\pi\epsilon_0 r)$, for the chosen external-potential reference. (b) $\vec{E} = -10\hat{i} \text{ N C}^{-1}$ and the total outward flux through the cube is zero.

Quick Tip to Solve: Count the mutual interaction energy once when assembling two charges. For a linear potential, differentiate its slope with a minus sign; opposite faces of a closed cube then cancel their fluxes.