

CBSE Class 12 Physics Question Paper 2020 with Solution PDF-Set 55/2/3



Question Paper with Solutions

Conducted by the Central Board of Secondary Education (CBSE)

Time Allowed: 3 hours

Maximum Marks: 70

General Instructions

- (i) This question paper comprises **four** Sections – **A, B, C and D**.
- (ii) There are **37** questions in the question paper. **All** questions are compulsory.
- (iii) **Section A** – Questions no. **1 to 20** are very short answer type questions, carrying **1** mark each.
- (iv) **Section B** – Questions no. **21 to 27** are short answer type questions, carrying **2** marks each.
- (v) **Section C** – Questions no. **28 to 34** are long answer type questions, carrying **3** marks each.
- (vi) **Section D** – Questions no. **35 to 37** are also long answer type questions, carrying **5** marks each.
- (vii) There is no overall choice in the question paper. However, an internal choice has been provided in **2** questions of **1** mark, **2** questions of **2** marks, **1** question of three marks and all the **3** questions of five marks. You have to attempt only one of the choices in such questions.
- (viii) In addition to this, separate instructions are given with each section and question, wherever necessary.
- (ix) Use of calculators and log tables is not permitted.

You may use the following values of physical constants wherever necessary :

$$c = 3 \times 10^8 \text{ m/s}$$

$$h = 6.63 \times 10^{-34} \text{ Js}$$

$$e = 1.6 \times 10^{-19} \text{ C}$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ T m A}^{-1}$$

$$\epsilon_0 = 8.854 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$$

$$\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$$

$$\text{Mass of electron } (m_e) = 9.1 \times 10^{-31} \text{ kg}$$

$$\text{Mass of neutron} = 1.675 \times 10^{-27} \text{ kg}$$

$$\text{Mass of proton} = 1.673 \times 10^{-27} \text{ kg}$$

$$\text{Avogadro's number} = 6.023 \times 10^{23} \text{ per gram mole}$$

$$\text{Boltzmann constant} = 1.38 \times 10^{-23} \text{ JK}^{-1}$$

Section A (20 marks)

Note : Select the most appropriate option from those given below each question :

1. For a fixed potential difference applied across a conductor, the drift speed of free electrons does not depend upon [1 mark]

- (A) free electron density in the conductor.
- (B) mass of the electrons.
- (C) length of the conductor.
- (D) temperature of the conductor.

Correct Answer: (A) free electron density in the conductor.

Solution:

Concept:

- The drift speed is $v_d = eE\tau/m$, where e is the elementary charge, τ the relaxation time and m the electron mass.
- For potential difference V across length L , $E = V/L$.

Step 1: Use the microscopic expression Substitution gives

$$v_d = \frac{eV\tau}{mL}.$$

Free electron number density n does not occur in this expression. It affects the current through $I = neAv_d$, where A is the cross-sectional area, rather than being an independent factor in the drift-speed formula.

Step 2: Check the other choices Electron mass appears in the denominator, and changing the conductor length changes the electric field for the specified fixed voltage. Temperature affects collision frequency and hence τ . Thus choices (B), (C) and (D) describe factors on which drift speed depends in the standard relaxation-time model.

Step 3: Conclusion The drift speed does not depend upon free electron density. Hence, the correct option is (A).

Quick Tip to Solve: For fixed voltage, substitute $E = V/L$ into $v_d = eE\tau/m$ before comparing the factors.

2. Ohm's law is obeyed by [1 mark]

- (A) extrinsic semiconductors.
- (B) intrinsic semiconductors.
- (C) metals at low temperature.
- (D) metals at high temperature.

Correct Answer: (C) metals at low temperature.

Solution:

Concept:

- Ohmic behaviour means $V = IR$, with resistance R independent of applied voltage under unchanged physical conditions.

- In the elementary metallic conduction model, small electric fields give $J = \sigma E$, with conductivity σ effectively constant.

Step 1: Identify the intended ohmic material Metals at low temperature, in the usual regime of ordinary metallic conduction, exhibit an approximately linear current-voltage relation. Weak thermal agitation and controlled temperature allow the resistance to remain effectively constant during the measurement.

Step 2: Compare the alternatives Intrinsic and extrinsic semiconductors are not the standard ohmic examples in this classification: carrier populations and transport can change with conditions, giving nonlinear response. The high-temperature choice is rejected in the intended textbook comparison because substantial heating changes metallic resistance. Ohm's law always presumes constant physical conditions; the answer is not a claim that every metal at any low temperature is ohmic, including its superconducting state.

Step 3: Conclusion The intended textbook answer is metals at low temperature. Hence, the correct option is (C).

Quick Tip to Solve: Look for an ordinary metal with controlled temperature when identifying the textbook ohmic example.

3. Electromagnetic waves used as a diagnostic tool in medicine are

[1 mark]

- (A) X-rays.
- (B) ultraviolet rays.
- (C) infrared radiation.
- (D) ultrasonic waves.

Correct Answer: (A) X-rays.

Solution:

Concept:

- X-rays are high-frequency electromagnetic radiation that can penetrate soft tissue.
- Different absorption by bone and soft tissue produces contrast in diagnostic radiographs.

Step 1: Connect the radiation to diagnosis In an X-ray examination, transmitted radiation is recorded after passing through the body. Bone absorbs X-rays more strongly than many soft tissues, so their differing transmission forms an image useful for identifying fractures and other internal conditions.

Step 2: Reject the competing options Ultraviolet radiation is associated with effects such as fluorescence and sterilisation, rather than the penetrating radiographic technique asked here. Infrared radiation is associated with thermal effects and thermal imaging, but X-rays are the standard diagnostic radiation intended in this spectrum question. Ultrasonic waves are also used for medical diagnosis, yet they are mechanical sound waves and are not electromagnetic waves.

Step 3: Conclusion X-rays are the electromagnetic radiation intended here. Hence, the correct option is (A).

Quick Tip to Solve: Ultrasound is diagnostic but mechanical; for an electromagnetic radiograph, select X-rays.

4. A straight current carrying conductor is placed inside a uniform magnetic field. The force per unit length acting on the conductor is [1 mark]

- (A) maximum when the conductor is perpendicular to the direction of magnetic field.
- (B) maximum when the conductor is along the direction of magnetic field.
- (C) minimum when the conductor is perpendicular to the direction of magnetic field.
- (D) minimum when the conductor makes an angle of 45° with the direction of magnetic field.

Correct Answer: (A) maximum when the conductor is perpendicular to the direction of magnetic field.

Solution:

Concept:

- The magnetic force on a straight segment is $\vec{F} = I\vec{L} \times \vec{B}$.
- Its magnitude per unit length is $F/L = IB \sin \theta$, where θ is the angle between current and field.

Step 1: Find the maximum For fixed current I and magnetic field magnitude B , the angle alone determines the force variation. Since $\sin \theta \leq 1$, the largest possible force per unit length is IB , reached at $\theta = 90^\circ$, when the conductor is perpendicular to the field.

Step 2: Test the remaining angles Along the field, $\theta = 0^\circ$ and $F/L = 0$, so (B) is false. Perpendicular orientation gives the maximum, making (C) false. At 45° , $F/L = IB/\sqrt{2}$, an intermediate nonzero value, not the minimum; hence (D) is also false.

Step 3: Conclusion The force per unit length is maximum for a perpendicular conductor. Hence, the correct option is (A).

Quick Tip to Solve: Use the sine of the angle between current and field: parallel gives zero, perpendicular gives maximum.

5. In a travelling plane electromagnetic wave, which of the following have zero average value? [1 mark]

- (A) Magnetic energy and electric energy
- (B) Magnetic field and electric field
- (C) Magnetic energy and magnetic field
- (D) Electric energy and electric field

Correct Answer: (B) Magnetic field and electric field

Solution:

Concept:

- The electric and magnetic fields of a sinusoidal travelling plane wave oscillate about zero.
- The energy densities are $u_E = \epsilon_0 E^2/2$ and $u_B = B^2/(2\mu_0)$, which depend on squared fields.

Step 1: Average the fields over a cycle At a fixed point write $E = E_0 \cos \phi$ and $B = B_0 \cos \phi$, where ϕ advances through 2π in one period. Positive and negative half cycles cancel, giving $\langle E \rangle = 0$ and $\langle B \rangle = 0$.

Step 2: Distinguish energy from field Squaring removes the alternating sign. Since $\langle \cos^2 \phi \rangle = 1/2$, the average energy densities are $\langle u_E \rangle = \epsilon_0 E_0^2/4$ and $\langle u_B \rangle = B_0^2/(4\mu_0)$. Both are positive for a nonzero wave. Options (A), (C) and (D) each include an energy whose average therefore is not zero.

Step 3: Conclusion Both magnetic field and electric field have zero cycle averages. Hence, the correct

option is (B).

Quick Tip to Solve: A sinusoidal field averages to zero, but its square averages to half the peak square.

6. In β^- decay, a

[1 mark]

- (A) neutron converts into a proton emitting antineutrino.
- (B) neutron converts into a proton emitting neutrino.
- (C) proton converts into a neutron emitting antineutrino.
- (D) proton converts into a neutron emitting neutrino.

Correct Answer: (A) neutron converts into a proton emitting antineutrino.

Solution:

Concept:

- In beta-minus decay, a neutron becomes a proton while emitting an electron and an electron antineutrino.
- Charge, energy, momentum and lepton number are conserved in the decay.

Step 1: Write the elementary decay The nuclear process is

$$n \longrightarrow p + e^- + \bar{\nu}_e.$$

The electron is the beta-minus particle. The emitted neutral lepton is the electron antineutrino, needed together with the electron to conserve lepton number.

Step 2: Compare the listed conversions The neutron-to-proton conversion increases the nuclear atomic number by one and leaves the mass number unchanged. This agrees with (A). Choice (B) names a neutrino instead of an antineutrino, so it is incorrect. Choices (C) and (D) reverse the nucleon conversion; proton-to-neutron conversion is associated with beta-plus decay or electron capture, rather than beta-minus decay.

Step 3: Conclusion A neutron converts into a proton, emitting an electron and an antineutrino. Hence, the correct option is (A).

Quick Tip to Solve: Remember beta-minus decay as $n \rightarrow p + e^- + \bar{\nu}_e$: atomic number rises by one.

7. An electron is released from rest in a region of uniform electric and magnetic fields acting parallel to each other. The electron will

[1 mark]

- (A) move in a straight line.
- (B) move in a circle.
- (C) remain stationary.
- (D) move in a helical path.

Correct Answer: (A) move in a straight line.

Solution:

Concept:

- The Lorentz force is $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$.

- For an electron, $q = -e$; its electric acceleration is opposite to $\vec{E}$.

Step 1: Determine the initial motion The electron is released from rest, so $\vec{v} = 0$ initially and the magnetic force is zero. The electric force $-e\vec{E}$ accelerates it along a line opposite to the electric field. Since the fields are parallel, this line is also parallel or antiparallel to the magnetic field.

Step 2: Check whether magnetic curvature develops At all later times the velocity remains along that same line, so $\vec{v} \times \vec{B} = 0$. There is no transverse magnetic force to produce a circle or the circular component of a helix. The nonzero electric force prevents the electron from remaining stationary. Thus (B), (C) and (D) are excluded by the given initial condition.

Step 3: Conclusion The electron moves in a straight line opposite to the electric field. Hence, the correct option is (A).

Quick Tip to Solve: For release from rest in parallel fields, electric acceleration creates no velocity perpendicular to the magnetic field.

8. At equilibrium, in a p-n junction diode the net current is

[1 mark]

- (A) due to diffusion of majority charge carriers.
- (B) due to drift of minority charge carriers.
- (C) zero as diffusion and drift currents are equal and opposite.
- (D) zero as no charge carriers cross the junction.

Correct Answer: (C) zero as diffusion and drift currents are equal and opposite.

Solution:

Concept:

- Diffusion carries majority carriers across a junction because their concentrations differ on its two sides.
- Drift is caused by the built-in electric field of the depletion region. At equilibrium, the two conventional currents cancel.

Step 1: Identify diffusion and drift Electrons diffuse from the n-side to the p-side, while holes diffuse from the p-side to the n-side. The depletion-region field drives minority carriers in the opposite sense, producing a drift current opposite to the diffusion current.

Step 2: Apply the equilibrium condition At thermal equilibrium there is no external bias and no net current:

$$I_{\text{net}} = I_{\text{diffusion}} + I_{\text{drift}} = 0.$$

Options (A) and (B) describe only one component. Option (D) is incorrect because carriers still cross the junction; their opposing currents are equal.

Step 3: Conclusion The net current is zero because diffusion and drift currents are equal and opposite. Correct option: (C).

Quick Tip to Solve: Equilibrium means cancellation of the two currents, not cessation of carrier motion.

9. In an n-type semiconductor, the donor energy level lies

[1 mark]

- (A) at the centre of the energy gap.
- (B) just below the conduction band.
- (C) just above the valance band.
- (D) in the conduction band.

Correct Answer: (B) just below the conduction band.

Solution:

Concept:

- An n-type semiconductor contains pentavalent donor impurities, each supplying an extra electron.
- The donor level lies in the forbidden gap close to the conduction-band edge, so little thermal energy is needed to free its electron.

Step 1: Locate the donor level Four of the five valence electrons of a donor form covalent bonds with neighbouring atoms. The fifth is weakly bound, so its energy level is just below the conduction band rather than deep in the forbidden gap.

Step 2: Distinguish the alternatives A small thermal excitation promotes this electron into the conduction band, where it contributes to conduction. A level just above the valence band characterises an acceptor impurity, so (C) is wrong. The donor level is neither at the centre of the gap nor inside the conduction band, eliminating (A) and (D).

Step 3: Conclusion The donor energy level lies just below the conduction band. Correct option: (B).

Quick Tip to Solve: Remember donor electrons enter the conduction band, so their level is immediately below it.

10. When two nuclei ($A \leq 10$) fuse together to form a heavier nucleus, the

[1 mark]

- (A) binding energy per nucleon increases.
- (B) binding energy per nucleon decreases.
- (C) binding energy per nucleon does not change.
- (D) total binding energy decreases.

Correct Answer: (A) binding energy per nucleon increases.

Solution:

Concept:

- Binding energy per nucleon is B/A , where B is the total binding energy and A the mass number.
- For very light nuclei, the binding-energy-per-nucleon curve rises as mass number increases toward the iron region.

Step 1: Use the light-nucleus region The printed condition $A \leq 10$ places the initial nuclei on the rising part of the binding-energy-per-nucleon curve. Fusion into a heavier light nucleus produces nucleons that are more tightly bound on average.

Step 2: Relate the increase to released energy The product has a larger binding energy per nucleon. Its larger total binding energy corresponds to a decrease in rest mass, with energy released according to $Q = \Delta mc^2$. Thus (B) and (C) contradict the rising curve, while (D) incorrectly predicts a decrease in total binding energy.

Step 3: Conclusion The binding energy per nucleon increases. Correct option: (A).

Quick Tip to Solve: For fusion of very light nuclei, move rightward on the rising binding-energy-per-nucleon curve.

11. The potential difference between two points in vacuum is V_0 . If vacuum is replaced by a medium of dielectric constant K , the new value of potential difference will be _____. [1 mark]

Solution:

Concept:

- The dielectric constant is $K = \epsilon/\epsilon_0$, with ϵ the permittivity of the medium.
- For an unchanged source-charge distribution in a uniform dielectric, the electrostatic field and potential difference are reduced by the factor K .

Step 1: Compare the electric fields For the same charges and geometry, Coulomb field expressions contain the factor $1/\epsilon$. Replacing vacuum by a dielectric therefore gives $\mathbf{E} = \mathbf{E}_0/K$. This is the fixed-source-charge interpretation of the stated replacement.

Step 2: Integrate between the two points Potential difference is the negative line integral of the field. Since K is constant throughout the medium,

$$V = - \int \mathbf{E} \cdot d\mathbf{l} = \frac{1}{K} \left(- \int \mathbf{E}_0 \cdot d\mathbf{l} \right) = \frac{V_0}{K}.$$

The two points remain the same, so only the permittivity changes the result.

Step 3: Conclusion The blank is V_0/K .

Quick Tip to Solve: For fixed charges, divide the vacuum potential difference by the dielectric constant.

12. Unpolarised light passes from a rarer into a denser medium. If the reflected and the refracted rays are mutually perpendicular, the reflected light is linearly polarised _____ to the plane of incidence. [1 mark]

Solution:

Concept:

- At the polarising angle (Brewster angle), the reflected and refracted rays are perpendicular.
- The plane of incidence contains the incident ray and the normal to the surface. Polarisation specifies the direction of the electric-field oscillation.

Step 1: Recognise Brewster reflection The given perpendicularity is the Brewster condition: $i_B + r = 90^\circ$. At this incidence angle, the component with its electric field parallel to the plane of incidence has zero reflected amplitude.

Step 2: Identify the surviving component Only the component whose electric field oscillates perpen-

pendicular to the plane of incidence remains in the reflected beam. Consequently the reflected light is completely linearly polarised in that direction. This statement concerns electric-field oscillation; it does not mean the reflected ray travels out of the plane of incidence.

Step 3: Conclusion The blank is perpendicular to the plane of incidence.

Quick Tip to Solve: At Brewster incidence, remember that the reflected electric field is perpendicular to the incidence plane.

13. The resolving power of a compound microscope can be _____ by using a medium (oil) of higher refractive index than air between the object and the objective lens. [1 mark]

Solution:

Concept:

- The numerical aperture is $NA = n \sin \theta$, where n is the refractive index around the objective and θ its collection half-angle.
- The minimum resolvable separation is $d_{\min} = 0.61\lambda / (n \sin \theta)$, so resolving power is proportional to $n \sin \theta$.

Step 1: Examine the effect of immersion oil Using oil of refractive index greater than that of air increases the numerical aperture for a given collection angle. The higher index allows finer object detail to be resolved by the objective.

Step 2: Convert separation into resolving power For a fixed wavelength,

$$d_{\min} = \frac{0.61\lambda}{n \sin \theta}, \quad RP = \frac{1}{d_{\min}}.$$

Increasing n reduces the minimum distinguishable separation and therefore increases the resolving power. This improvement is an ability to distinguish nearby points, rather than merely an increase in magnification.

Step 3: Conclusion The blank is increased.

Quick Tip to Solve: Higher immersion-medium index increases numerical aperture and decreases resolvable separation.

14. In Young's double slit experiment, the path difference between two interfering waves at a point on the screen is $\frac{5\lambda}{2}$, λ being the wavelength of the light used. The _____ dark fringe will lie at this point. [1 mark]

Solution:

Concept:

- In Young's double slit experiment, destructive interference occurs when the path difference is an odd multiple of half the wavelength: $\Delta = (2n - 1)\lambda/2$, where $n = 1, 2, 3, \dots$ counts dark fringes from the central bright fringe on either side.
- The first dark fringe has $\Delta = \lambda/2$; the central fringe has zero path difference and is bright.

Step 1: Match the path difference to the dark-fringe condition The given path difference is $\Delta = 5\lambda/2$.

Equating this to $(2n - 1)\lambda/2$ gives $2n - 1 = 5$, hence $2n = 6$ and $n = 3$.

Step 2: Check the counting from the centre The successive dark fringes have path differences $\lambda/2$, $3\lambda/2$ and $5\lambda/2$. Thus the specified point is the third dark fringe. The coefficient five counts half-wavelengths of path difference, rather than the fringe number; confusing these gives an incorrect fifth dark fringe.

Step 3: Conclusion The third dark fringe will lie at this point.

Quick Tip to Solve: For a dark fringe with path difference $k\lambda/2$, where k is odd, use $n = (k + 1)/2$ to find its order.

OR

14. If one of the slits in Young's double slit experiment is fully closed, the new pattern has _____ central maximum in angular size. [1 mark]

Solution:

Concept:

- A slit of finite width produces diffraction; its first minima satisfy $a \sin \theta = \lambda$, where a is the slit width.
- Interference requires two coherent waves, while a single open slit retains its diffraction envelope.

Step 1: Identify the new pattern Closing one slit removes the two-slit interference fringes. The remaining finite-width slit produces a single-slit diffraction pattern, whose bright central region extends between the first minima on either side.

Step 2: Compare angular widths The single-slit central maximum has angular width approximately $2\lambda/a$. The original interference maxima have widths governed by the slit separation d , which exceeds the slit width. Thus the surviving diffraction central maximum is broader in angular size than an individual interference maximum.

Step 3: Conclusion The blank is **wider** (or broader).

Quick Tip to Solve: Distinguish the narrow interference fringes from the broad diffraction envelope of one slit.

15. At sunset or sunrise, the _____ scattered coloured light from the sun reaches our eyes. [1 mark]

Solution:

Concept:

- For molecules much smaller than the wavelength, Rayleigh scattering varies approximately as $1/\lambda^4$.
- Red light has a longer wavelength than blue light and is scattered less strongly.

Step 1: Consider the atmospheric path At sunrise and sunset, sunlight travels a much longer distance

through the atmosphere before reaching an observer. The shorter wavelengths, especially blue and violet, are scattered strongly out of the direct beam along this path.

Step 2: Identify the surviving colour The longer red wavelengths undergo the least scattering and remain relatively abundant in the light reaching the eyes from the Sun. Hence the intended colour in the printed blank is red; the reddish appearance is caused by preferential removal of the shorter wavelengths.

Step 3: Conclusion The blank is red.

Quick Tip to Solve: For sunrise and sunset, a longer atmospheric path removes blue more strongly and leaves red.

16. Define the term ‘threshold frequency’ in photoelectric emission.

[1 mark]

Solution:

Concept:

- A photon has energy $h\nu$, where h is Planck's constant and ν is its frequency.
- The work function ϕ is the minimum energy required to release an electron from a surface.

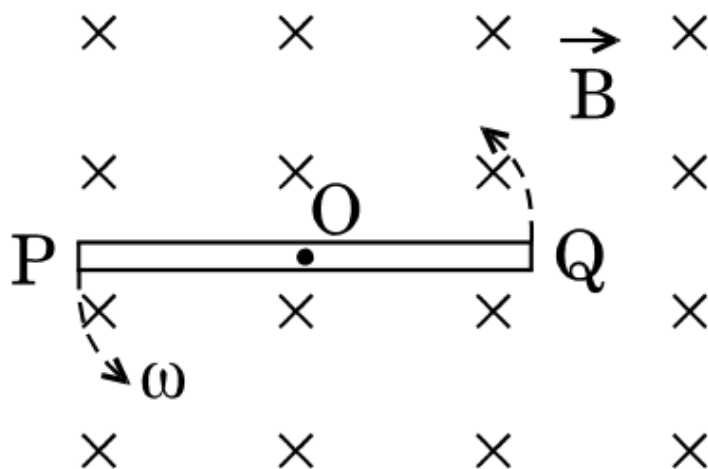
Step 1: Apply the emission condition Photoelectric emission can occur only when the energy supplied by an incident photon is at least the work function of the material: $h\nu \geq \phi$. At the limiting frequency, an emitted electron has zero maximum kinetic energy.

Step 2: State the defining frequency The threshold frequency ν_0 is therefore the minimum frequency of incident electromagnetic radiation that can cause photoelectric emission from the given surface. It satisfies $h\nu_0 = \phi$; radiation below this frequency cannot eject electrons even if its intensity is increased.

Step 3: Conclusion Threshold frequency is the minimum incident-radiation frequency required for photoelectric emission from a given surface: $\nu_0 = \phi/h$.

Quick Tip to Solve: Frequency sets photon energy; increasing intensity cannot overcome a frequency below threshold.

17. A metallic rod PQ of length l is rotated with an angular velocity ω in a magnetic field $\vec{B}$ about an axis passing through the mid-point O of the rod and perpendicular to the plane of paper as shown in the figure. What is the potential difference developed between the points O and Q of the rod? [1 mark]



Solution:

Concept:

- A point at distance r from the rotation axis has speed $v = \omega r$.
- The motional emf across a radial element dr in a perpendicular field has magnitude $d\varepsilon = B\omega r dr$.

Step 1: Choose the correct rotating length The axis passes through the midpoint O, so the segment OQ has length $l/2$, not l . The speed varies continuously from zero at O to $\omega l/2$ at Q, so the emf must be integrated over this half of the rod.

Step 2: Integrate and determine polarity The magnitude is

$$|V_Q - V_O| = \int_0^{l/2} B\omega r dr = \frac{B\omega l^2}{8}.$$

The arrows show anticlockwise rotation and the crosses show a field into the paper. At Q, $\vec{v} \times \vec{B}$ points towards O, driving positive charge inward; hence O is at the higher potential.

Step 3: Conclusion The potential-difference magnitude is $B\omega l^2/8$; for the shown rotation, $V_O - V_Q = B\omega l^2/8$.

Quick Tip to Solve: Use half the rod length for midpoint-to-end emf; the motional emf depends on its square.

18. What is the impedance of a capacitor of capacitance C in an ac circuit using source of frequency n Hz? [1 mark]

Solution:

Concept:

- The capacitive reactance is $X_C = 1/(\omega C)$, where C is capacitance.
- For source frequency n in hertz, angular frequency is $\omega = 2\pi n$.

Step 1: Relate frequency to angular frequency A sinusoidal source makes n complete cycles per

second. Each cycle spans 2π radians, so the angular frequency entering the reactance formula is $2\pi n$, rather than simply n .

Step 2: Evaluate the capacitor impedance An ideal capacitor has no resistive component. Its impedance magnitude equals its capacitive reactance, giving $|Z_C| = X_C = 1/(2\pi nC)$. In the usual phasor convention the complex impedance is $Z_C = 1/(j\omega C) = -j/(2\pi nC)$, expressing that voltage lags current by a quarter cycle.

Step 3: Conclusion The impedance magnitude is $\boxed{1/(2\pi nC)}$ ohm.

Quick Tip to Solve: Convert frequency in hertz to angular frequency before using the capacitor reactance formula.

OR

18. What is the value of impedance of a resonant series LCR circuit?

[1 mark]

Solution:

Concept:

- The series LCR impedance magnitude is $Z = \sqrt{R^2 + (X_L - X_C)^2}$.
- At resonance the inductive and capacitive reactances are equal: $X_L = X_C$.

Step 1: Use the resonance condition In a series LCR circuit, the inductor and capacitor contribute opposite reactive voltage drops. At the resonant frequency $\omega_0 = 1/\sqrt{LC}$, the condition $\omega_0 L = 1/(\omega_0 C)$ makes their net reactance zero.

Step 2: Reduce the impedance expression Substituting $X_L - X_C = 0$ into the impedance formula gives $Z = \sqrt{R^2} = R$, since resistance is nonnegative. The circuit then behaves as a purely resistive load, with current and applied voltage in phase. Individual reactances need not be zero; they cancel each other.

Step 3: Conclusion The impedance of a resonant series LCR circuit is $\boxed{Z = R}$.

Quick Tip to Solve: At series resonance cancel the two reactances, but retain the circuit resistance.

19. The threshold frequencies of two photosensitive surfaces are ν_1 and ν_2 , respectively. What is the ratio of the velocities of the photoelectrons emitted from these surfaces when light of frequency ν is incident on them and photoemission occurs?

[1 mark]

Solution:

Concept:

- Einstein's photoelectric equation is $K_{\max} = h(\nu - \nu_0)$.
- For an electron of mass m_e , $K_{\max} = \frac{1}{2}m_e v_{\max}^2$.

Step 1: Write equations for the two surfaces For the same incident frequency ν , the two work functions are $h\nu_1$ and $h\nu_2$. Therefore the maximum emitted speeds satisfy

$$\frac{1}{2}m_e v_1^2 = h(\nu - \nu_1), \quad \frac{1}{2}m_e v_2^2 = h(\nu - \nu_2).$$

Both differences are positive when photoemission with nonzero speeds occurs.

Step 2: Divide and take the square root Dividing cancels the identical electron mass and Planck constant: $v_1^2/v_2^2 = (\nu - \nu_1)/(\nu - \nu_2)$. Taking the positive square root gives the ratio of maximum speeds. Actual emitted electrons may have a range of energies, so the standard photoelectric velocity here means the maximum velocity.

Step 3: Conclusion

$$\frac{v_1}{v_2} = \sqrt{\frac{\nu - \nu_1}{\nu - \nu_2}}$$

Quick Tip to Solve: First subtract each threshold frequency, then take a square root because speed depends on kinetic energy.

20. Draw the graph showing variation of the value of the induced emf as a function of rate of change of current flowing through an ideal inductor. [1 mark]

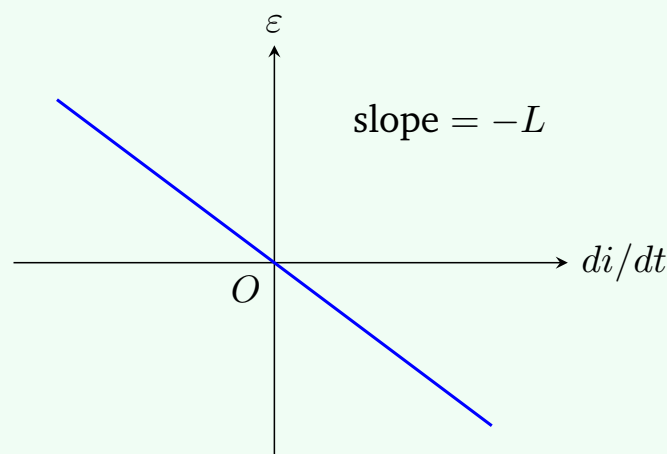
Solution:

Concept:

- For an ideal inductor of constant self-inductance L , the signed induced emf is $\varepsilon = -L di/dt$.
- The minus sign expresses Lenz's law: the emf opposes the change in current that produces it.

Step 1: Choose the axes Put the rate of change of current di/dt on the horizontal axis and signed induced emf ε on the vertical axis. Constant L makes this a linear relation.

Step 2: Determine slope and intercept At $di/dt = 0$, the induced emf is zero. A positive rate gives negative emf, and a negative rate gives positive emf. The line therefore passes through the origin with slope $-L$. If emf magnitude alone is plotted against a positive rate, the slope is $+L$.



Signed induced emf versus rate of change of current

Step 3: Conclusion The signed-emf graph is a straight line through the origin with slope $-L$.

Quick Tip to Solve: Use the minus sign in Lenz's law to choose the downward slope for signed emf.

Section B (14 marks)

21. Which of the following electromagnetic waves has (a) minimum wavelength, and (b) minimum frequency? Write one use of each of these two waves.

Infrared waves, Microwaves, γ -rays and X-rays

[2 marks]

Solution:

Concept:

- All electromagnetic waves travel with speed c in vacuum, with $c = \nu\lambda$, where ν is frequency and λ is wavelength.
- Shorter wavelength therefore corresponds to higher frequency; compare only the four waves listed in the question.

Step 1: Identify the minimum wavelength The order of increasing wavelength is gamma rays, X-rays, infrared waves and microwaves. Thus γ -rays have the minimum wavelength among these waves. Their highly penetrating, ionising radiation is used in radiotherapy to destroy cancer cells.

Step 2: Identify the minimum frequency Since frequency is inversely proportional to wavelength, the longest-wavelength member has the lowest frequency. Microwaves therefore have the minimum frequency in this list. One use is radar, where reflected microwave signals help locate objects and measure their distance. Infrared waves and X-rays lie between the two extremes and cannot give either requested minimum.

Step 3: Conclusion (a) Minimum wavelength: γ -rays; use: cancer radiotherapy. (b) Minimum frequency: microwaves; use: radar.

Quick Tip to Solve: Arrange the given waves by wavelength first; reverse that order for frequency.

22. A circular loop carrying a current 5 A, produces a magnetic field of π mT, at its centre. Find the value of the magnetic moment of the loop.

[2 marks]

Solution:

Concept:

- The field at the centre of a single circular current loop is $B = \mu_0 I / (2r)$, with current I and radius r .
- The magnitude of its magnetic moment is $m = IA = I\pi r^2$, where A is the enclosed area and $\mu_0 = 4\pi \times 10^{-7} \text{ T m A}^{-1}$.

Step 1: Calculate the loop radius The given field is $B = \pi \text{ mT} = \pi \times 10^{-3} \text{ T}$, and $I = 5 \text{ A}$. Rearranging the central-field relation gives

$$r = \frac{\mu_0 I}{2B} = \frac{(4\pi \times 10^{-7})(5)}{2\pi \times 10^{-3}} = 1.0 \times 10^{-3} \text{ m}.$$

The factor of π cancels; converting millitesla to tesla is essential.

Step 2: Calculate the magnetic moment The area is $A = \pi(10^{-3})^2 = \pi \times 10^{-6} \text{ m}^2$. Hence

$$m = IA = 5\pi \times 10^{-6} \text{ A m}^2.$$

The magnetic-moment direction is normal to the loop, given by the right-hand rule; its magnitude is what is requested because the orientation is unspecified.

Step 3: Conclusion The magnetic moment of the loop is $5\pi \times 10^{-6} \text{ A m}^2$, approximately $1.57 \times 10^{-5} \text{ A m}^2$.

Quick Tip to Solve: Find radius from the central field, then square it when calculating loop area.

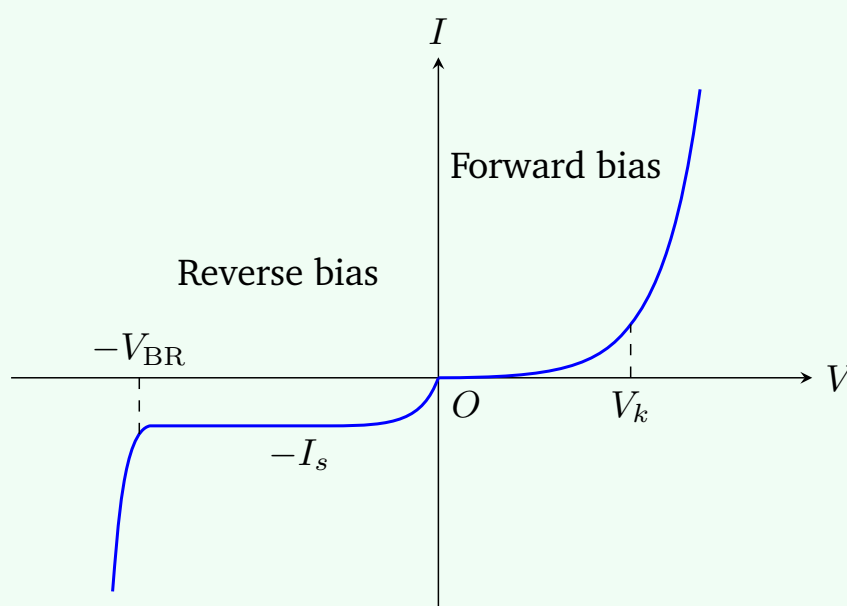
23. Draw V-I characteristics of a p-n junction diode. Explain, why the current under reverse bias is almost independent of the applied voltage up to the critical voltage. [2 marks]

Solution:

Concept:

- Forward bias lowers the junction barrier, allowing a rapidly increasing majority-carrier current beyond the knee voltage.
- Reverse current below breakdown is mainly due to thermally generated minority carriers; its value at a fixed temperature is nearly constant.

Step 1: Draw and interpret the characteristic The first-quadrant branch represents forward bias: current is initially small and rises steeply after the knee voltage V_k . The third-quadrant branch represents reverse bias: a small negative current $-I_s$ flows almost unchanged until the reverse breakdown voltage $-V_{BR}$, beyond which its magnitude rises sharply. The sketch is qualitative and the reverse-current scale is enlarged for visibility.



Qualitative diode V-I characteristic (reverse-current scale enlarged)

Step 2: Explain the nearly constant reverse current Under reverse bias, the electric field sweeps minority carriers across the depletion region. At a fixed temperature their supply is controlled mainly

by thermal generation, rather than by the reverse voltage. Once these carriers are collected, increasing reverse bias hardly changes the current; this gives the reverse saturation current. At the critical breakdown voltage, carrier multiplication or tunnelling creates many additional carriers, so the current then increases rapidly.

Step 3: Conclusion The V-I curve has a rapidly rising forward branch and an almost constant reverse saturation current up to breakdown; the latter is limited by minority-carrier generation.

Quick Tip to Solve: Attribute the flat reverse branch to the limited minority-carrier supply at fixed temperature.

24. Define the term ‘mobility’ of charge carriers in a current carrying conductor. Obtain the relation for mobility in terms of relaxation time. [2 marks]

Solution:

Concept:

- Mobility μ is the magnitude of the drift velocity acquired by a charge carrier per unit applied electric field: $\mu = v_d/E$. Its SI unit is $\text{m}^2 \text{V}^{-1} \text{s}^{-1}$.
- Relaxation time τ is the average time between successive collisions of a carrier. For an electron, charge is $-e$, where $e > 0$, and mass is m_e .

Step 1: Determine the acceleration in the field In a uniform electric field $\vec{E}$, the force on an electron is $\vec{F} = -e\vec{E}$. Newton’s second law therefore gives $\vec{a} = -e\vec{E}/m_e$. Thermal motion is random, so its mean velocity is zero; the applied field adds a systematic mean velocity opposite to the field.

Step 2: Average over the relaxation time Between collisions, the field accelerates the electron. Averaging the field-induced velocity change over the mean collision time gives

$$\vec{v}_d = -\frac{e\tau}{m_e}\vec{E}, \quad v_d = \frac{eE\tau}{m_e}.$$

Dividing the drift speed by the field magnitude yields $\mu = v_d/E = e\tau/m_e$. Mobility is positive; the minus sign belongs to the direction of electron drift, not to its mobility.

Step 3: Conclusion Mobility is drift speed per unit electric field, and for electrons $\mu = e\tau/m_e$.

Quick Tip to Solve: Use the magnitude of electron charge in mobility; retain the negative sign only in the vector drift-velocity relation.

OR

24. Define the term ‘drift velocity’ of electrons in a current carrying conductor. Obtain the relationship between the current density and the drift velocity of electrons. [2 marks]

Solution:

Concept:

- Electron drift velocity $\vec{v}_d$ is the average velocity with which free electrons drift through a conductor under an applied electric field, superposed on their random thermal motion.
- Current density $\vec{J}$ is conventional current per unit cross-sectional area normal to the current. Let n be the number of free electrons per unit volume and e the positive magnitude of electron charge.

Step 1: Count the electrons crossing a section Consider a conductor of uniform cross-sectional area S . In a time interval Δt , electrons with drift speed v_d travel an average distance $v_d \Delta t$. The volume whose electrons cross a chosen section is therefore $S v_d \Delta t$, containing $n S v_d \Delta t$ electrons.

Step 2: Relate charge flow to current density The magnitude of charge crossing the section is $\Delta Q = n e S v_d \Delta t$. Hence

$$I = \frac{\Delta Q}{\Delta t} = n e S v_d, \quad J = \frac{I}{S} = n e v_d.$$

Electrons carry negative charge, so conventional current points opposite to their drift velocity. The complete vector relation is therefore $\vec{J} = -n e \vec{v}_d$. Random thermal velocities contribute no net current because their average is zero.

Step 3: Conclusion Drift velocity is the mean field-induced velocity of electrons. The required relation is $\vec{J} = -n e \vec{v}_d$, or $J = n e v_d$ for magnitudes.

Quick Tip to Solve: First write current as charge crossing per second, then divide by area; electron drift and conventional current have opposite directions.

25. An object is placed in front of a convex mirror of focal length 15 cm. It produces an image that is half the size of the object. Find (a) position of the object, (b) nature of the image, and (c) draw the ray diagram of image formation. [2 marks]

Solution:

Concept:

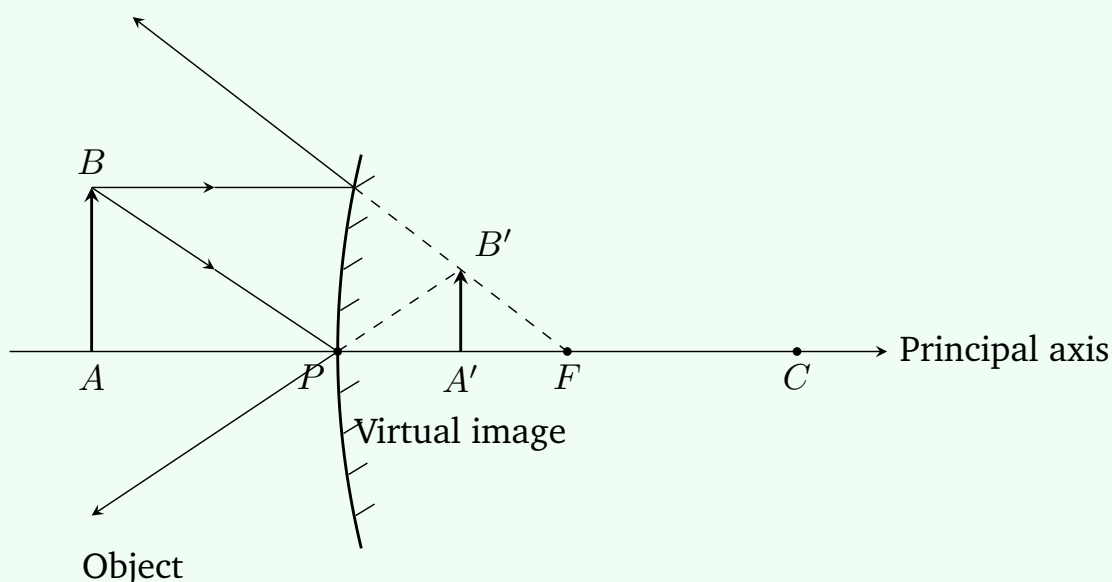
- Using the Cartesian sign convention, distances measured behind the mirror are positive. For a convex mirror, $f = +15$ cm, while a real object in front has $u < 0$.
- The mirror formula is $1/f = 1/v + 1/u$, and linear magnification is $m = h_i/h_o = -v/u$. A convex mirror forms an erect image, so half-size means $m = +1/2$.

Step 1: (a) Calculate the object position From magnification, $-v/u = 1/2$, so $v = -u/2$. Substituting this into the mirror formula gives

$$\frac{1}{15} = \frac{1}{-u/2} + \frac{1}{u} = -\frac{1}{u}.$$

Thus $u = -15$ cm, and $v = +7.5$ cm. The object must be placed 15 cm in front of the mirror.

Step 2: (b) Identify the image and (c) construct the rays Since $v > 0$, the image lies behind the mirror and is virtual. Positive magnification shows that it is erect, and $m = 1/2$ shows that it is diminished. Draw one incident ray parallel to the principal axis: its reflected ray diverges as though it came from F . Draw another ray to pole P , reflecting it at equal angles to the axis. Their backward extensions intersect between P and F , at the top of the half-height image.



Convex-mirror image formation; dashed lines are backward extensions of reflected rays.

Step 3: Conclusion (a) Object: 15 cm in front of the mirror. (b) Image: virtual, erect and half-sized, 7.5 cm behind the mirror. (c) The ray diagram shows its formation.

Quick Tip to Solve: For a convex mirror take half-size magnification as positive; setting it negative would incorrectly describe an inverted image.

26. (a) Write the SI unit of activity of a radioactive substance.

(b) Find the half-life of a radioactive substance if the activity of the substance drops to $\frac{1}{8}$ of its initial value in 15 years. [2 marks]

Solution:

Concept:

- Activity A is the number of radioactive disintegrations per unit time: $A = -dN/dt = \lambda N$, where N is the number of undecayed nuclei and λ is the decay constant.
- After time t , activity obeys $A/A_0 = (1/2)^{t/T_{1/2}}$, where A_0 is initial activity and $T_{1/2}$ is half-life. Activity and the number of undecayed nuclei fall by the same fraction.

Step 1: (a) State the SI unit of activity The SI unit is the becquerel, symbol Bq. An activity of 1 Bq means one nuclear disintegration per second, so $1 \text{ Bq} = 1 \text{ s}^{-1}$. Activity is a decay rate, not the total number of nuclei present.

Step 2: (b) Count the elapsed half-lives The printed final activity is $A_0/8$. Since $1/8 = (1/2)^3$, the

activity has undergone three successive halvings: A_0 to $A_0/2$, then $A_0/4$, and finally $A_0/8$. Therefore,

$$\left(\frac{1}{2}\right)^{15/T_{1/2}} = \left(\frac{1}{2}\right)^3, \quad \frac{15}{T_{1/2}} = 3.$$

Thus $T_{1/2} = 15/3 = 5$ years. Keeping both the elapsed time and half-life in years makes the exponent dimensionless.

Step 3: Conclusion (a) SI unit: becquerel (Bq). (b) Half-life: 5 years.

Quick Tip to Solve: Express the remaining activity fraction as a power of one-half; its exponent directly counts the number of half-lives.

27. (a) Explain the term ‘sharpness of resonance’ in ac circuit.

(b) In a series LCR circuit, $V_L = V_C \neq V_R$. What is the value of power factor for this circuit? [2 marks]

Solution:

Concept:

- In a series LCR circuit, resonance occurs when $X_L = X_C$, with $\omega_0 = 1/\sqrt{LC}$.
- The power factor is $\cos \phi = R/Z$, where $Z = \sqrt{R^2 + (X_L - X_C)^2}$ and ϕ is the phase difference between source voltage and current.

Step 1: (a) Explain sharpness of resonance Sharpness of resonance describes how rapidly the current falls from its maximum value when the applied frequency is changed slightly from the resonant frequency. A narrow current-frequency peak indicates sharp resonance and greater selectivity. Quantitatively, the quality factor is $Q = \omega_0/\Delta\omega = \omega_0 L/R$, where $\Delta\omega$ is the bandwidth between the two half-power frequencies. Thus, a smaller resistance gives a narrower resonance peak for fixed L and C .

Step 2: (b) Use the equality of the reactive voltages Since the same current flows through every series element, $V_L = IX_L$ and $V_C = IX_C$. Their equality implies $X_L = X_C$. These two voltages have opposite phases and cancel in the phasor sum, leaving the source voltage in phase with the current. Hence $Z = R$ and $\phi = 0$. The statement $V_L = V_C \neq V_R$ does not change this cancellation; the resistive voltage need not equal either reactive voltage.

Step 3: Conclusion Sharpness measures the narrowness of the resonance peak, or the rapid fall of current away from resonance. For the stated circuit, $\cos \phi = 1$.

Quick Tip to Solve: Equal series inductive and capacitive voltages cancel as phasors, giving unity power factor regardless of their individual magnitudes.

27. An ac source of emf $V = V_0 \sin \omega t$ is connected to a capacitor of capacitance C . Deduce the expression for the current (I) flowing in it. Plot the graph of (i) V vs. ωt , and (ii) I vs. ωt . [2 marks]

Solution:

Concept:

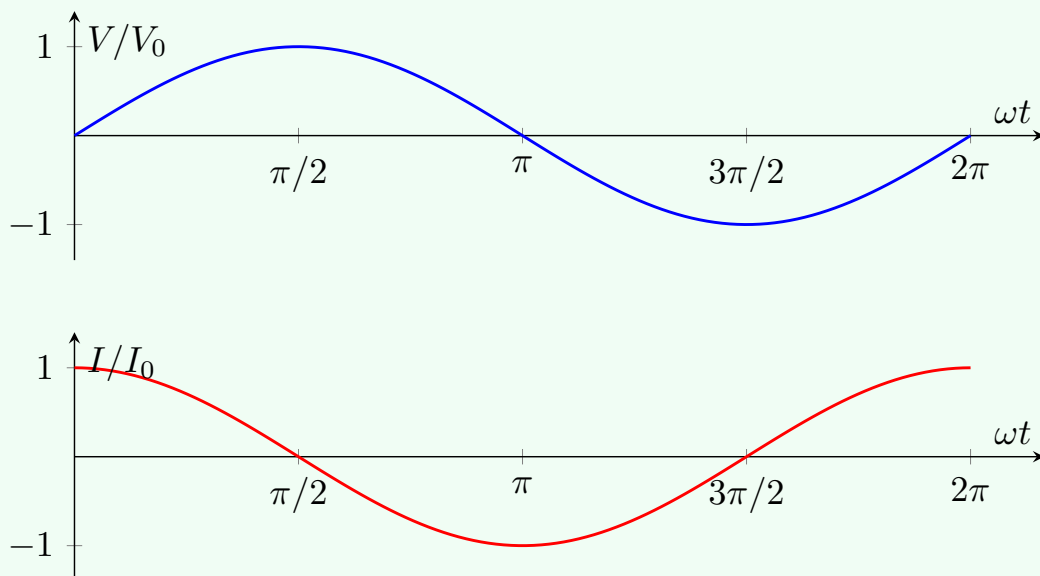
- For an ideal capacitor, the instantaneous charge is $q = CV$, where C is constant.
- Current is the rate of change of charge: $I = dq/dt = C dV/dt$. The capacitive reactance is $X_C = 1/(\omega C)$.

Step 1: Differentiate the instantaneous charge The applied emf is $V = V_0 \sin \omega t$. Therefore the charge on the capacitor is $q = CV_0 \sin \omega t$. Differentiating with respect to time gives

$$I = \frac{dq}{dt} = \omega CV_0 \cos \omega t = I_0 \sin \left(\omega t + \frac{\pi}{2} \right), \quad I_0 = \omega CV_0 = \frac{V_0}{X_C}.$$

This assumes an ideal capacitor, so there is no resistive voltage drop.

Step 2: Plot the voltage and current The voltage graph is a sine curve and the current graph is a cosine curve, each plotted against ωt . At $\omega t = 0$, the voltage is zero and increasing, whereas the current is at its positive maximum. At $\omega t = \pi/2$, voltage is maximum but current is zero. Thus corresponding current peaks occur $\pi/2$ radians earlier than voltage peaks. The normalized vertical axes below keep the different physical units and amplitudes explicit.



Voltage and current versus phase angle for an ideal capacitor.

Step 3: Conclusion $I = \omega CV_0 \cos \omega t$. The current amplitude is $I_0 = \omega CV_0$, and current leads voltage by $\pi/2$ radians.

Quick Tip to Solve: Differentiate the voltage sine wave: its cosine derivative immediately shows that capacitor current leads by a quarter cycle.

Section C (21 marks)

28. A capacitor of $4\ \mu\text{F}$ is charged by a battery of $12\ \text{V}$. The battery is disconnected and a dielectric slab of dielectric constant 8 is inserted in between the plates of the capacitor to fill the space completely. Find the change in the (a) charge stored in the capacitor, (b) potential difference between the plates of the capacitor, and (c) energy stored in the capacitor. [3 marks]

Solution:

Concept:

- An isolated capacitor retains its free charge because there is no conducting path to or from its plates.
- With the space fully filled by dielectric of relative permittivity K , capacitance becomes $C_f = KC_i$.
- For fixed charge, $V = Q/C$ and stored energy is $U = Q^2/(2C)$. Changes are defined as final minus initial values.

Step 1: (a) Determine the charge and its change Initially, $C_i = 4\ \mu\text{F}$ and $V_i = 12\ \text{V}$. Hence

$$Q_i = C_i V_i = (4 \times 10^{-6})(12) = 48 \times 10^{-6}\ \text{C} = 48\ \mu\text{C}.$$

The battery is disconnected before insertion. The plates cannot exchange free charge with a battery, so $Q_f = Q_i$. Dielectric polarization creates bound charges in the slab but does not change the free charge stored on the plates.

Step 2: (b) Calculate the final potential difference The slab completely fills the space, so

$$C_f = KC_i = 8(4\ \mu\text{F}) = 32\ \mu\text{F}.$$

Keeping the free charge fixed gives

$$V_f = \frac{Q_f}{C_f} = \frac{48\ \mu\text{C}}{32\ \mu\text{F}} = 1.5\ \text{V}.$$

Therefore $\Delta V = V_f - V_i = 1.5 - 12 = -10.5\ \text{V}$. The polarization field reduces the electric field between the plates and hence their potential difference.

Step 3: (c) Calculate the stored energies Initially,

$$U_i = \frac{1}{2} C_i V_i^2 = \frac{1}{2} (4 \times 10^{-6})(12)^2 = 288\ \mu\text{J}.$$

Since charge remains fixed and capacitance increases eightfold,

$$U_f = \frac{Q_i^2}{2C_f} = \frac{U_i}{8} = 36\ \mu\text{J}, \quad \Delta U = 36 - 288 = -252\ \mu\text{J}.$$

The reduction in electric-field energy can supply mechanical work as the dielectric is drawn into the capacitor; it is consistent with energy conservation.

Step 4: Conclusion $\Delta Q = 0$, with charge remaining $48 \mu\text{C}$; $\Delta V = -10.5 \text{ V}$, a decrease to 1.5 V ; and $\Delta U = -252 \mu\text{J}$, a decrease to $36 \mu\text{J}$.

Quick Tip to Solve: After the battery is disconnected, hold charge fixed: inserting a full dielectric divides both voltage and stored energy by its dielectric constant.

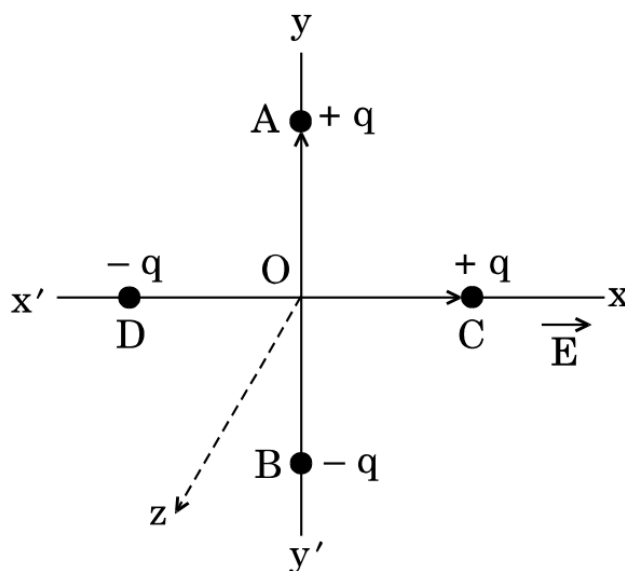
OR

28. Two small identical electric dipoles AB and CD, each of dipole moment $\vec{p}$ are kept at an angle of 90° to each other in an external electric field $\vec{E}$ pointing along the x -axis as shown in the figure. Find the

(a) dipole moment of the arrangement, and

(b) magnitude and direction of the net torque acting on it.

[3 marks]



Solution:

Concept:

- An electric dipole moment points from the negative charge to the positive charge; the resultant dipole moment is the vector sum of the individual moments.
- In a uniform electric field, a dipole experiences torque $\vec{\tau} = \vec{p} \times \vec{E}$, of magnitude $pE \sin \theta$, where θ is the angle between the moment and the field.

Step 1: (a) Identify and add the two moments In the printed figure, B is negative and A is positive, so $\vec{p}_{AB} = p\hat{j}$. D is negative and C is positive, so $\vec{p}_{CD} = p\hat{i}$. Therefore

$$\vec{p}_{\text{net}} = p\hat{i} + p\hat{j}, \quad |\vec{p}_{\text{net}}| = \sqrt{p^2 + p^2} = \sqrt{2}p.$$

Its direction satisfies $\tan \alpha = p/p = 1$. It is thus along the bisector in the first quadrant, at 45° to the positive x -axis towards the positive y -axis. The equal magnitudes add vectorially, rather than giving the scalar sum $2p$.

Step 2: (b) Calculate the individual torques Since $\vec{E} = E\hat{i}$, dipole CD is parallel to the field and has

zero torque. Dipole AB is perpendicular to the field, giving

$$\vec{\tau}_{AB} = p\hat{j} \times E\hat{i} = -pE\hat{k}, \quad \vec{\tau}_{CD} = 0.$$

Equivalently, the positive charge at A is pushed towards the right and the negative charge at B towards the left. These forces form a clockwise couple in the x-y plane. The net torque is therefore along the negative z-axis, opposite to the positive z direction labelled in the figure. As a check, $\sqrt{2}pE \sin 45^\circ = pE$ follows directly from the resultant moment.

Step 3: Conclusion (a) $\vec{p}_{\text{net}} = p(\hat{i} + \hat{j})$, with magnitude $\sqrt{2}p$, at 45° above the positive x-axis. (b) $\vec{\tau}_{\text{net}} = -pE\hat{k}$, of magnitude pE , along the negative z-axis (clockwise in the x-y plane).

Quick Tip to Solve: Read the charge signs first: CD is already aligned with the field, so only AB contributes to the torque.

29. What is the difference in the construction of an astronomical telescope and a compound microscope? The focal lengths of the objective and eyepiece of a compound microscope are 1.25 cm and 5.0 cm, respectively. Find the position of the object relative to the objective in order to obtain an angular magnification of 30 when the final image is formed at the near point. [3 marks]

Solution:

Concept:

- A telescope observes distant objects; a microscope observes a small nearby object. Both use an objective to form a real intermediate image and an eyepiece to magnify it.
- For a compound microscope with the final image at the near point, $|M| = |m_o|(1 + D/f_e)$, where $m_o = v_o/u_o$, $D = 25$ cm, and f_e is the eyepiece focal length.
- For the objective, the Cartesian lens formula is $1/f_o = 1/v_o - 1/u_o$; a real object on the incident side has $u_o < 0$.

Step 1: Compare the construction An astronomical telescope has an objective of large aperture and long focal length, with an eyepiece of much shorter focal length. The large objective collects light from faint distant objects, and its long focal length produces a sufficiently large intermediate image. A compound microscope has an objective of very short focal length and small aperture, placed close to the object. Its eyepiece has a longer focal length than the objective, though both focal lengths are short. The objective first gives a highly enlarged real image of the small object, and the eyepiece magnifies that image further.

Step 2: Obtain the objective magnification Taking the usual least distance of distinct vision as $D = 25$ cm, the near-point eyepiece magnification is

$$M_e = 1 + \frac{D}{f_e} = 1 + \frac{25}{5.0} = 6.$$

Hence $|m_o| = 30/6 = 5$. The objective forms an inverted real image, so $m_o = v_o/u_o = -5$, giving $v_o = -5u_o$. Using magnitudes alone in the signed lens equation would give the wrong object position.

Step 3: Find the object distance using the lens formula Substitute $f_o = 1.25$ cm and $v_o = -5u_o$:

$$\frac{1}{1.25} = \frac{1}{-5u_o} - \frac{1}{u_o} = -\frac{6}{5u_o},$$
$$u_o = -\frac{6(1.25)}{5} = -1.50 \text{ cm}.$$

Then $v_o = 7.50$ cm. The object is just beyond the objective focus, as required for an enlarged real intermediate image, and $(7.50/1.50) \times 6 = 30$ verifies the required angular magnification.

Step 4: Conclusion The telescope objective has a long focal length and large aperture; the microscope objective has a very short focal length and small aperture. The microscope object must be placed 1.50 cm in front of its objective ($u_o = -1.50$ cm).

Quick Tip to Solve: Divide the total magnification by the near-point eyepiece factor first, then apply the signed objective lens formula.

30. What is the function of a solar cell? Briefly explain its working and draw its I-V characteristic curve. [3 marks]

Solution:

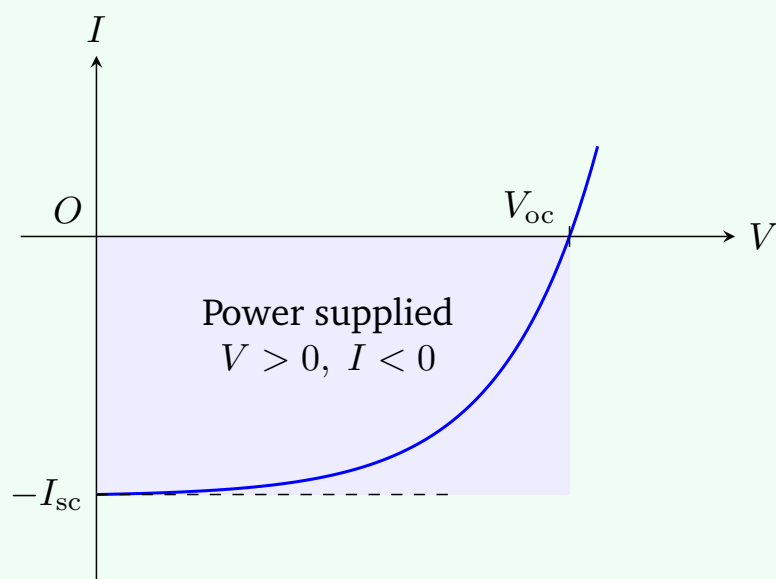
Concept:

- A solar cell is a p-n junction device that converts incident solar energy directly into electrical energy through the photovoltaic effect.
- Photons with energy $h\nu \geq E_g$, where E_g is the semiconductor band gap, create electron-hole pairs. The junction's built-in electric field separates these charges.
- The illuminated I-V curve enters the fourth quadrant when current is positive in the conventional forward-diode direction; negative current represents electrical power delivered by the cell.

Step 1: State the function and construction The function of a solar cell is to supply electrical energy from sunlight without an external bias. A typical silicon solar cell has a thin p-type layer over an n-type substrate, producing a p-n junction near the illuminated surface. The front metal contact is arranged as a grid so that light can enter, while a metal contact on the back completes the electrical connection. An external load connected between the contacts receives the generated electrical power.

Step 2: Explain generation and separation of carriers When sunlight falls on the junction, photons of sufficient energy produce electron-hole pairs in and near the depletion region. Its built-in electric field drives electrons towards the n-side and holes towards the p-side. The resulting charge separation makes the p-side positive relative to the n-side and establishes a photovoltage. On connecting a load, electrons pass externally from the n-side towards the p-side; conventional current in the load flows from the p-side to the n-side. Continued illumination replenishes the carriers, sustaining the current. The cell thus requires illumination but no externally applied voltage to operate as a source.

Step 3: Interpret the illuminated I-V characteristic In the graph, V is the p-side potential relative to the n-side, and I is measured in the forward-diode direction. At $V = 0$, the short-circuit current is $I = -I_{sc}$. At $I = 0$, the open-circuit voltage is V_{oc} . Between these intercepts, $V > 0$ and $I < 0$, so the cell supplies power to a load. Its output power is $P_{out} = -VI$. The fourth-quadrant portion is therefore the useful operating region; beyond V_{oc} , forward diode current becomes positive. The graph is schematic, with axis values normalized to the intercepts.



Illuminated solar-cell I-V curve; the shaded fourth-quadrant region supplies power.

Step 4: Conclusion A solar cell converts sunlight directly into electrical energy. Light creates electron-hole pairs and the junction field separates them, producing a photovoltage and load current. Its illuminated I-V curve has intercepts $(0, -I_{sc})$ and $(V_{oc}, 0)$, with power generation in the fourth quadrant.

Quick Tip to Solve: Label both intercepts and state the current convention: with forward diode current positive, the solar-cell generating curve lies below the voltage axis.

31. The maximum kinetic energy of the photoelectrons emitted is doubled when the wavelength of light incident on the photosensitive surface changes from λ_1 to λ_2 . Deduce expressions for the threshold wavelength and work function for the metal surface in terms of λ_1 and λ_2 . [3 marks]

Solution:

Concept:

- Einstein's photoelectric equation is $K_{\max} = hc/\lambda - \phi$, where h is Planck's constant, c is the speed of light, λ is the incident wavelength and ϕ is the metal's work function.
- At the threshold wavelength λ_0 , emitted electrons have zero maximum kinetic energy, so $\phi = hc/\lambda_0$. The work function depends on the surface and stays unchanged when the incident wavelength changes.

Step 1: Write the energy equations for the two wavelengths Let the initial maximum kinetic energy be K . The question states that the final maximum kinetic energy is $2K$. Thus, for the same metal,

$$K = \frac{hc}{\lambda_1} - \phi,$$

$$2K = \frac{hc}{\lambda_2} - \phi.$$

Photon energy supplies both the energy needed to escape the surface and the kinetic energy of the fastest electron. Therefore it is the excess above ϕ , rather than the full photon energy, that doubles.

Step 2: Eliminate the kinetic energy to determine the work function Substitute twice the first equation

into the second:

$$2\left(\frac{hc}{\lambda_1} - \phi\right) = \frac{hc}{\lambda_2} - \phi,$$

$$\phi = hc\left(\frac{2}{\lambda_1} - \frac{1}{\lambda_2}\right) = \frac{hc(2\lambda_2 - \lambda_1)}{\lambda_1\lambda_2}.$$

The positive work function requires $2\lambda_2 > \lambda_1$. Also the increase in kinetic energy implies an increase in photon energy, hence $\lambda_2 < \lambda_1$.

Step 3: Use the threshold condition Since $\phi = hc/\lambda_0$, dividing the preceding expression by hc gives

$$\frac{1}{\lambda_0} = \frac{2}{\lambda_1} - \frac{1}{\lambda_2}.$$

Taking the reciprocal expresses the threshold wavelength entirely in terms of the two incident wavelengths. The conditions above give $\lambda_0 > \lambda_1$, consistent with emission at the initial wavelength.

Step 4: Conclusion The threshold wavelength and work function are

$$\lambda_0 = \frac{\lambda_1\lambda_2}{2\lambda_2 - \lambda_1}, \quad \phi = \frac{hc(2\lambda_2 - \lambda_1)}{\lambda_1\lambda_2}.$$

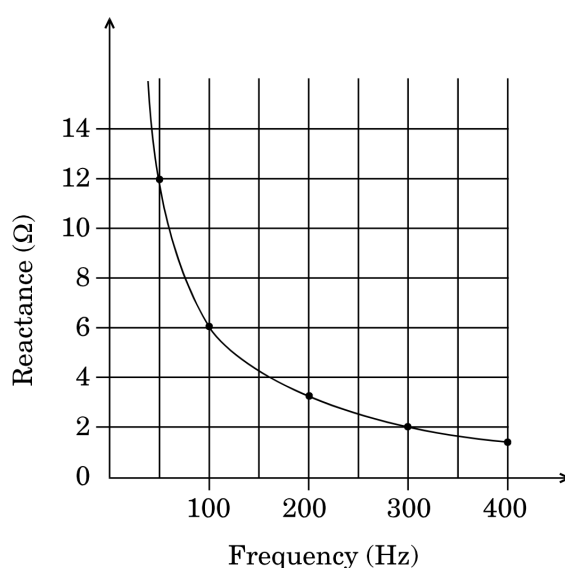
Quick Tip to Solve: Double the entire initial Einstein equation, including its work-function term, before eliminating the kinetic energy.

32. The figure shows the graphical variation of the reactance of a capacitor with frequency of ac source.

(a) Find the capacitance of the capacitor.

(b) An ideal inductor has the same reactance at 100 Hz frequency as the capacitor has at the same frequency. Find the value of inductance of the inductor.

(c) Draw the graph showing the variation of the reactance of this inductor with frequency. [3 marks]



Solution:

Concept:

- Capacitive reactance is $X_C = 1/(2\pi fC)$, where f is the frequency in hertz and C is capacitance in farads. It decreases inversely with frequency.
- Inductive reactance is $X_L = 2\pi fL$, where L is inductance in henry. For an ideal inductor of fixed inductance, reactance increases linearly with frequency and is zero at zero frequency.

Step 1: (a) Read the capacitor graph and calculate capacitance The marked point on the supplied graph gives $X_C = 6\ \Omega$ at $f = 100\text{ Hz}$. Rearranging the capacitive-reactance relation,

$$\begin{aligned}C &= \frac{1}{2\pi f X_C} \\&= \frac{1}{2\pi(100)(6)}\text{ F} \\&= 2.65 \times 10^{-4}\text{ F} \simeq 265\ \mu\text{F}.\end{aligned}$$

As a consistency check, the point $(300\text{ Hz}, 2\ \Omega)$ gives the same product $fX_C = 600$, and hence the same capacitance. Use a clear marked point rather than estimating between grid lines.

Step 2: (b) Determine the inductance at the common reactance At 100 Hz , the question specifies equality of the two reactances. Consequently the inductor has $X_L = 6\ \Omega$ at this frequency. Thus

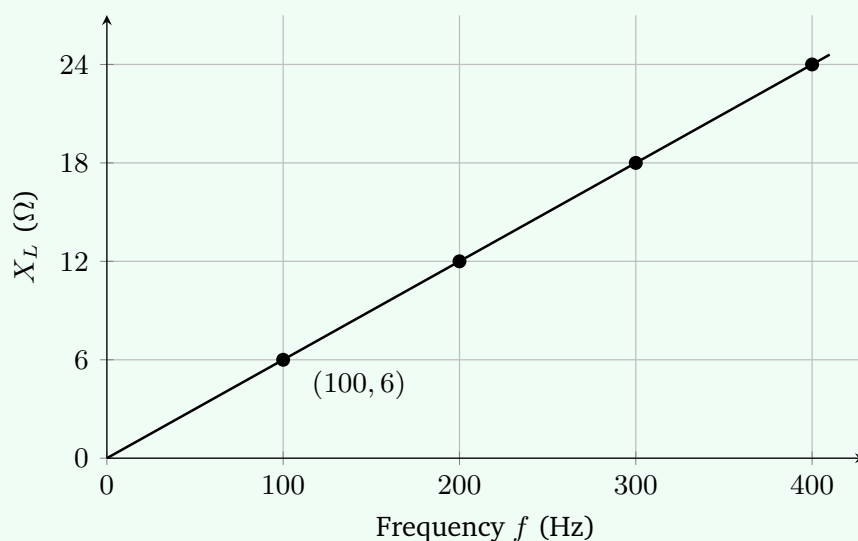
$$\begin{aligned}L &= \frac{X_L}{2\pi f} \\&= \frac{6}{2\pi(100)}\text{ H} \\&= 9.55 \times 10^{-3}\text{ H} \simeq 9.55\text{ mH}.\end{aligned}$$

The equality applies at the stated frequency; the capacitor and inductor do not have equal reactance at all frequencies, because their frequency dependences are opposite.

Step 3: (c) Plot inductive reactance against frequency Substituting the calculated inductance into $X_L = 2\pi Lf$ gives

$$X_L = (0.060\ \Omega\text{ Hz}^{-1})f.$$

Therefore the graph is a straight line through the origin, with frequency on the horizontal axis and inductive reactance on the vertical axis. It passes through $(100\text{ Hz}, 6\ \Omega)$, $(200\text{ Hz}, 12\ \Omega)$ and $(300\text{ Hz}, 18\ \Omega)$. Its positive slope is $2\pi L$, unlike the falling capacitor curve in the question.



Inductive reactance varies linearly with frequency.

Step 4: Conclusion (a) $C \simeq 265 \mu\text{F}$. (b) $L \simeq 9.55 \text{ mH}$. (c) The required inductor graph is the rising straight line through the origin shown above, obeying $X_L = 0.060f$ in SI units.

Quick Tip to Solve: Read the common point at 100 Hz first: use its 6-ohm value in both formulas, then draw the inductor line through that point and the origin.

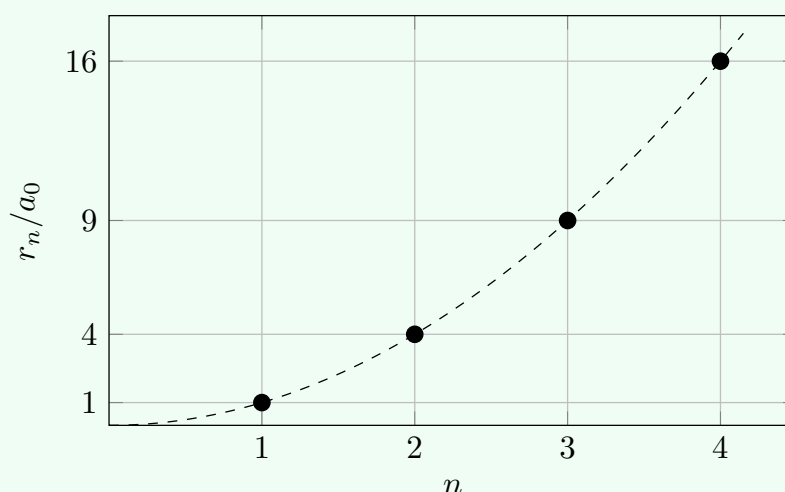
- 33. (a)** Draw the graph of radius of orbit (r_n) in hydrogen atom as a function of orbit number (n).
(b) In a hydrogen atom, find the ratio of the time taken by the electron to complete one revolution in the first excited and in the second excited states. [3 marks]

Solution:

Concept:

- For a hydrogen atom, Bohr's model gives $r_n = a_0 n^2$, where $a_0 = 0.529 \text{ \AA}$ is the ground-state radius and $n = 1, 2, 3, \dots$
- The electron speed is $v_n = v_1/n$. The period of revolution is $T_n = 2\pi r_n / v_n$, so both the orbital circumference and speed must be included.

Step 1: Part (a): Plot radius against orbit number The radii are $r_1 = a_0$, $r_2 = 4a_0$, $r_3 = 9a_0$ and $r_4 = 16a_0$. Hence radius increases as the square of orbit number. Plot n on the horizontal axis and r_n on the vertical axis. The allowed orbits are discrete, so the physical values are the marked points; the dotted parabola is only a guide showing the quadratic dependence.



Allowed hydrogen orbit radii; the dotted curve is a guide.

Step 2: Part (b): Obtain the period dependence Using the Bohr expressions,

$$T_n = \frac{2\pi a_0 n^2}{v_1/n} = \frac{2\pi a_0}{v_1} n^3 = T_1 n^3.$$

The factor n^2 comes from the increasing circumference, while the additional factor n comes from the decreasing speed. The first excited state is $n = 2$, and the second excited state is $n = 3$; they are not $n = 1$ and $n = 2$. Therefore

$$\frac{T_{\text{first excited}}}{T_{\text{second excited}}} = \frac{T_2}{T_3} = \frac{2^3}{3^3} = \frac{8}{27}.$$

Both periods have the same units and the common factor T_1 cancels.

Step 3: Conclusion (a) The allowed radii lie on $r_n = a_0 n^2$, an upward-opening quadratic graph. (b) The required time ratio is $T_2 : T_3 = 8 : 27$.

Quick Tip to Solve: Convert excited-state names to n first, then use $T_n \propto n^3$, not the radius law alone.

34. (a) Derive the condition of balance for Wheatstone bridge.

(b) Draw the circuit diagram of a meter bridge to explain how it is based on Wheatstone bridge.

[3 marks]

Solution:

Concept:

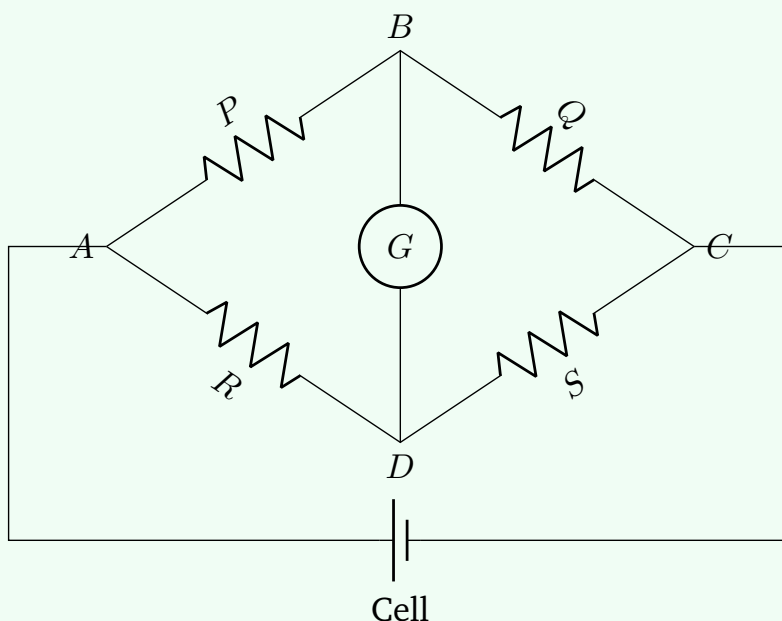
- A Wheatstone bridge compares four resistances P, Q, R, S . At balance no current flows through its galvanometer, so the two galvanometer terminals have equal potential.
- A uniform meter-bridge wire has resistance proportional to length: $R_{AJ}/R_{JB} = l/(100 - l)$, with lengths measured in centimetres.

Step 1: Part (a): Derive the balance condition Let the supply be connected between A and C , and the galvanometer between B and D . The resistances AB, BC, AD, DC are respectively P, Q, R, S . At balance $I_G = 0$, so $V_B = V_D$. If currents I_1 and I_2 flow in the upper and lower branches, equality of the drops from A to the equal-potential points gives $I_1 P = I_2 R$. Equality of the remaining drops to C

gives $I_1 Q = I_2 S$. Dividing these equations eliminates the currents:

$$\frac{P}{Q} = \frac{R}{S}, \quad PS = QR.$$

This condition does not require the two branch currents to be equal; it follows from the zero galvanometer current.

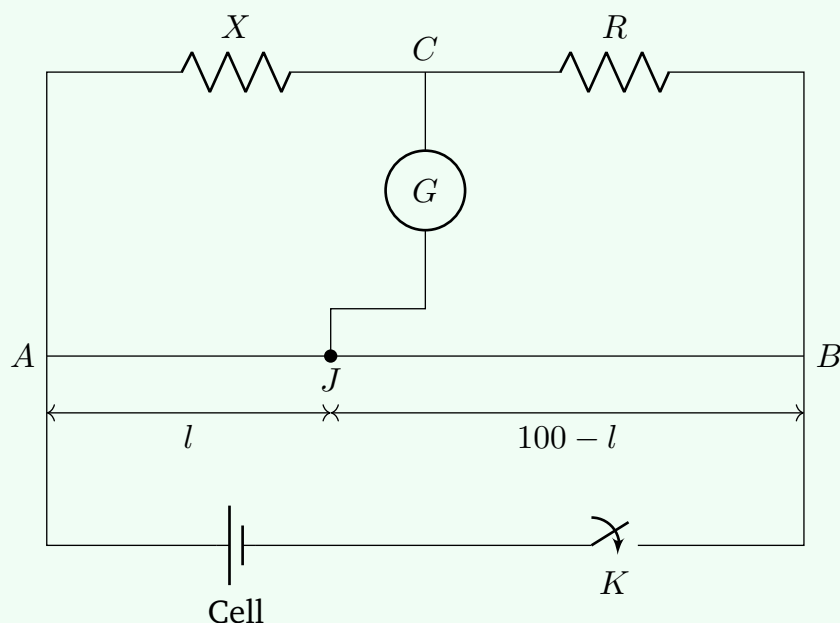


Wheatstone bridge with galvanometer between equal-potential points at balance.

Step 2: Part (b): Identify the four meter-bridge arms In the meter bridge, place an unknown resistance X and a known resistance R in the two gaps. Connect their common terminal C through the galvanometer G to a sliding jockey J on the uniform wire AB . The cell and key supply current between A and B . Move J until the galvanometer reads zero. Then C and J have the same potential. The four Wheatstone arms are X, R, R_{AJ}, R_{JB} . If $AJ = l$ cm, then $JB = (100 - l)$ cm, and the balance condition becomes

$$\frac{X}{R} = \frac{R_{AJ}}{R_{JB}} = \frac{l}{100 - l}.$$

Thus a resistance ratio is measured by a ratio of wire lengths. Uniform cross-section and material are essential for replacing resistance ratios by length ratios.



Meter bridge: X and R are the gaps; J is the jockey on the 100 cm wire AB .

Step 3: Conclusion The Wheatstone balance condition is $\boxed{P/Q = R/S}$. The meter bridge implements the same condition using a uniform wire, giving $\boxed{X = Rl/(100 - l)}$ at null deflection.

Quick Tip to Solve: Label the resistance above each wire segment consistently: the left gap corresponds to l , the right gap to $100 - l$.

Section D (15 marks)

35. (a) Derive lens maker's formula for a biconvex lens.

(b) A point object is placed at a distance of 12 cm on the principal axis of a convex lens of focal length 10 cm. A convex mirror is placed coaxially on the other side of the lens at a distance of 10 cm. If the final image coincides with the object, sketch the ray diagram and find the focal length of the convex mirror.

[5 marks]

Solution:

Concept:

- Refraction at a spherical surface obeys $\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$, using Cartesian distances measured from the pole in the incident-light direction.
- For a thin lens in air, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$. Its relative refractive index is μ , and R_1, R_2 are the signed radii of the two surfaces.
- A ray directed towards the centre of curvature of a spherical mirror strikes normally and retraces its path. The mirror focal length is half its radius of curvature.

Step 1: (a) Refraction at the first lens surface Consider a thin biconvex lens of refractive index μ in air. Let light travel from left to right and take this as the positive direction. The object distance is u ; the first surface forms an intermediate image at distance v_1 . Applying spherical refraction from air

into glass gives

$$\frac{\mu}{v_1} - \frac{1}{u} = \frac{\mu - 1}{R_1}. \quad (1)$$

This intermediate image acts as the object for the second surface, even when it is a virtual object there. We use paraxial rays so that the spherical-surface formula is valid.

Step 2: (a) Combine both refractions and identify the focal length Since the lens is thin, its surface separation can be neglected, and the second surface has object distance v_1 . Refraction from glass to air gives

$$\frac{1}{v} - \frac{\mu}{v_1} = \frac{1 - \mu}{R_2}. \quad (2)$$

Adding (1) and (2) cancels the intermediate image term:

$$\frac{1}{v} - \frac{1}{u} = (\mu - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right).$$

For an object at infinity, $1/u \rightarrow 0$ and the final image lies at $v = f$. Therefore the lens maker's formula is

$$\boxed{\frac{1}{f} = (\mu - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)}.$$

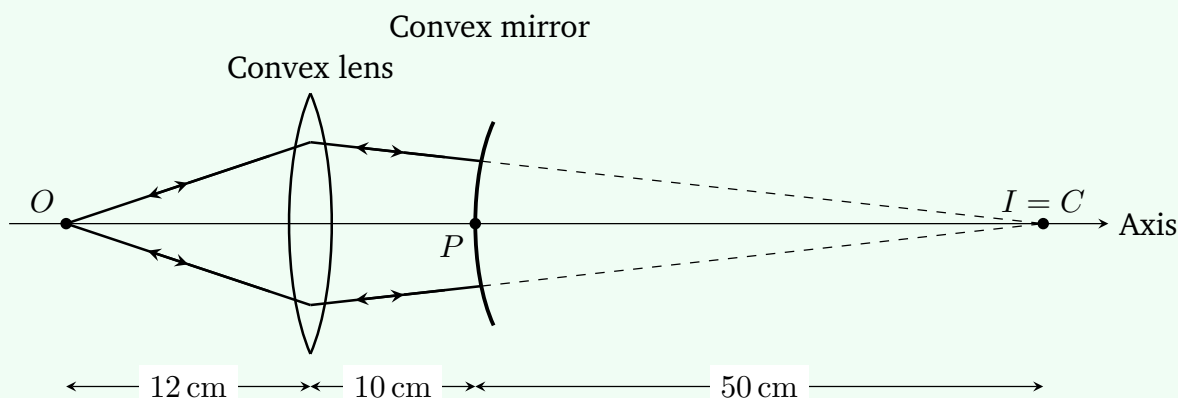
For the biconvex lens in this convention, $R_1 > 0$ and $R_2 < 0$, so both curved surfaces contribute positively to the converging power. The assumptions are a thin lens, air on both sides and paraxial refraction.

Step 3: (b) Find the image the lens would form without the mirror Place the lens at $x = 0$, the point object at $x = -12$ cm, and the mirror pole at $x = 10$ cm. For the initial left-to-right passage, $u = -12$ cm and $f = +10$ cm. Thus

$$\begin{aligned} \frac{1}{v} &= \frac{1}{f} + \frac{1}{u} = \frac{1}{10} - \frac{1}{12} = \frac{1}{60}, \\ v &= 60 \text{ cm.} \end{aligned}$$

The rays reaching the mirror are consequently converging towards point I , situated 60 cm to the right of the lens and 50 cm behind the mirror. This is a virtual object for the mirror; it is not a real image formed before reflection.

Step 4: (b) Apply the coincidence condition and draw the retracing rays To return through the lens and form an image at the original point object, the reflected beam must diverge as though it came from I . This follows from reversibility of the lens refraction: the lens maps O to I , and the reverse passage maps I to O . Hence the mirror must image its virtual object I back onto the same point I . This occurs when I is its centre of curvature C , because every incident ray is then normal to the spherical surface and retraces its path. The sketch shows two paraxial rays and their returning paths; the dashed extensions meet at $I = C$ behind the mirror. It is schematic, with the stated distances determining the calculation.



Schematic retracing rays: the converging incident beam is directed towards $I = C$, behind the convex mirror.

Step 5: (b) Calculate and check the mirror focal length The centre of curvature lies behind the convex mirror at distance

$$R = PC = 60 - 10 = 50 \text{ cm}, \quad f_m = \frac{R}{2} = 25 \text{ cm}.$$

Equivalently, using the incident direction for the mirror sign convention, both its virtual object and virtual image distances are positive: $u_m = v_m = +50 \text{ cm}$. The mirror equation confirms

$$\frac{1}{f_m} = \frac{1}{u_m} + \frac{1}{v_m} = \frac{1}{50} + \frac{1}{50} = \frac{1}{25}.$$

Thus the positive focal length is consistent with a convex mirror, whose focus is behind its reflecting surface.

Step 6: Conclusion The thin-lens maker's formula in air is $\frac{1}{f} = (\mu - 1)(\frac{1}{R_1} - \frac{1}{R_2})$. In the lens-mirror arrangement, the mirror centre of curvature coincides with the lens's would-be image, and the required convex-mirror focal length is $f_m = +25 \text{ cm}$.

Quick Tip to Solve: First locate the lens's would-be image; for exact return to the point object, place the mirror centre there and halve the pole-to-centre distance.

OR

35. (a) What is a wavefront? How does it propagate? Using Huygens' principle, explain reflection of a plane wavefront from a surface and verify the laws of reflection.

(b) A parallel beam of light of wavelength 500 nm falls on a narrow slit and the resulting diffraction pattern is obtained on a screen 1 m away. If the first minimum is formed at a distance of 2.5 mm from the centre of the screen, find the (i) width of the slit, and (ii) distance of first secondary maximum from the centre of the screen. [5 marks]

Solution:

Concept:

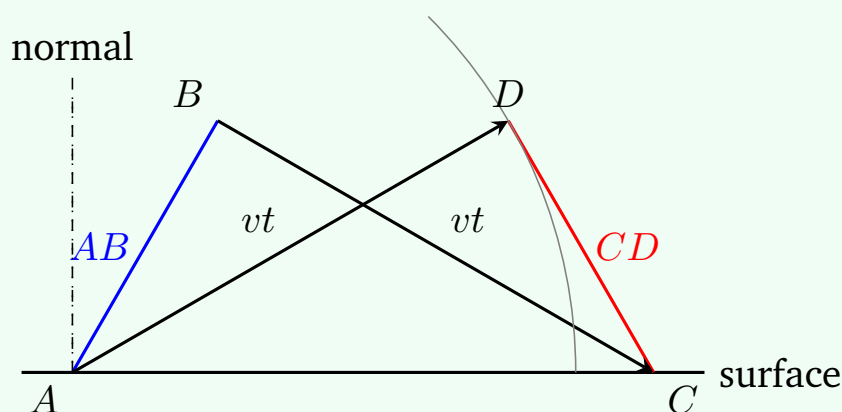
- A wavefront is the locus of points of a wave that are in the same phase. Rays in a homogeneous isotropic medium are normal to the wavefront.
- Huygens' principle: every point on a wavefront emits secondary wavelets; the forward envelope of these wavelets gives the wavefront at a later instant.
- For a slit of width a , diffraction minima obey $a \sin \theta = n\lambda$. At small angles the first minimum is at

$y_1 = D\lambda/a$, where D is the screen distance.

- In the usual school approximation the first secondary maximum lies midway between the first two minima, at $y_s \simeq 3D\lambda/(2a)$.

Step 1: (a) Define a wavefront and describe propagation Points on the same wavefront have the same phase of vibration. A point source produces spherical wavefronts, while a sufficiently distant source gives approximately plane wavefronts. During a time interval t , secondary wavelets advance a distance vt , where v is the wave speed. Their common forward tangent or envelope forms the new wavefront. Successive envelopes thus describe propagation, and the direction of propagation is perpendicular to the wavefront in an isotropic medium.

Step 2: Construct the reflected plane wavefront Let AB be an incident plane wavefront meeting a plane reflecting surface first at A . During time t , the disturbance at B travels along the incident ray to C on the surface, so $BC = vt$. Meanwhile the secondary wavelet originating at A has radius $AD = vt$ in the incident medium. Draw the tangent CD from C to this wavelet. The common envelope CD is the reflected wavefront, and AD , being normal to it, is a reflected ray. Hence $AD = BC$.



Huygens' construction: incident wavefront AB , reflected wavefront CD , and equal distances BC and AD .

Step 3: Verify both laws of reflection The triangles ABC and ADC are right angled at B and D , respectively, because rays are normal to their wavefronts. They have the same hypotenuse AC and equal sides $BC = AD$; therefore they are congruent. Consequently,

$$\angle BAC = \angle DCA.$$

The angle between the incident wavefront AB and the surface equals the angle i between the incident ray and the surface normal. Likewise the angle between reflected wavefront CD and the surface equals the reflection angle r . Thus $i = r$. The construction lies in the plane containing the incident ray and the normal: the reflected ray lies in this same plane. These are the two laws of reflection.

Step 4: (b)(i) Calculate the slit width Convert all quantities to SI units:

$$\lambda = 500 \times 10^{-9} \text{ m}, \quad D = 1 \text{ m}, \quad y_1 = 2.5 \times 10^{-3} \text{ m}.$$

Since $y_1/D = 0.0025 \ll 1$, the small-angle approximation is appropriate. For the first minimum,

$$a = \frac{D\lambda}{y_1} = \frac{1 \times 500 \times 10^{-9}}{2.5 \times 10^{-3}} = 2.0 \times 10^{-4} \text{ m}.$$

Step 5: (b)(ii) Locate the first secondary maximum The first and second minima occur at y_1 and $2y_1$.

Using the standard approximate position of the intervening secondary maximum,

$$y_s \simeq \frac{3D\lambda}{2a} = \frac{3}{2}y_1 = \frac{3}{2} \times 2.5 \text{ mm} = 3.75 \text{ mm}.$$

This is its distance from the centre on either side of the central maximum. The midpoint rule is an approximation: the exact intensity maximum obeys $\tan \beta = \beta$, with $\beta = \pi a \sin \theta / \lambda$. Its first nonzero root $\beta \simeq 4.493$ gives about 3.58 mm; the school-level midpoint approximation gives the expected value above.

Step 6: Conclusion Huygens' construction gives $i = r$ and places the incident ray, reflected ray and normal in one plane. The slit width is $2.0 \times 10^{-4} \text{ m} = 0.20 \text{ mm}$, and the first secondary maximum is approximately 3.75 mm from the centre using the standard CBSE approximation.

Quick Tip to Solve: Find the first-minimum distance first: the approximate first secondary maximum is 1.5 times that distance, so the second calculation needs no fresh SI substitution.

36. (a) Use Gauss's law to show that due to a uniformly charged spherical shell of radius R , the electric field at any point situated outside the shell at a distance r from its centre is equal to the electric field at the same point, when the entire charge on the shell were concentrated at its centre. Also plot the graph showing the variation of electric field with r , for $r < R$ and $r \geq R$.

(b) Two point charges of $+1 \mu\text{C}$ and $+4 \mu\text{C}$ are kept 30 cm apart. How far from the $+1 \mu\text{C}$ charge on the line joining the two charges, will the net electric field be zero? [5 marks]

Solution:

Concept:

- Gauss's law states that $\oint \vec{E} \cdot d\vec{A} = Q_{\text{enc}}/\epsilon_0$, where Q_{enc} is the charge enclosed by a closed Gaussian surface and ϵ_0 is the permittivity of free space.
- Spherical symmetry makes the field of a uniformly charged spherical shell radial and its magnitude constant at every point of a concentric spherical Gaussian surface. Let Q be the total shell charge and R its radius.
- Electric fields obey superposition. A positive point charge produces a field directed away from itself, with magnitude $E = kq/d^2$, where $k = 1/(4\pi\epsilon_0)$ and d is the distance from the charge.

Step 1: (a) Choose the Gaussian surface outside the shell Take a concentric spherical Gaussian surface of radius $r > R$. Its outward area vector is radial; hence it is parallel to $\vec{E}$ for a positively charged shell. Since the field magnitude is constant over this surface, the flux integral simplifies to

$$\oint \vec{E} \cdot d\vec{A} = E(r) \oint dA = E(r) 4\pi r^2.$$

The entire charge Q lies inside this Gaussian surface. Gauss's law therefore gives

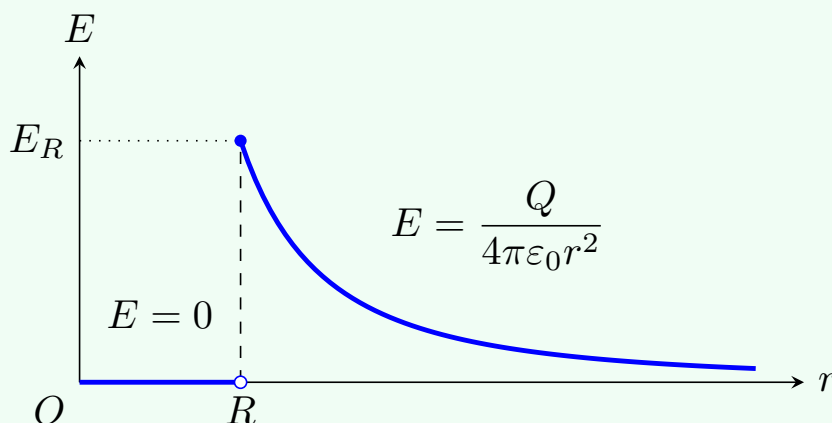
$$E(r) 4\pi r^2 = \frac{Q}{\epsilon_0}, \quad \vec{E}(r) = \frac{Q}{4\pi\epsilon_0 r^2} \hat{r}.$$

This is precisely the field of a point charge Q placed at the centre of the shell. The field is outward for $Q > 0$ and inward for $Q < 0$; the expression retains the sign of Q .

Step 2: (a) Obtain the interior field and draw the variation For a concentric Gaussian sphere of radius $r < R$, no shell charge is enclosed. Spherical symmetry still makes the field radial and constant in magnitude, so $E(r)4\pi r^2 = 0$, giving zero field everywhere inside; the centre also has zero field by symmetry. For a positive shell the field magnitude is

$$E(r) = \begin{cases} 0, & 0 \leq r < R, \\ \frac{Q}{4\pi\epsilon_0 r^2}, & r \geq R \quad (\text{exterior value at the surface}). \end{cases}$$

Thus the graph stays on the horizontal axis inside the shell, jumps at its surface from zero to $E_R = Q/(4\pi\epsilon_0 R^2)$, and then falls as $1/r^2$, approaching zero at large distance. For an ideal infinitesimally thin charged shell the two one-sided surface values differ; the plotted value at R uses the exterior convention requested in the question.



Field magnitude for a positively charged spherical shell; the dashed line indicates the surface discontinuity.

Step 3: (b) Locate the region where cancellation is possible Put the $+1\ \mu\text{C}$ charge at $x = 0$ and the $+4\ \mu\text{C}$ charge at $x = 30\text{ cm}$. Outside the segment joining them, both fields point in the same direction and cannot cancel. Between the charges, the first field points towards the right and the second towards the left. Therefore a finite zero-field point must lie between them, nearer the smaller charge.

Step 4: (b) Equate the opposing field magnitudes Let the required point be x centimetres from the $+1\ \mu\text{C}$ charge. Its distance from the other charge is $(30 - x)$ centimetres. Using the same distance unit on both sides, cancellation requires

$$\begin{aligned} \frac{k(1 \times 10^{-6})}{x^2} &= \frac{k(4 \times 10^{-6})}{(30 - x)^2}, \\ (30 - x)^2 &= 4x^2, \\ 30 - x &= 2x, \\ x &= 10\text{ cm}. \end{aligned}$$

The positive square root is used because both distances are positive inside the segment. As a check, the distances are 0.10 m and 0.20 m; the charge ratio is four and the squared-distance ratio is also four, so each field has magnitude $9.0 \times 10^5 \text{ N C}^{-1}$, with opposite directions.

Step 5: Conclusion (a) Outside the shell, $\vec{E} = \frac{Q}{4\pi\epsilon_0 r^2} \hat{r}$, identical to the field of its entire charge concentrated at the centre. Inside, $E = 0$; the graph jumps at R to the exterior value and then decreases as r^{-2} . (b) The net electric field is zero 10 cm from the $+1 \mu\text{C}$ charge towards the $+4 \mu\text{C}$ charge, between the charges.

Quick Tip to Solve: For two like charges, the zero-field point lies between them and its distances satisfy $d_1 : d_2 = \sqrt{q_1} : \sqrt{q_2}$; here the ratio is 1 : 2, dividing 30 cm into 10 cm and 20 cm.

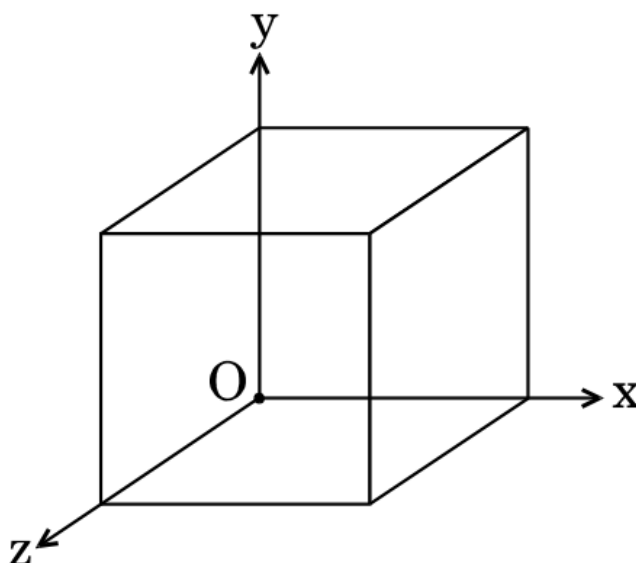
OR

36. (a) Two point charges q_1 and q_2 are kept r distance apart in a uniform external electric field $\vec{E}$. Find the amount of work done in assembling this system of charges.

(b) A cube of side 20 cm is kept in a region as shown in the figure. An electric field $\vec{E}$ exists in the region such that the potential at a point is given by $V = 10x + 5$, where V is in volt and x is in m.

Find the (i) electric field $\vec{E}$, and (ii) total electric flux through the cube.

[5 marks]



Solution:

Concept:

- The work done by an external agent in quasistatic assembly is the change in electrostatic potential energy. A charge q at position $\vec{r}$ in the external potential V_{ext} contributes $qV_{\text{ext}}(\vec{r})$.
- The mutual interaction energy of two point charges separated by r is $q_1 q_2 / (4\pi\epsilon_0 r)$. Count this pair interaction once; point-charge self-energies are excluded.
- Electric field is the negative potential gradient, $\vec{E} = -\nabla V$. Electric flux through a closed surface is $\Phi = \oint \vec{E} \cdot d\vec{A}$, with outward area vectors.

Step 1: (a) Bring the first charge to its specified position Let the final positions be $\vec{r}_1$ and $\vec{r}_2$, with $|\vec{r}_2 - \vec{r}_1| = r$. Choose a reference for the prescribed external potential. Bringing q_1 from that reference

to $\vec{r}_1$, before the second charge is present, requires

$$W_1 = q_1 V_{\text{ext}}(\vec{r}_1).$$

For a uniform field, one may write $V_{\text{ext}}(\vec{r}) = V_0 - \vec{E} \cdot (\vec{r} - \vec{r}_0)$. A globally uniform field does not admit a finite potential zero at infinity, so a finite reference must be specified.

Step 2: (a) Add the second charge and combine the work At $\vec{r}_2$, the potential already present is the sum of the external potential and the potential of q_1 . Hence

$$W_2 = q_2 \left[V_{\text{ext}}(\vec{r}_2) + \frac{q_1}{4\pi\epsilon_0 r} \right].$$

Adding the two contributions gives

$$W = q_1 V_{\text{ext}}(\vec{r}_1) + q_2 V_{\text{ext}}(\vec{r}_2) + \frac{q_1 q_2}{4\pi\epsilon_0 r}.$$

This is the conventional assembly energy in the chosen external-potential reference. The separation alone does not fix the two external-potential terms: positions and a reference are needed for a numerical value. The external field sources are treated as fixed.

Step 3: (b)(i) Differentiate the given potential Since $V = 10x + 5$ has no dependence on y or z ,

$$\vec{E} = - \left(\frac{\partial V}{\partial x} \hat{i} + \frac{\partial V}{\partial y} \hat{j} + \frac{\partial V}{\partial z} \hat{k} \right) = -10\hat{i} \text{ V m}^{-1}.$$

Thus the field is uniform, of magnitude 10 N C^{-1} , directed along the negative x -axis. The additive constant 5 V has no effect on the field.

Step 4: (b)(ii) Add the fluxes through the cube's faces The side is $a = 20 \text{ cm} = 0.20 \text{ m}$, so each face has area $a^2 = 0.040 \text{ m}^2$. The outward normal on the positive- x face is $+\hat{i}$, giving flux $-10a^2 = -0.40 \text{ N m}^2 \text{ C}^{-1}$. On the opposite face the normal is $-\hat{i}$, giving $+0.40 \text{ N m}^2 \text{ C}^{-1}$. On the four other faces the field is tangent and the flux is zero. Consequently the outward and inward fluxes cancel exactly. Equivalently, $\nabla \cdot \vec{E} = 0$ implies no enclosed net charge and hence zero closed-surface flux.

Step 5: Conclusion (a) $W = q_1 V_{\text{ext}}(\vec{r}_1) + q_2 V_{\text{ext}}(\vec{r}_2) + q_1 q_2 / (4\pi\epsilon_0 r)$, for a chosen external-potential reference. (b)(i) $\vec{E} = -10\hat{i} \text{ N C}^{-1}$; (ii) $\Phi_{\text{cube}} = 0$.

Quick Tip to Solve: In assembly work, include both external-potential energies but the mutual pair energy only once; for a uniform field, opposite cube-face fluxes cancel.

37. (a) A circular loop of radius R carries a current I . Obtain an expression for the magnetic field at a point on its axis at a distance x from its centre.

(b) A conducting rod of length 2 m is placed on a horizontal table in north-south direction. It carries a current of 5 A from south to north. Find the direction and magnitude of the magnetic force acting on the rod. Given that the Earth's magnetic field at the place is 0.6×10^{-4} T and angle of dip is $\frac{\pi}{6}$. [5 marks]

Solution:

Concept:

- Biot-Savart law gives $d\vec{B} = (\mu_0 I / 4\pi) d\vec{\ell} \times \hat{s} / s^2$, where $\vec{s}$ points from the current element to the observation point.
- Symmetry cancels components perpendicular to the axis of a circular loop; the axial components from all elements add in the same direction.
- A straight current-carrying rod experiences $\vec{F} = I\vec{L} \times \vec{B}$. Earth's field has northward horizontal component $B \cos \delta$ and downward vertical component $B \sin \delta$ at the stated positive dip δ .

Step 1: (a) Determine the field from a current element Take the loop in a plane perpendicular to its axis and choose an axial point P a distance x from its centre O . Every current element is the same distance $s = \sqrt{R^2 + x^2}$ from P . The tangent $d\vec{\ell}$ is perpendicular to the vector from the element to P , so Biot-Savart law gives

$$dB = \frac{\mu_0 I d\ell}{4\pi(R^2 + x^2)}.$$

Let α be the angle that the element-to-point vector makes with the loop axis. Geometry gives $\sin \alpha = R / \sqrt{R^2 + x^2}$. Since $d\vec{B}$ is perpendicular to that vector, the axial projection is $dB \sin \alpha$.

Step 2: (a) Sum the axial components around the loop Diametrically opposite current elements produce equal and opposite transverse field components. Their axial components reinforce, so

$$\begin{aligned} B_{\text{axis}} &= \oint dB \sin \alpha \\ &= \frac{\mu_0 IR}{4\pi(R^2 + x^2)^{3/2}} \oint d\ell \\ &= \frac{\mu_0 IR}{4\pi(R^2 + x^2)^{3/2}} (2\pi R) \\ &= \frac{\mu_0 IR^2}{2(R^2 + x^2)^{3/2}}. \end{aligned}$$

The field points along the loop axis in the direction indicated by the right-hand thumb when the fingers curl with the current. At the centre, $x = 0$, this expression reduces to $\mu_0 I / (2R)$, providing a consistency check. For a single loop there is no additional turn-count factor.

Step 3: (b) Resolve Earth's field relative to the rod The current flows horizontally northward. The horizontal field $B_H = B \cos \delta$ is northward and parallel to the rod, so it contributes no magnetic force. The downward vertical component is perpendicular to the rod:

$$B_V = B \sin \delta = (0.6 \times 10^{-4}) \sin(\pi/6) = 3.0 \times 10^{-5} \text{ T}.$$

Only this component enters the magnitude $F = ILB_V$; using the entire field as a perpendicular field would give an incorrect result.

Step 4: (b) Calculate the magnitude and fix the direction Substitute $I = 5 \text{ A}$ and $L = 2 \text{ m}$:

$$F = (5)(2)(3.0 \times 10^{-5}) = 3.0 \times 10^{-4} \text{ N}.$$

To verify the direction explicitly, take $\hat{x}$ east, $\hat{y}$ north and $\hat{z}$ upward. Then $\vec{L} = 2\hat{y} \text{ m}$ and $\vec{B} = B \cos \delta \hat{y} - B \sin \delta \hat{z}$. Thus

$$\vec{F} = I(2\hat{y}) \times (-B \sin \delta \hat{z}) = -ILB \sin \delta \hat{x}.$$

The negative eastward direction is west. This also follows from Fleming's left-hand rule with current northward and field downward.

Step 5: Conclusion (a) The axial field is $B = \mu_0 I R^2 / [2(R^2 + x^2)^{3/2}]$, directed along the axis by the right-hand rule. (b) The magnetic force is $3.0 \times 10^{-4} \text{ N}$, horizontally towards the west.

Quick Tip to Solve: For a northward horizontal rod, Earth's horizontal field is parallel to the current; use only $B \sin \delta$ and take north crossed with down to obtain west.

OR

37. (a) Obtain the expression for the deflecting torque acting on the current carrying rectangular coil of a galvanometer in a uniform magnetic field. Why is a radial magnetic field employed in the moving coil galvanometer?

(b) Particles of mass $1.6 \times 10^{-27} \text{ kg}$ and charge $1.6 \times 10^{-19} \text{ C}$ are accelerated in a cyclotron of dee radius 40 cm. It employs a magnetic field 0.4 T. Find the kinetic energy (in MeV) of the particle beam imparted by the accelerator. [5 marks]

Solution:

Concept:

- The magnetic force on a straight conductor carrying current I is $\vec{F} = I\vec{l} \times \vec{B}$, where $\vec{l}$ points along the current and $\vec{B}$ is the magnetic field.
- A coil of N turns and area A has magnetic moment $\vec{m} = NIA\hat{n}$. Its torque is $\vec{\tau} = \vec{m} \times \vec{B}$; $\hat{n}$ is the unit normal to its plane.
- In a cyclotron, the magnetic force supplies centripetal force: $qvB = mv^2/r$. The nonrelativistic kinetic energy is $K = mv^2/2$, and $1 \text{ MeV} = 1.6 \times 10^{-13} \text{ J}$.

Step 1: (a) Identify the force couple on the rectangular coil Consider a rectangular coil of N turns, length l , breadth b , and area $A = lb$, carrying current I . Let θ be the angle between the normal to the coil and the uniform field $\vec{B}$. Choose the pair of sides of length l perpendicular to the field and parallel to the rotation axis. Each of these sides experiences a force of magnitude $F = NI l B$. Their currents are opposite, so the forces are equal and opposite. The forces have no resultant but form a couple. The perpendicular separation of their lines of action is $b \sin \theta$. Forces on the other two sides do not contribute torque about this axis.

Step 2: (a) Obtain the deflecting torque and explain the radial field The moment of the force couple is therefore

$$\tau_d = F(b \sin \theta) = NI l B b \sin \theta = NIAB \sin \theta.$$

In vector form, $\vec{\tau}_d = \vec{m} \times \vec{B}$, with $m = NIA$. The torque tends to align the magnetic moment with the field. A radial magnetic field keeps the field in the plane of the rotating coil throughout its useful

motion, so that the coil normal remains perpendicular to the field. Thus $\theta = 90^\circ$ at every deflection and $\tau_d = NIAB$, independent of the angular position of the coil. If the restoring torsional couple of its suspension is $\kappa\phi$, equilibrium gives $\kappa\phi = NIAB$, or $\phi = (NAB/\kappa)I$. Consequently, deflection is directly proportional to current, giving a uniform scale and maximum magnetic torque for the given current.

Step 3: (b) Relate the exit radius to kinetic energy At the outer edge of the dees, use the maximum orbit radius $r = 40 \text{ cm} = 0.40 \text{ m}$. With $m = 1.6 \times 10^{-27} \text{ kg}$, $q = 1.6 \times 10^{-19} \text{ C}$, and $B = 0.4 \text{ T}$, the circular-motion equation gives

$$qvB = \frac{mv^2}{r}, \quad v = \frac{qBr}{m}, \quad K = \frac{q^2 B^2 r^2}{2m}.$$

The electric field across the dee gap supplies energy; the magnetic field bends the trajectory without doing work. The limiting radius therefore determines the final momentum $mv = qBr$.

Step 4: (b) Substitute and convert joules to MeV Substitution yields

$$\begin{aligned} K &= \frac{(1.6 \times 10^{-19})^2 (0.4)^2 (0.40)^2}{2(1.6 \times 10^{-27})} \text{ J} \\ &= 2.048 \times 10^{-13} \text{ J}, \\ K \text{ (in MeV)} &= \frac{2.048 \times 10^{-13}}{1.6 \times 10^{-13}} = 1.28. \end{aligned}$$

As a consistency check, the corresponding speed is $v = 1.6 \times 10^7 \text{ m s}^{-1}$, about $0.053c$, so the nonrelativistic treatment used in the cyclotron formula is appropriate.

Step 5: Conclusion (a) The deflecting torque is $\tau_d = NIAB \sin \theta$. A radial field makes it $\tau_d = NIAB$ for every deflection, producing a uniform current scale. (b) The beam kinetic energy is 1.28 MeV .

Quick Tip to Solve: Define the torque angle with the coil normal; for the cyclotron use the dee radius in metres and convert joules using $1 \text{ MeV} = 1.6 \times 10^{-13} \text{ J}$.