

CBSE Class 12 Physics Question Paper 2020 with Solution PDF-Set 55/3/1



Question Paper with Solutions

Conducted by the Central Board of Secondary Education (CBSE)

Time Allowed: 3 hours

Maximum Marks: 70

General Instructions

- (i) This question paper comprises **four** Sections – **A, B, C** and **D**.
- (ii) There are **37** questions in the question paper. **All** questions are compulsory.
- (iii) **Section A** – Questions no. **1 to 20** are very short answer type questions, carrying **one** mark each.
- (iv) **Section B** – Questions no. **21 to 27** are short answer type questions, carrying **two** marks each.
- (v) **Section C** – Questions no. **28 to 34** are long answer type questions, carrying **three** marks each.
- (vi) **Section D** – Questions no. **35 to 37** are also long answer type questions, carrying **five** marks each.
- (vii) There is no overall choice in the question paper. However, an internal choice has been provided in 2 questions of 1 mark, 2 questions of 2 marks, 1 question of three marks and all the 3 questions of five marks. You have to attempt only one of the choices in such questions.
- (viii) In addition to this, separate instructions are given with each section and question, wherever necessary.
- (ix) Use of calculators and log tables is **not** permitted.

You may use the following values of physical constants wherever necessary.

$$c = 3 \times 10^8 \text{ m/s}$$

$$h = 6.63 \times 10^{-34} \text{ Js}$$

$$e = 1.6 \times 10^{-19} \text{ C}$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ T m A}^{-1}$$

$$\epsilon_0 = 8.854 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$$

$$\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$$

$$\text{Mass of electron } (m_e) = 9.1 \times 10^{-31} \text{ kg}$$

$$\text{Mass of neutron} = 1.675 \times 10^{-27} \text{ kg}$$

$$\text{Mass of proton} = 1.673 \times 10^{-27} \text{ kg}$$

$$\text{Avogadro's number} = 6.023 \times 10^{23} \text{ per gram mole}$$

$$\text{Boltzmann constant} = 1.38 \times 10^{-23} \text{ JK}^{-1}$$

Section A (20 marks)

Note : Select the most appropriate option from those given below each question :

1. If a positive charge is displaced against the electric field in which it was situated, then [1 mark]

- (A) work will be done by the electric field on the charge.
- (B) the intensity of the electric field decreases.
- (C) energy of the system will decrease.
- (D) energy will be provided by external source displacing the charge.

Correct Answer: (D) energy will be provided by external source displacing the charge.

Solution:

Concept:

- An electric field exerts force $\vec{F} = q\vec{E}$ on a charge q . For a positive charge, force is along the field.
- For slow displacement, external work equals the increase in electric potential energy: $W_{\text{ext}} = \Delta U = -W_{\text{field}}$.

Step 1: Determine the sign of field work The displacement is opposite to the field and hence opposite to the force on the positive charge. The work of the electric field is therefore negative:

$$W_{\text{field}} = \int q\vec{E} \cdot d\vec{r} < 0.$$

The field opposes this displacement rather than supplying the energy needed to move the charge.

Step 2: Identify the energy source An external agent must work against the electric force. For displacement without a change in kinetic energy, $W_{\text{ext}} = -W_{\text{field}} > 0$, so the electric potential energy increases. Option (C) has the wrong energy change. The field intensity need not decrease, ruling out (B). Although the field does negative work, (A) does not identify the source providing energy for displacement against the force; the intended positive energy supply is described by (D).

Step 3: Conclusion Energy is supplied by the external source displacing the charge against the field. Hence, the correct option is (D).

Quick Tip to Solve: For a positive charge moved against the field, remember: external work is positive and potential energy increases.

2. The electric flux emerging out from 1 C charge is

[1 mark]

- (A) $\frac{1}{\varepsilon_0}$
(B) 4π
(C) $\frac{4\pi}{\varepsilon_0}$
(D) ε_0

Correct Answer: (A) $\frac{1}{\varepsilon_0}$

Solution:

Concept:

- Gauss's law gives total outward electric flux through any closed surface as $\Phi_E = q_{\text{enc}}/\varepsilon_0$.
- Here q_{enc} is the enclosed charge and ε_0 is the permittivity of free space; flux has unit $\text{N m}^2 \text{C}^{-1}$.

Step 1: Apply Gauss's law Choose a closed surface surrounding the given charge. The total electric flux emerging from the charge equals the flux through this surface and depends only on the enclosed charge, not on the shape or size of the surface. Thus

$$\Phi_E = \frac{q_{\text{enc}}}{\varepsilon_0}.$$

Step 2: Substitute the enclosed charge With $q_{\text{enc}} = 1 \text{ C}$, the flux is

$$\Phi_E = \frac{1}{\varepsilon_0} \text{ N m}^2 \text{C}^{-1}.$$

The factor 4π in the point-charge field cancels against the spherical area $4\pi r^2$. Therefore (B) and (C) incorrectly retain a geometric factor, while (D) incorrectly multiplies by permittivity instead of dividing by it.

Step 3: Conclusion The outward flux is $1/\varepsilon_0$ in SI flux units. Hence, the correct option is (A).

Quick Tip to Solve: For total flux from an enclosed charge, use q divided by permittivity directly; no extra factor of 4π remains.

3. Two capacitors of capacitances C_1 and C_2 are connected in parallel. If a charge Q is given to the combination, the ratio of the charge on the capacitor C_1 to the charge on C_2 will be [1 mark]

- (A) $\frac{C_1}{C_2}$
(B) $\sqrt{\frac{C_1}{C_2}}$
(C) $\sqrt{\frac{C_2}{C_1}}$
(D) $\frac{C_2}{C_1}$

Correct Answer: (A) $\frac{C_1}{C_2}$

Solution:

Concept:

- Capacitors connected in parallel have the same potential difference V .
- The charge on a capacitor is $Q_i = C_i V$, where C_i is its capacitance; the total charge is $Q = Q_1 + Q_2$.

Step 1: Express the charges at the common voltage Both capacitors are connected to the same two terminals. Their potential differences are therefore equal, even though their stored charges can differ. Using the capacitance definition gives

$$Q_1 = C_1 V, \quad Q_2 = C_2 V.$$

Also $V = Q/(C_1 + C_2)$, since the given charge is distributed between the two capacitors.

Step 2: Divide the two charge equations Taking the ratio in the order asked cancels the common voltage:

$$\frac{Q_1}{Q_2} = \frac{C_1 V}{C_2 V} = \frac{C_1}{C_2}.$$

The square-root expressions in (B) and (C) do not follow from the linear relation $Q = CV$. Option (D) reverses the numerator and denominator. The capacitor with larger capacitance stores more charge at the same voltage.

Step 3: Conclusion The required charge ratio is $Q_1/Q_2 = C_1/C_2$. Hence, the correct option is (A).

Quick Tip to Solve: In parallel, cancel the common voltage in Q_1/Q_2 ; charge divides in direct proportion to capacitance.

4. The electrical resistance of a conductor

[1 mark]

- (A) varies directly proportional to its area of cross-section.
- (B) decreases with increase in its temperature.
- (C) decreases with increase in its conductivity.
- (D) is independent of its shape but depends only on its volume.

Correct Answer: (C) decreases with increase in its conductivity.

Solution:

Concept:

- For a uniform conductor, resistance is $R = \rho L/A = L/(\sigma A)$, where L is length and A is cross-sectional area.
- Resistivity ρ and conductivity σ are reciprocals: $\rho = 1/\sigma$.

Step 1: Compare resistance with conductivity Hold the conductor's dimensions fixed. From

$$R = \frac{L}{\sigma A},$$

resistance is inversely proportional to conductivity. A material that conducts current more readily has lower resistance for the same length and area.

Step 2: Check the other statements Option (A) reverses the area dependence: $R \propto 1/A$, not A . For an ordinary metallic conductor, resistance increases as temperature increases, so (B) is not the standard behaviour. Resistance depends on dimensions and hence on shape; even at fixed volume $V = AL$, $R = \rho L^2/V$ changes with length. Thus (D) is also incorrect.

Step 3: Conclusion Resistance decreases when conductivity increases, with geometry fixed. Hence, the correct option is (C).

Quick Tip to Solve: Write $R = L/(\sigma A)$ before comparing material properties: conductivity belongs in the denominator.

5. $\text{m}^2 \text{V}^{-1} \text{s}^{-1}$ is the SI unit of which of the following?

[1 mark]

- (A) Drift velocity
- (B) Mobility
- (C) Resistivity
- (D) Potential gradient

Correct Answer: (B) Mobility

Solution:

Concept:

- Charge-carrier mobility is drift speed per unit electric field: $\mu = v_d/E$.
- The SI units of drift speed v_d and field magnitude E are m s^{-1} and V m^{-1} , respectively.

Step 1: Find the unit from the definition Mobility measures how readily a carrier acquires drift

Quick Tip to Solve: Divide the unit of drift velocity by V/m to identify mobility; the metre in the field denominator produces $\text{m}^2 \text{V}^{-1} \text{s}^{-1}$.

motion under an electric field. Dividing the SI units in its definition gives

$$[\mu] = \frac{\text{m s}^{-1}}{\text{V m}^{-1}} = \text{m}^2 \text{V}^{-1} \text{s}^{-1}.$$

This is exactly the unit stated in the question.

Step 2: Compare the remaining quantities Drift velocity itself has unit m s^{-1} , so it does not contain an inverse volt. Resistivity has unit $\Omega \text{ m}$, obtained from $\rho = RA/L$. Potential gradient has unit V m^{-1} . None matches the given unit; the extra metre and inverse volt arise specifically from dividing drift speed by electric field.

Step 3: Conclusion The given SI unit is that of mobility. Hence, the correct option is (B).

Quick Tip to Solve: Divide the unit of drift velocity by V/m to identify mobility; the metre in the field denominator produces m squared .

6. The element of a heater is rated (P, V) . If it is connected across a source of voltage $\frac{V}{2}$, then the power consumed by it will be [1 mark]

- (A) P
- (B) $2P$
- (C) $\frac{P}{2}$
- (D) $\frac{P}{4}$

Correct Answer: (D) $\frac{P}{4}$

Solution:

Concept:

- For a resistive heater, electrical power is $P = V^2/R$, where R is the element resistance.
- The rating (P, V) fixes the resistance as $R = V^2/P$; use this resistance when changing the applied voltage.

Step 1: Determine the element resistance The heater consumes its rated power P when the rated voltage V is applied. From Joule's heating relation,

$$R = \frac{V^2}{P}.$$

As usual for this rating question, the element is treated as having the same resistance at the new operating voltage.

Step 2: Evaluate power at half voltage At the new voltage $V' = V/2$,

$$P' = \frac{(V/2)^2}{R} = \frac{V^2}{4R} = \frac{P}{4}.$$

Quick Tip to Solve: For the same resistance, square the voltage ratio: half the voltage gives one quarter of the power.

The power depends on the square of voltage for fixed resistance. Thus (C), which halves the power, incorrectly assumes a linear dependence. Options (A) and (B) respectively leave power unchanged or increase it despite the reduced voltage.

Step 3: Conclusion The heater consumes $P/4$. Hence, the correct option is (D).

Quick Tip to Solve: For the same resistance, square the voltage ratio: half the voltage gives one quarter of the power.

7. In Bohr's model of hydrogen atom, the total energy of the electron in n^{th} discrete orbit is proportional to

[1 mark]

- (A) n
- (B) $\frac{1}{n}$
- (C) n^2
- (D) $\frac{1}{n^2}$

Correct Answer: (D) $\frac{1}{n^2}$

Solution:

Concept:

- Bohr's hydrogen energy levels are $E_n = -13.6/n^2$ eV, measured relative to a free electron at rest.
- The principal quantum number n labels the orbit, and the orbit radius varies as $r_n \propto n^2$.

Step 1: Relate total energy to orbit radius For an electron bound by the Coulomb force, potential energy is negative and twice the magnitude of its kinetic energy. Therefore

$$E_n = K_n + U_n = -\frac{e^2}{8\pi\epsilon_0 r_n}.$$

The total energy is negative and inversely proportional to the orbit radius.

Step 2: Use the Bohr radius dependence Since $r_n \propto n^2$, substitution gives

$$E_n = -\frac{13.6}{n^2} \text{ eV} \Rightarrow E_n \propto \frac{1}{n^2}.$$

The proportionality constant is negative; the energy magnitude decreases for larger n , while the signed energy approaches zero. Options (A) and (C) incorrectly grow with n , and (B) has only the first power in the denominator.

Step 3: Conclusion The total energy has inverse-square dependence on n . Hence, the correct option is (D).

Quick Tip to Solve: Keep the pair together: Bohr radius grows as n squared, whereas binding-energy magnitude falls as $1/n$ squared.

8. A zener diode has

[1 mark]

- (A) heavily doped p-side and lightly doped n-side.
- (B) heavily doped n-side and lightly doped p-side.
- (C) heavily doped n-side as well as p-side.
- (D) lightly doped n-side as well as p-side.

Correct Answer: (C) heavily doped n-side as well as p-side.

Solution:

Concept:

- A Zener diode is a heavily doped p-n junction designed to operate safely in reverse breakdown.
- Heavy doping on both sides produces a thin depletion layer and a large electric field at a relatively small reverse voltage.

Step 1: Connect doping to the depletion layer Increasing the acceptor concentration on the p-side and donor concentration on the n-side reduces the width of the depletion region. In a Zener diode, both sides are heavily doped, making this region sufficiently thin for a strong junction electric field under reverse bias.

Step 2: Compare the doping combinations The strong field permits Zener breakdown at the designed reverse voltage, so the diode can maintain an approximately constant voltage while current varies within its operating limits. Options (A) and (B) leave one side lightly doped; depletion then extends more into that side. Option (D) leaves both sides lightly doped and produces a wider depletion region. These do not describe the standard Zener junction.

Step 3: Conclusion A Zener diode has heavily doped n-side as well as p-side. Hence, the correct option is (C).

Quick Tip to Solve: Associate Zener operation with a thin depletion layer: achieving it requires heavy doping on both sides of the junction.

9. A region has a uniform magnetic field in it. A proton enters into the region with velocity making an angle of 45° with the direction of the magnetic field. In this region the proton will move on a path having the shape of a

[1 mark]

- (A) straight line
- (B) circle
- (C) spiral
- (D) helix

Correct Answer: (D) helix

Solution:

Concept:

- The magnetic force on a proton is $\vec{F} = q\vec{v} \times \vec{B}$. It acts perpendicular to the velocity and does no work.

Quick Tip to Solve: For magnetic motion, first check whether both $v \sin \theta$ and $v \cos \theta$ are nonzero: both together give a helix.

- Resolve the velocity into $v_{\parallel} = v \cos \theta$ along the field and $v_{\perp} = v \sin \theta$ perpendicular to it.

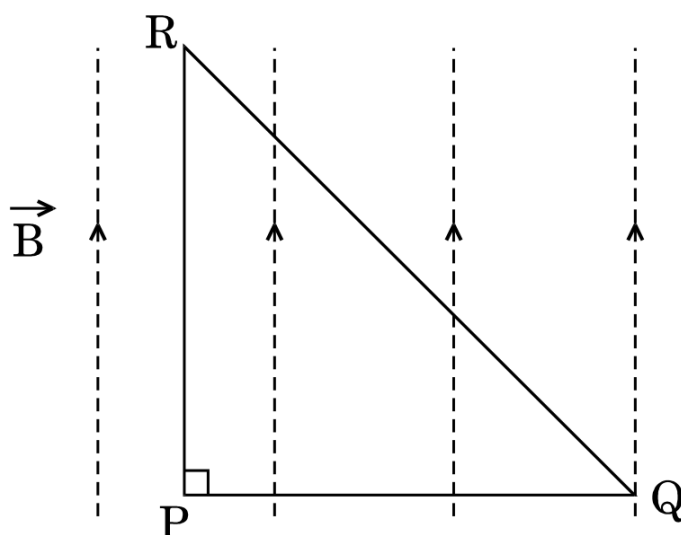
Step 1: Resolve the initial velocity At $\theta = 45^\circ$, both components are nonzero and equal to $v/\sqrt{2}$. The parallel component experiences no magnetic force and therefore remains constant.

Step 2: Combine the two motions The perpendicular component undergoes circular motion of fixed radius $r = mv_{\perp}/(qB)$, while the proton advances uniformly along the field. The radius does not grow or shrink, so a planar spiral is excluded. A circle would require zero parallel component; a straight line would require zero perpendicular component.

Step 3: Conclusion The superposition produces a helix of constant radius and pitch along the magnetic field. Hence, the correct option is (D).

Quick Tip to Solve: For magnetic motion, first check whether both $v \sin \theta$ and $v \cos \theta$ are nonzero: both together give a helix.

10. An isosceles right angled current carrying loop PQR is placed in a uniform magnetic field $\vec{B}$ pointing along PR. If the magnetic force acting on the arm PQ is F , then the magnetic force which acts on the arm QR will be [1 mark]



- (A) F
 (B) $\frac{F}{\sqrt{2}}$
 (C) $\sqrt{2} F$
 (D) $-F$

Correct Answer: (D) $-F$

Solution:

Concept:

- The force on a straight current-carrying arm is $\vec{F} = I\vec{L} \times \vec{B}$, with $\vec{L}$ directed along the current.

Quick Tip to Solve: In a uniform field, first check the arm parallel to the field. If its force is zero, the other two forces in the closed triangle must cancel.

- For a closed loop in a uniform field, $\sum \vec{L} = 0$, hence its net magnetic force is zero.

Step 1: Find the force on arm PR The field is along PR, so this arm is parallel or antiparallel to $\vec{B}$. Therefore $\vec{F}_{PR} = 0$, irrespective of the current direction.

Step 2: Compare the other two arms The loop must satisfy $\vec{F}_{PQ} + \vec{F}_{QR} + \vec{F}_{PR} = 0$. Thus the two remaining forces have equal magnitudes and opposite directions. This also follows from $PQ = a$, $QR = \sqrt{2}a$, and $\sin 45^\circ = 1/\sqrt{2}$: the longer arm has a smaller sine factor, so its force magnitude remains IBa . Options (B) and (C) omit one of these factors; option (A) expresses only the magnitude and misses the opposite direction.

Step 3: Conclusion Taking the direction of the force on PQ as positive, the force on QR is $-F$. Hence, the correct option is (D).

Quick Tip to Solve: In a uniform field, first check the arm parallel to the field. If its force is zero, the other two forces in the closed triangle must cancel.

11. The shape of the wavefront originating from a line source is _____.

[1 mark]

Solution:

Concept:

- A wavefront is a surface joining points of a wave that have the same phase at a given instant.
- For a long, uniform line source in a homogeneous medium, the wavefront symmetry is determined by equal perpendicular distances from the source line.

Step 1: Identify equal travel distances At a given time after emission, the disturbance has travelled the same distance in every direction perpendicular to the line source. Points at the same perpendicular distance receive the same phase.

Step 2: Construct the constant-phase surface In each plane normal to the line, these points form a circle centred on the source. Extending those circles along the source line generates a cylindrical surface. This differs from a point source, whose equal-distance surface is spherical.

Step 3: Conclusion The wavefront originating from a line source is **cylindrical**.

Quick Tip to Solve: Relate the source geometry to equal-distance surfaces: a point gives spheres, while a straight line gives cylinders.

12. The refractive index of the material of a converging lens is 1.5. If air is replaced by a medium of refractive index 1.6, then the lens will now behave as a _____ lens.

[1 mark]

Solution:

Concept:

- For a thin lens in a medium, $1/f = (n_\ell/n_m - 1)(1/R_1 - 1/R_2)$, where n_ℓ and n_m are the lens and medium refractive indices.

Quick Tip to Solve: Compare the lens index with the surrounding medium index before classifying its behaviour; the same shape can reverse its power.

- The lens shape is unchanged; its power changes sign if the relative refractive index passes from above unity to below unity.

Step 1: Calculate the relative refractive index The lens material has $n_\ell = 1.5$, and the replacement medium has $n_m = 1.6$. Therefore

$$\frac{n_\ell}{n_m} = \frac{1.5}{1.6} = \frac{15}{16} < 1.$$

Step 2: Check the sign of the power The factor $(n_\ell/n_m - 1)$ is now $-1/16$, whereas it was positive in air. Since the curvature factor is unchanged and the lens was converging in air, its power reverses sign and its focal length becomes negative. The surrounding medium being optically denser causes this reversal.

Step 3: Conclusion The lens now behaves as a **diverging lens**.

Quick Tip to Solve: Compare the lens index with the surrounding medium index before classifying its behaviour; the same shape can reverse its power.

13. In Young's double slit experiment, the separation between the two slits is halved. The new fringe width will be _____ times its initial value. [1 mark]

Solution:

Concept:

- In Young's double-slit experiment the fringe width is $\beta = \lambda D/d$, where λ is wavelength, D is screen distance and d is slit separation.
- With wavelength and screen distance fixed, fringe width is inversely proportional to slit separation.

Step 1: Write the new slit separation The separation is halved, so $d' = d/2$. The question changes neither the wavelength of light nor the distance of the screen from the slits.

Step 2: Compare the fringe widths Substitute the new separation into the same expression:

$$\beta' = \frac{\lambda D}{d/2} = 2 \frac{\lambda D}{d} = 2\beta.$$

Reducing the separation increases the distance between successive bright fringes because the path difference changes more slowly with position on the screen.

Step 3: Conclusion The new fringe width is **2 times** its initial value.

Quick Tip to Solve: Use the ratio $\beta'/\beta = d/d'$ when only the slit separation changes; halving the denominator doubles the width.

14. The value of Brewster's angle for air-glass interface is $\frac{\pi}{3}$; hence the refractive index of glass is _____. [1 mark]

Solution:

Concept:

- Brewster's law gives $\tan i_B = n_2/n_1$, where i_B is the polarising angle and n_1, n_2 are the refractive indices of the incident and refracting media.
- For incidence from air into glass, $n_1 \simeq 1$, so the refractive index of glass is $n_g = \tan i_B$.

Step 1: Express the given angle The Brewster angle is $i_B = \pi/3$ radians, or 60° . This is the angle of incidence measured from the normal, as used in Brewster's law.

Step 2: Apply Brewster's law For the air-glass interface, substitute the angle:

$$n_g = \tan\left(\frac{\pi}{3}\right) = \tan 60^\circ = \sqrt{3}.$$

The refractive index is a ratio of speeds and therefore has no unit. It is the tangent of the given angle, not its sine.

Step 3: Conclusion The refractive index of glass is $\sqrt{3}$ (approximately 1.73).

Quick Tip to Solve: For a Brewster-angle question, use the tangent and the relative index; $\pi/3$ is 60° , whose tangent is $\sqrt{3}$.

15. In photoelectric effect, the number of emitted photoelectrons is proportional to _____ of incident light. [1 mark]

Solution:

Concept:

- Photoelectric emission occurs when incident photon frequency exceeds the threshold frequency of the surface.
- At a fixed frequency, each photon has energy $h\nu$; increasing intensity increases the number of photons arriving per unit area per unit time.

Step 1: Relate intensity to photon arrival rate For a fixed illuminated area A and frequency ν , the incident photon rate is proportional to $IA/(h\nu)$, where I is the intensity. More intense light supplies more photons in the same time interval.

Step 2: Relate photons to emitted electrons A photon transfers its energy to a single electron in the elementary photoelectric process. With the same surface and frequency above threshold, increasing the photon flux increases the photoelectron emission rate proportionally. Intensity controls the number emitted, while frequency controls the maximum kinetic energy.

Step 3: Conclusion The number of emitted photoelectrons is proportional to the **intensity** of incident light (at fixed frequency above threshold).

Quick Tip to Solve: Distinguish the two effects: intensity changes the emission rate, whereas frequency changes the maximum kinetic energy.

15. Light of frequency ν is incident on a photosensitive surface of threshold frequency ν_0 ($\nu > \nu_0$). The value of kinetic energy of the emitted photoelectrons will be _____. [1 mark]

Solution:

Concept:

- Einstein's photoelectric equation is $h\nu = \phi + K_{\max}$, where ϕ is the surface work function and h is Planck's constant.
- The threshold frequency obeys $\phi = h\nu_0$; at this minimum frequency the maximum kinetic energy just vanishes.

Step 1: Identify the energy required for escape Each incident photon carries energy $h\nu$. An electron must use an amount $\phi = h\nu_0$ to escape from the photosensitive surface. The given condition $\nu > \nu_0$ ensures a positive energy surplus.

Step 2: Calculate the available kinetic energy Subtract the work function from the photon energy:

$$K_{\max} = h\nu - h\nu_0 = h(\nu - \nu_0).$$

This is the maximum kinetic energy intended by the fill-in-the-blank. Individual electrons can emerge with lower kinetic energies owing to energy losses before escape; the equation gives the upper limit.

Step 3: Conclusion The required expression is $K_{\max} = h(\nu - \nu_0)$.

Quick Tip to Solve: Convert threshold frequency into the work function $h\nu_0$, then subtract it from the incident photon energy $h\nu$.

16. An ac is passed through a series LCR circuit. What is the impedance of the circuit at resonance? [1 mark]

Solution:

Concept:

- In a series LCR circuit, the impedance is $Z = \sqrt{R^2 + (X_L - X_C)^2}$, where R is the resistance, $X_L = \omega L$ is the inductive reactance and $X_C = 1/(\omega C)$ is the capacitive reactance.
- Resonance occurs when the inductive and capacitive reactances have equal magnitudes, so their opposing contributions cancel.

Step 1: Apply the resonance condition At the resonant angular frequency ω_0 , $\omega_0 L = 1/(\omega_0 C)$. Hence $X_L - X_C = 0$: the circuit has no net reactance.

Step 2: Evaluate the remaining impedance Substituting this condition gives

$$Z_{\text{res}} = \sqrt{R^2 + 0} = R.$$

Thus the impedance is purely resistive and has its minimum value; it is not zero unless the resistance itself vanishes.

Quick Tip to Solve: For series resonance, cancel the two reactances before evaluating the impedance; the resistance remains.

Step 3: Conclusion At resonance, the impedance of the series LCR circuit is $Z = R$, equal to its resistance.

Quick Tip to Solve: For series resonance, cancel the two reactances before evaluating the impedance; the resistance remains.

17. Two identical coils, one of copper and the other of aluminium are rotated with the same angular speed in an external magnetic field. In which of the two coils will the induced current be more?

[1 mark]

Solution:

Concept:

- Faraday's law gives the induced emf as $\varepsilon = -N d\Phi/dt$, where N is the number of turns and Φ is the magnetic flux through one turn.
- For a coil of fixed dimensions, resistance is $R = \rho\ell/A_w$, where ρ is resistivity, ℓ is wire length and A_w is wire cross-sectional area.

Step 1: Compare the induced emfs The identical coils have the same number of turns and area. Rotation at the same angular speed in the same magnetic field gives the same rate of flux change, and therefore equal induced emfs. Changing the metal does not change this geometrical flux change.

Step 2: Compare their resistances and currents Copper has a smaller resistivity than aluminium. Since ℓ and A_w are the same, $R_{Cu} < R_{Al}$. With equal induced emf, $I = \varepsilon/R$ gives a larger induced current in the copper coil.

Step 3: Conclusion The induced current is greater in the copper coil, because its resistance is lower.

Quick Tip to Solve: First compare flux changes to establish equal emf; only then use the metal resistivity to compare currents.

18. In an ac circuit, the applied voltage and flowing current are $E = E_0 \sin \omega t$ and $I = I_0 \sin \left(\omega t + \frac{\pi}{2} \right)$ respectively. What is the average power consumed in one cycle in this circuit?

[1 mark]

Solution:

Concept:

- The average power in an ac circuit is $P_{av} = E_{rms} I_{rms} \cos \phi$, where ϕ is the phase difference between voltage and current.
- For sinusoidal signals, $E_{rms} = E_0/\sqrt{2}$ and $I_{rms} = I_0/\sqrt{2}$. The instantaneous power is the product $p(t) = E(t)I(t)$.

Step 1: Identify the phase difference The voltage is $E_0 \sin \omega t$, while the current is $I_0 \sin(\omega t + \pi/2)$. The current leads the voltage by $\pi/2$, so the power factor is $\cos(\pi/2) = 0$.

Quick Tip to Solve: A quarter-cycle phase difference makes the power factor zero even though both voltage and current are nonzero.

Step 2: Average the power over one cycle Therefore

$$P_{av} = \frac{E_0 I_0}{2} \cos \frac{\pi}{2} = 0.$$

Equivalently, $p(t) = (E_0 I_0 / 2) \sin 2\omega t$; its equal positive and negative portions cancel over a complete cycle.

Step 3: Conclusion The average power consumed in one complete cycle is 0 W.

Quick Tip to Solve: A quarter-cycle phase difference makes the power factor zero even though both voltage and current are nonzero.

OR

18. What happens when a block of metal is kept in a varying magnetic field?

[1 mark]

Solution:

Concept:

- A time-varying magnetic field changes the magnetic flux through closed conducting paths inside a metal block.
- By Faraday's law, the flux change induces an emf. The circulating currents produced within the bulk metal are called eddy currents; their direction obeys Lenz's law.

Step 1: Identify the induced currents The metal contains many possible closed conducting paths. When the magnetic field varies, the flux linked with these paths changes, inducing circulating eddy currents without any separate external circuit.

Step 2: State the physical effect These currents flow through the finite resistance of the metal and dissipate electrical energy as heat. Their magnetic field opposes the change that produces them, as required by Lenz's law. Thus the varying field can heat the block.

Step 3: Conclusion Eddy currents are induced in the metal block, causing heating.

Quick Tip to Solve: For a bulk metal in a changing magnetic field, name eddy currents and then mention their Joule-heating effect.

19. Mention the contribution of Indian physicist J.C. Bose in the production of electromagnetic waves.

[1 mark]

Solution:

Concept:

- Electromagnetic waves include microwaves, which have short wavelengths and can be generated and detected experimentally.
- J. C. Bose carried out pioneering experiments on short-wavelength electromagnetic radiation, demonstrating its production and wave properties.

Quick Tip to Solve: Associate J. C. Bose with millimetre waves and quote the approximate 25 mm to 5 mm wavelength range.

Step 1: Identify Bose's contribution J. C. Bose produced electromagnetic waves in the millimetre wavelength region. These were much shorter than the radio waves used in many early experiments and fall in the microwave region of the spectrum.

Step 2: Specify the wavelength scale His experiments produced waves with wavelengths ranging approximately from 25 mm down to 5 mm. This contribution concerns the experimental generation of short electromagnetic waves, rather than the theoretical prediction of their existence.

Step 3: Conclusion J. C. Bose produced short electromagnetic waves of wavelengths approximately 25 mm to 5 mm in the microwave region.

Quick Tip to Solve: Associate J. C. Bose with millimetre waves and quote the approximate 25 mm to 5 mm wavelength range.

20. Write one use of the electromagnetic waves of frequency range from 10^{16} Hz to 10^{20} Hz. [1 mark]

Solution:

Concept:

- Electromagnetic radiation is classified by frequency, with wavelength given by $\lambda = c/\nu$, where c is the speed of light and ν is frequency.
- The stated broad frequency interval includes the X-ray region. X-rays penetrate soft tissue more readily than dense bone, giving useful absorption contrast.

Step 1: Relate the interval to suitable radiation For example, radiation of frequency 10^{18} Hz lies within the stated interval. Its wavelength is

$$\lambda = \frac{3.0 \times 10^8}{10^{18}} = 3.0 \times 10^{-10} \text{ m},$$

which is in the X-ray region.

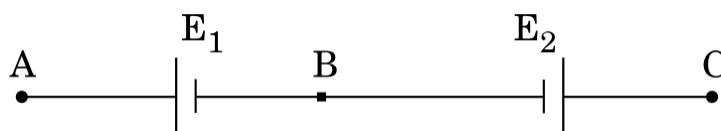
Step 2: State one application X-rays are used in medical radiography to examine bones and detect fractures. Bone absorbs X-rays more strongly than the surrounding soft tissue, so a radiograph reveals the bone outline and any break in it.

Step 3: Conclusion One use is medical X-ray imaging to detect bone fractures.

Quick Tip to Solve: The frequency interval is broad; select an X-ray frequency within it and give a clear X-ray application.

Section B (14 marks)

21. Two cells of emf E_1 and E_2 ($E_1 > E_2$) are connected as shown in the figure below. When a potentiometer is used to measure potential difference between the points A and B, the balancing length of the potentiometer wire is 300 cm. But the same potentiometer for the potential difference between points A and C, gives the balancing length 100 cm. Find $\frac{E_1}{E_2}$. [2 marks]



Solution:

Concept:

- At potentiometer balance, the measured potential difference is $V = k\ell$, where k is the uniform potential gradient and ℓ is the balancing length.
- The longer line of a cell symbol is its positive terminal. The two cells in the figure oppose each other, so the net emf between A and C is $E_1 - E_2$.

Step 1: Read the cell polarities For the first cell, A is on the positive side and B on the negative side. Thus $V_A - V_B = E_1$. For the second cell, C is on the positive side and B on the negative side, giving $V_C - V_B = E_2$. Therefore

$$V_A - V_C = (V_A - V_B) - (V_C - V_B) = E_1 - E_2.$$

The condition $E_1 > E_2$ ensures this difference is positive.

Step 2: Use the two balancing lengths The same potentiometer has the same potential gradient in both measurements. With lengths expressed in centimetres,

$$E_1 = k(300), \quad E_1 - E_2 = k(100).$$

Dividing eliminates the unknown gradient:

$$\frac{E_1}{E_1 - E_2} = \frac{300}{100} = 3.$$

Step 3: Solve for the emf ratio Rearranging, $E_1 = 3E_1 - 3E_2$, hence $2E_1 = 3E_2$. As a check, $E_2 = k(200)$, so the first cell emf is indeed greater than the second.

Step 4: Conclusion The required ratio is $\boxed{\frac{E_1}{E_2} = \frac{3}{2}}$.

Quick Tip to Solve: Read the long battery plates first: AB measures the first emf, while AC measures the difference of the emfs.

22. Two identical bars, one of paramagnetic material and other of diamagnetic material are kept in a uniform external magnetic field parallel to it. Draw diagrammatically the modifications in the magnetic field pattern in each case. [2 marks]

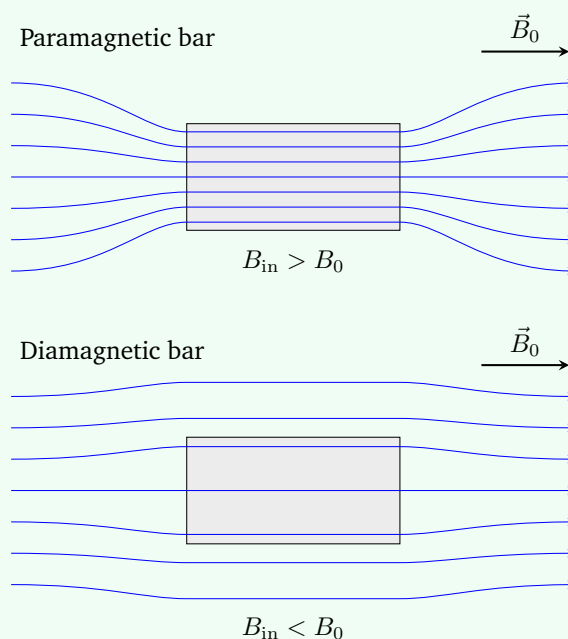
Solution:

Concept:

- Field-line density represents magnetic field strength. A paramagnetic material has positive magnetic susceptibility, $\chi_m > 0$, and its induced magnetisation is parallel to the applied field.
- A diamagnetic material has negative magnetic susceptibility, $\chi_m < 0$, and its induced magnetisation opposes the applied field. For weak materials, $B = \mu_0(1 + \chi_m)H$.

Step 1: Paramagnetic bar The bar is placed with its length parallel to the uniform applied field $\vec{B}_0$. Its magnetisation reinforces the applied field, so the field inside the bar is slightly stronger than outside. The field lines therefore bend towards the bar and become more closely spaced inside it. The upper pattern shows this increase in line density; the arrows point along the applied field.

Step 2: Diamagnetic bar The induced magnetisation is opposite to $\vec{B}_0$, reducing the resultant field inside the bar. Field lines tend to bend away from the material, leaving fewer lines through the bar and increasing the spacing of those that penetrate it. The lower pattern shows this partial expulsion. The field is reduced, rather than completely excluded, for an ordinary diamagnetic material.



Field patterns around bars parallel to the applied magnetic field (changes exaggerated for clarity).

Step 3: Conclusion Paramagnetic bar: field lines crowd inside, $B_{in} > B_0$. Diamagnetic bar: field lines spread away, $B_{in} < B_0$. Both patterns are shown with the bar parallel to the applied field.

Quick Tip to Solve: Remember: positive susceptibility draws field lines into the bar; negative susceptibility pushes them away.

23. Two coplanar and concentric coils 1 and 2 have respectively the number of turns N_1 and N_2 and radii r_1 and r_2 ($r_2 \gg r_1$). Deduce the expression for mutual inductance of this system. [2 marks]

Solution:

Concept:

- Mutual inductance M is the flux linkage in one coil per unit current in the other: $N_1\Phi_1 = MI_2$, where Φ_1 is the flux through each turn of coil 1.
- The field at the centre of a circular coil of radius r_2 , N_2 turns and current I_2 is $B_2 = \mu_0 N_2 I_2 / (2r_2)$.

Step 1: Choose current in the larger coil Let current I_2 flow in coil 2, the larger coil. Since the coils are concentric and coplanar, its field near the centre is perpendicular to the plane of the smaller coil. The condition $r_2 \gg r_1$ makes this field nearly uniform over the area of coil 1. Thus

$$B_2 \simeq \frac{\mu_0 N_2 I_2}{2r_2}.$$

Choosing the larger coil as the source avoids approximating its field far away from its centre.

Step 2: Find the linked flux and divide by current The area of each turn of coil 1 is $A_1 = \pi r_1^2$. The field is normal to this area, so the flux per turn is $\Phi_1 = B_2 A_1$. For N_1 turns the flux linkage is

$$N_1\Phi_1 \simeq N_1 \frac{\mu_0 N_2 I_2}{2r_2} \pi r_1^2.$$

Dividing the flux linkage by I_2 gives the mutual inductance. By reciprocity, the same value applies when the roles of the coils are interchanged; the result assumes an air or vacuum medium.

Step 3: Conclusion The mutual inductance is

$$M \simeq \frac{\mu_0 \pi N_1 N_2 r_1^2}{2r_2}.$$

The approximation is valid for $r_2 \gg r_1$; the SI unit is henry.

Quick Tip to Solve: Put current in the larger coil, use its central field, and multiply by the smaller coil area and turns.

24. How does an oscillating charge radiate an electromagnetic wave? Give the relation between the frequency of radiated wave and the frequency of oscillating charge. [2 marks]

Solution:

Concept:

- An oscillating charge is an accelerated charge. Accelerated charges emit electromagnetic radiation; a charge moving uniformly does not radiate merely because it has a velocity.
- An electromagnetic wave consists of time-dependent electric and magnetic fields perpendicular to each other and to the direction of propagation. In vacuum it travels at $c = 1/\sqrt{\mu_0 \epsilon_0}$.

Step 1: Explain the radiation mechanism As the charge oscillates, its electric field changes with time.

Quick Tip to Solve: Trace the repeating acceleration of the charge: its period is also the period of the emitted wave.

A time-varying electric field produces a time-varying magnetic field, and the changing magnetic field is associated with a changing electric field. The coupled disturbance propagates outward through space, carrying energy away from the accelerated charge. These mutually perpendicular changing fields form an electromagnetic wave, which does not require a material medium.

Step 2: Relate the frequencies If the charge performs sinusoidal oscillations with frequency f and period $T = 1/f$, the electric and magnetic disturbances repeat with that same period. For example, $x = x_0 \cos(2\pi ft)$ has acceleration $a = -(2\pi f)^2 x_0 \cos(2\pi ft)$; the radiated fields follow this oscillatory acceleration. Consequently the frequency of the emitted wave equals the frequency of the oscillating charge.

Step 3: Conclusion An oscillating charge radiates through its time-varying electric and magnetic fields. The frequency relation is

$$f_{\text{wave}} = f_{\text{charge}}.$$

Quick Tip to Solve: Trace the repeating acceleration of the charge: its period is also the period of the emitted wave.

OR

24. (a) Explain briefly the fact that electromagnetic waves carry energy.

(b) Why do we not feel the pressure due to sunshine?

[2 marks]

Solution:

Concept:

- Electric and magnetic fields store energy. For an electromagnetic wave their energy densities are $u_E = \frac{1}{2}\epsilon_0 E^2$ and $u_B = B^2/(2\mu_0)$.
- Electromagnetic waves also carry momentum. Radiation pressure on a perfectly absorbing surface is $p = I/c$, where I is the incident intensity; for normal incidence on a perfectly reflecting surface it is $2I/c$.

Step 1: (a) Explain energy transport An electromagnetic wave contains electric and magnetic fields, so it has total energy density

$$u = \frac{1}{2}\epsilon_0 E^2 + \frac{B^2}{2\mu_0}.$$

In vacuum $E = cB$, making the two contributions equal. As the wave propagates, this energy travels with it and can be transferred to matter when absorbed. For example, sunlight warms a surface and electromagnetic radiation can produce a photoelectric effect. Thus the wave carries energy from the source to the receiver.

Step 2: (b) Explain why sunshine pressure is imperceptible When sunlight is absorbed, its momentum is transferred to the surface, producing a force and hence radiation pressure. However, the pressure is very small because $p = I/c$ and $c \simeq 3 \times 10^8 \text{ m s}^{-1}$ is enormous. For an illustrative intensity of 10^3 W m^{-2} , absorption gives $p \simeq 3.3 \times 10^{-6} \text{ Pa}$. Even complete reflection only doubles this tiny

Quick Tip to Solve: Use $p = I/c$: dividing the solar intensity by the very large speed of light immediately shows why pressure is tiny.

pressure, so it is far below what our skin can ordinarily feel.

Step 3: Conclusion (a) Electromagnetic waves transport energy stored in their electric and magnetic fields. (b) Sunshine exerts radiation pressure, but its magnitude is too small for us to feel.

Quick Tip to Solve: Use $p = I/c$: dividing the solar intensity by the very large speed of light immediately shows why pressure is tiny.

25. A converging lens of focal length f_1 is placed coaxially in contact with a diverging lens of focal length f_2 ($f_1 > f_2$). Determine the power and nature of the combination in terms of f_1 and f_2 .

[2 marks]

Solution:

Concept:

- For thin lenses coaxially in contact, their powers add: $P = P_1 + P_2$, where $P = 1/F$ and F is the equivalent signed focal length in metres.
- A converging lens has positive power; a diverging lens has negative power. Here f_1 and f_2 denote the positive magnitudes of their respective focal lengths in the printed comparison $f_1 > f_2$.

Step 1: Add the signed powers Using the Cartesian sign convention, the focal lengths to substitute in the power formula are $+f_1$ for the converging lens and $-f_2$ for the diverging lens. Thus, when both magnitudes are expressed in metres,

$$P = \frac{1}{f_1} + \frac{1}{-f_2} = \frac{f_2 - f_1}{f_1 f_2} \text{ D.}$$

The contact condition lets us add powers directly, without a lens separation term.

Step 2: Determine the nature from the sign Since $f_1 > f_2 > 0$, the numerator $f_2 - f_1$ is negative and the denominator is positive. Equivalently, the diverging lens has the greater power magnitude because its focal-length magnitude is smaller. The equivalent focal length is

$$F = \frac{f_1 f_2}{f_2 - f_1} < 0.$$

Therefore a parallel incident beam emerges diverging from the combination.

Step 3: Conclusion $P = \frac{f_2 - f_1}{f_1 f_2}$ D for f_1, f_2 in metres; the combination is diverging.

Quick Tip to Solve: Assign the diverging lens a negative focal length before adding powers; compare focal-length magnitudes to identify the stronger lens.

25. How is the resolving power of a compound microscope affected if (a) wavelength of light used is decreased, and (b) the diameter of its objective lens is increased? Justify your answers. [2 marks]

Solution:

Concept:

- Resolving power is the reciprocal of the minimum separation $d_{\min}$ between two specimen points that can just be distinguished as separate.
- For a microscope, $d_{\min} = \lambda / (2\mu \sin \alpha)$, so $RP = 2\mu \sin \alpha / \lambda$. Here λ is the light wavelength, μ the medium's refractive index, and α the semi-angle of the cone accepted by the objective; $\mu \sin \alpha$ is its numerical aperture.

Step 1: Decrease the wavelength For part (a), keep the objective and medium unchanged. Its numerical aperture is then fixed, giving

$$RP \propto \frac{1}{\lambda}.$$

Decreasing λ decreases the diffraction-limited separation $d_{\min}$. Two closer points can therefore be seen separately, so resolving power increases.

Step 2: Increase the objective diameter For part (b), keep wavelength, medium and the other geometrical conditions unchanged. Increasing the clear diameter of the objective allows it to accept a wider cone of rays from a specimen point. Hence α , $\sin \alpha$ and the numerical aperture increase. Since

$$RP \propto \mu \sin \alpha,$$

the minimum resolvable separation decreases and resolving power increases. This is an improvement in the ability to distinguish details, rather than merely an increase in image size.

Step 3: Conclusion Resolving power increases in both cases: (a) it varies inversely with wavelength; (b) a larger objective diameter increases numerical aperture under otherwise unchanged conditions.

Quick Tip to Solve: Use $RP = 2NA/\lambda$: shorter wavelength or a larger accepted cone lets the microscope distinguish closer points.

26. Define the terms (a) threshold frequency, and (b) stopping potential. How were these terms incorporated in Einstein's photoelectric equation? [2 marks]

Solution:

Concept:

- Einstein's photoelectric equation expresses conservation of energy for one absorbed photon: $h\nu = \phi + K_{\max}$, where h is Planck's constant, ν the incident frequency, ϕ the metal's work function and $K_{\max}$ the maximum emitted-electron kinetic energy.
- The work function is $\phi = h\nu_0$, and the stopping-potential magnitude V_0 measures maximum kinetic energy through $K_{\max} = eV_0$, where e is the positive magnitude of electronic charge.

Quick Tip to Solve: Replace work function by $h\nu_0$ and maximum kinetic energy by eV_0 ; use a positive magnitude for the stopping potential.

Step 1: Define threshold frequency Part (a): The threshold frequency ν_0 is the minimum frequency of incident light required to just eject photoelectrons from a particular metal surface. Below ν_0 , no photoelectric emission occurs, however high the light intensity. At threshold, the most energetic electrons emerge with zero kinetic energy, so $h\nu_0 = \phi$.

Step 2: Define stopping potential and substitute Part (b): The stopping potential V_0 is the magnitude of the minimum retarding potential applied to the collector relative to the emitting surface that stops even the fastest photoelectrons, reducing photocurrent to zero. The collector is negative relative to the emitter. Its retarding work is $eV_0 = K_{\max}$. Incorporating both definitions into Einstein's equation gives

$$h\nu = h\nu_0 + eV_0,$$

$$eV_0 = h(\nu - \nu_0).$$

Thus the incident photon supplies the threshold energy first, and its excess energy appears as the maximum electron kinetic energy.

Step 3: Conclusion Threshold frequency is the minimum frequency for emission; stopping potential is the minimum retarding-potential magnitude that makes photocurrent zero. They enter Einstein's equation as $h\nu = h\nu_0 + eV_0$.

Quick Tip to Solve: Replace work function by $h\nu_0$ and maximum kinetic energy by eV_0 ; use a positive magnitude for the stopping potential.

27. A hydrogen atom is in its third excited state.

(a) How many spectral lines can be emitted by it before coming to the ground state? Show these transitions in the energy level diagram.

(b) In which of the above transitions will the spectral line of shortest wavelength be emitted? [2 marks]

Solution:

Concept:

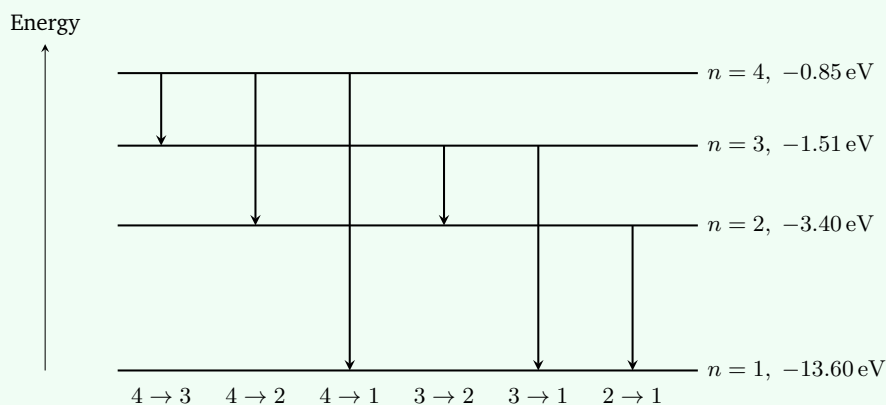
- The ground state of hydrogen is $n = 1$; successive excited states are $n = 2, 3, 4, \dots$. Thus the third excited state is $n = 4$.
- Each downward transition gives a photon with $hc/\lambda = E_i - E_f$. Hydrogen levels have energies $E_n = -13.6/n^2$ eV, where n is the principal quantum number.

Step 1: Count and draw the possible transitions Part (a): Starting from $n = 4$, the available levels before reaching the ground state are 4, 3, 2, 1. Counting all distinct pairs of levels gives the maximum number of possible spectral lines:

$$N = \frac{n(n-1)}{2} = \frac{4 \times 3}{2} = 6.$$

The transitions are $4 \rightarrow 3$, $4 \rightarrow 2$, $4 \rightarrow 1$, $3 \rightarrow 2$, $3 \rightarrow 1$ and $2 \rightarrow 1$, shown by the six downward arrows. These are possible lines across all decay paths. One atom in a single cascade emits at most three photons, rather than all six lines.

Quick Tip to Solve: Convert the third excited state to $n = 4$, then count level pairs. For shortest wavelength, select the largest downward energy jump.



Six possible downward transitions from levels up to $n = 4$; vertical spacing is schematic.

Step 2: Find the largest photon energy Part (b): Since $\lambda = hc/(E_i - E_f)$, the shortest wavelength corresponds to the greatest energy difference. The highest occupied initial level is $n = 4$, and the lowest final level is $n = 1$. Therefore the largest drop is

$$\Delta E_{4 \rightarrow 1} = 13.6 \left(1 - \frac{1}{16} \right) = 12.75 \text{ eV}.$$

Every other listed transition spans a smaller energy interval, so none gives a shorter wavelength.

Step 3: Conclusion (a) Six spectral lines are possible. (b) The shortest-wavelength line is emitted in the transition $n = 4 \rightarrow n = 1$.

Quick Tip to Solve: Convert the third excited state to $n = 4$, then count level pairs. For shortest wavelength, select the largest downward energy jump.

Section C (21 marks)

28. (a) Differentiate between the random velocity and the drift velocity of electrons in an electrical conductor. Give their order of magnitudes.

(b) A conductor of uniform cross-sectional area is connected across a dc source of variable voltage. Draw a graph showing variation of drift velocity of electrons (v_d) as a function of current density (J) in it.

[3 marks]

Solution:

Concept:

- Random motion is the irregular motion of conduction electrons in all directions; its vector average is zero in equilibrium.
- Drift velocity is the small average velocity superposed on random motion by an applied electric field. Electrons drift opposite to the field.
- For electron number density n , charge magnitude e , and drift-speed magnitude v_d , the current-density magnitude is $J = nev_d$.

Step 1: Compare the two electron velocities (a) Random velocity changes direction in collisions and

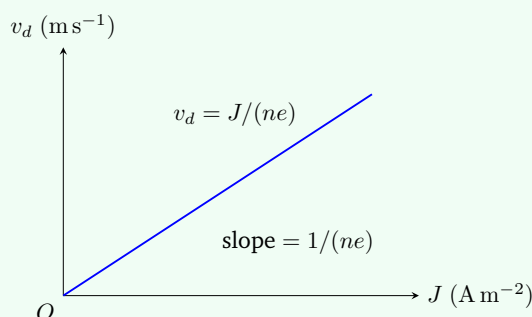
Quick Tip to Solve: State whether you are plotting speed magnitudes: $J = nev_d$ gives a positive slope, while electron drift and conventional current point in opposite directions.

exists without an applied voltage. For electrons in a metallic conductor its typical speed is of order 10^5 m s^{-1} . The velocities are distributed in all directions, so they do not produce a net electric current in equilibrium. Drift velocity is the directed average velocity acquired when an electric field acts on the conductor. In an ordinary wire its magnitude is typically of order 10^{-4} m s^{-1} , depending on the material and current density. It is much smaller than random speed. The electron drift is opposite to the applied field, whereas conventional current is along the field.

Step 2: Relate drift speed to current density (b) In a time Δt , the electrons crossing a section of area A occupy a volume $Av_d\Delta t$. The magnitude of charge transported is $\Delta Q = neAv_d\Delta t$. Therefore,

$$I = \frac{\Delta Q}{\Delta t} = neAv_d, \quad J = \frac{I}{A} = nev_d.$$

For the same conductor, n is constant. Hence $v_d = J/(ne)$: the plot of drift-speed magnitude on the vertical axis against current-density magnitude on the horizontal axis is a straight line through the origin, with positive slope $1/(ne)$. Here the plotted quantities are magnitudes; in vector form $\mathbf{J} = -nev_d\mathbf{\hat{v}}$.



Drift-speed magnitude versus current-density magnitude for a fixed conductor.

Step 3: Conclusion Random speed is typically of order 10^5 m s^{-1} , with zero mean velocity; directed drift speed is typically of order 10^{-4} m s^{-1} . The magnitude graph is linear through the origin: $v_d = J/(ne)$.

Quick Tip to Solve: State whether you are plotting speed magnitudes: $J = nev_d$ gives a positive slope, while electron drift and conventional current point in opposite directions.

29. A series LCR ac circuit has $L = 2.0 \text{ H}$, $C = 32 \mu\text{F}$ and $R = 10 \Omega$.

(a) At what angular frequency of ac will it resonate?

(b) Calculate the Q value of the circuit.

[3 marks]

Solution:

Concept:

- In a series LCR circuit, resonance occurs when the inductive and capacitive reactances are equal: $\omega_0 L = 1/(\omega_0 C)$.
- The resonant angular frequency is $\omega_0 = 1/\sqrt{LC}$, where L is inductance and C capacitance.
- The quality factor of a series resonant circuit is $Q = \omega_0 L/R = (1/R)\sqrt{L/C}$. It is dimensionless

Quick Tip to Solve: First compute ω_0 , then use $Q = \omega_0 L/R$; this avoids taking a second square root and makes the cancellation of units clear.

and measures the sharpness of resonance.

Step 1: Convert the capacitance to SI units The given circuit has $L = 2.0 \text{ H}$, $R = 10 \Omega$, and

$$C = 32 \mu\text{F} = 32 \times 10^{-6} \text{ F}.$$

Using farads rather than microfarads in the formula is essential. At resonance the net reactive part of the impedance vanishes, so the impedance becomes R and the current is maximum for a fixed supply voltage.

Step 2: Calculate the resonant angular frequency (a) Equating the two reactances gives $\omega_0^2 LC = 1$. Substitute the SI values:

$$\begin{aligned} LC &= (2.0)(32 \times 10^{-6}) = 64 \times 10^{-6}, \\ \sqrt{LC} &= 8.0 \times 10^{-3} \text{ s}, \\ \omega_0 &= \frac{1}{8.0 \times 10^{-3}} = 125 \text{ rad s}^{-1}. \end{aligned}$$

This is angular frequency, the quantity requested, so no extra division by 2π is made in the answer.

Step 3: Calculate the quality factor (b) Evaluate the ratio of the resonant inductive reactance to resistance:

$$X_L(\omega_0) = \omega_0 L = (125)(2.0) = 250 \Omega, \quad Q = \frac{250}{10} = 25.$$

The ohms cancel, leaving a pure number. As a check, $(1/R)\sqrt{L/C} = (1/10)\sqrt{2.0/(32 \times 10^{-6})} = 25$ gives the same value.

Step 4: Conclusion (a) The circuit resonates at $\omega_0 = 125 \text{ rad s}^{-1}$. (b) Its quality factor is $Q = 25$, with no units.

Quick Tip to Solve: First compute ω_0 , then use $Q = \omega_0 L/R$; this avoids taking a second square root and makes the cancellation of units clear.

OR

29. An ideal inductor of $\frac{5}{\pi} \text{ H}$ inductance is connected to a 200 V, 50 Hz ac supply.

(a) Calculate the rms and peak value of current in the inductor.

(b) What is the phase difference between current through the inductor and the applied voltage? How will it change if a small resistance is connected in series with this inductor in the circuit? [3 marks]

Solution:

Concept:

- An ideal inductor has reactance $X_L = \omega L = 2\pi fL$; its rms current is $I_{\text{rms}} = V_{\text{rms}}/X_L$.
- For a sinusoidal current, peak current $I_0 = \sqrt{2} I_{\text{rms}}$. The quoted ac supply voltage is an rms value.
- Current lags the applied voltage by $\pi/2$ in a pure inductor. In a series RL circuit the lag angle obeys $\tan \phi = \omega L/R$.

Step 1: Calculate the inductive reactance Use the printed inductance $L = 5/\pi \text{ H}$, the frequency

Quick Tip to Solve: Cancel the printed $1/\pi$ in the inductance against $2\pi f$ first; use the rms voltage for rms current and multiply only the current by $\sqrt{2}$ for its peak.

$f = 50 \text{ Hz}$, and the rms supply voltage $V_{\text{rms}} = 200 \text{ V}$. The angular frequency and reactance are

$$\omega = 2\pi f = 100\pi \text{ rad s}^{-1}, \quad X_L = (100\pi) \left(\frac{5}{\pi} \right) = 500 \Omega.$$

The factor π cancels because the specified inductance itself contains $1/\pi$. An ideal inductor has no resistive contribution to its impedance.

Step 2: Find the rms and peak currents (a) Apply the rms form of Ohm's law for the inductive impedance, then convert the current to its peak value:

$$I_{\text{rms}} = \frac{200}{500} = 0.40 \text{ A}, \quad I_0 = \sqrt{2}(0.40) = 0.40\sqrt{2} \text{ A} \simeq 0.566 \text{ A}.$$

The rms current is the effective current used with the quoted supply voltage; it is not the maximum instantaneous current.

Step 3: Determine the phase lag and its change (b) Since $v = L di/dt$, a sinusoidal current produces a voltage leading it by a quarter cycle. Thus current lags voltage by 90° , or $\pi/2$ rad. Adding a small positive resistance R in series produces an RL circuit with

$$\phi = \tan^{-1} \left(\frac{X_L}{R} \right) = \tan^{-1} \left(\frac{500}{R} \right).$$

The current still lags the applied voltage, but the lag decreases below 90° . For $R \ll 500 \Omega$, the angle remains close to 90° ; a numerical new angle cannot be specified without the resistance value.

Step 4: Conclusion The rms current is 0.40 A and the peak current is $0.40\sqrt{2} \text{ A} \simeq 0.566 \text{ A}$. Current initially lags voltage by 90° ; a small series resistance reduces this lag to $\tan^{-1}(500/R)$.

Quick Tip to Solve: Cancel the printed $1/\pi$ in the inductance against $2\pi f$ first; use the rms voltage for rms current and multiply only the current by $\sqrt{2}$ for its peak.

30. (a) Using the necessary ray diagram, derive the mirror formula for a concave mirror.

(b) In the magnified image of a measuring scale (with equidistant markings) lying along the principal axis of a concave mirror, the markings are not equidistant. Explain. [3 marks]

Solution:

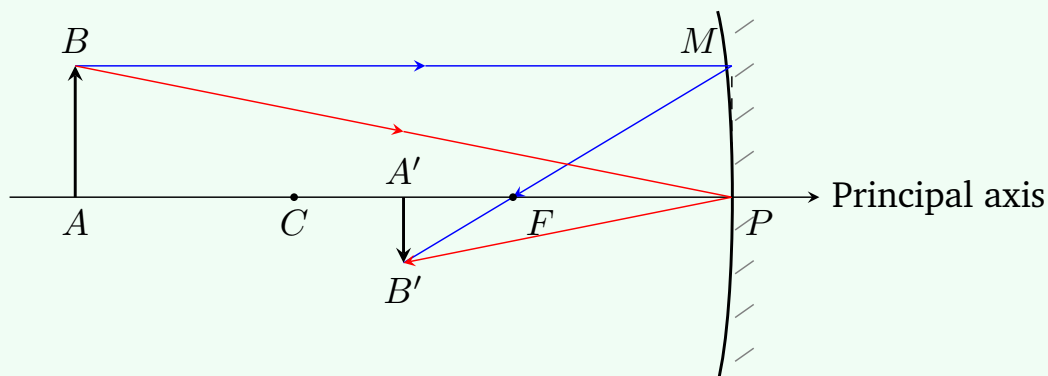
Concept:

- The mirror formula relates the signed object distance u , image distance v , and focal length f : $1/f = 1/u + 1/v$.
- The derivation uses paraxial rays, a small aperture, and similar triangles. Distances are measured from the pole P ; in the Cartesian convention, distances to the left of the mirror are negative.
- For a scale along the axis, the longitudinal image spacing depends on object position, since the relation between v and u is nonlinear.

Step 1: (a) Construct the paraxial ray diagram Place the upright object AB beyond the centre of curvature C . A ray from B parallel to the axis strikes the mirror at M and reflects through the focus F .

Quick Tip to Solve: For a scale along the axis, compare changes in image distance using $v = fu/(u - f)$, rather than using transverse magnification alone.

A second ray strikes the pole P , where its angles with the principal axis are equal. The reflected rays meet at B' , giving the inverted real image $A'B'$. Write the positive distance magnitudes as $U = AP$, $V = A'P$, and $F_0 = FP$, and the height magnitudes as $h = AB$, $h' = A'B'$. For a small aperture, the axial projection of M can be taken at P , and $MP \simeq h$.



Paraxial rays for a concave mirror: object beyond C , real image between C and F .

Step 2: (a) Use similar triangles to obtain the formula Equality of the incidence and reflection angles at the pole makes triangles ABP and $A'B'P$ similar. Therefore,

$$\frac{h'}{h} = \frac{V}{U}.$$

The parallel ray reflected through F gives similar triangles MPF and $B'A'F$. For the arrangement shown, $A'F = V - F_0$, so

$$\frac{h'}{h} = \frac{V - F_0}{F_0}.$$

Equating the ratios and rearranging,

$$\begin{aligned} \frac{V}{U} &= \frac{V - F_0}{F_0}, \\ VF_0 &= UV - UF_0, \\ \frac{1}{F_0} &= \frac{1}{U} + \frac{1}{V}. \end{aligned}$$

Here $u = -U$, $v = -V$, and $f = -F_0$. Substitution gives the Cartesian mirror formula $1/f = 1/u + 1/v$, which also applies to the other paraxial image positions with their appropriate signs.

Step 3: (b) Explain the unequal spacing of scale markings Different markings along the principal axis have different object distances u . Their image distances obey

$$v = \frac{fu}{u - f}, \quad \frac{dv}{du} = -\frac{f^2}{(u - f)^2}.$$

Thus, for a small equal separation $|\Delta u|$ between successive markings, the image separation is approximately $|\Delta v| = f^2|\Delta u|/(u - f)^2$. This factor changes along the scale and becomes larger for markings closer to the focus. Equal object intervals therefore produce unequal image intervals, even though the image is magnified.

Quick Tip to Solve: For a scale along the axis, compare changes in image distance using $v = fu/(u - f)$, rather than using transverse magnification alone.

Step 4: Conclusion The paraxial concave-mirror formula is $\boxed{1/f = 1/u + 1/v}$. The scale's image markings are unequally spaced because longitudinal magnification varies with the position of each marking along the principal axis.

Quick Tip to Solve: For a scale along the axis, compare changes in image distance using $v = fu/(u - f)$, rather than using transverse magnification alone.

31. (a) The density of the nuclear matter is tremendously larger than the physical density of the material. Explain.

(b) The nuclear forces are not coulomb forces between nucleons. Explain.

(c) Draw a plot of the potential energy between a pair of nucleons as a function of distance between them inside a nucleus. [3 marks]

Solution:

Concept:

- Nearly all atomic mass is concentrated in a nucleus whose radius is much smaller than the atomic radius. Nuclear radius obeys $R = R_0 A^{1/3}$, where A is mass number and $R_0 \simeq 1.2$ fm.
- The strong nuclear interaction acts between nucleons, including electrically neutral neutrons. It is attractive over typical nucleon separations, strongly repulsive at very short separations, and negligible beyond a few femtometres.

Step 1: (a) Compare nuclear and ordinary material densities With nucleon mass $m_N \simeq 1.67 \times 10^{-27}$ kg, nuclear mass is approximately Am_N . Therefore,

$$\rho_N = \frac{Am_N}{(4\pi/3)R_0^3 A} = \frac{3m_N}{4\pi R_0^3} \simeq 2.3 \times 10^{17} \text{ kg m}^{-3}.$$

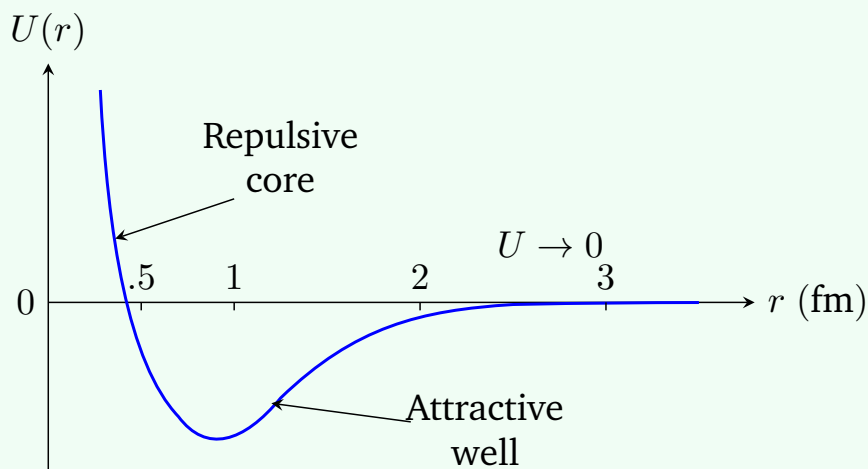
This is approximately independent of mass number. An atom occupies a region of radius roughly 10^{-10} m, whereas nuclear dimensions are of order 10^{-15} to 10^{-14} m. Almost all the mass is packed into this tiny central volume, while the surrounding atomic region contributes little mass. Ordinary material density averages the mass over the much larger volume occupied by atoms, explaining its enormously smaller value.

Step 2: (b) Distinguish nuclear attraction from the Coulomb force A Coulomb force depends on electric charge. Two protons repel electrostatically, while the Coulomb force between a proton and an uncharged neutron, or between two uncharged neutrons, is zero. Nuclear attraction nevertheless acts in proton-neutron and neutron-neutron pairs and binds nuclei despite proton-proton electrostatic repulsion. Nuclear forces are approximately charge independent and have a range of only a few femtometres. Coulomb forces are long range and vary as $1/r^2$. These facts show that the binding interaction cannot be a Coulomb force between nucleons.

Step 3: (c) Interpret the nuclear potential-energy curve Choose $U = 0$ at large separation. The plotted nuclear interaction potential rises steeply to positive values in the repulsive core, at separations less than about 0.5 fm. It has a negative well with a minimum at a separation of order 1 fm, and approaches zero from below beyond roughly 2 to 3 fm. The force is $F_r = -dU/dr$: the inner side of

Quick Tip to Solve: Read the force from the slope $F_r = -dU/dr$: a downward slope means repulsion, and an upward slope means attraction.

the well gives an outward repulsive force, whereas its outer side gives an inward attractive force. At the minimum, the force is zero. The graph is schematic and represents the short-range nuclear contribution; a proton pair also has a separate Coulomb repulsion.



Schematic nuclear potential between two nucleons; the zero is at large separation.

Step 4: Conclusion Nuclear density is enormous because nearly all atomic mass occupies the tiny nuclear volume. Nuclear binding is due to a short-range strong interaction rather than Coulomb attraction. Its potential has a repulsive core, a negative attractive well, and a negligible tail at large separation.

Quick Tip to Solve: Read the force from the slope $F_r = -dU/dr$: a downward slope means repulsion, and an upward slope means attraction.

32. What do you mean by wave nature of an electron? How was quantisation of angular momentum of the orbiting electron in Bohr's model of hydrogen atom explained by de Broglie hypothesis? [3 marks]

Solution:

Concept:

- De Broglie associates a matter wave with a moving electron of momentum p . Its wavelength is $\lambda = h/p$, where h is Planck's constant.
- A stable orbit in the de Broglie explanation of Bohr's model contains an integral number of wavelengths around its circumference, forming a standing wave.

Step 1: Explain the wave nature of an electron The wave nature of an electron means that its behaviour shows wave effects such as diffraction and interference. A moving electron is associated with a de Broglie wavelength $\lambda = h/p$; for a nonrelativistic electron of mass m and speed v , this becomes $\lambda = h/(mv)$. This matter wave is not an electromagnetic wave and does not mean that the electron is literally an extended vibrating string. Electron diffraction, such as diffraction by a crystal, gives experimental evidence of this wave character.

Step 2: Apply the standing-wave condition to an orbit In the de Broglie interpretation of Bohr's

Quick Tip to Solve: Start with circumference equal to an integer number of wavelengths, then substitute $\lambda = h/(mv)$ to obtain Bohr's rule directly.

circular orbit of radius r , the electron's associated wave must join smoothly after one complete circuit. The wave therefore fits around the circumference with a whole number of wavelengths:

$$2\pi r = n\lambda, \quad n = 1, 2, 3, \dots$$

A complete circuit then produces a phase change of $2\pi n$, so the returning wave is in phase with itself. Such a closed standing-wave pattern is allowed. A nonintegral number of wavelengths would not produce this self-consistent standing-wave pattern and is excluded in this explanation of the stable Bohr orbits.

Step 3: Derive the quantisation of angular momentum Substitute $\lambda = h/(mv)$ into the circumference condition:

$$2\pi r = n \frac{h}{mv},$$

$$mvr = n \frac{h}{2\pi} = n\hbar.$$

For circular motion the momentum is tangential, so the magnitude of orbital angular momentum is $L = rp = mvr$. Thus the standing-wave requirement gives exactly Bohr's angular-momentum rule. The integer n counts the de Broglie wavelengths accommodated by that orbit; continuous values of L are not permitted by this condition.

Step 4: Conclusion An electron has wave character with $\lambda = h/p$. The condition $2\pi r = n\lambda$ for a closed standing matter wave explains Bohr's quantisation $L = mvr = n\hbar/(2\pi) = n\hbar$.

Quick Tip to Solve: Start with circumference equal to an integer number of wavelengths, then substitute $\lambda = h/(mv)$ to obtain Bohr's rule directly.

33. Name the diode which can act as a voltage regulator. Explain its working with the help of its labelled circuit diagram. Draw its V-I characteristic. [3 marks]

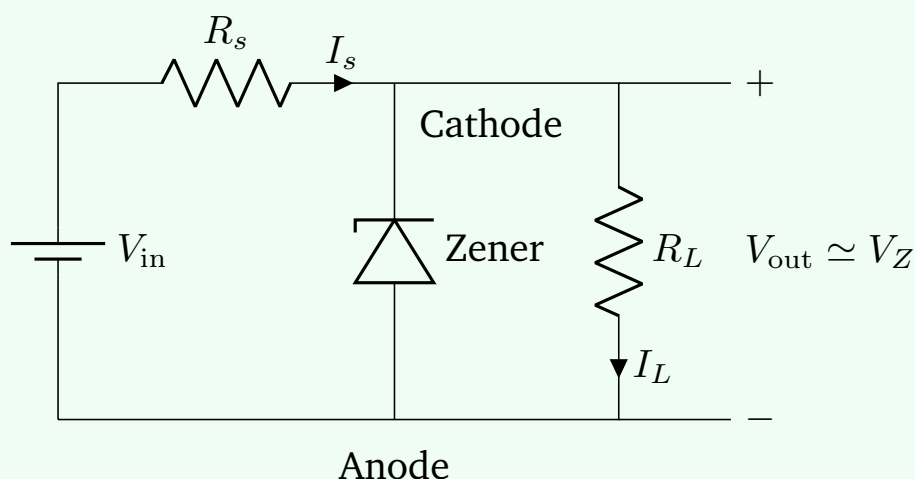
Solution:

Concept:

- A Zener diode is a heavily doped p-n junction diode designed to operate safely in reverse breakdown. Its breakdown voltage is denoted by V_Z .
- In the breakdown region, a large change in reverse current produces only a small change in voltage across the diode. A series resistor limits the current, while the load is connected in parallel with the Zener diode.

Step 1: Connect the voltage regulator Connect a series resistance R_s between the positive terminal of an unregulated dc supply V_{in} and the output terminal. Connect the Zener diode in reverse bias across the load resistance R_L : its cathode is connected to the positive output and its anode to the negative supply terminal. Choose $V_{in} > V_Z$, so that the diode operates in breakdown. The resistor drops the excess input voltage and protects the diode from excessive current.

Quick Tip to Solve: Put the Zener cathode at the positive output and draw the load in parallel with it; the series resistor belongs before that parallel combination.



Reverse-biased Zener diode used as a voltage regulator.

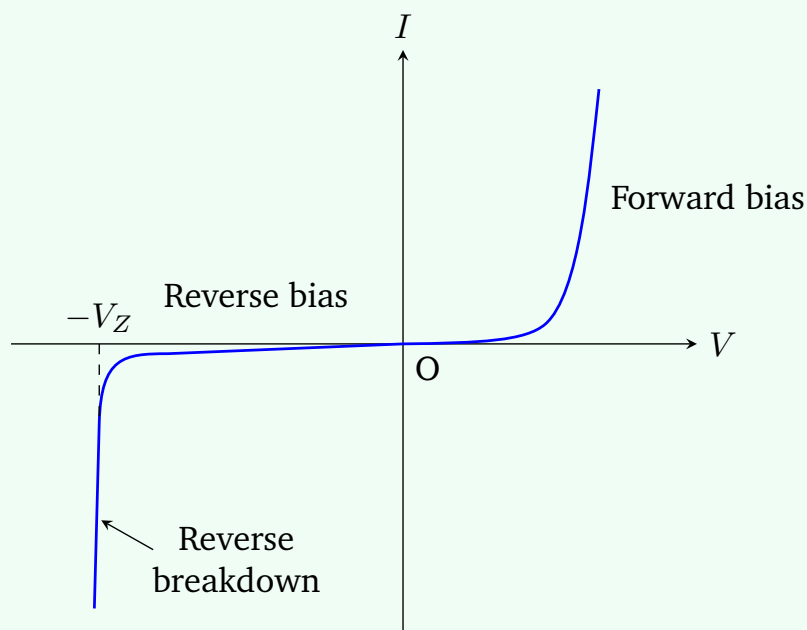
Step 2: Explain how the output remains steady The output is the voltage across both the load and the Zener diode, so $V_{\text{out}} \simeq V_Z$. The current through the series resistor divides into load current and Zener current:

$$I_s = \frac{V_{\text{in}} - V_Z}{R_s} = I_L + I_Z, \quad I_L = \frac{V_Z}{R_L}.$$

If the input voltage rises, the current through R_s rises; the extra current flows mainly through the Zener diode and its voltage remains nearly unchanged. If the input falls, the Zener current falls while the output stays close to V_Z . Similarly, an increase in load current is supplied by a decrease in Zener current, and a decrease in load current increases Zener current. Regulation holds only while the Zener remains in breakdown and its current stays within its permitted range.

Step 3: Interpret the V-I characteristic In forward bias the Zener diode behaves like an ordinary junction diode: current increases rapidly after the forward knee. In reverse bias only a small leakage current flows until the reverse voltage reaches V_Z . Beyond the reverse breakdown knee, the magnitude of reverse current rises sharply with very little further change in voltage. This almost vertical portion of the characteristic enables voltage regulation. On the signed axes shown, reverse voltage and reverse current are negative, so the breakdown knee lies near $-V_Z$.

Quick Tip to Solve: Put the Zener cathode at the positive output and draw the load in parallel with it; the series resistor belongs before that parallel combination.



Zener V-I characteristic; the reverse breakdown region is nearly vertical.

Step 4: Conclusion The diode is a Zener diode. Connected in reverse breakdown across the load with a series current-limiting resistor, it maintains an approximately constant output voltage $V_{\text{out}} = V_Z$.

Quick Tip to Solve: Put the Zener cathode at the positive output and draw the load in parallel with it; the series resistor belongs before that parallel combination.

34. (a) Why is an intrinsic semiconductor deliberately converted into an extrinsic semiconductor by adding impurity atoms?

(b) Explain briefly the two processes that occur in p-n junction region to create a potential barrier.

[3 marks]

Solution:

Concept:

- An intrinsic semiconductor has equal electron and hole concentrations and a relatively low conductivity. Controlled doping introduces additional mobile charge carriers.
- Diffusion is the movement of carriers from a region of higher concentration to a region of lower concentration. Drift is their movement under an electric field.

Step 1: (a) Explain the purpose of adding impurities The number of free charge carriers in a pure semiconductor is small at ordinary temperatures, so its conductivity is low. A small, controlled amount of suitable impurity greatly increases the majority-carrier concentration and hence increases conductivity. Pentavalent donor atoms provide extra electrons to form an n-type semiconductor; trivalent acceptor atoms create holes to form a p-type semiconductor. Doping therefore gives a useful, controllable conductivity and determines the type of majority carrier needed in semiconductor devices.

Step 2: (b) Describe diffusion and formation of the depletion layer When the p-type and n-type

Quick Tip to Solve: Remember the fixed-ion signs: positive donors remain on the n-side, negative acceptors on the p-side, so the barrier field points from n to p.

regions form a junction, electrons diffuse from the n-side to the p-side because their concentration is higher on the n-side. Holes diffuse from the p-side to the n-side for the same reason. Near the junction, diffusing electrons and holes recombine. This leaves immobile, positively charged donor ions on the n-side and negatively charged acceptor ions on the p-side. The region is depleted of mobile majority carriers and is called the depletion region. Its fixed charges set up an electric field directed from the n-side to the p-side and a built-in potential barrier that opposes further majority-carrier diffusion.

Step 3: (b) Describe drift and the equilibrium condition The electric field in the depletion region causes a drift of carriers: electrons move towards the n-side, opposite to the field, and holes towards the p-side, along the field. This drift opposes the corresponding diffusion across the junction. As charge separation builds up, the field and opposing drift current increase. At thermal equilibrium, the drift current equals the diffusion current in magnitude and is opposite in direction, so the net current is zero. Thus diffusion produces the space charge and potential barrier, while field-driven drift balances diffusion and establishes the equilibrium barrier.

Step 4: Conclusion (a) Doping increases and controls conductivity by adding majority charge carriers. (b) Diffusion creates the depletion layer and built-in potential barrier; drift caused by the resulting electric field opposes diffusion until the two currents balance at equilibrium.

Quick Tip to Solve: Remember the fixed-ion signs: positive donors remain on the n-side, negative acceptors on the p-side, so the barrier field points from n to p.

Section D (15 marks)

35. (a) An electric dipole of dipole moment $\vec{P}$ is placed in a uniform electric field $\vec{E}$ at an angle θ with it. Derive the expression for torque ($\vec{\tau}$) acting on it. Find the orientation of the dipole relative to the electric field for which torque on it is (i) maximum, and (ii) half of maximum.

(b) Two point charges $q_1 = +1 \mu\text{C}$ and $q_2 = +4 \mu\text{C}$ are placed 2 m apart in air. At what distance from q_1 along the line joining the two charges, will the net electric field be zero? [5 marks]

Solution:

Concept:

- An electric dipole consists of charges $-q$ and $+q$ separated by a distance $2a$. Its dipole moment is $\vec{p} = q(2\vec{a})$, directed from the negative charge to the positive charge.
- In a uniform electric field $\vec{E}$, the forces on the two charges are equal and opposite. Their resultant is zero, but their separated lines of action form a couple.
- Torque magnitude equals one force multiplied by the perpendicular separation of the two lines of action. Electric fields superpose vectorially, so cancellation requires equal magnitudes and opposite directions.

Step 1: (a) Derive the torque on the dipole Let the dipole axis make an angle θ with the uniform field. The positive charge experiences $\vec{F}_+ = q\vec{E}$, while the negative charge experiences $\vec{F}_- = -q\vec{E}$. Thus the net force is zero. The perpendicular distance between the parallel lines of action of these

Quick Tip to Solve: For two like charges, first place the null point between them; the distances from it are proportional to the square roots of their charge magnitudes.

forces is $2a \sin \theta$. The moment of the couple is therefore

$$\tau = (qE)(2a \sin \theta) = pE \sin \theta.$$

Equivalently, about the dipole centre each force contributes $aqE \sin \theta$ in the same rotational sense. With the torque direction specified by the right-hand rule, the vector expression is

$$\vec{\tau} = \vec{p} \times \vec{E}.$$

This torque tends to turn the dipole towards alignment with the electric field.

Step 2: (a) Determine the requested orientations For fixed p and E , the torque magnitude depends only on $\sin \theta$, with $0 \leq \theta \leq 180^\circ$. Its maximum occurs when $\sin \theta = 1$, so the dipole axis is perpendicular to the field at $\theta = 90^\circ$, and $\tau_{\max} = pE$. For half this magnitude,

$$pE \sin \theta = \frac{pE}{2} \implies \sin \theta = \frac{1}{2}.$$

Both $\theta = 30^\circ$ and $\theta = 150^\circ$ satisfy this condition. These angles refer to the direction of the dipole moment, from $-q$ to $+q$, relative to $\vec{E}$.

Step 3: (b) Locate the region in which cancellation is possible Both point charges are positive, so their fields point away from their respective charges. Outside the interval joining them, the two fields have the same direction and cannot cancel. Between the charges the fields oppose each other. Let the zero-field point be at a distance x from q_1 towards q_2 , where $0 < x < 2$ m. Its distance from q_2 is then $2 - x$, with distances measured in metres.

Step 4: (b) Equate the field magnitudes and solve Using Coulomb's field expression with $k = 1/(4\pi\epsilon_0)$, cancellation requires

$$\frac{k(1 \times 10^{-6})}{x^2} = \frac{k(4 \times 10^{-6})}{(2 - x)^2}.$$

Cancelling the common factors and taking positive square roots, since both distances are positive, gives

$$\frac{1}{x} = \frac{2}{2 - x} \implies 2 - x = 2x \implies x = \frac{2}{3} \text{ m}.$$

The distances from the charges are in the ratio 1 : 2, as required for fields from charges in the ratio 1 : 4 to have equal magnitude.

Step 5: Conclusion (a) $\vec{\tau} = \vec{p} \times \vec{E}$, with magnitude $pE \sin \theta$. The torque is maximum at 90° , and half of maximum at 30° or 150° . (b) The net field is zero at $\boxed{\frac{2}{3} \text{ m}}$ from q_1 towards q_2 , between the two charges.

Quick Tip to Solve: For two like charges, first place the null point between them; the distances from it are proportional to the square roots of their charge magnitudes.

35. (a) Derive an expression for the energy stored in a parallel plate capacitor of capacitance C when charged up to voltage V . How is this energy stored in the capacitor?

(b) A capacitor of capacitance $1\ \mu\text{F}$ is charged by connecting a battery of negligible internal resistance and emf 10V across it. Calculate the amount of charge supplied by the battery in charging the capacitor fully.

[5 marks]

Solution:

Concept:

- Capacitance is $C = q/v$, where q is the magnitude of charge on either plate and v is the instantaneous potential difference. For a fixed capacitor, C remains constant during charging.
- Transferring a small charge dq between plates against a potential difference v requires work $dW = v dq$. The total quasistatic charging work is stored as electrostatic potential energy.
- For an air or vacuum parallel plate capacitor with negligible fringing, $C = \epsilon_0 A/d$ and $E = V/d$. The energy density of its electric field is $u = \frac{1}{2}\epsilon_0 E^2$.

Step 1: (a) Express the work for an incremental charge transfer Consider a capacitor initially uncharged. At an intermediate stage of charging, let its plate charges be $+q$ and $-q$. Its potential difference is then $v = q/C$. To transfer a further small positive charge dq from the negative plate to the positive plate, an external source must do work against this potential difference:

$$dW = v dq = \frac{q}{C} dq.$$

The potential difference rises continuously as charge accumulates. Hence the work cannot be calculated by multiplying the final voltage by the total transferred charge; each increment is transferred against its own instantaneous voltage.

Step 2: (a) Integrate to obtain the stored energy Integrate the incremental work from $q = 0$ to the final charge $q = Q$. With zero energy assigned to the uncharged capacitor,

$$U = \int_0^Q \frac{q}{C} dq = \frac{1}{C} \left[\frac{q^2}{2} \right]_0^Q = \frac{Q^2}{2C}.$$

At the final voltage V , the plate charge is $Q = CV$. Substituting this relation gives the equivalent forms

$$U = \frac{Q^2}{2C} = \frac{1}{2} QV = \frac{1}{2} CV^2.$$

These expressions describe the electrostatic energy retained by the charged capacitor, irrespective of the particular charging circuit. If a constant-voltage battery charges the capacitor directly, its total energy supplied need not equal the stored energy; the remaining energy is dissipated or radiated during the charging transient.

Step 3: (a) Explain where the energy is stored The energy is stored in the electric field in the space between the plates. For parallel plates of area A separated by d , neglect edge effects and take air

Quick Tip to Solve: Integrate the rising capacitor voltage to find its stored energy, but use the final relation $Q = CV$ directly for the charge delivered by the battery.

approximately as vacuum. Using $C = \epsilon_0 A/d$ and $V = Ed$,

$$U = \frac{1}{2} \frac{\epsilon_0 A}{d} (Ed)^2 = \frac{1}{2} \epsilon_0 E^2 (Ad).$$

Since Ad is the volume occupied by the approximately uniform field, the energy per unit volume is $u = U/(Ad) = \frac{1}{2} \epsilon_0 E^2$. Charging establishes this field by separating equal and opposite charges on the plates.

Step 4: (b) Calculate the charge supplied by the battery After charging is complete, the capacitor's potential difference equals the battery emf, because the final current is zero. Thus $V = 10\text{ V}$, while $C = 1\text{ }\mu\text{F} = 1 \times 10^{-6}\text{ F}$. The total charge that flows through the battery during charging is the magnitude of charge accumulated on either plate:

$$Q = CV = (1 \times 10^{-6})(10) = 1 \times 10^{-5}\text{ C} = 10\text{ }\mu\text{C}.$$

One plate gains $+Q$ and the other $-Q$; these are produced by the same transferred charge. Their magnitudes must not be added to obtain the charge supplied by the battery.

Step 5: Conclusion (a) The capacitor stores $U = \frac{1}{2} CV^2 = \frac{Q^2}{2C}$ as energy of the electric field between its plates. (b) The charge supplied by the battery is $10\text{ }\mu\text{C} = 1 \times 10^{-5}\text{ C}$.

Quick Tip to Solve: Integrate the rising capacitor voltage to find its stored energy, but use the final relation $Q = CV$ directly for the charge delivered by the battery.

36. (a) Derive the expression for the force acting per unit length between two long straight parallel current carrying conductors. Hence define one ampere.

(b) Two long parallel straight conductors are placed 12 cm apart in air. They carry equal currents of 3 A each. Find the magnitude and direction of the magnetic field at a point midway between them (drawing a figure) when the currents in them flow in opposite directions. [5 marks]

Solution:

Concept:

- A long straight wire carrying current I produces a magnetic field $B = \mu_0 I / (2\pi r)$ at perpendicular distance r , where μ_0 is the permeability of free space. The direction follows the right-hand thumb rule.
- A straight segment of length L carrying current I in a field has force $\vec{F} = I\vec{L} \times \vec{B}$. When the field is perpendicular to the wire, $F = ILB$.
- Magnetic fields add vectorially. Between parallel wires carrying opposite currents, their fields point in the same direction and therefore add.

Step 1: (a) Obtain the field produced by the first conductor Consider two very long straight parallel conductors, labelled 1 and 2, separated by distance d in vacuum. Let their currents be I_1 and I_2 . The

Quick Tip to Solve: At the midpoint, use half the wire separation in each field formula. Opposite currents make the fields between the wires add; use the thumb rule before calculating.

field produced by conductor 1 at every point of conductor 2 has magnitude

$$B_1 = \frac{\mu_0 I_1}{2\pi d}.$$

This field is perpendicular to conductor 2 because its field lines are circles in planes normal to the wires. The long-wire assumption neglects end effects, so the field is uniform along the chosen length L of the second conductor.

Step 2: (a) Calculate the force per unit length and its direction The force on length L of conductor 2 due to the field of conductor 1 is

$$\begin{aligned} F_{21} &= I_2 L B_1 \sin 90^\circ \\ &= \frac{\mu_0 I_1 I_2 L}{2\pi d}, \\ \frac{F_{21}}{L} &= \frac{\mu_0 I_1 I_2}{2\pi d}. \end{aligned}$$

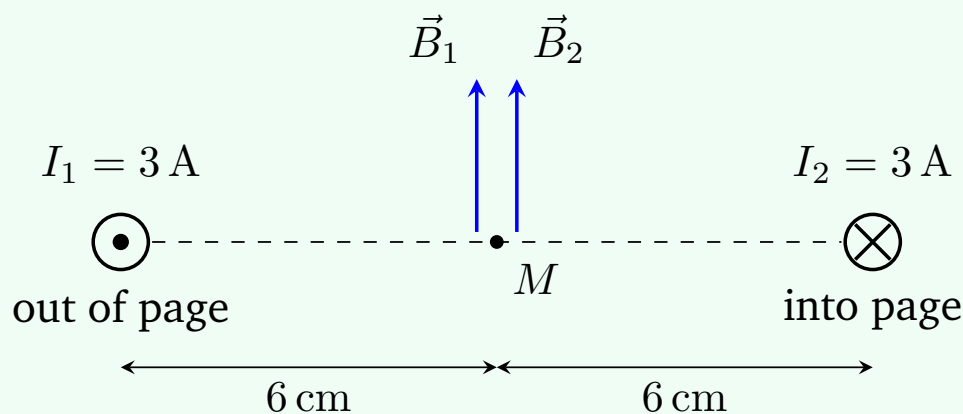
Interchanging the conductors gives the same force per unit length on conductor 1. The forces are equal and opposite. Applying the right-hand rule to $I\vec{L} \times \vec{B}$ shows that currents in the same direction attract, whereas currents in opposite directions repel. The expression above gives the magnitude in either case.

Step 3: (a) State the force-based definition of one ampere In the conventional force-based definition used in this question, one ampere is that constant current which, when maintained in each of two straight parallel conductors of infinite length and negligible circular cross-section, placed 1 m apart in vacuum, produces a force of 2×10^{-7} N per metre between them. Indeed, using $\mu_0 = 4\pi \times 10^{-7} \text{ N A}^{-2}$,

$$\frac{F}{L} = \frac{4\pi \times 10^{-7} \times 1 \times 1}{2\pi \times 1} = 2 \times 10^{-7} \text{ N m}^{-1}.$$

Step 4: (b) Draw the arrangement and determine both field directions Represent the wires by their cross-sections, with the left current coming out of the page ($\odot$) and the right current going into the page ($\otimes$). Their separation is 12 cm; midpoint M is 6 cm from each wire. The left wire's field circulates anticlockwise and points upward at M . The right wire's field circulates clockwise and also points upward there. Thus the resultant is perpendicular to the plane containing the two wires, and is upward in this cross-sectional sketch. Reversing both current directions reverses the resultant direction.

Quick Tip to Solve: At the midpoint, use half the wire separation in each field formula. Opposite currents make the fields between the wires add; use the thumb rule before calculating.



Cross-section of opposite currents: both magnetic fields at the midpoint point upward.

Step 5: (b) Substitute the distances and add the fields Convert the midpoint distance to SI units: $r = 6 \text{ cm} = 0.06 \text{ m}$. Each current is 3 A. In air, take the permeability as μ_0 . Each contribution is

$$B_1 = B_2 = \frac{4\pi \times 10^{-7} \times 3}{2\pi \times 0.06} = 1.0 \times 10^{-5} \text{ T}.$$

Since these two fields point in the same direction at the midpoint,

$$B_M = B_1 + B_2 = 2.0 \times 10^{-5} \text{ T}.$$

Equal magnitudes alone do not imply cancellation: the field directions, fixed by the opposite current directions, must be checked first.

Step 6: Conclusion (a) The force per unit length is $F/L = \mu_0 I_1 I_2 / (2\pi d)$, attractive for parallel currents and repulsive for antiparallel currents. One ampere gives $2 \times 10^{-7} \text{ N m}^{-1}$ between the specified conductors 1 m apart in vacuum. (b) The midpoint field is $2.0 \times 10^{-5} \text{ T}$, upward in the cross-sectional sketch for left current out of the page and right current into the page.

Quick Tip to Solve: At the midpoint, use half the wire separation in each field formula. Opposite currents make the fields between the wires add; use the thumb rule before calculating.

36. (a) Draw the schematic sketch of a cyclotron. Explain the shape of the path on which charged particle moves when the particle is accelerated by it.

(b) To convert a given galvanometer into a voltmeter of ranges $2V$, V and $\frac{V}{2}$ volt, resistances R_1 , R_2 and R_3 ohm respectively, are required to be connected in series with the galvanometer. Obtain the relationship between R_1 , R_2 and R_3 . [5 marks]

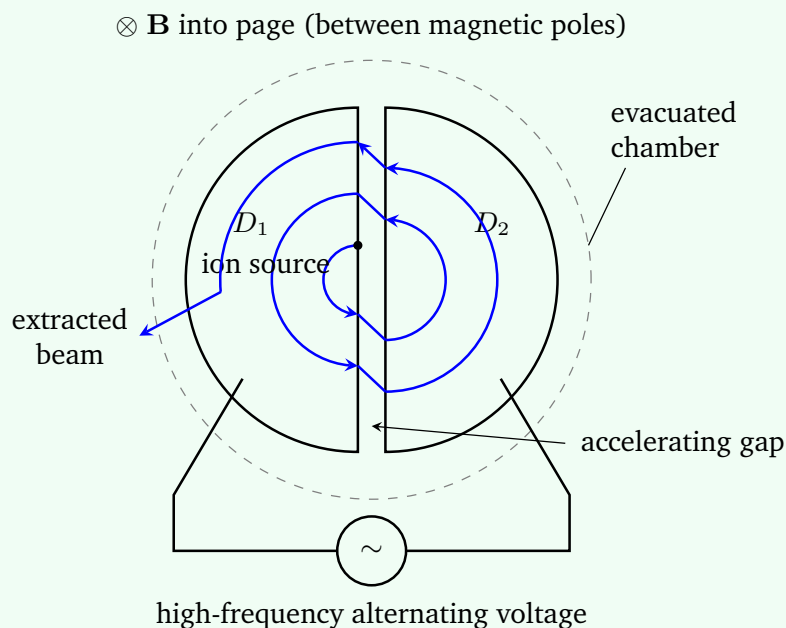
Solution:

Concept:

- A cyclotron accelerates a charged particle using an alternating electric field across the gap between two hollow D-shaped electrodes (dees). A uniform magnetic field B , perpendicular to their plane, bends the motion inside each dee.
- For a particle of mass m , charge magnitude q and speed v , magnetic force supplies centripetal force: $qvB = mv^2/r$, so $r = mv/(qB)$. The non-relativistic cyclotron frequency is $f = qB/(2\pi m)$.
- A galvanometer of resistance G and full-scale current I_g becomes a voltmeter of range U when a resistance R is connected in series: $U = I_g(G + R)$.

Step 1: (a) Identify the components of the cyclotron The schematic shows two hollow metal dees D_1 and D_2 , separated by a narrow gap inside an evacuated chamber. They connect to a high-frequency alternating voltage source. Magnetic pole pieces produce a uniform field perpendicular to the plane of the dees; crosses denote a field into the page in this top view. A positive ion is released near the centre. An extraction arrangement guides the accelerated particle out near the outer edge. The electric field acts mainly in the gap: inside the conducting dees the particle is shielded from it. Thus the gap increases the particle's kinetic energy, while the magnetic field determines its curved path inside a dee.

Quick Tip to Solve: For several ranges of the same galvanometer, write $U = I_g(G + R)$ first, then subtract pairs of range equations to remove G . In a cyclotron, radius increases with speed but the non-relativistic half-period stays constant.



Cyclotron schematic (top view): increasing semicircular arcs, accelerating gap, alternating source and perpendicular magnetic field.

Step 2: (a) Explain the expanding path and synchronisation Within a dee the magnetic force is perpendicular to velocity and does no work. The particle therefore moves at constant speed along a semicircular arc. Equating the magnetic and centripetal forces gives

$$qvB = \frac{mv^2}{r}, \quad r = \frac{mv}{qB}.$$

Each time the particle crosses the gap, the electric field accelerates it. Its speed rises, so its next semicircular orbit has a larger radius. Successive semicircles of increasing radius, joined by short gap crossings, form an outward spiral-like path. The path is not one circle of fixed radius. For non-relativistic motion, the time spent on each semicircle is independent of speed:

$$t_{1/2} = \frac{\pi r}{v} = \frac{\pi m}{qB}.$$

The voltage must reverse polarity during this interval so that the field accelerates the particle on every gap crossing. Consequently its frequency must be

$$f = \frac{1}{2t_{1/2}} = \frac{qB}{2\pi m}.$$

The particle eventually reaches the outer region and is extracted. The fixed-frequency argument assumes that its mass can be treated as constant.

Step 3: (b) Write the full-scale voltage equations The same galvanometer is used for all three ranges, so both G and I_g remain unchanged. At full-scale deflection, the potential difference across the entire

Quick Tip to Solve: For several ranges of the same galvanometer, write $U = I_g(G + R)$ first, then subtract pairs of range equations to remove G . In a cyclotron, radius increases with speed but the non-relativistic half-period stays constant.

series combination equals the specified range. Hence

$$2V = I_g(G + R_1),$$

$$V = I_g(G + R_2),$$

$$\frac{V}{2} = I_g(G + R_3).$$

The range therefore depends on total resistance $G + R$, rather than just the externally added resistance. Omitting G at this stage would incorrectly imply that the added resistances themselves have the range ratio.

Step 4: (b) Eliminate the galvanometer quantities Subtract the second voltage equation from the first, then the third from the second:

$$V = I_g(R_1 - R_2),$$

$$\frac{V}{2} = I_g(R_2 - R_3).$$

Dividing the two equations eliminates V and I_g :

$$R_1 - R_2 = 2(R_2 - R_3).$$

Thus the required relation can also be written as $R_1 + 2R_3 = 3R_2$. It correctly gives $R_1 > R_2 > R_3$: a larger voltmeter range needs a larger series resistance to limit the current to the same I_g .

Step 5: Conclusion (a) The charged particle follows successive semicircular arcs of increasing radius, forming an outward spiral-like path as it gains energy across the gap. The alternating voltage has frequency $qB/(2\pi m)$. (b) The series resistances satisfy $R_1 + 2R_3 = 3R_2$.

Quick Tip to Solve: For several ranges of the same galvanometer, write $U = I_g(G + R)$ first, then subtract pairs of range equations to remove G . In a cyclotron, radius increases with speed but the non-relativistic half-period stays constant.

37. (a) What is meant by plane polarised light? An unpolarised light is incident at an angle θ on the surface of glass of refractive index μ . If the reflected and refracted rays are perpendicular to each other, then obtain the relationship between μ and θ .

(b) Two polaroids P_1 and P_2 are placed in a crossed position. Unpolarised light of intensity I_0 is incident on P_1 . If P_2 is rotated through an angle θ about the direction of propagation of light, keeping P_1 fixed, plot the graph of intensity of light for $0^\circ < \theta < 360^\circ$ which is (i) transmitted by P_1 , and (ii) transmitted by P_2 . [5 marks]

Solution:

Concept:

- In plane polarised light the electric field oscillates along a fixed direction perpendicular to the direction of propagation. Unpolarised light has electric field vibrations in all directions in that transverse plane.

Quick Tip to Solve: Measure the analyser's angle from its initial crossed position: use sine squared for that rotation angle, while the first polaroid always transmits half of unpolarised intensity.

- Snell's law for light travelling from air into glass is $\mu = \sin i / \sin r$, where i and r are measured from the normal and air has refractive index approximately unity.
- An ideal polaroid transmits half the intensity of unpolarised light. Malus' law gives $I = I_{\text{in}} \cos^2 \alpha$ for incident plane polarised light, with α the angle between its vibration direction and the transmission axis.

Step 1: (a) Define plane polarised light and use the ray geometry Plane polarised light has its electric vector vibrating in a single fixed direction in the plane perpendicular to propagation. Suppose the incident ray makes angle θ with the normal to the glass, and the refracted ray makes angle r with that normal. By the law of reflection the reflected ray also makes angle θ with the normal. When the reflected and refracted rays are mutually perpendicular, their geometry gives

$$\theta + r = 90^\circ, \quad r = 90^\circ - \theta.$$

This incidence angle is the polarising angle, or Brewster angle. At this angle the reflected light is plane polarised; its electric vibrations are perpendicular to the plane of incidence.

Step 2: (a) Obtain the refractive index relation Applying Snell's law at the air-glass boundary and substituting the relation between the angles,

$$\begin{aligned} \mu &= \frac{\sin \theta}{\sin r} = \frac{\sin \theta}{\sin(90^\circ - \theta)} \\ &= \frac{\sin \theta}{\cos \theta} = \tan \theta. \end{aligned}$$

Thus the required relationship is Brewster's law. It applies here with the incidence angle measured from the normal, rather than from the surface.

Step 3: (b)(i) Intensity transmitted by the first polaroid The incident light on P_1 is unpolarised. Averaging $\cos^2 \alpha$ over all transverse vibration directions gives $1/2$, so the transmitted intensity is

$$I_1 = \frac{I_0}{2}.$$

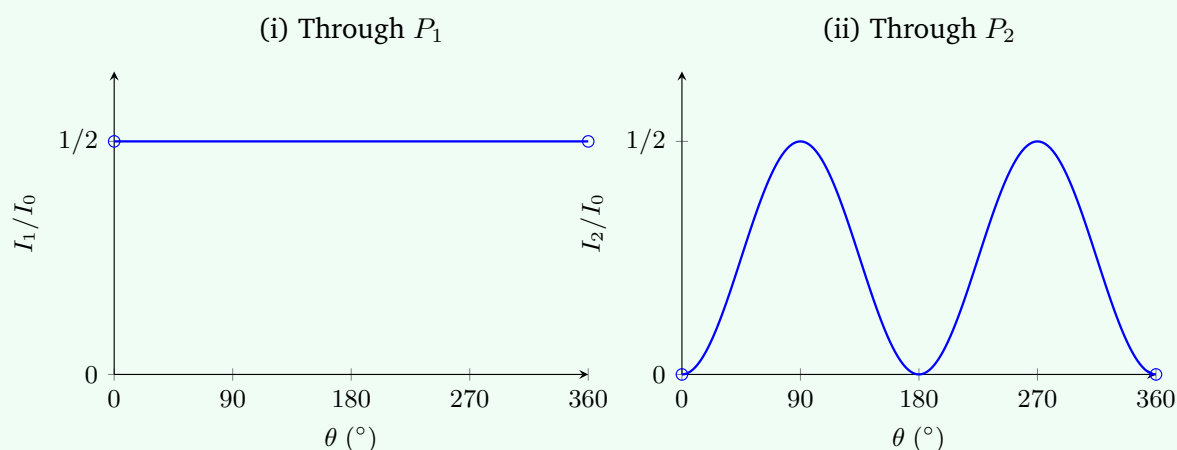
Rotating P_2 does not change the incident light or the fixed axis of P_1 . Consequently I_1 remains constant for the entire angular interval. Its graph is a horizontal line at $I_0/2$.

Step 4: (b)(ii) Intensity transmitted by the rotating second polaroid Initially the two transmission axes are crossed, so their relative angle is 90° . After rotating P_2 by θ , the relative angle can be taken as $90^\circ - \theta$; choosing the opposite direction of rotation gives the same intensity. The light incident on P_2 is already polarised along the fixed axis of P_1 . Therefore Malus' law gives

$$I_2 = I_1 \cos^2(90^\circ - \theta) = \frac{I_0}{2} \sin^2 \theta.$$

The intensity reaches $I_0/2$ at 90° and 270° , when the axes are parallel, and vanishes at 180° , when they are crossed again. It approaches zero at the excluded endpoints 0° and 360° . The intensity pattern repeats every 180° . The following graphs use the dimensionless vertical coordinate I/I_0 ; open circles denote the endpoints excluded by the printed interval.

Quick Tip to Solve: Measure the analyser's angle from its initial crossed position: use sine squared for that rotation angle, while the first polaroid always transmits half of unpolarised intensity.



Intensities transmitted by the fixed first polaroid and the rotating second polaroid.

Step 5: Conclusion Plane polarised light has its electric vibrations confined to a fixed transverse direction. The required relation is $\mu = \tan \theta$. For the polaroids, $I_1 = I_0/2$ and $I_2 = (I_0/2) \sin^2 \theta$: the first graph is constant and the second has two equal maxima over one full rotation.

Quick Tip to Solve: Measure the analyser's angle from its initial crossed position: use sine squared for that rotation angle, while the first polaroid always transmits half of unpolarised intensity.

OR

37. (a) Briefly describe the Young's double slit experiment of interference of light. Derive the expression for fringe width in the pattern.

(b) Monochromatic light of wavelength 588 nm is incident from air to water interface. Find the wavelength and speed of the refracted light. The refractive index of water is $\frac{4}{3}$. [5 marks]

Solution:

Concept:

- Coherent sources have the same frequency and a constant phase difference. Their superposition gives constructive interference for path difference $n\lambda$ and destructive interference for $(n + \frac{1}{2})\lambda$.
- In Young's double slit experiment, d is the slit separation, D is the slit-to-screen distance and y is the distance of a screen point from the central maximum. For $D \gg d$ and small angles, the path difference is $\Delta \simeq dy/D$.
- The fringe width β is the separation of successive bright fringes or successive dark fringes. At an interface, frequency f remains unchanged, while speed and wavelength obey $v = f\lambda$ and $\mu = c/v$ for incidence from air.

Step 1: (a) Describe the experimental arrangement A monochromatic source illuminates a narrow slit S . Light from S falls on two close, narrow slits S_1 and S_2 , placed symmetrically with respect to it. The two slits act as coherent secondary sources because both receive light from the same source. A screen is placed parallel to the slit plane at distance D , large compared with slit separation d . The

Quick Tip to Solve: Derive fringe width by subtracting two adjacent fringe positions. At an air-to-water interface, divide both the incident wavelength and speed by the refractive index; keep the frequency unchanged.

waves from S_1 and S_2 overlap on the screen. Where they arrive in phase, their amplitudes reinforce and a bright fringe appears; where their phase difference is π , they cancel and a dark fringe appears. Thus alternate bright and dark bands are observed. At the centre O , the paths are equal, so the central fringe is bright.

Step 2: Obtain the path difference at a screen point Let M be the midpoint between the slits and O the point where the normal MO meets the screen. Let P lie at distance $OP = y$ above O , and let θ be the angle made by MP with the normal MO . For a distant screen, the rays from the two slits reaching P are nearly parallel, giving the path difference in magnitude

$$\Delta = d \sin \theta.$$

Since $y \ll D$, the small-angle approximation gives $\sin \theta \simeq \tan \theta = OP/MO = y/D$. Therefore,

$$\Delta \simeq \frac{dy}{D}.$$

This approximation is responsible for the nearly uniform spacing of fringes near the centre.

Step 3: Locate successive fringes and derive their width For the bright fringe of order n , constructive interference requires $\Delta = n\lambda$, where n is an integer. Hence,

$$\frac{dy_n}{D} = n\lambda \quad \Rightarrow \quad y_n = \frac{n\lambda D}{d}.$$

Subtract the positions of adjacent bright fringes:

$$\begin{aligned} \beta &= y_{n+1} - y_n \\ &= \frac{(n+1)\lambda D}{d} - \frac{n\lambda D}{d} = \frac{\lambda D}{d}. \end{aligned}$$

Similarly, destructive interference occurs at $y_n^{\text{dark}} = (n + \frac{1}{2})\lambda D/d$; subtracting two successive dark-fringe positions gives the same width. Thus the bright fringes are equally spaced, as are the dark fringes, with a dark fringe midway between neighbouring bright fringes.

Step 4: (b) Find the speed of light in water The refractive index of water relative to air is $\mu = 4/3$. Taking the speed in air as $c = 3.00 \times 10^8 \text{ m s}^{-1}$, the refracted speed is

$$v = \frac{c}{\mu} = \frac{3.00 \times 10^8}{4/3} = 2.25 \times 10^8 \text{ m s}^{-1}.$$

Water slows the wave because its refractive index exceeds unity.

Step 5: Find the refracted wavelength using unchanged frequency The incident wavelength is $\lambda_{\text{air}} = 588 \text{ nm}$. Frequency is fixed by the source and does not change when light enters water. Dividing $v = f\lambda_{\text{water}}$ by $c = f\lambda_{\text{air}}$ therefore gives

$$\begin{aligned} \lambda_{\text{water}} &= \lambda_{\text{air}} \frac{v}{c} = \frac{\lambda_{\text{air}}}{\mu} \\ &= 588 \times \frac{3}{4} \text{ nm} = 441 \text{ nm} = 4.41 \times 10^{-7} \text{ m}. \end{aligned}$$

Quick Tip to Solve: Derive fringe width by subtracting two adjacent fringe positions. At an air-to-water interface, divide both the incident wavelength and speed by the refractive index; keep the frequency unchanged.

Both wavelength and speed decrease by the factor $3/4$, so their ratio, the frequency, stays constant.

Step 6: Conclusion (a) Young's experiment produces alternate bright and dark fringes from two coherent slits, with fringe width $\beta = \lambda D/d$. (b) In water, the refracted light has wavelength 441 nm and speed $2.25 \times 10^8 \text{ m s}^{-1}$, while its frequency remains unchanged.

Quick Tip to Solve: Derive fringe width by subtracting two adjacent fringe positions. At an air-to-water interface, divide both the incident wavelength and speed by the refractive index; keep the frequency unchanged.