

CBSE Class 12 Physics Question Paper 2020 with Solution PDF-Set 55/3/3



Question Paper with Solutions

Conducted by the Central Board of Secondary Education (CBSE)

Time Allowed: 3 hours

Maximum Marks: 70

General Instructions

- (i) This question paper comprises **four** Sections – **A, B, C** and **D**.
- (ii) There are **37** questions in the question paper. **All** questions are compulsory.
- (iii) **Section A** – Questions no. **1 to 20** are very short answer type questions, carrying **one** mark each.
- (iv) **Section B** – Questions no. **21 to 27** are short answer type questions, carrying **two** marks each.
- (v) **Section C** – Questions no. **28 to 34** are long answer type questions, carrying **three** marks each.
- (vi) **Section D** – Questions no. **35 to 37** are also long answer type questions, carrying **five** marks each.
- (vii) There is no overall choice in the question paper. However, an internal choice has been provided in 2 questions of 1 mark, 2 questions of 2 marks, 1 question of three marks and all the 3 questions of five marks. You have to attempt only one of the choices in such questions.
- (viii) In addition to this, separate instructions are given with each section and question, wherever necessary.
- (ix) Use of calculators and log tables is not permitted.

You may use the following values of physical constants wherever necessary.

$$c = 3 \times 10^8 \text{ m/s}$$

$$h = 6.63 \times 10^{-34} \text{ Js}$$

$$e = 1.6 \times 10^{-19} \text{ C}$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ T m A}^{-1}$$

$$\epsilon_0 = 8.854 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$$

$$\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$$

$$\text{Mass of electron } (m_e) = 9.1 \times 10^{-31} \text{ kg}$$

$$\text{Mass of neutron} = 1.675 \times 10^{-27} \text{ kg}$$

$$\text{Mass of proton} = 1.673 \times 10^{-27} \text{ kg}$$

$$\text{Avogadro's number} = 6.023 \times 10^{23} \text{ per gram mole}$$

$$\text{Boltzmann constant} = 1.38 \times 10^{-23} \text{ JK}^{-1}$$

Section A (20 marks)

Note : Select the most appropriate option from those given below each question :

1. A charge Q is uniformly distributed over the surface of a spherical shell of radius R . The work done in bringing a test charge Q_0 from its centre to its surface is [1 mark]

- (A) $\frac{1}{4\pi\epsilon_0} \frac{QQ_0}{R}$
(B) $\frac{1}{4\pi\epsilon_0} \frac{QQ_0}{2R}$
(C) $\frac{QQ_0}{\epsilon_0 R}$
(D) Zero

Correct Answer: (D) Zero

Solution:

Concept:

- Inside a uniformly charged spherical shell the electric field is zero, so its potential is constant.
- Work done by an external agent is $W = Q_0(V_f - V_i)$, where Q_0 is the test charge.

Step 1: Compare the two potentials The potential everywhere inside the shell equals its surface potential. Thus

$$V_{\text{centre}} = V_{\text{surface}} = \frac{1}{4\pi\epsilon_0} \frac{Q}{R}.$$

Moving from the centre to the surface therefore crosses no potential difference.

Step 2: Calculate the work Using the potential difference gives

$$W = Q_0 (V_{\text{surface}} - V_{\text{centre}}) = 0.$$

The field also does zero work since it vanishes throughout the interior. Options (A), (B) and (C) give nonzero energies; (A) corresponds to bringing the test charge from infinity to the shell, which is a different journey.

Step 3: Conclusion The required work is zero. Hence, the correct option is (D).

Quick Tip to Solve: For a charged spherical shell, compare endpoint potentials before using a Coulomb-force expression.

2. A zener diode has [1 mark]

- (A) heavily doped p-side and lightly doped n-side.
(B) heavily doped n-side and lightly doped p-side.
(C) heavily doped n-side as well as p-side.
(D) lightly doped n-side as well as p-side.

Correct Answer: (C) heavily doped n-side as well as p-side.

Solution:

Concept:

- A Zener diode is a specially designed p-n junction operated in reverse breakdown.

- Heavy doping on both sides produces a thin depletion region and a strong junction electric field.

Step 1: Identify the junction construction Both the p-side and the n-side of a Zener diode are heavily doped. The large concentration of charge carriers makes the depletion layer narrow. A large electric field can therefore develop across the junction even at a comparatively small reverse voltage.

Step 2: Check the alternatives This construction enables a sharp reverse breakdown at the designed Zener voltage, which is useful for voltage regulation when the current is limited externally. Options (A) and (B) describe unequal heavy and light doping, so neither gives the required heavy doping of both sides. Option (D), with both sides lightly doped, gives a wider depletion region and is also incorrect.

Step 3: Conclusion Both the n-side and p-side are heavily doped. Hence, the correct option is (C).

Quick Tip to Solve: Remember Zener construction as heavy doping on both sides, giving a thin depletion layer.

3. Two large conducting spheres carrying charges Q_1 and Q_2 are kept with their centres r distance apart. The magnitude of electrostatic force between them is not exactly $\frac{1}{4\pi\epsilon_0} \frac{Q_1 Q_2}{r^2}$ because [1 mark]

- (A) these are not point charges.
- (B) charge distribution on the spheres is not uniform.
- (C) charges on spheres will shift towards the centres of their respective spheres.
- (D) charges will shift towards the portions of the spheres which are closer and facing towards each other.

Correct Answer: (B) charge distribution on the spheres is not uniform.

Solution:

Concept:

- Coulomb's centre-to-centre formula is exact for point charges and for suitable spherically symmetric charge distributions.
- Free charges on conductors redistribute under the electric field produced by a nearby charged conductor.

Step 1: Examine the charge distribution Each conducting sphere is influenced by the electric field of the other sphere. Its mobile surface charges rearrange to make its surface equipotential. Consequently the surface charge density is generally nonuniform, and the original spherical symmetry of the distribution is lost.

Step 2: Distinguish the proposed reasons The resultant force is then obtained from the actual distributed charges and need not equal the point-charge expression using the centre separation. Option (A) alone is insufficient: an extended but spherically symmetric distribution can produce the same exterior field as a central point charge. Option (C) is false because excess charge remains on the surface. Option (D) is not general; like charges tend to move away from the facing regions, whereas unlike charges concentrate there.

Step 3: Conclusion The charge distribution on the spheres is not uniform. Hence, the correct option is (B).

Quick Tip to Solve: For nearby conductors, ask whether induction has broken spherical symmetry before treating charges as central points.

4. A charged particle is placed between the two plates of a charged parallel plate capacitor. It experiences a force F . If one plate is removed, then the force on the particle will be [1 mark]

- (A) $2F$
- (B) F
- (C) $\frac{F}{2}$
- (D) Zero

Correct Answer: (C) $\frac{F}{2}$

Solution:

Concept:

- An ideal large charged plane produces field magnitude $E_1 = \sigma/(2\epsilon_0)$, where σ is its surface charge density.
- Between equal oppositely charged capacitor plates the two fields add, and a charge q experiences force magnitude $F = |q|E$.

Step 1: Add the fields of both plates Neglecting edge effects, each plate contributes a field of equal magnitude in the same direction within the gap. The initial field is therefore

$$E = 2E_1 = \frac{\sigma}{\epsilon_0}, \quad F = |q|\frac{\sigma}{\epsilon_0}.$$

Step 2: Remove one contribution With one plate removed, the field of the remaining charged plate is E_1 , half the previous field, under the usual fixed-charge ideal-plane assumption of this question. Thus

$$F_{\text{new}} = |q|E_1 = \frac{F}{2}.$$

Doubling the force in (A) has no basis, (B) overlooks the removed plate contribution, and (D) overlooks the nonzero field of a single charged plate.

Step 3: Conclusion The force becomes $F/2$. Hence, the correct option is (C).

Quick Tip to Solve: Treat the capacitor field as the sum of two equal plate fields; removing one removes half the field.

5. In Bohr's model of hydrogen atom, the total energy of the electron in n^{th} discrete orbit is proportional to

[1 mark]

- (A) n
- (B) $\frac{1}{n}$
- (C) n^2
- (D) $\frac{1}{n^2}$

Correct Answer: (D) $\frac{1}{n^2}$

Solution:

Concept:

- For hydrogen in Bohr's model, the permitted orbit radius is $r_n = a_0 n^2$, with a_0 the Bohr radius.
- The bound electron has total energy $E_n = -13.6/n^2$ eV, measured relative to a free electron at infinity.

Step 1: Relate kinetic and potential energies The Coulomb attraction supplies the centripetal force. Hence $K = ke^2/(2r_n)$ and $U = -ke^2/r_n$, where $k = 1/(4\pi\epsilon_0)$. Their sum is

$$E_n = K + U = -\frac{ke^2}{2r_n}.$$

Step 2: Use the allowed orbit radius Substituting $r_n = a_0 n^2$ gives

$$E_n = -\frac{ke^2}{2a_0} \frac{1}{n^2} = -\frac{13.6}{n^2} \text{ eV}.$$

The negative constant indicates a bound state and does not change the inverse-square dependence on n . Options (A), (B) and (C) give respectively linear, inverse-linear and square dependence, none of which follows from the allowed radii.

Step 3: Conclusion The total energy is proportional to $1/n^2$. Hence, the correct option is (D).

Quick Tip to Solve: Bohr radius grows as n squared, while total energy varies inversely with radius.

6. A uniform wire of $16\ \Omega$ resistance is bent round in the form of a circle. When connected in a circuit between its two diametrically opposite points, its effective resistance will be

[1 mark]

- (A) $8\ \Omega$
- (B) $16\ \Omega$
- (C) $4\ \Omega$
- (D) $2\ \Omega$

Correct Answer: (C) $4\ \Omega$

Solution:

Concept:

- For a uniform wire, resistance is proportional to length: $R = \rho L/A$.
- Two resistors between the same pair of terminals are in parallel: $1/R_{eq} = 1/R_1 + 1/R_2$.

Step 1: Find each branch resistance Diametrically opposite connection points divide the circle into two equal semicircles. Each semicircle contains half of the total wire length, so each has resistance

$$R_1 = R_2 = \frac{16}{2} = 8 \Omega.$$

Step 2: Combine the two branches Current can travel from one terminal to the other through either semicircle; both experience the same potential difference. They are therefore in parallel, giving

$$R_{\text{eq}} = \frac{8 \times 8}{8 + 8} = 4 \Omega.$$

Option (A) counts only one branch, (B) treats the entire wire as one series path, and (D) halves the resistance once too often. The equal paths must first be identified and then combined.

Step 3: Conclusion The effective resistance is 4Ω . Hence, the correct option is (C).

Quick Tip to Solve: Opposite points on a uniform wire circle create two half-resistance branches in parallel: total resistance divided by four.

7. A region has a uniform magnetic field in it. A proton enters into the region with velocity making an angle of 45° with the direction of the magnetic field. In this region the proton will move on a path having the shape of a [1 mark]

- (A) straight line
- (B) circle
- (C) spiral
- (D) helix

Correct Answer: (D) helix

Solution:

Concept:

- The magnetic force is $\mathbf{F} = q\mathbf{v} \times \mathbf{B}$, perpendicular to velocity, so it does no work.
- The velocity component parallel to a uniform field is constant; the perpendicular component produces uniform circular motion.

Step 1: Resolve the entry velocity At 45° , the proton has both nonzero components:

$$v_{\parallel} = v \cos 45^\circ = \frac{v}{\sqrt{2}}, \quad v_{\perp} = v \sin 45^\circ = \frac{v}{\sqrt{2}}.$$

The parallel component is unaffected because its cross product with the magnetic field vanishes.

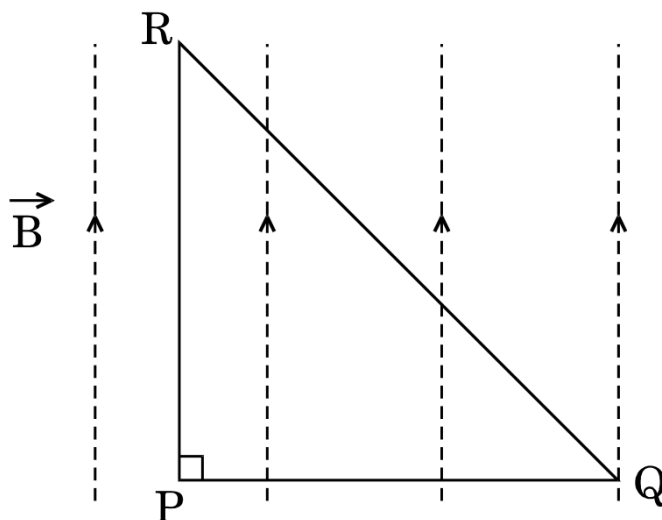
Step 2: Combine the two motions The perpendicular component rotates at constant speed with radius $r = mv_{\perp}/(qB)$, while the proton advances steadily along the field. The combination has constant radius and constant pitch, forming a helix. A straight line in (A) would require no perpendicular component; a circle in (B) would require no parallel component. A spiral in (C) would have changing

radius, which does not occur here because the magnetic force does no work.

Step 3: Conclusion The proton follows a helix. Hence, the correct option is (D).

Quick Tip to Solve: Nonzero parallel and perpendicular velocity components in a uniform magnetic field give a constant-pitch helix.

8. An isosceles right angled current carrying loop PQR is placed in a uniform magnetic field $\vec{B}$ pointing along PR. If the magnetic force acting on the arm PQ is F , then the magnetic force which acts on the arm QR will be [1 mark]



- (A) F
- (B) $\frac{F}{\sqrt{2}}$
- (C) $\sqrt{2}F$
- (D) $-F$

Correct Answer: (D) $-F$

Solution:

Concept:

- The force on a straight current-carrying segment is $\vec{F} = I\vec{L} \times \vec{B}$, where $\vec{L}$ points along its current.
- A closed loop in a uniform magnetic field has zero resultant magnetic force.

Step 1: Apply the force law to PR The arm PR is parallel to the field, so its magnetic force is zero: $F_{PR} = IBL \sin 0 = 0$. The remaining two forces must therefore sum to zero.

Step 2: Compare PQ and QR Let $PQ = PR = a$. Then $QR = \sqrt{2}a$, but it makes 45° with the field, so its force magnitude is $IB\sqrt{2}a \sin 45^\circ = IBa$, equal to that on PQ. The currents have opposite components perpendicular to the field, giving opposite force directions. Thus the force on QR is the negative of the force on PQ. The factors in (B) and (C) ignore the angle; (A) gives the magnitude but

misses the opposite direction.

Step 3: Conclusion $\vec{F}_{QR} = -\vec{F}_{PQ}$. Interpreting the signed force as printed, the answer is $-F$, option (D).

Quick Tip to Solve: For a closed loop in a uniform field, use zero net force after finding any force-free arm.

9. The electric flux emerging out from 1 C charge is

[1 mark]

- (A) $\frac{1}{\epsilon_0}$
- (B) 4π
- (C) $\frac{4\pi}{\epsilon_0}$
- (D) ϵ_0

Correct Answer: (A) $\frac{1}{\epsilon_0}$

Solution:

Concept:

- Gauss's law gives total outward electric flux $\Phi_E = \oint \vec{E} \cdot d\vec{A} = q_{\text{enclosed}}/\epsilon_0$.

Step 1: Identify the enclosed charge Take a closed surface surrounding the given isolated charge. The enclosed charge is $q = 1$ C. Flux depends on this enclosed charge, regardless of the radius or shape of the surface.

Step 2: Substitute in Gauss law The outward flux is $\Phi_E = (1 \text{ C})/\epsilon_0$. The 4π in the field of a point charge cancels the $4\pi r^2$ surface-area factor when flux is calculated. Therefore neither 4π nor $4\pi/\epsilon_0$ is correct; multiplication by ϵ_0 is also inconsistent with the law.

Step 3: Conclusion $\Phi_E = 1/\epsilon_0$ in $\text{N m}^2/\text{C}$: option (A).

Quick Tip to Solve: For total flux from a charge, directly divide the enclosed charge by permittivity.

10. $\text{m}^2 \text{V}^{-1} \text{s}^{-1}$ is the SI unit of which of the following?

[1 mark]

- (A) Drift velocity
- (B) Mobility
- (C) Resistivity
- (D) Potential gradient

Correct Answer: (B) Mobility

Solution:

Concept:

- Mobility μ is the magnitude of drift velocity per unit electric field: $\mu = v_d/E$.

Step 1: Calculate the unit from the definition Drift speed has unit m s^{-1} , and electric field has unit V m^{-1} . Thus $[\mu] = (\text{m s}^{-1})/(\text{V m}^{-1}) = \text{m}^2 \text{V}^{-1} \text{s}^{-1}$.

Step 2: Check the other quantities Drift velocity itself has unit m s^{-1} . Resistivity has unit Ωm . Potential gradient has unit V m^{-1} . None of these matches the given unit, whereas mobility matches

every exponent.

Step 3: Conclusion The required quantity is mobility, option (B).

Quick Tip to Solve: Divide the unit of speed by the unit of electric field to recognise mobility immediately.

11. The shape of the wavefront originating from a point source of light is _____.

[1 mark]

Solution:

Concept:

- A wavefront is the locus of points having the same phase of vibration.
- In a homogeneous isotropic medium, light from a point source travels equally fast in all directions.

Step 1: Compare travel times from the source At a fixed time after emission, all points reached by the disturbance are at the same distance $r = vt$ from the point source, where v is the speed of light in the medium.

Step 2: Identify the equal-phase surface The surface made by points at a fixed distance from one point is a sphere. Every point on this surface has the same propagation time and hence the same phase. A circular cross-section in a drawing represents the spherical wavefront in three dimensions. A plane wavefront is only the distant-source approximation.

Step 3: Conclusion The wavefront is spherical.

Quick Tip to Solve: A point source gives a sphere of equal radius; use the full three-dimensional surface name.

12. In photoelectric effect, the number of emitted photoelectrons is proportional to _____ of incident light.

[1 mark]

Solution:

Concept:

- For frequency ν above threshold, a photon of energy $h\nu$ can eject a photoelectron.
- At fixed frequency, increasing intensity increases the number of incident photons per unit area per unit time.

Step 1: Relate intensity to photon number If N photons strike area A in time t , the incident intensity is $I = Nh\nu/(At)$. With ν , A and t fixed, the number of incident photons is proportional to intensity.

Step 2: Relate photons to emitted electrons For a fixed photosensitive surface and frequency above threshold, a greater photon arrival rate gives a proportionately greater rate of photoelectron emission. The emission efficiency is taken as constant. Frequency controls the photon energy and maximum electron kinetic energy, whereas intensity controls the number emitted under these conditions.

Step 3: Conclusion The blank is intensity of incident light.

Quick Tip to Solve: In photoelectric questions, intensity changes electron number; frequency changes maximum energy.

12. Light of frequency ν is incident on a photosensitive surface of threshold frequency ν_0 ($\nu > \nu_0$). The value of kinetic energy of the emitted photoelectrons will be _____. [1 mark]

Solution:

Concept:

- Einstein's photoelectric equation is $h\nu = \phi + K_{\max}$, where ϕ is the work function.
- The threshold frequency satisfies $\phi = h\nu_0$.

Step 1: Express the work function At threshold the photon has just enough energy to liberate an electron with zero maximum kinetic energy. Therefore the minimum escape energy is $\phi = h\nu_0$.

Step 2: Use conservation of energy For incident frequency $\nu > \nu_0$, subtract the escape energy from the incident photon energy: $K_{\max} = h\nu - h\nu_0 = h(\nu - \nu_0)$. The printed blank refers to this maximum kinetic energy. Electrons emitted from deeper inside the surface may lose energy before escape, so individual emitted electrons can have smaller kinetic energies.

Step 3: Conclusion $K_{\max} = h(\nu - \nu_0)$.

Quick Tip to Solve: Subtract threshold photon energy from incident photon energy to obtain the maximum kinetic energy.

13. In Young's double slit experiment, if the monochromatic source of light is replaced by another monochromatic source of shorter wavelength, the value of fringe width will _____. [1 mark]

Solution:

Concept:

- Young's double-slit fringe width is $\beta = \lambda D/d$, where D is screen distance and d is slit separation.

Step 1: Hold the apparatus geometry fixed Only the monochromatic source is replaced, so the slit separation d and screen distance D stay constant. Consequently the fringe width is directly proportional to wavelength λ .

Step 2: Compare the two wavelengths If the new wavelength is $\lambda_2 < \lambda_1$, then $\beta_2/\beta_1 = \lambda_2/\lambda_1 < 1$. Hence adjacent bright fringes and adjacent dark fringes become closer together. The central bright fringe remains at zero path difference; changing wavelength changes the spacing, not this central condition.

Step 3: Conclusion The fringe width will decrease.

Quick Tip to Solve: When geometry is unchanged, compare fringe widths using the ratio of the two wavelengths.

14. The refractive index of the material of a converging lens is 1.5. If air is replaced by a medium of refractive index 1.6, then the lens will now behave as a _____ lens. [1 mark]

Solution:

Concept:

- For a lens in a surrounding medium, its power is $P = (n_l/n_m - 1)(1/R_1 - 1/R_2)$, where n_l and n_m are the lens and medium refractive indices.
- The sign of the power determines whether parallel rays converge or diverge.

Step 1: Compare the refractive indices The relative refractive index is $n_l/n_m = 1.5/1.6 = 0.9375$. Thus $n_l/n_m - 1 = -0.0625$, whereas in air this factor was $1.5 - 1 = 0.5$, a positive value.

Step 2: Determine the sign of the power The radii and shape of the lens remain unchanged. Since its power was positive in air, changing the refractive-index factor from positive to negative reverses the sign of its power. A lens with negative power makes incident parallel rays spread out.

Step 3: Conclusion The lens now behaves as a diverging lens.

Quick Tip to Solve: Compare the lens index with the surrounding index before using its shape to decide its action.

15. The value of Brewster's angle for air-glass interface is $\frac{\pi}{3}$; hence the refractive index of glass is _____. [1 mark]

Solution:

Concept:

- Brewster's law is $\tan i_B = n_g/n_a$, where i_B is the polarising angle and n_g, n_a are the refractive indices of glass and air.
- For air, the refractive index is taken as $n_a = 1$.

Step 1: Apply Brewster's law Light is incident from air on glass, so the required glass refractive index is $n_g = n_a \tan i_B$. The specified angle is $i_B = \pi/3 = 60^\circ$. This law uses the tangent of the incidence angle, rather than its sine.

Step 2: Evaluate the trigonometric ratio Substituting the given angle gives

$$n_g = 1 \times \tan(\pi/3) = \sqrt{3}.$$

A refractive index is a ratio of speeds and has no unit. The value exceeds unity, as expected for glass relative to air.

Step 3: Conclusion The refractive index of glass is $\sqrt{3}$ (approximately 1.73).

Quick Tip to Solve: For incidence from air, write the index directly as the tangent of the Brewster angle.

16. What is the value of power factor of a series LCR circuit at resonance?

[1 mark]

Solution:

Concept:

- The power factor is $\cos \phi = R/Z$, where ϕ is the voltage-current phase difference, R is resistance and Z is impedance.
- At series resonance, the inductive and capacitive reactances satisfy $X_L = X_C$.

Step 1: Use the resonance condition The impedance of a series LCR circuit is $Z = \sqrt{R^2 + (X_L - X_C)^2}$. At resonance the two reactances cancel, so the reactive term vanishes and the impedance reduces to $Z = R$.

Step 2: Find the phase relation The net circuit impedance is purely resistive at resonance. Therefore the applied voltage and current are in phase, giving $\phi = 0$. Using either the phase relation or the impedance ratio gives $\cos \phi = \cos 0 = R/R$.

Step 3: Conclusion The power factor at resonance is $\boxed{1}$ (unity).

Quick Tip to Solve: At series resonance cancel the two reactances first; the remaining impedance is simply R .

17. In which orientation of the armature coil of an ac generator relative to the magnetic field, will the induced emf be maximum?

[1 mark]

Solution:

Concept:

- The magnetic flux linkage of a rotating coil is $N\Phi = NBA \cos \theta$, where θ is the angle between the coil normal and magnetic field.
- Faraday's law gives the induced emf $\varepsilon = NBA\omega \sin \theta$ for uniform rotation.

Step 1: Identify the condition for maximum emf For fixed N , B , A and angular speed ω , the magnitude of emf is greatest when $|\sin \theta| = 1$. This occurs at $\theta = 90^\circ$ or 270° : the normal to the coil is perpendicular to the magnetic field.

Step 2: Translate the normal into the coil plane The normal is perpendicular to the plane of the coil. Thus when the normal is perpendicular to the field, the field lies parallel to the coil plane. At this orientation the flux itself is zero, but its rate of change is greatest; emf depends on that rate.

Step 3: Conclusion The induced emf is maximum when the plane of the coil is parallel to the magnetic field.

Quick Tip to Solve: Distinguish the coil plane from its normal: maximum emf occurs at zero flux, not maximum flux.

18. Mention the contribution of Indian physicist J.C. Bose in the production of electromagnetic waves.

[1 mark]

Solution:

Concept:

- Electromagnetic waves include the microwave region, with wavelengths shorter than ordinary radio waves.

- J. C. Bose experimentally produced short-wavelength electromagnetic radiation.

Step 1: State Bose's experimental contribution J. C. Bose succeeded in producing electromagnetic waves of very short wavelength, in the millimetre-wave or microwave region. His work extended experimental production of electromagnetic radiation to wavelengths much smaller than the long radio waves used in early experiments.

Step 2: Give the characteristic wavelength The standard description of this contribution is production of electromagnetic waves with wavelengths in the range 5 mm to 25 mm. This identifies the short-wavelength radiation involved and makes the answer more specific than simply saying that he produced radio waves.

Step 3: Conclusion J. C. Bose produced short-wavelength electromagnetic waves of about 5 mm to 25 mm.

Quick Tip to Solve: Remember Bose with millimetre waves: specify the short wavelength range when stating his contribution.

19. In an ac circuit, the applied voltage and flowing current are $E = E_0 \sin \omega t$ and $I = I_0 \sin \left(\omega t + \frac{\pi}{2} \right)$ respectively. What is the average power consumed in one cycle in this circuit? [1 mark]

Solution:

Concept:

- The average power in a sinusoidal ac circuit is $P_{av} = E_{rms} I_{rms} \cos \phi$, where ϕ is the phase difference.
- The rms values of the specified sinusoids are $E_0/\sqrt{2}$ and $I_0/\sqrt{2}$.

Step 1: Read the phase difference The voltage is $E_0 \sin \omega t$, whereas the current is $I_0 \sin(\omega t + \pi/2)$. Hence the current leads the voltage by $\pi/2$, and the power factor is $\cos(\pi/2) = 0$.

Step 2: Average the instantaneous power Using $\sin(\omega t + \pi/2) = \cos \omega t$, the instantaneous power is

$$p(t) = E_0 I_0 \sin \omega t \cos \omega t = \frac{E_0 I_0}{2} \sin 2\omega t.$$

Its positive and negative contributions cancel over one complete cycle, consistent with the zero power factor.

Step 3: Conclusion The average power consumed in one cycle is 0 W.

Quick Tip to Solve: A quarter-cycle voltage-current phase difference makes the average power zero, even if both amplitudes are nonzero.

19. What happens when a block of metal is kept in a varying magnetic field?

[1 mark]

Solution:**Concept:**

- A changing magnetic flux induces an emf according to Faraday's law: $\varepsilon = -d\Phi/dt$.
- In a bulk conductor, induced currents can circulate in closed paths within the material; these are eddy currents.

Step 1: Apply electromagnetic induction When the magnetic field through a metal block varies, the magnetic flux through conducting loops within the block changes. An induced emf is therefore produced in these loops. No external wire or separate circuit needs to be connected because the metal itself provides conducting paths.

Step 2: Describe the physical effect The induced currents circulate within the block and are called eddy currents. By Lenz's law, their magnetic effect opposes the change in flux producing them. Since the metal has resistance, the currents dissipate electrical energy as heat, so the block may become heated.

Step 3: Conclusion Eddy currents are induced in the metal block, producing a heating effect.

Quick Tip to Solve: A varying magnetic field in a solid conductor induces circulating eddy currents within the conductor itself.

20. Name the electromagnetic radiations which may cause sunburn. Write their wavelength range.

[1 mark]

Solution:**Concept:**

- Ultraviolet (UV) radiation lies beyond violet in the electromagnetic spectrum and has shorter wavelengths than visible light.
- The ultraviolet band extends approximately from 1 nm to 400 nm; wavelength boundaries between spectral bands are conventional.

Step 1: Identify the radiation Sunburn is caused by ultraviolet radiation from the Sun, particularly the UV-B component. These photons have sufficient energy to damage skin cells. Visible light is not the expected answer, while infrared radiation mainly produces heating.

Step 2: State its wavelength range The wavelength range of ultraviolet radiation is approximately 1 nm to 400 nm, or 10^{-9} m to 4×10^{-7} m. The question asks for the ultraviolet spectral band, rather than only the narrower sunburn-producing UV-B sub-band.

Step 3: Conclusion Ultraviolet radiation may cause sunburn. Its wavelength range is approximately 1 nm to 400 nm.

Quick Tip to Solve: For sunburn recall UV; distinguish the whole UV band from its narrower UV-B component.

Section B (14 marks)

21. The number densities of free electrons in three conductors X , Y and Z are in the ratio $4 : 1 : 8$, and their electrical conductivities are in the ratio $2 : 1 : 2$. For which conductor will the relaxation time of free electrons be (a) maximum, and (b) minimum? [2 marks]

Solution:

Concept:

- In the free-electron model, conductivity is $\sigma = ne^2\tau/m$, where n is electron number density, e is electron charge magnitude, m is electron mass and τ is relaxation time.
- Thus $\tau = m\sigma/(ne^2)$. Since m and e are common, compare σ/n , not conductivity alone.

Step 1: Form the relaxation-time ratio Let the number densities be $4n_0, n_0, 8n_0$ and the conductivities be $2\sigma_0, \sigma_0, 2\sigma_0$ for X, Y, Z , respectively. Substituting in the conductivity relation gives

$$\tau_X : \tau_Y : \tau_Z = \frac{2}{4} : \frac{1}{1} : \frac{2}{8} = \frac{1}{2} : 1 : \frac{1}{4} = 2 : 4 : 1.$$

Step 2: Compare the three values The largest entry is that for Y , and the smallest is that for Z . Although X and Z have equal conductivity, Z has twice the free-electron density of X , so its relaxation time is half that of X . This checks the comparison independently.

Step 3: Conclusion (a) Relaxation time is maximum for conductor Y . (b) It is minimum for conductor Z ; the ratio is $\tau_X : \tau_Y : \tau_Z = 2 : 4 : 1$.

Quick Tip to Solve: Divide each conductivity ratio entry by its corresponding number-density entry before comparing.

22. A converging lens of focal length f_1 is placed coaxially in contact with a diverging lens of focal length f_2 ($f_1 > f_2$). Determine the power and nature of the combination in terms of f_1 and f_2 . [2 marks]

Solution:

Concept:

- The power of a thin lens is $P = 1/f$, with signed focal length f in metres and power in dioptres.
- For two thin coaxial lenses in contact, powers add: $P = P_1 + P_2$. A converging lens has positive power and a diverging lens has negative power.

Step 1: Apply the signs to the focal lengths The inequality $f_1 > f_2$ is used here for the positive magnitudes of the two focal lengths. Consequently the signed focal lengths are $+f_1$ for the converging lens and $-f_2$ for the diverging lens. Thus

$$P_1 = \frac{1}{f_1}, \quad P_2 = -\frac{1}{f_2}.$$

Step 2: Add the powers and determine the sign For contact lenses,

$$P = \frac{1}{f_1} - \frac{1}{f_2} = \frac{f_2 - f_1}{f_1 f_2}.$$

Since both magnitudes are positive and $f_1 > f_2$, the numerator is negative. The diverging lens therefore has the larger power magnitude. The equivalent focal length is $F = f_1 f_2 / (f_2 - f_1) < 0$, confirming the nature of the combination.

Step 3: Conclusion The combination has power $P = (f_2 - f_1) / (f_1 f_2) \text{ D}$, with f_1, f_2 expressed in metres as magnitudes. Its power is negative, so it is diverging.

Quick Tip to Solve: Attach the negative sign to the diverging lens before adding reciprocals; a shorter focal-length magnitude means stronger power.

OR

22. How is the resolving power of a compound microscope affected if (a) wavelength of light used is decreased, and (b) the diameter of its objective lens is increased? Justify your answers. [2 marks]

Solution:

Concept:

- The minimum separation resolved by a microscope is $d_{\min} = \lambda / (2\mu \sin \theta)$, where λ is wavelength, μ is the medium refractive index and θ is the objective half-angle of acceptance.
- Resolving power is $1/d_{\min} = 2\mu \sin \theta / \lambda$. The quantity $\mu \sin \theta$ is the numerical aperture.

Step 1: Effect of decreasing wavelength With the objective and the medium unchanged, $\mu \sin \theta$ remains fixed. Reducing λ reduces $d_{\min}$ in the same proportion, allowing two more closely spaced points to be distinguished. Because resolving power is inversely proportional to wavelength, the resolving power increases.

Step 2: Effect of increasing objective diameter For the same microscope geometry and medium, a larger objective diameter accepts rays over a larger cone. The half-angle θ , and hence $\sin \theta$, increases. The numerical aperture therefore increases and the diffraction-limited separation decreases. From $\text{RP} = 2\mu \sin \theta / \lambda$, the resolving power increases. This is an improvement in resolving detail, distinct from simply increasing magnification.

Step 3: Conclusion (a) Decreasing wavelength increases resolving power. (b) Increasing objective diameter also increases resolving power by increasing the numerical aperture.

Quick Tip to Solve: Use $\text{RP} \propto \text{NA} / \lambda$: shorter wavelength and a wider acceptance cone both improve resolution.

23. (a) Define the SI unit of self-inductance.

(b) The self-inductance of a solenoid is L . If the number of turns per unit length in it is doubled and the area of cross-section is halved, find the new inductance of the solenoid. [2 marks]

Solution:

Concept:

- Self-induced emf is $\varepsilon = -L \, dI/dt$. The negative sign expresses opposition to the change in current; L is self-inductance.
- For a long solenoid, $L = \mu n^2 A \ell$, where μ is permeability, n is turns per unit length, A is cross-sectional area and ℓ is length.

Step 1: Define the SI unit The SI unit of self-inductance is the henry (H). A coil has self-inductance 1 H if a current changing at 1 A s^{-1} induces an emf of magnitude 1 V in it. Thus

$$1 \text{ H} = 1 \text{ V s A}^{-1}.$$

The definition uses emf magnitude because its polarity opposes the current change.

Step 2: Calculate the changed inductance The stated changes give $n' = 2n$ and $A' = A/2$, while the solenoid length and core permeability remain unchanged. Consequently

$$L' = \mu(2n)^2 \frac{A}{2} \ell = 2\mu n^2 A \ell = 2L.$$

Doubling turns per unit length supplies a factor of four, while halving the area supplies a factor of one-half.

Step 3: Conclusion (a) One henry is the self-inductance producing 1 V of induced emf for a current-change rate of 1 A s^{-1} . (b) The new self-inductance is $2L$.

Quick Tip to Solve: For a solenoid keep the square on turns per unit length: multiply L by $2^2 \times \frac{1}{2}$.

24. A hydrogen atom is in its third excited state.

(a) How many spectral lines can be emitted by it before coming to the ground state? Show these transitions in the energy level diagram.

(b) In which of the above transitions will the spectral line of shortest wavelength be emitted? [2 marks]

Solution:

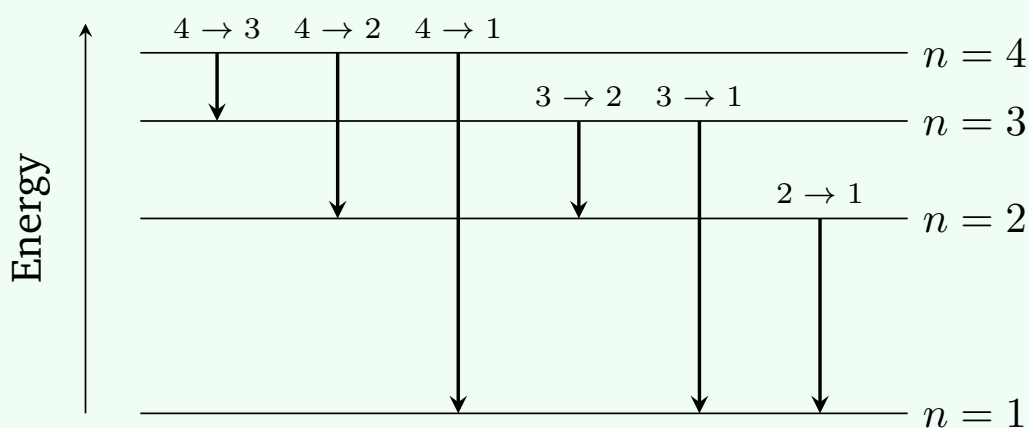
Concept:

- The ground state of hydrogen is $n = 1$; therefore its third excited state is $n = 4$. A downward transition emits a photon of energy $h\nu = E_i - E_f = hc/\lambda$.
- The number of distinct downward transitions among levels 1 to n is $N = n(n-1)/2$.

Step 1: Count and draw the possible transitions For $n = 4$, the possible distinct lines number

$$N = \frac{4(4-1)}{2} = 6.$$

The transitions are $4 \rightarrow 3$, $4 \rightarrow 2$, $4 \rightarrow 1$, $3 \rightarrow 2$, $3 \rightarrow 1$, and $2 \rightarrow 1$. The diagram shows all six downward transitions. This counts all possible spectral lines from atoms initially in this state; any one atom follows a particular decay route and need not emit all six photons.



All six possible downward transitions (energy spacing schematic)

Step 2: Identify the largest energy difference Since $\lambda = hc/(E_i - E_f)$, the shortest wavelength corresponds to the largest energy drop. Among these levels, E_4 is the highest and E_1 the lowest, so the transition $4 \rightarrow 1$ has the largest drop. Using $E_n = -13.6/n^2$ eV, this drop is $13.6(1 - 1/16) = 12.75$ eV.

Step 3: Conclusion (a) Six possible spectral lines are emitted, with transitions shown above. (b) The shortest-wavelength line corresponds to $n = 4 \rightarrow n = 1$.

Quick Tip to Solve: Convert the excited-state number to n first: the third excited state is $n=4$, then count all pairs of levels.

25. How does an oscillating charge radiate an electromagnetic wave? Give the relation between the frequency of radiated wave and the frequency of oscillating charge. [2 marks]

Solution:

Concept:

- An accelerated charge radiates electromagnetic waves. An oscillating charge has continuously changing velocity and hence acceleration.
- A changing electric field produces a magnetic field, and a changing magnetic field produces an electric field; the coupled disturbance propagates outward.

Step 1: Explain the production of radiation As a charge oscillates about its mean position, its electric field changes periodically with time. The changing electric field is accompanied by a changing magnetic field. The mutually coupled electric and magnetic disturbances propagate away from the charge as an electromagnetic wave and carry energy supplied by the agent maintaining the oscillation. In the radiation region, the electric and magnetic fields are perpendicular to one another and to the direction of propagation.

Step 2: Relate the frequencies For a charge oscillating periodically at frequency f , the radiated fields

repeat with the same period $T = 1/f$. Thus the emitted electromagnetic wave has frequency

$$f_{\text{wave}} = f_{\text{charge}}.$$

Its vacuum wavelength is consequently $\lambda = c/f$, where c is the speed of light in vacuum.

Step 3: Conclusion An oscillating charge radiates through its time-varying, mutually coupled electric and magnetic fields. The radiation frequency equals the oscillation frequency: $f_{\text{wave}} = f_{\text{charge}}$.

Quick Tip to Solve: Identify acceleration as the source of radiation, then use the same oscillation period for the radiated fields.

OR

25. (a) Explain briefly the fact that electromagnetic waves carry energy.

(b) Why do we not feel the pressure due to sunshine?

[2 marks]

Solution:

Concept:

- Electromagnetic waves contain electric and magnetic fields with energy density $u = \frac{1}{2}\epsilon_0 E^2 + \frac{B^2}{2\mu_0}$. Here ϵ_0 and μ_0 are the vacuum permittivity and permeability.
- Radiation transports momentum as well as energy. For normal incidence, the pressure is $P = I/c$ for complete absorption and $P = 2I/c$ for complete reflection, where I is intensity.

Step 1: Explain energy transport The electric and magnetic fields of an electromagnetic wave store energy. As the wave propagates, this energy is carried from its source to the receiving surface. On absorption, it can do work on charges or become heat; for example, sunlight warms an absorbing surface. Thus the energy in the wave is physically transferred rather than being a disturbance with no energy content.

Step 2: Explain why sunshine pressure is imperceptible The absorbed momentum flux of sunlight gives pressure $P = I/c$. Since $c \simeq 3 \times 10^8 \text{ m s}^{-1}$ is extremely large, even an intensity of order 10^3 W m^{-2} gives only $P \sim 3 \times 10^{-6} \text{ Pa}$. Reflection can at most double this estimate. The resulting force on the body is extremely small compared with ordinary mechanical forces, so we do not feel it.

Step 3: Conclusion (a) Electromagnetic waves transport the energy stored in their electric and magnetic fields. (b) Sunshine exerts a very small radiation pressure because its momentum flux is intensity divided by the very large speed of light.

Quick Tip to Solve: Use $P=I/c$ to judge radiation pressure: the large denominator makes sunlight pressure too small to feel.

26. (a) In what ways are matter waves different from electromagnetic waves?

(b) If the kinetic energy of a particle is reduced to one-fourth $\left(\frac{1}{4}\right)$ of its initial value, how many times will the de Broglie wavelength associated with it become? [2 marks]

Solution:

Concept:

- Matter waves describe the quantum wave behaviour of material particles and have de Broglie wavelength $\lambda = h/p$, where p is particle momentum.
- For a non-relativistic particle of mass m and kinetic energy K , $p = \sqrt{2mK}$, so $\lambda = h/\sqrt{2mK}$.

Step 1: Distinguish the two kinds of waves Matter waves are associated with moving material particles such as electrons and describe their probability amplitude; they are not oscillations of electric and magnetic fields. They occur for charged as well as neutral particles. Electromagnetic waves consist of coupled oscillating electric and magnetic fields and are produced by accelerated charges. Electromagnetic waves propagate in vacuum at speed c , whereas a matter-wave packet has group velocity equal to the velocity of its particle, which need not be c .

Step 2: Calculate the wavelength ratio For the same non-relativistic particle, its mass remains constant and $\lambda \propto K^{-1/2}$. If $K_f = K_i/4$, then

$$\frac{\lambda_f}{\lambda_i} = \sqrt{\frac{K_i}{K_f}} = \sqrt{4} = 2.$$

Equivalently its momentum is halved, and the inverse relation $\lambda = h/p$ doubles its wavelength.

Step 3: Conclusion (a) Matter waves describe particle probability amplitudes, whereas electromagnetic waves are oscillating electric and magnetic fields. (b) The de Broglie wavelength becomes twice its initial value, $\lambda_f = 2\lambda_i$, in the non-relativistic regime.

Quick Tip to Solve: For fixed mass use wavelength proportional to the inverse square root of kinetic energy, not its inverse.

27. Two identical bars, one of paramagnetic material and other of diamagnetic material are kept in a uniform external magnetic field parallel to it. Draw diagrammatically the modifications in the magnetic field pattern in each case. [2 marks]

Solution:

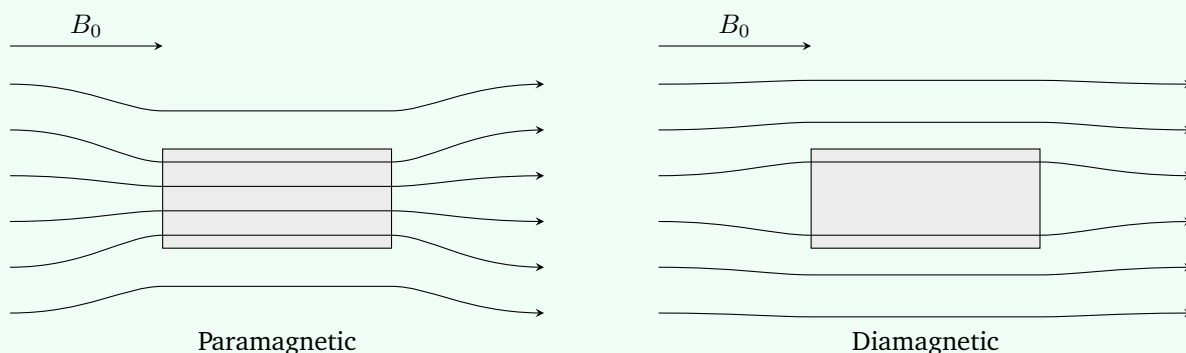
Concept:

- Paramagnetic susceptibility $\chi_m > 0$ means induced magnetisation reinforces the applied field; $\mu_r = 1 + \chi_m > 1$.
- Diamagnetic susceptibility $\chi_m < 0$ means induced magnetisation opposes the applied field; $\mu_r < 1$. Field-line density represents field strength.

Step 1: Paramagnetic bar Take the external field horizontally to the right, parallel to the long axis of the bar. The induced magnetisation reinforces this field. Neighbouring magnetic field lines bend towards the bar, becoming more closely spaced within it. The field inside is therefore stronger than the external field. All arrows continue to point in the applied-field direction.

Step 2: Diamagnetic bar The magnetisation induced in this bar is opposite to the applied field. The

nearby lines bend away from the bar, leaving a lower density of lines inside it. Thus the internal field is weaker. Ordinary diamagnetism does not completely exclude the magnetic field. The sketches show identical bar dimensions, and the spacing changes are exaggerated to make this weak effect visible.



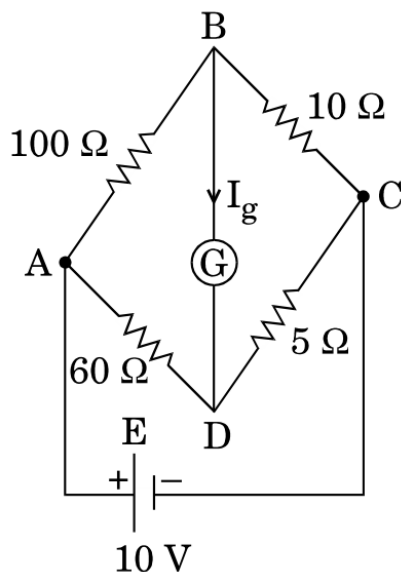
Field-line patterns in identical bars parallel to the external field.

Step 3: Conclusion Magnetic field lines concentrate inside the paramagnetic bar and spread away from the diamagnetic bar, giving greater and smaller internal field strengths respectively.

Quick Tip to Solve: Keep arrows in the applied-field direction in both sketches; change the line spacing inside each bar to show its magnetic response.

Section C (21 marks)

28. Using Kirchhoff's rules, calculate the current (I_g) that flows through the galvanometer of resistance $15\ \Omega$ in the circuit diagram shown in the figure. [3 marks]

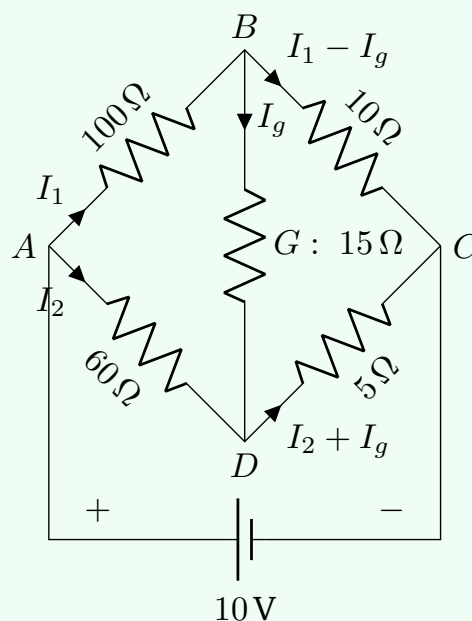


Solution:

Concept:

- Kirchhoff's junction rule follows from charge conservation: currents entering a junction equal currents leaving it.
- Kirchhoff's loop rule follows from energy conservation: the algebraic sum of potential changes around a closed loop is zero. A resistor traversed along its current gives a drop IR .

Step 1: Assign branch currents using the junction rule The positive terminal of the 10V cell is connected to A and its negative terminal to C. Let I_1 flow from A to B through $100\ \Omega$, and I_2 from A to D through $60\ \Omega$. Take I_g from B to D, as indicated in the question. At B the current towards C is $I_1 - I_g$; at D the current towards C is $I_2 + I_g$. The galvanometer resistance is $15\ \Omega$.



Branch-current assignment for applying Kirchhoff's rules.

Step 2: Apply the loop rule to the two source loops Traversing A-B-C and returning through the cell gives

$$100I_1 + 10(I_1 - I_g) = 10, \quad 110I_1 - 10I_g = 10.$$

Traversing A-D-C and returning through the cell similarly gives

$$60I_2 + 5(I_2 + I_g) = 10, \quad 65I_2 + 5I_g = 10.$$

Currents in these equations are in amperes, resistances in ohms and the right-hand side in volts.

Step 3: Use the bridge loop and eliminate the other currents Around A-B-D-A, the first two branches are traversed along their assumed currents and D-A opposite to I_2 . Thus

$$100I_1 + 15I_g - 60I_2 = 0.$$

Substituting $I_1 = (10 + 10I_g)/110$ and $I_2 = (10 - 5I_g)/65$,

$$\begin{aligned} \frac{100(10 + 10I_g)}{110} + 15I_g - \frac{60(10 - 5I_g)}{65} &= 0, \\ \frac{100}{11} - \frac{120}{13} + \left(\frac{100}{11} + 15 + \frac{60}{13} \right) I_g &= 0, \\ -\frac{20}{143} + \frac{4105}{143} I_g &= 0. \end{aligned}$$

The positive result confirms the assumed B-to-D direction. As a check, the branch equations give $I_1 \simeq 0.09135$ A and $I_2 \simeq 0.15347$ A, consistent with the potential drops around the bridge.

Step 4: Conclusion

$$I_g = \frac{4}{821} \text{ A} \simeq 4.87 \text{ mA, from B to D.}$$

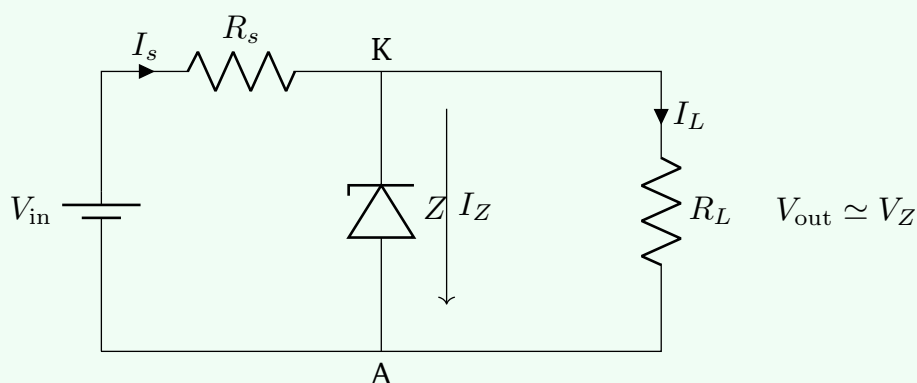
Quick Tip to Solve: At the two galvanometer junctions use opposite signs: the upper right branch carries $I_1 - I_g$, while the lower right carries $I_2 + I_g$.

29. Name the diode which can act as a voltage regulator. Explain its working with the help of its labelled circuit diagram. Draw its V-I characteristic. [3 marks]

Solution:**Concept:**

- A Zener diode is a heavily doped junction designed to operate safely in reverse breakdown, where its voltage V_Z is nearly constant despite changes in current.
- The series resistor R_s limits current. The reverse-biased Zener is parallel to the load R_L , so $V_{\text{out}} \simeq V_Z$ and $I_s = I_Z + I_L$.

Step 1: Connect the regulator The required diode is a Zener diode. Its cathode is connected to the positive output node and its anode to the negative common terminal. The load is connected across the diode, and this combination receives the input through a series resistor. The input exceeds the breakdown voltage, while the resistor limits the diode current to a safe value. The reverse-biased diode is therefore operated in its breakdown region.



Zener regulator, with cathode K at the positive output node.

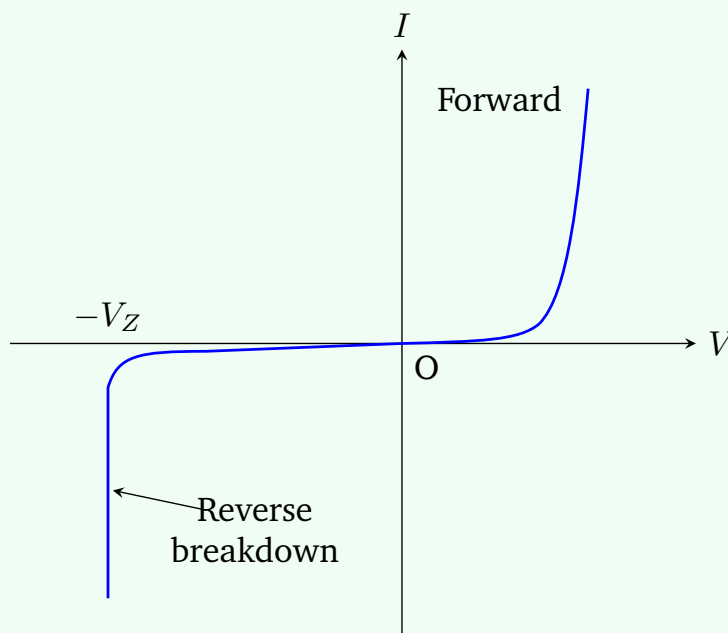
Step 2: Explain regulation The currents obey

$$I_s = \frac{V_{\text{in}} - V_Z}{R_s}, \quad I_L = \frac{V_Z}{R_L}, \quad I_Z = I_s - I_L.$$

If the input increases, the Zener current increases substantially while its voltage changes very little. The additional input voltage appears mainly across the series resistor. If the input decreases, the Zener current decreases. If the load draws more current, the Zener current decreases to supply the additional load demand; if the load draws less, the excess flows through the Zener. Thus the load voltage remains approximately constant, provided the Zener current stays above its breakdown knee and below the maximum safe current.

Step 3: Describe the characteristic In forward bias, current rises rapidly beyond the forward knee, as for an ordinary junction diode. In reverse bias a small leakage current flows until the voltage reaches

$-V_Z$. Beyond this point the magnitude of reverse current increases sharply for only a small voltage change. The steep reverse-breakdown branch is responsible for regulation. Signed voltage and current axes place this branch in the third quadrant; the forward conducting branch lies in the first quadrant.



V-I characteristic with reverse breakdown at negative voltage.

Step 4: Conclusion A Zener diode connected in reverse breakdown across the load regulates the output at approximately V_Z .

Quick Tip to Solve: Connect the cathode to the positive load node and show breakdown at negative voltage on signed axes.

30. (a) The density of the nuclear matter is tremendously larger than the physical density of the material. Explain.

(b) The nuclear forces are not coulomb forces between nucleons. Explain.

(c) Draw a plot of the potential energy between a pair of nucleons as a function of distance between them inside a nucleus. [3 marks]

Solution:

Concept:

- Nuclear mass is approximately Am_N , and nuclear radius is $R = R_0 A^{1/3}$, where $R_0 \simeq 1.2 \times 10^{-15}$ m and m_N is the nucleon mass.
- The strong nuclear force is attractive at ordinary nucleon separations, strongly repulsive at very short separation, and negligible beyond a few femtometres. The potential is taken as zero at infinite separation.

Step 1: (a) Compare nuclear density with material density Nearly all the atom's mass is concentrated in its tiny nucleus. An atomic radius is of order 10^{-10} m, whereas nuclear dimensions are of order 10^{-15} to 10^{-14} m. Most atomic volume is therefore outside the nucleus. The same mass confined to

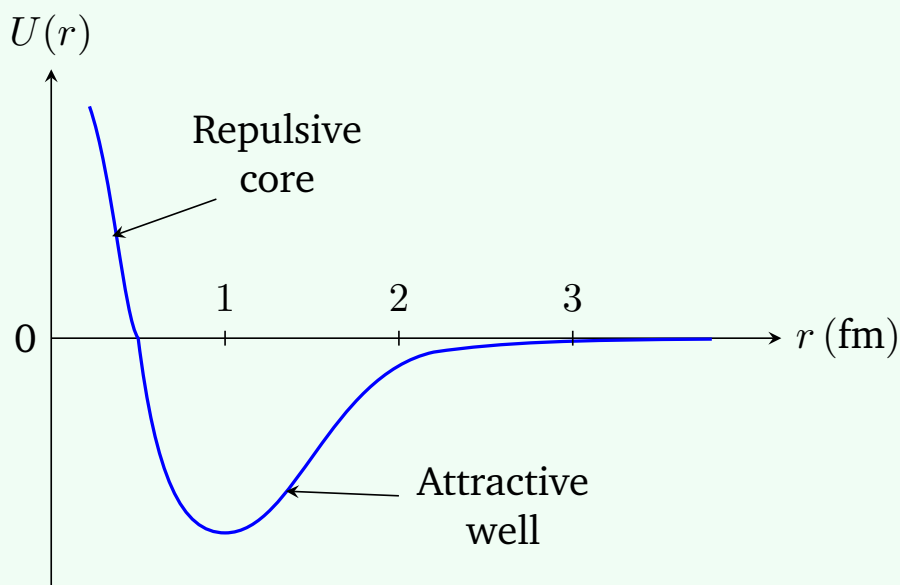
this much smaller volume gives a tremendously greater density. More quantitatively,

$$\rho_N = \frac{Am_N}{(4\pi/3)R_0^3 A} = \frac{3m_N}{4\pi R_0^3} \simeq 2.3 \times 10^{17} \text{ kg m}^{-3}.$$

This is approximately independent of mass number and vastly exceeds ordinary material densities, typically of order 10^3 to 10^4 kg m^{-3} .

Step 2: (b) Distinguish nuclear and Coulomb forces Coulomb forces act between charges; they cannot explain attraction between two electrically neutral neutrons or between a neutron and a proton. Moreover, the Coulomb force between two protons is repulsive, whereas nuclear attraction can bind them in a nucleus despite that repulsion. Nuclear forces are approximately charge independent, acting between proton-proton, neutron-neutron and proton-neutron pairs. They have a very short range, unlike the long-range inverse-square Coulomb force. These observations show that nuclear binding arises from a separate strong interaction.

Step 3: (c) Sketch and interpret the potential Take $U = 0$ for widely separated nucleons. At separations smaller than roughly 0.5 fm, the potential rises sharply to positive values, representing a repulsive core. At intermediate separations it forms a negative well, with a minimum around 1 fm. Beyond the minimum, the potential rises towards zero and becomes negligible beyond a few femtometres. Since $F_r = -dU/dr$, the descending inner branch corresponds to outward repulsion, while the rising outer branch corresponds to inward attraction. The plot is qualitative; its depth depends on the nucleon state.



Qualitative nucleon-nucleon potential: repulsive core and attractive well.

Step 4: Conclusion Nuclear density is enormous because nearly all atomic mass occupies a minute volume. Nuclear binding is due to short-range strong forces, with a potential showing a repulsive core and an attractive well.

Quick Tip to Solve: Sketch the potential above zero at tiny separation, below zero in the well, and approaching zero at large separation.

31. What are matter waves? Find the ratio of de Broglie wavelengths associated with proton and alpha particles when both particles

(a) are accelerated through the same potential difference.

(b) have same velocity.

[3 marks]

Solution:

Concept:

- Matter waves are de Broglie waves associated with moving material particles. Their wavelength is $\lambda = h/p$, where h is Planck's constant and p is momentum. They express the wave nature of matter and are not electromagnetic waves.
- For a non-relativistic particle of mass m and charge magnitude q , accelerated from rest through potential difference V , $qV = p^2/(2m)$.
- A proton has mass m_p and charge e ; an alpha particle has approximately mass $4m_p$ and charge $2e$.

Step 1: (a) Obtain the wavelength after acceleration Conservation of energy gives $qV = p^2/(2m)$. Hence

$$p = \sqrt{2mqV}, \quad \lambda = \frac{h}{\sqrt{2mqV}}.$$

Both mass and charge must be retained. Equal potentials do not give equal kinetic energies: the alpha particle gains twice as much energy because its charge is twice as large.

Step 2: (a) Form the ratio at the same potential The common factors cancel to give

$$\frac{\lambda_p}{\lambda_\alpha} = \sqrt{\frac{m_\alpha q_\alpha}{m_p q_p}} = \sqrt{\frac{(4m_p)(2e)}{m_p e}} = \sqrt{8}.$$

The alpha particle has greater momentum, so its wavelength is shorter. This also checks that the proton-to-alpha wavelength ratio must exceed unity.

Step 3: (b) Form the ratio at the same velocity For equal non-relativistic velocities use $p = mv$ directly. Thus

$$\frac{\lambda_p}{\lambda_\alpha} = \frac{h/(m_p v)}{h/(m_\alpha v)} = \frac{m_\alpha}{m_p} = 4.$$

Charge does not enter this comparison because the velocity is already specified. The fourfold mass gives the alpha particle fourfold momentum and one-fourth the proton wavelength.

Step 4: Conclusion Matter waves accompany moving particles and obey $\lambda = h/p$. The ratios $\lambda_p : \lambda_\alpha$ are (a) $2\sqrt{2} : 1$ for the same potential and (b) $4 : 1$ for the same velocity.

Quick Tip to Solve: For equal potentials compare mq under a square root; for equal velocities compare masses directly.

32. A series LCR ac circuit has $L = 2.0 \text{ H}$, $C = 32 \mu\text{F}$ and $R = 10 \Omega$.

(a) At what angular frequency of ac will it resonate?

(b) Calculate the Q value of the circuit.

[3 marks]

Solution:

Concept:

- In a series LCR circuit, resonance occurs when inductive reactance $X_L = \omega L$ equals capacitive reactance $X_C = 1/(\omega C)$. The net reactance then vanishes.
- The resonant angular frequency is $\omega_0 = 1/\sqrt{LC}$. The quality factor is $Q = \omega_0 L/R = (1/R)\sqrt{L/C}$, a dimensionless measure of resonance sharpness.

Step 1: Convert the data into SI units The given values are

$$L = 2.0 \text{ H}, \quad C = 32 \times 10^{-6} \text{ F}, \quad R = 10 \Omega.$$

The microfarad conversion is essential because the resonance formula uses henries and farads. Resistance affects the quality factor but not the ideal series-resonance condition.

Step 2: (a) Calculate the resonant angular frequency Equating the reactances gives $\omega_0 L = 1/(\omega_0 C)$, so

$$\begin{aligned}\omega_0 &= \frac{1}{\sqrt{LC}} \\ &= \frac{1}{\sqrt{(2.0)(32 \times 10^{-6})}} \\ &= \frac{1}{8.0 \times 10^{-3}} = 125 \text{ rad s}^{-1}.\end{aligned}$$

This is angular frequency, not ordinary frequency. At this value the impedance is purely resistive, equal to R , and current is in phase with the supply voltage.

Step 3: (b) Calculate and check the quality factor Using the resonant angular frequency,

$$Q = \frac{\omega_0 L}{R} = \frac{125 \times 2.0}{10} = 25.$$

As a check, the reactances at resonance are $X_L = 125 \times 2 = 250 \Omega$ and $X_C = 1/(125 \times 32 \times 10^{-6}) = 250 \Omega$. Their ratio to resistance is $250/10 = 25$, consistent with the calculated quality factor. The ratio of two resistances is dimensionless.

Step 4: Conclusion The circuit resonates at $\omega_0 = 125 \text{ rad s}^{-1}$, and its quality factor is $Q = 25$.

Quick Tip to Solve: Convert microfarads to farads first and use angular frequency, not frequency in hertz, in $Q = \omega L/R$.

32. An ideal inductor of $\frac{5}{\pi}$ H inductance is connected to a 200 V, 50 Hz ac supply.

(a) Calculate the rms and peak value of current in the inductor.

(b) What is the phase difference between current through the inductor and the applied voltage? How will it change if a small resistance is connected in series with this inductor in the circuit? [3 marks]

Solution:

Concept:

- For an ideal inductor, $X_L = \omega L = 2\pi fL$, where f is supply frequency and L is inductance. Its rms current is $I_{\text{rms}} = V_{\text{rms}}/X_L$.
- For sinusoidal current, $I_0 = \sqrt{2}I_{\text{rms}}$. In a pure inductor, current lags voltage by $\pi/2$. With series resistance, the lag angle obeys $\tan \phi = X_L/R$.

Step 1: (a) Find the inductive reactance The printed inductance is $L = 5/\pi$ H, with frequency $f = 50$ Hz. The stated ac supply voltage is its rms value, $V_{\text{rms}} = 200$ V. Thus

$$X_L = 2\pi fL = 2\pi(50)\frac{5}{\pi} = 500 \Omega.$$

Retaining the denominator π in the inductance makes the cancellation exact.

Step 2: (a) Calculate rms and peak current Apply the ac form of Ohm's law to the reactance:

$$I_{\text{rms}} = \frac{200}{500} = 0.40 \text{ A}.$$

For a sinusoidal waveform the peak exceeds the rms value by $\sqrt{2}$, so

$$I_0 = \sqrt{2}(0.40) = 0.40\sqrt{2} \text{ A} \approx 0.566 \text{ A}.$$

The supply specification must not be treated as peak voltage; doing so would introduce an incorrect extra factor of $\sqrt{2}$.

Step 3: (b) Describe the phase and the effect of resistance For the ideal inductor, current lags the applied voltage by 90° , or $\pi/2$ radians. When a small positive resistance is added in series,

$$Z = \sqrt{R^2 + X_L^2}, \quad \phi = \tan^{-1}\left(\frac{X_L}{R}\right) = \tan^{-1}\left(\frac{500 \Omega}{R}\right).$$

The current still lags, but $0 < \phi < 90^\circ$. If $R \ll 500 \Omega$, the lag remains close to 90° while becoming slightly smaller. A numerical new angle cannot be given without the resistance value.

Step 4: Conclusion $I_{\text{rms}} = 0.40$ A and $I_0 = 0.40\sqrt{2} \text{ A} \approx 0.566$ A. Current initially lags voltage by 90° ; adding series resistance reduces this lag below 90° .

Quick Tip to Solve: Cancel pi in $2\pi fL$ using the printed $5/\pi$ H; then distinguish rms current from peak current by $\sqrt{2}$.

33. (a) Using the necessary ray diagram, derive the mirror formula for a concave mirror.

(b) In the magnified image of a measuring scale (with equidistant markings) lying along the principal axis of a concave mirror, the markings are not equidistant. Explain. [3 marks]

Solution:

Concept:

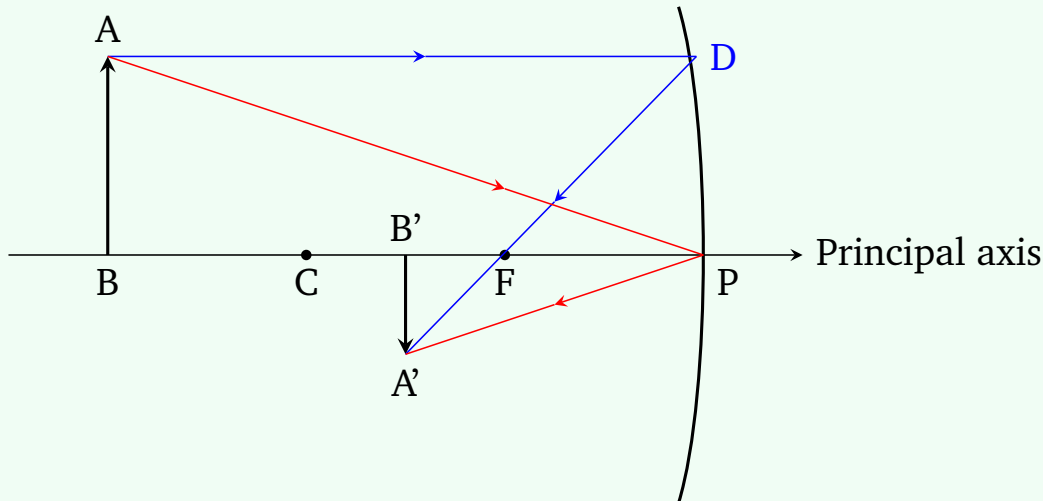
- A concave spherical mirror obeys the mirror formula in the paraxial approximation: rays make small angles with the principal axis and strike close to the pole.
- In the Cartesian convention, distances are measured from the pole P , positive along incident light. Thus a real object, real image and the focus of a concave mirror have $u < 0$, $v < 0$ and $f < 0$.

Step 1: (a) Construct the ray diagram and compare triangles Let the upright object be AB , with B on the axis, and its real inverted image be $A'B'$. Write the positive distance magnitudes as $PB = U$, $PB' = V$, $PF = F_0$, with object height h and image-height magnitude h' . A ray from A parallel to the axis strikes the mirror at D and reflects through F ; another strikes the pole and reflects with equal angles to the axis. They meet at A' . The pole-ray triangles are similar because the angles of incidence and reflection are equal. Therefore,

$$\frac{h'}{h} = \frac{V}{U}.$$

For a small aperture, D is approximately in the transverse plane through P , so its height is h . Comparing the similar triangles along the reflected ray through F gives

$$\frac{h'}{h} = \frac{B'F}{PF} = \frac{V - F_0}{F_0}.$$



Paraxial rays for a concave mirror; the real image lies between C and F.

Step 2: (a) Obtain the mirror formula with signed distances Equating the two height ratios and rearranging,

$$\begin{aligned} \frac{V}{U} &= \frac{V - F_0}{F_0}, \\ VF_0 &= UV - UF_0, \\ \frac{1}{F_0} &= \frac{1}{U} + \frac{1}{V}. \end{aligned}$$

For the real-object, real-image configuration shown, $u = -U$, $v = -V$, $f = -F_0$. Substituting these into the last equation yields $1/f = 1/u + 1/v$. The signed formula also applies to virtual images when

the appropriate Cartesian signs are used.

Step 3: (b) Explain the unequal image spacing Each marking along the scale has a different object distance. The mirror formula gives its image position as

$$v = \frac{fu}{u-f}, \quad \frac{dv}{du} = -\frac{f^2}{(u-f)^2}.$$

Thus the longitudinal magnification in magnitude, $|dv/du| = f^2/(u-f)^2$, varies with the position of a marking. Equal small object intervals Δu consequently have image intervals approximately $|dv/du| \Delta u$, which differ from one part of the scale to another. The change is particularly large for markings near the focus, so the magnified image does not have equidistant markings.

Step 4: Conclusion The paraxial mirror formula is $1/f = 1/u + 1/v$. Markings along the principal axis have unequal image spacings because longitudinal magnification depends on object distance.

Quick Tip to Solve: For a scale along the axis, compare changes in image distance, not the transverse image heights: differentiate the mirror formula.

34. (a) How are energy bands formed in a crystalline solid?

(b) Draw the energy band diagrams for p-type and n-type semiconductors. Depict the donor/acceptor energy levels in these diagrams and write their significance. [3 marks]

Solution:

Concept:

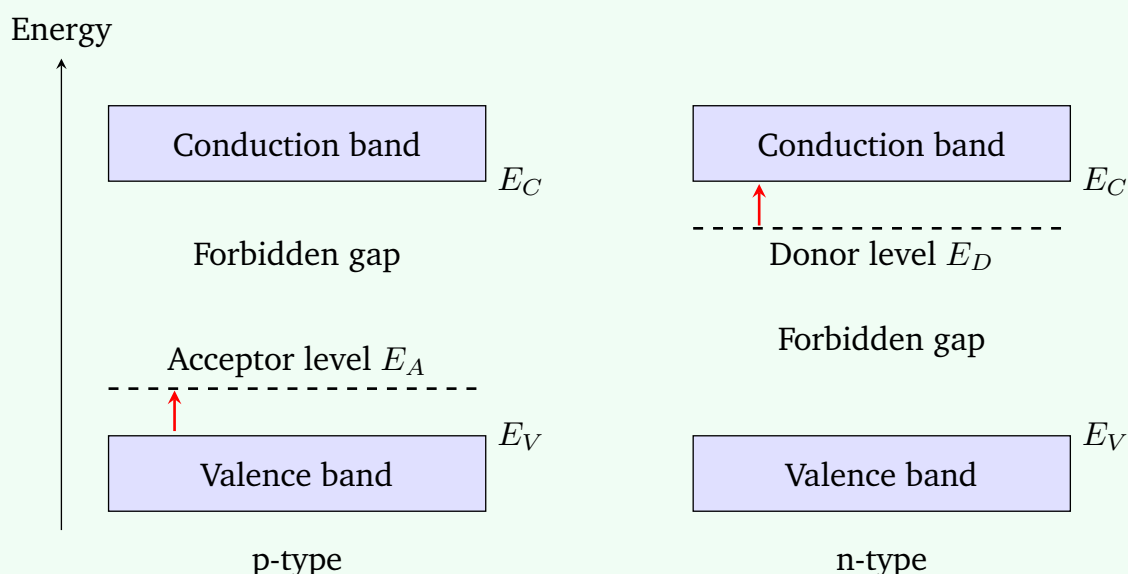
- An isolated atom has discrete allowed electron energies. In a crystal, interaction between neighbouring atoms splits these levels into closely spaced levels that form allowed energy bands.
- The valence band and conduction band are separated by a forbidden energy gap. Doping introduces donor levels near the conduction band or acceptor levels near the valence band.

Step 1: (a) Explain the formation of energy bands When a large number N of atoms come together to form a crystalline solid, their outer-electron wavefunctions overlap and electrons interact with neighbouring atoms. A level characteristic of an isolated atom splits into a very large number of closely spaced allowed levels. The Pauli exclusion principle prevents all the electrons from occupying the same quantum state. Since N is extremely large, these closely spaced levels are effectively continuous and constitute an allowed energy band. Regions between allowed bands in which no electron energy states are available are called forbidden energy gaps. The highest occupied band at zero temperature is the valence band; the next higher available band is the conduction band.

Step 2: (b) Locate the acceptor level in a p-type semiconductor A trivalent impurity such as boron in silicon has one fewer valence electron than needed to complete the four covalent bonds. It introduces an acceptor energy level E_A slightly above the valence-band edge E_V , within the forbidden gap. Only a small energy $E_A - E_V$ is needed for a valence electron to occupy this level. The impurity then becomes a negatively charged ion and a mobile hole is left in the valence band. Hence holes are the majority carriers and electrons are the minority carriers in p-type material.

Step 3: (b) Locate the donor level in an n-type semiconductor A pentavalent impurity such as phosphorus in silicon provides an extra, weakly bound electron after forming four covalent bonds. It introduces a donor energy level E_D slightly below the conduction-band edge E_C , within the forbidden gap. A small energy $E_C - E_D$ promotes this electron to the conduction band, leaving a

positively charged, immobile donor ion. Thus electrons are the majority carriers and holes are the minority carriers. Because these impurity ionisation energies are much smaller than the full band gap, thermal energy can readily produce the additional carriers without lifting an electron across the entire forbidden gap.



Energy increases upward. Impurity levels lie within the forbidden gap; spacings are schematic.

Step 4: Conclusion Closely spaced split atomic levels form energy bands. The p-type acceptor level lies just above the valence band and supplies holes; the n-type donor level lies just below the conduction band and supplies electrons.

Quick Tip to Solve: Remember the level positions by the required transition: valence band to acceptor for holes, and donor to conduction band for electrons.

Section D (15 marks)

35. (a) What is meant by plane polarised light? An unpolarised light is incident at an angle θ on the surface of glass of refractive index μ . If the reflected and refracted rays are perpendicular to each other, then obtain the relationship between μ and θ .

(b) Two polaroids P_1 and P_2 are placed in a crossed position. Unpolarised light of intensity I_0 is incident on P_1 . If P_2 is rotated through an angle θ about the direction of propagation of light, keeping P_1 fixed, plot the graph of intensity of light for $0^\circ \leq \theta \leq 360^\circ$ which is (i) transmitted by P_1 , and (ii) transmitted by P_2 . [5 marks]

Solution:

Concept:

- Plane polarised light has its electric-field vibrations confined to one fixed direction perpendicular to the direction of propagation. Unpolarised light has electric-field vibrations in all transverse directions.
- Snell's law for incidence from air on glass is $\mu = \sin i / \sin r$, where i and r are the angles with the normal. At Brewster incidence the reflected and refracted rays are perpendicular.
- An ideal polaroid transmits half the intensity of unpolarised incident light. Malus's law gives

$I = I_{\text{in}} \cos^2 \alpha$ for plane polarised incident light, where α is the angle between its electric-field direction and the transmission axis.

Step 1: (a) Define plane polarised light and use the ray geometry In plane polarised light, the electric field oscillates along a single fixed transverse direction; the light still propagates perpendicular to that direction. Let the incidence angle on the glass be $i = \theta$ and the refraction angle be r , both measured from the normal. By the law of reflection, the reflected ray makes angle θ with the normal above the interface. The angle between the reflected and refracted rays is $180^\circ - (\theta + r)$. Since this angle is 90° ,

$$\theta + r = 90^\circ, \quad r = 90^\circ - \theta.$$

The angles used in Snell's law are angles with the normal, not with the glass surface.

Step 2: (a) Apply Snell's law Taking the refractive index of air as unity, Snell's law gives

$$\begin{aligned} \sin \theta &= \mu \sin r, \\ \mu &= \frac{\sin \theta}{\sin(90^\circ - \theta)} = \frac{\sin \theta}{\cos \theta}. \end{aligned}$$

This special incidence angle is the polarising or Brewster angle. The reflected light is plane polarised under this condition.

Step 3: (b)(i) Find the intensity after the first polaroid The incident light is unpolarised. Averaging the squared projection of its electric field onto the fixed transmission axis of P_1 gives $\langle \cos^2 \alpha \rangle = 1/2$. Thus

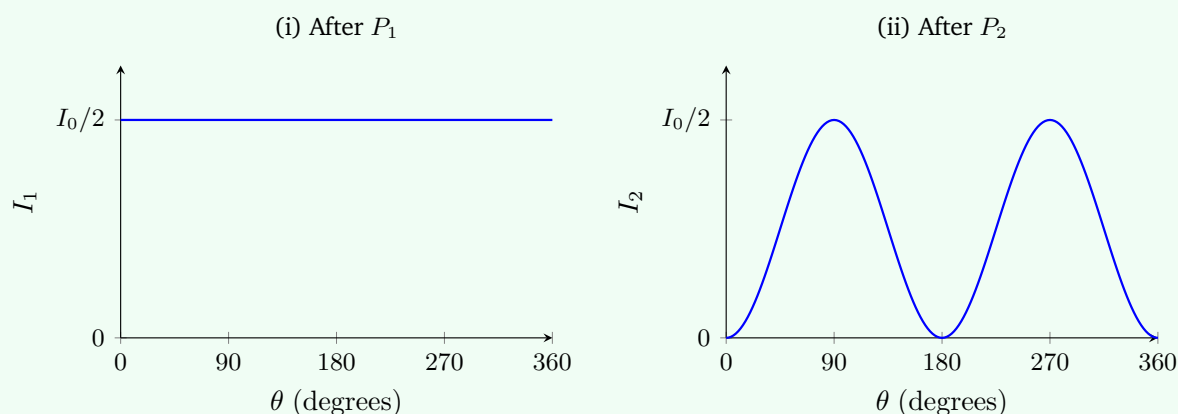
$$I_1 = \frac{I_0}{2}.$$

Rotating P_2 cannot change the light already transmitted by P_1 . Therefore the first graph is a horizontal straight line at $I_0/2$ throughout the specified interval. The light emerging from P_1 is plane polarised along its transmission axis.

Step 4: (b)(ii) Find the intensity after the rotating second polaroid Initially the two transmission axes are crossed, so their relative angle is 90° . After rotating P_2 through θ , the angle may be written as $\alpha = 90^\circ - \theta$; choosing the opposite rotation direction gives the same squared projection. Malus's law therefore gives

$$I_2 = I_1 \cos^2(90^\circ - \theta) = \frac{I_0}{2} \sin^2 \theta.$$

The graph starts at zero, rises smoothly to $I_0/2$ at 90° , returns to zero at 180° , reaches $I_0/2$ again at 270° , and returns to zero at 360° . Its period is 180° , because reversing a transmission axis does not change its physical direction. At the two maxima the axes are parallel; at the three zeros they are crossed.



Intensities transmitted by the fixed first polaroid and the rotating second polaroid.

Step 5: Conclusion $\mu = \tan \theta$ when the reflected and refracted rays are perpendicular. The required intensity graphs are $I_1 = I_0/2$ and $I_2 = (I_0/2) \sin^2 \theta$ for $0^\circ \leq \theta \leq 360^\circ$.

Quick Tip to Solve: The rotation is measured from the initially crossed position: check that the second graph starts at zero before choosing sine squared or cosine squared.

OR

35. (a) Briefly describe the Young's double slit experiment of interference of light. Derive the expression for fringe width in the pattern.

(b) Monochromatic light of wavelength 588 nm is incident from air to water interface. Find the wavelength and speed of the refracted light. The refractive index of water is $\frac{4}{3}$. [5 marks]

Solution:

Concept:

- Coherent sources maintain a constant phase difference and have the same frequency. Their overlapping waves produce sustained interference.
- For slit separation d , screen distance D and transverse coordinate y , the path difference is approximately $\Delta = dy/D$, when $D \gg d$ and $y \ll D$.
- On refraction frequency f remains unchanged; refractive index is $\mu = c/v$ and wavelength in a medium is $\lambda = \lambda_0/\mu$.

Step 1: Describe the experiment (a) A narrow slit illuminated with monochromatic light illuminates two closely spaced narrow slits S_1 and S_2 . Both receive light from the same source and therefore act as coherent secondary sources. Their light overlaps on a screen placed at distance D , much larger than their separation d . Alternate bright and dark parallel bands are observed: reinforcement produces bright fringes and cancellation produces dark fringes. At the central point the two paths are equal, so the central fringe is bright. The source slit and double slits are parallel, giving a fringe pattern parallel to the slits.

Step 2: Obtain the path difference For a point P on the screen at displacement y from the centre, let the observation angle be α . The geometrical path difference is $\Delta = S_2P - S_1P \simeq d \sin \alpha$. Under the

small-angle approximation, $\sin \alpha \simeq \tan \alpha = y/D$. Hence

$$\Delta \simeq \frac{dy}{D}.$$

The slits are in phase, so this path difference alone determines reinforcement or cancellation.

Step 3: Derive the fringe width Constructive interference occurs for $\Delta = n\lambda$, giving $y_n = n\lambda D/d$, where n is an integer. Thus the separation of consecutive bright fringes is

$$\beta = y_{n+1} - y_n = \frac{\lambda D}{d}.$$

Dark fringes occur for $\Delta = (n + \frac{1}{2})\lambda$, so their coordinates are $y_n^{\text{dark}} = (n + \frac{1}{2})\lambda D/d$. Consecutive dark fringes have the same separation β . The distance between an adjacent bright and dark fringe is $\beta/2$; fringe width refers to successive fringes of the same kind.

Step 4: Calculate wavelength and speed in water (b) With $\lambda_0 = 588 \text{ nm}$ and $\mu = 4/3$, the frequency is unchanged at the interface. Taking the speed of light in air as $c = 3.00 \times 10^8 \text{ m s}^{-1}$,

$$\lambda_{\text{water}} = \frac{588}{4/3} \text{ nm} = 441 \text{ nm},$$

$$v_{\text{water}} = \frac{3.00 \times 10^8}{4/3} \text{ m s}^{-1} = 2.25 \times 10^8 \text{ m s}^{-1}.$$

Both wavelength and speed decrease by the same factor, preserving $v/\lambda = f$.

Step 5: Conclusion The fringe width is $\beta = \lambda D/d$. In water the wavelength is 441 nm and speed is $2.25 \times 10^8 \text{ m s}^{-1}$.

Quick Tip to Solve: Find adjacent bright-fringe coordinates before subtracting; on refraction divide both wavelength and speed by the refractive index.

36. (a) An electric dipole of dipole moment $\vec{P}$ is placed in a uniform electric field $\vec{E}$ at an angle θ with it. Derive the expression for torque ($\vec{\tau}$) acting on it. Find the orientation of the dipole relative to the electric field for which torque on it is (i) maximum, and (ii) half of maximum.

(b) Two point charges $q_1 = +1 \mu\text{C}$ and $q_2 = +4 \mu\text{C}$ are placed 2 m apart in air. At what distance from q_1 along the line joining the two charges, will the net electric field be zero? [5 marks]

Solution:

Concept:

- A dipole consists of charges $-q$ and $+q$ separated by $2a$. Its dipole moment $\vec{p}$ points from negative to positive charge, with magnitude $p = 2aq$.
- In a uniform field the net dipole force is zero but the forces form a couple: $\vec{\tau} = \vec{p} \times \vec{E}$.
- A point charge produces field magnitude $E = kq/r^2$, where $k = 1/(4\pi\epsilon_0)$. Fields add vectorially.

Step 1: Derive the torque on the dipole (a) In the uniform field, the positive charge experiences $+q\vec{E}$ and the negative charge experiences $-q\vec{E}$. These forces are equal and opposite, so the resultant force vanishes. The perpendicular separation of their lines of action is $2a \sin \theta$, where θ is the angle from $\vec{E}$

to the dipole axis. The torque magnitude of this couple is therefore

$$\tau = (qE)(2a \sin \theta) = pE \sin \theta.$$

To establish its direction, let the displacement from the negative charge to the positive charge be $\vec{\ell}$. Taking moments about the negative charge gives $\vec{\tau} = \vec{\ell} \times (q\vec{E}) = \vec{p} \times \vec{E}$. Since the net force is zero, the couple has the same torque about any origin. Its sense tends to align the dipole moment with the field.

Step 2: Find the requested orientations The magnitude is maximum when $\sin \theta = 1$. Thus $\theta = 90^\circ$, meaning the dipole axis is perpendicular to the field, and $\tau_{\max} = pE$. For half the maximum magnitude,

$$pE \sin \theta = \frac{pE}{2} \Rightarrow \sin \theta = \frac{1}{2}.$$

In the physical range $0^\circ \leq \theta \leq 180^\circ$, both $\theta = 30^\circ$ and $\theta = 150^\circ$ satisfy this condition. The second orientation must also be included because torque magnitude depends on the sine of the angle. Parallel and antiparallel orientations have zero torque.

Step 3: Locate the possible zero-field region (b) Both charges are positive, so each field points away from its source charge. Outside the segment joining them, the fields point in the same direction and cannot cancel. Between them they point in opposite directions. Let the zero-field point be at distance x from $q_1 = +1 \mu\text{C}$ towards $q_2 = +4 \mu\text{C}$. Its distances from the charges are x and $2 - x$ metres, with $0 < x < 2$.

Step 4: Equate the field magnitudes For cancellation,

$$\frac{k(1 \times 10^{-6})}{x^2} = \frac{k(4 \times 10^{-6})}{(2 - x)^2}.$$

Cancelling common factors and taking positive square roots of the positive distances gives

$$\frac{1}{x} = \frac{2}{2 - x} \Rightarrow 2 - x = 2x \Rightarrow x = \frac{2}{3} \text{ m}.$$

The other distance is $4/3$ m. The stronger charge is twice as far away, so its fourfold charge is exactly offset by the fourfold squared distance. This confirms equal field magnitudes with opposite directions.

Step 5: Conclusion $\vec{\tau} = \vec{p} \times \vec{E}$, with magnitude $pE \sin \theta$. Torque is maximum at 90° and half maximum at 30° or 150° . The net field vanishes $\boxed{2/3 \text{ m}}$ from q_1 , between the charges towards q_2 .

Quick Tip to Solve: For two like charges, search between them and take positive square roots: the distances are in the ratio of square roots of the charges.

36. (a) Derive an expression for the energy stored in a parallel plate capacitor of capacitance C when charged up to voltage V . How is this energy stored in the capacitor?

(b) A capacitor of capacitance $1\ \mu\text{F}$ is charged by connecting a battery of negligible internal resistance and emf 10 V across it. Calculate the amount of charge supplied by the battery in charging the capacitor fully.

[5 marks]

Solution:**Concept:**

- The capacitance is $C = q/v$, where q is the instantaneous plate charge and v is the potential difference. For a fixed parallel plate geometry, C is constant during charging.
- The work needed to transfer an additional charge dq between plates is $dW = v dq$. This work becomes electrostatic potential energy in the electric field between the plates.

Step 1: (a) Work done in charging the capacitor Initially the capacitor is uncharged, so its potential difference is zero. As charge accumulates, the opposing potential difference increases; it is therefore incorrect to use the final voltage for every increment of charge. When the plate charges are $+q$ and $-q$, the potential difference is $v = q/C$. To transfer a small further charge dq from one plate to the other, the external agent must do work against this existing potential difference:

$$dW = v dq = \frac{q}{C} dq.$$

Integrating from the initially uncharged state to the final charge Q ,

$$U = \int_0^Q \frac{q}{C} dq = \frac{Q^2}{2C}.$$

Since $Q = CV$, this gives the equivalent expressions

$$U = \frac{1}{2} CV^2 = \frac{1}{2} QV.$$

The factor $1/2$ reflects the increase in voltage from zero to V ; the average potential difference during this ideal charging process is $V/2$.

Step 2: (a) Identify where the energy is stored The energy is stored in the electric field in the region between the capacitor plates. For a parallel plate capacitor in vacuum or approximately in air, neglecting fringing, let plate area be A and separation be d . Then $C = \epsilon_0 A/d$ and $V = Ed$, so

$$U = \frac{1}{2} \frac{\epsilon_0 A}{d} (Ed)^2 = \frac{1}{2} \epsilon_0 E^2 (Ad).$$

The quantity Ad is the field-filled volume. Hence the electrostatic energy per unit volume is $u = \frac{1}{2} \epsilon_0 E^2$. The stored energy can be released when the plates discharge through an external circuit.

Step 3: (b) Calculate the charge supplied by the battery On full charging, the capacitor voltage equals the battery emf because no steady charging current flows. The given capacitance and final voltage are $C = 1 \times 10^{-6}\text{ F}$ and $V = 10\text{ V}$. Therefore the charge transferred through the battery is

$$Q = CV = (1 \times 10^{-6})(10) = 1 \times 10^{-5}\text{ C} = 10\ \mu\text{C}.$$

The battery transfers this amount of charge from one plate to the other, producing equal and opposite

plate charges. Their absolute values must not be added to obtain the charge supplied through the battery.

Step 4: Conclusion (a) The stored energy is $U = \frac{1}{2}CV^2 = Q^2/(2C)$, stored in the electric field between the plates. (b) The charge supplied by the battery is $10\ \mu\text{C}$.

Quick Tip to Solve: Integrate the changing capacitor voltage during charging; for the battery's transferred charge use $Q = CV$, not twice the plate charge.

37. (a) Derive the expression for the force acting per unit length between two long straight parallel current carrying conductors. Hence define one ampere.

(b) Two long parallel straight conductors are placed 12 cm apart in air. They carry equal currents of 3 A each. Find the magnitude and direction of the magnetic field at a point midway between them (drawing a figure) when the currents in them flow in opposite directions. [5 marks]

Solution:

Concept:

- A long straight conductor carrying current I produces magnetic field $B = \mu_0 I / (2\pi r)$ at perpendicular distance r in vacuum or approximately in air. Its direction follows the right-hand thumb rule.
- A straight current-carrying length L in a magnetic field experiences force $\vec{F} = I\vec{L} \times \vec{B}$. Fields from different conductors add vectorially.

Step 1: (a) Derive the force per unit length Consider two infinitely long straight parallel conductors separated by distance d , carrying currents I_1 and I_2 . The magnetic field produced by conductor 1 at conductor 2 is

$$B_1 = \frac{\mu_0 I_1}{2\pi d}.$$

This field is perpendicular to the current in conductor 2. A length L of conductor 2 therefore experiences force of magnitude $F_{21} = I_2 L B_1$, giving

$$\frac{F_{21}}{L} = \frac{\mu_0 I_1 I_2}{2\pi d}.$$

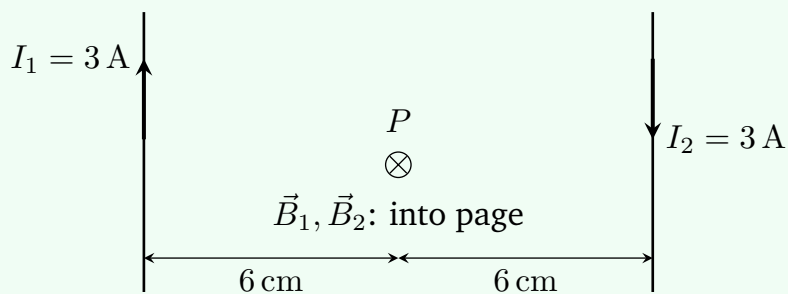
Similarly, the field of conductor 2 exerts an equal force per unit length on conductor 1, in the opposite direction. The right-hand rule for $\vec{L} \times \vec{B}$ shows that currents in the same direction attract, while currents in opposite directions repel. The formula gives the magnitude in either case.

Step 2: (a) Define one ampere using parallel conductors In the force-based definition expected for this question, one ampere is the constant current which, when maintained in each of two straight parallel conductors of infinite length and negligible circular cross-section placed one metre apart in vacuum, produces a force of 2×10^{-7} N per metre of length between them. Indeed, putting $I_1 = I_2 = 1$ A, $d = 1$ m, and $\mu_0 = 4\pi \times 10^{-7}$ N A⁻² into the derived formula gives

$$\frac{F}{L} = \frac{4\pi \times 10^{-7}}{2\pi} = 2 \times 10^{-7} \text{ N m}^{-1}.$$

Step 3: (b) Determine the field directions at the midpoint Choose the left conductor's current upwards and the right conductor's current downwards, as shown. The midpoint P lies to the right of the left wire and to the left of the right wire. By the right-hand thumb rule, the left wire produces a field into

the plane of the page at P ; the right wire also produces a field into the plane there. Thus the two fields add. Reversing both chosen current directions would reverse the net field, while keeping its magnitude unchanged.



Opposite currents give reinforcing fields into the page at midpoint P .

Step 4: (b) Calculate the resultant magnitude The distance of the midpoint from each conductor is half the separation:

$$r = \frac{12}{2} \text{ cm} = 0.06 \text{ m}.$$

With $I = 3 \text{ A}$, each conductor produces

$$B_1 = B_2 = \frac{(4\pi \times 10^{-7})(3)}{2\pi(0.06)} = 1.0 \times 10^{-5} \text{ T}.$$

Since these vectors point in the same direction at the midpoint,

$$B_{\text{net}} = B_1 + B_2 = 2.0 \times 10^{-5} \text{ T}.$$

Although the currents are opposite, their fields between the conductors reinforce; they would cancel at the midpoint for equal currents in the same direction.

Step 5: Conclusion (a) $F/L = \mu_0 I_1 I_2 / (2\pi d)$; one ampere gives $2 \times 10^{-7} \text{ N m}^{-1}$ between the specified conductors one metre apart in vacuum. (b) $B_{\text{net}} = 2.0 \times 10^{-5} \text{ T}$, into the page for the current directions drawn, and out of the page if both currents reverse.

Quick Tip to Solve: Apply the right-hand rule separately at the midpoint: opposite currents reinforce their fields between the wires, then use half the separation for each distance.

37. (a) Draw the schematic sketch of a cyclotron. Explain the shape of the path on which charged particle moves when the particle is accelerated by it.

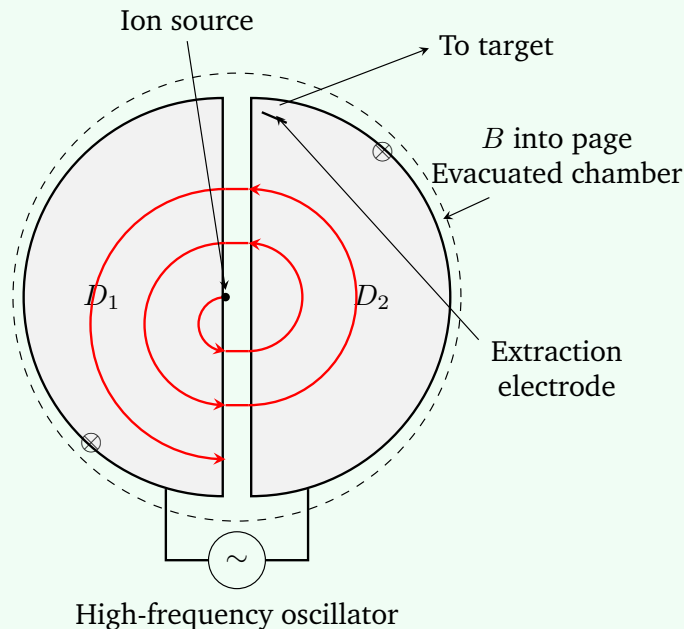
(b) To convert a given galvanometer into a voltmeter of ranges $2V$, V and $\frac{V}{2}$ volt, resistances R_1 , R_2 and R_3 ohm respectively, are required to be connected in series with the galvanometer. Obtain the relationship between R_1 , R_2 and R_3 . [5 marks]

Solution:

Concept:

- A cyclotron accelerates a charged particle using the alternating electric field across the gap between two hollow D-shaped electrodes (dees). A uniform magnetic field perpendicular to the dee plane bends its path.
- For a nonrelativistic particle of mass m , charge magnitude $|q|$ and speed v , $|q|vB = mv^2/r$, so $r = mv/(|q|B)$ and the revolution frequency is $f = |q|B/(2\pi m)$.
- A galvanometer of resistance G and full-scale current I_g , connected to a series resistance R , has voltmeter range $U = I_g(G + R)$.

Step 1: (a) Schematic arrangement of the cyclotron The two hollow metal dees D_1 and D_2 lie inside an evacuated chamber between the poles of an electromagnet. The magnetic field is perpendicular to their plane. A high-frequency alternating voltage is applied between the dees; an ion source near the centre supplies the charged particles. The diagram is a top view, with crosses indicating magnetic field into the page. An extraction electrode near the outer edge guides the accelerated particles towards a target.



Cyclotron schematic (top view); successive semicircles grow as the particle is accelerated across the gap.

Step 2: (a) Explain the successive semicircles Inside a metal dee, the electric field is zero, so the magnetic force alone acts on the particle. It is perpendicular to velocity and does no work; hence the speed stays constant during each semicircle. It supplies the centripetal force:

$$|q|vB = \frac{mv^2}{r}, \quad r = \frac{mv}{|q|B}.$$

On reaching the gap, the particle is accelerated by the electric field and gains kinetic energy. The voltage reverses during the next half revolution, so the electric field again accelerates it at its next gap crossing. Since its speed increases while m , $|q|$, and B remain constant, each subsequent semicircle has a larger radius. Thus the overall path is an outward spiral made of successive semicircular arcs, with acceleration occurring only in the gap.

Step 3: (a) Synchronisation with the alternating voltage The time spent in one dee is

$$t_{1/2} = \frac{\pi r}{v} = \frac{\pi m}{|q|B}.$$

This time is independent of speed in the nonrelativistic approximation. Therefore a fixed alternating voltage of frequency $f = |q|B/(2\pi m)$ reverses its polarity at each arrival at the gap, maintaining acceleration. The growing orbit finally reaches the extraction region. At relativistic speeds this simple synchronisation no longer holds.

Step 4: (b) Write the three full-scale voltage equations The same galvanometer is used for all ranges, so its G and I_g are unchanged. At full-scale deflection the entire voltage appears across the series combination. Hence

$$2V = I_g(G + R_1),$$

$$V = I_g(G + R_2),$$

$$V/2 = I_g(G + R_3).$$

The first equation minus the second gives $V = I_g(R_1 - R_2)$. The second minus the third gives $V/2 = I_g(R_2 - R_3)$. Taking their ratio eliminates both the unknown galvanometer resistance and its full-scale current.

Step 5: (b) Eliminate the galvanometer parameters Dividing the two differences yields

$$\frac{R_1 - R_2}{R_2 - R_3} = 2.$$

Therefore $R_1 - R_2 = 2R_2 - 2R_3$. The larger range requires the larger series resistance, consistent with $R_1 > R_2 > R_3$. It is the total resistances $G + R_i$, rather than the added resistances alone, that are proportional to the respective voltage ranges.

Step 6: Conclusion (a) A cyclotron produces an outward spiral of semicircular arcs of increasing radius, with energy gained at each gap crossing. (b) The required relationship is $R_1 + 2R_3 = 3R_2$.

Quick Tip to Solve: Subtract adjacent voltmeter-range equations first: this cancels the galvanometer resistance and leaves a simple ratio of resistance differences.