

CBSE Class 12 Physics Question Paper 2020 with Solution PDF-Set 55/4/1



Question Paper with Solutions

Conducted by the Central Board of Secondary Education (CBSE)

Time Allowed: 3 hours

Maximum Marks: 70

General Instructions

- (i) This question paper comprises **four** sections – A, B, C and D.
- (ii) There are **37** questions in the question paper. **All** questions are compulsory.
- (iii) Section A : Q. no. **1** to **20** are very short-answer type questions carrying **1** mark each.
- (iv) Section B : Q. no. **21** to **27** are short-answer type questions carrying **2** marks each.
- (v) Section C : Q. no. **28** to **34** are long-answer type questions carrying **3** marks each.
- (vi) Section D : Q. no. **35** to **37** are also long answer type questions carrying **5** marks each.
- (vii) There is no overall choice in the question paper. However, an internal choice has been provided in **two** questions of **one** mark, **two** questions of **two** marks, **one** question of **three** marks and all the **three** questions **five** marks. You have to attempt only one of the choices in such questions.
- (viii) However, separate instructions are given with each section and question, wherever necessary.
- (ix) Use of calculators and log tables is not permitted.

You may use the following values of physical constants wherever necessary :

$$c = 3 \times 10^8 \text{ m/s}$$

$$h = 6.63 \times 10^{-34} \text{ Js}$$

$$e = 1.6 \times 10^{-19} \text{ C}$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ T m A}^{-1}$$

$$\epsilon_0 = 8.854 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$$

$$\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$$

$$\text{Mass of electron } (m_e) = 9.1 \times 10^{-31} \text{ kg}$$

$$\text{Mass of neutron} = 1.675 \times 10^{-27} \text{ kg}$$

$$\text{Mass of proton} = 1.673 \times 10^{-27} \text{ kg}$$

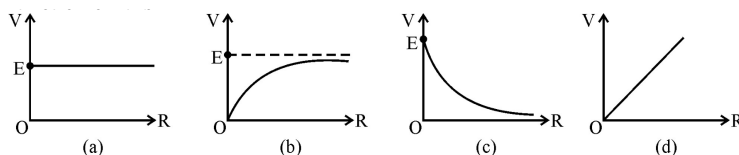
$$\text{Avogadro's number} = 6.023 \times 10^{23} \text{ per gram mole}$$

$$\text{Boltzmann constant} = 1.38 \times 10^{-23} \text{ JK}^{-1}$$

Section A (20 marks)

Select the most appropriate option from those given below each question.

1. A cell of emf (E) and internal resistance r is connected across a variable external resistance R . The graph of terminal potential difference V as a function of R is — [1 mark]



(A) Graph (a): $V = E$, independent of R .

(B) Graph (b): V rises from zero and approaches E asymptotically.

(C) Graph (c): V decreases from E towards zero.

(D) Graph (d): V increases linearly from the origin.

Correct Answer: (B) Graph (b): V rises from zero and approaches E asymptotically.

Solution:

Concept:

- The terminal potential difference is the voltage across the external resistance: $V = IR$. Here I is the circuit current and R is the external resistance.
- For a cell of emf E and internal resistance r , $I = E/(R + r)$; hence $V = ER/(R + r)$.

Step 1: Find the limiting voltages For $R = 0$, the terminals are short-circuited and $V = 0$. As R becomes very large, I tends to zero, so the internal voltage drop Ir vanishes and V approaches E .

Step 2: Identify the curve For finite positive R , $V < E$, and $dV/dR = Er/(R + r)^2 > 0$. The slope decreases as R increases, giving a rising curve that flattens towards E . Graph (a) ignores internal resistance, graph (c) decreases, and graph (d) rises without a limiting value.

Step 3: Conclusion The required curve is graph (b). Hence, the correct option is (B).

Quick Tip to Solve: Check the two endpoints first: a short circuit gives zero terminal voltage, while an open circuit gives the emf.

2. A uniform wire of resistance $2R$ is bent in the form of a circle. The effective resistance between the ends of any diameter of the circle is:

[1 mark]

(A) $2R$

(B) R

(C) $\frac{R}{2}$

(D) $\frac{R}{4}$

Correct Answer: (C) $\frac{R}{2}$

Solution:

Concept:

- For a uniform wire, resistance is proportional to its length. Each semicircle therefore has half of the total resistance $2R$.
- Two resistances between the same pair of terminals are in parallel: $1/R_{\text{eq}} = 1/R_1 + 1/R_2$.

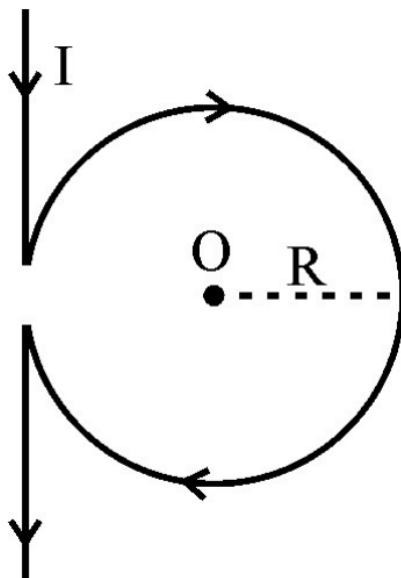
Step 1: Identify the two paths The endpoints of a diameter divide the circular wire into equal semicircles. Each semicircle has resistance R . Current can travel from one terminal to the other through either semicircle, so these paths are in parallel, not in series.

Step 2: Combine the resistances and check the options Thus $R_{\text{eq}} = R \times R / (R + R) = R/2$. Option (A) is the total wire resistance before allowing parallel paths; option (B) counts only one semicircle. Option (D) would require each branch to have resistance $R/2$, which is inconsistent with the given total $2R$.

Step 3: Conclusion The effective resistance is $R/2$. Hence, the correct option is (C).

Quick Tip to Solve: Between diametrically opposite terminals, halve the wire resistance for each branch and then halve it again for their parallel combination.

3. A current I flows through a long straight conductor which is bent into a circular loop of radius R in the middle as shown in the figure. The magnitude of the net magnetic field at point O will be [1 mark]



- (A) Zero
- (B) $\frac{\mu_0 I}{2R}(1 + \pi)$
- (C) $\frac{\mu_0 I}{4\pi R}$
- (D) $\frac{\mu_0 I}{2R}\left(1 - \frac{1}{\pi}\right)$

Correct Answer: (D) $\frac{\mu_0 I}{2R}\left(1 - \frac{1}{\pi}\right)$

Solution:

Concept:

- At the centre of a circular loop of radius R , the magnetic field is $B_{\text{loop}} = \mu_0 I / (2R)$.
- The two long straight portions together give $B_{\text{straight}} = \mu_0 I / (2\pi R)$. Field directions follow the right-hand rule.

Step 1: Determine the field directions The arrows show clockwise current around the circular loop, so its magnetic field at O points into the page. The straight portions carry current downwards, with O to their right. Their field points out of the page. The two contributions therefore oppose each other.

Step 2: Subtract the field magnitudes The straight portions are each effectively semi-infinite at perpendicular distance R , giving $\mu_0 I / (4\pi R)$ each. Therefore

$$B_{\text{net}} = \frac{\mu_0 I}{2R} - \frac{\mu_0 I}{2\pi R} = \frac{\mu_0 I}{2R} \left(1 - \frac{1}{\pi}\right).$$

The fields cannot cancel because their magnitudes differ. Printed option (B), containing $1 + \pi$, is inconsistent with these fields, while option (C) includes only one straight contribution.

Step 3: Conclusion The magnitude is $\frac{\mu_0 I}{2R} \left(1 - \frac{1}{\pi}\right)$. Hence, the correct option is (D).

Quick Tip to Solve: Use the arrows to decide whether loop and straight-wire fields add or subtract before substituting their magnitudes.

4. A circular loop of radius r , carrying a current I lies in y - z plane with its centre at the origin. The net magnetic flux through the loop is: [1 mark]

- (A) directly proportional to r
- (B) zero
- (C) inversely proportional to r
- (D) directly proportional to I

Correct Answer: (D) directly proportional to I

Solution:

Concept:

- The magnetic field produced by a current-carrying loop is proportional to its current for a fixed geometry: $\vec{B} \propto I$.
- Magnetic flux through the area bounded by a loop is $\Phi = \int \vec{B} \cdot d\vec{A}$; the self-flux is proportional to I .

Step 1: Identify the flux-producing field The loop lies in the y - z plane, so its area normal is along the x -axis. Its own magnetic field has a normal component through the bounded area; at the centre this field is along the x -axis. The flux through this open surface is therefore not forced to vanish.

Step 2: Determine the dependence and check the options For the given loop geometry, increasing I increases every field contribution in the same proportion. Integrating the normal component over the same area gives $\Phi = LI$, where L is the self-inductance for the fixed loop. Gauss's law gives zero flux through a closed surface, not through the open area of this loop. Neither a direct nor an inverse dependence on radius alone describes the stated general result.

Step 3: Conclusion The net magnetic flux through the loop is directly proportional to I . Hence, the correct option is (D).

Quick Tip to Solve: For self-flux through a fixed current loop, use $\Phi = LI$; reserve the zero-net-flux rule for closed surfaces.

5. The kinetic energy of a proton and that of an α -particle are 4 eV and 1 eV, respectively. The ratio of the de-Broglie wavelengths associated with them, will be [1 mark]

- (A) 2 : 1
- (B) 1 : 1
- (C) 1 : 2
- (D) 4 : 1

Correct Answer: (B) 1 : 1

Solution:

Concept:

- The de Broglie wavelength is $\lambda = h/p = h/\sqrt{2mK}$ for a non-relativistic particle of mass m and kinetic energy K .
- An alpha particle has approximately four times the proton mass: $m_\alpha = 4m_p$.

Step 1: Form the wavelength ratio Using $K_p = 4$ eV and $K_\alpha = 1$ eV,

$$\frac{\lambda_p}{\lambda_\alpha} = \sqrt{\frac{m_\alpha K_\alpha}{m_p K_p}}.$$

The common factor h , the factor 2 and the conversion from electron-volts to joules all cancel.

Step 2: Substitute and compare the options The product $m_p K_p$ equals $4m_p$ eV, while $m_\alpha K_\alpha$ also equals $4m_p$ eV. Hence the momenta, and therefore wavelengths, are equal. Ratios 2 : 1 or 1 : 2 result from neglecting one of the mass or energy factors; 4 : 1 neglects the square root as well.

Step 3: Conclusion The ratio is $\lambda_p : \lambda_\alpha = 1 : 1$. Hence, the correct option is (B).

Quick Tip to Solve: Compare the products mK , not energies alone: equal products mean equal de Broglie wavelengths.

6. A photocell connected in an electrical circuit is placed at a distance d from a source of light. As a result, current I flows in the circuit. What will be the current in the circuit when the distance is reduced to $d/2$? [1 mark]

- (A) I
- (B) $2I$
- (C) $4I$
- (D) $I/2$

Correct Answer: (C) $4I$

Solution:

Concept:

- For a point source, the light intensity varies inversely as the square of the distance: $\mathcal{I} \propto 1/d^2$.
- At fixed light frequency and the same collection conditions, photocurrent is proportional to incident intensity, since intensity controls the number of emitted photoelectrons per second.

Step 1: Calculate the change in intensity When the distance changes from d to $d/2$,

$$\frac{I_{\text{new}}}{I_{\text{old}}} = \frac{d^2}{(d/2)^2} = 4.$$

The frequency and energy of individual photons do not change; the photon arrival rate increases.

Step 2: Relate intensity to current and check the options The number of photoelectrons collected per unit time increases by the same factor, so $I_{\text{new}} = 4I$. Option (A) would require unchanged illumination. Option (B) incorrectly uses an inverse-distance dependence rather than an inverse-square dependence. Option (D), $I/2$, predicts a decrease despite moving the photocell closer.

Step 3: Conclusion The new current is $4I$. Hence, the correct option is (C).

Quick Tip to Solve: Halving the source distance multiplies intensity and the corresponding photocurrent by four, under unchanged collection conditions.

7. A current of 10 A is flowing from east to west in a long straight wire kept on a horizontal table. The magnetic field developed at a distance of 10 cm due north on the table is: [1 mark]

- (A) 2×10^{-5} T, acting downwards
- (B) 2×10^{-5} T, acting upwards
- (C) 4×10^{-5} T, acting downwards
- (D) 4×10^{-5} T, acting upwards

Correct Answer: (A) 2×10^{-5} T, acting downwards

Solution:

Concept:

- At perpendicular distance r from a long straight wire, $B = \mu_0 I / (2\pi r)$, where $\mu_0 = 4\pi \times 10^{-7} \text{ T m A}^{-1}$.
- The magnetic field direction follows the right-hand rule, with the thumb pointing along the conventional current.

Step 1: Calculate the magnetic field Convert the distance: $r = 10 \text{ cm} = 0.10 \text{ m}$. With $I = 10 \text{ A}$,

$$B = \frac{4\pi \times 10^{-7} \times 10}{2\pi \times 0.10} = 2 \times 10^{-5} \text{ T}.$$

Options (C) and (D) have twice this correct magnitude.

Step 2: Determine the vertical direction Take east as $+x$, north as $+y$, and upward as $+z$. The current is westward, along $-x$, and the observation point is northward, along $+y$. The cross product $(-\hat{x}) \times \hat{y} = -\hat{z}$ gives a downward field. Thus option (B) has the correct magnitude but the wrong direction.

Step 3: Conclusion The field is $2 \times 10^{-5} \text{ T}$, acting downwards. Hence, the correct option is (A).

Quick Tip to Solve: After calculating the magnitude, use west cross north to get downward; always convert centimetres to metres first.

8. When a wave undergoes reflection at an interface from rarer to denser medium, adhoc change in its phase is: [1 mark]

- (A) $\frac{\pi}{2}$
- (B) 0
- (C) π
- (D) $\frac{\pi}{4}$

Correct Answer: (C) π

Solution:

Concept:

- Reflection at a boundary from an optically rarer medium to an optically denser medium reverses the sign of the reflected wave displacement.
- A sign reversal corresponds to a phase change of π rad, because $\sin(\phi + \pi) = -\sin \phi$.

Step 1: Apply the boundary rule The incident wave reaches a rarer-to-denser interface. The reflected wave undergoes phase reversal, so a crest in the incident disturbance corresponds to a trough in the reflected disturbance at the boundary. This is a change of half a cycle, not a change of frequency.

Step 2: Compare the printed choices A complete cycle has phase 2π , so half a cycle gives $\Delta\phi = \pi$. Option (B), zero phase change, belongs to reflection at a denser-to-rarer boundary in the usual optical convention. Options (A) and (D) represent quarter-cycle and eighth-cycle changes and do not describe the phase reversal here.

Step 3: Conclusion The phase change is π rad. Hence, the correct option is (C).

Quick Tip to Solve: For rarer-to-denser reflection, recall crest becomes trough: half a cycle means pi radians.

9. Paschen series of atomic spectrum of hydrogen gas lies in: [1 mark]

- (A) Infrared region
- (B) Ultraviolet region
- (C) Visible region
- (D) Partly in ultraviolet and partly in visible region

Correct Answer: (A) Infrared region

Solution:

Concept:

- The Paschen series of hydrogen consists of transitions from higher levels $n = 4, 5, \dots$ to the final level $n_f = 3$.
- The Rydberg relation is $1/\lambda = R_H(1/n_f^2 - 1/n_i^2)$, where R_H is the hydrogen Rydberg constant and $n_i > n_f$.

Step 1: Identify the final energy level All Paschen lines terminate at the third energy level. These transitions release less energy than transitions to the first or second level, so their wavelengths are longer than the visible Balmer wavelengths.

Step 2: Locate the wavelength range At the series limit, $n_i \rightarrow \infty$, giving $\lambda_{\min} = 9/R_H \approx 820$ nm. The first line, $4 \rightarrow 3$, is approximately 1875 nm. Thus even the shortest Paschen wavelength is beyond the red end of the visible spectrum. All lines are infrared; ultraviolet, visible and mixed ultraviolet-visible

choices are excluded.

Step 3: Conclusion The Paschen series lies in the infrared region. Hence, the correct printed option is (A).

Quick Tip to Solve: Match hydrogen series to its final level: Paschen ends at $n = 3$ and is infrared.

10. In the α -particle scattering experiment, the shape of the trajectory of the scattered α -particles depend upon: [1 mark]

- (A) only on impact parameter.
- (B) only on the source of α -particles.
- (C) both impact parameter and source of α -particles.
- (D) impact parameter and the screen material of the detector.

Correct Answer: (A) only on impact parameter.

Solution:

Concept:

- The impact parameter b is the perpendicular distance between the initial direction of an incident alpha particle and the centre of the target nucleus.
- In the standard scattering experiment, the alpha-particle beam energy and target are fixed; varying b changes the Coulomb deflection and trajectory.

Step 1: Relate approach distance to deflection An alpha particle and the nucleus are both positively charged, so their interaction is repulsive. For a small impact parameter the particle approaches the nucleus closely and is strongly deflected. For a large impact parameter it remains farther away and suffers a smaller deflection. A head-on approach can lead to backward scattering.

Step 2: Compare the alternatives in the stated experiment The printed question concerns different trajectories in the same experimental arrangement, with the incident beam and target fixed. Their differing shapes are determined by their impact parameters. The source supplies the particles and the detector screen records their arrival; neither is the geometrical variable determining these different paths. Thus the source-only, source-plus-impact-parameter, and screen-material alternatives do not give the intended dependence.

Step 3: Conclusion For the stated scattering experiment, the trajectory depends only on the impact parameter. Hence, the correct printed option is (A).

Quick Tip to Solve: For particles in the same Rutherford beam, closer initial aim at the nucleus means greater deflection.

11. Fill in the blank with appropriate answer.

Torque acting on an electric dipole placed in an electric field is maximum when the angle between the electric field and the dipole moment is _____. [1 mark]

Solution:

Concept:

- For an electric dipole in a uniform electric field, the torque is $\tau = \mathbf{p} \times \mathbf{E}$.

- Its magnitude is $\tau = pE \sin \theta$, where p is the dipole moment, E the field magnitude and θ the angle between them.

Step 1: Use the angular dependence For fixed dipole moment and field strength, only the factor $\sin \theta$ changes the torque. Its largest possible value is one. Hence the magnitude of the torque is greatest when the dipole moment is perpendicular to the electric field.

Step 2: Determine the required angle Within the angle range from 0 to π between two vectors, $\sin \theta = 1$ at $\theta = \pi/2 = 90^\circ$. In contrast, at parallel or antiparallel orientations, $\sin \theta = 0$ and the torque vanishes. The maximum magnitude is therefore pE .

Step 3: Conclusion The blank is 90° ($\pi/2$ rad).

Quick Tip to Solve: Use $\sin(\theta)$, not $\cos(\theta)$, for dipole torque; it peaks for perpendicular vectors.

12. Fill in the blank with appropriate answer.

A proton released from rest in an electric field, will start moving towards a region of _____ potential in the field. [1 mark]

Solution:

Concept:

- A proton has positive charge $q = +e$, so the electric force on it is $\mathbf{F} = e\mathbf{E}$.
- The electric field points towards decreasing potential: $\mathbf{E} = -\nabla V$. The electric potential energy is $U = qV$.

Step 1: Determine the initial motion The proton is released from rest, so its initial acceleration follows the electric force. Since its charge is positive, it accelerates along the electric field, rather than against it. Thus its initial displacement is in the direction in which the field points.

Step 2: Connect the field direction to potential Along the electric field the electric potential decreases. Equivalently, the field does positive work on the moving proton: $\Delta K = -\Delta U = -e\Delta V$. Starting from zero kinetic energy, the proton gains kinetic energy, so $\Delta V < 0$. Therefore the region it starts moving towards has lower potential.

Step 3: Conclusion The blank is lower potential.

Quick Tip to Solve: A positive charge released from rest moves along the electric field and initially towards lower potential.

13. Fill in the blank with appropriate answer.

To minimize the percentage error in the determination of unknown resistance of a conductor in meter bridge experiment, the balance point is adjusted near _____ of the wire. [1 mark]

Solution:

Concept:

- At balance in a metre bridge, $X/R = l/(L - l)$, where X is the unknown resistance, R the comparison resistance and $L = 100$ cm the wire length.
- The fractional uncertainty caused by a small balance-length uncertainty δl is approximately

$$\delta X/X = [1/l + 1/(L - l)]\delta l, \text{ with } R \text{ fixed.}$$

Step 1: Examine sensitivity to balance length The resistance ratio depends on both wire segments. If the balance point lies close to an end, one segment is short, so the same absolute error in locating the balance point produces a relatively large error in that segment and hence in the calculated resistance.

Step 2: Minimise the relative uncertainty The length-dependent uncertainty factor is $L/[l(L - l)]$. It is smallest when the product $l(L - l)$ is greatest. For a fixed total length this product reaches its maximum at $l = L/2 = 50 \text{ cm}$. Adjusting the known resistance so the two segments are nearly equal therefore minimises the percentage error.

Step 3: Conclusion The balance point should be near the middle (centre) of the wire, at about 50 cm.

Quick Tip to Solve: Choose the comparison resistance close to the unknown so the metre-bridge balance lies near 50 cm.

OR

13. In potentiometer, a long uniform wire is used to _____ potential gradient along the wire.

[1 mark]

Solution:

Concept:

- The potential gradient of a potentiometer is $k = V/L$, where V is the potential drop across its wire and L is its length.
- A uniform wire carrying steady current has a constant potential drop per unit length; a smaller gradient makes the instrument more sensitive.

Step 1: Relate wire length to gradient For a given potential difference across the potentiometer wire, spreading that drop over a larger length reduces the drop per unit length: $k = V/L$. Thus using a long wire gives a small potential gradient. Uniformity ensures that this gradient is the same at different positions along the wire.

Step 2: Explain the measurement advantage An unknown emf ε is balanced by a length l such that $\varepsilon = kl$. With smaller k , a small change in emf produces a larger change in balancing length, making it easier to detect and reducing the relative effect of a small length-reading error. In the printed blank, the action of increasing wire length is therefore to reduce the gradient.

Step 3: Conclusion The blank is reduce (decrease) the potential gradient.

Quick Tip to Solve: For a fixed voltage across the wire, longer length means smaller V/L and greater potentiometer sensitivity.

14. Fill in the blank with appropriate answer.

Unpolarised light of intensity I_0 is incident on two crossed polaroids. The intensity of light transmitted by the combination will be _____. [1 mark]

Solution:

Concept:

- An ideal polaroid transmits half the incident intensity of unpolarised light: $I_1 = I_0/2$.
- Malus's law is $I_2 = I_1 \cos^2 \theta$, where θ is the angle between the transmission axes.

Step 1: Transmission through the first polaroid The incident light has electric-field vibrations in all transverse directions. The first polaroid selects the component along its transmission axis, producing plane-polarised light with intensity $I_1 = I_0/2$.

Step 2: Apply the crossed-axis condition Crossed polaroids have transmission axes perpendicular to one another, so $\theta = 90^\circ$. Therefore

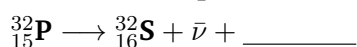
$$I_2 = \frac{I_0}{2} \cos^2 90^\circ = 0.$$

The second polaroid rejects the vibration direction selected by the first.

Step 3: Conclusion The intensity transmitted by the combination is $\boxed{0}$.

Quick Tip to Solve: For crossed ideal polaroids, insert 90° in Malus's law; the first halving does not change the zero result.

15. Name the particle emitted spontaneously in the following nuclear reaction:



[1 mark]

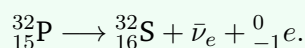
Solution:

Concept:

- In a nuclear reaction, both nucleon number A and total charge number Z are conserved.
- Beta-minus emission produces an electron ${}_{-1}^0e$ and an electron antineutrino $\bar{\nu}_e$; the antineutrino carries no electric charge.

Step 1: Balance the printed reaction The parent ${}_{15}^{32}\text{P}$ and daughter ${}_{16}^{32}\text{S}$ have the same nucleon number, 32. Thus the missing particle has nucleon number zero. Charge conservation gives $15 = 16 + 0 + Z$, so its charge number is $Z = -1$.

Step 2: Identify the emitted particle The particle with nucleon number zero and charge $-e$ is an electron, called a β^- particle. The complete decay is



A neutron in the nucleus changes into a proton, accounting for the increase of atomic number without a change in mass number.

Step 3: Conclusion The missing spontaneously emitted particle is a $\boxed{\beta^- \text{ particle (electron)}}$.

Quick Tip to Solve: An unchanged mass number together with an atomic-number increase of one identifies beta-minus emission.

16. The work done in moving a charged particle between two points in a uniform electric field does not depend on the path followed by the particle. Why? [1 mark]

Solution:

Concept:

- An electrostatic field is conservative: its line integral around any closed path is zero.
- For a charge q in a uniform field $\vec{E}$, the field work is $W = q\vec{E} \cdot (\vec{r}_B - \vec{r}_A) = q(V_A - V_B)$.

Step 1: Write the work integral The electric force is $\vec{F} = q\vec{E}$, so work done by the field between positions A and B is $W = \int_A^B q\vec{E} \cdot d\vec{r}$. Since the electrostatic field is uniform, $\vec{E}$ is constant and can be taken outside the integral.

Step 2: Reduce the integral to the endpoints The integral of the displacement element is the net displacement, whatever curved or straight path is used. Hence

$$W = q\vec{E} \cdot (\vec{r}_B - \vec{r}_A) = q(V_A - V_B).$$

Only the initial and final positions, or their potential difference, enter this expression; intermediate positions do not.

Step 3: Conclusion The work is path independent because the electrostatic field is conservative and the work depends only on the potential difference between the two endpoints.

Quick Tip to Solve: For a uniform electrostatic field, replace the path integral by the dot product of the field and net displacement.

17. Define 'magnetic declination' at a place on earth. [1 mark]

Solution:

Concept:

- The geographic meridian is the vertical plane through the geographic north-south direction at the place.
- The magnetic meridian is the vertical plane containing the local magnetic-field direction, whose horizontal projection gives magnetic north.

Step 1: Identify the two reference directions A freely suspended magnetic compass aligns with the horizontal component of Earth's magnetic field. Its magnetic north-south direction need not coincide with geographic north-south. The corresponding vertical planes are the magnetic and geographic meridians.

Step 2: Specify which angle is declination The angle between these meridians is measured in the horizontal plane, equivalently between geographic north and magnetic north. It can be described as eastward or westward according to the direction of magnetic north. The vertical angle made by the magnetic field with the horizontal is a different quantity, called inclination or dip.

Step 3: Conclusion Magnetic declination at a place is the angle between the geographic meridian

and the magnetic meridian at that place.

Quick Tip to Solve: Declination compares the two north directions horizontally; dip compares the magnetic field with the horizontal vertically.

18. An A.C. source with variable frequency is connected to a parallel plate capacitor. How will the displacement current be affected with the decrease in frequency of the source? [1 mark]

Solution:

Concept:

- Displacement current between capacitor plates is $I_d = \epsilon_0 d\Phi_E/dt = C dV/dt$, with capacitance C fixed.
- For a sinusoidal voltage $V = V_0 \sin(\omega t)$, the current amplitude is $I_{d0} = \omega CV_0 = 2\pi f CV_0$.

Step 1: Express the varying electric field For fixed plate area A and separation d , the electric field is $E = V/d$ and the flux is $\Phi_E = AV/d$. Thus $I_d = (\epsilon_0 A/d) dV/dt = C dV/dt$. Faster voltage variation produces a larger displacement-current amplitude.

Step 2: Relate the current to frequency Keeping the source-voltage amplitude unchanged, differentiation gives

$$I_d = \omega CV_0 \cos(\omega t), \quad I_{d0} = 2\pi f CV_0.$$

Therefore the amplitude, and likewise the rms displacement current, is directly proportional to frequency. A decrease in frequency reduces the rate at which electric flux changes.

Step 3: Conclusion The displacement current decreases when the source frequency decreases, for the same applied voltage amplitude; its amplitude is proportional to f .

Quick Tip to Solve: Use $I_{d0} = 2\pi f CV_0$: with fixed voltage and capacitor geometry, current scales directly with frequency.

19. An astronomical telescope may be a refracting type or a reflecting type. Which of the two produces image of better quality? Justify your answer. [1 mark]

Solution:

Concept:

- A refracting telescope uses a lens objective, whose refractive index generally varies with wavelength.
- A reflecting telescope uses a mirror objective; reflection focuses different wavelengths without chromatic aberration.

Step 1: Compare the objectives In a refracting telescope, different colours undergo different amounts of refraction and have different focal positions. This chromatic aberration can create coloured fringes and reduce the sharpness of the image unless corrected by suitable compound lenses.

Step 2: Explain the advantage of reflection The reflecting telescope uses a concave mirror instead of an objective lens. Its focal length does not depend on the colour of the light in the way a refracting lens's does, so the objective does not produce chromatic aberration. A properly shaped mirror, such as a parabolic primary, can also reduce spherical aberration for distant axial objects.

Step 3: Conclusion A reflecting telescope produces an image of better quality because its mirror objective is free from chromatic aberration.

Quick Tip to Solve: For telescope image quality, identify the objective: a mirror avoids the colour-dependent refraction of a lens.

20. Can a slab of p-type semi-conductor be physically joined to another n-type semiconductor slab to form p-n junction? Justify your answer. [1 mark]

Solution:

Concept:

- A p-n junction is formed in a single continuous semiconductor crystal by creating neighbouring p-type and n-type regions through controlled doping.
- The depletion layer and built-in barrier arise when mobile carriers diffuse across a continuous junction and recombine, leaving immobile ionised dopants.

Step 1: Examine what happens when slabs are joined Merely pressing or joining two separate doped semiconductor slabs does not make a proper p-n junction. Their contacting surfaces contain defects, impurities and an interrupted crystal lattice. Consequently, the interface cannot be treated as the continuous semiconductor region required for normal junction behaviour.

Step 2: State how the junction is formed A practical junction is made by forming p-type and n-type regions within the same crystal, for example by suitable doping. Electrons diffuse from the n-side and holes from the p-side across the boundary and recombine. The resulting depletion region has a definite internal electric field and potential barrier, which are essential to diode action.

Step 3: Conclusion No. Physical joining of separate p-type and n-type slabs does not form a proper p-n junction because the lattice is discontinuous and the interface has defects and impurities; the junction must be formed within a continuous crystal.

Quick Tip to Solve: Think of a junction as a change of doping within one continuous crystal, rather than as a mechanical contact between two slabs.

OR

20. In a p-n junction diode the forward bias resistance is low as compared to the reverse bias resistance. Give reason. [1 mark]

Solution:

Concept:

- A p-n junction has a depletion region and a built-in potential barrier opposing diffusion of majority carriers.
- Bias changes the barrier height and hence the current through the junction; resistance measures

the opposition to current.

Step 1: Examine forward bias The applied voltage opposes the built-in barrier, reducing its height and narrowing the depletion layer. Majority carriers can cross the junction readily, producing a large forward current even for a modest applied voltage.

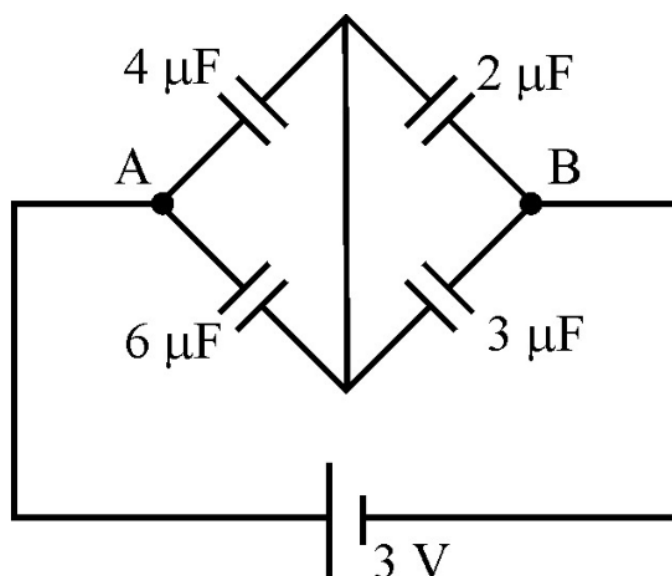
Step 2: Examine reverse bias The applied voltage reinforces the barrier and widens the depletion layer. Majority-carrier flow is suppressed, leaving only a very small minority-carrier current before breakdown. Thus the same order of applied voltage produces a much smaller reverse current.

Step 3: Conclusion Forward bias offers low resistance because it lowers the potential barrier and permits a large majority-carrier current. Reverse bias raises the barrier, so its tiny current corresponds to a much higher resistance, before breakdown.

Quick Tip to Solve: Link forward bias to a narrower depletion layer and reverse bias to a wider one; compare the resulting currents.

Section B (14 marks)

21. Find the total charge stored in the network of capacitors connected between A and B as shown in figure: [2 marks]



Solution:

Concept:

- Points connected by an ideal wire have the same potential; capacitors between the same two nodes are in parallel and their capacitances add.
- For two capacitors in series, $C_{eq} = C_1 C_2 / (C_1 + C_2)$. The charge supplied by a battery of voltage V is $Q = C_{eq} V$.

Step 1: Reduce the two parallel groups The vertical wire joins the top and bottom vertices of the diamond into a single node, say M . Therefore the $4 \mu\text{F}$ and $6 \mu\text{F}$ capacitors both lie between A and M , while the $2 \mu\text{F}$ and $3 \mu\text{F}$ capacitors both lie between M and B . Their parallel equivalents are

$$C_{AM} = 4 + 6 = 10 \mu\text{F}, \quad C_{MB} = 2 + 3 = 5 \mu\text{F}.$$

Step 2: Use the series equivalent and battery voltage These two equivalent groups are in series across the 3 V battery. Thus

$$C_{AB} = \frac{10 \times 5}{10 + 5} = \frac{10}{3} \mu\text{F}, \quad Q = C_{AB}V = \frac{10}{3} \times 3 \mu\text{C}.$$

The group voltages are $V_{AM} = Q/C_{AM} = 1 \text{ V}$ and $V_{MB} = Q/C_{MB} = 2 \text{ V}$, whose sum equals the battery voltage. The individual charge magnitudes are consequently 4, 6, 4, 6 μC , respectively. Adding charges of successive series groups would count the same transferred charge twice.

Step 3: Conclusion The equivalent capacitance is $\frac{10}{3} \mu\text{F}$. Thus the charge supplied by the 3 V battery to the network is $Q = 10 \mu\text{C}$; this is the magnitude of the charge at either external terminal.

Quick Tip to Solve: Merge vertices linked by the central wire before deciding which capacitors are in series or parallel.

22. A wire of length L_0 has a resistance R_0 . It is gradually stretched till its length becomes $2L_0$.

(a) Plot a graph showing variation of its resistance R with its length l during stretching.

(b) What will be its resistance when its length becomes $2L_0$?

[2 marks]

Solution:

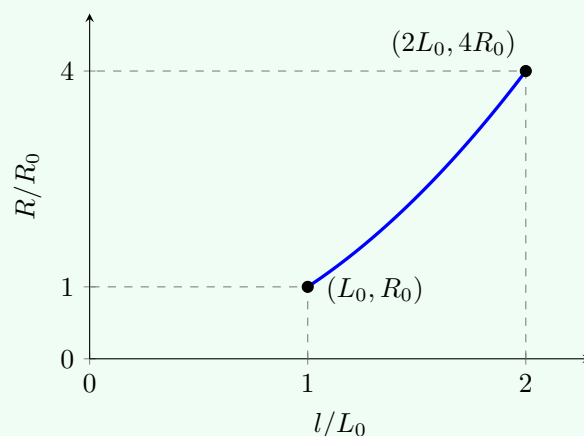
Concept:

- A uniform wire has resistance $R = \rho l/A$, where ρ is resistivity, l is length and A is cross-sectional area.
- During uniform stretching, assume constant volume and temperature, so $Al = A_0L_0$ and ρ remains constant.

Step 1: (a) Derive the resistance-length relation Let the original area be A_0 . Conservation of volume gives $A = A_0L_0/l$. Substituting this reduced area into the resistance formula,

$$R = \frac{\rho l^2}{A_0L_0}, \quad R_0 = \frac{\rho L_0}{A_0}, \quad \boxed{R = R_0 \left(\frac{l}{L_0} \right)^2}.$$

Hence R varies quadratically with length. The graph curves upwards: its slope increases as the wire is stretched. For the actual stretching process, the relevant segment begins at (L_0, R_0) and ends at $(2L_0, 4R_0)$; the axes below use normalized variables to show those values clearly.



Resistance during stretching, with normalized length and resistance axes.

Step 2: (b) Evaluate resistance at twice the original length Setting $l = 2L_0$ in the derived relation gives

$$R = R_0 \left(\frac{2L_0}{L_0} \right)^2 = 4R_0.$$

This also follows directly: doubling length halves the area because the volume is unchanged. Thus the factor of two from increased length and the factor of two from decreased area multiply. Treating the cross-sectional area as constant would give the wrong result.

Step 3: Conclusion (a) The R -versus- l graph is an upward-curving parabola, $R = R_0 l^2 / L_0^2$, over the stretching interval $L_0 \leq l \leq 2L_0$. (b) The final resistance is $4R_0$.

Quick Tip to Solve: For a stretched wire, use constant volume first: resistance scales as length squared, not merely as length.

23. A resistor R and an inductor L are connected in series to a source of voltage $V = V_0 \sin \omega t$. The voltage is found to lead current in phase by $\pi/4$. If the inductor is replaced by a capacitor C , the voltage lags behind current in phase by $\pi/4$. When L , C and R are connected in series with the same source, find the:

- average power dissipated and
- instantaneous current in the circuit.

[2 marks]

Solution:

Concept:

- For a series RL circuit, $\tan \phi = X_L/R$, where $X_L = \omega L$; for a series RC circuit the magnitude of the phase angle satisfies $\tan |\phi| = X_C/R$, where $X_C = 1/(\omega C)$.
- In a series RLC circuit, $Z = \sqrt{R^2 + (X_L - X_C)^2}$. The average power is $P_{av} = V_{rms} I_{rms} \cos \phi$, with $V_{rms} = V_0/\sqrt{2}$.

Step 1: Determine both reactances from the measured phase angles In the RL circuit, voltage leads current by $\pi/4$, so

$$\frac{X_L}{R} = \tan \frac{\pi}{4} = 1, \quad \omega L = R.$$

In the RC circuit, voltage lags current by the same magnitude, giving

$$\frac{X_C}{R} = \tan \frac{\pi}{4} = 1, \quad \frac{1}{\omega C} = R.$$

Thus the inductive and capacitive reactances are equal at the given source frequency. These phase measurements determine their magnitudes even though the phase signs differ.

Step 2: (i) Find average power in the combined circuit Connecting all three components in series gives $X_L - X_C = 0$, so $Z = R$ and $\phi = 0$. The circuit is at series resonance and its power factor is unity. Therefore

$$P_{av} = \frac{V_{rms}^2}{R} = \frac{(V_0/\sqrt{2})^2}{R} = \frac{V_0^2}{2R}.$$

Only the resistor dissipates average power; the ideal inductor and capacitor exchange stored energy with the source.

Step 3: (ii) Write the instantaneous current The current amplitude is $I_0 = V_0/Z = V_0/R$. Since the net reactance vanishes, current and source voltage are in phase. Using the same sine convention as the given voltage,

$$i(t) = I_0 \sin(\omega t - \phi) = \frac{V_0}{R} \sin \omega t.$$

No residual phase of $\pi/4$ remains after both reactive components are present.

Step 4: Conclusion At the given frequency the circuit is resonant: $P_{av} = V_0^2/(2R)$ and $i(t) = (V_0/R) \sin \omega t$.

Quick Tip to Solve: Equal opposite phase angles of $\pi/4$ imply each reactance equals R ; together they cancel, leaving a purely resistive impedance.

24. Light of same wavelength is incident on three photo-sensitive surfaces A , B and C . The following observations are recorded.

(i) From surface A , photo electrons are not emitted.

(ii) From surface B , photo electrons are just emitted.

(iii) From surface C , photo electrons with some kinetic energy are emitted.

Compare the threshold frequencies of the three surfaces and justify your answer.

[2 marks]

Solution:

Concept:

- The incident frequency is $\nu = c/\lambda$; equal wavelengths imply the same frequency and photon energy $h\nu$ for all three surfaces.
- The threshold frequency ν_0 is the minimum frequency for emission; the work function is $\phi = h\nu_0$, and Einstein's equation is $K_{max} = h(\nu - \nu_0)$.

Step 1: Compare surfaces A and B For surface A , no photoelectrons are emitted, so the photon energy is below the work function: $h\nu < h\nu_{0A}$, or $\nu < \nu_{0A}$. On B , electrons are just emitted, meaning the incident photons only supply the work function and the maximum kinetic energy is zero. Thus $h\nu = h\nu_{0B}$, or $\nu = \nu_{0B}$.

Step 2: Compare surface C with the same incident frequency On C , photoelectrons have positive kinetic energy. Therefore $h\nu - h\nu_{0C} > 0$, giving $\nu > \nu_{0C}$. The three observations compare each

threshold with the very same incident frequency; they are not observations made with different photon energies. A larger threshold frequency therefore means a more strongly bound electron, as also expressed by $\phi = h\nu_0$.

Step 3: Conclusion Combining the inequalities,

$$\nu_{0A} > \nu_{0B} > \nu_{0C}, \quad \nu_{0B} = \nu.$$

Surface A has the highest threshold, B is exactly at threshold, and C has the lowest threshold.

Quick Tip to Solve: Use the common incident frequency as a reference: no emission lies above it, just emission equals it, and positive kinetic energy lies below it.

OR

24. If the frequency of light incident on the cathode of a photo-cell is increased, how will the following be affected? Justify your answer.

(i) Energy of the photo electrons.

(ii) Photo current.

[2 marks]

Solution:

Concept:

- Einstein's equation gives the maximum photoelectron kinetic energy: $K_{\max} = h\nu - \phi$, where h is Planck's constant and ϕ is the fixed cathode work function.
- Photocurrent measures the rate of collected electrons, $i = e\dot{N}_e$; the usual comparison assumes the illumination intensity and collecting conditions are unchanged.

Step 1: Effect on energy For light already above the threshold frequency $\nu_0 = \phi/h$, increasing ν raises the energy of each photon. The work function of the cathode remains unchanged, so the extra photon energy becomes extra electron kinetic energy:

$$\Delta K_{\max} = h \Delta \nu > 0.$$

Thus the maximum kinetic energy, and hence the stopping potential $V_s = K_{\max}/e$, increases.

Step 2: Effect on photocurrent In the standard photoelectric-effect comparison, the photocurrent is regarded as unchanged when only frequency changes and the illumination intensity and collection conditions are held fixed. Increasing photon energy increases electron energy; it does not by itself establish a higher emission rate. The current depends on the number of electrons emitted and collected per second. More precisely, for an ideal detector of fixed quantum efficiency η at fixed optical power P , the photon arrival rate is $P/(h\nu)$, giving $i = e\eta P/(h\nu)$. Thus an exact quantitative change requires specifying photon flux, efficiency and collection conditions; the expected qualitative answer uses the usual textbook convention.

Step 3: Conclusion (i) The maximum energy of the photoelectrons increases. (ii) Under the standard

textbook assumption of unchanged illumination intensity and collection, photocurrent is taken as unchanged; it is governed by emission rate, not electron energy.

Quick Tip to Solve: Separate energy per electron from electrons per second: frequency directly changes the former through Einstein's equation.

25. Briefly explain how a potential barrier is set up across a p-n junction as a result of diffusion and drift of the charge carriers. [2 marks]

Solution:

Concept:

- Diffusion transports majority carriers down their concentration gradients: electrons move from the n -side to the p -side, while holes move from the p -side to the n -side.
- Fixed ionised donors and acceptors in the depletion layer produce an electric field; drift is carrier motion caused by that field.

Step 1: Diffusion creates the depletion layer Immediately after the junction is formed, electrons diffuse across it from the electron-rich n -region, and holes diffuse oppositely from the hole-rich p -region. They recombine near the junction. Loss of mobile electrons leaves positively charged donor ions on the n -side; loss of holes leaves negatively charged acceptor ions on the p -side. These ions are fixed in the crystal, forming a space-charge region depleted of mobile majority carriers.

Step 2: Drift opposes further diffusion The uncovered ions produce an electric field directed from the positive n -side to the negative p -side. This field opposes further majority-carrier diffusion and drives electrons toward the n -side and holes toward the p -side, establishing a drift current opposite to the diffusion current. The associated potential difference increases as more charge is uncovered. At thermal equilibrium, drift and diffusion currents are equal in magnitude and opposite in direction, so the net junction current is zero.

Step 3: Conclusion The fixed space charge creates a built-in potential barrier across the depletion layer. It opposes majority-carrier diffusion, and reaches equilibrium when the opposing drift and diffusion currents balance.

Quick Tip to Solve: Remember the exposed ion signs: positive donors on n , negative acceptors on p ; the electric field therefore points from n to p .

26. (a) Why is a photo diode operated under reverse bias condition?

(b) Draw V - I characteristic curves of photo diode for incident light of intensities I_1 and I_2 ($I_1 > I_2$).

[2 marks]

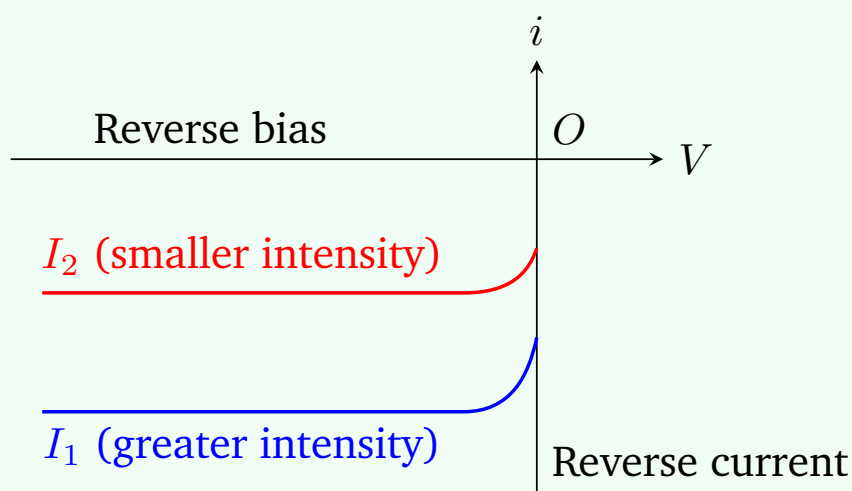
Solution:

Concept:

- A photodiode converts incident light into electron-hole pairs; its reverse photocurrent measures the generation rate of these carriers.
- Reverse bias increases the depletion-region width and junction electric field, sweeping electrons and holes to opposite sides before they recombine.

Step 1: Explain reverse-bias operation (a) The reverse electric field rapidly separates light-generated electron-hole pairs in the depletion region and collects them as reverse current. The reverse dark current is small, so the increase caused by illumination is readily measurable. Reverse bias also widens the active depletion region and gives a fast response. In forward bias, the large current due to majority-carrier injection would obscure the comparatively small light-induced change.

Step 2: Draw and interpret the characteristics (b) The official question specifies $I_1 > I_2$. Use signed diode voltage and current: reverse voltage and reverse current are both negative, so the curves lie in the third quadrant. Away from the small-voltage transition and below breakdown, the reverse current is almost independent of reverse voltage. Greater light intensity generates more electron-hole pairs per second and therefore a larger reverse-current magnitude. Accordingly, the curve for I_1 lies farther below the voltage axis than the curve for I_2 ; the current should not be drawn less negative for the brighter illumination.



Reverse-bias characteristics: the larger intensity $I_1 > I_2$ gives the larger magnitude of reverse photocurrent.

Step 3: Conclusion (a) Reverse bias gives efficient collection of light-generated carriers against a small dark-current background. (b) The two characteristic curves show $|i_{\text{rev},1}| > |i_{\text{rev},2}|$ because the printed intensities satisfy $I_1 > I_2$.

Quick Tip to Solve: On signed axes, brighter illumination means a curve farther below the voltage axis; compare magnitudes rather than signed negative currents.

26. (a) State the level of doping and biasing condition used in light emitting diode (LED).

(b) Write any two advantages of LED over the conventional low power lamps.

[2 marks]

Solution:

Concept:

- A light emitting diode is a heavily doped p-n junction that emits light when electrons and holes recombine near the junction.
- Forward bias injects majority carriers across the junction. In a suitable semiconductor, their recombination releases energy as photons.

Step 1: (a) Specify doping and bias An LED is made from a heavily doped p-n junction. It is operated under forward bias: the p side is connected to the positive terminal and the n side to the negative terminal of the supply. Forward bias reduces the junction barrier and allows electrons and holes to enter the junction region, where radiative recombination produces light. Heavy doping helps obtain a sufficiently large carrier population and useful light output.

Step 2: (b) Give two advantages First, LEDs consume less electrical power for useful illumination than conventional low power lamps, so they are more energy efficient. Second, LEDs have a much longer operational life, reducing the frequency of replacement. These are distinct advantages concerning power consumption and durability.

Step 3: Conclusion An LED uses a heavily doped p-n junction under forward bias. Two advantages are low power consumption and long operational life.

Quick Tip to Solve: Remember that a light emitting diode uses forward bias to inject carriers for light emission.

27. (a) Explain the formation of energy bands in crystalline solids.

(b) Draw the energy band diagrams of (i) a metal and (ii) a semiconductor.

[2 marks]

Solution:

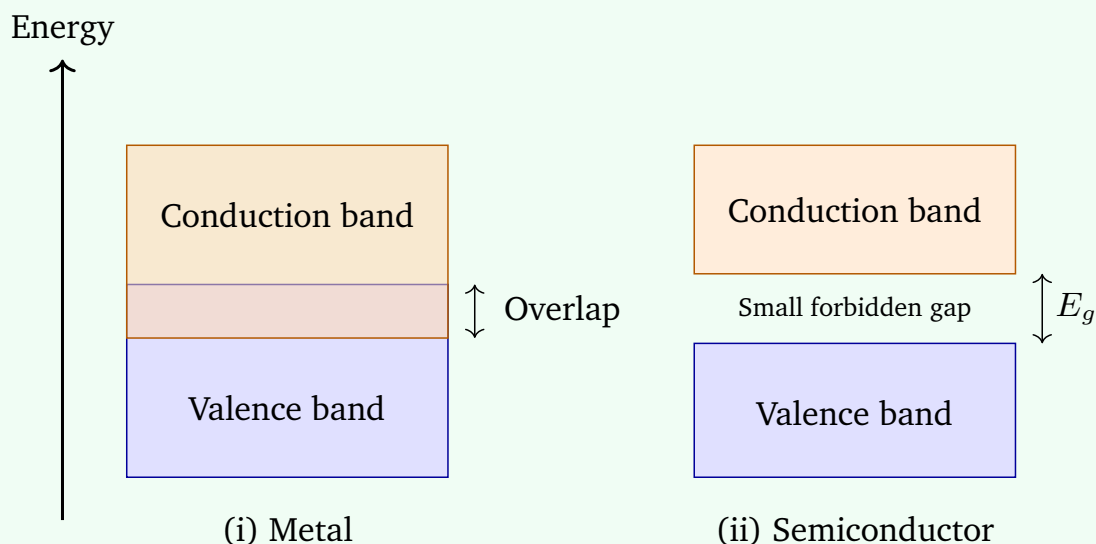
Concept:

- An isolated atom has discrete electron energy levels. Interactions between neighbouring atoms split these levels when atoms form a crystal.
- A large number of closely spaced allowed levels form an energy band; a forbidden energy gap contains no allowed electron states.

Step 1: (a) Explain how bands form When many atoms are brought close together in a crystalline solid, the outer electron wavefunctions overlap and the electrons interact with neighbouring atoms. Each atomic energy level splits into a large number of slightly different levels. Since a crystal contains an enormous number of atoms, these levels lie extremely close together and form nearly continuous allowed energy bands. The intervals between allowed bands are forbidden gaps. The highest occupied band associated with valence electrons is the valence band; the next higher band is the conduction

band.

Step 2: (b) Compare a metal with a semiconductor In a metal, the valence and conduction bands may overlap, as shown, so electrons have available nearby states and can conduct readily. A partially filled allowed band is another metallic arrangement. In a semiconductor the valence and conduction bands are separated by a small forbidden gap, typically of order 1 eV. At 0 K, the valence band of an intrinsic semiconductor is full and the conduction band empty; at higher temperatures some electrons can cross this small gap.



Schematic energy bands: overlapping bands in a metal and a small forbidden gap in a semiconductor.

Step 3: Conclusion Closely spaced split atomic levels form energy bands. A metal has an overlapping or partially filled band, whereas a semiconductor has a small gap between valence and conduction bands.

Quick Tip to Solve: Draw the semiconductor's small forbidden gap clearly; show overlap for the metallic example.

Section C (21 marks)

28. A hollow conducting sphere of inner radius r_1 and outer radius r_2 has a charge Q on its surface. A point charge $-q$ is also placed at the centre of the sphere.

- (a) What is the surface charge density on the (i) inner and (ii) outer surface of the sphere?
- (b) Use Gauss' law of electrostatics to obtain the expression for the electric field at a point lying outside the sphere.

[3 marks]

Solution:

Concept:

- In electrostatic equilibrium the electric field inside the material of a conductor is zero. Gauss' law is $\oint \vec{E} \cdot d\vec{A} = Q_{\text{enc}}/\epsilon_0$.
- For a spherical surface of radius r , uniform surface charge density is $\sigma = Q_s/(4\pi r^2)$, where Q_s is its charge.
- The central location of the point charge preserves spherical symmetry; the shell's total charge

remains Q .

Step 1: (a)(i) Find the inner surface charge density Choose a spherical Gaussian surface lying in the conducting material, with radius between r_1 and r_2 . Since $\vec{E} = 0$ there, its electric flux and hence its enclosed charge vanish. If Q_{in} is the charge induced on the inner surface, then

$$-q + Q_{\text{in}} = 0, \quad Q_{\text{in}} = q.$$

Spherical symmetry makes this induced charge uniform. Thus

$$\sigma_{\text{in}} = \frac{q}{4\pi r_1^2}.$$

Step 2: (a)(ii) Find the outer surface charge density The shell retains its net charge Q . Charge conservation gives

$$Q_{\text{in}} + Q_{\text{out}} = Q, \quad Q_{\text{out}} = Q - q.$$

By spherical symmetry the outer charge is uniform, so

$$\sigma_{\text{out}} = \frac{Q - q}{4\pi r_2^2}.$$

Step 3: (b) Apply Gauss' law outside the sphere At a point a distance $r > r_2$ from the centre, choose a concentric spherical Gaussian surface of radius r . The field is radial and its signed radial component E_r is constant on this surface. The total enclosed charge is

$$Q_{\text{enc}} = -q + q + (Q - q) = Q - q.$$

Gauss' law gives

$$E_r(4\pi r^2) = \frac{Q - q}{\epsilon_0}, \quad \vec{E}(r) = \frac{Q - q}{4\pi\epsilon_0 r^2} \hat{r}.$$

The field points outward if $Q > q$, inward if $Q < q$, and vanishes if $Q = q$. Its magnitude is $|Q - q|/(4\pi\epsilon_0 r^2)$.

Step 4: Conclusion $\sigma_{\text{in}} = q/(4\pi r_1^2)$, $\sigma_{\text{out}} = (Q - q)/(4\pi r_2^2)$, and for $r > r_2$, $\vec{E} = (Q - q)\hat{r}/(4\pi\epsilon_0 r^2)$.

Quick Tip to Solve: First force the enclosed charge to zero inside the metal, then conserve the shell's own charge Q .

OR

- 28. (a)** An infinitely long thin straight wire has a uniform linear charge density λ . Obtain the expression for the electric field E at a point lying at a distance x from the wire, using Gauss' law.
- (b)** Show graphically the variation of this electric field E as a function of distance x from the wire.

[3 marks]

Solution:

Concept:

- Gauss' law states that $\oint \vec{E} \cdot d\vec{A} = Q_{\text{enc}}/\epsilon_0$, where Q_{enc} is the charge enclosed by the closed surface and ϵ_0 is the permittivity of free space.

- An infinite uniform line charge has cylindrical symmetry: the electric field is radial, independent of the axial coordinate, and has the same magnitude at every point a fixed distance from the wire.

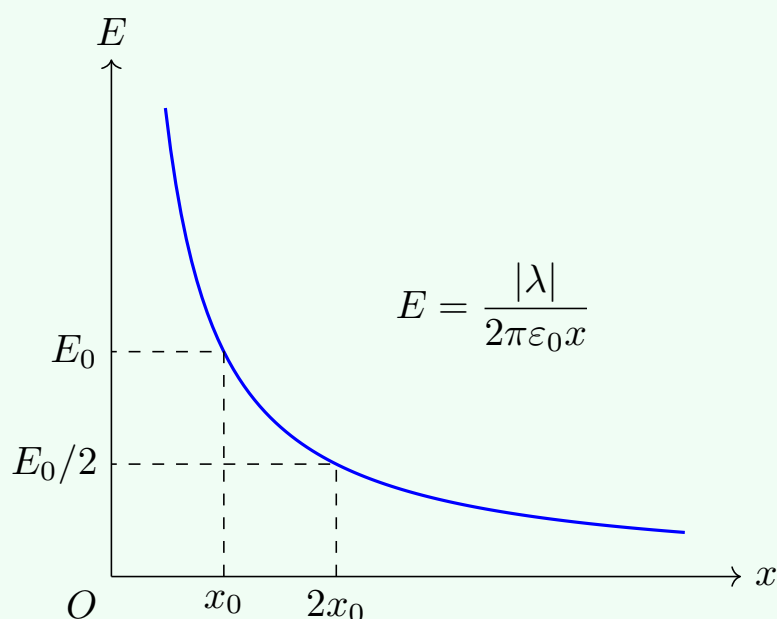
Step 1: (a) Choose a cylindrical Gaussian surface Take a closed cylinder of radius x and length l , coaxial with the wire. Its enclosed charge is $Q_{\text{enc}} = \lambda l$. For a positive line charge, $\vec{E}$ points radially outward and is parallel to the outward normal on the curved surface. Thus the curved-surface flux is $E(2\pi x l)$. On either flat end cap, the area vector is along the wire whereas the field is perpendicular to the wire, so $\vec{E} \cdot d\vec{A} = 0$. The caps therefore contribute no flux.

Step 2: Apply Gauss' law Adding the flux through all portions of the closed cylinder gives

$$E(2\pi x l) = \frac{\lambda l}{\epsilon_0}, \quad E = \frac{\lambda}{2\pi\epsilon_0 x}.$$

The length l cancels, as expected for an infinite wire. More generally, the magnitude is $|\lambda|/(2\pi\epsilon_0 x)$; the direction is outward for $\lambda > 0$ and inward for $\lambda < 0$.

Step 3: (b) Plot the dependence on distance For fixed positive λ , the product Ex is constant, so the E -versus- x curve is a decreasing rectangular hyperbola in the first quadrant. Doubling the distance halves the field. In the ideal infinitely thin-wire model, E grows without bound as $x \rightarrow 0^+$, and approaches zero as $x \rightarrow \infty$; neither axis is crossed. The graph below uses scaled axes, retaining this inverse-distance dependence.



Electric field magnitude decreases inversely with distance from an infinite line charge.

Step 4: Conclusion The field magnitude is $E = |\lambda|/(2\pi\epsilon_0 x)$, radial from the wire. Its magnitude varies as $1/x$, giving a decreasing rectangular hyperbola.

Quick Tip to Solve: Use a coaxial cylinder: only its curved surface contributes flux, and its arbitrary length cancels.

29. (a) Explain the principle of working of a potentiometer.

(b) In a potentiometer, a standard source of emf 5 V and negligible internal resistance maintains a steady current through the potentiometer wire of length 10 m. Two primary cells of emf E_1 and E_2 are joined together in a series with (i) same polarity and (ii) opposite polarity. The combination is connected to the potentiometer circuit in each case. The balancing length of the wire in the two cases are found to be 700 cm and 100 cm, respectively. Find the values of emf of the two cells. [3 marks]

Solution:

Concept:

- The potentiometer principle is that, for a uniform wire carrying a steady current, the potential difference across any length is directly proportional to that length: $V = kl$, where k is the potential gradient.
- At the null point the galvanometer current is zero, so no current is drawn from the cell being measured. Its emf therefore equals the potential drop along its balancing length, without an internal-resistance correction.

Step 1: (a) Explain the principle For wire of uniform cross-sectional area A and resistivity ρ , the resistance of length l is $R_l = \rho l/A$. With steady current I , its potential drop is $V_l = IR_l = (I\rho/A)l = kl$. Thus the potential gradient k is constant. To measure an unknown emf, balance it against the drop along a suitable length of the wire. At balance, the galvanometer detects no current in the test-cell branch, and $E = kl$. The driving source must provide a sufficiently large wire drop to obtain a balance.

Step 2: (b) Calculate the potential gradient The standard source maintains a potential difference of 5 V across the 10 m = 1000 cm potentiometer wire. Its negligible internal resistance permits use of this voltage for the wire drop. Hence

$$k = \frac{5}{1000} = 0.005 \text{ V cm}^{-1}.$$

Use centimetres for both given balancing lengths to keep units consistent.

Step 3: Form the aiding and opposing equations With the cells in series aiding, their emfs add. With their polarities opposed, the net emf is the magnitude of their difference. Therefore

$$\begin{aligned} E_1 + E_2 &= k(700) = 3.5 \text{ V}, \\ |E_1 - E_2| &= k(100) = 0.5 \text{ V}. \end{aligned}$$

Let E_1 denote the larger emf. Adding these equations gives $2E_1 = 4.0 \text{ V}$, so $E_1 = 2.0 \text{ V}$. Subtraction gives $2E_2 = 3.0 \text{ V}$, so $E_2 = 1.5 \text{ V}$. Interchanging the cell labels does not alter the measured lengths.

Step 4: Conclusion A uniform potentiometer wire carrying steady current obeys $V \propto l$. The two cell emfs are $\boxed{2.0 \text{ V and } 1.5 \text{ V}}$, with $E_1 = 2.0 \text{ V}$ if E_1 is chosen as the larger emf.

Quick Tip to Solve: Convert both balancing lengths to the same unit; then take half the sum and half the difference of the aiding and opposing voltages.

30. (a) Differentiate between self inductance and mutual inductance.

(b) The mutual inductance of two coaxial coils is 2 H. The current in one coil is changed uniformly from zero to 0.5 A in 100 ms. Find the:

(i) change in magnetic flux through the other coil.

(ii) emf induced in the other coil during the change.

[3 marks]

Solution:

Concept:

- Self inductance L relates a coil's own current to its flux linkage: $N\Phi = LI$. A changing current produces emf $\varepsilon = -L dI/dt$ in that same coil.
- Mutual inductance M relates current in one coil to flux linkage in another: $N_2\Phi_{21} = MI_1$. The emf in the second coil is $\varepsilon_2 = -M dI_1/dt$. Both L and M have SI unit henry.

Step 1: (a) Distinguish self and mutual inductance Self inductance concerns a single coil: a change in its own current changes its own magnetic flux linkage and induces an opposing emf in that coil. Its coefficient is $L = N\Phi/I$ for a linear magnetic system. Mutual inductance concerns a pair of magnetically coupled coils: a change in current in the first changes the flux linked with the second and induces an emf in the second. Its coefficient is $M = N_2\Phi_{21}/I_1$. Thus the source of the changing flux and the location of the induced emf differ in the two effects. The negative signs in the induced-emf formulae express Lenz's law.

Step 2: (b)(i) Calculate the change in linked magnetic flux Given $M = 2$ H and a current increase from zero to 0.5 A, we have $\Delta I_1 = 0.5$ A. The change in linked flux is

$$\Delta(N_2\Phi_{21}) = M\Delta I_1 = 2 \times 0.5 = 1.0 \text{ Wb}.$$

This is the total flux linkage of the other coil, which is the linked magnetic flux intended in the numerical question. If flux per turn is required, it is $\Delta\Phi_{21} = 1.0/N_2$ Wb; the number of turns N_2 has not been supplied. No numerical flux per turn can therefore be inferred independently.

Step 3: (b)(ii) Calculate the induced emf The change occurs uniformly in $\Delta t = 100$ ms = 0.100 s, so the current-change rate and the induced-emf magnitude are constant during the interval:

$$\begin{aligned}\frac{\Delta I_1}{\Delta t} &= \frac{0.5}{0.100} = 5 \text{ A s}^{-1}, \\ |\varepsilon_2| &= M \frac{\Delta I_1}{\Delta t} = 2 \times 5 = 10 \text{ V}.\end{aligned}$$

Equivalently, dividing the flux-linkage change 1.0 Wb by 0.100 s gives the same emf. Its polarity opposes the increasing linked flux; an absolute polarity requires the coil winding directions to be specified.

Step 4: Conclusion Self inductance produces emf in the same coil; mutual inductance produces emf in another coupled coil. The change in total linked flux is 1.0 Wb, and the induced-emf magnitude is 10 V.

Quick Tip to Solve: First calculate $M\Delta I$, then divide by the time in seconds; remember that inductance relates current to total flux linkage.

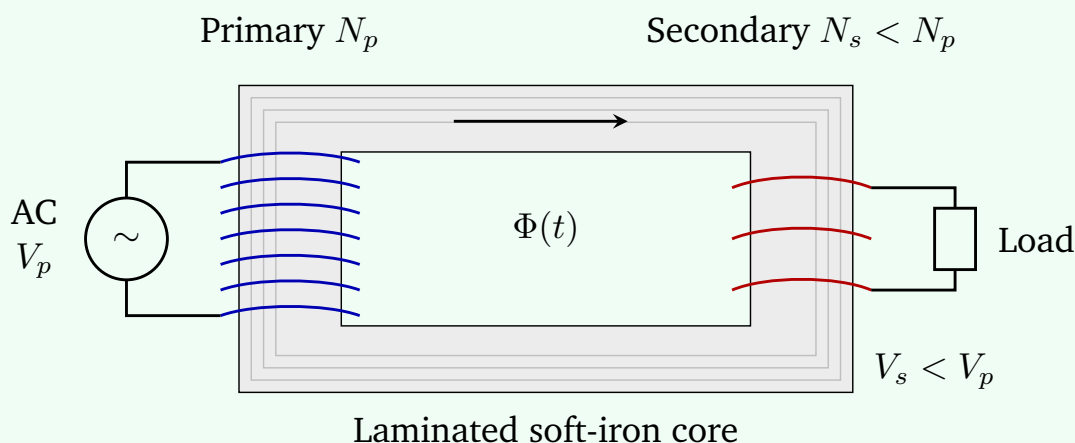
31. Explain with the help of a diagram, the working of a step-down transformer. Why is a laminated iron core used in a transformer? [3 marks]

Solution:

Concept:

- A transformer works by mutual induction: a time-varying magnetic flux in the primary winding links the secondary winding and induces an emf in it.
- For an ideal transformer, $V_s/V_p = N_s/N_p$, where N_p, N_s are the primary and secondary turns and V_p, V_s are their rms voltages. A step-down transformer has $N_s < N_p$.
- Eddy currents are circulating currents induced within a conducting core. Insulated laminations interrupt their paths and reduce the energy lost as heat.

Step 1: Arrangement and changing flux Two insulated windings are wound on a common laminated soft-iron core. The primary has more turns than the secondary and is connected to an alternating-voltage supply. Its alternating current produces an alternating magnetic flux Φ in the core. The core provides a path of low magnetic reluctance so that nearly the same flux links both windings. The secondary is connected to the load, as shown below.



Step-down transformer: the common laminated core links a primary with more turns to a secondary with fewer turns.

Step 2: Voltage transformation by mutual induction By Faraday's law, the induced emfs are $e_p = -N_p d\Phi/dt$ and $e_s = -N_s d\Phi/dt$. Dividing their magnitudes, and neglecting winding resistance and leakage flux, gives

$$\frac{V_s}{V_p} = \frac{N_s}{N_p} < 1.$$

Thus the secondary supplies a smaller alternating voltage at the same frequency. When a load is connected, current flows through it. For an ideal transformer, $V_p I_p = V_s I_s$, so a decrease in voltage is accompanied by an increase in current. A steady direct current cannot maintain the changing flux needed for continuous transformer action.

Step 3: Why the core is laminated The alternating core flux also induces eddy currents in the iron itself. A solid core would permit large circulating currents and substantial heating. Making the core from thin iron sheets separated by insulating layers breaks up the large current loops and increases

the resistance of their paths. This reduces eddy-current loss and improves efficiency. The soft-iron material also has a small hysteresis loss; lamination specifically addresses eddy-current heating.

Step 4: Conclusion A step-down transformer uses mutual induction and fewer secondary turns, giving $V_s < V_p$. Its laminated iron core reduces eddy currents and the associated heating loss.

Quick Tip to Solve: Decide the voltage change from the turns ratio first: fewer secondary turns mean a lower secondary voltage; associate laminations with eddy-current loss.

32. Name the electro-magnetic waves with their frequency range, produced in

(a) some radioactive decay

(b) sparks during electric welding

(c) TV remote

[3 marks]

Solution:

Concept:

- Electromagnetic waves are classified by their frequency ν or vacuum wavelength λ , related by $c = \nu\lambda$, where $c = 3.0 \times 10^8 \text{ m s}^{-1}$.
- In increasing frequency, the relevant order is infrared, visible, ultraviolet, X-rays and gamma rays. Photon energy is $E = h\nu$, where h is Planck's constant.
- Frequency ranges for spectral bands are approximate; transitions between neighbouring bands are conventional rather than sharp physical discontinuities.

Step 1: (a) Radiation from some radioactive decays An excited daughter nucleus can lose its excess energy by emitting a gamma-ray photon. Thus the electromagnetic radiation associated with such radioactive decays is **gamma radiation**, with a typical approximate frequency range

$$\nu \sim 10^{18} \text{ to } 10^{22} \text{ Hz.}$$

These photons have very high energy and very short wavelengths. Alpha and beta emissions, which can also occur in radioactive processes, are material particles and are not electromagnetic waves; the question asks for the electromagnetic emission.

Step 2: (b) Radiation from electric-welding sparks The intense electric arc and sparks in welding emit **ultraviolet radiation**, whose approximate frequency range is

$$\nu \sim 10^{15} \text{ to } 10^{17} \text{ Hz.}$$

Ultraviolet has a higher frequency and shorter wavelength than visible light. Welding also produces visible light and thermal radiation, but ultraviolet is the characteristic electromagnetic band sought for this source.

Step 3: (c) Radiation from a TV remote A conventional television remote uses an infrared light-emitting diode to transmit coded pulses to the receiver. The waves are **infrared radiation**, with the

broad approximate frequency range

$$\nu \sim 10^{11} \text{ to } 10^{14} \text{ Hz.}$$

Infrared lies below visible light in frequency. The remote uses a narrow part of the infrared band; the quoted interval describes the band, rather than a frequency sweep emitted by one remote.

Step 4: Conclusion (a) Gamma rays: approximately 10^{18} – 10^{22} Hz. (b) Ultraviolet rays: approximately 10^{15} – 10^{17} Hz. (c) Infrared rays: approximately 10^{11} – 10^{14} Hz. These are conventional broad, approximate ranges.

Quick Tip to Solve: Match the source first: nuclear transition means gamma, welding arc means ultraviolet, and a conventional TV remote means infrared; then order their frequencies.

33. Two coherent light waves of intensity $5 \times 10^{-2} \text{ W m}^{-2}$ each super-impose and produce the interference pattern on a screen. At a point where the path difference between the waves is $\lambda/6$, λ being the wavelength of the wave, find the

(a) phase difference between the waves.

(b) resultant intensity at the point.

(c) resultant intensity in terms of the intensity at the maximum.

[3 marks]

Solution:

Concept:

- A path difference Δx corresponds to phase difference $\phi = 2\pi\Delta x/\lambda$, where λ is the wavelength.
- For two coherent waves of intensities I_1 and I_2 , the resultant intensity is $I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \phi$.
- For equal individual intensities I_0 , $I = 4I_0 \cos^2(\phi/2)$ and the maximum intensity is $I_{\max} = 4I_0$.

Step 1: Identify the data and convert path difference to phase Each wave has intensity $I_0 = 5 \times 10^{-2} \text{ W m}^{-2}$, and the path difference at the specified point is $\Delta x = \lambda/6$. For coherent waves producing the stated interference pattern, the phase difference due to this path difference is

$$\phi = \frac{2\pi}{\lambda} \Delta x = \frac{2\pi}{\lambda} \frac{\lambda}{6} = \frac{\pi}{3} \text{ rad} = 60^\circ.$$

The wavelength cancels, so its numerical value is unnecessary. A full wavelength corresponds to 2π radians, making one-sixth of a wavelength correspond to one-sixth of a full phase cycle.

Step 2: Calculate the resultant intensity at this point Since the two incident intensities are equal, the

interference term is $2I_0 \cos \phi$. Substitution gives

$$\begin{aligned} I &= 2I_0(1 + \cos \phi) \\ &= 2(5 \times 10^{-2}) \left(1 + \cos \frac{\pi}{3}\right) \\ &= 0.10 \left(1 + \frac{1}{2}\right) \\ &= 0.15 \text{ W m}^{-2}. \end{aligned}$$

Adding the separate intensities alone would give 0.10 W m^{-2} , which misses the coherent interference term. Here that term is positive because $\cos 60^\circ > 0$.

Step 3: Express the result relative to the maximum intensity At a bright-fringe maximum the two equal waves are in phase, so

$$I_{\max} = (\sqrt{I_0} + \sqrt{I_0})^2 = 4I_0 = 0.20 \text{ W m}^{-2}.$$

Therefore,

$$\frac{I}{I_{\max}} = \frac{0.15}{0.20} = \frac{3}{4}.$$

Equivalently, the equal-intensity formula gives $I/I_{\max} = \cos^2(\phi/2) = \cos^2 30^\circ = 3/4$. This confirms that the point is brighter than the sum of the individual intensities but is below a full constructive-interference maximum.

Step 4: Conclusion (a) The phase difference is $\pi/3 \text{ rad} = 60^\circ$. (b) The resultant intensity is 0.15 W m^{-2} . (c) It is $I = \frac{3}{4} I_{\max}$, where $I_{\max} = 0.20 \text{ W m}^{-2}$.

Quick Tip to Solve: For equal incident intensities, use $I/I_{\max} = \cos^2(\pi \Delta x / \lambda)$ to obtain the fraction of maximum directly from the path difference.

34. Two objects P and Q when placed at different positions in front of a concave mirror of focal length 20 cm, form real images of equal size. Size of object P is three times size of object Q . If the distance of P is 50 cm from the mirror, find the distance of Q from the mirror. [3 marks]

Solution:

Concept:

- The mirror formula is $1/f = 1/v + 1/u$, where u , v and f are signed distances measured from the pole. With incident light travelling towards the mirror, a concave mirror and a real object in front of it have negative f and u .
- Linear magnification is $m = h_i/h_o = -v/u$. Real images formed by a concave mirror are inverted; equal image sizes mean equal magnitudes $|h_i|$, rather than equal object magnifications.

Step 1: Find the image distance and magnification for object P For object P , $u_P = -50 \text{ cm}$ and $f = -20 \text{ cm}$. Hence

$$\begin{aligned} \frac{1}{v_P} &= \frac{1}{f} - \frac{1}{u_P} = -\frac{1}{20} + \frac{1}{50} = -\frac{3}{100}, \\ v_P &= -\frac{100}{3} \text{ cm}, \quad m_P = -\frac{v_P}{u_P} = -\frac{2}{3}. \end{aligned}$$

The negative magnification confirms that the image is real and inverted; its size is $(2/3)h_P$.

Step 2: Use the equality of image sizes to find the magnification for Q The printed relation is $h_P = 3h_Q$. Therefore equal image sizes require

$$|m_P|h_P = |m_Q|h_Q \Rightarrow |m_Q| = \frac{2}{3} \times 3 = 2.$$

Since the image of Q is also real and inverted, $m_Q = -2$. Thus $-v_Q/u_Q = -2$, or $v_Q = 2u_Q$. The smaller object must have the larger magnification to produce the same image size.

Step 3: Substitute into the mirror formula and check the positions Substituting $v_Q = 2u_Q$ gives

$$-\frac{1}{20} = \frac{1}{2u_Q} + \frac{1}{u_Q} = \frac{3}{2u_Q}, \quad u_Q = -30 \text{ cm}, \quad v_Q = -60 \text{ cm}.$$

Object Q lies between the focus at 20 cm and the centre of curvature at 40 cm, so its real image is enlarged. Object P lies beyond the centre of curvature, so its real image is diminished. These positions agree with both magnifications.

Step 4: Conclusion Object Q is 30 cm in front of the concave mirror. Its signed object distance is $u_Q = -30 \text{ cm}$.

Quick Tip to Solve: For equal image sizes, first use $|m_P|h_P = |m_Q|h_Q$; equal image size does not imply equal magnification when the objects have different sizes.

Section D (15 marks)

35. (a) Show that a current carrying solenoid behaves like a small bar magnet. Obtain the expression for the magnetic field at an external point lying on its axis.

(b) A steady current of 2 A flows through a circular coil having 5 turns of radius 7 cm. The coil lies in X - Y plane with its centre at the origin. Find the magnitude and direction of the magnetic dipole moment of the coil. [5 marks]

Solution:

Concept:

- A circular loop of radius a , carrying current I , produces the axial field $B = \mu_0 I a^2 / [2(a^2 + x^2)^{3/2}]$ at distance x from its centre. Fields of the turns of a solenoid add along its axis.
- The magnetic dipole moment of a coil is $\mathbf{m} = NIA\hat{n}$, where N is its number of turns, A its area and $\hat{n}$ the unit normal given by the right-hand rule. Far from a small magnetic dipole, the axial field magnitude is $\mu_0(2m)/(4\pi r^3)$.

Step 1: (a) Identify the poles and magnetic behaviour of the solenoid A solenoid is a closely wound helical coil. The fields of successive turns reinforce one another inside the coil. Outside it, the field lines emerge from one end and enter the other, as for a bar magnet. Viewed from an end, anticlockwise current makes that end a north pole, while clockwise current makes it a south pole. A freely suspended current-carrying solenoid aligns with an external magnetic field and experiences a torque, just as a bar magnet does. Its dipole moment points from its south end to its north end along its axis. The following axial-field calculation establishes the dipole equivalence quantitatively.

Step 2: (a) Sum the fields of the turns at an external axial point Consider a thin, uniformly wound solenoid of radius a , length $2l$, and n turns per unit length. Take its centre as the origin and its magnetic moment along the positive axis. Let the external observation point be at $r > l$ on this axis.

A slice of thickness dx , at coordinate x , contains $n dx$ turns, and its separation from the observation point is $r - x$. Thus

$$dB = \frac{\mu_0 n I a^2 dx}{2[a^2 + (r - x)^2]^{3/2}}.$$

Integration over all the turns gives

$$B = \frac{\mu_0 n I}{2} \int_{-l}^l \frac{a^2 dx}{[a^2 + (r - x)^2]^{3/2}}.$$

Using $t = r - x$ and the antiderivative $\int a^2(a^2 + t^2)^{-3/2} dt = t/\sqrt{a^2 + t^2}$, we obtain the external axial field:

$$B = \frac{\mu_0 n I}{2} \left[\frac{r + l}{\sqrt{(r + l)^2 + a^2}} - \frac{r - l}{\sqrt{(r - l)^2 + a^2}} \right].$$

On the north-side axis its direction is away from the north end, parallel to the solenoid's magnetic moment.

Step 3: (a) Obtain the small-bar-magnet form of the field For a distant point, $r \gg l, a$, every turn is approximately at distance r , and the loop expression reduces to $\mu_0 I a^2 / (2r^3)$ per turn. Since the total number of turns is $N = 2ln$,

$$B \simeq \frac{\mu_0 N I a^2}{2r^3} = \frac{\mu_0}{4\pi} \frac{2m}{r^3}, \quad m = N I \pi a^2.$$

This is precisely the axial-field expression for a small bar magnet of dipole moment m . Thus the current-carrying solenoid behaves as a magnetic dipole. The inverse-cube expression applies in the distant-point approximation; the preceding expression retains the finite length and radius.

Step 4: (b) Calculate the coil's magnetic moment and specify its direction The coil has $N = 5$, $I = 2$ A, and radius $R = 7$ cm = 0.07 m. Therefore

$$\begin{aligned} A &= \pi R^2 = \pi(0.07)^2 \text{ m}^2, \\ m &= N I A = 5 \times 2 \times \pi(0.07)^2 \\ &= 0.049\pi \text{ A m}^2 \simeq 0.154 \text{ A m}^2. \end{aligned}$$

The coil lies in the X - Y plane, so its moment is perpendicular to that plane, along the Z -axis. Curl the fingers of the right hand in the current direction: the thumb gives the moment direction. It is $+\hat{z}$ if the current is anticlockwise as seen from the positive Z -axis, and $-\hat{z}$ if it is clockwise from that viewpoint. The printed question supplies no current sense, so a unique sign cannot be chosen.

Step 5: Conclusion (a) A solenoid has north and south poles and magnetic moment $m = N I \pi a^2$; at a distant external axial point, $B = \mu_0(2m)/(4\pi r^3)$. (b) The coil's magnetic moment has magnitude 0.154 A m^2 and lies along the Z -axis, with its sense fixed by the right-hand rule and the current direction.

Quick Tip to Solve: Convert the radius to metres before squaring it, and use the right-hand rule to select the normal to the coil; its plane alone fixes an axis, not the sign of the moment.

35. (a) Derive the expression for the force acting between two long parallel current carrying conductors. Hence, define 1 A current.

(b) A bar magnet of dipole moment 3 A m^2 rests with its centre on a frictionless pivot. A force F is applied at right angles to the axis of the magnet, 10 cm from the pivot. It is observed that an external magnetic field of 0.25 T is required to hold the magnet in equilibrium at an angle of 30° with the field. Calculate the value of F . How will the equilibrium be effected if F is withdrawn? [5 marks]

Solution:

Concept:

- A long straight conductor carrying current I produces a magnetic field $B = \mu_0 I / (2\pi d)$ at perpendicular distance d , where μ_0 is the permeability of free space.
- A straight wire segment of length L carrying current I in a perpendicular magnetic field experiences force $F = ILB$. Its direction follows $\mathbf{F} = I\mathbf{L} \times \mathbf{B}$.
- A magnetic dipole of moment m in a uniform field B experiences torque $\tau = mB \sin \theta$. A force perpendicular to the magnet at distance r from its pivot supplies torque Fr .

Step 1: (a) Find the magnetic field of the first conductor Consider two infinitely long straight parallel conductors in vacuum, separated by distance d , carrying steady currents I_1 and I_2 . By Ampere's circuital law, for a circular path of radius d centred on conductor 1,

$$B_1(2\pi d) = \mu_0 I_1, \quad B_1 = \frac{\mu_0 I_1}{2\pi d}.$$

The magnetic field at conductor 2 is perpendicular to its length. The field direction is obtained using the right-hand thumb rule around conductor 1.

Step 2: (a) Obtain the force and define one ampere The force on length L of conductor 2 is therefore

$$F_{21} = I_2 L B_1 = \frac{\mu_0 I_1 I_2 L}{2\pi d}, \quad \frac{F}{L} = \frac{\mu_0 I_1 I_2}{2\pi d}.$$

Conductor 1 experiences an equal and opposite force due to the field of conductor 2. Applying the right-hand rule for the field followed by the magnetic-force rule shows that currents in the same direction attract, whereas currents in opposite directions repel. For the force-based definition requested in this paper, one ampere is that constant current which, when maintained in each of two straight parallel conductors of infinite length and negligible circular cross-section placed 1 m apart in vacuum, produces a force of $2 \times 10^{-7} \text{ N}$ per metre of length between them. Indeed, putting $I_1 = I_2 = 1 \text{ A}$, $d = 1 \text{ m}$ and $\mu_0 = 4\pi \times 10^{-7} \text{ N A}^{-2}$ in the force expression gives this value.

Step 3: (b) Balance the torques about the pivot The pivot force acts through the centre and contributes no torque. The applied force acts perpendicular to the axis at $r = 10 \text{ cm} = 0.10 \text{ m}$, so its torque magnitude is Fr . To hold the magnet at $\theta = 30^\circ$, this torque must oppose and equal the magnetic torque:

$$Fr = mB \sin \theta, \\ F = \frac{(3)(0.25) \sin 30^\circ}{0.10} = \frac{0.375}{0.10} = 3.75 \text{ N}.$$

The force lever arm is the full 0.10 m, since the force is perpendicular to the magnet; the sine factor belongs to the magnetic torque alone.

Step 4: (b) Explain the motion after the force is withdrawn On withdrawing F , its balancing torque

disappears. At the initial angle the magnetic torque is still 0.375 N m , directed so as to reduce the angle between the dipole moment and the field. Thus the magnet is no longer in equilibrium at 30° and starts turning towards alignment. Parallel alignment, $\theta = 0$, is the stable equilibrium orientation, since the potential energy $U = -mB \cos \theta$ is then minimum. With a frictionless pivot and no damping specified, a magnet released from rest overshoots alignment and oscillates about it; it would settle along the field only if energy were dissipated.

Step 5: Conclusion The force per unit length between the conductors is $F/L = \mu_0 I_1 I_2 / (2\pi d)$, attractive for parallel currents in the same direction and repulsive for opposite currents. The required applied force is $[F = 3.75 \text{ N}]$. On withdrawing it, the 30° equilibrium is lost and the magnet turns towards, and in the ideal undamped case oscillates about, its stable orientation along the field.

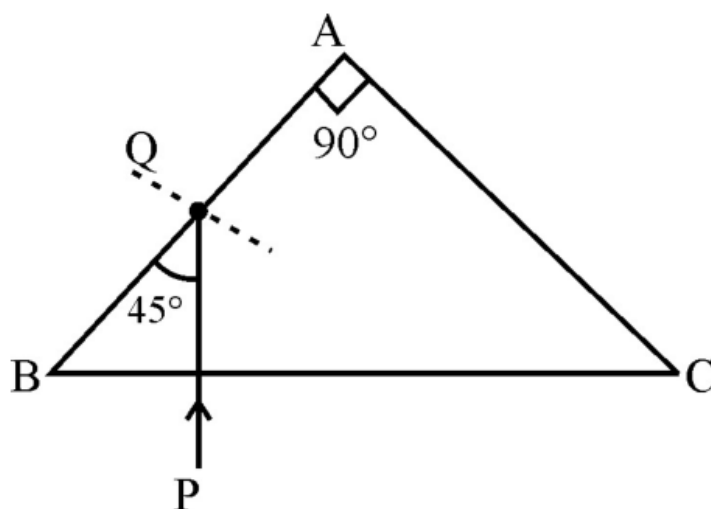
Quick Tip to Solve: Write the torque balance about the pivot first: use Fr , not $Fr \sin 30^\circ$, because the applied force is already perpendicular to the magnet.

36. (a) Draw the ray diagram showing refraction of ray of light through a glass prism. Derive the expression for the refractive index μ of the material of prism in terms of the angle of prism A and angle of minimum deviation δ_m .

(b) A ray of light PQ enters an isosceles right angled prism ABC of refractive index 1.5 as shown in figure.

(i) Trace the path of the ray through the prism.

(ii) What will be the effect on the path of the ray if the refractive index of the prism is 1.4? [5 marks]



Solution:

Concept:

- For refraction from air into glass, Snell's law gives $\mu = \sin i / \sin r$, where i and r are the angles measured from the normal. At the second face, $\mu \sin r_2 = \sin e$, where e is the angle of emergence.
- In a prism of apex angle A , the internal angles obey $r_1 + r_2 = A$. The total deviation is $\delta = i + e - A$. At minimum deviation, the ray follows a symmetric path: $i = e$ and $r_1 = r_2 = A/2$.
- At a glass-to-air boundary, the critical angle C is defined by $\sin C = 1/\mu$. Total internal reflection requires incidence in the denser medium with $i > C$.

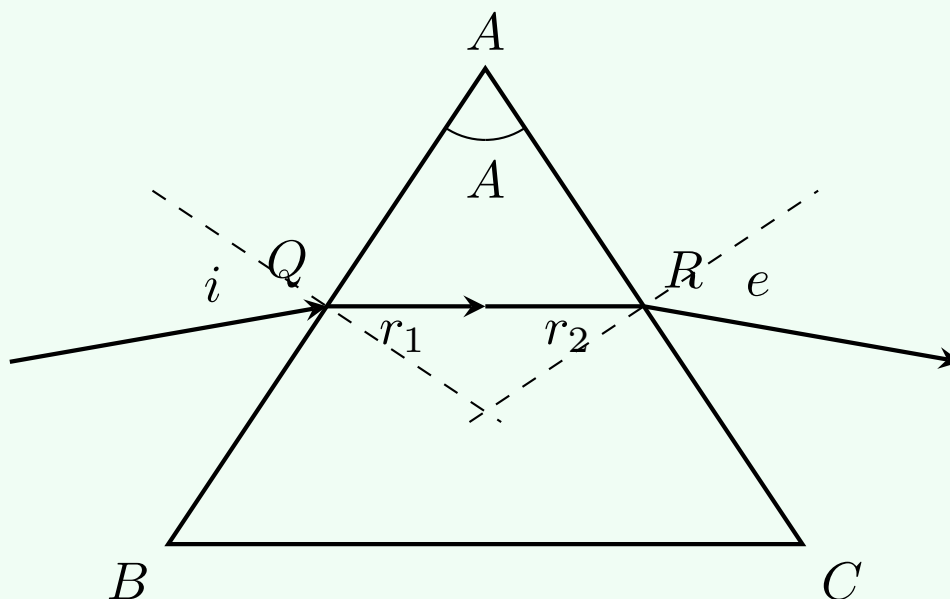
Step 1: (a) Ray diagram and prism-angle relation The diagram shows a ray incident on the first face,

refracted inside the prism, and emerging from the second face. The dashed lines are normals; all refraction angles are measured from them. If the normals meet inside the prism, their included angle is $180^\circ - A$. Applying the angle sum to the triangle formed by these normals and the internal ray gives

$$r_1 + r_2 + (180^\circ - A) = 180^\circ, \quad r_1 + r_2 = A.$$

At the first face the ray turns through $i - r_1$, and at the second face it turns through $e - r_2$. Both turns contribute to the net deviation in the same sense, hence

$$\delta = (i - r_1) + (e - r_2) = i + e - A.$$



Refraction through a prism: normals and the angles used in the derivation.

Step 2: (a) Derive the minimum-deviation expression As the prism is rotated, the deviation first decreases and then increases. At its minimum the forward and reversed ray paths coincide by reversibility of light, giving equal incidence and emergence angles: $i = e$. Snell's law at the two faces then gives equal internal refraction angles, $r_1 = r_2$. Therefore,

$$r_1 = r_2 = \frac{A}{2}, \quad \delta_m = 2i - A, \quad i = \frac{A + \delta_m}{2}.$$

Substituting these angles into Snell's law for the air-to-prism boundary yields

$$\mu = \frac{\sin i}{\sin r_1} = \frac{\sin((A + \delta_m)/2)}{\sin(A/2)}.$$

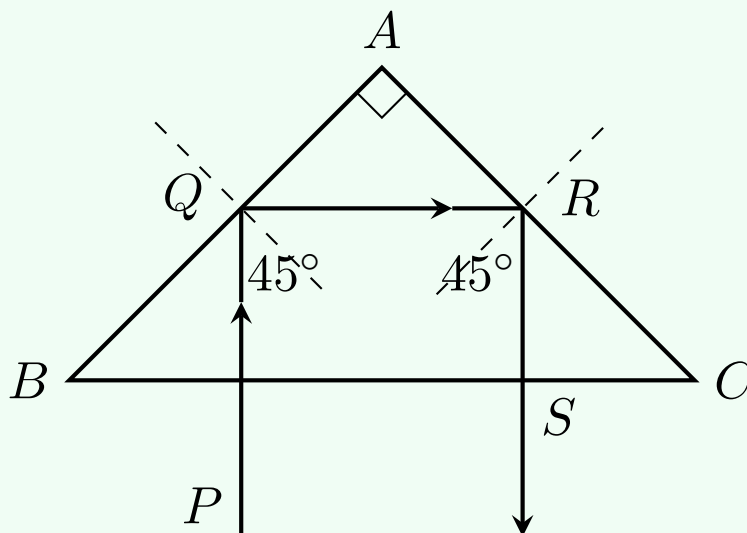
This expression is the refractive index of the prism material relative to the surrounding air. The minimum condition must be used; a general measured deviation cannot simply replace δ_m .

Step 3: (b)(i) Trace the ray for refractive index 1.5 The ray PQ enters through the horizontal face BC normally, so its direction does not change at entry. Both base angles of the isosceles right angled prism are 45° . The upward ray therefore strikes face AB at Q with incidence 45° to the normal. For

this glass,

$$C = \sin^{-1}\left(\frac{1}{1.5}\right) \simeq 41.8^\circ.$$

Since $45^\circ > C$, it is totally internally reflected at AB . The reflected ray travels horizontally to the right and strikes face AC at R , again with incidence 45° . It undergoes total internal reflection there as well, then travels vertically downward to BC . At S , it meets BC normally and emerges without further deviation. The emergent ray is parallel to, and opposite in direction to, the incident ray: the net directional reversal is 180° .



For $\mu = 1.5$: normal entry, two total internal reflections, and normal emergence.

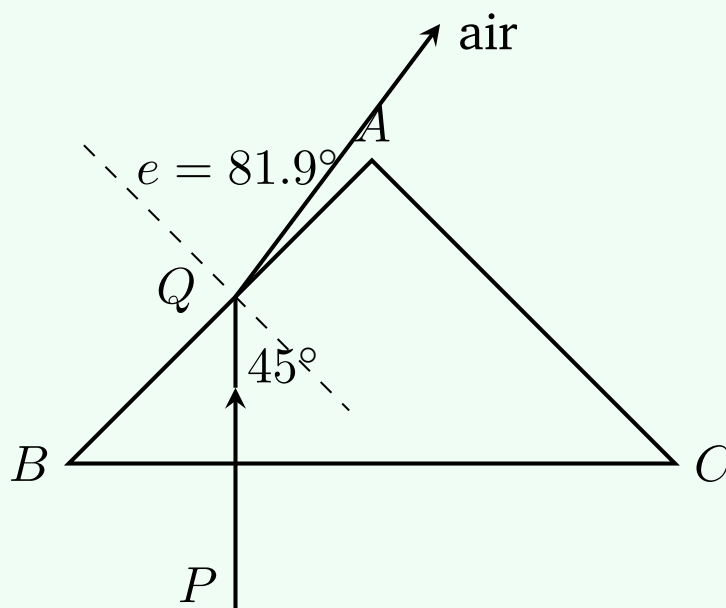
Step 4: (b)(ii) Trace the change for refractive index 1.4 Entry through BC remains normal and the incidence at Q remains 45° , because neither the geometry nor the incident direction changes. However, the critical angle becomes

$$C' = \sin^{-1}\left(\frac{1}{1.4}\right) \simeq 45.6^\circ.$$

Now $45^\circ < C'$, so total internal reflection does not occur at AB . The transmitted ray emerges from AB , bending away from the normal. Snell's law gives its angle in air:

$$\sin e = 1.4 \sin 45^\circ = \frac{1.4}{\sqrt{2}}, \quad e \simeq 81.9^\circ.$$

Thus the refracted ray makes approximately 8.1° with face AB on its air side, as shown. The principal transmitted path leaves at Q , instead of undergoing the two total internal reflections and emerging through BC . Ordinary partial reflection may also occur at Q ; it is not total internal reflection.



For $\mu = 1.4$: the refracted ray exits through AB at Q.

Step 5: Conclusion (a) $\mu = \frac{\sin((A + \delta_m)/2)}{\sin(A/2)}$. (b)(i) For $\mu = 1.5$, the ray suffers two total internal reflections at AB and AC, then emerges normally through BC, opposite to PQ. (ii) For $\mu = 1.4$, it refracts out through AB at Q, at approximately 81.9° to the normal.

Quick Tip to Solve: For a right angled isosceles prism entered normally through its hypotenuse, compare μ with $\sqrt{2}$: $\mu > \sqrt{2}$ permits total internal reflection at the 45° faces.

OR

36. (a) Two thin lenses are placed coaxially in contact. Obtain the expression for the focal length of this combination in terms of the focal lengths of the two lenses.

(b) A converging lens of refractive index 1.5 has a power of 10 D. When it is completely immersed in a liquid, it behaves as a diverging lens of focal length 50 cm. Find the refractive index of the liquid. [5 marks]

Solution:

Concept:

- For a thin lens with signed object distance u , image distance v and focal length f , the Cartesian lens formula is $1/v - 1/u = 1/f$. All distances are measured from the lens along the same axis.
- For coaxial thin lenses in contact, their powers add. For a lens of refractive index n_g surrounded by a medium of index n_m , the lens-maker formula is $P = (n_g/n_m - 1)(1/R_1 - 1/R_2)$, with radii in metres.

Step 1: (a) Refraction at the first lens Let an object be at signed distance u from the two lenses. Denote the focal lengths by f_1 and f_2 . If the first lens alone produces an intermediate image at v_1 , then

$$\frac{1}{v_1} - \frac{1}{u} = \frac{1}{f_1}.$$

This intermediate image acts as the object for the second lens. The contact separation is negligible, so its signed object distance for the second lens is also v_1 . In particular, an intermediate image to the right of the second lens is a virtual object with positive object distance; the same signed formula still applies.

Step 2: (a) Combine the two lens equations If the final image is at v , the second lens gives

$$\frac{1}{v} - \frac{1}{v_1} = \frac{1}{f_2}.$$

Adding the two equations cancels the intermediate-image term and gives

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f_1} + \frac{1}{f_2}.$$

Defining F as the focal length of the equivalent single lens means that its left-hand side equals $1/F$. This derivation assumes thin lenses, paraxial rays and negligible separation; the signs of both focal lengths are retained, so it applies to converging and diverging lenses.

Step 3: (b) Determine the curvature factor from the power in air Write the unchanged curvature factor as $K = 1/R_1 - 1/R_2$. In air, $n_m = 1$, $n_g = 1.5$, and the converging lens has positive power $P_a = +10$ D. Thus

$$10 = (1.5 - 1)K, \quad K = 20 \text{ m}^{-1}.$$

Immersion changes the relative refractive index but does not change the two surface radii. Therefore this same K must be used for the immersed lens.

Step 4: (b) Apply the negative focal length in the liquid The immersed lens is diverging, so its focal length is $f_l = -50 \text{ cm} = -0.50 \text{ m}$ and its power is $P_l = 1/f_l = -2$ D. If the liquid has refractive index n_l , then

$$\begin{aligned} -2 &= \left(\frac{1.5}{n_l} - 1 \right) 20, \\ \frac{1.5}{n_l} - 1 &= -0.10, \\ \frac{1.5}{n_l} &= 0.90. \end{aligned}$$

The negative power is physically consistent with a liquid whose refractive index exceeds that of the glass: a lens with this converging shape then behaves as a diverging lens.

Step 5: Conclusion (a) The focal length of the contact combination is $\boxed{1/F = 1/f_1 + 1/f_2}$, or $\boxed{F = f_1 f_2 / (f_1 + f_2)}$ when $f_1 + f_2 \neq 0$. If the powers cancel, F is infinite. (b) The liquid's refractive index is $\boxed{n_l = 1.5/0.90 = 5/3 \approx 1.67}$.

Quick Tip to Solve: Give the immersed lens a negative focal length first; then divide its power by the air power to cancel the unchanged curvature factor.

37. (a) Derive the law of radioactive decay $N = N_0 e^{-\lambda t}$.

(b) The half life of ${}^{238}_{92}\text{U}$ undergoing α -decay is 4.5×10^9 years. Find its mean life.

(c) What fraction of the initial mass of a radioactive substance will decay in five half-life periods?

[5 marks]

Solution:

Concept:

- Each undecayed nucleus has a constant probability per unit time of decaying, described by the decay constant λ . Thus the number of decays per unit time is proportional to the number N of parent nuclei still present.
- The half-life $T_{1/2}$ satisfies $N(T_{1/2}) = N_0/2$, and the mean life is $\tau = 1/\lambda$. The parent isotope's remaining mass is proportional to its remaining number of nuclei.

Step 1: (a) Write and separate the decay equation Let N_0 be the number of undecayed nuclei at $t = 0$, and N their number at time t . In a small interval dt , the number that decay is $\lambda N dt$. The population of parent nuclei decreases, so its change is negative:

$$dN = -\lambda N dt, \quad \frac{dN}{N} = -\lambda dt.$$

Here $\lambda > 0$ is a characteristic constant for the isotope, with units inverse to the time unit used. The differential equation expresses the statistical decay law for a large population rather than a prediction of the decay time of a particular nucleus.

Step 2: (a) Integrate with the initial condition Integrating from the initial population N_0 to the remaining population N , and from 0 to t , gives

$$\int_{N_0}^N \frac{dN'}{N'} = -\lambda \int_0^t dt', \quad \ln \left(\frac{N}{N_0} \right) = -\lambda t.$$

Exponentiating gives an exponential decrease. The sign is essential: a positive exponent would wrongly predict a growing population of undecayed nuclei. The initial condition is satisfied since the exponential factor equals unity at zero time.

Step 3: (b) Relate half-life and mean life At one half-life, substitution into the decay law gives

$$\frac{1}{2} = e^{-\lambda T_{1/2}}, \quad \lambda T_{1/2} = \ln 2.$$

The mean lifetime is the average decay time of the initial nuclei. Using the decay-time probability density $\lambda e^{-\lambda t}$, it is

$$\tau = \int_0^\infty t \lambda e^{-\lambda t} dt = \frac{1}{\lambda} = \frac{T_{1/2}}{\ln 2}.$$

For ${}^{238}_{92}\text{U}$, the stated half-life is 4.5×10^9 years. Keeping years throughout avoids an unnecessary conversion:

$$\tau = \frac{4.5 \times 10^9}{0.693} \text{ years} \approx 6.49 \times 10^9 \text{ years}.$$

As a check, mean life exceeds half-life because $1/\ln 2 > 1$. The fact that uranium undergoes alpha decay does not change the exponential population relation.

Step 4: (c) Distinguish the remaining fraction from the decayed fraction After every half-life, half of the parent nuclei that are still present remain. Therefore after five half-life periods the remaining

parent-isotope mass m obeys

$$\frac{m}{m_0} = \frac{N}{N_0} = \left(\frac{1}{2}\right)^5 = \frac{1}{32}.$$

The question asks for the fraction that has decayed, so subtract this remaining fraction from the initial fraction of unity. This counts the original radioactive parent nuclei that have transformed; it does not mean the decay products have disappeared.

Step 5: Conclusion (a) The radioactive decay law is $N = N_0 e^{-\lambda t}$. (b) The mean life of $^{238}_{92}\text{U}$ is $\tau \approx 6.49 \times 10^9$ years. (c) The fraction of initial radioactive mass that decays in five half-lives is $1 - 1/32 = 31/32$, or 96.875%.

Quick Tip to Solve: After five half-lives, first write the remaining fraction as 2^{-5} ; subtract it from one because the question asks how much has decayed.

OR

37. (a) State the postulates of Bohr's model of hydrogen atom and derive the expression for Bohr radius.

(b) Find the ratio of the longest and the shortest wavelengths amongst the spectral lines of Balmer series in the spectrum of hydrogen atom. [5 marks]

Solution:

Concept:

- An electron in a hydrogen atom is attracted to its proton by the Coulomb force. In a circular orbit this supplies the centripetal force: $m_e v^2 / r = e^2 / (4\pi\epsilon_0 r^2)$, where m_e is electron mass and e is the magnitude of its charge.
- Bohr's angular-momentum condition is $m_e v r = n\hbar = nh / (2\pi)$, with $n = 1, 2, 3, \dots$. A transition between stationary states produces a photon whose energy equals the energy difference.
- For hydrogen, $1/\lambda = R_H (1/n_f^2 - 1/n_i^2)$. All Balmer lines terminate at $n_f = 2$; their initial levels are $n_i = 3, 4, 5, \dots$

Step 1: (a) State Bohr's postulates An electron revolves around the nucleus only in certain permitted circular orbits, called stationary states. While remaining in such an orbit it does not radiate energy, so each permitted orbit has a definite, constant energy.

Only those orbits are allowed for which the electron's orbital angular momentum is an integral multiple of $h/(2\pi)$:

$$m_e v r = \frac{n\hbar}{2\pi}, \quad n = 1, 2, 3, \dots$$

Radiation is emitted or absorbed only when the electron jumps between two stationary states. The photon frequency satisfies $h\nu = |E_i - E_f|$. A downward transition emits a photon; an upward transition requires absorption of the corresponding energy.

Step 2: (a) Derive the radius of an allowed orbit For a hydrogen atom, equate the attractive electrostatic force to the required centripetal force:

$$\frac{m_e v_n^2}{r_n} = \frac{e^2}{4\pi\epsilon_0 r_n^2}.$$

From angular-momentum quantisation, $v_n = nh/(2\pi m_e r_n)$. Substitute this into the force equation:

$$\begin{aligned}\frac{n^2 h^2}{4\pi^2 m_e r_n^3} &= \frac{e^2}{4\pi\epsilon_0 r_n^2}, \\ \frac{n^2 h^2}{4\pi^2 m_e} &= \frac{e^2 r_n}{4\pi\epsilon_0}, \\ r_n &= \frac{\epsilon_0 n^2 h^2}{\pi m_e e^2} = \frac{4\pi\epsilon_0 \hbar^2}{m_e e^2} n^2.\end{aligned}$$

Thus the allowed radii increase as n^2 . The first orbit has radius $a_0 = r_1 = 5.29 \times 10^{-11}$ m, known as the Bohr radius. Here the standard school-level approximation treats the proton as stationary; m_e is the electron mass.

Step 3: (b) Identify the longest and shortest Balmer wavelengths The Balmer wavenumber is

$$\frac{1}{\lambda} = R_H \left(\frac{1}{4} - \frac{1}{n_i^2} \right).$$

The longest wavelength corresponds to the smallest energy difference, namely $n_i = 3 \rightarrow 2$. Hence

$$\frac{1}{\lambda_{\max}} = R_H \left(\frac{1}{4} - \frac{1}{9} \right) = \frac{5R_H}{36}, \quad \lambda_{\max} = \frac{36}{5R_H}.$$

As n_i increases, the wavenumber increases and the wavelength decreases. The shortest wavelength is the series-limit value, obtained as $n_i \rightarrow \infty$:

$$\frac{1}{\lambda_{\min}} = \frac{R_H}{4}, \quad \lambda_{\min} = \frac{4}{R_H}.$$

Therefore, on dividing the wavelengths in the order requested,

$$\frac{\lambda_{\max}}{\lambda_{\min}} = \frac{36/(5R_H)}{4/R_H} = \frac{9}{5}.$$

The ratio is independent of the numerical value of R_H . Reversing the order would give 5/9, which is not the requested longest-to-shortest ratio.

Step 4: Conclusion The allowed orbit radii are $r_n = n^2 a_0$, where $a_0 = \epsilon_0 h^2 / (\pi m_e e^2) = 5.29 \times 10^{-11}$ m. For the Balmer series, $\lambda_{\text{longest}} : \lambda_{\text{shortest}} = 9 : 5$.

Quick Tip to Solve: For any hydrogen series, use the first allowed transition for the longest wavelength and the infinite initial level for the shortest, then divide wavelengths by inverting their wavenumbers.