

CBSE Class 12 Physics Question Paper 2020 with Solution PDF-Set 55/4/2



Question Paper with Solutions

Conducted by the Central Board of Secondary Education (CBSE)

Time Allowed: 3 hours

Maximum Marks: 70

General Instructions

- (i) This question paper comprises **four** sections – A, B, C and D.
- (ii) There are **37** questions in the question paper. **All** questions are compulsory.
- (iii) Section A : Q. no. **1** to **20** are very short-answer type questions carrying **1** mark each.
- (iv) Section B : Q. no. **21** to **27** are short-answer type questions carrying **2** marks each.
- (v) Section C : Q. no. **28** to **34** are long-answer type questions carrying **3** marks each.
- (vi) Section D : Q. no. **35** to **37** are also long answer type questions carrying **5** marks each.
- (vii) There is no overall choice in the question paper. However, an internal choice has been provided in **two** questions of **one** mark, **two** questions of **two** marks, **one** question of **three** marks and all the **three** questions **five** marks. You have to attempt only one of the choices in such questions.
- (viii) However, separate instructions are given with each section and question, wherever necessary.
- (ix) Use of calculators and log tables is not permitted.

You may use the following values of physical constants wherever necessary :

$$c = 3 \times 10^8 \text{ m/s}$$

$$h = 6.63 \times 10^{-34} \text{ Js}$$

$$e = 1.6 \times 10^{-19} \text{ C}$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ T m A}^{-1}$$

$$\epsilon_0 = 8.854 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$$

$$\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$$

$$\text{Mass of electron } (m_e) = 9.1 \times 10^{-31} \text{ kg}$$

$$\text{Mass of neutron} = 1.675 \times 10^{-27} \text{ kg}$$

$$\text{Mass of proton} = 1.673 \times 10^{-27} \text{ kg}$$

$$\text{Avogadro's number} = 6.023 \times 10^{23} \text{ per gram mole}$$

$$\text{Boltzmann constant} = 1.38 \times 10^{-23} \text{ JK}^{-1}$$

Section A (20 marks)

Select the most appropriate option from those given below each question :

1. An electron and a proton are moving along the same direction with the same kinetic energy. They enter a uniform magnetic field acting perpendicular to their velocities. The dependence of radius of their paths on their masses is: [1 mark]

- (A) $r \propto m$
(B) $r \propto \sqrt{m}$
(C) $r \propto \frac{1}{m}$
(D) $r \propto \frac{1}{\sqrt{m}}$

Correct Answer: (B) $r \propto \sqrt{m}$

Solution:

Concept:

- The magnetic force supplies centripetal force: $|q|vB = mv^2/r$, where B is perpendicular to velocity.
- Non-relativistic kinetic energy is $K = mv^2/2$, hence momentum $p = \sqrt{2mK}$.

Step 1: Express the radius From the force equation, $r = mv/(|q|B) = p/(|q|B)$. Electron and proton have opposite charges, but equal charge magnitude e ; their bending directions differ, while radius depends on the magnitude.

Step 2: Compare equal kinetic energies Substitute $p = \sqrt{2mK}$ to obtain $r = \sqrt{2mK}/(eB)$. Since K , e and B are the same, the only varying factor is $\sqrt{m}$. Option (A) would require equal speeds, not equal energies; (C) and (D) wrongly make the heavier particle follow a smaller radius.

Step 3: Conclusion Thus $r \propto \sqrt{m}$. The correct option is (B).

Quick Tip to Solve: For equal kinetic energy, replace momentum by $\sqrt{2mK}$ before comparing magnetic radii.

2. A circular loop of radius r , carrying a current I lies in y - z plane with its centre at the origin. The net magnetic flux through the loop is: [1 mark]

- (A) directly proportional to r
(B) zero
(C) inversely proportional to r
(D) directly proportional to I

Correct Answer: (D) directly proportional to I

Solution:

Concept:

- Magnetic flux through a surface is $\Phi_B = \int \mathbf{B} \cdot d\mathbf{A}$.
- For a fixed loop geometry, its own magnetic field and self-flux are proportional to the current:

Quick Tip to Solve: Distinguish flux through an open loop surface from the zero net flux through a closed surface.

$\Phi_B = LI$, where L is self-inductance.

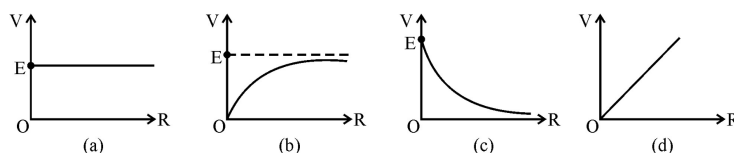
Step 1: Identify the relevant magnetic field The question specifies a current-carrying loop and gives no external magnetic field. The loop therefore produces its own field. Its field threads the surface bounded by the loop; the area normal lies along the x -axis because the loop is in the y - z plane.

Step 2: Compare the options At fixed radius and wire geometry, the Biot-Savart law makes every contribution to $\mathbf{B}$ proportional to I , so their surface integral is also proportional to I . The zero-flux rule applies to a closed surface, whereas the surface spanning a circular loop is open. Neither an inverse-radius nor a universal direct-radius dependence follows without specifying the wire geometry; self-inductance also depends on the wire thickness.

Step 3: Conclusion The self-flux is $\Phi_B = LI$, directly proportional to I . The correct option is (D).

Quick Tip to Solve: Distinguish flux through an open loop surface from the zero net flux through a closed surface.

3. A cell of emf (E) and internal resistance r is connected across a variable external resistance R . The graph of terminal potential difference V as a function of R is — [1 mark]



(A) Graph (a): $V = E$, constant for all R .

(B) Graph (b): V rises from zero and approaches E asymptotically.

(C) Graph (c): V decreases from E towards zero.

(D) Graph (d): V rises linearly from zero.

Correct Answer: (B) Graph (b): V rises from zero and approaches E asymptotically.

Solution:

Concept:

- For a cell of emf E and internal resistance r , the current is $I = E/(R + r)$.
- The terminal potential difference across the load is $V = IR = E - Ir$.

Step 1: Obtain the function Substitution gives $V = ER/(R + r)$. For a short circuit, $R = 0$, the terminal voltage is zero. As R increases, the current and the internal voltage loss Ir decrease.

Step 2: Match the limiting shape For $r > 0$, $dV/dR = Er/(R + r)^2 > 0$, and $d^2V/dR^2 = -2Er/(R + r)^3 < 0$. Thus the curve rises with decreasing slope and approaches E from below as $R \rightarrow \infty$. Graph (a) ignores internal resistance, graph (c) decreases, and graph (d) has no limiting value; only graph (b) has all the required features.

Step 3: Conclusion The correct graph is (b), so the correct option is (B).

Quick Tip to Solve: Check $R = 0$ and $R \rightarrow \infty$: the voltage must start at zero and approach the emf.

4. A photocell connected in an electrical circuit is placed at a distance d from a source of light. As a result, current I flows in the circuit. What will be the current in the circuit when the distance is reduced to $d/3$? [1 mark]

- (A) I
- (B) $6I$
- (C) $9I$
- (D) $\frac{I}{3}$

Correct Answer: (C) $9I$

Solution:

Concept:

- At fixed frequency above threshold, photoelectric current is proportional to incident light intensity.
- For the source geometry assumed here, intensity at distance d varies as $1/d^2$.

Step 1: Use the printed distance The official question reduces the source distance from d to $d/3$. With the same source output and photocell conditions, the light falls on a smaller spherical area, so the photon arrival rate per unit area increases.

Step 2: Compute the current ratio Using inverse-square variation, $I_{\text{new}}/I = [d/(d/3)]^2 = 3^2 = 9$. More incident photons liberate more electrons per second; the maximum electron energy remains set by frequency. Option (A) ignores the intensity change, (B) uses an incorrect factor of six, and (D) reverses the increase. The distance factor must be squared.

Step 3: Conclusion The new current is $9I$. The correct option is (C).

Quick Tip to Solve: When the source distance is divided by three, multiply photocurrent by the square, nine.

5. A uniform wire of resistance $2R$ is bent in the form of a circle. The effective resistance between the ends of any diameter of the circle is: [1 mark]

- (A) $2R$
- (B) R
- (C) $\frac{R}{2}$
- (D) $\frac{R}{4}$

Correct Answer: (C) $\frac{R}{2}$

Solution:

Concept:

- A uniform wire has resistance proportional to its length.
- Two resistors between the same terminals are in parallel: $1/R_{\text{eq}} = 1/R_1 + 1/R_2$.

Step 1: Split the circular wire The endpoints of any diameter divide the circle into equal semicircles. Each semicircle has half the total wire length, so each has resistance $(2R)/2 = R$. Both connect the

Quick Tip to Solve: First halve the wire resistance for each semicircle, then halve again for two equal parallel paths.

same diametrically opposite terminals.

Step 2: Combine the two paths The current divides between the semicircles, which are parallel rather than series. Therefore $R_{\text{eq}} = R \cdot R/(R + R) = R/2$. Option (A) is the original unbent-wire resistance and would wrongly count both paths in series. Option (B) considers only one semicircle, and option (D) halves the resistance one time too many.

Step 3: Conclusion The effective resistance is $\frac{R}{2}$. The correct option is (C).

Quick Tip to Solve: First halve the wire resistance for each semicircle, then halve again for two equal parallel paths.

6. A current of 5 A is flowing from east to west in a long straight wire kept on a horizontal table. The magnetic field developed at a distance of 10 cm due south on the table is: [1 mark]

- (A) 1×10^{-5} T acting downwards
- (B) 1×10^{-5} T acting upwards
- (C) 2×10^{-5} T acting downwards
- (D) 2×10^{-5} T acting upwards

Correct Answer: (B) 1×10^{-5} T acting upwards

Solution:

Concept:

- For a long straight wire, $B = \mu_0 I / (2\pi a)$, where a is perpendicular distance.
- The direction follows the right-hand grip rule, or $\mathbf{I} \times \hat{\mathbf{r}}$.

Step 1: Calculate the magnitude The current is 5 A, the distance is $10 \text{ cm} = 0.10 \text{ m}$, and $\mu_0 = 4\pi \times 10^{-7} \text{ T m A}^{-1}$. Thus $B = (4\pi \times 10^{-7} \times 5) / (2\pi \times 0.10) = 1.0 \times 10^{-5} \text{ T}$. The values $2 \times 10^{-5} \text{ T}$ in (C) and (D) are incorrect.

Step 2: Determine the direction Take east as $+x$, north as $+y$, and upward as $+z$. The current points west, along $-\hat{x}$, and the observation point lies south, along $-\hat{y}$. Hence $(-\hat{x}) \times (-\hat{y}) = +\hat{z}$: the field points vertically upward. Option (A) has the correct magnitude but the wrong direction.

Step 3: Conclusion The field is $1 \times 10^{-5} \text{ T}$, acting upwards. The correct option is (B).

Quick Tip to Solve: Set east, north and up as a right-handed axis system to avoid reversing the field direction.

7. The kinetic energy of a proton and that of an α -particle are 4 eV and 1 eV, respectively. The ratio of the de-Broglie wavelengths associated with them, will be [1 mark]

- (A) 2 : 1
- (B) 1 : 1
- (C) 1 : 2
- (D) 4 : 1

Correct Answer: (B) 1 : 1

Solution:

Concept:

- The de Broglie wavelength is $\lambda = h/p = h/\sqrt{2mK}$.
- An alpha particle has mass approximately four times the proton mass: $m_\alpha \simeq 4m_p$.

Step 1: Compare the mass-energy products The proton has $K_p = 4$ eV, while the alpha particle has $K_\alpha = 1$ eV. Thus $m_p K_p = 4m_p$ eV and $m_\alpha K_\alpha \simeq (4m_p)(1 \text{ eV}) = 4m_p$ eV. Their momentum magnitudes are equal.

Step 2: Form the wavelength ratio Therefore $\lambda_p/\lambda_\alpha = \sqrt{m_\alpha K_\alpha / (m_p K_p)} = \sqrt{4/4} = 1$. Options (A) and (C) would arise from considering only the energy or only the mass factor; option (D) misses the square root as well. Particle charge does not enter the de Broglie wavelength when kinetic energy is already specified.

Step 3: Conclusion The wavelength ratio is 1 : 1. The correct option is (B).

Quick Tip to Solve: Compare mK , not mass or kinetic energy alone, when calculating de Broglie wavelength ratios.

8. In the α -particle scattering experiment, the shape of the trajectory of the scattered α -particles depend upon: [1 mark]

- (A) only on impact parameter.
- (B) only on the source of α -particles.
- (C) both impact parameter and source of α -particles.
- (D) impact parameter and the screen material of the detector.

Correct Answer: (C) both impact parameter and source of α -particles.

Solution:

Concept:

- For a fixed target nucleus, the Coulomb force is repulsive and the trajectory depends on the incident kinetic energy and impact parameter.
- The physical principle must be applied to the particle or wave specified in the printed question.

Step 1: Apply the governing principle The impact parameter b is the perpendicular distance between the initial line of motion and the nuclear centre. A smaller b brings the particle closer to the nucleus, increasing its deflection.

Quick Tip to Solve: For scattering questions, identify both the closest approach geometry and the incident particle energy.

Step 2: Interpret the result The source determines the incident alpha-particle energy K . For a given target, Rutherford scattering gives $\tan(\theta/2) = k(2e)(Ze)/(2Kb)$, where Z is the atomic number of the target and k is the electrostatic constant. Thus changing either b or K changes the orbit. Options (A) and (B) each omit one dependence; the detector screen in (D) only records the scattered particle.

Step 3: Conclusion The trajectory depends on both impact parameter and the source of alpha-particles. Hence, the correct option is (C).

Quick Tip to Solve: For scattering questions, identify both the closest approach geometry and the incident particle energy.

9. When a wave undergoes reflection at an interface from rarer to denser medium, adhoc change in its phase is: [1 mark]

- (A) $\frac{\pi}{2}$
- (B) 0
- (C) π
- (D) $\frac{\pi}{4}$

Correct Answer: (C) π

Solution:

Concept:

- Reflection from an optically denser medium reverses the sign of the wave displacement or electric-field amplitude.
- The physical principle must be applied to the particle or wave specified in the printed question.

Step 1: Apply the governing principle If the incident oscillation is $E_i = E_0 \cos(\omega t)$, the reflected wave at the boundary has a negative reflection coefficient for incidence from a rarer medium into a denser one. The sign change is represented by $-\cos(\omega t) = \cos(\omega t + \pi)$.

Step 2: Interpret the result Therefore the abrupt phase change on reflection is half a cycle. Zero phase change would correspond to reflection from a denser to a rarer medium in the usual normal-incidence description. The quarter-cycle and eighth-cycle values in (A) and (D) do not represent the sign reversal.

Step 3: Conclusion The phase change is π radians. Hence, the correct option is (C).

Quick Tip to Solve: Remember that reflection at a denser boundary gives a sign reversal, equivalent to half a cycle.

10. Paschen series of atomic spectrum of hydrogen gas lies in:

[1 mark]

- (A) Infrared region
- (B) Ultraviolet region
- (C) Visible region
- (D) Partly in ultraviolet and partly in visible region

Correct Answer: (A) Infrared region

Solution:

Concept:

- The Paschen series consists of hydrogen transitions from higher levels $n > 3$ to the final level $n = 3$.
- The physical principle must be applied to the particle or wave specified in the printed question.

Step 1: Apply the governing principle The wavelength follows $1/\lambda = R_H(1/3^2 - 1/n^2)$. Even the shortest wavelength, obtained as $n \rightarrow \infty$, is $\lambda_{\min} = 9/R_H$, approximately 820 nm.

Step 2: Interpret the result All finite transitions have wavelengths longer than this limit, so every Paschen line lies beyond the visible red end in the infrared region. The ultraviolet choice corresponds to the Lyman series, while the visible lines belong mainly to the Balmer series. Neither visible nor mixed ultraviolet-visible regions describe Paschen transitions.

Step 3: Conclusion The Paschen series lies in the infrared region. Hence, the correct option is (A).

Quick Tip to Solve: Identify the final level first: Lyman ends at 1, Balmer at 2, and Paschen at 3.

11. Fill in the blank with appropriate answer.

In β -decay, a neutron is converted into a proton, e^- and _____.

[1 mark]

Solution:

Concept:

- In beta-minus decay, a neutron changes into a proton with the emission of an electron and an electron antineutrino.
- The physical principle must be applied to the particle or wave specified in the printed question.

Step 1: Apply the governing principle The complete decay equation is

$$n \longrightarrow p + e^- + \bar{\nu}_e.$$

Charge is conserved: the proton charge $+e$ and electron charge $-e$ sum to the initial zero charge of the neutron.

Step 2: Interpret the result The electron has lepton number $+1$, so an electron antineutrino with lepton number -1 accompanies it. This neutral particle also carries part of the released energy and momentum, explaining why beta electrons are emitted with a continuous range of energies rather than a single fixed energy.

Step 3: Conclusion The blank is an electron antineutrino, $\bar{\nu}_e$.

Quick Tip to Solve: An emitted electron in beta-minus decay is accompanied by an antineutrino, not a neutrino.

12. Fill in the blank with appropriate answer.

In a linearly polarised light, the electric vector oscillates along a direction perpendicular to the aligned molecules of the polaroid, called _____ axis. [1 mark]

Solution:

Concept:

- A polaroid transmits the component of the electric field along its transmission axis and absorbs the perpendicular component.
- The physical principle must be applied to the particle or wave specified in the printed question.

Step 1: Apply the governing principle The aligned molecules of the polaroid preferentially absorb the electric-field component parallel to their alignment. Consequently the emerging light has its electric vector oscillating perpendicular to the aligned molecules.

Step 2: Interpret the result The permitted direction of electric-field vibration is called the transmission axis, also known as the pass axis. For linearly polarised light this electric-field direction stays fixed while the wave propagates. The molecular alignment direction is the absorption direction and must not be confused with the direction that is transmitted.

Step 3: Conclusion The blank is transmission: the direction is called the transmission axis.

Quick Tip to Solve: The transmission axis is the direction of the electric field that passes through the polaroid.

13. Fill in the blank with appropriate answer.

A proton released from rest in an electric field, will start moving towards a region of _____ potential in the field. [1 mark]

Solution:

Concept:

- The electric force on a proton is $\mathbf{F} = e\mathbf{E}$, since its charge e is positive.
- The physical principle must be applied to the particle or wave specified in the printed question.

Step 1: Apply the governing principle A proton released from rest initially accelerates in the direction of the electric field. The electric field points towards decreasing electric potential, as expressed by $\mathbf{E} = -\nabla V$.

Step 2: Interpret the result The same conclusion follows from energy conservation. Its electric potential energy is $U = eV$. As the proton gains kinetic energy from rest, its potential energy decreases; because $e > 0$, its potential V decreases as well. A negative charge would respond in the opposite direction, but the particle specified here is a proton.

Step 3: Conclusion The blank is lower: the proton starts moving towards a region of lower potential.

Quick Tip to Solve: Use the charge sign: a positive charge accelerates along the field towards decreasing potential.

14. Fill in the blank with appropriate answer.

To minimize the percentage error in the determination of unknown resistance of a conductor in meter bridge experiment, the balance point is adjusted near _____ of the wire. [1 mark]

Solution:

Concept:

- At the balance point of a metre bridge, the Wheatstone-bridge condition is $X/R = l/(L - l)$, where $L = 100$ cm, X is the unknown resistance and R is the known resistance.
- The percentage uncertainty due to a small balance-length error is smallest when the two wire segments have equal lengths.

Step 1: Relate resistance uncertainty to balance length Taking the fractional variation of $X = Rl/(L - l)$, with the known resistance fixed, gives

$$\frac{|\delta X|}{X} \simeq \left(\frac{1}{l} + \frac{1}{L - l} \right) |\delta l| = \frac{L|\delta l|}{l(L - l)}.$$

Thus, for a fixed small uncertainty in locating the null point, the error decreases as the product of the two segment lengths increases.

Step 2: Choose the most favourable balance point Since $l(L - l) = L^2/4 - (l - L/2)^2$, this product is greatest at $l = L/2 = 50$ cm. Adjust the known resistance until the null point is near the middle of the wire; a null point near either end gives a small segment length and magnifies the relative error.

Step 3: Conclusion The blank is the middle, or midpoint, of the wire, near the 50 cm position.

Quick Tip to Solve: Choose a resistance comparable to the unknown one to obtain a balance length near 50 cm.

OR

14. In potentiometer, a long uniform wire is used to _____ potential gradient along the wire. [1 mark]

Solution:

Concept:

- Potential gradient is the potential difference per unit length: $k = V/L$.
- A uniform wire has constant resistance per unit length, so a steady current produces equal potential drops across equal lengths.

Step 1: Use the uniformity of the wire For a wire of constant area and resistivity, the resistance of a length l is $R_l = \rho l/A$. With steady current I , the potential drop is $V_l = IR_l = I\rho l/A$; hence $V_l/l = I\rho/A$ is constant along the wire.

Step 2: Use a long length For a given potential difference across the potentiometer wire, increasing its length reduces $k = V/L$. A small potential difference then corresponds to a longer balancing length, making its measurement more sensitive. Thus uniformity gives a constant gradient, while

Quick Tip to Solve: Remember both roles: uniform wire gives constant gradient; longer wire gives a smaller gradient.

length makes that gradient small.

Step 3: Conclusion The blank is **provide a small and uniform** potential gradient along the wire.

Quick Tip to Solve: Remember both roles: uniform wire gives constant gradient; longer wire gives a smaller gradient.

15. Fill in the blank with appropriate answer.

Torque acting on an electric dipole placed in an electric field is maximum when the angle between the electric field and the dipole moment is _____. [1 mark]

Solution:

Concept:

- The dipole moment $\vec{p}$ points from negative to positive charge.
- The torque magnitude in a uniform electric field is $\tau = pE \sin \theta$, where θ is the angle between $\vec{p}$ and $\vec{E}$.

Step 1: Apply the torque relation The equal and opposite electric forces on the two dipole charges form a couple. The perpendicular separation of their lines of action gives the factor $\sin \theta$, so the magnitude of torque is $pE \sin \theta$.

Step 2: Find the maximum factor For fixed dipole moment and field strength, the largest possible value of $\sin \theta$ is 1, attained at $\theta = 90^\circ$. At 0° or 180° , the dipole is parallel or antiparallel to the field and the torque vanishes.

Step 3: Conclusion The required angle is 90° ($\pi/2$ radians); the maximum torque is pE .

Quick Tip to Solve: Torque uses sine, whereas dipole potential energy uses cosine; maximum torque occurs perpendicular to the field.

16. An astronomical telescope may be a refracting type or a reflecting type. Which of the two produces image of better quality? Justify your answer. [1 mark]

Solution:

Concept:

- A refracting telescope uses a lens as its objective; a reflecting telescope uses a concave mirror.
- Chromatic aberration occurs because the refractive index of a lens material depends on wavelength.

Step 1: Compare the objective action In a refracting telescope, different colours are refracted through different angles and are focused at slightly different positions. This chromatic aberration can produce coloured fringes and reduce the sharpness of the image.

Step 2: Explain the reflecting telescope advantage Reflection obeys the same geometrical law for all colours, so the mirror objective does not produce chromatic aberration. A suitably shaped reflecting objective can therefore bring the colours to the same focus and yield a clearer image.

Quick Tip to Solve: Compare the objectives: reflection is independent of colour, whereas a refracting lens disperses white light.

Step 3: Conclusion A reflecting telescope produces the better-quality image because its mirror objective is free from chromatic aberration.

Quick Tip to Solve: Compare the objectives: reflection is independent of colour, whereas a refracting lens disperses white light.

17. An A.C. source with variable frequency is connected to a parallel plate capacitor. How will the displacement current be affected with the decrease in frequency of the source? [1 mark]

Solution:

Concept:

- Displacement current is $I_d = \epsilon_0 d\Phi_E/dt$.
- For an ideal parallel-plate capacitor of capacitance C , $I_d = C dV/dt$.

Step 1: Write the alternating voltage Taking the source voltage amplitude to remain constant, let $V(t) = V_0 \sin(\omega t)$, where $\omega = 2\pi f$. The electric flux between the plates varies with this voltage, so its rate of change determines the displacement current.

Step 2: Compare current amplitudes Differentiation gives $I_d(t) = \omega C V_0 \cos(\omega t)$. Hence its amplitude is $I_{d0} = 2\pi f C V_0$. For the same capacitor and voltage amplitude, this amplitude is directly proportional to frequency; lowering frequency makes the field change more slowly.

Step 3: Conclusion The displacement-current amplitude **decreases** when the source frequency decreases, with $I_{d0} \propto f$ at fixed voltage amplitude.

Quick Tip to Solve: For a capacitor at fixed AC voltage, use current amplitude $I_0 = 2\pi f C V_0$ to see the frequency dependence.

18. Can a slab of p-type semi-conductor be physically joined to another n-type semiconductor slab to form p-n junction? Justify your answer. [1 mark]

Solution:

Concept:

- A p-n junction is formed within a single continuous semiconductor crystal by creating adjacent p-type and n-type regions.
- A suitable junction requires a clean crystal interface that allows carrier diffusion and formation of a depletion layer.

Step 1: Consider physically joining separate slabs When separately prepared p-type and n-type slabs are pressed together, their surfaces do not create one continuous crystal lattice. Surface contamination, oxide films and imperfect atomic contact can remain at the interface, so it does not behave like a proper p-n junction.

Step 2: State the correct formation method Instead, one region of the same semiconductor crystal is doped with acceptors and the adjacent region with donors. Electrons and holes can then diffuse

Quick Tip to Solve: A p-n junction needs continuity of the crystal lattice, not merely physical contact between two doped pieces.

across the internal boundary, leaving fixed ions that form the depletion layer and built-in potential barrier.

Step 3: Conclusion No. Simply joining separate p-type and n-type slabs does not form a proper p-n junction; the two regions must be produced in a single continuous semiconductor crystal.

Quick Tip to Solve: A p-n junction needs continuity of the crystal lattice, not merely physical contact between two doped pieces.

OR

18. In a p-n junction diode the forward bias resistance is low as compared to the reverse bias resistance. Give reason. [1 mark]

Solution:

Concept:

- Forward bias reduces the junction potential barrier and narrows the depletion layer.
- Reverse bias increases the potential barrier and widens the depletion layer.

Step 1: Explain conduction in forward bias Connecting the p side to the positive terminal and the n side to the negative terminal opposes the built-in electric field. The reduced barrier allows majority carriers to cross the junction readily, giving a comparatively large current for a small applied voltage.

Step 2: Explain conduction in reverse bias With the connections reversed, the barrier increases and majority-carrier transport is suppressed. Below breakdown, only a very small current due mainly to minority carriers flows. Thus the ratio of applied voltage to current is much larger in reverse bias than in forward bias.

Step 3: Conclusion Forward-bias resistance is low because the barrier is reduced and majority carriers conduct readily; reverse-bias resistance is high because the barrier increases and only a tiny minority-carrier current flows.

Quick Tip to Solve: Compare depletion layers: forward bias narrows the barrier and aids majority carriers; reverse bias widens it.

19. Depict equipotential surfaces due to an electric dipole. [1 mark]

Solution:

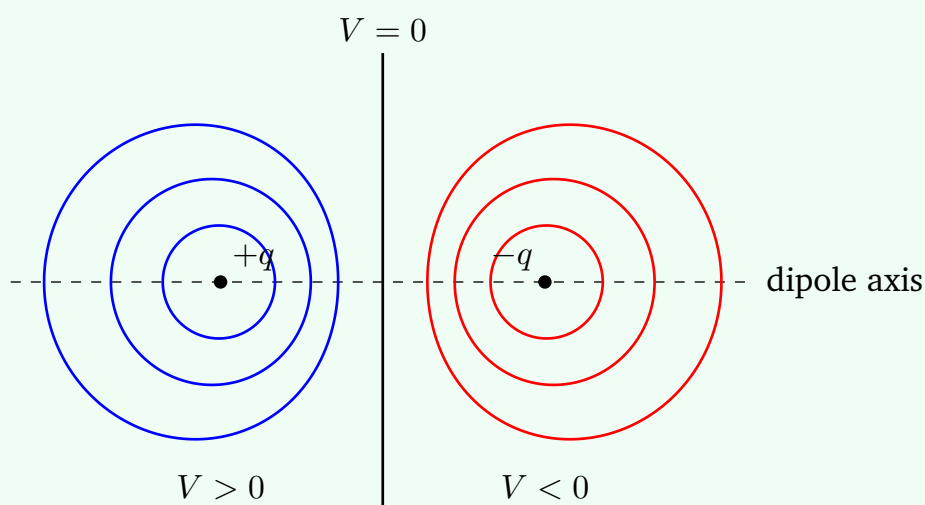
Concept:

- The potential of an electric dipole is $V = kq(1/r_+ - 1/r_-)$, where r_+ and r_- are distances from $+q$ and $-q$, and $k = 1/(4\pi\epsilon_0)$.
- An equipotential surface has constant potential; electric field lines cross it normally.

Step 1: Locate the zero-potential surface On the perpendicular bisector, $r_+ = r_-$, so the two potentials cancel. In three dimensions this is the entire equatorial plane.

Quick Tip to Solve: Draw the zero-potential perpendicular bisector first, then mirror the curved surfaces and reverse their potential signs.

Step 2: Depict the remaining surfaces The sketch shows meridional sections of nested positive and negative equipotential surfaces surrounding the respective charges. Rotating these sections about the dipole axis gives the surfaces. They are approximately spherical close to each charge and become distorted farther away.



Meridional sections of dipole equipotential surfaces; revolve about the horizontal dipole axis.

Step 3: Conclusion The equatorial plane has $V = 0$; the nested surfaces on the $+q$ side have $V > 0$, and those on the $-q$ side have $V < 0$.

Quick Tip to Solve: Draw the zero-potential perpendicular bisector first, then mirror the curved surfaces and reverse their potential signs.

20. Define 'angle of dip' at a place on earth.

[1 mark]

Solution:

Concept:

- The Earth's magnetic field $\mathbf{B}$ generally has horizontal and vertical components, B_H and B_V .
- The magnetic meridian is the vertical plane containing the local magnetic field direction.

Step 1: Specify the reference direction Take the horizontal direction in the magnetic meridian at the given place. The dip is measured from this horizontal to the direction of the Earth's total magnetic field, not from the geographical meridian.

Step 2: Define the inclination The angle of dip, or magnetic inclination I , is the angle made by the Earth's total magnetic field with the horizontal in the magnetic meridian. Its magnitude satisfies $\tan I = |B_V|/B_H$.

Step 3: Conclusion Angle of dip is the angle between the Earth's magnetic field and the horizontal, measured in the magnetic meridian at that place.

Quick Tip to Solve: For dip, use the horizontal reference in a vertical plane; declination instead compares directions in a horizontal plane.

Section B (14 marks)

21. A uniform wire is cut into three parts with their lengths in the ratio 2 : 3 : 6. The ends of each of these three parts are connected across an ideal battery of 10 V. If a current of 5 A is drawn from the battery, find the initial resistance of the wire. [2 marks]

Solution:

Concept:

- For a uniform wire, $R = \rho \ell / A$; unchanged resistivity ρ and cross-sectional area A make resistance proportional to length.
- Each piece connected across the same battery forms a parallel branch: $1/R_{\text{eq}} = \sum_i 1/R_i$.

Step 1: Express the resistances of the pieces Let the resistance of the original wire be R . Since the length ratio totals $2 + 3 + 6 = 11$, the three lengths are $2\ell/11$, $3\ell/11$, and $6\ell/11$. Hence

$$R_1 = \frac{2R}{11}, \quad R_2 = \frac{3R}{11}, \quad R_3 = \frac{6R}{11}.$$

Step 2: Find the parallel equivalent The total current drawn is 5 A, while each branch has 10 V across it. Thus

$$R_{\text{eq}} = \frac{10}{5} = 2 \Omega.$$

Using the branch resistances,

$$\frac{1}{R_{\text{eq}}} = \frac{11}{R} \left(\frac{1}{2} + \frac{1}{3} + \frac{1}{6} \right) = \frac{11}{R}.$$

Therefore $R = 11R_{\text{eq}}$.

Step 3: Conclusion The initial resistance of the uncut wire is $R = 22 \Omega$.

Quick Tip to Solve: Convert the length ratio to resistance fractions, then use reciprocals because all three pieces share the battery voltage.

22. A resistor R and an inductor L are connected in series to a source of voltage $V = V_0 \sin \omega t$. The voltage is found to lead current in phase by $\pi/4$. If the inductor is replaced by a capacitor C , the voltage lags behind current in phase by $\pi/4$. When L , C and R are connected in series with the same source, find the:

- average power dissipated and
- instantaneous current in the circuit.

[2 marks]

Solution:

Concept:

- For a series RL circuit, $\tan \phi = X_L/R$, with $X_L = \omega L$; for an RC circuit the voltage lag has magnitude $\tan |\phi| = X_C/R$, where $X_C = 1/(\omega C)$.
- For series RLC, $Z = \sqrt{R^2 + (X_L - X_C)^2}$ and $P_{\text{av}} = V_{\text{rms}} I_{\text{rms}} \cos \phi$.

Quick Tip to Solve: Equal RL lead and RC lag angles at the same frequency imply equal reactances; their cancellation makes the series RLC circuit resistive.

Step 1: Infer the two reactances The voltage leads current by $\pi/4$ in RL, so

$$\frac{X_L}{R} = \tan \frac{\pi}{4} = 1, \quad X_L = R.$$

On replacing the inductor with the capacitor, voltage lags current by the same angle. Thus

$$\frac{X_C}{R} = 1, \quad X_C = R.$$

The same source means the angular frequency remains ω .

Step 2: Find average power and instantaneous current With all three elements in series, $X_L - X_C = 0$, so the circuit is at resonance: $Z = R$ and voltage and current are in phase. The source amplitude is V_0 , hence $V_{\text{rms}} = V_0/\sqrt{2}$ and $I_{\text{rms}} = V_0/(\sqrt{2}R)$. Therefore

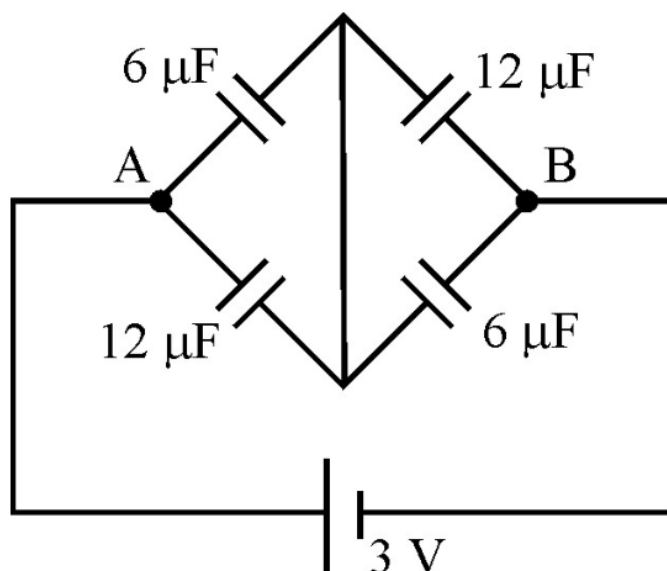
$$P_{\text{av}} = \frac{V_0^2}{2R}, \quad i(t) = \frac{V_0}{R} \sin \omega t.$$

Only the resistor dissipates net energy over a complete cycle.

Step 3: Conclusion (i) $P_{\text{av}} = V_0^2/(2R)$. (ii) $i(t) = (V_0/R) \sin \omega t$, in phase with the applied voltage.

Quick Tip to Solve: Equal RL lead and RC lag angles at the same frequency imply equal reactances; their cancellation makes the series RLC circuit resistive.

23. Find the total charge stored in the network of capacitors connected between A and B as shown in figure: [2 marks]



Solution:**Concept:**

- Capacitors sharing the same pair of nodes are parallel: $C_p = C_1 + C_2$. For series capacitors, $1/C_s = 1/C_1 + 1/C_2$.
- The terminal charge stored by a capacitor network is $Q = C_{eq}V$, where V is the voltage across its terminals.

Step 1: Identify the common central node The vertical conducting wire joins the top and bottom corners of the diamond, making them one node M . Thus the left $6\ \mu\text{F}$ and $12\ \mu\text{F}$ capacitors are parallel between A and M . Similarly, the right pair is parallel between M and B :

$$C_{AM} = 6 + 12 = 18\ \mu\text{F}, \quad C_{MB} = 12 + 6 = 18\ \mu\text{F}.$$

Step 2: Reduce the series combination and calculate charge The two equivalent capacitors are in series across the printed 3 V battery. Hence

$$C_{eq} = \frac{18 \times 18}{18 + 18} = 9\ \mu\text{F}.$$

The charge supplied to terminal A , with equal opposite charge at B , is

$$Q = C_{eq}V = (9\ \mu\text{F})(3\text{ V}) = 27\ \mu\text{C}.$$

The equal series groups each have a potential difference of 1.5 V .

Step 3: Conclusion The total charge stored by the network between A and B is $\boxed{27\ \mu\text{C}}$.

Quick Tip to Solve: Join points linked by a plain wire into one node before deciding which capacitors are parallel or series.

24. (a) Explain the formation of energy bands in crystalline solids.

(b) Draw the energy band diagrams of (i) a metal and (ii) a semiconductor.

[2 marks]

Solution:**Concept:**

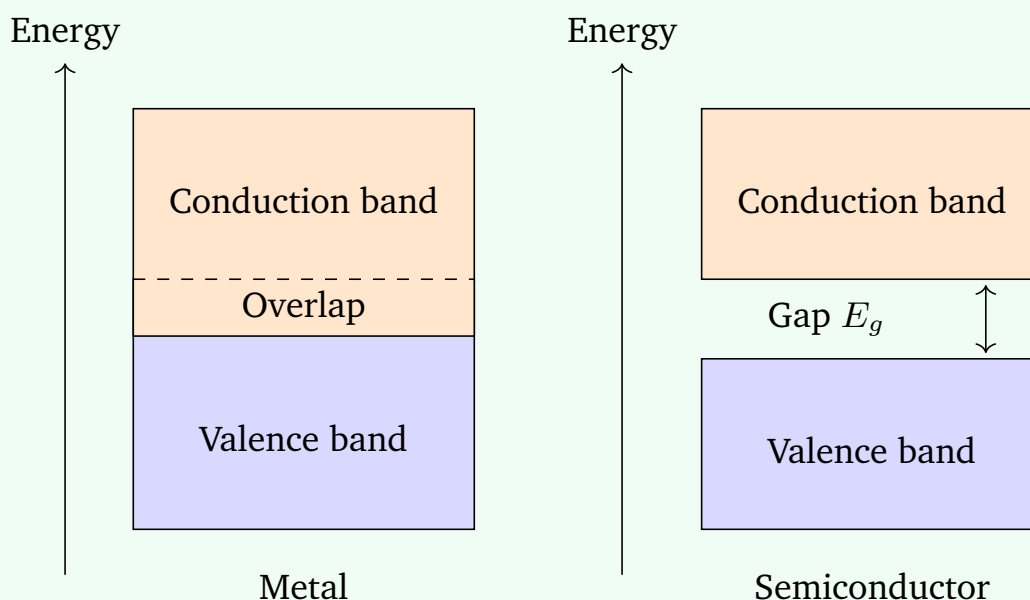
- An isolated atom has discrete electron energies; closely spaced atoms in a crystal interact and split these levels into many closely spaced allowed energies.
- The valence band contains valence-electron states, while the conduction band contains states available for mobile electrons. The forbidden energy gap E_g contains no allowed electron states.

Step 1: (a) Formation of allowed bands As a large number N of atoms approach one another to form a crystal, their outer electron wavefunctions overlap. Interatomic interaction and the Pauli exclusion principle split each atomic energy level into a large number of closely spaced levels. For a macroscopic crystal these levels are so close that they form an almost continuous allowed energy band. Adjacent allowed bands may be separated by a forbidden energy gap. This explains why a solid has bands

Quick Tip to Solve: Label both allowed bands and the semiconductor gap; show an actual overlap for the metal rather than a small positive gap.

rather than the isolated levels of a free atom.

Step 2: (b) Compare a metal and a semiconductor In a metal, the valence and conduction bands overlap (or the highest occupied band is partially filled), so electrons can access nearby empty states without crossing a forbidden gap. In an intrinsic semiconductor the valence band is full and the conduction band empty at 0 K, separated by a small positive gap of order 1 eV. At ordinary temperature some electrons acquire enough energy to cross this gap, leaving holes in the valence band. The diagrams show the overlapping-band metal and the small-gap semiconductor models.



Allowed energy bands: overlap in a metal and a small forbidden gap in a semiconductor.

Step 3: Conclusion Energy bands arise from the splitting of atomic levels in a crystal. A metal has accessible states without a band gap, whereas a semiconductor has a small forbidden gap between its valence and conduction bands.

Quick Tip to Solve: Label both allowed bands and the semiconductor gap; show an actual overlap for the metal rather than a small positive gap.

25. (a) Why is a photo diode operated under reverse bias condition?

(b) Draw V - I characteristic curves of photo diode for incident light of intensities I_1 and I_2 ($I_1 > I_2$).

[2 marks]

Solution:

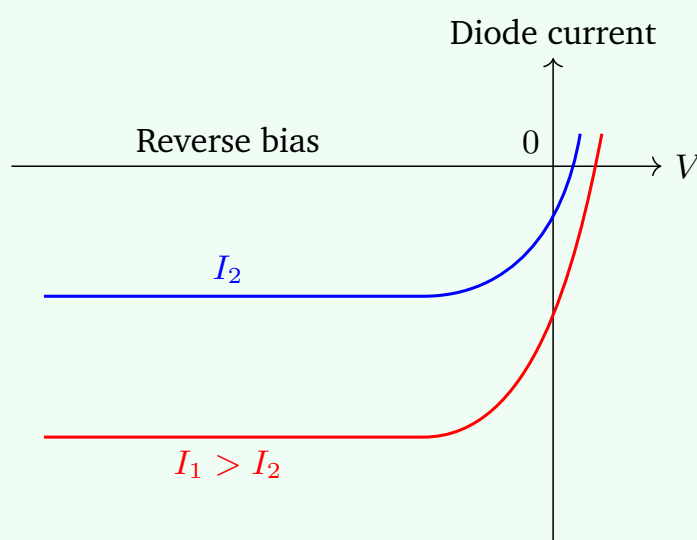
Concept:

- Incident photons create electron-hole pairs in the depletion region of a photodiode; its electric field separates these carriers and produces a photocurrent.
- Reverse bias widens the depletion region and efficiently collects photogenerated carriers. For fixed wavelength, photocurrent magnitude increases with incident light intensity.

Quick Tip to Solve: First read the printed intensity inequality; a larger light intensity puts the signed reverse-current curve farther below the voltage axis.

Step 1: (a) Reason for reverse-bias operation Reverse bias strengthens the junction electric field and widens the depletion region. Electron-hole pairs generated by light are rapidly separated and swept to the contacts, producing a reverse photocurrent. The dark reverse current is small, so a change in illumination produces a readily measurable change in current. Under forward bias the much larger injection current would obscure this small light-dependent signal. Thus reverse bias provides sensitive light detection.

Step 2: (b) Characteristics for the two printed intensities The paper gives $I_1 > I_2$, where these symbols denote incident light intensities, not diode currents. Greater intensity generates more electron-hole pairs each second; therefore the magnitude of reverse current is larger for I_1 . With signed diode current on the vertical axis, both reverse characteristics lie below the voltage axis and the I_1 curve lies below the I_2 curve. Before reverse breakdown the reverse photocurrent is nearly independent of reverse voltage; the sketch therefore shows approximately horizontal portions in the reverse-bias region. The joining portions close to zero voltage are schematic.



Schematic photodiode characteristics before breakdown; incident light intensities satisfy $I_1 > I_2$.

Step 3: Conclusion A photodiode is reverse biased for efficient, sensitive collection of photogenerated carriers. For the printed relation $I_1 > I_2$, the reverse-current magnitude is larger for illumination I_1 , as shown below the I_2 curve.

Quick Tip to Solve: First read the printed intensity inequality; a larger light intensity puts the signed reverse-current curve farther below the voltage axis.

25. (a) State the level of doping and biasing condition used in light emitting diode (LED).

(b) Write any two advantages of LED over the conventional low power lamps.

[2 marks]

Solution:

Concept:

- A light emitting diode is a heavily doped p-n junction made from a suitable direct-band-gap semiconductor.
- In forward bias, injected electrons and holes recombine and release energy as photons. Light output follows the forward current.

Step 1: (a) State doping and operating bias An LED uses a heavily doped p-n junction and is operated under forward bias: its p-side is connected to the positive terminal and n-side to the negative terminal. The reduced junction barrier allows electrons and holes to enter the junction region and recombine. In a suitable LED material, recombination releases light, with photon energy approximately equal to the band gap, $h\nu \simeq E_g$. Reverse bias is not the normal light-emitting operating condition.

Step 2: (b) Two advantages over conventional low-power lamps First, an LED has low power consumption and higher light-producing efficiency: a larger fraction of electrical input can produce useful light instead of heating a filament. Second, it has a long operating life and high reliability because it has no fragile heated filament that readily burns out. These are two distinct advantages; the first concerns electrical energy use and the second concerns durability in operation. They make LEDs suitable for indicators and frequently used low-power lighting.

Step 3: Conclusion An LED is heavily doped and forward biased. Two advantages over conventional low-power lamps are low power consumption (high efficiency) and long operating life.

Quick Tip to Solve: Pair heavy doping with forward bias, and give two distinct benefits such as efficiency and lifetime rather than repeating low power in different words.

26. Briefly explain how a potential barrier is set up across a p-n junction as a result of diffusion and drift of the charge carriers.

[2 marks]

Solution:

Concept:

- Diffusion carries majority charge carriers from the region of high concentration towards the region of low concentration.
- An electric field produces drift of charge carriers. At equilibrium, drift and diffusion currents across an unbiased p-n junction are equal and opposite.

Step 1: Explain diffusion and depletion Immediately after the junction is formed, electrons diffuse from the n side to the p side and holes diffuse from the p side to the n side. They recombine near the junction. This leaves positively charged, immobile donor ions on the n side and negatively charged, immobile acceptor ions on the p side. The region depleted of mobile majority carriers is the depletion

Quick Tip to Solve: First identify the fixed ions: positive on n, negative on p. Their field points n to p and opposes majority-carrier diffusion.

layer.

Step 2: Explain the barrier and opposing drift The uncovered ions set up an electric field directed from the n side to the p side and hence a potential barrier across the junction. This field opposes further majority-carrier diffusion. It drives electrons towards the n side and holes towards the p side, giving a drift current opposite to the diffusion current. As the barrier grows, diffusion decreases until the two currents balance; the net current is then zero.

Step 3: Conclusion Diffusion and recombination create the charged depletion layer, whose electric field establishes the potential barrier. Equilibrium is reached when drift balances diffusion.

Quick Tip to Solve: First identify the fixed ions: positive on n, negative on p. Their field points n to p and opposes majority-carrier diffusion.

27. Light of same wavelength is incident on three photo-sensitive surfaces A, B and C. The following observations are recorded.

(i) From surface A, photo electrons are not emitted.

(ii) From surface B, photo electrons are just emitted.

(iii) From surface C, photo electrons with some kinetic energy are emitted.

Compare the threshold frequencies of the three surfaces and justify your answer.

[2 marks]

Solution:

Concept:

- The threshold frequency ν_0 is the minimum incident frequency required for photoelectric emission; the work function is $\phi = h\nu_0$.
- Einstein's equation is $K_{\max} = h\nu - h\nu_0$, where ν is the incident frequency and h is Planck's constant.

Step 1: Use the absence of emission from A The incident wavelength is the same for all three surfaces, so the incident frequency $\nu = c/\lambda$ and photon energy $h\nu$ are the same. No electrons are emitted from A: its work function exceeds the photon energy. Thus $h\nu < h\nu_{0A}$, or $\nu < \nu_{0A}$.

Step 2: Compare the emission thresholds of B and C Electrons are just emitted from B, so the incident photon supplies exactly the minimum energy required to escape. Their maximum kinetic energy is zero and $\nu = \nu_{0B}$. Electrons from C have positive kinetic energy, so $h\nu - h\nu_{0C} > 0$, giving $\nu > \nu_{0C}$. Combining these inequalities compares the three threshold frequencies directly, without requiring their numerical values.

Step 3: Conclusion The threshold frequencies satisfy

$$\nu_{0A} > \nu_{0B} > \nu_{0C}.$$

In particular, $\nu_{0B} = \nu$, while A has a higher and C a lower threshold than the incident frequency.

Quick Tip to Solve: At a fixed wavelength, translate no emission, just emission and energetic emission into $\nu < \nu_0$, $\nu = \nu_0$ and $\nu > \nu_0$, respectively.

27. If the frequency of light incident on the cathode of a photo-cell is increased, how will the following be affected? Justify your answer.

(i) Energy of the photo electrons.

(ii) Photo current.

[2 marks]

Solution:

Concept:

- Einstein's photoelectric equation gives $K_{\max} = h\nu - \phi$, where ϕ is the cathode work function, h is Planck's constant and ν is the incident frequency.
- Photocurrent measures the charge collected per second. In the usual school-level comparison it depends on illumination intensity and collection conditions, whereas frequency controls the maximum electron kinetic energy.

Step 1: Determine the change in electron energy For a given cathode, ϕ remains constant. Increasing the incident frequency from ν_1 to ν_2 therefore increases the maximum photoelectron kinetic energy by

$$\Delta K_{\max} = h(\nu_2 - \nu_1) > 0.$$

This statement assumes that the light is above the threshold frequency so emission occurs. The emitted electrons have a range of energies; Einstein's equation specifies their maximum energy.

Step 2: State the photocurrent comparison and its condition With the intensity and collection conditions unchanged, the standard photoelectric-effect comparison gives no change in photocurrent simply because the frequency increases. Frequency changes the energy of the emitted electrons rather than the emission rate in this comparison. The question does not specify a change in intensity, so an increase or decrease of photocurrent cannot be inferred from a frequency change alone; the usual unchanged-current answer assumes fixed illumination and collection conditions.

Step 3: Conclusion (i) The maximum photoelectron kinetic energy increases. (ii) Photocurrent remains unchanged under the usual assumption of unchanged intensity and collection conditions; frequency alone does not determine its change.

Quick Tip to Solve: Use $K_{\max} = h\nu - \phi$ for electron energy, and check intensity and collection conditions separately before predicting a photocurrent change.

Section C (21 marks)

28. (a) Explain the principle of working of a potentiometer.

(b) In a potentiometer, a standard source of emf 5 V and negligible internal resistance maintains a steady current through the potentiometer wire of length 10 m. Two primary cells of emf E_1 and E_2 are joined together in a series with (i) same polarity and (ii) opposite polarity. The combination is connected to the potentiometer circuit in each case. The balancing length of the wire in the two cases are found to be 700 cm and 100 cm, respectively. Find the values of emf of the two cells. [3 marks]

Solution:

Concept:

- For a uniform potentiometer wire carrying a steady current, the potential drop is proportional to its length: $V = k\ell$, where k is the potential gradient.
- At the balance point the galvanometer current is zero, so the emf of the test source equals the wire's potential drop and is measured without drawing current from that source.
- Series cells with aiding polarities give $E_1 + E_2$; opposing polarities give the magnitude $|E_1 - E_2|$.

Step 1: (a) Explain the potentiometer principle Let the wire have resistivity ρ , uniform cross-sectional area A , and steady current I . A length ℓ has resistance $R = \rho\ell/A$. By Ohm's law, its potential drop is

$$V = IR = \frac{I\rho}{A}\ell = k\ell.$$

Thus the potential drop along a uniform wire is directly proportional to length when its current is constant. At balance, $E = k\ell$. No current flows through the test cell, so its internal resistance does not affect the emf measurement.

Step 2: (b) Calculate the potential gradient The negligible internal resistance of the standard source makes the potential difference across the full wire 5 V. Its length is 10 m = 1000 cm, so

$$k = \frac{5}{1000} = 0.005 \text{ V cm}^{-1}.$$

This gradient applies to both combinations.

Step 3: Use the aiding and opposing balance lengths Aiding cells balance at 700 cm and opposing cells at 100 cm. Thus

$$\begin{aligned}E_1 + E_2 &= k(700) = 3.5 \text{ V}, \\|E_1 - E_2| &= k(100) = 0.5 \text{ V}.\end{aligned}$$

Calling the larger emf E_1 , add and subtract the equations:

$$\begin{aligned}2E_1 &= 3.5 + 0.5 = 4.0 \text{ V}, \\2E_2 &= 3.5 - 0.5 = 3.0 \text{ V}.\end{aligned}$$

Interchanging the cell labels gives the same pair of values.

Step 4: Conclusion The principle is $V \propto \ell$ for a uniform wire carrying steady current. The two cell emfs are 2.0 V and 1.5 V; taking $E_1 > E_2$, $E_1 = 2.0 \text{ V}$ and $E_2 = 1.5 \text{ V}$.

Quick Tip to Solve: Convert the full wire length to the balance-length unit, then use half the sum and half the difference of the two balanced voltages.

29. (a) Differentiate between self inductance and mutual inductance.

(b) The mutual inductance of two coaxial coils is 2 H. The current in one coil is changed uniformly from zero to 0.5 A in 100 ms. Find the:

(i) change in magnetic flux through the other coil.

(ii) emf induced in the other coil during the change.

[3 marks]

Solution:

Concept:

- Self inductance L measures the flux linkage of a coil due to its own current: $N\Phi = LI$. A changing current induces a back emf $\varepsilon = -L dI/dt$ in that same coil.
- Mutual inductance M measures the flux linkage of one coil due to current in another: $N_2\Phi_{21} = MI_1$. The induced emf in coil 2 is $\varepsilon_2 = -M dI_1/dt$. Both coefficients have SI unit henry.

Step 1: (a) Distinguish the two inductances In self induction, the changing current and the induced emf belong to the same coil. Its inductance depends on its turns, dimensions and core permeability. In mutual induction, a change of current in one coil induces emf in a separate, magnetically coupled coil. Mutual inductance depends on both coils and their relative position, as well as the core. In either case the minus sign expresses Lenz's law: the induced effect opposes the change producing it.

Step 2: (b)(i) Calculate the change in linked magnetic flux The current change is $\Delta I_1 = 0.5 - 0 = 0.5$ A, and $M = 2$ H. Thus

$$\Delta(N_2\Phi_{21}) = M\Delta I_1 = 2 \times 0.5 = 1.0 \text{ Wb.}$$

This is the change in total flux linked with the other coil, conventionally called the magnetic flux through the coil in this question. If flux per turn is intended, it is $1.0/N_2$ Wb; the number of turns is not specified.

Step 3: (b)(ii) Calculate the induced emf Convert the duration before substituting: $100 \text{ ms} = 0.100 \text{ s}$. Since the current changes uniformly, the emf magnitude is constant throughout this interval:

$$|\varepsilon_2| = M \frac{\Delta I_1}{\Delta t} = 2 \frac{0.5}{0.100} = 10 \text{ V.}$$

Its polarity opposes the increasing flux; an absolute sign cannot be assigned without a chosen winding direction and terminal convention.

Step 4: Conclusion Self inductance concerns one coil's own current; mutual inductance concerns coupling between two coils. The change in flux linkage is 1.0 Wb, and the induced emf has magnitude 10 V.

Quick Tip to Solve: Calculate $M\Delta I$ first, then divide by the time in seconds to obtain the emf; do not treat milliseconds as seconds.

30. Explain the principle of the device with diagram, which is used to provide electricity at the proper voltage for household purposes.

Briefly discuss loss of energy in it due to flux leakage and its minimization.

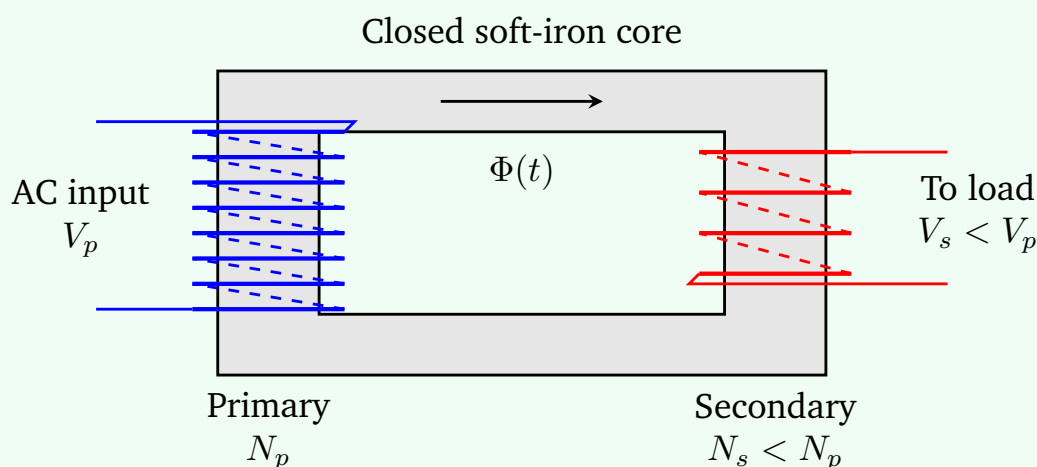
[3 marks]

Solution:

Concept:

- A transformer transfers electrical energy between two coils through mutual induction. A changing primary current produces changing magnetic flux in a common magnetic core.
- For an ideal transformer, $V_s/V_p = N_s/N_p$, where V_p, V_s are the primary and secondary rms voltages and N_p, N_s are the respective numbers of turns.

Step 1: Identify and describe the device A step-down transformer reduces the distribution voltage to the appropriate household supply voltage. It has a primary winding connected to the higher-voltage AC supply and a secondary winding connected to the domestic load. Both insulated windings are wound on a common soft-iron core. The secondary has fewer turns than the primary, $N_s < N_p$, so its output voltage is smaller. The diagram shows the essential winding and core arrangement; the winding counts are schematic.



Schematic step-down transformer with a common closed magnetic core.

Step 2: Explain the principle of operation Alternating current in the primary produces an alternating flux Φ in the core. This flux links both coils and, by Faraday's law, induces emfs

$$\varepsilon_p = -N_p \frac{d\Phi}{dt}, \quad \varepsilon_s = -N_s \frac{d\Phi}{dt}.$$

Neglecting winding resistance and other losses, the ratio of rms voltages equals the turns ratio: $V_s/V_p = N_s/N_p < 1$. Electrical energy is transferred by the changing magnetic field. A steady DC input cannot sustain this operation because, after the initial change, it produces no changing flux and hence no continuous secondary emf.

Step 3: Describe flux leakage and its minimization Some flux produced by the primary passes through the surrounding air instead of linking the secondary. This leakage reduces magnetic coupling, so less of the primary flux contributes to the secondary emf and useful energy transfer is reduced. Flux leakage is minimized by placing the two windings close together, often winding them over one another

Quick Tip to Solve: For a household step-down transformer, label the primary as having more turns; explain leakage using flux that misses the secondary.

with suitable insulation, and using a closed core of high-permeability soft iron to provide an easy magnetic path linking both windings. Laminating the core controls eddy-current loss; close coupling and a closed magnetic path specifically address flux leakage.

Step 4: Conclusion The device is a step-down transformer working on mutual induction. With $N_s < N_p$, it delivers a lower AC voltage. Closely coupled windings on a closed high-permeability core minimize flux leakage.

Quick Tip to Solve: For a household step-down transformer, label the primary as having more turns; explain leakage using flux that misses the secondary.

31. (a) We feel the warmth of the sun light but not the pressure on our hands. Explain.
(b) Which out of wavelength, frequency and speed of an electro-magnetic wave does not change on passing from one medium to another?
(c) A thin ozone layer in the upper atmosphere is crucial for human survival on earth, why? [3 marks]

Solution:

Concept:

- Electromagnetic radiation transports energy and momentum. Absorbed radiant energy raises the temperature of matter, while transferred momentum produces radiation pressure.
- At a stationary boundary the wave frequency ν remains fixed by the source. The relation $v = \nu\lambda$ connects speed v , frequency and wavelength λ .
- Atmospheric ozone absorbs much of the harmful ultraviolet radiation from the Sun and thereby protects living organisms.

Step 1: (a) Compare heating and radiation pressure Sunlight falling on the hand transfers energy to the skin when it is absorbed. This energy increases the internal energy of the skin and produces a perceptible warming effect. Light also carries momentum, so absorption exerts a force on the hand. For incident intensity I , normal incidence and complete absorption, the radiation pressure is $p = I/c$, where c is the speed of light in vacuum. Because c is very large, this pressure is extremely small at ordinary sunlight intensities and cannot be felt by our hands. Reflection can increase the momentum transfer, but even the pressure $2I/c$ for complete normal reflection remains very small. Thus the pressure exists although our sense of touch does not detect it.

Step 2: (b) Identify the unchanged quantity The frequency of an electromagnetic wave does not change on passing from one medium to another. The oscillations in the second medium must match the incident oscillations at the boundary, so their rate is the same. The wave speed generally changes because the optical properties of the medium change. Since $\lambda = v/\nu$, the wavelength changes in the same proportion as the speed while the frequency remains unchanged.

Step 3: (c) Explain the protective role of ozone The thin ozone layer in the upper atmosphere absorbs much of the Sun's harmful ultraviolet radiation before it reaches Earth's surface. Excessive ultraviolet exposure damages biological molecules, including DNA, and can cause skin cancer and eye damage. By greatly reducing this exposure, the ozone layer protects human health and other forms of life, making its presence crucial for survival on Earth.

Quick Tip to Solve: At a medium boundary keep frequency fixed, then use wavelength equals speed divided by frequency to decide which other quantities change.

Step 4: Conclusion (a) Absorbed solar energy causes detectable warmth, whereas radiation pressure is too small to feel. (b) Frequency remains unchanged. (c) Ozone absorbs harmful ultraviolet radiation and protects life on Earth.

Quick Tip to Solve: At a medium boundary keep frequency fixed, then use wavelength equals speed divided by frequency to decide which other quantities change.

32. Two coherent light waves of intensity $5 \times 10^{-2} \text{ W m}^{-2}$ each super-impose and produce the interference pattern on a screen. At a point where the path difference between the waves is $\lambda/6$, λ being the wavelength of the wave, find the

(a) phase difference between the waves.

(b) resultant intensity at the point.

(c) resultant intensity in terms of the intensity at the maximum.

[3 marks]

Solution:

Concept:

- For waves of wavelength λ , path difference Δx produces phase difference $\phi = 2\pi\Delta x/\lambda$.
- For coherent waves, $I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \phi$. For equal individual intensities I_0 , this becomes $I = 2I_0(1 + \cos \phi)$.
- The maximum occurs for waves in phase: $I_{\max} = 4I_0$. Intensity is proportional to the square of resultant amplitude.

Step 1: Part (a): convert path difference into phase difference The given path difference is $\Delta x = \lambda/6$. One full wavelength corresponds to a phase change of 2π radians, so

$$\phi = \frac{2\pi}{\lambda} \frac{\lambda}{6} = \frac{\pi}{3} \text{ rad} = 60^\circ.$$

The wavelength cancels; its numerical value is unnecessary.

Step 2: Part (b): evaluate the resultant intensity Each wave has intensity $I_0 = 5 \times 10^{-2} \text{ W m}^{-2}$. Since $\cos(\pi/3) = 1/2$, the interference formula gives

$$I = 2(5 \times 10^{-2})(1 + 1/2) = 3(5 \times 10^{-2}) = 1.5 \times 10^{-1} \text{ W m}^{-2}.$$

Adding the individual intensities alone would omit the interference term. Here that term is positive, so the result exceeds $2I_0$.

Step 3: Part (c): compare with maximum intensity At constructive interference the amplitudes add fully, giving

$$I_{\max} = 4I_0 = 2.0 \times 10^{-1} \text{ W m}^{-2}, \quad \frac{I}{I_{\max}} = \frac{3I_0}{4I_0} = \frac{3}{4}.$$

Equivalently, the equal-intensity relation $I/I_{\max} = \cos^2(\phi/2)$ gives $\cos^2 30^\circ = 3/4$.

Step 4: Conclusion (a) $\phi = \pi/3 \text{ rad}$. (b) $I = 0.15 \text{ W m}^{-2}$. (c) $I = \frac{3}{4}I_{\max}$.

Quick Tip to Solve: For equal coherent waves, first find the phase and use $I/I_{\max} = \cos^2(\phi/2)$.

33. An object is placed in front of a concave mirror of focal length 12 cm. There are two possible positions of the object for which the image formed is three times the size of the object.

(a) Draw the ray diagram for each case, and

(b) Find the distance between the two positions of the object.

[3 marks]

Solution:

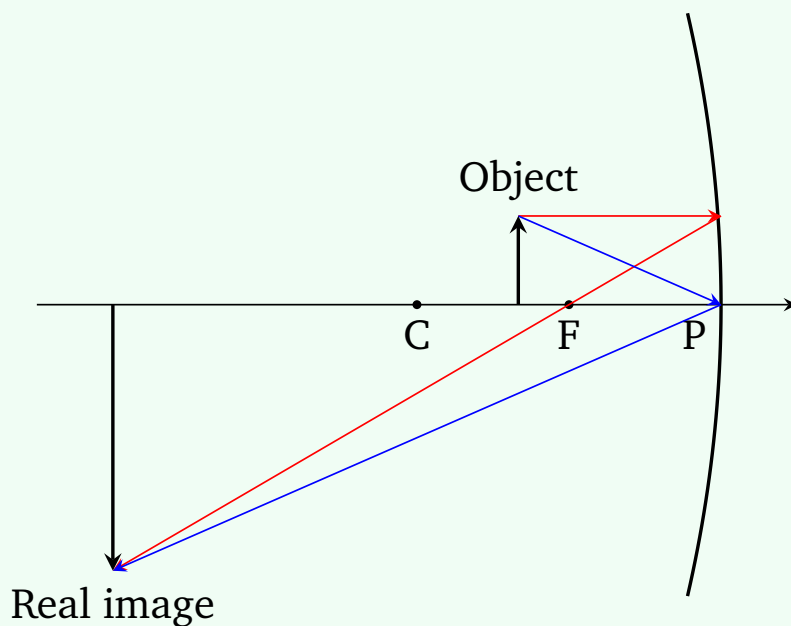
Concept:

- With Cartesian signs, a concave mirror has $f = -12$ cm and a real object has $u < 0$. The mirror equation is $1/f = 1/v + 1/u$.
- Transverse magnification is $m = -v/u$. Threefold image size permits $m = -3$ for an inverted real image or $m = +3$ for an upright virtual image.
- A ray parallel to the principal axis reflects through the focus. A ray incident at the pole reflects with equal angles to the principal axis.

Step 1: Part (a): real, inverted image and its object position For the real image, $m = -3$, so $v = 3u$. Substituting in the mirror equation,

$$\frac{1}{f} = \frac{1}{3u} + \frac{1}{u} = \frac{4}{3u}, \quad u = \frac{4f}{3} = -16 \text{ cm}, \quad v = -48 \text{ cm}.$$

The object lies between F at 12 cm and C at 24 cm in front of the mirror. The reflected rays meet beyond C , giving an inverted image three times as high as the object.



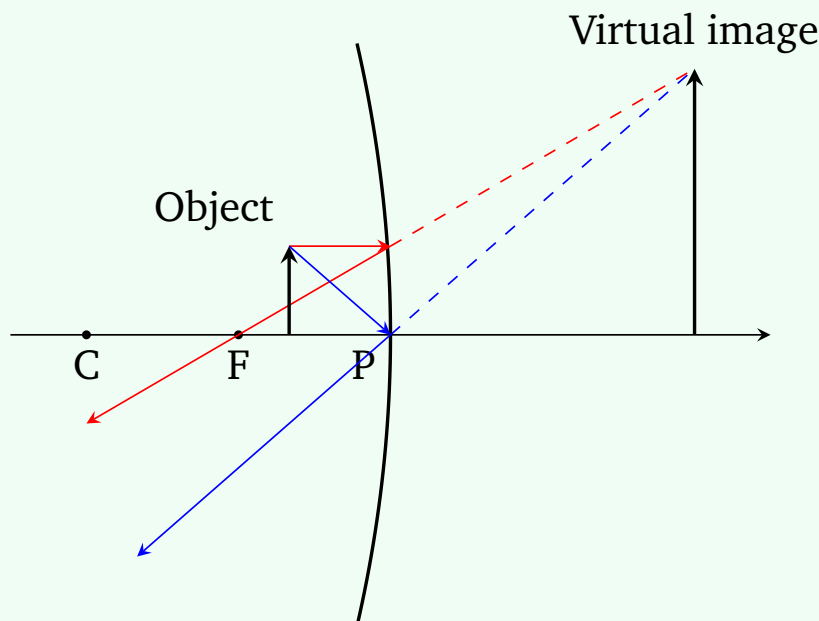
Inverted real image: object between focus and centre of curvature.

Step 2: Part (a): virtual, upright image and its object position For the virtual image, $m = +3$, so $v = -3u$. Therefore

$$\frac{1}{f} = -\frac{1}{3u} + \frac{1}{u} = \frac{2}{3u}, \quad u = \frac{2f}{3} = -8 \text{ cm}, \quad v = +24 \text{ cm}.$$

Quick Tip to Solve: For a stated image-size ratio, solve for both signs of magnification before comparing object distances.

The object is between F and the pole P . The reflected rays diverge; their backward extensions intersect behind the mirror, giving an upright image three times the object height. The dashed lines in the diagram represent these extensions.



Upright virtual image: object between focus and pole; dashed rays are backward extensions.

Step 3: Part (b): separate the two positions Both objects lie on the same side of the mirror, at distances 16 cm and 8 cm from its pole. Their separation is

$$d = |(-16) - (-8)| \text{ cm} = 8 \text{ cm}.$$

Taking both signs of magnification is essential: the phrase three times the size specifies magnitude, while orientation distinguishes the two cases.

Step 4: Conclusion The object positions are 16 cm and 8 cm in front of the mirror, separated by 8 cm.

Quick Tip to Solve: For a stated image-size ratio, solve for both signs of magnification before comparing object distances.

34. A hollow conducting sphere of inner radius r_1 and outer radius r_2 has a charge Q on its surface. A point charge $-q$ is also placed at the centre of the sphere.

- (a) What is the surface charge density on the (i) inner and (ii) outer surface of the sphere?
 (b) Use Gauss' law of electrostatics to obtain the expression for the electric field at a point lying outside the sphere. [3 marks]

Solution:

Concept:

- Inside a conductor in electrostatic equilibrium, $\mathbf{E} = 0$. Gauss's law is $\oint \mathbf{E} \cdot d\mathbf{A} = Q_{\text{enc}}/\epsilon_0$.

Quick Tip to Solve: First find zero enclosed charge for a Gaussian surface in the metal, then conserve the shell's net charge.

- The charge Q is the net charge of the conducting shell. Induction redistributes this charge between its two surfaces without changing the shell's total.
- The central charge and concentric spherical surfaces give spherical symmetry, so each surface charge density is uniform and the exterior field is radial.

Step 1: Part (a)(i): find the inner-surface charge Take a spherical Gaussian surface entirely inside the metal, with radius between r_1 and r_2 . Its electric flux is zero because the field in the metal vanishes. If the inner-surface charge is Q_{in} , Gauss's law gives

$$0 = \frac{-q + Q_{\text{in}}}{\epsilon_0}, \quad Q_{\text{in}} = +q.$$

Dividing by the inner area therefore gives $\sigma_{\text{in}} = q/(4\pi r_1^2)$.

Step 2: Part (a)(ii): conserve the charge of the shell The net charge carried by the conductor remains Q , so

$$Q_{\text{out}} + Q_{\text{in}} = Q, \quad Q_{\text{out}} = Q - q.$$

The outer area is $4\pi r_2^2$, hence $\sigma_{\text{out}} = (Q - q)/(4\pi r_2^2)$. The central point charge belongs to the cavity, not to the conductor's specified net charge.

Step 3: Part (b): apply Gauss's law outside For a concentric Gaussian sphere of radius $r > r_2$, the total enclosed charge is

$$-q + Q_{\text{in}} + Q_{\text{out}} = Q - q.$$

The radial field has the same magnitude at every point of this sphere. Thus

$$E_r(4\pi r^2) = \frac{Q - q}{\epsilon_0}, \quad \mathbf{E}(r) = \frac{Q - q}{4\pi\epsilon_0 r^2} \hat{\mathbf{r}}.$$

Here $\hat{\mathbf{r}}$ points radially outward. The field points outward for $Q > q$, inward for $Q < q$, and vanishes outside for $Q = q$.

Step 4: Conclusion $\sigma_{\text{in}} = q/(4\pi r_1^2)$, $\sigma_{\text{out}} = (Q - q)/(4\pi r_2^2)$, and $\mathbf{E} = (Q - q)\hat{\mathbf{r}}/(4\pi\epsilon_0 r^2)$ for $r > r_2$.

Quick Tip to Solve: First force zero enclosed charge for a Gaussian surface in the metal, then conserve the shell's net charge.

34. (a) An infinitely long thin straight wire has a uniform linear charge density λ . Obtain the expression for the electric field E at a point lying at a distance x from the wire, using Gauss' law.
 (b) Show graphically the variation of this electric field E as a function of distance x from the wire.

[3 marks]

Solution:**Concept:**

- Gauss' law states that the electric flux through a closed surface is $\oint \mathbf{E} \cdot d\mathbf{A} = Q_{\text{enc}}/\epsilon_0$, where ϵ_0 is the permittivity of free space.
- An infinite uniformly charged straight wire has cylindrical symmetry. Its field is radial, and its magnitude depends only on the perpendicular distance x . For positive linear charge density λ , the field points away from the wire.

Step 1: (a) Choose a cylindrical Gaussian surface Take a cylinder of radius x and length l , coaxial with the wire. The charge enclosed is $Q_{\text{enc}} = \lambda l$. On its curved surface, the field is perpendicular to the surface and has the same magnitude everywhere. On both end caps, the field lies parallel to their planes, so their flux is zero. Thus the total flux is $E(2\pi xl)$; it is not the field multiplied by the total area including the caps.

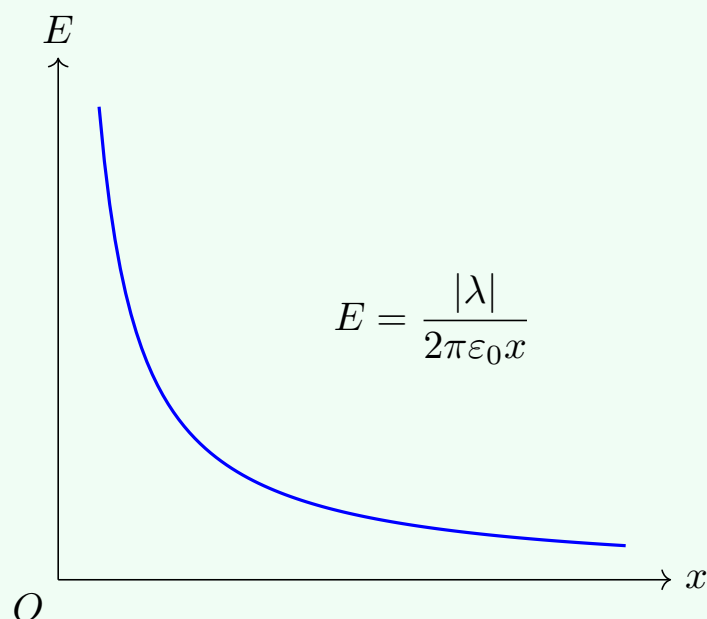
Step 2: Apply Gauss' law and obtain the field For $\lambda > 0$, Gauss' law gives

$$E(2\pi xl) = \frac{\lambda l}{\epsilon_0}, \quad E(x) = \frac{\lambda}{2\pi\epsilon_0 x}.$$

The length l cancels, as required for an infinitely long wire. More generally, the vector field is $\mathbf{E} = \lambda \hat{\mathbf{r}}/(2\pi\epsilon_0 x)$, and its magnitude is $|\lambda|/(2\pi\epsilon_0 x)$. A negatively charged wire therefore produces an inward field.

Step 3: (b) Plot the dependence on distance For a fixed nonzero charge density, $E \propto 1/x$. Hence doubling the distance halves the field. The magnitude decreases continuously with distance, diverges as $x \rightarrow 0^+$ in the ideal thin-wire model, and approaches zero as $x \rightarrow \infty$. Its graph is a rectangular hyperbola in the first quadrant; it does not intersect either axis.

Quick Tip to Solve: Choose a coaxial cylinder: only its curved surface carries flux, and the arbitrary cylinder length cancels.



Electric field magnitude versus distance from an infinite charged wire.

Step 4: Conclusion The field magnitude is $E = |\lambda|/(2\pi\epsilon_0 x)$, directed radially outward for positive charge and inward for negative charge. The E -versus- x graph decreases as $1/x$.

Quick Tip to Solve: Choose a coaxial cylinder: only its curved surface carries flux, and the arbitrary cylinder length cancels.

Section D (15 marks)

35. (a) Derive the law of radioactive decay $N = N_0 e^{-\lambda t}$.

(b) The half life of ${}^{238}_{92}\text{U}$ undergoing α -decay is 4.5×10^9 years. Find its mean life.

(c) What fraction of the initial mass of a radioactive substance will decay in five half-life periods?

[5 marks]

Solution:

Concept:

- Radioactive decay is spontaneous and random. Each undecayed nucleus has a constant probability λ of decaying per unit time, where λ is the decay constant.
- The half-life $T_{1/2}$ is the time for the number of undecayed nuclei to halve. The mean life τ is their average lifetime: $\tau = 1/\lambda = T_{1/2}/\ln 2$.
- The mass of the remaining parent isotope is proportional to its number of undecayed nuclei, so the surviving parent-mass fraction equals N/N_0 .

Step 1: (a) Set up the differential decay equation Let N be the number of undecayed nuclei at time t , with $N = N_0$ at $t = 0$. In a short interval dt , the expected number that decay is $\lambda N dt$. Since decay

Quick Tip to Solve: After n half-lives, first find the surviving fraction 2 to the power minus n , then subtract it from one to obtain the decayed fraction.

reduces the parent population, its change obeys

$$dN = -\lambda N dt, \quad \frac{dN}{dt} = -\lambda N.$$

The minus sign describes the decrease of N ; the decay rate or activity $-dN/dt$ is positive. This macroscopic law applies to a large population of identical unstable nuclei with a constant decay probability.

Step 2: Integrate with the initial condition Separating variables and integrating between the initial and final populations gives

$$\begin{aligned} \int_{N_0}^N \frac{dN'}{N'} &= -\lambda \int_0^t dt', \\ \ln \left(\frac{N}{N_0} \right) &= -\lambda t, \\ N &= N_0 e^{-\lambda t}. \end{aligned}$$

Thus the surviving population decreases exponentially. At $t = 0$ this expression reproduces N_0 , and for positive λ it decreases with increasing time, consistent with the physical process.

Step 3: (b) Relate half-life to mean life and substitute Set $N = N_0/2$ at $t = T_{1/2}$ in the decay law:

$$\frac{1}{2} = e^{-\lambda T_{1/2}}, \quad \lambda T_{1/2} = \ln 2.$$

Hence the mean life of uranium-238 is

$$\tau = \frac{1}{\lambda} = \frac{T_{1/2}}{\ln 2} = \frac{4.5 \times 10^9}{0.693} \text{ years} \simeq 6.49 \times 10^9 \text{ years}.$$

The mean life is about 1.443 times the half-life. The fact that this isotope undergoes alpha decay does not change the half-life-to-mean-life relation.

Step 4: (c) Distinguish the remaining fraction from the decayed fraction Each half-life leaves half the preceding parent population. After five half-lives,

$$\frac{N}{N_0} = \left(\frac{1}{2} \right)^5 = \frac{1}{32}.$$

Therefore the fraction of the initial parent mass that has decayed is

$$\frac{m_0 - m}{m_0} = 1 - \frac{m}{m_0} = 1 - \frac{1}{32} = \frac{31}{32} = 0.96875.$$

This is the fraction transformed from the original radioactive parent isotope, rather than a claim that this fraction of the sample's total matter disappears.

Step 5: Conclusion (a) $N = N_0 e^{-\lambda t}$. (b) The mean life of uranium-238 is 6.49×10^9 years, approximately 6.5×10^9 years. (c) The fraction of the initial parent mass decayed is $31/32$, or 96.875%.

Quick Tip to Solve: After n half-lives, first find the surviving fraction 2 to the power minus n , then subtract it from one to obtain the decayed fraction.

35. (a) State the postulates of Bohr's model of hydrogen atom and derive the expression for Bohr radius.

(b) Find the ratio of the longest and the shortest wavelengths amongst the spectral lines of Balmer series in the spectrum of hydrogen atom. [5 marks]

Solution:

Concept:

- In Bohr's model the electron of mass m_e moves in permitted stationary circular orbits under the Coulomb attraction of the proton. The angular momentum is quantised: $m_e v r = n\hbar$, where $n = 1, 2, \dots$ and $\hbar = h/(2\pi)$.
- The hydrogen spectrum obeys $1/\lambda = R_H(1/n_f^2 - 1/n_i^2)$. In the Balmer series the final level is $n_f = 2$, and the initial level is $n_i = 3, 4, \dots$

Step 1: (a) State Bohr's postulates An electron revolves around the nucleus only in certain allowed stationary orbits. It does not radiate energy while remaining in such an orbit, so each orbit has a definite energy. The allowed orbits satisfy the angular-momentum condition $m_e v r = n\hbar/(2\pi)$. Radiation is emitted or absorbed only when the electron jumps between two stationary orbits. The frequency satisfies $h\nu = |E_i - E_f|$: a transition to a lower energy emits a photon, whereas a transition to a higher energy requires absorption.

Step 2: (a) Obtain the radius of an allowed orbit For a hydrogen atom, the electric attraction supplies the centripetal force:

$$\frac{m_e v^2}{r} = \frac{e^2}{4\pi\epsilon_0 r^2}.$$

From angular-momentum quantisation, $v = n\hbar/(2\pi m_e r)$. Substituting this velocity into the force equation gives

$$\frac{n^2 \hbar^2}{4\pi^2 m_e r^3} = \frac{e^2}{4\pi\epsilon_0 r^2}, \quad r_n = \frac{\epsilon_0 n^2 \hbar^2}{\pi m_e e^2} = n^2 a_0.$$

Here $a_0 = \epsilon_0 \hbar^2 / (\pi m_e e^2)$ is the Bohr radius, the radius of the ground-state orbit. Using $\hbar = 6.626 \times 10^{-34}$ J s, $m_e = 9.109 \times 10^{-31}$ kg, and $e = 1.602 \times 10^{-19}$ C, one obtains $a_0 = 5.29 \times 10^{-11}$ m. The radii grow as the square of the principal quantum number.

Step 3: (b) Identify the two limiting Balmer transitions The longest wavelength corresponds to the smallest photon energy, namely the first Balmer transition $3 \rightarrow 2$. Thus

$$\frac{1}{\lambda_{\max}} = R_H \left(\frac{1}{4} - \frac{1}{9} \right) = \frac{5R_H}{36}, \quad \lambda_{\max} = \frac{36}{5R_H}.$$

As the initial level increases, the energy difference increases and the wavelength decreases. The shortest wavelength is the series limit $n_i \rightarrow \infty$, for which

$$\frac{1}{\lambda_{\min}} = \frac{R_H}{4}, \quad \lambda_{\min} = \frac{4}{R_H}.$$

Therefore $\lambda_{\max}/\lambda_{\min} = (36/(5R_H))/(4/R_H) = 9/5$. The Rydberg constant cancels, so its numerical value is unnecessary.

Quick Tip to Solve: For any series, use the first allowed transition for the longest wavelength and the infinite initial level for the shortest wavelength.

Step 4: Conclusion The permitted hydrogen orbit radii are $r_n = n^2 a_0$, with $a_0 = 5.29 \times 10^{-11}$ m. The ratio of the longest to the shortest Balmer wavelengths is $\boxed{9 : 5}$.

Quick Tip to Solve: For any series, use the first allowed transition for the longest wavelength and the infinite initial level for the shortest wavelength.

36. (a) Show that a current carrying solenoid behaves like a small bar magnet. Obtain the expression for the magnetic field at an external point lying on its axis.

(b) A steady current of 2 A flows through a circular coil having 5 turns of radius 7 cm. The coil lies in X - Y plane with its centre at the origin. Find the magnitude and direction of the magnetic dipole moment of the coil. [5 marks]

Solution:

Concept:

- A current loop has magnetic dipole moment $\mathbf{m} = I\mathbf{A}$; for N identical turns, $\mathbf{m} = N I \mathbf{A}$, directed by the right-hand rule.
- The axial field of a circular turn of radius a , at axial distance x , is $B = \mu_0 I a^2 / [2(a^2 + x^2)^{3/2}]$. Superposition gives the field of a solenoid.
- Far from a localized magnetic dipole, its axial field is $B = \mu_0 (2m) / (4\pi r^3)$, where r is measured from its centre.

Step 1: (a) Identify the magnetic poles of the solenoid The magnetic fields of all turns reinforce one another inside a solenoid and form closed field lines outside. Viewed from an end, anticlockwise current makes that end a north pole, while clockwise current makes it a south pole. Outside, field lines emerge from the north end and enter the south end, just as for a bar magnet. A freely suspended solenoid aligns with an external magnetic field and experiences the torque $\boldsymbol{\tau} = \mathbf{m} \times \mathbf{B}$. Thus it behaves as a magnetic dipole with moment $m = N I A$, where A is the area of each turn.

Step 2: (a) Integrate the field of the turns at an external axial point Consider a thin, uniformly wound solenoid of radius a , length $2l$, total turns N , and turn density $n = N/(2l)$. Put its centre at $z = 0$, and the external observation point at $z = r > l$. A slice of thickness dz at z has $n dz$ turns, each at distance $r - z$ from the observation point. Hence

$$B = \frac{\mu_0 n I a^2}{2} \int_{-l}^l \frac{dz}{[a^2 + (r - z)^2]^{3/2}} = \frac{\mu_0 n I}{2} \left[\frac{r + l}{\sqrt{a^2 + (r + l)^2}} - \frac{r - l}{\sqrt{a^2 + (r - l)^2}} \right].$$

This is the axial field for the ideal thin winding, without assuming a distant observation point. Its direction on the external axis is parallel to the solenoid's magnetic moment.

Step 3: (a) Obtain the small-bar-magnet form For distances from both ends large compared with the

Quick Tip to Solve: Convert the coil radius to metres before squaring it, and use the curled fingers of your right hand along the current to determine the sign of the axial moment.

radius, use $x/\sqrt{x^2 + a^2} \simeq 1 - a^2/(2x^2)$. The preceding result becomes

$$B \simeq \frac{\mu_0 n I a^2}{4} \left[\frac{1}{(r-l)^2} - \frac{1}{(r+l)^2} \right] \\ = \frac{\mu_0}{4\pi} \frac{2mr}{(r^2 - l^2)^2}, \quad m = (2ln)I\pi a^2 = NIA.$$

For a small solenoid observed far from its centre, $r \gg l, a$, this reduces to

$$B_{\text{axial}} \simeq \frac{\mu_0}{4\pi} \frac{2m}{r^3}.$$

This is exactly the distant axial field of a small bar magnet with the same dipole moment. The approximations are essential: the dipole expression is not the exact field immediately next to a finite solenoid.

Step 4: (b) Calculate the coil's magnetic dipole moment and direction With $N = 5$, $I = 2 \text{ A}$, and $a = 7 \text{ cm} = 0.07 \text{ m}$,

$$m = N I \pi a^2 = 5 \times 2 \times \pi (0.07)^2 = 0.049\pi \text{ A m}^2 \simeq 0.154 \text{ A m}^2.$$

The area vector is perpendicular to the X - Y plane, so the moment lies along the Z axis. It points along $+Z$ if the current is anticlockwise when viewed from the $+Z$ side, and along $-Z$ if the current is clockwise from that side. The printed question supplies no sense of current, so a unique choice between these two directions cannot be inferred.

Step 5: Conclusion A solenoid behaves as a bar magnet with $m = NIA$; its distant external axial field is $B = \mu_0(2m)/(4\pi r^3)$. The coil's moment is 0.154 A m^2 , perpendicular to the X - Y plane along either $+Z$ or $-Z$, as determined by the current sense.

Quick Tip to Solve: Convert the coil radius to metres before squaring it, and use the curled fingers of your right hand along the current to determine the sign of the axial moment.

OR

36. (a) Derive the expression for the force acting between two long parallel current carrying conductors. Hence, define 1 A current.

(b) A bar magnet of dipole moment 3 A m^2 rests with its centre on a frictionless pivot. A force F is applied at right angles to the axis of the magnet, 10 cm from the pivot. It is observed that an external magnetic field of 0.25 T is required to hold the magnet in equilibrium at an angle of 30° with the field. Calculate the value of F . How will the equilibrium be effected if F is withdrawn? [5 marks]

Solution:

Concept:

- A long straight conductor carrying current I produces a magnetic field $B = \mu_0 I / (2\pi d)$ at perpendicular distance d , where μ_0 is the permeability of free space.

Quick Tip to Solve: Balance torques using the force's lever arm in metres: here $0.10F = 3(0.25) \sin 30^\circ$. A frictionless pivot permits oscillation after the balancing force is removed.

- A conductor segment of length L carrying current I in a perpendicular magnetic field experiences force $F = ILB$. The field direction follows the right-hand thumb rule and the force direction follows $\mathbf{F} = I\mathbf{L} \times \mathbf{B}$.
- A magnetic dipole of moment m at angle θ to a uniform field B experiences torque $\tau = mB \sin \theta$. A perpendicular force applied at distance r from a pivot produces torque Fr .

Step 1: (a) Obtain the magnetic field of the first conductor Consider two straight, very long, parallel conductors in vacuum, separated by distance d , carrying steady currents I_1 and I_2 . For a circular Amperian path of radius d about conductor 1, the magnetic field has constant magnitude and is tangential to the path. Ampere's circuital law gives

$$B_1(2\pi d) = \mu_0 I_1, \quad B_1 = \frac{\mu_0 I_1}{2\pi d}.$$

At conductor 2 this field is perpendicular to the conductor. End effects are neglected because the conductors are very long compared with their separation.

Step 2: (a) Calculate the force and specify its direction A length L of conductor 2 therefore experiences the force

$$F_{21} = I_2 L B_1 = \frac{\mu_0 I_1 I_2 L}{2\pi d}, \quad \frac{F}{L} = \frac{\mu_0 I_1 I_2}{2\pi d}.$$

Interchanging the conductors gives an equal force on the same length of conductor 1, in the opposite direction. Applying the right-hand thumb rule to the magnetic field and the vector cross product to the force shows that currents in the same direction attract, whereas currents in opposite directions repel.

Step 3: (a) State the ampere definition used in this paper In the force-based definition intended by this question, one ampere is the constant current which, when maintained in each of two straight, parallel conductors of infinite length and negligible circular cross-section, placed one metre apart in vacuum, produces a force of 2×10^{-7} N per metre of length between them. Indeed, with $I_1 = I_2 = 1$ A, $d = 1$ m, and $\mu_0 = 4\pi \times 10^{-7}$ N A⁻², the expression above gives

$$\frac{F}{L} = \frac{4\pi \times 10^{-7}}{2\pi} = 2 \times 10^{-7} \text{ N m}^{-1}.$$

Step 4: (b) Balance the torques about the pivot The pivot passes through the centre, so its reaction produces no torque about that point. The applied force is perpendicular to the magnet's axis and acts $r = 10 \text{ cm} = 0.10 \text{ m}$ from the pivot. Its torque must oppose the magnetic torque for equilibrium. Thus

$$Fr = mB \sin 30^\circ, \\ F = \frac{(3)(0.25)(1/2)}{0.10} = 3.75 \text{ N}.$$

The distance in this calculation is the perpendicular lever arm of the applied force, not the full length of the magnet.

Step 5: (b) Examine what happens when the force is withdrawn Immediately after withdrawal, the magnetic torque is still $mB \sin 30^\circ = 0.375 \text{ N m}$, so the magnet is no longer in equilibrium at 30° . It turns towards alignment of its dipole moment with the field. The stable equilibrium orientation is

Quick Tip to Solve: Balance torques using the force's lever arm in metres: here $0.10F = 3(0.25) \sin 30^\circ$. A frictionless pivot permits oscillation after the balancing force is removed.

$\theta = 0$, where its potential energy $U = -mB \cos \theta$ is minimum. Since the pivot is frictionless, in the ideal absence of other damping the magnet overshoots this position and oscillates about the field direction; it does not simply come to rest there. With damping it eventually settles parallel to the field.

Step 6: Conclusion The force per unit length is $F/L = \mu_0 I_1 I_2 / (2\pi d)$, attractive for parallel currents in the same direction and repulsive for opposite currents. The force-based ampere definition corresponds to $2 \times 10^{-7} \text{ N m}^{-1}$ at one metre separation. For the magnet, $[F = 3.75 \text{ N}]$. With F withdrawn, the 30° equilibrium is lost and the undamped magnet oscillates about its stable orientation parallel to the field.

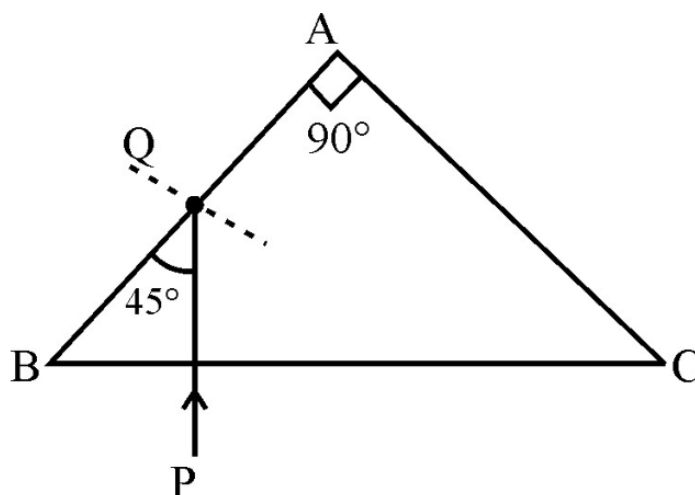
Quick Tip to Solve: Balance torques using the force's lever arm in metres: here $0.10F = 3(0.25) \sin 30^\circ$. A frictionless pivot permits oscillation after the balancing force is removed.

37. (a) Draw the ray diagram showing refraction of ray of light through a glass prism. Derive the expression for the refractive index μ of the material of prism in terms of the angle of prism A and angle of minimum deviation δ_m .

(b) A ray of light PQ enters an isosceles right angled prism ABC of refractive index 1.5 as shown in figure.

(i) Trace the path of the ray through the prism.

(ii) What will be the effect on the path of the ray if the refractive index of the prism is 1.4? [5 marks]



Solution:

Concept:

- Snell's law at an air-glass boundary is $\sin i = \mu \sin r$, where μ is the refractive index of glass relative to air.
- For a prism, the internal refraction angles satisfy $r_1 + r_2 = A$, and the deviation is $\delta = i + e - A$.
- At minimum deviation the ray path is symmetric: $i = e$ and $r_1 = r_2 = A/2$. Total internal reflection requires incidence from glass to air with $i > C$, where $\sin C = 1/\mu$.

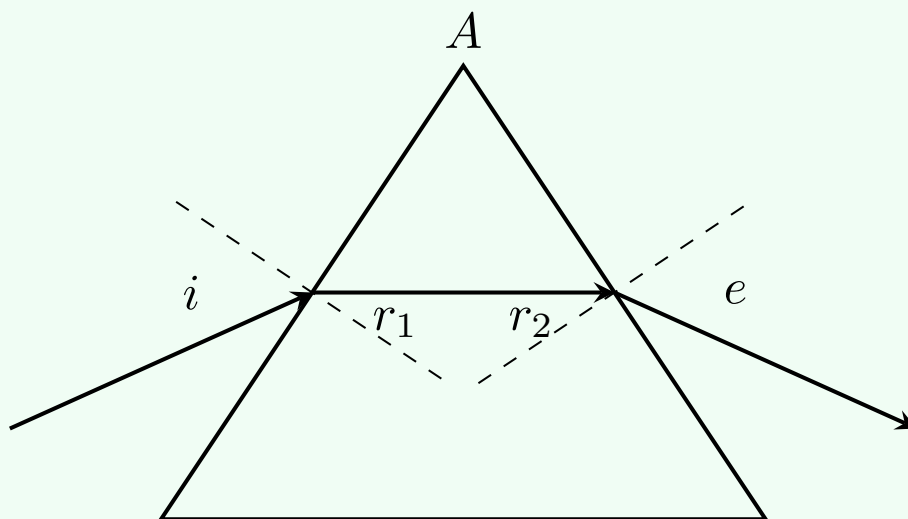
Step 1: (a) Ray diagram and prism geometry Let a ray enter one face at incidence angle i , refract at angle r_1 , reach the second face at internal incidence angle r_2 , and emerge at angle e . All four angles

Quick Tip to Solve: Compare the fixed internal incidence of 45° with $C = \sin^{-1}(1/\mu)$ before tracing any reflected ray.

are measured from the respective normals. The two normals and the internal ray give $r_1 + r_2 = A$. The first surface turns the ray through $i - r_1$, and the second through $e - r_2$. Therefore the total deviation is

$$\delta = (i - r_1) + (e - r_2) = i + e - A.$$

The diagram below shows the symmetric path at minimum deviation; the dashed lines are the surface normals.



Minimum deviation: $i = e, r_1 = r_2$

Symmetric ray path at minimum deviation; angles are measured from dashed normals.

Step 2: (a) Apply the minimum-deviation condition At minimum deviation, reversibility and symmetry of the ray path give $i = e$ and $r_1 = r_2$. Hence

$$r_1 = r_2 = \frac{A}{2}, \quad \delta_m = 2i - A, \quad i = \frac{A + \delta_m}{2}.$$

Applying Snell's law at the first face, with air of refractive index approximately unity, gives

$$\mu = \frac{\sin i}{\sin r_1} = \frac{\sin[(A + \delta_m)/2]}{\sin(A/2)}.$$

This expression uses the prism angle and the least deviation, rather than the deviation of an arbitrary incident ray.

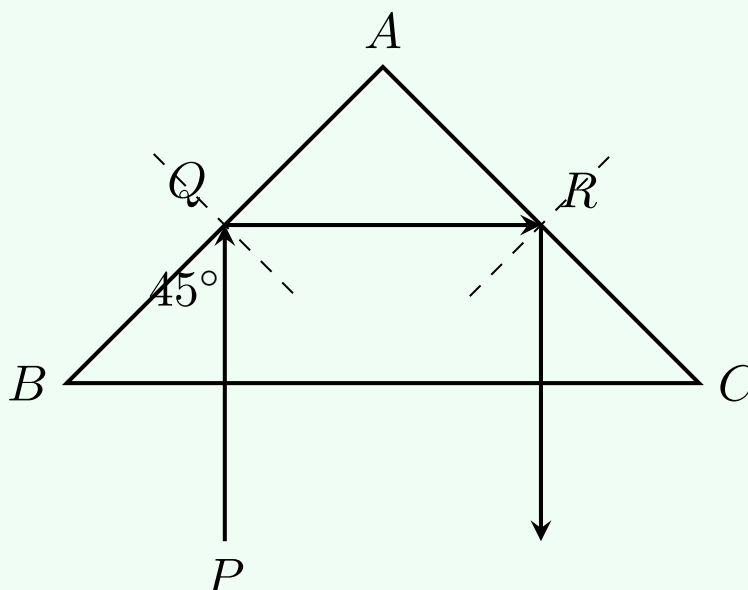
Step 3: (b)(i) Trace the ray for refractive index 1.5 The ray PQ enters through BC normally, so it continues vertically without deviation. Since ABC is an isosceles right-angled prism with $A = 90^\circ$, the angles at B and C are 45° . At Q on AB , the internal incidence is therefore 45° . For glass of index 1.5,

$$C = \sin^{-1}(1/1.5) \simeq 41.8^\circ.$$

Since $45^\circ > C$, the ray undergoes total internal reflection at AB , travels horizontally to AC , and again strikes that face at 45° . A second total internal reflection directs it vertically downwards. It then reaches BC normally and emerges without refraction, parallel to but opposite in direction to the

Quick Tip to Solve: Compare the fixed internal incidence of 45° with $C = \sin^{-1}(1/\mu)$ before tracing any reflected ray.

incident ray. Thus the two reflections reverse its direction.



For index 1.5, total internal reflections at Q and R reverse the ray.

Step 4: (b)(ii) Trace the change for refractive index 1.4 The entry through BC remains normal, and the incidence at Q remains 45° . However, the new critical angle is

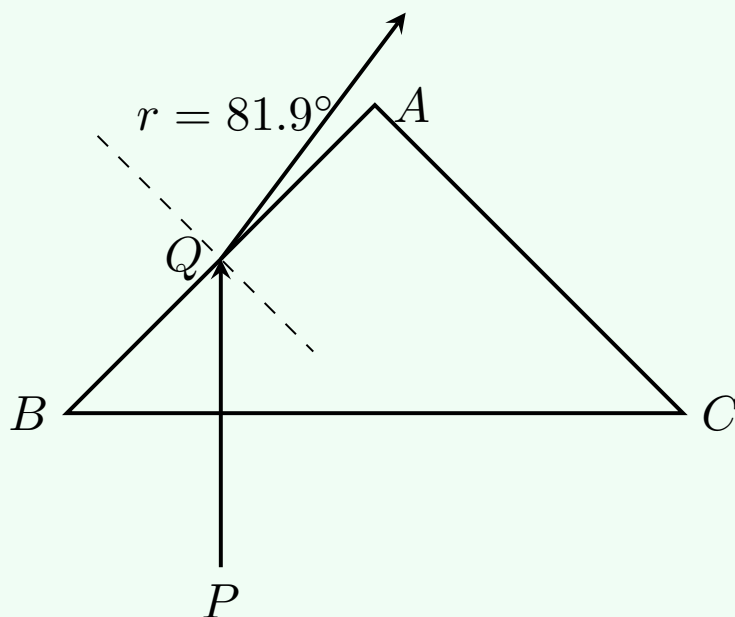
$$C = \sin^{-1}(1/1.4) \simeq 45.6^\circ.$$

Now $45^\circ < C$, so total internal reflection does not occur at AB . The transmitted ray emerges into air through AB , bending away from the normal. Snell's law gives its angle r to the outward normal:

$$\sin r = 1.4 \sin 45^\circ \simeq 0.98995, \quad r \simeq 81.9^\circ.$$

The emergent ray is nearly along the face AB , making about 8.1° with it. The two-total-reflection path is consequently lost; the principal transmitted ray exits at the first inclined face.

Quick Tip to Solve: Compare the fixed internal incidence of 45° with $C = \sin^{-1}(1/\mu)$ before tracing any reflected ray.



For index 1.4, the transmitted ray emerges through AB near the surface.

Step 5: Conclusion (a) $\mu = \frac{\sin[(A + \delta_m)/2]}{\sin(A/2)}$. (b) For $\mu = 1.5$, two total internal reflections reverse the ray and it exits through BC. For $\mu = 1.4$, it refracts out through AB at $r \simeq 81.9^\circ$ to the normal.

Quick Tip to Solve: Compare the fixed internal incidence of 45° with $C = \sin^{-1}(1/\mu)$ before tracing any reflected ray.

OR

37. (a) Two thin lenses are placed coaxially in contact. Obtain the expression for the focal length of this combination in terms of the focal lengths of the two lenses.

(b) A converging lens of refractive index 1.5 has a power of 10 D. When it is completely immersed in a liquid, it behaves as a diverging lens of focal length 50 cm. Find the refractive index of the liquid. [5 marks]

Solution:

Concept:

- The thin-lens formula in the Cartesian sign convention is $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$, where distances are measured from the optical centre and positive distances point along the incident light.
- For coaxial thin lenses in contact, separation is negligible, so the intermediate image distance for the first lens is also the object distance for the second lens. Lens powers add algebraically.
- The lens-maker formula in a surrounding medium is $\frac{1}{f} = \left(\frac{n_g}{n_m} - 1\right) \left(\frac{1}{R_1} - \frac{1}{R_2}\right)$, where n_g and n_m are the absolute refractive indices of glass and medium.

Quick Tip to Solve: Assign the immersed diverging lens a negative focal length before using lens-maker's formula; then cancel the unchanged curvature factor using the air power.

Step 1: (a) Apply the lens formula to the first lens Let the two focal lengths be f_1 and f_2 . An object at distance u would form an image at distance v_1 through the first lens alone. Therefore,

$$\frac{1}{v_1} - \frac{1}{u} = \frac{1}{f_1}.$$

The second lens intercepts the light from the first lens before this intermediate image is formed. That intermediate image acts as the object for the second lens; it may be real or virtual depending on the signed value of v_1 .

Step 2: (a) Eliminate the intermediate image distance Since the lenses are in contact, their optical centres coincide to the thin-lens approximation. If the final image is at v , the second lens gives

$$\frac{1}{v} - \frac{1}{v_1} = \frac{1}{f_2}.$$

Adding the two lens equations cancels $1/v_1$:

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f_1} + \frac{1}{f_2}.$$

Comparing this with the formula for one equivalent lens of focal length F identifies its reciprocal focal length. The focal lengths must retain their signs; this derivation applies to both converging and diverging lenses under the paraxial approximation.

Step 3: (b) Find the fixed curvature factor in air Write $K = 1/R_1 - 1/R_2$, with radii in metres. In air, $n_m \simeq 1$, so the given power is

$$10 = (1.5 - 1)K, \quad K = 20 \text{ m}^{-1}.$$

The shape and hence K are unchanged when the lens is immersed. Its new power changes because the relative refractive index of glass with respect to the surrounding liquid changes.

Step 4: (b) Use the negative focal length in the liquid A diverging lens has a negative focal length, so in the liquid

$$f_\ell = -50 \text{ cm} = -0.50 \text{ m}, \quad P_\ell = -2 \text{ D}.$$

Let the liquid's absolute refractive index be n_ℓ . Substituting in the lens-maker formula gives

$$\begin{aligned} -2 &= \left(\frac{1.5}{n_\ell} - 1 \right) 20, \\ \frac{1.5}{n_\ell} - 1 &= -0.10, \\ \frac{1.5}{n_\ell} &= 0.90. \end{aligned}$$

The negative power requires the liquid to be optically denser than glass. This provides a useful physical check on the computed index.

Step 5: Conclusion (a) $\boxed{\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}}$, or $\boxed{F = \frac{f_1 f_2}{f_1 + f_2}}$ for a nonzero sum; if $f_1 + f_2 = 0$, the

Quick Tip to Solve: Assign the immersed diverging lens a negative focal length before using lens-maker's formula; then cancel the unchanged curvature factor using the air power.

equivalent power is zero and F is infinite. (b) $n_\ell = \frac{1.5}{0.90} = \frac{5}{3} \simeq 1.67$.

Quick Tip to Solve: Assign the immersed diverging lens a negative focal length before using lens-maker's formula; then cancel the unchanged curvature factor using the air power.