

CBSE Class 12 Physics Question Paper 2020 with Solution PDF-Set 55/4/3



Question Paper with Solutions

Conducted by the Central Board of Secondary Education (CBSE)

Time Allowed: 3 hours

Maximum Marks: 70

General Instructions

- (i) This question paper comprises **four** sections – A, B, C and D.
- (ii) There are **37** questions in the question paper. **All** questions are compulsory.
- (iii) Section A : Q. no. **1 to 20** are very short-answer type questions carrying **1** mark each.
- (iv) Section B : Q. no. **21 to 27** are short-answer type questions carrying **2** marks each.
- (v) Section C : Q. no. **28 to 34** are long-answer type questions carrying **3** marks each.
- (vi) Section D : Q. no. **35 to 37** are also long answer type questions carrying **5** marks each.
- (vii) There is no overall choice in the question paper. However, an internal choice has been provided in **two** questions of **one** mark, **two** questions of **two** marks, **one** question of **three** marks and all the **three** questions **five** marks. You have to attempt only one of the choices in such questions.
- (viii) However, separate instructions are given with each section and question, wherever necessary.
- (ix) Use of calculators and log tables is not permitted.

You may use the following values of physical constants wherever necessary :

$$c = 3 \times 10^8 \text{ m/s}$$

$$h = 6.63 \times 10^{-34} \text{ Js}$$

$$e = 1.6 \times 10^{-19} \text{ C}$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ T m A}^{-1}$$

$$\epsilon_0 = 8.854 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$$

$$\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$$

$$\text{Mass of electron } (m_e) = 9.1 \times 10^{-31} \text{ kg}$$

$$\text{Mass of neutron} = 1.675 \times 10^{-27} \text{ kg}$$

$$\text{Mass of proton} = 1.673 \times 10^{-27} \text{ kg}$$

$$\text{Avogadro's number} = 6.023 \times 10^{23} \text{ per gram mole}$$

$$\text{Boltzmann constant} = 1.38 \times 10^{-23} \text{ JK}^{-1}$$

Section A (20 marks)

Select the most appropriate option from those given below each question.

1. A photocell connected in an electrical circuit is placed at a distance d from a source of light. As a result, current I flows in the circuit. What will be the current in the circuit when the distance is increased to $2d$? [1 mark]

- (A) $2I$
- (B) $I/4$
- (C) $I/2$
- (D) $4I$

Correct Answer: (B) $I/4$

Solution:

Concept:

- For a point source, intensity J at distance d obeys $J \propto 1/d^2$.
- At fixed frequency above threshold, photocurrent is proportional to incident intensity because the number of photoelectrons emitted per second follows the photon arrival rate.

Step 1: Apply the inverse-square law On increasing the source distance from d to $2d$, the same emitted light spreads over four times the area. Therefore

$$\frac{J_2}{J_1} = \frac{d^2}{(2d)^2} = \frac{1}{4}.$$

Step 2: Relate intensity to current The frequency and photocell circuit are unchanged, so the emitted charge per unit time decreases in the same ratio: $I_2/I = 1/4$. Option (A) incorrectly increases the current; (C) uses an inverse-distance law instead of an inverse-square law, and (D) would correspond to halving the distance.

Step 3: Conclusion The new photocurrent is $I/4$. The correct option is (B).

Quick Tip to Solve: Square the distance ratio first; doubling distance reduces intensity and photocurrent to one quarter.

2. There are uniform electric and magnetic fields in a region pointing along X -axis. An α -particle is projected along Y -axis with a velocity v . The shape of the trajectory will be [1 mark]

- (A) circular in XZ plane
- (B) circular in YZ plane
- (C) helical with its axis parallel to X-axis
- (D) helical with its axis parallel to Y-axis

Correct Answer: (C) helical with its axis parallel to X-axis

Solution:

Concept:

- The Lorentz force is $\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$. An alpha particle has charge $q = +2e$.
- A magnetic field rotates the transverse velocity while a parallel electric field accelerates the particle

along the field axis.

Step 1: Resolve the motion Initially the velocity is along Y, perpendicular to the magnetic field along X. The magnetic force produces circular transverse motion in the YZ plane with cyclotron angular frequency qB/m . It does no work and leaves the transverse speed unchanged.

Step 2: Include the electric force The electric force qE is along X and produces $a_x = qE/m$, so $x = \frac{1}{2}(qE/m)t^2$ for zero initial X velocity. Combining this increasing axial displacement with transverse rotation gives a helix of increasing pitch about X. A purely circular trajectory omits the electric force, while a Y-axis helix assigns the wrong common-field direction.

Step 3: Conclusion The trajectory is helical with its axis parallel to the X-axis (with nonuniform pitch). The correct option is (C).

Quick Tip to Solve: Find the common field axis first; the electric field drives axial motion and the magnetic field rotates transverse motion.

3. A circular loop of radius r , carrying a current I lies in y - z plane with its centre at the origin. The net magnetic flux through the loop is: [1 mark]

- (A) directly proportional to r
- (B) zero
- (C) inversely proportional to r
- (D) directly proportional to I

Correct Answer: (D) directly proportional to I

Solution:

Concept:

- Magnetic flux through an open surface is $\Phi_B = \int_S \mathbf{B} \cdot d\mathbf{A}$.
- For a fixed loop geometry in a linear medium, its self-produced field is proportional to current, hence self-flux satisfies $\Phi_B = LI$, with L the self-inductance.

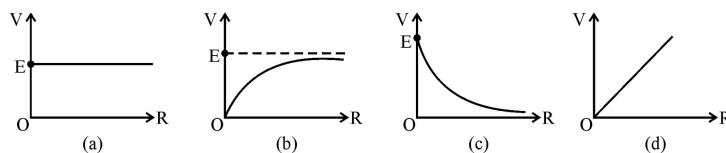
Step 1: Identify the relevant flux The loop lies in the YZ plane, so its area normal is along X. Its own magnetic field has a component along this normal and threads the loop. There is no external field stated, so the flux refers to that associated with the current in the loop.

Step 2: Compare the proposed dependences Increasing the current by a factor multiplies the magnetic field everywhere by that same factor for unchanged geometry. The surface integral therefore scales linearly with I . Gauss's law makes net flux zero through a closed surface, not through the open disk bounded by this loop. The radius options do not express the general self-flux relation; precise radius dependence also involves conductor geometry.

Step 3: Conclusion The magnetic flux is directly proportional to I . The correct option is (D).

Quick Tip to Solve: Use zero magnetic flux only for a closed surface; a current loop bounds an open surface with self-flux.

4. A cell of emf (E) and internal resistance r is connected across a variable external resistance R . The graph of terminal potential difference V as a function of R is — [1 mark]



- (A) Constant terminal voltage equal to E .
 (B) Terminal voltage rises from zero and approaches E asymptotically.
 (C) Terminal voltage decreases from E towards zero.
 (D) Terminal voltage rises linearly from zero.

Correct Answer: (B) Terminal voltage rises from zero and approaches E asymptotically.

Solution:

Concept:

- For emf E , internal resistance r , and external resistance R , the circuit current is $I = E/(R + r)$.
- The terminal potential difference is $V = IR = E - Ir = ER/(R + r)$.

Step 1: Determine the limiting voltages At $R = 0$, the external circuit is short-circuited and $V = 0$. As R becomes very large, the current tends to zero and so does the internal voltage drop Ir . Therefore V approaches E from below. For positive r , it remains less than E at every finite R .

Step 2: Test the shape and each option Differentiating with respect to R ,

$$\frac{dV}{dR} = \frac{Er}{(R + r)^2} > 0, \quad \frac{d^2V}{dR^2} = -\frac{2Er}{(R + r)^3} < 0.$$

Thus the graph rises from the origin with decreasing slope and approaches the horizontal asymptote $V = E$. Graph (A) incorrectly gives a constant voltage even at zero load resistance. Graph (C) decreases rather than increases. Graph (D) grows linearly without approaching a finite limiting voltage. Graph (B) alone has all the required properties.

Step 3: Conclusion The correct option is **(B)**: terminal voltage rises from zero and approaches E asymptotically.

Quick Tip to Solve: Test $R = 0$ and very large R first; choose the rising curve that flattens towards E .

5. A current of 10 A is flowing from east to west in a long straight wire kept on a horizontal table. The magnetic field developed at a distance of 10 cm vertically above the wire is: [1 mark]

- (A) 1.2×10^{-5} T, acting towards south
 (B) 2×10^{-5} T, acting towards north
 (C) 3×10^{-5} T, acting downwards
 (D) 2×10^{-5} T, acting upwards

Correct Answer: (B) 2×10^{-5} T, acting towards north

Solution:

Concept:

- For a long straight wire, $B = \mu_0 I / (2\pi a)$, where a is perpendicular distance.

- The right-hand rule gives the magnetic field direction around the current; $\mu_0 = 4\pi \times 10^{-7} \text{ T m A}^{-1}$.

Step 1: Calculate the magnitude The perpendicular distance is $a = 10 \text{ cm} = 0.10 \text{ m}$, and $I = 10 \text{ A}$. Thus

$$B = \frac{4\pi \times 10^{-7} \times 10}{2\pi \times 0.10} = 2 \times 10^{-5} \text{ T}.$$

Step 2: Determine the direction Take east, north and vertically upward as positive X, Y and Z directions respectively. The current is along $-\hat{x}$, and the point above the wire is along $+\hat{z}$. The field direction is $(-\hat{x}) \times \hat{z} = +\hat{y}$, toward north. Option (D) has the right magnitude but the field circles the wire and does not point radially upward; (A) and (C) also have incorrect magnitudes.

Step 3: Conclusion The magnetic field is $2 \times 10^{-5} \text{ T}$, acting towards north. The correct option is (B).

Quick Tip to Solve: Convert centimetres to metres and use the current direction crossed with the radial direction to fix the field direction.

6. A uniform wire of resistance $4R$ is bent in the form of a circle. The effective resistance between the ends of any diameter of the circle is: [1 mark]

- (A) $2R$
- (B) R
- (C) $R/2$
- (D) $4R$

Correct Answer: (B) R

Solution:

Concept:

- Resistance of a uniform wire is proportional to its length: $R_w = \rho \ell / A$.
- Two resistances connected between the same pair of terminals are in parallel, with $R_{\text{eq}} = R_1 R_2 / (R_1 + R_2)$.

Step 1: Split the circle into two paths The ends of a diameter divide the circular wire into equal semicircles. Each semicircle is half the total wire length, so each has resistance $4R/2 = 2R$. Bending the wire preserves its material, cross section and total length.

Step 2: Combine the semicircles Current entering one diameter end can travel along either semicircle and rejoins at the other end. They are therefore parallel, not series:

$$R_{\text{eq}} = \frac{(2R)(2R)}{2R + 2R} = R.$$

Option (A) gives only one path; (C) incorrectly starts with total resistance $2R$; (D) treats the whole wire as one series path despite the second available branch.

Step 3: Conclusion The effective resistance is R . The correct option is (B).

Quick Tip to Solve: For diametrically opposite terminals, divide total wire resistance by four: two equal half-wire resistances are in parallel.

7. In the α -particle scattering experiment, the shape of the trajectory of the scattered α -particles depend upon: [1 mark]

- (A) only on impact parameter.
- (B) only on the source of α -particles.
- (C) both impact parameter and source of α -particles.
- (D) impact parameter and the screen material of the detector.

Correct Answer: (C) both impact parameter and source of α -particles.

Solution:

Concept:

- Rutherford scattering arises from repulsive Coulomb force between an alpha particle and a positive nucleus.
- For fixed charges, the deflection obeys $\tan(\theta/2) = kq_{\alpha}Q/(2Kb)$, where K is incident kinetic energy and b is impact parameter.

Step 1: Examine the impact parameter The impact parameter is the perpendicular separation between the initial straight-line direction and the nuclear centre. A smaller b brings the particle into a stronger field and produces a greater deflection. The resulting orbit shape therefore depends on b .

Step 2: Examine the source and detector The alpha source sets the initial kinetic energy of the particles. At the same impact parameter, more energetic particles are deflected less, as the scattering relation shows through its dependence on K . Therefore the source also affects the trajectory. The detector screen records arrivals after scattering and does not determine the Coulomb trajectory. Options (A) and (B) each omit one controlling factor, while (D) incorrectly includes screen material.

Step 3: Conclusion The trajectory depends on both impact parameter and source of alpha particles. The correct option is (C).

Quick Tip to Solve: When comparing Rutherford trajectories, hold both impact parameter and incident kinetic energy in mind; the detector only records them.

8. Paschen series of atomic spectrum of hydrogen gas lies in: [1 mark]

- (A) Infrared region
- (B) Ultraviolet region
- (C) Visible region
- (D) Partly in ultraviolet and partly in visible region

Correct Answer: (A) Infrared region

Solution:

Concept:

- Hydrogen spectral series group transitions ending at the same principal quantum number. Paschen transitions end at $n_f = 3$.
- The Rydberg formula is $1/\lambda = R_H(1/n_f^2 - 1/n_i^2)$, where $n_i > n_f$.

Step 1: Identify the series For the Paschen series, electrons fall from $n_i = 4, 5, 6, \dots$ to $n_f = 3$. The upper levels change, but the final level remains fixed.

Step 2: Check its spectral region The shortest wavelength occurs at the series limit, $n_i \rightarrow \infty$:

$\lambda_{\min} = 9/R_H \approx 820 \text{ nm}$. Every other Paschen wavelength is longer. Thus all lines lie beyond visible red, in the infrared. Ultraviolet describes the Lyman series, while visible lines belong chiefly to the Balmer series; neither visible nor a mixed ultraviolet-visible region fits Paschen.

Step 3: Conclusion Paschen series lies in the infrared region. The correct option is (A).

Quick Tip to Solve: Identify the final level first: Lyman 1, Balmer 2, Paschen 3; Paschen is infrared.

9. The kinetic energy of a proton and that of an α -particle are 4 eV and 1 eV, respectively. The ratio of the de-Broglie wavelengths associated with them, will be [1 mark]

- (A) 2 : 1
(B) 1 : 1
(C) 1 : 2
(D) 4 : 1

Correct Answer: (B) 1 : 1

Solution:

Concept:

- The de Broglie wavelength is $\lambda = h/p$, where h is Planck's constant and p is momentum.
- For nonrelativistic kinetic energy K , $p = \sqrt{2mK}$. An alpha particle has approximately four times the proton mass.

Step 1: Form the wavelength ratio The energies are tiny compared with the rest energies, so $\lambda_p/\lambda_\alpha = \sqrt{m_\alpha K_\alpha/(m_p K_p)}$. Planck's constant and the factor 2 cancel.

Step 2: Substitute both mass and energy Using $m_\alpha \simeq 4m_p$, $K_p = 4 \text{ eV}$ and $K_\alpha = 1 \text{ eV}$, the ratio is $\sqrt{(4m_p)(1)/(m_p)(4)} = 1$. The products mK are equal, so the momenta and wavelengths are equal. Ratios 2 : 1, 1 : 2 and 4 : 1 result from ignoring either the mass change or the square root.

Step 3: Conclusion $\lambda_p : \lambda_\alpha = 1 : 1$. The correct option is (B).

Quick Tip to Solve: Compare the products mK before taking square roots; equal mK gives equal wavelengths.

10. When a wave undergoes reflection at an interface from rarer to denser medium, adhoc change in its phase is: [1 mark]

- (A) $\frac{\pi}{2}$
(B) 0
(C) π
(D) $\frac{\pi}{4}$

Correct Answer: (C) π

Solution:

Concept:

- Reflection from an optically denser medium reverses the sign of the reflected electric-field amplitude.

- A sign reversal corresponds to a phase difference of π radians, or 180° .

Step 1: Identify the reflecting interface The incident wave travels in the rarer medium and encounters the denser medium. In the usual dielectric reflection convention, this reflected wave undergoes phase reversal.

Step 2: Convert reversal to phase If the incident oscillation is represented by $E_0 \cos \omega t$, a reversed oscillation is $-E_0 \cos \omega t = E_0 \cos(\omega t + \pi)$. Hence the added phase is π . Zero phase applies to reflection from the denser side toward a rarer medium. The values $\pi/2$ and $\pi/4$ are partial phase changes and do not represent reversal.

Step 3: Conclusion The phase change is $\pi \text{ rad}$. The correct option is (C).

Quick Tip to Solve: For rarer-to-denser reflection, remember a half-cycle reversal: phase π and path equivalent $\lambda/2$.

11. To minimize the percentage error in the determination of unknown resistance of a conductor in meter bridge experiment, the balance point is adjusted near _____ of the wire. [1 mark]

Solution:

Concept:

- At balance in a metre bridge, $X/R = l/(L - l)$, where $L = 100 \text{ cm}$ and l is measured from one end.
- For a fixed small uncertainty δl , the fractional resistance error is proportional to $L\delta l/[l(L - l)]$.

Step 1: Relate balance length to error Write $X = Rl/(L - l)$, with the known resistance R fixed. Differentiation gives $\delta X/X \simeq (1/l + 1/(L - l))\delta l = L\delta l/[l(L - l)]$. Thus positions near either end amplify a small length-reading error.

Step 2: Find the least sensitive position The product $l(L - l)$ is largest at $l = L/2$. For the metre wire this is 50 cm , the midpoint. Choosing the known resistance close to the unknown resistance brings the balance point near this location and reduces the percentage error.

Step 3: Conclusion The blank is $\text{the middle (midpoint)}$ of the wire, near the 50 cm position.

Quick Tip to Solve: Choose the resistance box value close to the unknown resistance so balance occurs near 50 cm .

OR

11. In potentiometer, a long uniform wire is used to _____ potential gradient along the wire. [1 mark]

Solution:

Concept:

- The potential gradient is the potential drop per unit length: $k = V/L$, with V across wire length L .
- For a uniform wire carrying steady current I , $k = I\rho/A$; uniform resistivity ρ and area A make the gradient uniform.

Step 1: Explain uniformity A wire with uniform material and cross-sectional area has equal resistance in equal lengths. With steady current, equal lengths therefore have equal potential drops, giving a

uniform potential gradient along the wire.

Step 2: Explain the long length For a given applied potential difference V , increasing L reduces $k = V/L$. A small uniform gradient gives a longer measurable balance length for a small emf, increasing sensitivity and reducing the fractional error in reading that length. Thus the long uniform wire provides a small, uniform gradient.

Step 3: Conclusion The blank is provide a small and uniform potential gradient along the wire.

Quick Tip to Solve: At fixed wire voltage, a longer potentiometer wire lowers V/L and increases the balance length.

12. Fill in the blank with appropriate answer.

Torque acting on an electric dipole placed in an electric field is maximum when the angle between the electric field and the dipole moment is _____. [1 mark]

Solution:

Concept:

- The torque on a dipole in a uniform electric field is $\tau = \mathbf{p} \times \mathbf{E}$.
- Its magnitude is $\tau = pE \sin \theta$, where θ is the angle between dipole moment $\mathbf{p}$ and electric field $\mathbf{E}$.

Step 1: Identify what varies For a dipole of fixed moment p placed in a field of fixed magnitude E , only the factor $\sin \theta$ controls the torque magnitude. The equal and opposite forces act as a couple.

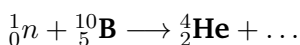
Step 2: Maximise the sine factor The maximum possible value of $\sin \theta$ is 1, attained at $\theta = 90^\circ = \pi/2$. The torque is then pE . When the dipole is parallel or antiparallel to the field, $\sin \theta = 0$ and the torque vanishes; therefore alignment cannot give the maximum.

Step 3: Conclusion The required angle is 90° , or $\pi/2 \text{ rad}$.

Quick Tip to Solve: Torque uses $\sin \theta$: perpendicular gives maximum torque, while parallel gives zero torque.

13. Fill in the blank with appropriate answer.

A neutron is bombarded on a ${}^{10}_5\text{B}$ nucleus and an alpha particle is emitted. The nuclear reaction involved is



[1 mark]

Solution:

Concept:

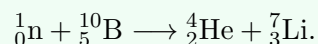
- In a nuclear reaction, total mass number A and total atomic number Z are conserved.
- A neutron has $A = 1, Z = 0$, and an alpha particle is ${}^4_2\text{He}$.

Step 1: Conserve mass number The initial mass number is $1 + 10 = 11$. After emission of an alpha particle of mass number 4, the residual nucleus must have $A = 11 - 4 = 7$.

Step 2: Conserve atomic number and identify the nucleus The initial atomic number is $0 + 5 = 5$. Since the alpha particle carries atomic number 2, the residual nucleus has $Z = 5 - 2 = 3$. The element with atomic number 3 is lithium. Hence the product is lithium-7. Checking the full equation gives

mass numbers $11 = 4 + 7$ and atomic numbers $5 = 2 + 3$, so both conservation requirements are satisfied.

Step 3: Conclusion The missing nucleus is ${}^7_3\text{Li}$:



Quick Tip to Solve: Subtract the emitted particle numbers separately: A gives the isotope, Z gives the element.

14. Fill in the blank with appropriate answer.

A proton released from rest in an electric field, will start moving towards a region of _____ potential in the field. [1 mark]

Solution:

Concept:

- The electric force on a proton is $\vec{F} = e\vec{E}$, since its charge e is positive.
- Electric field points in the direction of decreasing potential: $\vec{E} = -\nabla V$.

Step 1: Determine the initial direction A proton released from rest initially accelerates along the electric field because its positive charge makes its force parallel to $\vec{E}$. It therefore begins moving in the direction in which electric potential decreases.

Step 2: Check using energy The potential energy is $U = eV$. As the proton gains kinetic energy from rest, its potential energy decreases. Since $e > 0$, this decrease requires a decrease in V , consistent with the force argument.

Step 3: Conclusion The blank is lower potential.

Quick Tip to Solve: For a positive charge, follow the electric field toward lower potential; reverse this direction for a negative charge.

15. Fill in the blank with appropriate answer.

Unpolarised light of intensity I_0 is incident on two crossed polaroids. The intensity of light transmitted by the combination will be _____. [1 mark]

Solution:

Concept:

- An ideal polaroid transmits half the intensity of incident unpolarised light.
- Malus' law is $I = I_1 \cos^2 \theta$, where θ is the angle between the polarisation direction and the analyser axis.

Step 1: Apply the first polaroid The first polaroid produces plane-polarised light along its transmission axis. Its transmitted intensity is $I_1 = I_0/2$. This is the intensity incident on the second polaroid, rather than the original unpolarised intensity.

Step 2: Apply the crossed analyser Crossed polaroids have transmission axes at 90° . Hence the second

polaroid transmits

$$I = \frac{I_0}{2} \cos^2 90^\circ = 0.$$

The factor of one-half from the first polaroid does not alter this zero result.

Step 3: Conclusion The blank is $\boxed{0}$: no light is transmitted by the ideal crossed combination.

Quick Tip to Solve: Crossed means an angle of 90 degrees between transmission axes, so use the zero cosine factor after the first polaroid.

16. An A.C. source with variable frequency is connected to a parallel plate capacitor. How will the displacement current be affected with the decrease in frequency of the source? [1 mark]

Solution:

Concept:

- Displacement current is $I_d = \epsilon_0 d\Phi_E/dt = C dV/dt$ for an ideal parallel-plate capacitor.
- For sinusoidal voltage of fixed amplitude V_0 , the current amplitude is $I_{d0} = \omega CV_0 = 2\pi f CV_0$.

Step 1: Express the displacement current Write the applied voltage as $V = V_0 \sin(2\pi ft)$. Since the capacitor charge is $Q = CV$, differentiation gives

$$I_d = C \frac{dV}{dt} = 2\pi f CV_0 \cos(2\pi ft).$$

The electric flux changes more slowly when the source oscillates more slowly.

Step 2: Compare current amplitudes With the same capacitor and unchanged voltage amplitude, $I_{d0} = 2\pi f CV_0$ is directly proportional to frequency. The rms displacement current also obeys $I_{d,\text{rms}} = 2\pi f CV_{\text{rms}}$. Thus the comparison concerns current amplitude or rms value, not values at different phases.

Step 3: Conclusion The displacement current decreases when the frequency decreases, for fixed source voltage amplitude.

Quick Tip to Solve: For a capacitor at fixed voltage, remember the current amplitude is proportional to frequency, not inversely proportional.

17. Can a slab of p-type semi-conductor be physically joined to another n-type semiconductor slab to form p-n junction? Justify your answer. [1 mark]

Solution:

Concept:

- A p-n junction is formed by doping adjacent regions of the same semiconductor crystal.
- An effective junction requires a continuous crystal across which carriers can diffuse and establish a depletion layer.

Step 1: Consider physical contact Physically placing separate p-type and n-type slabs together does not create the required continuous semiconductor lattice. The contacting faces contain surface irregularities, impurities and broken bonds, so their contact is not equivalent to the internal boundary of a single crystal.

Step 2: Identify the proper junction formation In a real p-n junction, adjacent parts of one crystal are made p-type and n-type by suitable doping. Electrons and holes then diffuse across the common boundary and recombine, leaving ionised donors and acceptors that form the depletion region and its potential barrier.

Step 3: Conclusion No. A proper p-n junction is made within a single semiconductor crystal; merely joining two separate slabs does not form it.

Quick Tip to Solve: Distinguish an internal doping boundary in one crystal from mechanical contact between two independently prepared slabs.

OR

17. In a p-n junction diode the forward bias resistance is low as compared to the reverse bias resistance. Give reason. [1 mark]

Solution:

Concept:

- Forward bias opposes the built-in barrier potential and narrows the depletion layer.
- Reverse bias increases the barrier and widens the depletion layer; below breakdown only a small minority-carrier current flows.

Step 1: Explain forward conduction In forward bias the reduced barrier allows majority carriers to cross the junction easily: electrons from the n-side and holes from the p-side. A substantial current flows for a small applied forward voltage, giving a low effective junction resistance.

Step 2: Explain reverse conduction In reverse bias the increased barrier prevents majority carriers from crossing, while the widened depletion region further restricts their motion. Only a very small reverse saturation current due to minority carriers flows before breakdown, so the ratio of applied voltage to current is large.

Step 3: Conclusion Forward resistance is low because the barrier is reduced and majority carriers conduct; reverse resistance is high because the barrier increases and only a tiny minority-carrier current flows.

Quick Tip to Solve: Compare barrier height and depletion width first; low forward resistance and high reverse resistance then follow directly.

18. An astronomical telescope may be a refracting type or a reflecting type. Which of the two produces image of better quality? Justify your answer. [1 mark]

Solution:

Concept:

- A reflecting telescope uses a mirror as its objective, whereas a refracting telescope uses a lens.
- Chromatic aberration occurs because the refractive index of a lens material depends on wavelength; reflection does not cause this colour dispersion.

Step 1: Compare the focusing mechanisms A refracting objective bends different wavelengths by different amounts. Consequently, the focal lengths for different colours differ, causing coloured fringes

or loss of sharpness unless correction is provided. This is chromatic aberration of the lens objective.

Step 2: Explain the better image A reflecting objective focuses by reflection. The law of reflection is the same for all visible wavelengths, so the mirror brings different colours to the same focus and avoids chromatic aberration. A properly shaped reflecting objective therefore gives a sharper image with better colour fidelity.

Step 3: Conclusion The reflecting telescope produces the better-quality image because its objective is free from chromatic aberration.

Quick Tip to Solve: For image quality in this comparison, use the absence of chromatic aberration in the mirror objective as the decisive reason.

19. What is the change in the value of angle of dip when one goes from the equator to the north pole of earth? [1 mark]

Solution:

Concept:

- Angle of dip δ is the angle that the earth's magnetic field makes with the horizontal.
- Its components satisfy $\tan \delta = B_V/B_H$, where B_V and B_H are vertical and horizontal magnetic-field components.

Step 1: Identify the equatorial value At the magnetic equator, the earth's magnetic field is horizontal, so its vertical component is zero. Hence $\tan \delta = 0$ and the angle of dip is 0° . This is the standard idealised equator used in the question.

Step 2: Identify the polar value and change At the magnetic north pole, the field is vertically downward, and the horizontal component vanishes. Therefore the dip is 90° . On travelling from the equator towards the north magnetic pole, the inclination increases from horizontal to vertical.

Step 3: Conclusion The angle of dip increases from 0° to 90° , a total increase of 90° .

Quick Tip to Solve: Remember the field is horizontal at the magnetic equator and vertical at the magnetic poles; dip is measured from the horizontal.

20. A charged particle ($+q$) moves in a uniform electric field ($\vec{E}$) in the direction opposite to $\vec{E}$. What will be the effect on its electrostatic potential energy during its motion? [1 mark]

Solution:

Concept:

- The electric force on a positive charge is $\vec{F} = q\vec{E}$, so it acts in the direction of the electric field.
- The change in electrostatic potential energy is the negative of work done by the electric force: $\Delta U = -W_E$.

Step 1: Determine the sign of electric work Let the particle move a distance s opposite to the uniform field. The angle between its displacement and the electric force is 180° . Thus $W_E = qEs \cos 180^\circ = -qEs$: the electric field does negative work on the particle.

Step 2: Relate work to potential energy Using $\Delta U = -W_E$, we obtain $\Delta U = qEs > 0$. Equivalently, electric potential increases on moving opposite to $\vec{E}$; since $U = qV$ and $q > 0$, the potential energy

also increases. This conclusion concerns electrostatic potential energy irrespective of how the motion is produced.

Step 3: Conclusion The electrostatic potential energy of the positive charge increases during its motion.

Quick Tip to Solve: First identify the sign of the charge: moving a positive charge against the field increases its potential energy.

Section B (14 marks)

21. Briefly explain how a potential barrier is set up across a p-n junction as a result of diffusion and drift of the charge carriers. [2 marks]

Solution:

Concept:

- Diffusion is carrier motion caused by a concentration gradient; drift is carrier motion under an electric field.
- The depletion region contains fixed ionised donors and acceptors, with very few mobile majority carriers.

Step 1: Diffusion creates a depletion region Immediately after forming the junction, electrons diffuse from the n-side into the p-side and holes diffuse from the p-side into the n-side because of their concentration differences. They recombine near the junction. This leaves immobile positive donor ions on the n-side and immobile negative acceptor ions on the p-side. The region depleted of mobile carriers is called the depletion region.

Step 2: The internal field produces drift and equilibrium The exposed ions set up an electric field directed from the n-side to the p-side and hence a potential difference across the junction. This field opposes further diffusion of majority carriers: its force on electrons is towards the n-side, while its force on holes is towards the p-side. The field also drives minority carriers across the junction, producing a drift current opposite to the diffusion current. As the barrier develops, diffusion decreases until the drift and diffusion currents become equal in magnitude. The net current is then zero in equilibrium.

Step 3: Conclusion The potential barrier is established by the space charge left by diffusion and recombination; its electric field causes opposing drift until drift and diffusion currents balance.

Quick Tip to Solve: Remember the fixed-ion signs: n-side positive and p-side negative, so the junction field points from n to p.

22. Light of same wavelength is incident on three photo-sensitive surfaces A , B and C . The following observations are recorded.

(i) From surface A , photo electrons are not emitted.

(ii) From surface B , photo electrons are just emitted.

(iii) From surface C , photo electrons with some kinetic energy are emitted.

Compare the threshold frequencies of the three surfaces and justify your answer.

[2 marks]

Solution:

Concept:

- For a surface with threshold frequency ν_0 , the work function is $\phi = h\nu_0$. Photoemission requires $\nu \geq \nu_0$.
- Einstein's equation is $K_{\max} = h\nu - \phi = h(\nu - \nu_0)$, where h is Planck's constant.

Step 1: Use the common incident frequency Since the wavelength is the same for all three surfaces, their incident frequency is the same, $\nu = c/\lambda$. Surface A emits no photoelectrons, so the incident photon energy is below its work function: $h\nu < h\nu_{0A}$. Therefore $\nu < \nu_{0A}$.

Step 2: Apply the threshold and kinetic-energy conditions Photoelectrons are just emitted from B , meaning that the incident frequency equals its threshold and the maximum kinetic energy is zero: $\nu = \nu_{0B}$. Electrons from C have some kinetic energy, so $h(\nu - \nu_{0C}) > 0$, giving $\nu > \nu_{0C}$. Combining these three observations orders the threshold frequencies directly. The absence of emission from A is a threshold condition, not a statement that the common light has lower intensity on A .

Step 3: Conclusion The threshold frequencies satisfy $\nu_{0A} > \nu_{0B} > \nu_{0C}$, with $\nu_{0B} = \nu$.

Quick Tip to Solve: Translate no emission, just emission and positive kinetic energy into above, equal to and below the incident frequency.

OR

22. If the frequency of light incident on the cathode of a photo-cell is increased, how will the following be affected? Justify your answer.

(i) Energy of the photo electrons.

(ii) Photo current.

[2 marks]

Solution:

Concept:

- Einstein's photoelectric equation is $K_{\max} = h\nu - \phi$, where ϕ is the fixed work function of the cathode.
- Photocurrent is $I = e\dot{N}_e$, where $\dot{N}_e$ is the number of emitted electrons collected per second; it depends on illumination intensity and collection conditions.

Step 1: Effect on photoelectron energy For the same cathode, the work function does not change. Increasing the incident frequency from ν_1 to ν_2 therefore increases the maximum kinetic energy by $\Delta K_{\max} = h(\nu_2 - \nu_1)$. The emitted electrons have a range of kinetic energies, so the precise statement refers to their maximum kinetic energy. If the initial frequency was below threshold, emission starts only when the increased frequency reaches the threshold.

Step 2: Effect on photocurrent In the usual CBSE comparison, the intensity of illumination and the

collection conditions are held fixed, and the light is already above threshold. Under this convention the photocurrent remains unchanged: changing frequency changes the energy of the photoelectrons, whereas intensity determines their emission rate. An increase in electron energy by itself does not imply that more electrons are emitted per second. Since the question does not specify a change in intensity or photon arrival rate, no increase in photocurrent follows merely from increasing frequency. More precisely, if fixed optical power and a constant quantum efficiency are imposed, the photon rate is $P/(h\nu)$ and the saturation current can decrease; this is a different specified comparison.

Step 3: Conclusion The maximum photoelectron kinetic energy increases. In the standard fixed-intensity, above-threshold comparison, the photocurrent is unchanged.

Quick Tip to Solve: Use frequency to determine maximum electron energy and intensity to determine photocurrent; state the fixed-illumination assumption.

23. A wire of length L_0 has a resistance R_0 . It is gradually stretched till its length becomes $1.5 L_0$.

(a) Plot the graph showing variation of its resistance R with its length l during stretching.

(b) What will be its resistance when its length becomes $1.5 L_0$?

[2 marks]

Solution:

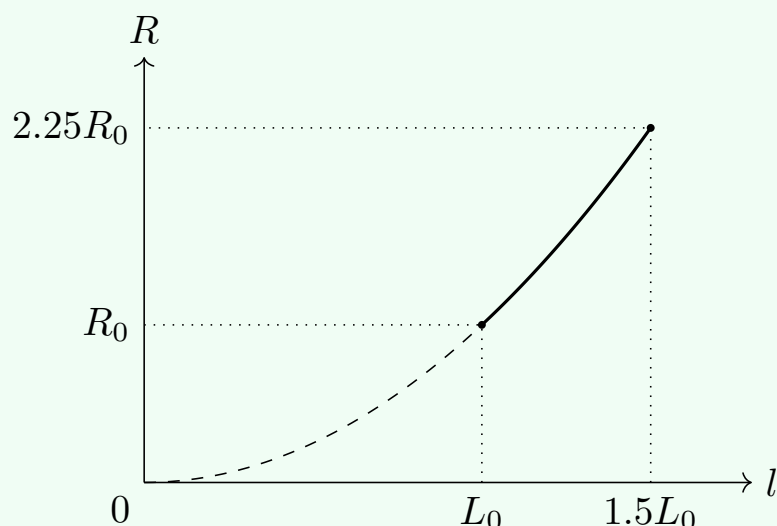
Concept:

- The resistance of a uniform wire is $R = \rho l/A$, where ρ is resistivity, l is length and A is its cross-sectional area.
- During stretching, take the volume of the wire and its resistivity as constant. Thus $Al = A_0 L_0$; the wire becomes thinner as it becomes longer.

Step 1: (a) Derive the resistance-length relation Initially, $R_0 = \rho L_0/A_0$. At length l , conservation of volume gives $A = A_0 L_0/l$. Substitution therefore gives

$$R = \frac{\rho l}{A_0 L_0/l} = \frac{\rho l^2}{A_0 L_0}, \quad \frac{R}{R_0} = \left(\frac{l}{L_0}\right)^2.$$

Hence the graph is an upward-curving parabola, with increasing slope. The actual stretching interval begins at (L_0, R_0) and ends at $(1.5L_0, 2.25R_0)$. The lower dashed part shows only the mathematical continuation of the relation.



Resistance versus length; the solid part is the stretching interval.

Step 2: (b) Substitute the final length At $l = 1.5L_0$, the area becomes $A = A_0/1.5$. Both the increase of length and decrease of area raise the resistance:

$$R = R_0(1.5)^2 = 2.25R_0 = \frac{9}{4}R_0.$$

Keeping the area fixed would incorrectly predict only $1.5R_0$; stretching changes the area as well.

Step 3: Conclusion $R = R_0(l/L_0)^2$, so the resistance-length graph is parabolic. The final resistance is $2.25R_0$.

Quick Tip to Solve: For a stretched wire of constant volume, square the length multiplication factor to obtain the resistance multiplication factor.

24. You are given three capacitors of $2\ \mu\text{F}$, $3\ \mu\text{F}$ and $4\ \mu\text{F}$, respectively.

(a) Form a combination of all these capacitors of equivalent capacitance $\frac{13}{3}\ \mu\text{F}$.

(b) What is the maximum and minimum value of the equivalent capacitance that can be obtained by connecting these capacitors? [2 marks]

Solution:

Concept:

- Parallel capacitances add: $C_p = C_1 + C_2$, because the potential difference is common and their stored charges add.
- For series capacitors, $1/C_s = 1/C_1 + 1/C_2$, because the charge is common and potential differences add.

Step 1: (a) Make the required combination Connect the $2\ \mu\text{F}$ and $4\ \mu\text{F}$ capacitors in series. Their equivalent capacitance is

$$C_{24} = \frac{2 \times 4}{2 + 4} = \frac{4}{3}\ \mu\text{F}.$$

Connect this series branch in parallel with the $3\ \mu\text{F}$ capacitor: both ends of the series branch go to the

corresponding terminals of the $3\ \mu\text{F}$ capacitor. Thus all three capacitors are used and

$$C_{\text{eq}} = 3 + \frac{4}{3} = \frac{13}{3}\ \mu\text{F}.$$

Step 2: (b) Find the two extreme capacitances The maximum is obtained with all three in parallel, so that each is connected directly across the same two terminals:

$$C_{\text{max}} = 2 + 3 + 4 = 9\ \mu\text{F}.$$

The minimum is obtained with all three in series:

$$\frac{1}{C_{\text{min}}} = \frac{1}{2} + \frac{1}{3} + \frac{1}{4} = \frac{13}{12}\ (\mu\text{F})^{-1}, \quad C_{\text{min}} = \frac{12}{13}\ \mu\text{F}.$$

Parallel connection increases the effective capacitance by adding charge capacity at a common voltage; series connection reduces it because the applied voltage is shared.

Step 3: Conclusion Use $2\ \mu\text{F}$ and $4\ \mu\text{F}$ in series, with this branch parallel to $3\ \mu\text{F}$. The maximum is $9\ \mu\text{F}$, and the minimum is $12/13\ \mu\text{F}$.

Quick Tip to Solve: Seek the fractional part of the target with a series pair, then add the remaining capacitor in parallel.

25. (a) Explain the formation of energy bands in crystalline solids.

(b) Draw the energy band diagrams of (i) a metal and (ii) a semiconductor.

[2 marks]

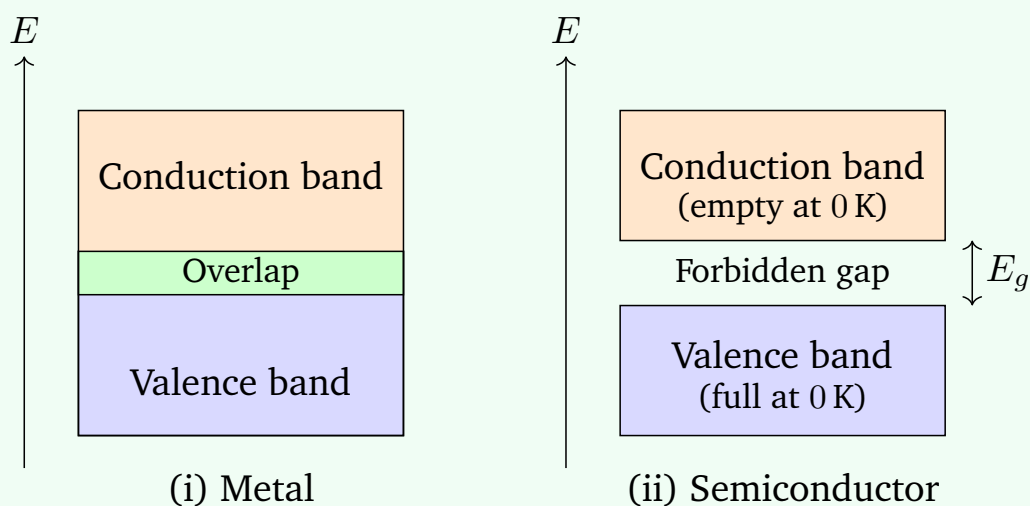
Solution:

Concept:

- An isolated atom has discrete allowed electron energies. In a crystal, interaction between neighbouring atoms splits these levels into many closely spaced allowed levels.
- The valence band contains the valence-electron states; the conduction band contains states in which electrons can move through the crystal. A forbidden energy gap contains no allowed electron states.

Step 1: (a) Explain the formation of bands When a large number of atoms come close together to form a crystalline solid, their outer-electron wave functions overlap and the atoms interact. Each atomic energy level consequently splits into a large number of slightly different levels, consistent with the Pauli exclusion principle. Since the crystal contains an enormous number of atoms, these levels are so closely spaced that they form an effectively continuous allowed energy band. Adjacent allowed bands may be separated by a forbidden energy gap, where an electron cannot have an allowed energy.

Step 2: (b) Compare the two energy-band diagrams In a metal, the valence and conduction bands may overlap, as shown here; equivalently, a metal may have a partially filled allowed band. Nearby empty states are available to electrons, so a small applied electric field produces conduction. In a semiconductor, the valence band is full and the conduction band is empty at 0 K. They are separated by a small forbidden gap E_g , typically of order 1 eV. At ordinary temperatures, some electrons gain enough thermal energy to reach the conduction band and leave holes in the valence band. The energy axis increases upward in both drawings.



Schematic energy bands of a metal and a semiconductor.

Step 3: Conclusion Crystal interactions split discrete atomic levels into closely spaced bands. A metal has overlapping bands or a partially filled band; a semiconductor has a small forbidden gap between its valence and conduction bands.

Quick Tip to Solve: Label both bands and the energy direction; show overlap for a metal and a clear small forbidden gap for a semiconductor.

26. A resistor R and an inductor L are connected in series to a source of voltage $V = V_0 \sin \omega t$. The voltage is found to lead current in phase by $\pi/4$. If the inductor is replaced by a capacitor C , the voltage lags behind current in phase by $\pi/4$. When L , C and R are connected in series with the same source, find the:

(i) average power dissipated and

(ii) instantaneous current in the circuit.

[2 marks]

Solution:

Concept:

- For a series resistor-inductor circuit, $\tan \phi = X_L/R$, where $X_L = \omega L$ is the inductive reactance.
- For a series resistor-capacitor circuit, $\tan \phi = -X_C/R$, where $X_C = 1/(\omega C)$. The sign describes the voltage phase relative to current.
- For series LCR, $Z = \sqrt{R^2 + (X_L - X_C)^2}$ and $P_{av} = V_{rms} I_{rms} \cos \phi$.

Step 1: Use the two measured phase angles The voltage leads the current by $\pi/4$ in the RL circuit. Hence $X_L/R = \tan(\pi/4) = 1$, giving $X_L = R$. In the RC circuit the voltage lags current by the same angle, so $X_C/R = 1$, giving $X_C = R$. Both measurements refer to the same source frequency; therefore $\omega L = 1/(\omega C)$.

Step 2: (i) Calculate average power When all three elements are in series, their reactances cancel: $X_L - X_C = 0$. Thus $Z = R$, the phase difference is zero, and the power factor is unity. Since V_0 is the peak voltage, $V_{rms} = V_0/\sqrt{2}$. Therefore

$$P_{av} = \frac{V_{rms}^2}{R} = \frac{V_0^2}{2R}.$$

Only the resistor dissipates average power; the ideal inductor and capacitor exchange energy with the source.

Step 3: (ii) Find instantaneous current The peak current is $I_0 = V_0/Z = V_0/R$. Because the net reactance vanishes, current has the same phase as the applied voltage $V_0 \sin \omega t$. Hence

$$i(t) = \frac{V_0}{R} \sin \omega t.$$

This is the steady-state current for ideal circuit elements.

Step 4: Conclusion $P_{\text{av}} = V_0^2/(2R)$, and $i(t) = (V_0/R) \sin \omega t$. The series circuit is at resonance.

Quick Tip to Solve: Equal and opposite phase angles of 45° imply $X_L = X_C = R$; use RMS voltage for power.

27. (a) Why is a photo diode operated under reverse bias condition?

(b) Draw V - I characteristic curves of photo diode for incident light of intensities I_1 and I_2 ($I_1 > I_2$).

[2 marks]

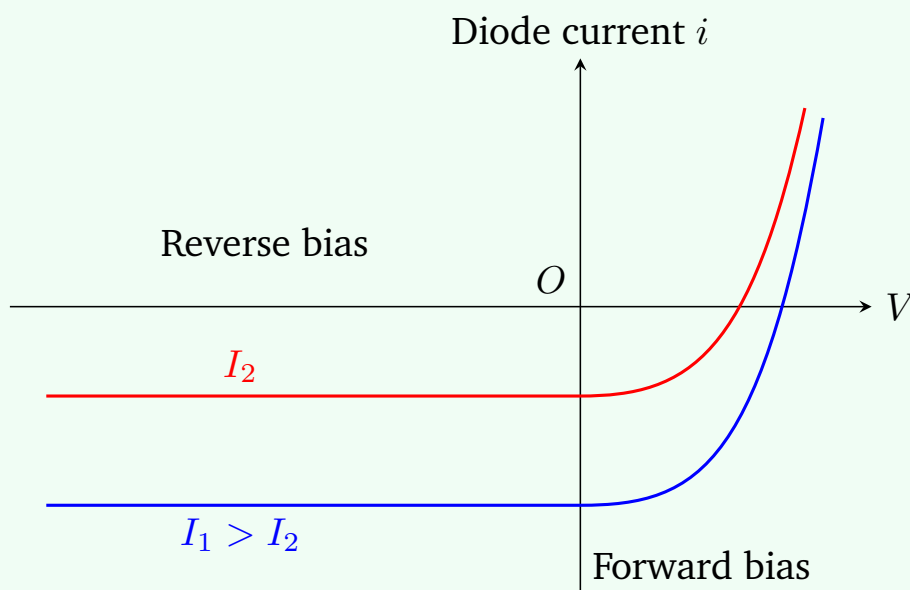
Solution:

Concept:

- A reverse-biased p-n junction has a widened depletion region and an electric field that separates photogenerated electron-hole pairs.
- Photocurrent increases with incident light intensity. In the reverse-bias region its magnitude is nearly independent of applied voltage before breakdown.

Step 1: (a) Explain reverse-bias operation Incident photons generate electron-hole pairs in and near the depletion region. The reverse-bias electric field sweeps electrons towards the n side and holes towards the p side, producing a reverse photocurrent. The ordinary reverse dark current is small, so the change caused by illumination is readily measured. Under forward bias, the large majority-carrier current would mask this small light-induced change. Reverse bias therefore gives sensitive light detection and efficient collection of photogenerated carriers.

Step 2: (b) Compare the two characteristic curves The official paper specifies $I_1 > I_2$, where I_1, I_2 denote incident light intensities, not diode current. More intense light generates more charge carriers per second. Therefore the reverse current magnitude is greater for I_1 : $|i_{\text{reverse},1}| > |i_{\text{reverse},2}|$. With signed voltage and diode current, reverse bias lies to the left of the origin and reverse current below the horizontal axis; the I_1 curve must lie below the I_2 curve. The sketch shows the nearly constant reverse-current region before breakdown, and the rising forward branches.



Photodiode V-I curves for incident intensities $I_1 > I_2$; reverse breakdown is outside the plotted range.

Step 3: Conclusion The photodiode is reverse biased to detect the small light-generated current sensitively. For $I_1 > I_2$, the I_1 curve has the larger reverse-current magnitude.

Quick Tip to Solve: On signed V-I axes, stronger illumination puts the reverse-current plateau farther below the voltage axis.

OR

27. (a) State the level of doping and biasing condition used in light emitting diode (LED).

(b) Write any two advantages of LED over the conventional low power lamps.

[2 marks]

Solution:

Concept:

- An LED is a heavily doped p-n junction that emits light when operated in forward bias.
- Electrons and holes injected across the junction recombine; in an appropriate semiconductor, their released energy emerges as photons.

Step 1: (a) State doping and bias conditions The LED uses a heavily doped p-n junction. It is operated under forward bias: the p side is connected to the positive terminal and the n side to the negative terminal of the supply. Forward bias reduces the junction barrier and permits injection of electrons and holes. Their radiative recombination produces the emitted light. Reverse bias is not the operating condition for normal light emission.

Step 2: (b) State two advantages First, LEDs consume low electrical power for useful light output, giving higher efficiency than conventional low-power filament lamps. Second, LEDs have a long operating life and high reliability because there is no heated filament to burn out. These are two distinct advantages: the first concerns energy consumption and the second concerns durability. A

suitable series current-limiting resistance is used in practice to keep the forward current within its rating.

Step 3: Conclusion An LED is heavily doped and forward biased. Two advantages are low power consumption and long operating life.

Quick Tip to Solve: Remember that forward bias injects carriers for LED emission; reverse bias is used for photodiode detection.

Section C (21 marks)

28. Two coherent light waves of intensity $5 \times 10^{-2} \text{ W m}^{-2}$ each super-impose and produce the interference pattern on a screen. At a point where the path difference between the waves is $\lambda/6$, λ being the wavelength of the wave, find the

(a) phase difference between the waves.

(b) resultant intensity at the point.

(c) resultant intensity in terms of the intensity at the maximum.

[3 marks]

Solution:

Concept:

- For coherent waves of wavelength λ , a path difference Δ produces phase difference $\phi = 2\pi\Delta/\lambda$.
- The resultant intensity is $I = I_a + I_b + 2\sqrt{I_a I_b} \cos \phi$, provided the waves overlap with matching polarization.
- For equal individual intensities I_0 , $I = 2I_0(1 + \cos \phi) = 4I_0 \cos^2(\phi/2)$, and $I_{\max} = 4I_0$.

Step 1: (a) Convert path difference to phase difference Each wave has intensity $I_0 = 5 \times 10^{-2} \text{ W m}^{-2}$, and the specified path difference is $\Delta = \lambda/6$. One full wavelength corresponds to a phase change of 2π radians. Thus

$$\phi = \frac{2\pi}{\lambda} \frac{\lambda}{6} = \frac{\pi}{3} \text{ rad} = 60^\circ.$$

The wavelength cancels, so a numerical wavelength is not needed.

Step 2: (b) Substitute into the intensity formula Coherent waves interfere through their amplitudes, so their intensities cannot simply be added without the interference term. As both intensities equal I_0 ,

$$\begin{aligned} I &= 2I_0(1 + \cos \phi) \\ &= 2(5 \times 10^{-2})(1 + \cos 60^\circ) \\ &= 0.10(1 + 0.5) \\ &= 0.15 \text{ W m}^{-2}. \end{aligned}$$

The positive interference term means this point is brighter than the incoherent sum 0.10 W m^{-2} , but it is not a maximum because the phase difference is nonzero.

Step 3: (c) Express the intensity relative to the maximum At a maximum the phase difference is an integral multiple of 2π , so $\cos \phi = 1$. For these equal waves,

$$I_{\max} = 4I_0 = 4(5 \times 10^{-2}) = 0.20 \text{ W m}^{-2}.$$

Consequently

$$\frac{I}{I_{\max}} = \frac{0.15}{0.20} = \frac{3}{4}.$$

Equivalently, $I/I_{\max} = \cos^2(\phi/2) = \cos^2 30^\circ = 3/4$, which independently checks the ratio.

Step 4: Conclusion (a) $\phi = \pi/3$ rad; (b) $I = 1.5 \times 10^{-1} \text{ W m}^{-2}$; (c) $I = \frac{3}{4} I_{\max}$.

Quick Tip to Solve: For equal wave intensities, use $I/I_{\max} = \cos^2(\pi\Delta/\lambda)$ to obtain the fraction directly.

29. (a) How are electro-magnetic waves produced? Depict an electro-magnetic wave propagating in Z-direction with its magnetic field $\vec{B}$ oscillating along X-direction.

(b) Write two characteristics of electro-magnetic waves.

[3 marks]

Solution:

Concept:

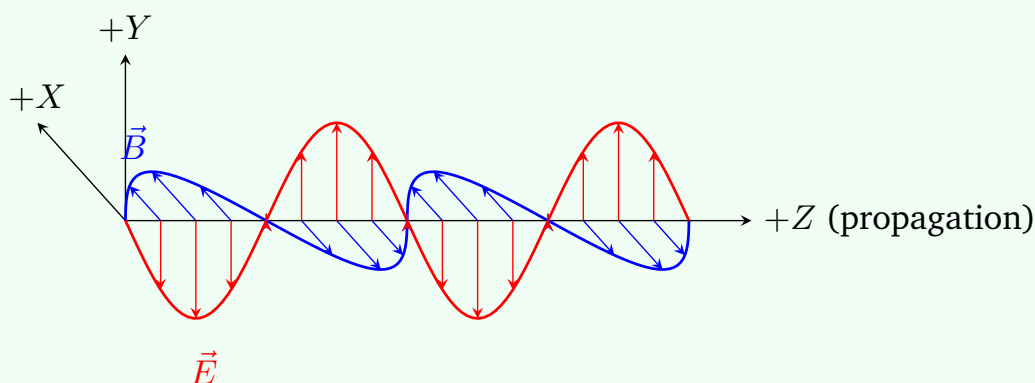
- An accelerating electric charge produces changing electric and magnetic fields, which travel outward as an electromagnetic wave.
- In a plane wave, $\vec{E}$, $\vec{B}$ and the direction of propagation are mutually perpendicular; $\vec{E} \times \vec{B}$ points along propagation.
- In vacuum the speed is $c = 1/\sqrt{\mu_0\epsilon_0}$, and the amplitudes obey $E_0 = cB_0$.

Step 1: (a) Production of the wave Electromagnetic waves are produced by accelerated charges, for example charges oscillating in an antenna. Their time-varying electric field is accompanied by a time-varying magnetic field. The changing magnetic field induces an electric field, while the changing electric field produces a magnetic field. These coupled disturbances propagate away from the source and carry energy. A stationary charge produces a static electric field, so it does not radiate an electromagnetic wave.

Step 2: (a) Orient the fields for propagation along the positive Z-axis With $\vec{B}$ along the X-axis, $\vec{E}$ must be along the Y-axis. To obtain propagation in $+Z$, one possible choice is

$$\vec{B} = B_0 \sin(kz - \omega t) \hat{x}, \quad \vec{E} = -E_0 \sin(kz - \omega t) \hat{y}.$$

Since $(-\hat{y}) \times \hat{x} = \hat{z}$, the energy flows along $+Z$. The fields oscillate at the same frequency and have simultaneous zero crossings and maxima in magnitude; their signed components differ because of the chosen axis directions.



Electromagnetic wave at an instant: B along X, E along Y, propagation along +Z.

Step 3: (b) Two characteristics First, electromagnetic waves are transverse: both oscillating fields are perpendicular to propagation and to each other. Second, they require no material medium and travel through vacuum at $c \simeq 3.0 \times 10^8 \text{ m s}^{-1}$. These statements distinguish the geometry of the fields from the ability of the wave to propagate through empty space.

Step 4: Conclusion Accelerated charges produce electromagnetic radiation. For propagation along $+Z$, $\vec{B}$ oscillates along X and $\vec{E}$ along Y with orientations giving $\vec{E} \times \vec{B} \parallel +Z$. The waves are transverse and propagate in vacuum at speed c .

Quick Tip to Solve: Check the field arrows with the right-hand rule for $\vec{E} \times \vec{B}$ before fixing the propagation direction.

30. A concave mirror forms a real image of an object kept at a distance 9 cm from it. If the object is taken away from the mirror by 6 cm, the image size reduces to $\frac{1}{4}$ of its previous size. Find the focal length of the mirror. [3 marks]

Solution:

Concept:

- The mirror formula is $1/f = 1/v + 1/u$, where distances are measured from the pole using the Cartesian sign convention.
- Linear magnification is $m = h_i/h_o = -v/u$. For a fixed object height, the ratio of image sizes is the ratio of magnification magnitudes.

Step 1: Set the two object distances and express magnification The object is initially 9 cm in front of the concave mirror and is then moved 6 cm farther away. Thus $u_1 = -9 \text{ cm}$ and $u_2 = -15 \text{ cm}$. Both images are real: the initial real image requires the object to be beyond the focus, and moving it farther away preserves that condition. The mirror formula gives

$$v = \frac{fu}{u - f}, \quad m = -\frac{f}{u - f}.$$

The two magnifications have the same negative sign because both real images are inverted.

Step 2: Use the printed image-size ratio The official paper states that the new image is one-quarter of the original size. Since the object height is unchanged, $|m_2|/|m_1| = 1/4$. The same signs allow us to use m_2/m_1 :

$$\begin{aligned} \frac{m_2}{m_1} &= \frac{u_1 - f}{u_2 - f} = \frac{1}{4}, \\ \frac{-9 - f}{-15 - f} &= \frac{1}{4}, \\ -36 - 4f &= -15 - f, \\ -21 &= 3f, \quad f = -7 \text{ cm}. \end{aligned}$$

The negative sign is consistent with a concave mirror under the Cartesian convention.

Step 3: Check the result against both real images Substituting $f = -7 \text{ cm}$, the first image distance is $v_1 = -31.5 \text{ cm}$, giving $m_1 = -3.5$. The second is $v_2 = -13.125 \text{ cm}$, giving $m_2 = -0.875$. Hence $|m_2|/|m_1| = 0.875/3.5 = 1/4$, exactly as printed. Both image distances are negative, confirming that both images lie in front of the mirror and are real.

Step 4: Conclusion The focal length is $f = -7 \text{ cm}$ in the Cartesian sign convention; its magnitude is 7 cm.

Quick Tip to Solve: For an unchanged object height, use the magnification ratio directly; moving away changes u from -9 cm to -15 cm.

31. (a) Differentiate between self inductance and mutual inductance.

(b) The mutual inductance of two coaxial coils is 2 H. The current in one coil is changed uniformly from zero to 0.5 A in 100 ms. Find the:

(i) change in magnetic flux through the other coil.

(ii) emf induced in the other coil during the change.

[3 marks]

Solution:

Concept:

- Self inductance L relates a coil's own flux linkage to its current: $N\Phi = LI$, with induced emf $\varepsilon = -L dI/dt$.
- Mutual inductance M relates the flux linkage of one coil to current in another: $N_2\Phi_{21} = MI_1$, with $\varepsilon_2 = -M dI_1/dt$.

Step 1: (a) Distinguish self and mutual inductance Self inductance is the property by which a change of current in a coil induces an emf in that same coil. Its coefficient L is the flux linkage of the coil per unit current flowing through it. Mutual inductance is the property by which a change of current in one coil induces an emf in a second nearby coil. Its coefficient M is the flux linkage of the second coil per unit current in the first. Self inductance depends on the geometry, turns and core of one coil; mutual inductance also depends on the relative position and magnetic coupling of the two coils. Both coefficients are measured in henry. In either case the minus sign in the induced-emf relation expresses Lenz's law.

Step 2: (b)(i) Calculate the change in linked magnetic flux Here $M = 2 \text{ H}$ and the source-coil current changes from zero to 0.5 A, so $\Delta I_1 = 0.5 \text{ A}$. Therefore

$$\Delta(N_2\Phi_{21}) = M\Delta I_1 = 2 \times 0.5 = 1 \text{ Wb}.$$

Thus the magnetic flux linked with the other coil changes by 1 Wb, understood as its total flux linkage. Strictly, if that coil has N_2 turns, the change of flux per turn is $1/N_2 \text{ Wb}$; its turn count is not supplied. The mutual-inductance relation determines the total linked flux directly.

Step 3: (b)(ii) Calculate the induced emf Convert the time interval before substitution: $100 \text{ ms} = 0.100 \text{ s}$. The change is uniform, so the induced emf is constant throughout this interval. Its magnitude is

$$|\varepsilon_2| = M \frac{\Delta I_1}{\Delta t} = 2 \frac{0.5}{0.100} = 10 \text{ V}.$$

Equivalently it is the linked-flux change divided by elapsed time, $1/0.100 = 10 \text{ V}$. With a positive linked-flux direction chosen for the increasing current, the signed emf is -10 V ; physically its direction

opposes the increasing magnetic flux.

Step 4: Conclusion Self inductance concerns emf in the same coil; mutual inductance concerns emf in another coil. The change in total linked flux is 1 Wb , and the induced emf has magnitude 10 V , opposing the flux increase.

Quick Tip to Solve: Convert milliseconds to seconds first, and remember that M times current gives total linked flux, not flux per turn.

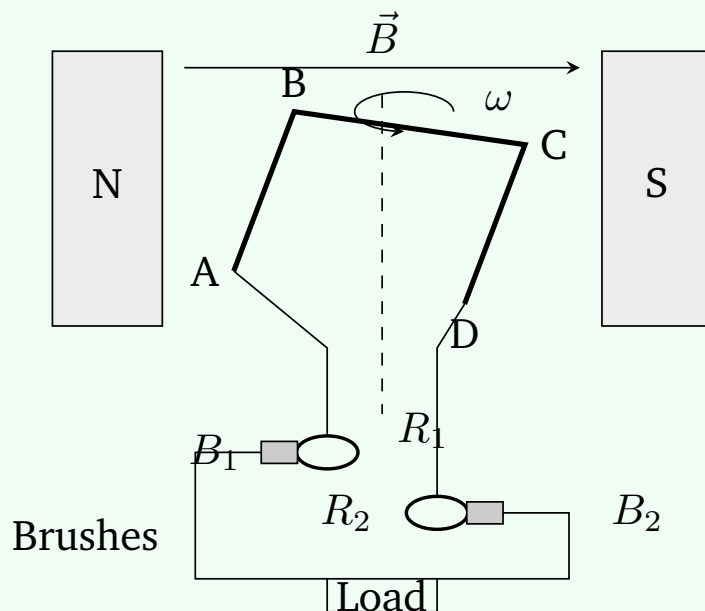
32. Draw the labelled diagram of an AC generator. Briefly explain its working and obtain the expression for the emf produced in the coil. [3 marks]

Solution:

Concept:

- An AC generator converts mechanical energy into electrical energy by electromagnetic induction: a changing magnetic flux through a coil induces an emf.
- Faraday's law is $\varepsilon = -d\Phi_B/dt$, where Φ_B is the total flux linkage. For N turns of area A in a uniform field B , $\Phi_B = NBA \cos \theta$, with θ the angle between the field and the coil's normal.

Step 1: Construction and labelled diagram A rectangular armature coil $ABCD$, containing N insulated turns, rotates between the north and south poles of a magnet. Its two ends connect to two separate slip rings R_1 and R_2 , which rotate with the coil. Stationary carbon brushes B_1 and B_2 maintain contact with the rings and connect the coil to the external load. An external mechanical drive rotates the shaft.



AC generator: rotating armature, separate slip rings and stationary brushes

Step 2: Working and flux linkage As the coil rotates, the magnetic flux through it changes continuously, inducing an emf. The slip rings transfer this emf to the external circuit without interchanging the coil connections. The induced current reverses after every half revolution because the direction of motion of each active side reverses relative to the magnetic field. Thus the output is alternating. Let the coil rotate uniformly with angular speed ω , and choose $t = 0$ when its normal is parallel to the magnetic

field. Then $\theta = \omega t$, and the flux linkage is

$$\Phi_B(t) = NBA \cos(\omega t).$$

Step 3: Derive the induced emf Applying Faraday's law gives

$$\begin{aligned}\varepsilon(t) &= -\frac{d}{dt}[NBA \cos(\omega t)] \\ &= NBA\omega \sin(\omega t).\end{aligned}$$

Hence the peak emf is $\varepsilon_0 = NBA\omega$. The emf vanishes when the coil's normal is parallel to the field and is maximum in magnitude when the normal is perpendicular to it. The negative sign in Faraday's law specifies the direction according to Lenz's law; the displayed sine corresponds to the chosen terminal polarity and initial orientation.

Step 4: Conclusion The generator produces a sinusoidal alternating emf $\varepsilon(t) = \varepsilon_0 \sin(\omega t)$, with $\varepsilon_0 = NBA\omega$, while the mechanical drive supplies the electrical energy delivered to the load.

Quick Tip to Solve: Measure the flux angle from the coil's normal: differentiate the cosine flux to obtain the sine emf.

33. A hollow conducting sphere of inner radius r_1 and outer radius r_2 has a charge Q on its surface. A point charge $-q$ is also placed at the centre of the sphere.

(a) What is the surface charge density on the (i) inner and (ii) outer surface of the sphere?

(b) Use Gauss' law of electrostatics to obtain the expression for the electric field at a point lying outside the sphere. [3 marks]

Solution:

Concept:

- In electrostatic equilibrium, the electric field within the material of a conductor is zero, and its net excess charge lies on its surfaces.
- Gauss's law states $\oint \vec{E} \cdot d\vec{A} = Q_{\text{enc}}/\varepsilon_0$, where Q_{enc} is the algebraic charge enclosed by a closed Gaussian surface.
- A charge at the centre preserves spherical symmetry. The induced charge is therefore uniform on each spherical surface, whose area is $4\pi r^2$.

Step 1: (a)(i) Find the charge on the inner surface Choose a concentric Gaussian sphere of radius s lying inside the conducting material, so that $r_1 < s < r_2$. Since the field is zero everywhere on this surface, its flux is zero. If the inner surface carries charge Q_{in} , Gauss's law gives

$$0 = \frac{-q + Q_{\text{in}}}{\varepsilon_0}, \quad Q_{\text{in}} = +q.$$

By symmetry this charge is uniformly distributed, giving the inner surface charge density

$$\sigma_{\text{in}} = \frac{q}{4\pi r_1^2}.$$

Step 2: (a)(ii) Find the charge on the outer surface The net charge of the conducting shell itself

remains Q ; placing the separate point charge in the cavity redistributes this shell charge. Hence

$$Q_{\text{in}} + Q_{\text{out}} = Q, \quad Q_{\text{out}} = Q - q.$$

Its uniform outer surface charge density is therefore

$$\sigma_{\text{out}} = \frac{Q - q}{4\pi r_2^2}.$$

The sign of the outer charge depends on the relative values of Q and q ; it need not be positive.

Step 3: (b) Apply Gauss's law outside the sphere For an external point at distance $r > r_2$ from the centre, take a concentric spherical Gaussian surface of radius r . Spherical symmetry makes the field radial and its radial component E_r constant on this surface. The enclosed charge includes the central point charge and both shell surfaces:

$$Q_{\text{enc}} = -q + q + (Q - q) = Q - q.$$

Gauss's law now yields

$$E_r 4\pi r^2 = \frac{Q - q}{\epsilon_0}, \quad \vec{E}(r) = \frac{Q - q}{4\pi\epsilon_0 r^2} \hat{r}.$$

Here $\hat{r}$ points outward. The field is outward for $Q > q$, inward for $Q < q$, and zero for $Q = q$. Its magnitude is $|Q - q|/(4\pi\epsilon_0 r^2)$.

Step 4: Conclusion The surface charge densities are $\sigma_{\text{in}} = q/(4\pi r_1^2)$ and $\sigma_{\text{out}} = (Q - q)/(4\pi r_2^2)$.

Outside, $\vec{E} = (Q - q)\hat{r}/(4\pi\epsilon_0 r^2)$ for $r > r_2$.

Quick Tip to Solve: First cancel the cavity charge using a Gaussian surface in the metal; then conserve the shell's own charge Q .

OR

33. (a) An infinitely long thin straight wire has a uniform linear charge density λ . Obtain the expression for the electric field E at a point lying at a distance x from the wire, using Gauss' law.

(b) Show graphically the variation of this electric field E as a function of distance x from the wire.

[3 marks]

Solution:

Concept:

- Gauss' law states that $\oint \vec{E} \cdot d\vec{A} = Q_{\text{enc}}/\epsilon_0$, where Q_{enc} is the charge enclosed by a closed surface.
- An infinite uniform line charge has cylindrical symmetry: the electric field is radial and its magnitude depends only on the perpendicular distance x from the wire.

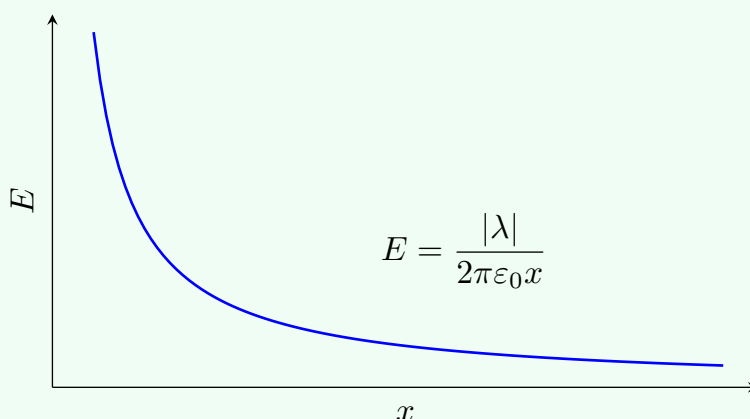
Step 1: (a) Choose a cylindrical Gaussian surface Take a cylinder of radius x and length L , coaxial with the wire. The enclosed charge is $Q_{\text{enc}} = \lambda L$. At every point of the curved surface, the field has the same magnitude E and is normal to the surface. On the two flat end faces the field lies parallel to their planes, so its scalar product with the outward area vector is zero. Thus only the curved surface contributes to the flux.

Step 2: Apply Gauss' law The curved area is $2\pi xL$. Therefore,

$$E(2\pi xL) = \frac{|\lambda|L}{\epsilon_0}, \quad E = \frac{|\lambda|}{2\pi\epsilon_0 x}.$$

For a positively charged wire, the usual expression is $E = \lambda/(2\pi\epsilon_0 x)$, directed radially away from the wire. For negative λ , the direction is radially inward; the magnitude still uses $|\lambda|$. The arbitrary cylinder length cancels, as required by translational symmetry.

Step 3: (b) Plot the variation with distance Since $E \propto 1/x$, the graph of field magnitude against positive distance is a decreasing rectangular hyperbola. Doubling x halves E . In the ideal thin-wire model the magnitude becomes unbounded as $x \rightarrow 0^+$ and tends to zero as $x \rightarrow \infty$. Neither axis is crossed; both are asymptotes.



Field magnitude decreases inversely with distance from the wire.

Step 4: Conclusion The field magnitude is $E = |\lambda|/(2\pi\epsilon_0 x)$; its E -versus- x graph is a decreasing rectangular hyperbola for $x > 0$.

Quick Tip to Solve: Choose the Gaussian cylinder coaxial with the wire: its end caps contribute no flux, leaving only the curved area.

34. (a) Explain the principle of working of a potentiometer.

(b) In a potentiometer, a standard source of emf 5 V and negligible internal resistance maintains a steady current through the potentiometer wire of length 10 m. Two primary cells of emf E_1 and E_2 are joined together in a series with (i) same polarity and (ii) opposite polarity. The combination is connected to the potentiometer circuit in each case. The balancing length of the wire in the two cases are found to be 700 cm and 100 cm, respectively. Find the values of emf of the two cells. [3 marks]

Solution:

Concept:

- For a uniform wire carrying a constant current, the potential drop is proportional to its length: $V = kl$, where k is the potential gradient.
- At potentiometer balance the galvanometer current is zero, so the measured cell supplies no current and its emf equals the potential drop along the balancing length.
- Series-aiding cells have resultant emf $E_1 + E_2$; series-opposing cells have resultant magnitude $|E_1 - E_2|$.

Step 1: (a) Explain the working principle For a wire of uniform cross-sectional area A and uniform resistivity ρ , a length l has resistance $R = \rho l/A$. If a steady current I flows through it, the potential drop is $V = IR = (I\rho/A)l$. Thus $V \propto l$, with constant potential gradient $k = I\rho/A$. The unknown cell is connected so that its emf opposes the drop along the selected length. The contact is moved until the galvanometer shows zero current; then the cell emf is $E = kl$. Because no current is drawn from the test cell at balance, its internal resistance causes no voltage drop in this measurement.

Step 2: (b) Calculate the potential gradient The driving source places 5 V across a wire of length 10 m = 1000 cm. Hence

$$k = \frac{5}{1000} = 0.005 \text{ V cm}^{-1}.$$

Both measurements use the same wire and steady current, so the potential gradient is unchanged.

Step 3: Form and solve the two emf equations For the same polarity, the emfs add and the balancing length is 700 cm:

$$E_1 + E_2 = k(700) = 3.5 \text{ V}.$$

For opposite polarity, the difference balances at 100 cm:

$$|E_1 - E_2| = k(100) = 0.5 \text{ V}.$$

Denote the larger emf by E_1 . Adding and subtracting these equations gives

$$E_1 = \frac{3.5 + 0.5}{2} = 2.0 \text{ V}, \quad E_2 = \frac{3.5 - 0.5}{2} = 1.5 \text{ V}.$$

These values give a sum of 3.5 V and a difference of 0.5 V, matching both measured lengths. If the labels of the cells are interchanged, the numerical pair remains the same.

Step 4: Conclusion The potentiometer works on $V \propto l$ for a uniform wire carrying steady current. The two cell emfs are 2.0 V and 1.5 V, with $E_1 = 2.0 \text{ V}$ if E_1 denotes the larger emf.

Quick Tip to Solve: Use the aiding length for the sum and the opposing length for the difference; half their sum and difference gives the two cell emfs.

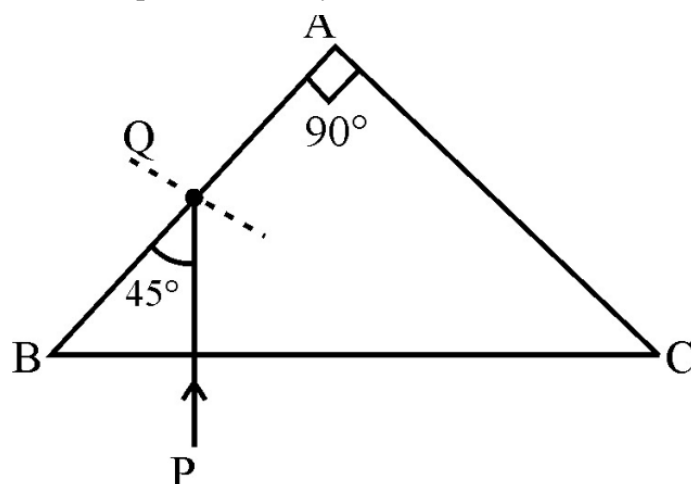
Section D (15 marks)

35. (a) Draw the ray diagram showing refraction of ray of light through a glass prism. Derive the expression for the refractive index μ of the material of prism in terms of the angle of prism A and angle of minimum deviation δ_m .

(b) A ray of light PQ enters an isosceles right angled prism ABC of refractive index 1.5 as shown in figure.

(i) Trace the path of the ray through the prism.

(ii) What will be the effect on the path of the ray if the refractive index of the prism is 1.4? [5 marks]



Solution:

Concept:

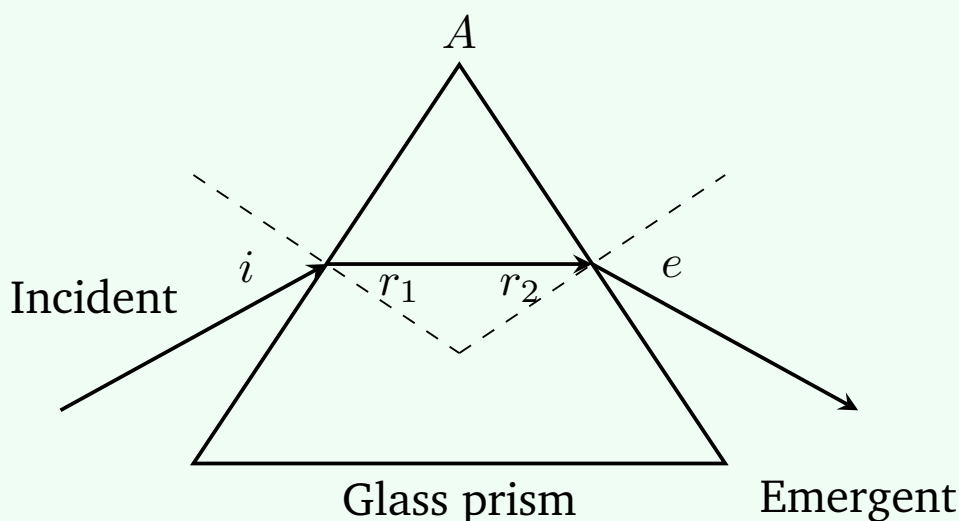
- Snell's law for refraction from air into glass is $\mu = \sin i / \sin r$, where i and r are measured from the normal.
- For a prism of apex angle A , its internal refraction angles satisfy $r_1 + r_2 = A$, and its deviation is $\delta = i + e - A$.
- At minimum deviation the ray path is symmetric: $i = e$ and $r_1 = r_2 = A/2$. Total internal reflection at a glass-air surface requires incidence greater than the critical angle C , where $\sin C = 1/\mu$.

Step 1: (a) Establish the prism relations The figure shows an incident ray refracting towards the normal at the first face and an emergent ray refracting away from the normal at the second face. Here i, e are the external angles and r_1, r_2 are the internal angles. The normals to the faces make an angle $180^\circ - A$ inside the prism. Applying the angle sum to the triangle formed by the internal ray and these normals gives

$$r_1 + r_2 = A.$$

The deviation at entry is $i - r_1$, and the deviation at emergence is $e - r_2$. Hence the total deviation is

$$\delta = (i - r_1) + (e - r_2) = i + e - A.$$



Refraction through a prism; all four angles are measured from the dashed normals.

Step 2: (a) Use the minimum-deviation condition On rotating the prism, the deviation first decreases, reaches a minimum δ_m , and then increases. At that minimum the incident and emergent paths are symmetric, so $i = e$. By Snell's law and reversibility of light, the internal angles are also equal. Therefore

$$r_1 = r_2 = \frac{A}{2}, \quad \delta_m = 2i - A, \quad i = \frac{A + \delta_m}{2}.$$

Substituting these angles into Snell's law at the first face yields

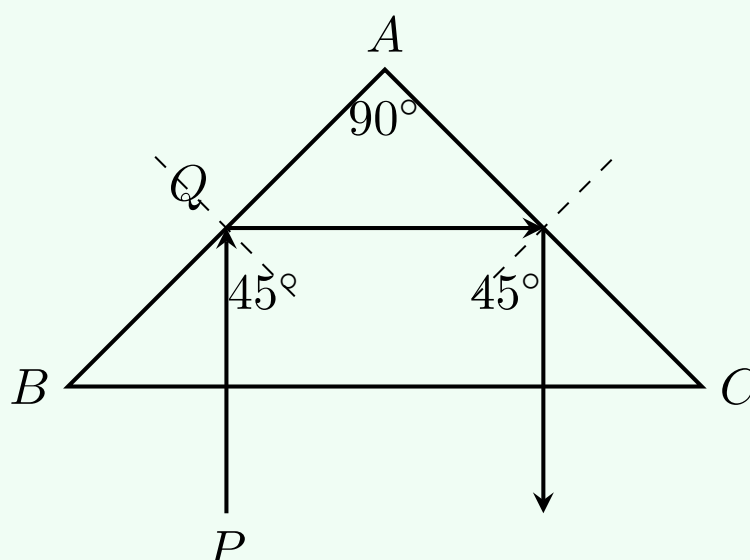
$$\mu = \frac{\sin i}{\sin r_1} = \frac{\sin[(A + \delta_m)/2]}{\sin(A/2)}.$$

This is the refractive index relative to air, whose refractive index is taken as unity.

Step 3: (b)(i) Trace the ray for refractive index 1.5 The ray enters through BC normally, so it suffers no deviation at entry and reaches AB at Q . Since the prism is isosceles and right angled, each base angle is 45° . The incidence at AB is therefore 45° . For $\mu = 1.5$,

$$C = \sin^{-1}\left(\frac{1}{1.5}\right) = 41.81^\circ.$$

Since $45^\circ > C$, total internal reflection occurs at AB . The reflected ray travels horizontally to AC , where it again has incidence 45° and undergoes total internal reflection. It then travels vertically downwards and emerges normally through BC . Thus the emergent ray is parallel to the incident ray but travels in the opposite direction; the net change of direction is 180° .



For refractive index 1.5, two total internal reflections reverse the ray direction.

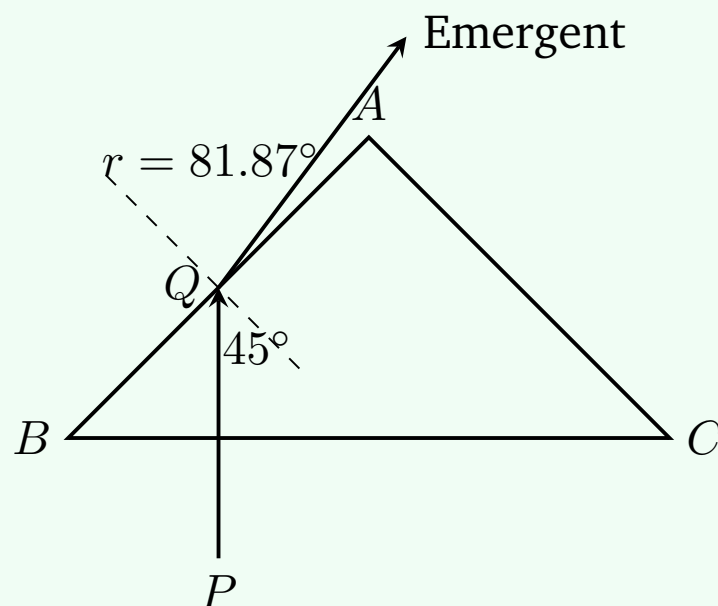
Step 4: (b)(ii) Trace the ray for refractive index 1.4 Normal entry at BC and incidence 45° at AB remain unchanged. However, the new critical angle is

$$C = \sin^{-1}\left(\frac{1}{1.4}\right) = 45.58^\circ.$$

Now the incidence is smaller than the critical angle, so total internal reflection does not occur at AB . The transmitted ray emerges through AB , bending away from its outward normal. Applying Snell's law gives

$$\sin r = 1.4 \sin 45^\circ = 0.98995, \quad r = 81.87^\circ.$$

It emerges nearly along the face, making about 8.13° with AB , as shown. The two-total-reflection path of part (i) is therefore lost. Ordinary partial reflection may still occur at the surface, but the main refracted ray leaves at AB .



For refractive index 1.4, the transmitted ray leaves through AB close to its surface.

Step 5: Conclusion $\mu = \frac{\sin[(A + \delta_m)/2]}{\sin(A/2)}$. For $\mu = 1.5$, the ray undergoes two total internal reflections and returns through BC in the reverse direction. For $\mu = 1.4$, it refracts out through AB, at 81.87° to the normal.

Quick Tip to Solve: For this right-angled prism, compare the refractive index with $\sqrt{2}$: total internal reflection at incidence 45° requires $\mu > \sqrt{2}$.

OR

35. (a) Two thin lenses are placed coaxially in contact. Obtain the expression for the focal length of this combination in terms of the focal lengths of the two lenses.

(b) A converging lens of refractive index 1.5 has a power of 10 D. When it is completely immersed in a liquid, it behaves as a diverging lens of focal length 50 cm. Find the refractive index of the liquid. [5 marks]

Solution:

Concept:

- For a thin lens, the Cartesian lens formula is $1/v - 1/u = 1/f$, where distances are measured from the optical centre with the direction of incident light positive.
- Thin lenses in contact have the same object and final-image reference plane; their powers add algebraically.
- In a surrounding medium of refractive index n_m , the lens-maker formula is $P = (n_g/n_m - 1)(1/R_1 - 1/R_2)$, where n_g is the glass index and the radii carry Cartesian signs.

Step 1: (a) Apply the lens formula to each lens Let the object distance from the first lens be u , its intermediate image distance be v_1 , and the final image distance after the second lens be v . Denote the

focal lengths by f_1 and f_2 . The first lens gives

$$\frac{1}{v_1} - \frac{1}{u} = \frac{1}{f_1}.$$

Since the lenses are thin and in contact, their optical centres have negligible separation. The intermediate image serves as the object for the second lens, with signed object distance v_1 . Thus

$$\frac{1}{v} - \frac{1}{v_1} = \frac{1}{f_2}.$$

These signed equations also apply when the intermediate image is a virtual object for the second lens; no separate change of sign is imposed.

Step 2: (a) Obtain the equivalent focal length Adding the two equations eliminates the intermediate image:

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f_1} + \frac{1}{f_2}.$$

If F is the focal length of the equivalent single lens, its lens formula has the same left-hand side and right-hand side $1/F$. Therefore

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}, \quad F = \frac{f_1 f_2}{f_1 + f_2}.$$

All focal lengths must use the same unit. The formula is algebraic: a diverging lens has negative focal length. If $f_1 + f_2 = 0$, the combination has zero power and infinite focal length.

Step 3: (b) Determine the curvature factor in air Put $K = 1/R_1 - 1/R_2$. The shape of the glass lens does not change on immersion. Taking the refractive index of air as unity, its given power gives

$$10 = (1.5 - 1)K, \quad K = 20 \text{ m}^{-1}.$$

Here the power is expressed in dioptres, so the radii and focal lengths are in metres. The positive value of K is consistent with the lens being converging in air.

Step 4: (b) Use the negative immersed focal length In the liquid the lens is diverging, so its signed focal length is $f_m = -50 \text{ cm} = -0.50 \text{ m}$, giving power $P_m = -2 \text{ D}$. If n_m is the liquid index, then

$$\begin{aligned} -2 &= \left(\frac{1.5}{n_m} - 1 \right) 20, \\ \frac{1.5}{n_m} - 1 &= -0.10, \\ n_m &= \frac{1.5}{0.90} = \frac{5}{3} \simeq 1.67. \end{aligned}$$

The liquid index exceeds the glass index, which reverses the sign of the relative refractive-index factor and explains why the same lens becomes diverging.

Step 5: Conclusion (a) $\boxed{1/F = 1/f_1 + 1/f_2}$, or $\boxed{F = f_1 f_2 / (f_1 + f_2)}$. (b) The refractive index of the liquid is $\boxed{5/3 \simeq 1.67}$.

Quick Tip to Solve: On immersion use the relative index of glass to liquid, and put a minus sign on the focal length when the lens becomes diverging.

36. (a) Derive the law of radioactive decay $N = N_0 e^{-\lambda t}$.

(b) The half life of ${}^{238}_{92}\text{U}$ undergoing α -decay is 4.5×10^9 years. Find its mean life.

(c) What fraction of the initial mass of a radioactive substance will decay in five half-life periods?

[5 marks]

Solution:

Concept:

- Radioactive decay is spontaneous and random: each surviving nucleus has the same constant probability per unit time of decaying. The decay constant λ has units inverse to the unit of time.
- Activity is $A = -dN/dt = \lambda N$, where N is the number of undecayed parent nuclei. The minus sign distinguishes their decreasing number from positive activity.
- Half-life and mean life are $T_{1/2} = \ln 2/\lambda$ and $\tau = 1/\lambda$. The remaining parent mass is proportional to the remaining number of parent nuclei.

Step 1: (a) Form the differential decay equation Let N_0 be the initial number of radioactive parent nuclei at $t = 0$, and let N survive at time t . In a small interval dt , the number that decays is proportional to both the number currently present and the duration of the interval. With positive proportionality constant λ , the change in the surviving population is

$$dN = -\lambda N dt, \quad \frac{dN}{N} = -\lambda dt.$$

This is the radioactive decay law in differential form. It describes the statistical behaviour of a large ensemble, without predicting when an individual nucleus will decay.

Step 2: (a) Integrate with the initial condition Integrate from the initial population N_0 at zero time to N at time t :

$$\begin{aligned} \int_{N_0}^N \frac{dN'}{N'} &= -\lambda \int_0^t dt', \\ \ln N - \ln N_0 &= -\lambda t, \\ \ln \left(\frac{N}{N_0} \right) &= -\lambda t. \end{aligned}$$

Exponentiating gives $N = N_0 e^{-\lambda t}$. The product λt is dimensionless, and the solution satisfies $N(0) = N_0$. For positive λ , it decreases continuously with time, as required for the undecayed parent population.

Step 3: (b) Relate mean life to half-life At $t = T_{1/2}$, half the initial nuclei remain. Hence

$$\frac{1}{2} = e^{-\lambda T_{1/2}}, \quad \lambda T_{1/2} = \ln 2.$$

Mean life is the average lifetime of a nucleus, $\tau = 1/\lambda$. Equivalently, averaging over the decay-time distribution gives

$$\tau = \int_0^\infty t \lambda e^{-\lambda t} dt = \frac{1}{\lambda}.$$

For the given uranium isotope,

$$\tau = \frac{T_{1/2}}{\ln 2} = \frac{4.5 \times 10^9}{0.693} \text{ years} \simeq 6.49 \times 10^9 \text{ years}.$$

The mean life is about 1.443 times the half-life. The fact that the process is alpha decay does not change this relation between its decay constant and lifetime.

Step 4: (c) Distinguish mass remaining from mass decayed Each half-life multiplies the surviving parent population by $1/2$. After five half-life periods, the remaining fraction of the initial radioactive parent mass is

$$\frac{m}{m_0} = \frac{N}{N_0} = \left(\frac{1}{2}\right)^5 = \frac{1}{32}.$$

The question asks for the fraction that has decayed, so subtract this remaining fraction from unity:

$$\frac{m_0 - m}{m_0} = 1 - \frac{1}{32} = \frac{31}{32} = 0.96875.$$

This refers to parent nuclei transformed into decay products; it does not imply that the decay products have vanished from the sample.

Step 5: Conclusion (a) $N = N_0 e^{-\lambda t}$. (b) The mean life of ^{238}U is 6.49×10^9 years. (c) The fraction of initial radioactive parent mass that decays is $31/32$, or 96.875%.

Quick Tip to Solve: After n half-lives, first find the remaining fraction as 2 to the power minus n ; subtract it from one only when the question asks for the decayed fraction.

OR

36. (a) State the postulates of Bohr's model of hydrogen atom and derive the expression for Bohr radius.

(b) Find the ratio of the longest and the shortest wavelengths amongst the spectral lines of Balmer series in the spectrum of hydrogen atom. [5 marks]

Solution:

Concept:

- Bohr's stationary orbits have definite energies and do not radiate energy; allowed angular momenta satisfy $m_e v r = n \hbar$.
- The electrostatic attraction supplies centripetal force. Radiation is emitted or absorbed only during transitions, with $h\nu = |E_i - E_f|$.
- In the Balmer series, transitions terminate at $n_f = 2$; the wavenumber is $1/\lambda = R_H(1/4 - 1/n_i^2)$.

Step 1: (a) State Bohr's postulates An electron revolves around the nucleus only in certain permitted stationary circular orbits. While remaining in a stationary orbit, it neither emits nor absorbs electromagnetic radiation, so its energy stays constant. The permitted orbits are selected by quantisation of orbital angular momentum:

$$m_e v r = n \frac{h}{2\pi} = n \hbar, \quad n = 1, 2, 3, \dots$$

Radiation is emitted when an electron jumps from a higher-energy stationary orbit to a lower-energy one, and absorbed for the reverse transition. Its frequency obeys $h\nu = |E_i - E_f|$, where h is Planck's constant.

Step 2: (a) Derive the radius of the permitted orbit For a hydrogen atom, the proton has charge $+e$ and the electron has charge $-e$. Neglecting nuclear motion, Coulomb attraction provides the

centripetal force:

$$\frac{e^2}{4\pi\epsilon_0 r_n^2} = \frac{m_e v_n^2}{r_n}.$$

Here m_e is the electron mass, r_n the orbital radius and ϵ_0 the vacuum permittivity. The angular-momentum condition gives $v_n = n\hbar/(m_e r_n)$. Substitution eliminates the unknown speed:

$$\frac{e^2}{4\pi\epsilon_0 r_n^2} = \frac{n^2 \hbar^2}{m_e r_n^3}, \quad r_n = \frac{4\pi\epsilon_0 \hbar^2}{m_e e^2} n^2.$$

Thus the radius grows as n^2 . The ground-state radius is $a_0 = 4\pi\epsilon_0 \hbar^2 / (m_e e^2) = \epsilon_0 \hbar^2 / (\pi m_e e^2)$, approximately 5.29×10^{-11} m.

Step 3: (b) Identify the two extreme Balmer wavelengths For Balmer radiation the initial levels are $n_i = 3, 4, 5, \dots$. The longest wavelength has the smallest photon energy, hence is the transition $3 \rightarrow 2$:

$$\frac{1}{\lambda_{\max}} = R_H \left(\frac{1}{4} - \frac{1}{9} \right) = \frac{5R_H}{36}, \quad \lambda_{\max} = \frac{36}{5R_H}.$$

As the initial level increases, the wavenumber increases and the wavelength decreases. The shortest wavelength is the series limit $n_i \rightarrow \infty$:

$$\frac{1}{\lambda_{\min}} = \frac{R_H}{4}, \quad \lambda_{\min} = \frac{4}{R_H}.$$

Taking the ratio cancels the Rydberg constant: $\lambda_{\max}/\lambda_{\min} = (36/5)/4 = 9/5$. The series limit is approached by the spectral lines, rather than attained by a finite initial level.

Step 4: Conclusion The permitted radius is $r_n = a_0 n^2$, with $a_0 = 5.29 \times 10^{-11}$ m. The ratio of the longest to the shortest Balmer wavelength is $\boxed{9 : 5}$.

Quick Tip to Solve: For a hydrogen series, use the first allowed transition for the longest wavelength and the infinite initial level for the shortest.

37. (a) Show that a current carrying solenoid behaves like a small bar magnet. Obtain the expression for the magnetic field at an external point lying on its axis.

(b) A steady current of 2 A flows through a circular coil having 5 turns of radius 7 cm. The coil lies in X - Y plane with its centre at the origin. Find the magnitude and direction of the magnetic dipole moment of the coil. [5 marks]

Solution:

Concept:

- A current loop has magnetic dipole moment $\mathbf{m} = IA\hat{n}$; for N turns, $\mathbf{m} = NIA\hat{n}$, with direction fixed by the right-hand rule.
- A solenoid is a stack of coaxial current loops. Its axial field is the sum of the individual loop fields, and its distant field has the magnetic dipole form.

Step 1: (a) Explain the equivalence to a bar magnet Viewed from an end of a solenoid, anticlockwise current makes that end a north pole, whereas clockwise current makes it a south pole. Magnetic field lines emerge from the north end and enter the south end outside the solenoid; inside they return from south to north, forming closed loops. This field pattern resembles that of a bar magnet. In an external

uniform field the solenoid experiences a torque $\tau = \mathbf{m} \times \mathbf{B}$, just as a bar magnet does. Each of its N turns contributes a parallel moment IA , giving $m = NIA$. The axial-field derivation below makes the equivalence quantitative at distant external points.

Step 2: (a) Obtain the axial field by summing the loop fields Consider a thin solenoid of length $2l$, radius a , turn density n , and current I . Let its centre be the origin and let the external axial point be at $r > l$, measured towards its north end. A slice of thickness dx at position x contains $n dx$ turns. The axial field of that slice is

$$dB = \frac{\mu_0 n I a^2 dx}{2[a^2 + (r - x)^2]^{3/2}}.$$

All transverse contributions cancel by symmetry. Integrating from $-l$ to $+l$, with $u = r - x$, gives

$$B = \frac{\mu_0 n I}{2} \left[\frac{r + l}{\sqrt{a^2 + (r + l)^2}} - \frac{r - l}{\sqrt{a^2 + (r - l)^2}} \right].$$

This expression gives the field at any external axial point of the ideal thin winding, without assuming that the point is far away.

Step 3: (a) Reduce the field to the small-magnet expression When the distances from the point to both ends are large compared with a , use $u/\sqrt{u^2 + a^2} \simeq 1 - a^2/(2u^2)$. Then

$$\begin{aligned} B &\simeq \frac{\mu_0 n I a^2}{4} \left[\frac{1}{(r - l)^2} - \frac{1}{(r + l)^2} \right] \\ &= \frac{\mu_0 n I a^2 r l}{(r^2 - l^2)^2} = \frac{\mu_0}{4\pi} \frac{2mr}{(r^2 - l^2)^2}, \end{aligned}$$

where $m = NIA = (2ln)I\pi a^2$. This is the familiar axial expression for an equivalent slender bar magnet. For a small solenoid observed at a distance $r \gg l, a$, it reduces further to

$$B_{\text{axis}} \simeq \frac{\mu_0}{4\pi} \frac{2m}{r^3}.$$

Thus the current-carrying solenoid produces the same distant axial field as a magnetic dipole of moment NIA , directed along the solenoid's axis towards its north end.

Step 4: (b) Calculate the coil's magnetic moment and state its direction The coil has $N = 5$, $I = 2\text{ A}$, and radius $a = 7\text{ cm} = 0.07\text{ m}$. Its area is $A = \pi a^2$. Therefore

$$m = NIA = 5 \times 2 \times \pi \times (0.07)^2 = 0.049\pi\text{ A m}^2 \simeq 0.154\text{ A m}^2.$$

Since the coil lies in the X - Y plane, its moment is perpendicular to that plane, along the Z -axis. Curling the fingers of the right hand in the direction of conventional current makes the thumb point along the moment. If the current is anticlockwise as viewed from the positive Z -axis, the moment points along $+Z$; if clockwise, it points along $-Z$. The printed question does not specify the current sense, so a unique sign of the axial direction cannot be assigned.

Step 5: Conclusion A small solenoid has dipole moment $m = NIA$ and distant axial field $B = \mu_0(2m)/(4\pi r^3)$. The coil's moment has magnitude 0.154 A m^2 , normal to the X - Y plane along $+Z$ or $-Z$ according to the current sense.

Quick Tip to Solve: Convert the radius to metres before squaring it, and use the right-hand rule to decide the sign of the normal direction.

37. (a) Derive the expression for the force acting between two long parallel current carrying conductors. Hence, define 1 A current.

(b) A bar magnet of dipole moment 3 A m^2 rests with its centre on a frictionless pivot. A force F is applied at right angles to the axis of the magnet, 10 cm from the pivot. It is observed that an external magnetic field of 0.25 T is required to hold the magnet in equilibrium at an angle of 30° with the field. Calculate the value of F . How will the equilibrium be effected if F is withdrawn? [5 marks]

Solution:

Concept:

- A long straight wire carrying current I produces a magnetic field $B = \mu_0 I / (2\pi d)$ at perpendicular distance d ; μ_0 is the permeability of vacuum.
- A straight conductor of length L , carrying current I perpendicular to a magnetic field B , experiences a force of magnitude ILB .
- A magnetic dipole of moment m in a uniform field experiences torque $\tau_B = mB \sin \theta$, where θ is the angle between its moment and the field. A perpendicular force at distance r from a pivot supplies torque Fr .

Step 1: (a) Find the field of the first conductor Consider two very long, straight, parallel conductors in vacuum, separated by distance d , carrying currents I_1 and I_2 . Away from their ends, the field of conductor 1 at conductor 2 has magnitude

$$B_1 = \frac{\mu_0 I_1}{2\pi d}.$$

Its direction follows the right-hand thumb rule. This field is perpendicular to the second conductor because its field lines circle the first conductor.

Step 2: (a) Obtain the force and its direction For a length L of conductor 2, the force due to conductor 1 is given by $\mathbf{F}_{21} = I_2 \mathbf{L} \times \mathbf{B}_1$. Since $\mathbf{L}$ is perpendicular to $\mathbf{B}_1$,

$$F_{21} = I_2 L B_1 = \frac{\mu_0 I_1 I_2 L}{2\pi d}, \quad \frac{F}{L} = \frac{\mu_0 I_1 I_2}{2\pi d}.$$

Applying the right-hand rule to the cross product shows that currents flowing in the same direction attract: each conductor is pulled towards the other. Oppositely directed currents repel. Interchanging the conductors gives an equal magnitude of force on conductor 1 in the opposite direction. The quoted force per unit length neglects end effects, consistent with the conductors being very long.

Step 3: (a) State the ampere definition used in this paper In the conventional force-based definition requested here, one ampere is that constant current which, when maintained in each of two straight parallel conductors of infinite length and negligible circular cross-section, placed one metre apart in vacuum, produces a force of $2 \times 10^{-7} \text{ N}$ per metre of length between them. Indeed, using the conventional value $\mu_0 = 4\pi \times 10^{-7} \text{ N A}^{-2}$,

$$\frac{F}{L} = \frac{(4\pi \times 10^{-7})(1)(1)}{2\pi(1)} = 2 \times 10^{-7} \text{ N m}^{-1}.$$

This is the historical mechanical definition; the present SI fixes the elementary charge instead.

Step 4: (b) Balance torques about the pivot The magnet has $m = 3 \text{ A m}^2$, the field is $B = 0.25 \text{ T}$, and $\theta = 30^\circ$. The force acts at right angles to the magnet at a distance $r = 10 \text{ cm} = 0.10 \text{ m}$ from its centre,

so its moment arm is exactly r . In equilibrium its torque opposes and balances the magnetic torque:

$$\begin{aligned}Fr &= mB \sin \theta, \\ F(0.10) &= (3)(0.25) \sin 30^\circ = 0.375 \text{ N m}, \\ F &= \frac{0.375}{0.10} = 3.75 \text{ N}.\end{aligned}$$

The pivot's reaction passes through the centre and therefore contributes no torque about it.

Step 5: (b) Examine withdrawal of the force On removing F , its opposing torque vanishes. At 30° the magnetic torque is still 0.375 N m , so the magnet is no longer in equilibrium. It rotates towards alignment of its magnetic moment with the field. The magnetic potential energy $U = -mB \cos \theta$ is minimum at $\theta = 0$, making the parallel orientation stable; the antiparallel orientation is unstable. At a frictionless pivot, with no other damping specified, the magnet gains angular speed as it approaches alignment, overshoots it, and oscillates about the field direction. It cannot simply come to rest there without dissipation; with damping, it would eventually settle parallel to the field.

Step 6: Conclusion (a) The force per unit length is $F/L = \mu_0 I_1 I_2 / (2\pi d)$: like-directed currents attract and opposite-directed currents repel. The conventional one-ampere current gives $2 \times 10^{-7} \text{ N m}^{-1}$ at one metre separation in vacuum. (b) $F = 3.75 \text{ N}$. Removing the force destroys the equilibrium at 30° ; the magnet oscillates about its stable orientation parallel to the field when damping is absent.

Quick Tip to Solve: Convert the 10 cm lever arm to metres and use $Fr = mB \sin(\theta)$; the given force is perpendicular, so no extra sine belongs in its torque.