

CBSE Class 12 Physics Question Paper 2020 with Solution PDF-Set 55/5/1



Question Paper with Solutions

Conducted by the Central Board of Secondary Education (CBSE)

Time Allowed: 3 hours

Maximum Marks: 70

General Instructions

- (i) This question paper comprises **four** sections – A, B, C and D.
- (ii) There are **37** questions in the question paper. **All** questions are compulsory.
- (iii) Section A : Q. no. **1** to **20** are very short-answer type questions carrying **1** mark each.
- (iv) Section B : Q. no. **21** to **27** are short-answer type questions carrying **2** marks each.
- (v) Section C : Q. no. **28** to **34** are long-answer type questions carrying **3** marks each.
- (vi) Section D : Q. no. **35** to **37** are also long answer type questions carrying **5** marks each.
- (vii) There is no overall choice in the question paper. However, an internal choice has been provided in **two** questions of **one** mark, **two** questions of **two** marks, **one** question of **three** marks and all the **three** questions of **five** marks. You have to attempt **only one** of the choices in such questions.
- (viii) However, separate instructions are given with each section and question, wherever necessary.
- (ix) Use of calculators and log tables is not permitted.

You may use the following values of physical constants wherever necessary.

$$c = 3 \times 10^8 \text{ m/s}$$

$$h = 6.63 \times 10^{-34} \text{ Js}$$

$$e = 1.6 \times 10^{-19} \text{ C}$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ T m A}^{-1}$$

$$\epsilon_0 = 8.854 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$$

$$\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$$

$$\text{Mass of electron } (m_e) = 9.1 \times 10^{-31} \text{ kg}$$

$$\text{Mass of neutron} = 1.675 \times 10^{-27} \text{ kg}$$

$$\text{Mass of proton} = 1.673 \times 10^{-27} \text{ kg}$$

$$\text{Avogadro's number} = 6.023 \times 10^{23} \text{ per gram mole}$$

$$\text{Boltzmann constant} = 1.38 \times 10^{-23} \text{ JK}^{-1}$$

Section A (20 marks)

Note : Select the most appropriate option from those given below each question.

1. The relationship between Brewster angle θ and the speed of light v in the denser medium is

[1 mark]

- (A) $v \tan \theta = c$
- (B) $c \tan \theta = v$
- (C) $v \sin \theta = c$
- (D) $c \sin \theta = v$

Correct Answer: (A) $v \tan \theta = c$

Solution:

Concept:

- Brewster's law for light incident from air on a transparent medium is $\tan \theta = n$, where θ is the polarising angle and n is the medium's refractive index relative to air.
- Taking the speed in air as c , the refractive index is $n = c/v$, where v is the speed in the denser medium.

Step 1: Express Brewster's law using speeds Substitute the speed definition of refractive index into Brewster's law:

$$\tan \theta = \frac{c}{v}.$$

This uses the usual assumption that the incident medium is air.

Step 2: Compare the printed relations Multiplying both sides by v gives the required product. Option (B) reverses the speed ratio. Options (C) and (D) use sine, whereas Brewster's law involves tangent; they therefore do not express the polarising-angle condition.

Step 3: Conclusion The relationship is $v \tan \theta = c$. Hence, the correct option is (A).

Quick Tip to Solve: At Brewster's angle use $\tan \theta = n = c/v$; cross-multiply only after writing the speed ratio.

2. Photo diodes are used to detect

[1 mark]

- (A) radio waves
- (B) gamma rays
- (C) IR rays
- (D) optical signals

Correct Answer: (D) optical signals

Solution:

Concept:

- A photodiode converts incident optical radiation into an electrical signal. It is generally operated under reverse bias.
- Absorbed photons create electron-hole pairs in or near the depletion region; the junction field separates these carriers and produces photocurrent.

Step 1: Identify the detected signal When light intensity changes, the number of photogenerated

carriers and hence the reverse photocurrent change. Measuring this current allows the device to detect an optical signal, as in optical communication receivers.

Step 2: Compare the choices Optical signals is the broad intended answer. Suitable photodiodes can also detect infrared radiation, so option (C) names a possible spectral range rather than the general application asked here. The ordinary photodiode application in this context is neither radio-wave reception nor gamma-ray detection; options (A) and (B) do not describe it.

Step 3: Conclusion Photodiodes are used to detect optical signals. Hence, the correct option is (D).

Quick Tip to Solve: Link a photodiode with reverse photocurrent: changing light intensity is converted into a changing electrical signal.

3. The selectivity of a series LCR a.c. circuit is large, when

[1 mark]

- (A) L is large and R is large
- (B) L is small and R is small
- (C) L is large and R is small
- (D) $L = R$

Correct Answer: (C) L is large and R is small

Solution:

Concept:

- Selectivity is the ability of a resonant circuit to respond strongly to a narrow range of frequencies near resonance. A larger quality factor means sharper resonance.
- For a series LCR circuit, $Q = \omega_0 L / R = (1/R) \sqrt{L/C}$, where L , C , R and ω_0 are inductance, capacitance, resistance and resonant angular frequency.

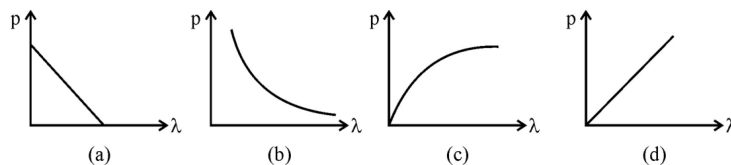
Step 1: Identify changes that sharpen resonance For the same capacitance, increasing L increases $\sqrt{L/C}$, while decreasing R increases its ratio to resistance. Both changes raise Q . The angular-frequency bandwidth $\Delta\omega = R/L$ also decreases.

Step 2: Test the printed combinations A large resistance in (A) opposes high selectivity. In (B), small inductance opposes the beneficial effect of small resistance, so that combination does not ensure high selectivity. Option (D) equates inductance with resistance, which have different dimensions and provides no physical selectivity criterion.

Step 3: Conclusion Large L and small R give high selectivity. Hence, the correct option is (C).

Quick Tip to Solve: Remember $Q = \sqrt{L/C}/R$: at fixed capacitance, larger inductance and smaller resistance sharpen resonance.

4. The graph showing the correct variation of linear momentum (p) of a charge particle with its de-Broglie wavelength (λ) is — [1 mark]



(A) (a)

(B) (b)

(C) (c)

(D) (d)

Correct Answer: (B) (b)

Solution:

Concept:

- The de Broglie relation is $\lambda = h/p$, where h is Planck's constant and p is the magnitude of the particle's linear momentum.
- Thus $p\lambda = h$: momentum and wavelength are inversely proportional, and their product remains constant.

Step 1: Determine the shape of the curve Rearrange the relation as $p = h/\lambda$. For positive wavelength, momentum is positive and decreases as wavelength increases. The decrease is nonlinear: doubling the wavelength halves the momentum. The graph is a rectangular hyperbola approaching both axes asymptotically.

Step 2: Match the printed graphs Graph (b) shows a decreasing hyperbolic curve. Graph (a) is a straight decreasing line and incorrectly reaches zero momentum at a finite wavelength. Graphs (c) and (d) both show momentum increasing with wavelength, contrary to the inverse relation.

Step 3: Conclusion Graph (b) represents $p = h/\lambda$. Hence, the correct option is (B).

Quick Tip to Solve: Rewrite the de Broglie relation as $p\lambda = h$; a constant product identifies the decreasing hyperbola immediately.

5. The wavelength and intensity of light emitted by a LED depend upon [1 mark]

(A) forward bias and energy gap of the semiconductor

(B) energy gap of the semiconductor and reverse bias

(C) energy gap only

(D) forward bias only

Correct Answer: (A) forward bias and energy gap of the semiconductor

Solution:

Concept:

- In a forward-biased light-emitting diode, electron-hole recombination releases photons. The semiconductor band gap E_g largely sets the photon energy.
- The emitted wavelength approximately satisfies $hc/\lambda = E_g$. The light intensity depends on the rate of radiative recombination, which increases with forward current in normal operation.

Step 1: Separate the two dependencies Changing the semiconductor's energy gap changes the characteristic wavelength through $\lambda \approx hc/E_g$. Increasing the forward bias increases carrier injection and forward current, producing more photons per second and greater intensity within the operating range.

Step 2: Compare the available pairs Option (A) includes both required factors: forward bias and energy gap. The factors are listed in reverse order to the corresponding quantities in the question, but the question does not say respectively. Reverse bias in (B) is not normal LED operation. Energy gap alone in (C) omits intensity control; forward bias alone in (D) omits the material's role in setting wavelength.

Step 3: Conclusion Wavelength depends primarily on the energy gap, while intensity depends on forward bias through the forward current. Hence, the correct option is (A).

Quick Tip to Solve: For an LED, pair colour with band gap and brightness with forward current; check that the option includes both factors.

6. A charge particle after being accelerated through a potential difference V enters in a uniform magnetic field and moves in a circle of radius r . If V is doubled, the radius of the circle will become

[1 mark]

- (A) $2r$
- (B) $\sqrt{2}r$
- (C) $4r$
- (D) $r/\sqrt{2}$

Correct Answer: (B) $\sqrt{2}r$

Solution:

Concept:

- A nonrelativistic particle of mass m and charge magnitude $|q|$, accelerated from rest through potential difference V , gains kinetic energy $\frac{1}{2}mv^2 = |q|V$.
- For circular motion perpendicular to a uniform magnetic field of magnitude B , magnetic force supplies centripetal force: $|q|vB = mv^2/r$, giving $r = mv/(|q|B)$.

Step 1: Express the radius in terms of voltage The speed after acceleration is $v = \sqrt{2|q|V/m}$. Substituting into the magnetic radius formula gives

$$r = \frac{1}{B} \sqrt{\frac{2mV}{|q|}}.$$

With the same particle and field, r is proportional to $\sqrt{V}$.

Step 2: Form the radius ratio For the new voltage $V' = 2V$,

$$\frac{r'}{r} = \sqrt{\frac{V'}{V}} = \sqrt{2}.$$

Doubling the voltage doubles kinetic energy, not speed. Thus neither $2r$ nor $4r$ follows, and the radius cannot decrease to $r/\sqrt{2}$ when the speed increases.

Step 3: Conclusion The new radius is $r' = \sqrt{2}r$. Hence, the correct option is (B).

Quick Tip to Solve: Combine $v \propto \sqrt{V}$ with $r \propto v$: multiplying the accelerating voltage by k multiplies the radius by $\sqrt{k}$.

7. The electric flux through a closed Gaussian surface depends upon

[1 mark]

- (A) Net charge enclosed and permittivity of the medium
- (B) Net charge enclosed, permittivity of the medium and the size of the Gaussian surface
- (C) Net charge enclosed only
- (D) Permittivity of the medium only

Correct Answer: (A) Net charge enclosed and permittivity of the medium

Solution:

Concept:

- Gauss's law relates the net electric flux through a closed surface to its enclosed charge: $\Phi_E = \oint \vec{E} \cdot d\vec{A} = Q_{\text{enc}}/\epsilon$, for a uniform medium of permittivity ϵ .
- Electric flux is an algebraic surface integral; outward flux is positive and inward flux is negative.

Step 1: Identify the quantities in Gauss's law For a given net enclosed charge, the flux is inversely proportional to the permittivity. For a given medium, it is proportional to the net charge enclosed. Thus both charge and permittivity determine the flux.

Step 2: Check the remaining options The size and shape of a Gaussian surface do not change its net flux provided it encloses the same net charge, so option (B) adds an irrelevant dependence. Option (C) omits the medium's permittivity and option (D) omits the enclosed charge. Neither alone describes the general dependence asked here.

Step 3: Conclusion The flux depends on the net enclosed charge and the permittivity of the medium. Hence, the correct option is (A).

Quick Tip to Solve: Write $\Phi_E = Q_{\text{enc}}/\epsilon$ first; surface size does not appear in the expression.

8. If photons of frequency ν are incident on the surfaces of metals A & B of threshold frequencies $\nu/2$ and $\nu/3$ respectively, the ratio of the maximum kinetic energy of electrons emitted from A to that from B is [1 mark]

- (A) 2 : 3
- (B) 3 : 4
- (C) 1 : 3
- (D) $\sqrt{3} : \sqrt{2}$

Correct Answer: (B) 3 : 4

Solution:

Concept:

- Einstein's photoelectric equation is $K_{\max} = h(\nu - \nu_0)$, where h is Planck's constant and ν_0 is the threshold frequency.
- The same incident frequency supplies the same photon energy, but different work functions leave different electron kinetic energies.

Step 1: Calculate the two maximum kinetic energies For metal A , the work function is $h\nu/2$; for metal B , it is $h\nu/3$. Both thresholds lie below the incident frequency, so both metals emit electrons. Thus

$$K_A = h\nu - h\nu/2 = h\nu/2, \quad K_B = h\nu - h\nu/3 = 2h\nu/3.$$

Step 2: Compare the energies and options Taking the ratio cancels the common factor $h\nu$:

$$\frac{K_A}{K_B} = \frac{1/2}{2/3} = \frac{3}{4}.$$

Options (A) and (C) give ratios of $2/3$ and $1/3$, neither equal to the calculated $3/4$. Option (D) is also incorrect: square-root relations concern speed, whereas kinetic energy varies directly with excess frequency.

Step 3: Conclusion The required ratio is $K_A : K_B = 3 : 4$. Hence, the correct option is (B).

Quick Tip to Solve: Subtract each threshold frequency from the common incident frequency before taking the ratio.

9. The power factor of a series LCR circuit at resonance will be: [1 mark]

- (A) 1
- (B) 0
- (C) $1/2$
- (D) $1/\sqrt{2}$

Correct Answer: (A) 1

Solution:

Concept:

- The power factor is $\cos \phi = R/Z$, where ϕ is the phase difference between current and applied voltage, R is resistance and Z is impedance.
- In a series LCR circuit, $Z = \sqrt{R^2 + (X_L - X_C)^2}$, with $X_L = \omega L$ and $X_C = 1/(\omega C)$.

Step 1: Apply the resonance condition At resonance, the inductive and capacitive reactances are equal: $X_L = X_C$. Their opposing reactive voltage contributions cancel, leaving the impedance purely resistive. Therefore $Z = R$, and the current is in phase with the source voltage.

Step 2: Evaluate the power factor Substitute the resonant impedance in the power-factor expression:

$$\cos \phi = \frac{R}{R} = 1.$$

The value zero corresponds to a purely reactive circuit. The values $1/2$ and $1/\sqrt{2}$ require nonzero phase differences, unlike resonance. This is the usual series LCR circuit with finite resistance.

Step 3: Conclusion The power factor at resonance is unity. Hence, the correct option is (A).

Quick Tip to Solve: At series resonance cancel the two reactances first; the remaining impedance is just the resistance.

10. A biconcave lens of power P vertically splits into two identical plano concave parts. The power of each part will be [1 mark]

- (A) $2P$
- (B) $P/2$
- (C) P
- (D) $P/\sqrt{2}$

Correct Answer: (B) $P/2$

Solution:

Concept:

- The thin-lens maker formula in air is $P = (\mu - 1)(1/R_1 - 1/R_2)$, where μ is refractive index and R_1, R_2 are signed surface radii.
- A plane refracting surface has infinite radius and contributes zero curvature to the lens power.

Step 1: Express the original lens power The two identical plano-concave parts imply equal magnitudes R of the original curved-surface radii. For light travelling left to right, a biconcave lens has $R_1 = -R$ and $R_2 = +R$. Its power is

$$P = (\mu - 1) \left(-\frac{1}{R} - \frac{1}{R} \right) = -\frac{2(\mu - 1)}{R}.$$

Step 2: Find the power after the split Each part retains one concave surface and gains one plane surface. Its power is therefore $P_{\text{part}} = -(\mu - 1)/R = P/2$. Both halves remain diverging. Option (A) would incorrectly double curvature, and (D) has no basis in the lens-maker relation. Option (C) would apply to merely reducing the aperture while retaining both original curved surfaces, which does not produce the stated plano-concave halves.

Step 3: Conclusion Each plano-concave part has power $P/2$. Hence, the correct option is (B).

Quick Tip to Solve: Count the curved surfaces retained: one of two equal concave contributions gives half the original power.

11. The physical quantity having SI unit NC^{-1}m is _____ .

[1 mark]

Solution:

Concept:

- Electric potential is potential energy per unit charge, $V = U/q$, measured in J C^{-1} .
- One joule is one newton metre: $1 \text{ J} = 1 \text{ N m}$.

Step 1: Rearrange the given unit The printed unit is $\text{N C}^{-1}\text{m}$, with the metre multiplying the electric-field unit. Regroup the factors:

$$\text{N C}^{-1}\text{m} = \text{N m C}^{-1} = \text{J C}^{-1}.$$

Step 2: Identify the physical quantity A joule per coulomb is a volt, the SI unit of electric potential and potential difference. This differs from electric field, which has unit N C^{-1} without the additional length factor. Multiplying a uniform field component by a displacement gives a potential difference.

Step 3: Conclusion The blank is electric potential (or potential difference); the given unit equals the volt.

Quick Tip to Solve: Combine newton and metre into joule, then recognise joule per coulomb as volt.

12. A copper wire of non-uniform area of cross-section is connected to a d.c. battery. The physical quantity which remains constant along the wire is _____ .

[1 mark]

Solution:

Concept:

- Steady current obeys conservation of charge: charge cannot continuously accumulate at any cross-section of a wire.
- Current is $I = JA = neAv_d$, where J is current density, A is area, n is carrier density and v_d is drift speed.

Step 1: Apply charge conservation to the wire After the brief switching transient, the d.c. battery establishes steady flow. The charge entering any segment per second must equal the charge leaving it per second; otherwise charge would build up indefinitely. Hence every cross-section carries the same current.

Step 2: Distinguish current from local quantities Although I is constant, the area varies along the wire. Thus $J = I/A$ is larger in narrower sections. For the same copper carrier density, $v_d = I/(neA)$ also varies inversely with area. The local electric field generally varies as well because $E = \rho J$, where ρ is the resistivity.

Step 3: Conclusion The physical quantity that remains constant along the wire is electric current, I .

Quick Tip to Solve: For a single steady-current path, keep current constant and use the local area to compare current densities.

13. A point charge is placed at the centre of a hollow conducting sphere of internal radius r and outer radius $2r$. The ratio of the surface charge density of the inner surface to that of the outer surface will be _____ . [1 mark]

Solution:

Concept:

- In electrostatic equilibrium the field inside conducting material is zero. A Gaussian surface in the metal must enclose zero net charge.
- For a spherical surface of radius a carrying uniform charge Q , its surface charge density is $\sigma = Q/(4\pi a^2)$.

Step 1: Find the induced surface charges Let the central point charge be q . As conventionally assumed when no initial charge is specified, the hollow conductor is initially neutral. The inner surface acquires $-q$ so that the net charge enclosed by a Gaussian surface in the metal is zero. Charge conservation leaves $+q$ on the outer surface. The central placement makes each surface distribution uniform.

Step 2: Use the two spherical areas The inner radius is r , whereas the outer radius is $2r$. Therefore

$$\sigma_{\text{in}} = -\frac{q}{4\pi r^2}, \quad \sigma_{\text{out}} = \frac{q}{4\pi(2r)^2}.$$

Dividing gives $\sigma_{\text{in}}/\sigma_{\text{out}} = -4$. The minus sign is essential because the induced inner and outer charges have opposite signs; only their magnitudes have ratio 4 : 1.

Step 3: Conclusion The signed ratio is $\sigma_{\text{in}} : \sigma_{\text{out}} = -4 : 1$, for the initially neutral sphere.

Quick Tip to Solve: First determine the opposite surface-charge signs, then use the square of the outer-to-inner radius ratio.

14. The _____ , a property of materials C, Si and Ge depends upon the energy gap between their conduction and valence bands. [1 mark]

Solution:

Concept:

- The forbidden energy gap E_g separates the valence and conduction bands of a solid.
- Electrical conductivity depends on the number of mobile charge carriers; thermal excitation across the band gap can supply conduction electrons and holes.

Step 1: Relate the gap to charge-carrier availability Electrons must gain sufficient energy to cross from the valence band into the conduction band. At a fixed temperature, a smaller energy gap generally permits more electrons to be thermally excited, increasing the number of mobile electrons and holes available to conduct current.

Step 2: Identify the material property Thus the energy-band gap helps determine electrical conductivity. In the usual band comparison, diamond-form carbon has a large gap and is insulating, whereas silicon and germanium have smaller gaps and behave as semiconductors. The question concerns the

ability of the materials to carry electric current, rather than a mechanical property.

Step 3: Conclusion The blank is electrical conductivity.

Quick Tip to Solve: Connect band gap to thermal carrier generation: easier excitation generally means higher electrical conductivity.

15. The ability of a junction diode to _____ an alternating voltage, is based on the fact that it allows current to pass only when it is forward biased. [1 mark]

Solution:

Concept:

- Rectification is the conversion of an alternating electrical signal into a unidirectional signal.
- A junction diode conducts readily in forward bias and offers very high resistance in reverse bias.

Step 1: Identify the conducting half-cycle When the alternating input makes the diode forward biased, its barrier is reduced and appreciable current flows through the load.

Step 2: Examine the opposite half-cycle When the input reverses, the diode becomes reverse biased and blocks appreciable current. Thus the load current retains only one direction; the output is pulsating rather than steady dc.

Step 3: Conclusion The missing word is **rectify**: a junction diode can rectify an alternating voltage because of its directional conduction.

Quick Tip to Solve: Link forward-only conduction to rectification; filtering is the separate process that smooths the pulsating output.

16. Define the term 'current sensitivity' of a moving coil galvanometer. [1 mark]

Solution:

Concept:

- Current sensitivity measures the angular deflection produced per unit current through a galvanometer.
- In a radial magnetic field, magnetic torque is $NIAB$ and restoring torque is $C\theta$, where N is the number of turns, A the coil area, B the field and C the torsional constant.

Step 1: Relate deflection to current At equilibrium the two torques balance: $NIAB = C\theta$. Hence the deflection θ is proportional to the current I through the coil.

Step 2: Express sensitivity as a ratio Divide the equilibrium relation by CI : $\theta/I = NAB/C$. The ratio compares how much angular movement a given current produces, so a larger ratio means greater sensitivity.

Step 3: Conclusion Current sensitivity is the angular deflection per unit current: $S_I = \theta/I = NAB/C$, measured in rad A^{-1} when deflection is in radians.

Quick Tip to Solve: For current sensitivity divide deflection by current; dividing by voltage instead defines voltage sensitivity.

17. Depict the fields diagram of an electromagnetic wave propagating along positive X -axis with its electric field along Y -axis. [1 mark]

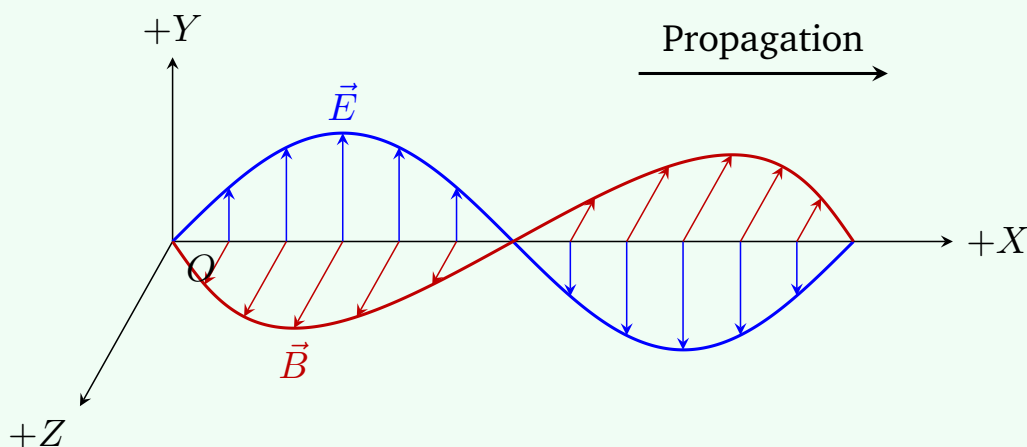
Solution:

Concept:

- An electromagnetic wave is transverse: $\vec{E}$, $\vec{B}$ and the direction of propagation are mutually perpendicular.
- Propagation is along $\vec{E} \times \vec{B}$. The electric and magnetic fields oscillate in phase, with $E_0 = cB_0$ in vacuum.

Step 1: Choose the magnetic-field axis With propagation along $+X$ and the electric field along Y , the magnetic field lies along Z . When $\vec{E}$ points along $+Y$, $\vec{B}$ points along $+Z$, since $\hat{y} \times \hat{z} = \hat{x}$.

Step 2: Show the fields in phase A consistent pair of waves is $\vec{E} = E_0 \sin(kx - \omega t)\hat{y}$ and $\vec{B} = B_0 \sin(kx - \omega t)\hat{z}$. Their zeros and maxima occur at the same positions. Both reverse together in the next half-cycle, keeping propagation along $+X$.



In-phase electric and magnetic fields; the two amplitudes are drawn on separate scales.

Step 3: Conclusion The fields diagram has $\vec{E} \parallel Y$, $\vec{B} \parallel Z$ and $\vec{E} \times \vec{B} \parallel +X$, as shown.

Quick Tip to Solve: Use $\hat{y} \times \hat{z} = \hat{x}$ to choose the magnetic-field direction, and draw both waves with matching zeros.

18. Write the conditions on path difference under which (i) constructive (ii) destructive interference occur in Young's double slit experiment. [1 mark]

Solution:

Concept:

- In Young's experiment the two coherent slits act as sources with the same initial phase.
- A path difference Δ produces phase difference $\phi = 2\pi\Delta/\lambda$, where λ is the wavelength.

Step 1: Set the condition for constructive interference The waves reinforce when they arrive in phase, so $\phi = 2n\pi$. Substitution in the phase-path relation gives $\Delta = n\lambda$, with n an integer. Zero path difference gives the central bright fringe.

Step 2: Set the condition for destructive interference The waves oppose when they arrive with an odd multiple of π phase difference: $\phi = (2n + 1)\pi$. Thus $\Delta = (2n + 1)\lambda/2$. For equal amplitudes the

resultant intensity at these points is zero.

Step 3: Conclusion (i) Constructive: $\Delta = n\lambda$. (ii) Destructive: $\Delta = (2n + 1)\lambda/2$, where $n \in \mathbb{Z}$ for signed path difference.

Quick Tip to Solve: Whole wavelengths give bright fringes; odd half-wavelengths give dark fringes in the usual in-phase Young's setup.

19. Plot a graph showing variation of induced e.m.f. with the rate of change of current flowing through a coil. [1 mark]

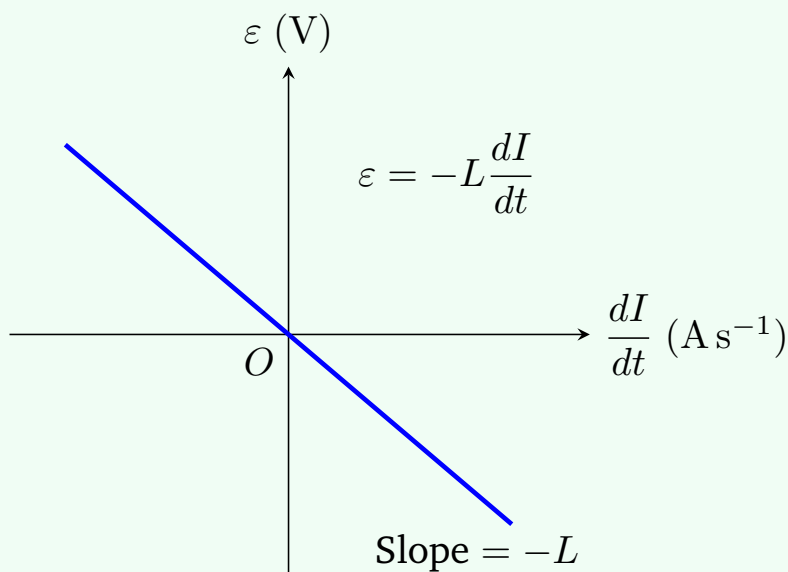
Solution:

Concept:

- For a coil of constant self-inductance L , the flux linkage is $N\Phi = LI$, where I is the current.
- Faraday's law gives the signed induced emf $\varepsilon = -L dI/dt$; the minus sign expresses Lenz's law.

Step 1: Identify the graph variables Put the rate of change of current dI/dt on the horizontal axis and signed induced emf ε on the vertical axis. For a fixed coil, L is constant, so the relation has the form $y = mx$.

Step 2: Determine the intercept and slope When $dI/dt = 0$, the induced emf is zero. The graph therefore passes through the origin, and its slope is $-L$. Increasing positive dI/dt gives a more negative emf. If only magnitudes are plotted, $|\varepsilon| = L|dI/dt|$ has positive slope L .



Signed induced emf versus rate of change of current, for constant self-inductance.

Step 3: Conclusion The signed ε -versus- dI/dt graph is a straight line through the origin with slope $-L$, as shown.

Quick Tip to Solve: Keep the minus sign for a signed-emf graph; a positive slope applies when plotting the magnitudes of both quantities.

19. A series combination of an inductor (L), capacitor (C) and a resistor (R) is connected across an ac source of emf of peak value E_0 and angular frequency (ω). Plot a graph to show variation of impedance of the circuit with angular frequency (ω). [1 mark]

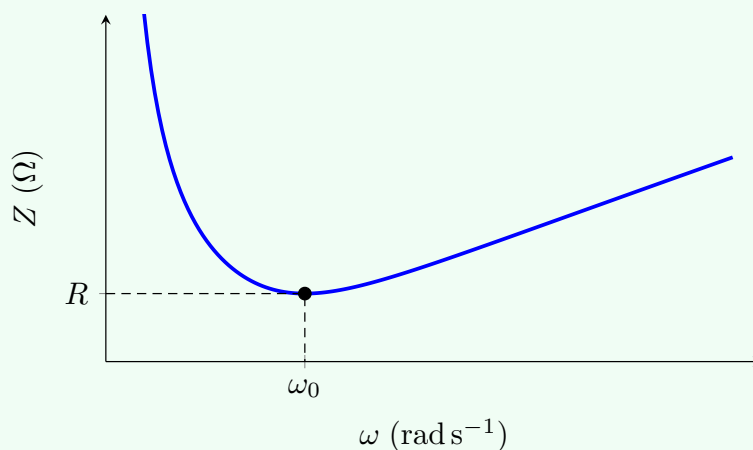
Solution:

Concept:

- The impedance of a series LCR circuit is $Z = \sqrt{R^2 + (X_L - X_C)^2}$, where $X_L = \omega L$ and $X_C = 1/(\omega C)$.
- At resonance the reactances cancel: $\omega_0 = 1/\sqrt{LC}$, giving the minimum impedance $Z = R$.

Step 1: Locate the minimum Since the squared reactance difference cannot be negative, the impedance cannot fall below R . It reaches R only when $\omega L = 1/(\omega C)$, at the resonant angular frequency ω_0 . The source peak emf E_0 does not affect this impedance curve.

Step 2: Sketch the limiting behaviour As $\omega \rightarrow 0^+$, the capacitive reactance becomes very large and $Z \rightarrow \infty$. Increasing frequency reduces Z until resonance; above resonance the inductive reactance dominates and Z increases without bound. Thus the curve is not symmetric about its minimum and does not meet the horizontal axis.

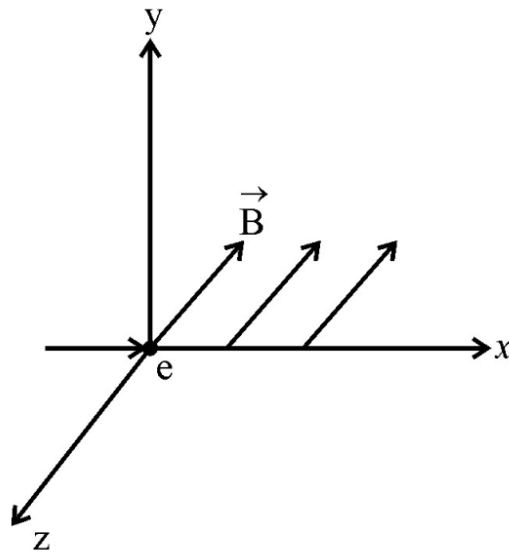


Impedance versus angular frequency (schematic scale).

Step 3: Conclusion The graph falls to its minimum R at $\omega_0 = 1/\sqrt{LC}$, then rises, as shown.

Quick Tip to Solve: Mark the nonzero minimum (ω_0, R) first; draw a steep capacitive branch on the left and a rising inductive branch on the right.

20. An electron moves along $+x$ direction. It enters into a region of uniform magnetic field B directed along $-z$ direction as shown in fig. Draw the shape of trajectory followed by the electron after entering the field. [1 mark]



Solution:

Concept:

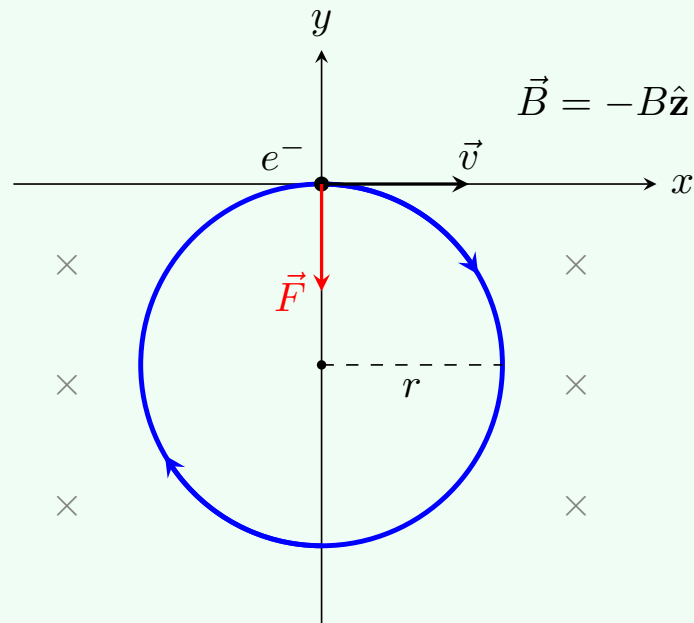
- Magnetic force is $\vec{F} = q(\vec{v} \times \vec{B})$. The electron has charge $q = -e$, with $e > 0$.
- For velocity perpendicular to a uniform magnetic field, the magnetic force supplies centripetal acceleration, giving a circular trajectory of radius $r = m_e v / (eB)$.

Step 1: Determine the initial force Here $\vec{v} = v\hat{x}$ and $\vec{B} = -B\hat{z}$. Their cross product points along $+y$, but the negative electron charge reverses the force:

$$\vec{F} = -e[v\hat{x} \times (-B\hat{z})] = -evB\hat{y}.$$

Therefore the path initially bends towards $-y$.

Step 2: Trace the trajectory The force always remains perpendicular to the instantaneous velocity, so it changes the direction but not the speed. Since v and B are constant, the radius is constant. The motion lies in the xy -plane and is clockwise when viewed from the $+z$ side. The sketch shows the circular path while the electron remains in the field.



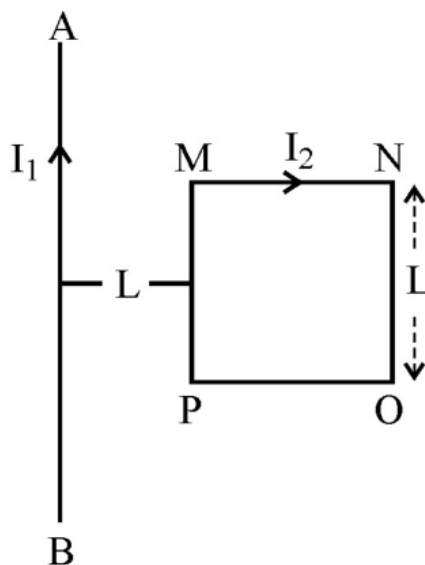
View from the positive z -axis: crosses denote the field into the page.

Step 3: Conclusion The electron follows a circular path in the xy -plane, initially curving towards $-y$, with radius $r = m_e v / (eB)$.

Quick Tip to Solve: Find $\vec{v} \times \vec{B}$ first, then reverse that direction for an electron; forgetting its negative charge reverses the curvature.

OR

20. A square shaped current carrying loop $MNOP$ is placed near a straight long current carrying wire AB as shown in the fig. The wire and the loop lie in the same plane. If the loop experiences a net force F towards the wire, find the magnitude of the force on the side NO of the loop. [1 mark]



Solution:

Concept:

- The field at distance d from a long wire carrying current I_1 is $B(d) = \mu_0 I_1 / (2\pi d)$.
- A parallel side of length L , carrying current I_2 , experiences force $I_2 L B(d)$. Parallel currents attract; antiparallel currents repel.

Step 1: Compare the two vertical sides The nearer side PM is at distance L from the wire and carries current upwards, parallel to I_1 ; its force is towards the wire. The side NO is at distance $2L$ and carries current downwards, so its force is away from the wire. Calling these magnitudes F_{PM} and F_{NO} ,

$$F_{PM} = \frac{\mu_0 I_1 I_2 L}{2\pi L}, \quad F_{NO} = \frac{\mu_0 I_1 I_2 L}{2\pi(2L)}.$$

Thus $F_{PM} = 2F_{NO}$.

Step 2: Use the given resultant force The forces on the horizontal sides MN and OP are equal and opposite: each spans the same range of distances from the wire and carries the same current in opposite directions. They cancel. The net force towards the wire is therefore

$$F = F_{PM} - F_{NO} = 2F_{NO} - F_{NO}.$$

Step 3: Conclusion The magnitude of the force on side NO is $\boxed{F}$. This individual force acts away from the wire.

Quick Tip to Solve: The two vertical sides are at distances L and $2L$, so their forces are in the ratio 2 : 1; their difference equals the smaller force.

Section B (14 marks)

21. Derive the expression for the torque acting on an electric dipole, when it is held in a uniform electric field. Identify the orientation of the dipole in the electric field, in which it attains a stable equilibrium.

[2 marks]

Solution:

Concept:

- An electric dipole consists of charges $-q$ and $+q$, separated by $2a$. Its dipole moment $\vec{p}$ points from $-q$ to $+q$, with magnitude $p = 2aq$.
- A couple has zero resultant force but a torque equal to either force multiplied by the perpendicular distance between the forces' lines of action.
- Stable equilibrium corresponds to minimum potential energy; for a dipole in a uniform field, $U = -\vec{p} \cdot \vec{E}$.

Step 1: Identify the electric-force couple Let the dipole axis make an angle θ with a uniform electric field $\vec{E}$. The positive charge experiences $+q\vec{E}$, and the negative charge experiences $-q\vec{E}$. These forces have equal magnitude qE , so the net force is zero. Their separated parallel lines of action form a couple, whose perpendicular arm is $2a \sin \theta$.

Step 2: Derive the torque Multiplying either force by the couple arm gives the torque magnitude:

$$\tau = (qE)(2a \sin \theta) = pE \sin \theta.$$

Its direction is perpendicular to the plane containing $\vec{p}$ and $\vec{E}$, in the sense that rotates the dipole towards alignment with the field. Thus the vector expression is $\vec{\tau} = \vec{p} \times \vec{E}$.

Step 3: Test the equilibrium orientations The torque vanishes at $\theta = 0$ and $\theta = \pi$. At $\theta = 0$, the energy $U = -pE \cos \theta$ is minimum, $-pE$, and a small angular displacement produces a restoring torque. At $\theta = \pi$, the energy is maximum and a small displacement drives the dipole farther away, so that orientation is unstable.

Step 4: Conclusion The torque is $\vec{\tau} = \vec{p} \times \vec{E}$, of magnitude $pE \sin \theta$. Stable equilibrium occurs when $\vec{p}$ is parallel to $\vec{E}$, i.e. $\theta = 0$.

Quick Tip to Solve: Zero torque alone does not prove stability: compare the energy or the restoring response. Parallel alignment minimizes $-pE \cos \theta$.

OR

21. Obtain the expression for the energy stored in a capacitor connected across a dc battery. Hence define energy density of the capacitor. [2 marks]

Solution:

Concept:

- The potential difference of a capacitor of capacitance C , when it carries charge q , is $v = q/C$. Its capacitance is assumed constant during charging.
- The electrostatic work required to add a small charge dq is $dU = v dq$. Energy density means field energy stored per unit volume.

Step 1: Integrate the work of charging Initially the capacitor is uncharged. If its final charge is Q after connection to a dc battery of voltage V , then $Q = CV$. Since its own potential rises from zero to V , the incremental work stored as field energy is

$$U = \int_0^Q \frac{q}{C} dq = \frac{Q^2}{2C} = \frac{1}{2}QV = \frac{1}{2}CV^2.$$

Thus the stored energy is obtained using the changing capacitor potential. For ordinary resistive charging, the battery supplies QV ; only half of this is stored, and the remainder is dissipated in the charging circuit.

Step 2: Express the energy per unit volume For a parallel-plate capacitor with plate area A , separation d , and vacuum between its plates, neglect fringing. The field occupies volume Ad ; using $C = \epsilon_0 A/d$ and $V = Ed$,

$$u = \frac{U}{Ad} = \frac{\frac{1}{2}(\epsilon_0 A/d)(Ed)^2}{Ad} = \frac{1}{2}\epsilon_0 E^2.$$

Here E is the uniform electric field between the plates. The SI unit of energy density is J m^{-3} .

Step 3: Conclusion The capacitor stores $U = \frac{1}{2}CV^2 = Q^2/(2C)$. Energy density is energy stored per unit volume; in vacuum, $u = \frac{1}{2}\epsilon_0 E^2$.

Quick Tip to Solve: Integrate the capacitor's changing potential q/C ; using the final battery voltage throughout would give twice the stored energy.

22. Gamma rays and radio waves travel with the same velocity in free space. Distinguish between them in terms of their origin and the main application. [2 marks]

Solution:

Concept:

- All electromagnetic waves travel in vacuum with speed $c = 1/\sqrt{\mu_0\epsilon_0}$, regardless of their frequency. Equal speeds therefore do not imply identical sources or uses.
- The photon energy is $E = h\nu$, where h is Planck's constant and ν is frequency. Gamma rays have much higher frequencies and photon energies than radio waves.

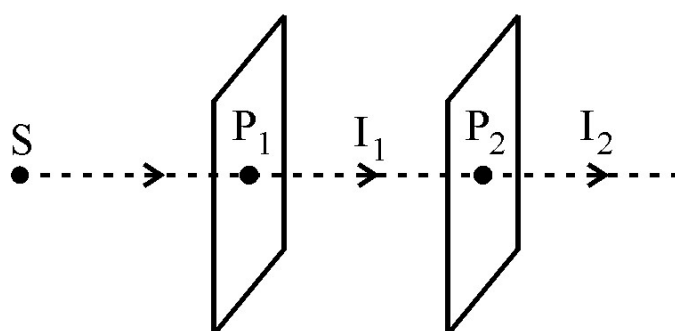
Step 1: Identify the origin and use of gamma rays Gamma rays originate in nuclear transitions: an excited atomic nucleus loses energy by emitting a high-energy photon, often following radioactive decay. Their important application is cancer treatment by radiotherapy. Their penetrating, ionising radiation can damage and destroy malignant cells when directed at the tumour in a controlled manner.

Step 2: Identify the origin and use of radio waves Radio waves are produced by accelerated electric charges, commonly electrons oscillating in a transmitting antenna driven by an alternating current. Their principal application is wireless communication, including radio and television broadcasting. Information is carried by modulating a radio-frequency carrier and recovered by a receiving antenna and receiver.

Step 3: Conclusion Gamma rays arise from nuclear transitions and are used in cancer radiotherapy. Radio waves arise from oscillating accelerated charges in antennas and are used in broadcasting and wireless communication. Both travel with speed c in free space.

Quick Tip to Solve: Pair each source with its use: nuclear transitions with radiotherapy, and oscillating antenna charges with radio communication.

23. Light from a sodium lamp (S) passes through two polaroid sheets P_1 and P_2 as shown in fig. What will be the effect on the intensity of the light transmitted (i) by P_1 and (ii) by P_2 on rotating polaroid P_1 about the direction of propagation of light? Justify your answer in both cases. [2 marks]



Solution:

Concept:

- Light emitted by a sodium lamp is unpolarised: its electric-field vibrations have no preferred transverse direction. An ideal polaroid transmits half the incident unpolarised intensity.
- Malus's law gives $I_2 = I_1 \cos^2 \theta$, where I_1 is the polarised intensity incident on the analyser and θ is the angle between the transmission axes of the two polaroids.

Step 1: Examine the light transmitted by the rotating first polaroid Let the incident intensity at P_1 be

I_0 . Because the lamp light is unpolarised, all orientations of the transmission axis of P_1 are equivalent. Rotating P_1 about the propagation direction therefore leaves its transmitted intensity unchanged:

$$I_1 = \frac{I_0}{2}.$$

The transmitted light is plane polarised. Its polarisation direction rotates with P_1 , although its intensity stays constant.

Step 2: Apply Malus's law at the second polaroid The transmission axis of P_2 is fixed. Rotation of P_1 changes the angle θ between that axis and the incident polarisation direction. Consequently,

$$I_2 = \frac{I_0}{2} \cos^2 \theta.$$

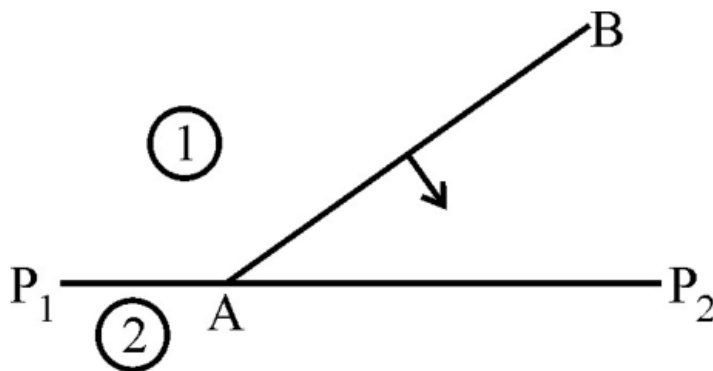
The intensity is maximum, $I_0/2$, when the two axes are parallel and zero when they are perpendicular. During one complete revolution of P_1 , the transmitted intensity through P_2 passes through two maxima and two minima.

Step 3: Conclusion (i) Intensity through P_1 remains constant at $I_0/2$. (ii) Intensity through P_2 varies between zero and $I_0/2$ according to Malus's law as P_1 rotates.

Quick Tip to Solve: First decide whether the incident light is unpolarised; then track the changing relative angle of the two transmission axes.

OR

23. Define the term 'wave front of light'. A plane wave front AB propagating from denser medium (1) into a rarer medium (2) is incident on the surface P_1P_2 separating the two media as shown in fig. Using Huygen's principle, draw the secondary wavelets and obtain the refracted wave front in the diagram. [2 marks]



Solution:

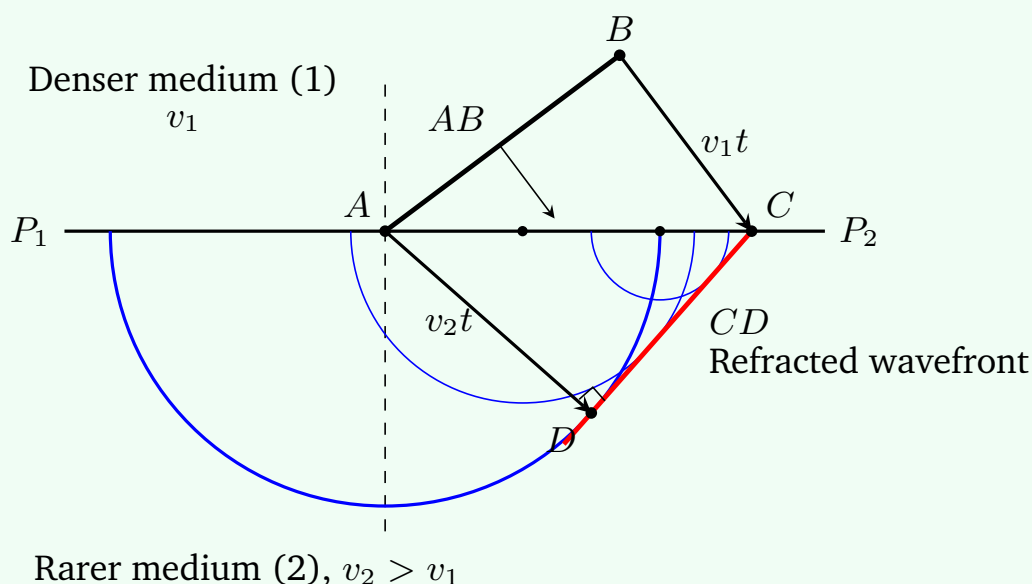
Concept:

- A wavefront of light is the locus of all points in a medium that vibrate in the same phase at a given instant. Rays in an isotropic medium are normal to the wavefront.
- Huygens' principle states that every point on a wavefront emits secondary wavelets. Their forward common envelope gives the wavefront at a later time. In the rarer medium, the speed v_2 exceeds the speed v_1 in the denser medium.

Step 1: Construct the secondary wavelets at the interface In the given geometry, A reaches the interface first. Let the point B of the incident plane wavefront reach C on the interface after time t , along the incident ray perpendicular to AB . Then $BC = v_1 t$. In the same time, the secondary wavelet from A advances a distance $AD = v_2 t$ into medium (2). Draw this wavelet with centre A and radius $v_2 t$. Other interface points, reached later by the incident wavefront, produce smaller wavelets in the rarer medium.

Step 2: Draw the common tangent as the refracted wavefront Draw the tangent CD from C to the wavelet centred at A ; the radius AD is perpendicular to CD . This tangent is also the forward common envelope of the smaller secondary wavelets. Hence it is the refracted plane wavefront. Since $AD = v_2 t > BC = v_1 t$, the refracted ray bends away from the normal. With incidence angle i and refraction angle r , the right triangles give

$$\frac{\sin i}{\sin r} = \frac{BC}{AD} = \frac{v_1}{v_2} < 1, \quad r > i.$$



Huygens' construction: AB is incident and CD is the refracted wavefront. The sketch is schematic.

Step 3: Conclusion A wavefront is a surface of constant phase. The forward common tangent CD to the secondary wavelets in medium (2) is the required refracted wavefront, and AD gives its normal ray direction.

Quick Tip to Solve: Draw the wavelet in the rarer medium larger than BC , then draw its tangent from C ; the tangent is the wavefront, not the ray.

24. A heavy nucleus P of mass number 240 and binding energy 7.6 MeV per nucleon splits in to two nuclei Q and R of mass numbers 110, 130 and binding energy per nucleon 8.5 MeV and 8.4 MeV, respectively. Calculate the energy released in the fission. [2 marks]

Solution:

Concept:

- The total binding energy of a nucleus is its mass number times its binding energy per nucleon:
 $B = A\varepsilon_b$.
- In fission, the energy released equals the increase in total binding energy, $E = B_{\text{products}} - B_{\text{parent}}$, because more strongly bound products have less rest-mass energy.

Step 1: Calculate the parent's total binding energy The parent nucleus P has 240 nucleons, each contributing an average binding energy of 7.6 MeV. Thus

$$B_P = 240 \times 7.6 = 1824 \text{ MeV}.$$

The two fragment mass numbers add to $110 + 130 = 240$, so the stated reaction conserves nucleon number without any additional fragment to include.

Step 2: Calculate the total binding energy of the fragments Multiply each fragment's binding energy per nucleon by its own mass number:

$$B_Q = 110 \times 8.5 = 935 \text{ MeV},$$

$$B_R = 130 \times 8.4 = 1092 \text{ MeV},$$

$$B_Q + B_R = 2027 \text{ MeV}.$$

The relevant comparison is between these total binding energies, rather than a direct subtraction of the per-nucleon values.

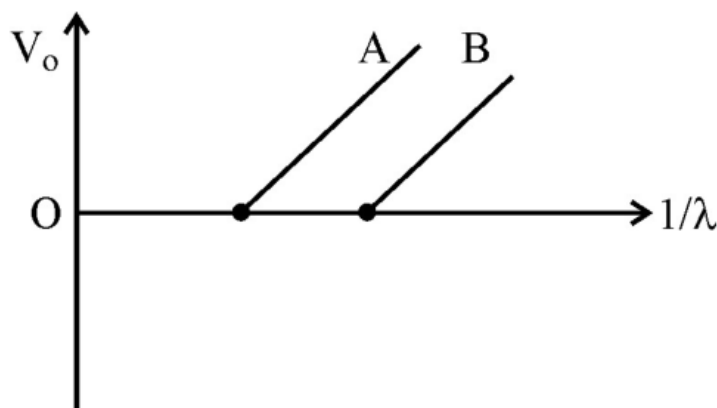
Step 3: Conclusion The energy released is $E = 2027 - 1824 = 203 \text{ MeV}$ per fission. The positive increase in binding energy represents the decrease in rest-mass energy available as released energy.

Quick Tip to Solve: Multiply each binding energy per nucleon by its mass number first; subtract the parent's total from the sum for the fragments.

25. Figure shows the stopping potential (V_0) for the photo electron versus $(1/\lambda)$ graph, for two metals A and B, λ being the wavelength of incident light.

(a) How is the value of Planck's constant determined from the graph ?

(b) If the distance between the light source and the surface of metal A is increased, how will the stopping potential for the electrons emitted from it be effected ? Justify your answer. [2 marks]



Solution:

Concept:

- Einstein's photoelectric equation is $eV_0 = h\nu - \phi = hc/\lambda - \phi$, where e is the magnitude of electronic charge, h is Planck's constant, c is the speed of light, and ϕ is the metal's work function.
- The stopping potential measures the maximum photoelectron kinetic energy: $K_{\max} = eV_0$. At a fixed wavelength it is independent of the incident intensity.

Step 1: (a) Relate the slope to Planck's constant Rearrange the photoelectric equation into the straight-line form

$$V_0 = \frac{hc}{e} \left(\frac{1}{\lambda} \right) - \frac{\phi}{e}.$$

Select two well-separated points on either straight line and calculate the slope using the actual axis scales:

$$m = \frac{\Delta V_0}{\Delta(1/\lambda)} = \frac{hc}{e}.$$

Both metals have the same slope because h , c and e are universal constants; different work functions change the intercepts. Use $1/\lambda$ in m^{-1} and V_0 in volts to obtain h in Js.

Step 2: (b) Examine the effect of increasing the distance Moving the light source farther from metal A decreases the light intensity at its surface while leaving the wavelength unchanged. Fewer photons then reach the metal per unit time, so the photoelectric current decreases. Each photon still carries the same energy hc/λ , and the work function of metal A is unchanged. Hence the maximum kinetic energy $hc/\lambda - \phi_A$ does not change.

Step 3: Conclusion (a) Planck's constant is $h = (e/c) [\Delta V_0 / \Delta(1/\lambda)]$. (b) The stopping potential is unchanged, because increasing the distance changes photon flux but not photon energy.

Quick Tip to Solve: Check the horizontal axis: for a plot against inverse wavelength the slope is hc/e , whereas a plot against frequency has slope h/e .

26. Use Bohr's model of hydrogen atom to obtain the relationship between the angular momentum and the magnetic moment of the revolving electron. [2 marks]

Solution:

Concept:

- In Bohr's model an electron of mass m_e revolves in a circular orbit of radius r with speed v ; its orbital angular momentum has magnitude $L = m_e v r = n\hbar$.
- A charge revolving periodically constitutes a current loop. Its magnetic moment has magnitude $\mu = IA$, where I is the current magnitude and A is the loop area.

Step 1: Obtain the equivalent orbital current The electron takes a time $T = 2\pi r/v$ to complete one revolution. A charge of magnitude e passes a fixed point once each period, so the equivalent current magnitude is

$$I = \frac{e}{T} = \frac{ev}{2\pi r}.$$

Since the electron has charge $-e$, the conventional current flows opposite to the direction of the electron's motion.

Step 2: Relate the magnetic moment to angular momentum The circular orbit encloses area $A = \pi r^2$. Therefore

$$\mu = I\pi r^2 = \frac{ev}{2\pi r} \pi r^2 = \frac{evr}{2}.$$

Comparing this expression with $L = m_e v r$ gives the magnitude ratio $\mu/L = e/(2m_e)$. The angular momentum direction follows the right-hand rule for the electron's motion, while the magnetic moment follows the right-hand rule for conventional current. These directions are opposite.

Step 3: Conclusion The required vector relation is $\mu = -\frac{e}{2m_e} \mathbf{L}$. Its magnitude is $\mu = eL/(2m_e) = ne\hbar/(2m_e) = n\mu_B$, where $\mu_B = e\hbar/(2m_e)$ is the Bohr magneton. The magnetic moment is antiparallel to the orbital angular momentum.

Quick Tip to Solve: Use current e/T and loop area πr^2 , then replace vr by L/m ; retain the minus sign in the vector relation for an electron.

27. In a single slit diffraction experiment, the width of the slit is increased. How will the (i) size and (ii) intensity of central bright band be affected? Justify your answer. [2 marks]

Solution:

Concept:

- For a slit of width a illuminated by light of wavelength λ , the first diffraction minima satisfy $a \sin \theta = \pm \lambda$. The central bright band lies between these minima.
- At the centre of the pattern, secondary waves from every point across the slit arrive in phase. For unchanged, uniform incident illumination, the resultant amplitude is proportional to the slit width and the intensity is proportional to the square of the amplitude.

Step 1: (i) Determine the change in the band's size If the screen is at distance D , the small-angle approximation gives the positions of the first minima as $y = \pm D\lambda/a$. Thus the full linear width and angular width of the central band are

$$W = \frac{2D\lambda}{a}, \quad \Delta\theta \simeq \frac{2\lambda}{a}.$$

With D and λ fixed, the width varies inversely with a . Increasing the slit width brings the first minima closer to the centre of the pattern.

Step 2: (ii) Determine the change in central intensity A wider slit admits more light under the same incident illumination. At zero diffraction angle there is no path difference between contributions from different parts of the slit, so they add constructively. If the width changes from a_1 to a_2 , then

$$\frac{E_{\text{centre},2}}{E_{\text{centre},1}} = \frac{a_2}{a_1}, \quad \frac{I_{\text{centre},2}}{I_{\text{centre},1}} = \left(\frac{a_2}{a_1}\right)^2.$$

Here E_{centre} denotes the wave's electric-field amplitude. This comparison holds with the incident intensity and other apparatus settings unchanged.

Step 3: Conclusion On increasing the slit width, (i) the central bright band's size decreases, since $W \propto 1/a$; (ii) its central intensity increases, since the in-phase amplitude increases and $I_{\text{centre}} \propto a^2$ for fixed incident illumination.

Quick Tip to Solve: Track two separate effects of a wider slit: the first minima move inward, while more in-phase light raises the intensity at the centre.

Section C (21 marks)

28. (a) Differentiate between electrical resistance and resistivity of a conductor.

(b) Two metallic rods, each of length L , area of cross A_1 and A_2 , having resistivities ρ_1 and ρ_2 are connected in parallel across a d.c. battery. Obtain the expression for the effective resistivity of this combination. [3 marks]

Solution:

Concept:

- Resistance $R = V/I$ measures a particular conductor's opposition to current, whereas resistivity ρ characterises its material at a specified temperature.
- For a uniform rod of length L and cross-sectional area A , $R = \rho L/A$. Parallel rods have the same potential difference and their currents add.

Step 1: (a) Distinguish resistance from resistivity The resistance of a conductor depends on its material, length, cross-sectional area and temperature. Its SI unit is the ohm, Ω . For a fixed material and temperature, increasing length increases resistance and increasing area decreases it. Resistivity is defined by $\rho = RA/L$; it is the resistance of a specimen of unit length and unit cross-sectional area, with the appropriate dimensions retained. Its SI unit is $\Omega \text{ m}$. Resistivity depends on the nature of the material and temperature, but not on the conductor's length or area.

Step 2: (b) Combine the two rod resistances Each rod spans the same length L , so its resistance is

$$R_1 = \frac{\rho_1 L}{A_1}, \quad R_2 = \frac{\rho_2 L}{A_2}.$$

The common battery voltage gives total current $I = V/R_1 + V/R_2$. Hence

$$\frac{1}{R_{\text{eq}}} = \frac{1}{L} \left(\frac{A_1}{\rho_1} + \frac{A_2}{\rho_2} \right), \quad R_{\text{eq}} = \frac{\rho_1 \rho_2 L}{\rho_2 A_1 + \rho_1 A_2}.$$

Step 3: Define the equivalent composite conductor Treat the parallel combination as one conductor of length L and total conducting cross-sectional area $A_1 + A_2$. By the definition of effective resistivity,

$$R_{\text{eq}} = \rho_{\text{eff}} \frac{L}{A_1 + A_2}.$$

Equate this resistance to the parallel result and cancel the common length. This identifies the material parameter of the composite, rather than just its resistance.

Step 4: Conclusion Resistance is a geometry-dependent quantity measured in Ω ; resistivity is a material property at fixed temperature measured in $\Omega \text{ m}$. For the parallel rods,

$$\rho_{\text{eff}} = \frac{\rho_1 \rho_2 (A_1 + A_2)}{\rho_2 A_1 + \rho_1 A_2}.$$

Quick Tip to Solve: First combine the resistances in parallel, then use the total area $A_1 + A_2$ when converting the result to effective resistivity.

29. Calculate the de-Broglie wavelength associated with the electron revolving in the first excited state of hydrogen atom. The ground state energy of the hydrogen atom is -13.6 eV . [3 marks]

Solution:

Concept:

- Hydrogen energy levels obey $E_n = E_1/n^2$, where n is the principal quantum number. The first excited state is $n = 2$.
- For an electron bound by the Coulomb force in the Bohr model, $U = -2K$, so the total energy is $E = K + U = -K$.
- The de-Broglie wavelength is $\lambda = h/p = h/\sqrt{2m_e K}$, where h is Planck's constant and m_e is the electron mass.

Step 1: Determine the energy in the first excited state The ground state corresponds to $n = 1$; therefore the next level, rather than the ground state, must be used. With the given ground-state energy,

$$E_2 = \frac{-13.6}{2^2} \text{ eV} = -3.4 \text{ eV}.$$

The negative sign signifies a bound electron. It is not the sign of its kinetic energy, which must be positive.

Step 2: Convert the kinetic energy to SI units Using $K = -E_2$, the electron's kinetic energy is 3.4 eV . Thus

$$K = 3.4(1.602 \times 10^{-19}) \text{ J} = 5.4468 \times 10^{-19} \text{ J}.$$

This is the energy to substitute into the nonrelativistic momentum relation $p^2 = 2m_e K$. Substituting the negative total energy directly in this relation would be incorrect.

Step 3: Evaluate the electron momentum and wavelength With $m_e = 9.11 \times 10^{-31} \text{ kg}$ and $h =$

$$6.626 \times 10^{-34} \text{ J s},$$

$$\begin{aligned} p &= \sqrt{2(9.11 \times 10^{-31})(5.4468 \times 10^{-19})} \\ &= 9.96 \times 10^{-25} \text{ kg m s}^{-1}, \\ \lambda &= \frac{6.626 \times 10^{-34}}{9.96 \times 10^{-25}} \text{ m}. \end{aligned}$$

This is the matter wavelength associated with the orbiting electron; no transition or emitted-photon energy is involved.

Step 4: Conclusion The de-Broglie wavelength of the electron in the first excited state is

$$\lambda \simeq 6.65 \times 10^{-10} \text{ m} = 0.665 \text{ nm}.$$

Quick Tip to Solve: For hydrogen, first convert the specified state to n , then use positive kinetic energy $K = 13.6/n^2 \text{ eV}$ in $h/\sqrt{2m_e K}$.

30. (a) Define the term decay constant of a radioactive substance.

(b) The half life of ${}_{92}^{238}\text{U}$ undergoing α decay is 4.5×10^9 years. Calculate the activity of 10 g sample of ${}_{92}^{238}\text{U}$.

[3 marks]

Solution:

Concept:

- Radioactive decay obeys $-dN/dt = \lambda N$, where N is the number of undecayed nuclei and λ is the decay constant.
- Half-life and decay constant are related by $\lambda = \ln 2/T_{1/2}$. Activity is $\mathcal{A} = \lambda N$, measured in becquerels, with $1 \text{ Bq} = 1 \text{ s}^{-1}$.
- A pure sample of mass m and molar mass M contains $N = (m/M)N_A$ nuclei, where N_A is Avogadro's constant.

Step 1: (a) Define the decay constant The decay constant is the probability per unit time that an individual radioactive nucleus decays. Equivalently, it is the fractional rate of decrease of the undecayed nuclei:

$$\lambda = -\frac{1}{N} \frac{dN}{dt}.$$

For a short interval dt , the fraction that decays is λdt . Its SI unit is s^{-1} , and its value is characteristic of the radioactive nuclide.

Step 2: (b) Calculate the decay constant in seconds Taking one year as $365 \times 24 \times 3600 \text{ s}$,

$$\begin{aligned} T_{1/2} &= 4.5 \times 10^9 (365 \times 24 \times 3600) \text{ s} \\ &= 1.41912 \times 10^{17} \text{ s}, \\ \lambda &= \frac{0.693}{1.41912 \times 10^{17}} = 4.883 \times 10^{-18} \text{ s}^{-1}. \end{aligned}$$

Converting the time first ensures that the resulting activity will be decays per second rather than decays per year.

Step 3: Count the uranium nuclei and apply the activity law For the given 10 g of uranium-238, use

$M = 238 \text{ g mol}^{-1}$ and $N_A = 6.022 \times 10^{23} \text{ mol}^{-1}$. The number of parent nuclei and their decay rate are

$$N = \frac{10}{238} (6.022 \times 10^{23}) = 2.530 \times 10^{22},$$

$$\mathcal{A} = (4.883 \times 10^{-18}) (2.530 \times 10^{22}) \text{ s}^{-1}.$$

Each uranium parent decay contributes one decay event to this activity. The emitted alpha particle's mass number does not multiply the number of decays.

Step 4: Conclusion The decay constant is the decay probability per nucleus per unit time. The activity of the given uranium-238 sample is

$$\mathcal{A} \simeq 1.24 \times 10^5 \text{ Bq},$$

or approximately $1.2 \times 10^5 \text{ Bq}$ to two significant figures.

Quick Tip to Solve: Use $\mathcal{A} = (0.693/T_{1/2})(m/M)N_A$, converting the half-life to seconds and keeping mass and molar mass in matching units.

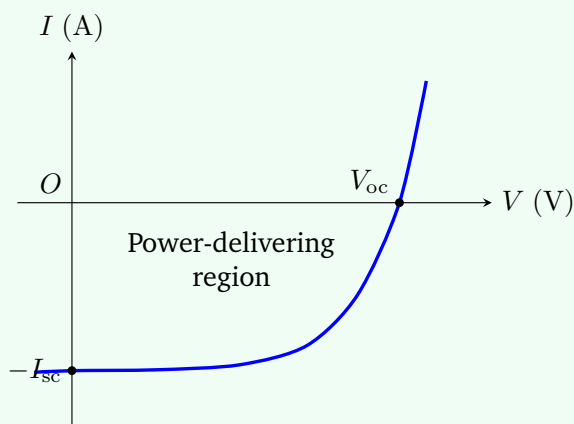
31. What is a solar cell ? Draw its V - I characteristics. Explain the three processes involved in its working. [3 marks]

Solution:

Concept:

- A solar cell is a large-area p - n junction device that converts light energy directly into electrical energy by the photovoltaic effect, without an external bias.
- Absorbed photons of energy $h\nu \geq E_g$, where E_g is the semiconductor band gap, generate electron-hole pairs. The junction's built-in electric field separates these carriers.

Step 1: Draw and interpret the illuminated characteristic Take voltage $V = V_p - V_n$ positive along the horizontal axis and current I positive in the forward-diode direction. Under illumination the useful power-generating part of the characteristic lies in the fourth quadrant: $V > 0$ and $I < 0$. At zero terminal voltage, the current is $-I_{sc}$, where I_{sc} is the magnitude of the short-circuit current. At zero terminal current, the voltage is the open-circuit voltage V_{oc} . A connected load operates between these intercepts; the negative current sign indicates that the cell delivers electrical power.



Illuminated solar-cell characteristic; I is positive in the forward-diode direction.

Step 2: Explain generation Light enters through the thin upper layer. Photons with sufficient energy

are absorbed in or near the depletion region and excite electrons into the conduction band, leaving holes behind. Thus incident radiation produces excess electron-hole pairs rather than merely heating the junction.

Step 3: Explain separation Photogenerated minority carriers outside the depletion layer diffuse towards the junction. Inside the depletion region, the built-in electric field points from the n -side to the p -side. It drives electrons towards the n -side and holes towards the p -side. This separation reduces recombination and builds a photovoltage, with the p -side positive relative to the n -side.

Step 4: Explain collection Metal contacts collect electrons at the n -side and holes at the p -side. When a load is connected, electrons flow through the external circuit from n to p ; conventional load current flows from p to n . Continued illumination supplies carriers, so electrical energy is delivered to the load.

Step 5: Conclusion A solar cell converts sunlight into electrical energy through **generation, separation and collection** of photocarriers. Its illuminated V - I curve has a power-delivering region in the fourth quadrant under the stated current convention.

Quick Tip to Solve: Label both V_{oc} and $-I_{sc}$; remember that electrons go to the n -side and holes to the p -side during separation.

OR

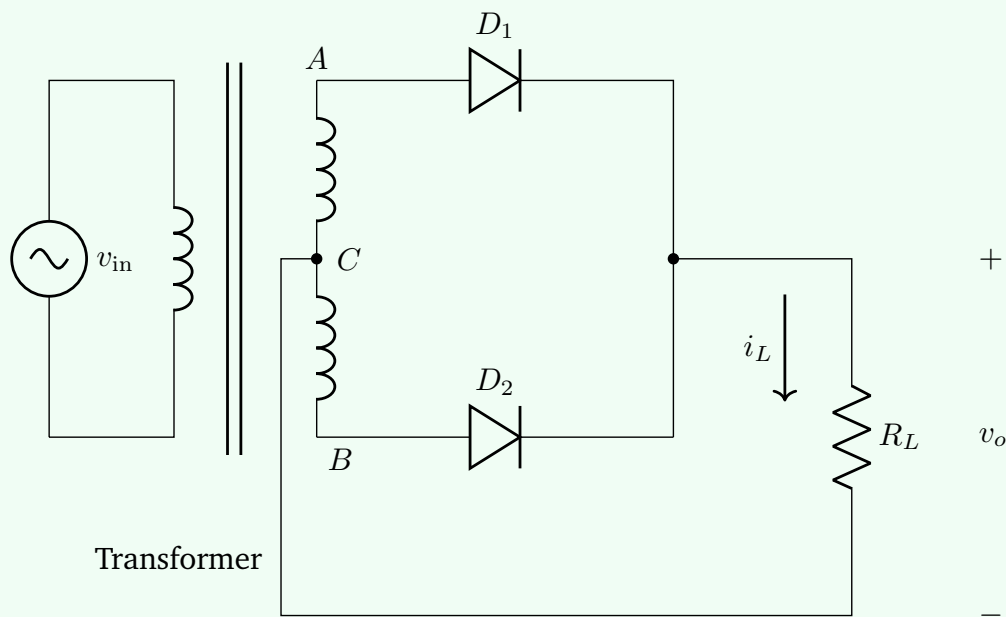
31. Draw the circuit diagram of a full wave rectifier. Explain its working showing its input and output waveforms. [3 marks]

Solution:

Concept:

- A diode conducts when forward biased and blocks when reverse biased. Rectification uses this one-way conduction to obtain a unidirectional current from an alternating input.
- A centre-tapped transformer supplies equal secondary voltages of opposite phase, measured from the centre tap C . Two diodes conduct alternately so that the current in the load resistance R_L always has the same direction.

Step 1: Connect the full-wave rectifier Connect secondary ends A and B to the anodes of D_1 and D_2 , respectively. Join their cathodes to the positive output terminal. Connect R_L between this junction and the centre tap C , the output return. The circuit below uses a centre-tapped secondary; the two equal half-windings supply $v_{AC} = V_m \sin \omega t$ and $v_{BC} = -V_m \sin \omega t$.



Centre-tapped full-wave rectifier; the joined cathodes provide positive output.

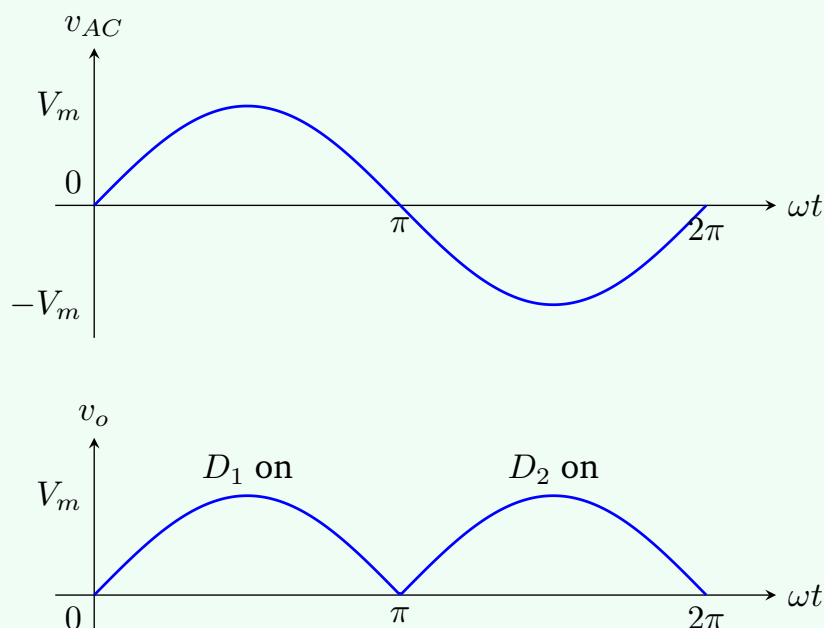
Step 2: Follow the positive half-cycle When A is positive relative to C , B is negative. Hence D_1 is forward biased and conducts, while D_2 is reverse biased and blocks. Conventional current follows $A \rightarrow D_1 \rightarrow R_L \rightarrow C$, returning through the upper half of the secondary. The upper end of the load is therefore positive.

Step 3: Follow the negative half-cycle In the next half-cycle, B becomes positive relative to C , while A is negative. Now D_2 conducts and D_1 blocks. Current follows $B \rightarrow D_2 \rightarrow R_L \rightarrow C$, returning through the lower secondary half. It passes down through R_L in the same direction as before.

Step 4: Compare the input and output waveforms Neglecting the diode's forward voltage drop, each negative input half-cycle is inverted at the load:

$$v_o = V_m |\sin \omega t|, \quad i_L = \frac{v_o}{R_L}.$$

Both half-cycles contribute positive output pulses. If the alternating input frequency is f , two pulses occur during every input period, giving output ripple frequency $2f$. The output is pulsating direct voltage; a smoothing filter would be needed for nearly constant DC.



Sinusoidal secondary input and full-wave output for ideal diodes.

Step 5: Conclusion The two diodes conduct on alternate half-cycles, producing load current in one fixed direction during both halves of the input cycle. Thus the circuit provides full-wave rectification with ripple frequency $2f$.

Quick Tip to Solve: Trace each conducting loop back to the centre tap: both paths must traverse R_L in the same direction, and only one diode conducts per half-cycle.

32. An optical instrument uses a lens of power 100 D for objective lens and 50 D for its eyepiece. When the tube length is kept at 25 cm, the final image is formed at infinity.

(a) Identify the optical instrument.

(b) Calculate the magnification produced by the instrument.

[3 marks]

Solution:

Concept:

- Lens power and focal length are related by $P = 1/f$, where P is in dioptres and f is in metres. A compound microscope uses a short-focal-length objective and an eyepiece of somewhat greater focal length.
- For normal adjustment, the final image is at infinity. The usual board-level magnifying-power formula is $M \simeq (L/f_o)(D/f_e)$, where L is the tube length, f_o, f_e are the objective and eyepiece focal lengths, and $D = 25$ cm is the least distance of distinct vision.

Step 1: (a) Identify the instrument from the focal lengths The objective and eyepiece have focal lengths

$$f_o = \frac{1}{100} \text{ m} = 1 \text{ cm}, \quad f_e = \frac{1}{50} \text{ m} = 2 \text{ cm}.$$

Both lenses are converging and have short focal lengths, with $f_o < f_e$. This combination is characteristic of a compound microscope: the objective produces an enlarged real intermediate image, which the eyepiece acts on as a magnifier. An astronomical telescope instead uses an objective of much longer

focal length than its eyepiece.

Step 2: (b) Use the magnifying power for a final image at infinity In normal adjustment, the intermediate image is at the first focal plane of the eyepiece. The eyepiece angular magnification is therefore D/f_e , without the additional unity used for a final image at the near point. In the standard long-tube approximation, the magnitude of the objective magnification is L/f_o . Substitution gives

$$|M| \simeq \frac{25}{1} \times \frac{25}{2} = 312.5.$$

All four lengths have been expressed in centimetres, so the magnifying power is dimensionless. The objective inverts the intermediate image, and the eyepiece preserves that orientation.

Step 3: Conclusion (a) The instrument is a **compound microscope**. (b) Its magnifying power in the standard tube-length approximation is 312.5. The final image is inverted relative to the object; with an orientation sign, $M \simeq -312.5$.

Quick Tip to Solve: Convert powers into focal lengths first; for a final image at infinity use D/f_e , not $1 + D/f_e$.

33. (a) Two point charges q_1 and q_2 are kept at a distance of r_{12} in air. Deduce the expression for the electrostatic potential energy of this system.

(b) If an external electric field ($\vec{E}$) is applied on the system, write the expression for the total energy of this system. [3 marks]

Solution:

Concept:

- Electrostatic potential energy is the work done by an external agent in assembling stationary charges quasistatically from a chosen reference configuration. For an isolated pair, take the interaction energy as zero at infinite separation.
- The potential at distance r from a point charge q in air is $V = kq/r$, where $k = 1/(4\pi\epsilon_0)$. Bringing another charge q' to that point requires work $q'V$.
- In a prescribed external electrostatic potential $V_{\text{ext}}(\mathbf{r})$, each charge contributes $q_i V_{\text{ext}}(\mathbf{r}_i)$, in addition to the mutual interaction energy. The external field satisfies $\mathbf{E} = -\nabla V_{\text{ext}}$.

Step 1: (a) Assemble the two-charge system First bring q_1 from infinity to its assigned position. Since no other charge is initially present, no work against another charge's field is required. Now hold q_1 fixed and bring q_2 from infinity to a point at distance r_{12} . The potential there due to q_1 is

$$V_1 = \frac{1}{4\pi\epsilon_0} \frac{q_1}{r_{12}}.$$

The work in this second stage, and therefore the interaction potential energy of the pair, is

$$U_{12} = q_2 V_1 = \frac{q_1 q_2}{4\pi\epsilon_0 r_{12}}.$$

The sign of $q_1 q_2$ is retained: like charges have positive interaction energy, whereas unlike charges have negative interaction energy relative to infinite separation.

Step 2: (b) Include the applied external field Let $V_{\text{ext}}(\mathbf{r}_1)$ and $V_{\text{ext}}(\mathbf{r}_2)$ be the external potentials at the two charge positions. Work must now also be done to place each charge in this prescribed field.

Thus the total potential energy associated with the two charges is

$$U = q_1 V_{\text{ext}}(\mathbf{r}_1) + q_2 V_{\text{ext}}(\mathbf{r}_2) + \frac{q_1 q_2}{4\pi\epsilon_0 r_{12}}.$$

The mutual term is counted once. These potentials exclude the potential of the two charges themselves. The expression does not include their divergent point-charge self-energies or the constant energy of the external field's sources. If the applied field is uniform, one may choose a finite reference origin with $V_{\text{ext}}(\mathbf{r}) = -\mathbf{E} \cdot \mathbf{r}$; the same formula then applies with that reference.

Step 3: Conclusion $U_{12} = q_1 q_2 / (4\pi\epsilon_0 r_{12})$. In an external electrostatic field, add the two external-potential contributions: $U = U_{12} + q_1 V_{\text{ext}}(\mathbf{r}_1) + q_2 V_{\text{ext}}(\mathbf{r}_2)$.

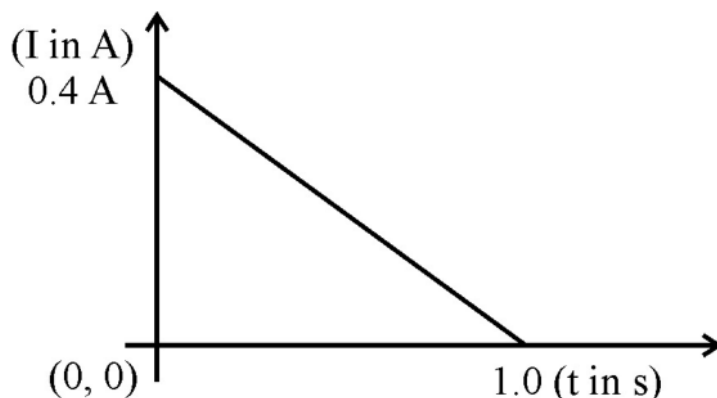
Quick Tip to Solve: Count each mutual charge pair once, then add qV_{ext} separately for every charge in the applied field.

34. When a conducting loop of resistance $10\ \Omega$ and area $10\ \text{cm}^2$ is removed from an external magnetic field acting normally, the variation of induced current in the loop with time is shown in the figure.

Find the

- total charge passed through the loop.
- change in magnetic flux through the loop.
- magnitude of the magnetic field applied.

[3 marks]



Solution:

Concept:

- Current is the rate of charge flow, so the charge that passes during an interval is $Q = \int I dt$, the area under its current-time graph.
- For a single conducting loop, Faraday's law gives $\mathcal{E} = -d\Phi_B/dt$. Combining its magnitude with Ohm's law gives $|\Delta\Phi_B| = RQ$ when the flux changes monotonically and the current does not reverse.
- The magnetic flux is $\Phi_B = BA \cos \theta$, where θ is the angle between the magnetic field and the area normal. Here the field is normal to the loop, so initially $|\Phi_B| = BA$.

Step 1: (i) Find the charge from the triangular graph The plotted current decreases linearly from 0.4 A

at $t = 0$ to zero at $t = 1.0$ s. Its graph encloses a triangle of height 0.4 A and base 1.0 s. Consequently,

$$Q = \frac{1}{2} \times 1.0 \times 0.4 = 0.20 \text{ C}.$$

Equivalently, the average current is $(0.4 + 0)/2 = 0.2$ A. The peak current must not be treated as constant over the full interval.

Step 2: (ii) Relate charge to the change in flux Integrating $RI = |d\Phi_B/dt|$ over the removal interval gives

$$|\Delta\Phi_B| = R \int I dt = RQ = 10 \times 0.20 = 2.0 \text{ Wb}.$$

The final flux is zero because the loop is completely removed from the external field. If the area normal is chosen along the initial field, the initial flux is positive and the signed change $\Phi_{B,f} - \Phi_{B,i}$ is negative.

Step 3: (iii) Calculate the initial field magnitude Convert the loop area carefully:

$$A = 10 \text{ cm}^2 = 10 \times 10^{-4} \text{ m}^2 = 1.0 \times 10^{-3} \text{ m}^2.$$

Since the field acts normally and the final flux is zero, $|\Delta\Phi_B| = BA$. Hence

$$B = \frac{|\Delta\Phi_B|}{A} = \frac{2.0}{1.0 \times 10^{-3}} = 2.0 \times 10^3 \text{ T}.$$

The large value follows directly from the printed current, time and small loop area; no change of the stated units is justified.

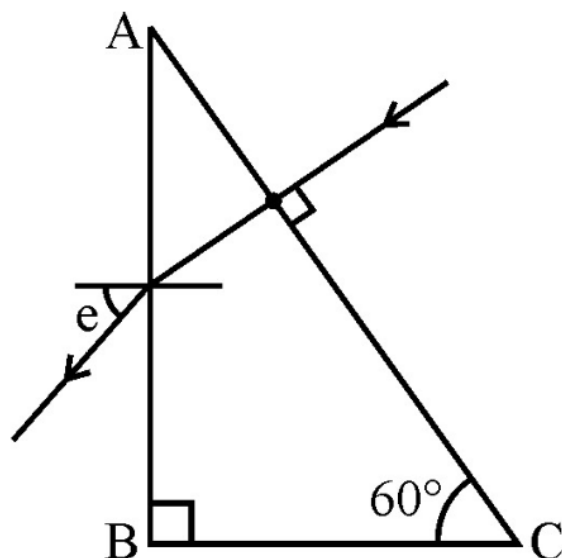
Step 4: Conclusion (i) $Q = 0.20 \text{ C}$. (ii) The flux decreases by 2.0 Wb , so $\Delta\Phi_B = -2.0 \text{ Wb}$ with the initial flux taken positive. (iii) $B = 2.0 \times 10^3 \text{ T}$.

Quick Tip to Solve: Use the triangular graph area first, then RQ for flux change; remember that $1 \text{ cm}^2 = 10^{-4} \text{ m}^2$.

Section D (15 marks)

35. (a) Define the term ‘focal length of a mirror’. With the help of a ray diagram, obtain the relation between its focal length and radius of curvature.

(b) Calculate the angle of emergence (e) of the ray of light incident normally on the face AC of a glass prism ABC of refractive index $\sqrt{3}$. How will the angle of emergence change qualitatively, if the ray of light emerges from the prism into a liquid of refractive index 1.3 instead of air? [5 marks]

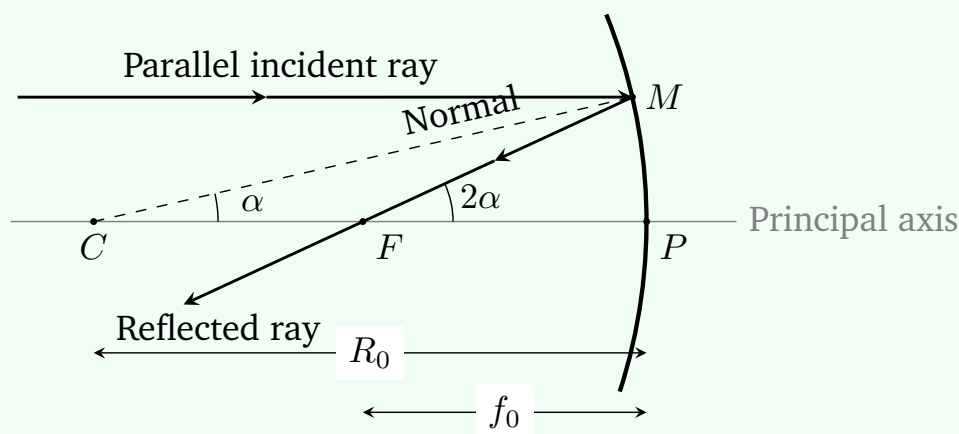


Solution:

Concept:

- The principal focus of a spherical mirror is the point on its principal axis where paraxial rays initially parallel to the axis converge, or appear to diverge from, after reflection.
- The normal at any point of a spherical mirror passes through its centre of curvature. Reflection obeys $i = r$. For paraxial rays, small angles satisfy $\tan \theta \simeq \theta$ in radians.
- Snell's law at a glass–external-medium boundary is $n_g \sin i = n_m \sin e$, where i and e are measured from the normal and n_g, n_m are refractive indices.

Step 1: (a) Define focal length and construct the rays The focal length of a mirror is the distance between its pole P and principal focus F , measured along the principal axis. Consider a concave spherical mirror with centre of curvature C . A paraxial ray parallel to the principal axis strikes the mirror at M , a small height h above the axis, and is reflected through F . Join CM , the normal to the mirror at M . In the schematic below the angles are enlarged for visibility.



Paraxial reflection at a concave spherical mirror; CM is the normal and $i = r = \alpha$.

Step 2: Relate the reflection angles to the mirror distances Let CM make angle α with the principal axis. The incident ray is parallel to this axis, so its angle of incidence is α . The reflected ray makes the same angle with CM , and hence makes angle 2α with the axis. For a narrow aperture, the axial projection of M is very close to P . Writing $R_0 = |CP|$ and $f_0 = |FP|$, elementary right-triangle geometry gives

$$\tan \alpha \simeq \frac{h}{R_0}, \quad \tan(2\alpha) \simeq \frac{h}{f_0}.$$

In the paraxial limit, $\tan \alpha \simeq \alpha$ and $\tan 2\alpha \simeq 2\alpha$. Dividing the second relation by the first,

$$2 \simeq \frac{R_0}{f_0}, \quad f_0 = \frac{R_0}{2}.$$

The same construction with backward extensions applies to a convex mirror. With the Cartesian sign convention, f and R are both negative for a concave mirror and both positive for a convex mirror, so the signed relation is also $f = R/2$. This relation assumes a spherical mirror of small aperture and paraxial rays.

Step 3: (b) Find the internal incidence angle at the exit face The ray is normal to AC , so its angle of incidence there is zero and it enters without deviation. The printed triangle has $\angle B = 90^\circ$ and $\angle C = 60^\circ$; hence

$$\angle A = 180^\circ - 90^\circ - 60^\circ = 30^\circ.$$

The angle between the normals to faces AC and AB equals the angle between those faces. Thus the ray strikes the exit face AB at $i = 30^\circ$ to its horizontal normal. The emergence angle must be measured from that normal, not from the vertical face.

Step 4: Apply Snell's law for emergence into air and liquid For air, take $n_m = 1$ and use $n_g = \sqrt{3}$:

$$\sin e = \sqrt{3} \sin 30^\circ = \frac{\sqrt{3}}{2}, \quad e = 60^\circ.$$

This sine is less than unity, so the ray emerges rather than undergoing total internal reflection. Replacing only the external air by the stated liquid does not change the internal incidence angle: normal entry through AC still causes no deviation. At the exit face,

$$1.3 \sin e_{\text{liquid}} = \sqrt{3} \sin 30^\circ, \quad \sin e_{\text{liquid}} = \frac{\sqrt{3}}{2.6} \simeq 0.6662.$$

Thus $e_{\text{liquid}} \simeq 41.8^\circ$. Qualitatively, a liquid with index closer to that of glass reduces bending away

from the normal; the emergence angle therefore decreases while remaining greater than the internal incidence angle 30° .

Step 5: Conclusion (a) The focal length is the pole-to-principal-focus distance, and $f = R/2$ for a spherical mirror in the paraxial approximation. (b) The emergence angle in air is 60° ; it decreases in the liquid of refractive index 1.3 (to approximately 41.8°).

Quick Tip to Solve: For normal entry into a prism, the internal ray follows the first-face normal. Use the angle between the two faces to find its incidence at the exit face, and measure emergence from the exit normal.

OR

35. (a) Define the term ‘resolving power of a telescope’. How will the resolving power be effected with the increase in

(i) Wavelength of light used.

(ii) Diameter of the objective lens.

Justify your answers.

(b) A screen is placed 80 cm from an object. The image of the object on the screen is formed by a convex lens placed between them at two different locations separated by a distance 20 cm. Determine the focal length of the lens.

[5 marks]

Solution:

Concept:

- The resolving power of a telescope is the reciprocal of the minimum angular separation of two distant point objects that can just be distinguished as separate.
- For a circular objective, Rayleigh’s criterion gives $\theta_{\min} = 1.22\lambda/D_o$, so the resolving power is $D_o/(1.22\lambda)$. Here D_o is the objective diameter and λ is the wavelength.
- For a convex lens producing a real image, $1/f = 1/u_o + 1/v_i$, where u_o and v_i denote positive object and image distances. The displacement method uses two conjugate lens positions.

Step 1: (a) Define and interpret resolving power Light from each distant point object forms a diffraction pattern through the finite circular aperture of the objective. Two objects are just resolved when the central maximum of one pattern falls on the first minimum of the other. If their minimum resolvable angular separation is $\theta_{\min}$, then

$$\text{Resolving power} = \frac{1}{\theta_{\min}} = \frac{D_o}{1.22\lambda}.$$

A smaller limiting angle therefore corresponds to a greater ability to distinguish nearby objects; resolving power is different from magnifying power.

Step 2: (a)(i) Increase the wavelength Keep the objective diameter fixed. Since $\theta_{\min} \propto \lambda$, increasing wavelength broadens the diffraction pattern and increases the limiting angular separation. Thus the resolving power decreases in inverse proportion to wavelength. For example, doubling λ halves the resolving power.

Step 3: (a)(ii) Increase the objective diameter Keep the wavelength fixed. Since $\theta_{\min} \propto 1/D_o$, a larger objective narrows the diffraction pattern and permits two objects with a smaller angular separation to be distinguished. Hence resolving power increases directly with the objective diameter.

Step 4: (b) Relate the two lens positions Let the fixed object-screen separation be $s = 80$ cm and the separation between the two lens positions be $d = 20$ cm. At one position the distances are u_o and v_i , whereas at the other they are interchanged. Therefore

$$u_o + v_i = s, \quad |v_i - u_o| = d.$$

Taking $v_i > u_o$ at the first position gives

$$u_o = \frac{s-d}{2} = 30 \text{ cm}, \quad v_i = \frac{s+d}{2} = 50 \text{ cm}.$$

Step 5: (b) Apply the lens formula In the Cartesian convention, the same position has $u = -30$ cm and $v = +50$ cm. Hence

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u} = \frac{1}{50} + \frac{1}{30},$$

$$f = \frac{30 \times 50}{30 + 50} = \frac{s^2 - d^2}{4s} = 18.75 \text{ cm}.$$

The positive focal length is consistent with a converging lens forming a real image on the screen.

Step 6: Conclusion (a) Resolving power is $1/\theta_{\min} = D_o/(1.22\lambda)$: it decreases when wavelength increases and increases when the objective diameter increases. (b) The focal length of the convex lens is 18.75 cm.

Quick Tip to Solve: In the displacement method, use the fixed object-screen distance as s and the lens-position separation as d ; then $f = (s^2 - d^2)/(4s)$, not $s/4$ unless the positions coincide.

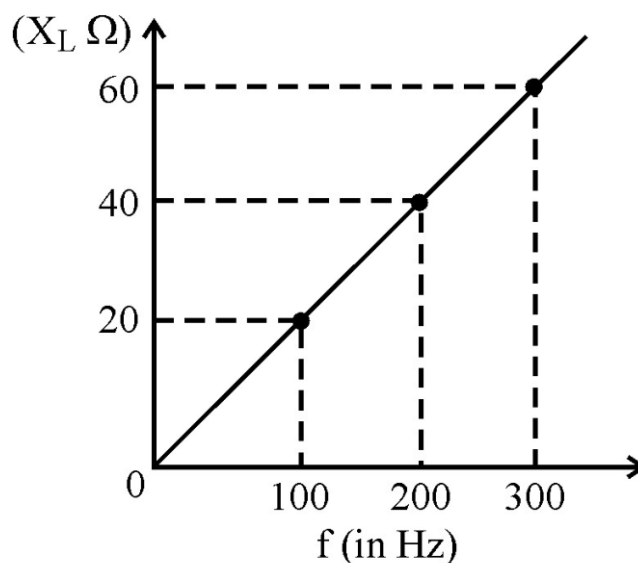
36. (a) Show that an ideal inductor does not dissipate power in an ac circuit.

(b) The variation of inductive reactance (X_L) of an inductor with the frequency (f) of the ac source of 100 V and variable frequency is shown in the fig.

(i) Calculate the self-inductance of the inductor.

(ii) When this inductor is used in series with a capacitor of unknown value and a resistor of 10Ω at 300 s^{-1} , maximum power dissipation occurs in the circuit. Calculate the capacitance of the capacitor.

[5 marks]



Solution:

Concept:

- An ideal inductor has zero resistance and obeys $v = L di/dt$; its current lags the applied sinusoidal voltage by $\pi/2$.
- The average ac power is $P_{av} = V_{rms} I_{rms} \cos \phi$, where ϕ is the voltage-current phase difference.
- Inductive and capacitive reactances are $X_L = 2\pi fL$ and $X_C = 1/(2\pi fC)$. A series LCR circuit absorbs maximum average power at resonance, when $X_L = X_C$.

Step 1: (a) Establish the phase difference Apply $v = V_m \sin \omega t$ to an ideal inductor. Using the sinusoidal steady-state solution of $L di/dt = v$, we obtain

$$i = -\frac{V_m}{\omega L} \cos \omega t = I_m \sin \left(\omega t - \frac{\pi}{2} \right), \quad I_m = \frac{V_m}{\omega L}.$$

Thus the current lags the voltage by a quarter cycle, with $\phi = \pi/2$.

Step 2: (a) Average the instantaneous power The instantaneous input power is

$$p(t) = vi = -V_m I_m \sin \omega t \cos \omega t = -\frac{V_m I_m}{2} \sin 2\omega t.$$

Over a full period T , the positive and negative contributions cancel:

$$P_{av} = \frac{1}{T} \int_0^T p(t) dt = 0.$$

Equivalently, $P_{av} = V_{rms} I_{rms} \cos(\pi/2) = 0$. Energy is stored in the magnetic field as $U = Li^2/2$ while the current magnitude increases and is returned to the source while it decreases. With no resistance, none is dissipated as heat.

Step 3: (b)(i) Obtain inductance from the graph slope The straight-line graph passes through the origin and the point $(f, X_L) = (300 \text{ Hz}, 60 \Omega)$. Its slope is $2\pi L$, so

$$2\pi L = \frac{60}{300} = 0.20 \Omega \text{ Hz}^{-1},$$
$$L = \frac{0.20}{2\pi} = \frac{1}{10\pi} \text{ H} \simeq 0.0318 \text{ H}.$$

The other marked points, $(100, 20)$ and $(200, 40)$, give the same slope. The source voltage does not enter this determination of L .

Step 4: (b)(ii) Identify the condition for maximum power For fixed rms source voltage and series resistance,

$$P = I_{rms}^2 R = \frac{V_{rms}^2 R}{R^2 + (X_L - X_C)^2}.$$

This is greatest when the net reactance vanishes, giving $X_L = X_C$. The stated 300 s^{-1} is the frequency $f = 300 \text{ Hz}$, consistent with the graph's frequency axis; hence $\omega = 2\pi f$, and the graph gives $X_L = 60 \Omega$ there. The resistance 10Ω sets the maximum current and power, but does not change this resonance condition.

Step 5: (b)(ii) Calculate the capacitance Set the capacitive reactance equal to the graph's inductive

reactance at resonance:

$$\frac{1}{2\pi fC} = 60 \Omega,$$
$$C = \frac{1}{2\pi \times 300 \times 60} \text{ F}$$
$$= 8.84 \times 10^{-6} \text{ F} = 8.84 \mu\text{F}.$$

This also satisfies $\omega^2 LC = 1$, as required for series resonance.

Step 6: Conclusion (a) An ideal inductor absorbs zero average power and dissipates no energy. (b)(i) $L = 0.0318 \text{ H} = 31.8 \text{ mH}$. (ii) Maximum power occurs at resonance, requiring $C = 8.84 \mu\text{F}$.

Quick Tip to Solve: Read $X_L = 60 \Omega$ directly at 300 Hz, then use $C = 1/(2\pi f X_L)$ at resonance; remember to convert ordinary frequency to angular frequency with 2π .

OR

36. (a) A conductor of length l is rotated about one of its ends at a constant angular speed ω in a plane perpendicular to a uniform magnetic field B . Plot graphs to show variations of the emf induced across the ends of the conductor with (i) angular speed ω and (ii) length of the conductor l .

(b) Two concentric circular loops of radius 1 cm and 20 cm are placed coaxially.

(i) Find mutual inductance of the arrangement.

(ii) If the current passed through the outer loop is changed at a rate of 5 A/ms, find the emf induced in the inner loop. Assume the magnetic field on the inner loop to be uniform. [5 marks]

Solution:

Concept:

- Motional emf in a short radial element of a rotating rod is $d\varepsilon = Bv dr$, where B is the perpendicular magnetic field and $v = \omega r$ is its speed at distance r from the pivot.
- A single circular loop of radius R , carrying current I , produces central field $B = \mu_0 I / (2R)$. The flux through the small coaxial loop of radius a is approximately $\Phi = B\pi a^2$.
- Mutual inductance is $M = \Phi / I$ for single-turn loops. The emf induced in the inner loop is $\varepsilon_{\text{in}} = -M dI/dt$; its direction opposes the change of linked flux.

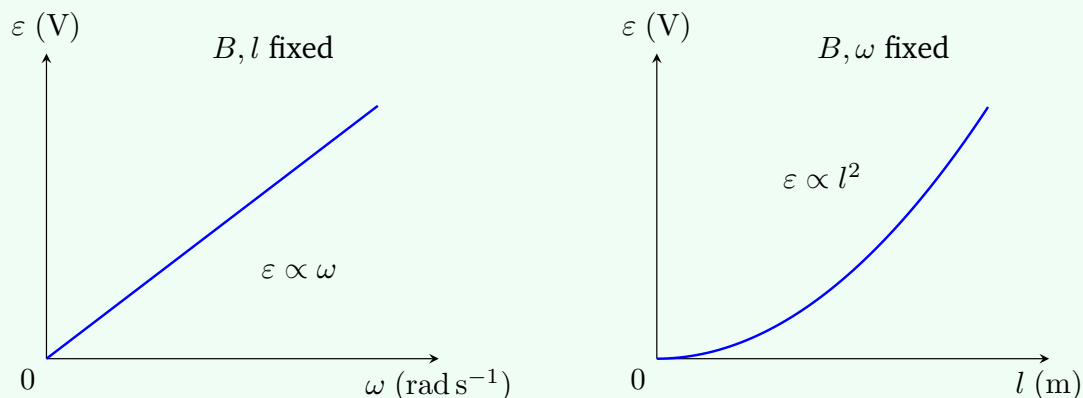
Step 1: (a) Derive the emf of the rotating conductor The rod rotates in a plane perpendicular to the field. At distance r from its fixed end, its velocity is tangential, so $\mathbf{v} \times \mathbf{B}$ lies along the rod. Every radial element therefore contributes to the emf between the same two ends. Using the emf magnitude,

$$d\varepsilon = B\omega r dr,$$
$$\varepsilon = \int_0^l B\omega r dr = \frac{1}{2} B\omega l^2.$$

The end at the pivot has zero speed and the free end has speed ωl . Thus using the free-end speed for the entire conductor would overestimate the emf by a factor of two.

Step 2: (a)(i), (ii) Plot the two dependences For fixed B and l , $\varepsilon = (Bl^2/2)\omega$. Hence the emf versus angular speed graph is a straight line through the origin, of slope $Bl^2/2$. For fixed B and ω , $\varepsilon = (B\omega/2)l^2$. Hence the emf versus length graph is an upward-opening parabola in the physical region $l \geq 0$, starting with zero slope at the origin. Doubling the angular speed doubles the emf,

whereas doubling the rod length makes the emf four times as large. Both plots show emf magnitudes.



Induced emf magnitude versus angular speed and rod length, with the other quantities fixed.

Step 3: (b)(i) Calculate the mutual inductance Let the outer-loop radius be $R = 20 \text{ cm} = 0.20 \text{ m}$ and the inner-loop radius be $a = 1 \text{ cm} = 0.010 \text{ m}$. The loops are concentric and coaxial, so the central field of the outer loop is normal to the inner loop. Using the stipulated uniform-field approximation over the inner loop,

$$\begin{aligned}\Phi &= \frac{\mu_0 I}{2R} \pi a^2, \\ M &= \frac{\Phi}{I} = \frac{\mu_0 \pi a^2}{2R} \\ &= \frac{(4\pi \times 10^{-7})\pi(0.010)^2}{2(0.20)} \text{ H} \\ &= \pi^2 \times 10^{-10} \text{ H} \simeq 9.87 \times 10^{-10} \text{ H}.\end{aligned}$$

Only the area of the inner loop enters the linked flux; the outer radius determines the magnetic field strength.

Step 4: (b)(ii) Find the emf in the inner loop First convert milliseconds to seconds:

$$\left| \frac{dI}{dt} \right| = 5 \text{ A/ms} = 5 \times 10^3 \text{ A s}^{-1}.$$

Applying the law of mutual induction gives

$$|\varepsilon_{\text{in}}| = M \left| \frac{dI}{dt} \right| = (9.87 \times 10^{-10})(5 \times 10^3) \text{ V} = 4.94 \times 10^{-6} \text{ V}.$$

A unique clockwise or anticlockwise direction cannot be named without the direction and sense of change of the outer current. Lenz's law fixes the induced polarity so as to oppose that flux change.

Step 5: Conclusion (a) $\varepsilon = \frac{1}{2} B \omega l^2$: the ε -versus- ω plot is linear through the origin and the ε -versus- l plot is parabolic. (b) $M = 9.87 \times 10^{-10} \text{ H}$, and the induced emf magnitude is $4.94 \times 10^{-6} \text{ V} = 4.94 \mu\text{V}$.

Quick Tip to Solve: Integrate the rotating rod's local speed ωr . For nested loops use the outer radius in B , the inner area in Φ , and multiply A/ms by 10^3 before finding the emf.

37. (a) Write two important characteristics of equipotential surfaces.

(b) A thin circular ring of radius r is charged uniformly so that its linear charge density becomes λ . Derive an expression for the electric field at a point P at a distance x from it along the axis of the ring. Hence, prove that at large distances ($x \gg r$), the ring behaves as a point charge. [5 marks]

Solution:

Concept:

- An equipotential surface has the same electric potential at every point. Hence the work done in moving a test charge along it is $W = q\Delta V = 0$.
- For an element dq , Coulomb's law gives $dE = k dq/R^2$, where $k = 1/(4\pi\epsilon_0)$, R is the distance from the element and ϵ_0 is the permittivity of vacuum.
- Superposition adds electric fields as vectors. For a uniformly charged ring, components perpendicular to the axis cancel in diametrically opposite pairs.

Step 1: (a) State two characteristics of equipotential surfaces First, moving a charge between any two points on the same equipotential surface requires no work against the electrostatic force, since the potential difference is zero. Second, the electric field is perpendicular to an equipotential surface wherever the field is nonzero. If the field had a tangential component, displacement along the surface would involve nonzero work and a change of potential, contradicting the definition of an equipotential surface.

Step 2: (b) Set up the geometry of the charged ring Let O be the centre of the ring and let P lie on its axis with $OP = x$. Take the positive axial direction from O towards P . The ring has radius r , circumference $2\pi r$, uniform linear charge density λ , and total charge

$$Q = 2\pi r\lambda.$$

Every element $dq = \lambda dl$ is at the same distance $R = \sqrt{x^2 + r^2}$ from P . If the line joining an element to P makes angle θ with the axis, then $\cos \theta = x/\sqrt{x^2 + r^2}$.

Step 3: Add the axial components of the elemental fields For positive charge, the field due to an element points from the element towards P . A diametrically opposite element supplies an equal and opposite transverse component but an equal axial component. Thus only the axial components survive the summation:

$$dE_x = \frac{k dq}{x^2 + r^2} \cos \theta = \frac{kx dq}{(x^2 + r^2)^{3/2}},$$
$$E_x = \frac{kx}{(x^2 + r^2)^{3/2}} \oint dq = \frac{kQx}{(x^2 + r^2)^{3/2}}.$$

Substituting the total charge expresses the result in terms of the given linear charge density. For negative λ , the signed expression reverses the field direction. At the centre, $x = 0$, symmetry correctly gives zero field.

Step 4: Obtain the field far from the ring For $x \gg r$, factor x^3 out of the denominator:

$$E_x = \frac{kQ}{x^2} \left(1 + \frac{r^2}{x^2}\right)^{-3/2}.$$

Since $r^2/x^2 \ll 1$, the bracket tends to unity. This limiting expression is Coulomb's field for a point charge of value Q placed at the centre of the ring; the spatial spread of charge becomes negligible compared with the observation distance.

Step 5: Conclusion Equipotential surfaces require zero work for motion along them and are normal

to the electric field. The axial field of the ring is

$$\vec{E}(P) = \frac{1}{4\pi\epsilon_0} \frac{2\pi r \lambda x}{(x^2 + r^2)^{3/2}} \hat{x}.$$

For $x \gg r$, $\vec{E} \simeq Q\hat{x}/(4\pi\epsilon_0 x^2)$, where $Q = 2\pi r \lambda$. Thus the ring behaves as a point charge at its centre.

Quick Tip to Solve: For a ring, first cancel the transverse components and then integrate the axial component; forgetting the projection factor loses the numerator x .

OR

37. (a) State Gauss's law on electrostatics and derive an expression for the electric field due to a long straight thin uniformly charged wire (linear charge density λ) at a point lying at a distance r from the wire.

(b) The magnitude of electric field (in NC^{-1}) in a region varies with the distance r (in m) as

$$E = 10r + 5$$

By how much does the electric potential increase in moving from point at $r = 1$ m to a point at $r = 10$ m. [5 marks]

Solution:

Concept:

- Gauss's law states that the net outward electric flux through a closed surface equals the enclosed charge divided by the permittivity of vacuum: $\oint_S \vec{E} \cdot d\vec{A} = Q_{\text{enc}}/\epsilon_0$.
- A sufficiently long uniformly charged wire has cylindrical symmetry. Its field is radial and depends only on the perpendicular distance r from the wire, when end effects can be neglected.
- Electric potential difference is $V_B - V_A = -\int_A^B \vec{E} \cdot d\vec{l}$. The minus sign expresses that potential decreases in the direction of the electric field.

Step 1: (a) State Gauss's law and choose a Gaussian surface The total electric flux through any closed surface is $1/\epsilon_0$ times the net charge enclosed by it:

$$\oint_S \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enc}}}{\epsilon_0}.$$

Take a cylindrical Gaussian surface of radius r and length L , coaxial with the uniformly charged wire. The length L is arbitrary and the enclosed charge is $Q_{\text{enc}} = \lambda L$. For the long-wire approximation, the observation region is far from either end.

Step 2: Evaluate the flux through the cylinder Cylindrical symmetry makes the field radial and equal in magnitude at every point on the curved surface. There it is parallel to the outward area normal. On both plane end caps, the area normals are along the wire and perpendicular to the field, so the cap fluxes vanish. For positive λ ,

$$\begin{aligned}\Phi_E &= E(2\pi r L) + 0 + 0, \\ E(2\pi r L) &= \frac{\lambda L}{\epsilon_0}, \\ E &= \frac{\lambda}{2\pi\epsilon_0 r}.\end{aligned}$$

The arbitrary length cancels, as expected. In signed vector form the result is $\vec{E} = \lambda \hat{r} / (2\pi\epsilon_0 r)$: outward for positive charge and inward for negative charge. Its magnitude is $|\lambda| / (2\pi\epsilon_0 r)$.

Step 3: (b) Integrate the field over the stated interval Part (b) gives a separate field law, $E = 10r + 5$, rather than the inverse-distance field of part (a). Under the usual interpretation that the field points along increasing r , its radial component is $E_r = 10r + 5$. Therefore

$$\begin{aligned} V(10) - V(1) &= - \int_1^{10} (10r + 5) dr \\ &= - [5r^2 + 5r]_1^{10} \\ &= - [(500 + 50) - (5 + 5)] \\ &= -540 \text{ V.} \end{aligned}$$

The unit is volt because $(\text{N C}^{-1})\text{m} = \text{J C}^{-1}$. Thus motion in the field direction produces a decrease, despite the printed word “increase.”

Step 4: Explain the direction needed for an increase Strictly, the printed question specifies the magnitude of the field without its direction. If the field instead points towards decreasing r , then $E_r = -(10r + 5)$ and the same outward displacement gives $V(10) - V(1) = +540 \text{ V}$. Hence an increase of 540 V is consistent with an inward field; an outward field gives a decrease of the same amount. The magnitude alone does not fix the sign of the potential change.

Step 5: Conclusion For the long uniformly charged wire,

$$\vec{E} = \frac{\lambda}{2\pi\epsilon_0 r} \hat{r}.$$

For part (b), the potential change has magnitude 540 V. It is -540 V for a field along increasing r , or an increase of $+540 \text{ V}$ for an inward field. A positive increase requires the latter direction, which the printed magnitude alone does not specify.

Quick Tip to Solve: Keep the minus sign in $\Delta V = - \int E_r dr$; distinguish the positive field magnitude from its signed radial component before assigning a potential rise or fall.