

CBSE Class 12 Physics Question Paper 2020 with Solution PDF-Set 55/5/2

Question Paper with Solutions

Conducted by the Central Board of Secondary Education (CBSE)



Time Allowed: 3 hours

Maximum Marks: 70

General Instructions

- (i) This question paper comprises **four** sections – A, B, C and D.
- (ii) There are **37** questions in the question paper. All questions are compulsory.
- (iii) Section A : Q. no. **1** to **20** are very short-answer type questions carrying **1** mark each.
- (iv) Section B : Q. no. **21** to **27** are short-answer type questions carrying **2** marks each.
- (v) Section C : Q. no. **28** to **34** are long-answer type questions carrying **3** marks each.
- (vi) Section D : Q. no. **35** to **37** are also long answer type questions carrying **5** marks each.
- (vii) There is no overall choice in the question paper. However, an internal choice has been provided in **two** questions of **one** mark, **two** questions of **two** marks, **one** question of **three** marks and all the **three** questions of **five** marks. You have to attempt **only one** of the choices in such questions.
- (viii) However, separate instructions are given with each section and question, wherever necessary.
- (ix) Use of calculators and log tables is not permitted.

You may use the following values of physical constants wherever necessary.

$$c = 3 \times 10^8 \text{ m/s}$$

$$h = 6.63 \times 10^{-34} \text{ Js}$$

$$e = 1.6 \times 10^{-19} \text{ C}$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ T m A}^{-1}$$

$$\epsilon_0 = 8.854 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$$

$$\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$$

$$\text{Mass of electron } (m_e) = 9.1 \times 10^{-31} \text{ kg}$$

$$\text{Mass of neutron} = 1.675 \times 10^{-27} \text{ kg}$$

$$\text{Mass of proton} = 1.673 \times 10^{-27} \text{ kg}$$

$$\text{Avogadro's number} = 6.023 \times 10^{23} \text{ per gram mole}$$

$$\text{Boltzmann constant} = 1.38 \times 10^{-23} \text{ JK}^{-1}$$

Section A (20 marks)

Note : Select the most appropriate option from those given below each question.

1. The wavelength and intensity of light emitted by a LED depend upon

[1 mark]

- (A) forward bias and energy gap of the semiconductor
- (B) energy gap of the semiconductor and reverse bias
- (C) energy gap only
- (D) forward bias only

Correct Answer: (A) forward bias and energy gap of the semiconductor

Solution:

Concept:

- An LED emits light when electron-hole recombination occurs across the energy gap E_g of a forward-biased semiconductor junction.
- The photon energy is approximately $hc/\lambda = E_g$, where h is Planck's constant and c is the speed of light; the emission intensity depends on the forward current.

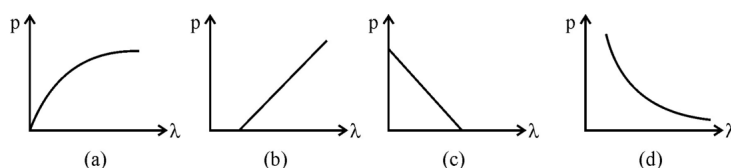
Step 1: Identify the wavelength dependence The wavelength is selected mainly by the semiconductor energy gap: $\lambda \simeq hc/E_g$. A larger energy gap produces a shorter wavelength. Thus an answer about wavelength must include the energy gap.

Step 2: Identify the intensity dependence and check the options Increasing forward bias increases the junction current and the rate of radiative recombination, increasing light intensity within the operating range. Reverse bias does not describe normal LED emission, so (B) is unsuitable. Energy gap alone in (C) omits intensity control; forward bias alone in (D) omits wavelength selection. Option (A) lists the two relevant factors, although its order is opposite to their respective effects.

Step 3: Conclusion Wavelength depends on the semiconductor energy gap, while intensity depends on forward bias (through forward current). Hence the correct option is (A).

Quick Tip to Solve: For an LED, pair colour with energy gap and brightness with forward current.

2. The graph showing the correct variation of linear momentum (p) of a charge particle with its de-Broglie wavelength (λ) is — [1 mark]



- (A) (a)
(B) (b)
(C) (c)
(D) (d)

Correct Answer: (D) (d)

Solution:

Concept:

- The de Broglie relation is $\lambda = h/p$, where p is momentum magnitude and h is Planck's constant.
- Consequently $p\lambda = h$ is constant: momentum and wavelength vary inversely.

Step 1: Establish the graph shape Rearranging gives $p = h/\lambda$. For positive wavelength, momentum is positive and decreases as wavelength increases. The slope is $dp/d\lambda = -h/\lambda^2$, so the curve falls continuously.

Step 2: Compare all four printed graphs The curve approaches each positive coordinate axis without meeting it: as $\lambda \rightarrow 0^+$, $p \rightarrow \infty$, and as $\lambda \rightarrow \infty$, $p \rightarrow 0^+$. Graphs (a) and (b) increase and contradict inverse proportionality. Graph (c) decreases linearly and reaches zero at a finite wavelength, so it also fails. Graph (d) is the required decreasing rectangular hyperbola.

Step 3: Conclusion The correct graph is (d), so the correct option is (D).

Quick Tip to Solve: Test the product: a correct momentum-wavelength graph must keep p times lambda constant.

3. The selectivity of a series LCR a.c. circuit is large, when [1 mark]

- (A) L is large and R is large
(B) L is small and R is small
(C) L is large and R is small
(D) $L = R$

Correct Answer: (C) L is large and R is small

Solution:

Concept:

- Selectivity is the ability of a resonant series LCR circuit to distinguish its resonant frequency from nearby frequencies.
- The quality factor is $Q = \omega_0 L/R$, with $\omega_0 = 1/\sqrt{LC}$; the bandwidth in angular frequency is $\Delta\omega = R/L$.

Step 1: Relate selectivity to circuit parameters A more selective circuit has a narrower resonance peak.

A small value of R/L reduces the bandwidth $\Delta\omega$, so increasing inductance and reducing resistance favour large selectivity. With capacitance fixed, $Q = \sqrt{L/C}/R$ gives the same conclusion.

Step 2: Compare the options Option (A) increases resistance as well as inductance, so both changes do not favour high selectivity. Option (B) decreases inductance as well as resistance, again giving competing changes. Option (C) increases L and decreases R , both narrowing the resonance peak. Option (D), $L = R$, is not a meaningful equality of physical quantities because inductance and resistance have different units.

Step 3: Conclusion Large inductance and small resistance give large selectivity. Hence the correct option is (C).

Quick Tip to Solve: Remember the bandwidth R/L : make this ratio small for a sharp resonance peak.

4. Photo diodes are used to detect

[1 mark]

- (A) radio waves
- (B) gamma rays
- (C) IR rays
- (D) optical signals

Correct Answer: (D) optical signals

Solution:

Concept:

- A photodiode converts incident light into an electrical signal by generating electron-hole pairs in a semiconductor junction.
- It is normally reverse biased; its photocurrent responds to the intensity of incident optical radiation.

Step 1: Identify the detected quantity When a suitable photon is absorbed near the depletion region, it creates an electron-hole pair. The junction electric field separates these carriers, producing a photocurrent. Variations in incident light intensity therefore become variations in electrical current, which allow optical signals to be detected.

Step 2: Distinguish the printed alternatives Radio waves in (A) are not detected by the ordinary photodiode mechanism used in this syllabus, and (B) is not the standard application of an optical photodiode. Some photodiodes do respond to infrared radiation, so (C) names a possible spectral region for suitable devices. However, the general function asked here is detection of optical signals, which includes light carrying information and is stated by (D).

Step 3: Conclusion Photodiodes are used to detect optical signals. Hence the intended correct option is (D).

Quick Tip to Solve: Identify the device by its output: incident light produces photocurrent in a reverse-biased photodiode.

5. The relationship between Brewster angle θ and the speed of light v in the denser medium is

[1 mark]

(A) $c \cos \theta = v$

(B) $v \tan \theta = c$

(C) $c = v \tan \theta$

(D) $v \cos \theta = c$

Correct Answer: (B) $v \tan \theta = c$

Solution:

Concept:

- Brewster's law for light incident from air is $\tan \theta = \mu$, where θ is the polarising angle and μ is the refractive index of the denser medium relative to air.
- The refractive index is $\mu = c/v$, where c is the speed of light in vacuum and v is its speed in the denser medium.

Step 1: Combine the two relations Substituting the speed relation in Brewster's law gives

$$\tan \theta = \frac{c}{v} \implies v \tan \theta = c.$$

This is the required relation; the tangent, rather than the cosine, is the defining ratio in Brewster's law.

Step 2: Check the printed options carefully Option (B) reads $v \tan \theta = c$ and is correct. In the official paper, option (C) reads $c = v \tan \theta$; this is the very same equation with its sides interchanged, so it is also mathematically correct. Options (A) and (D) use cosine and do not give Brewster's law in general. Thus the printed paper contains duplicate correct alternatives; this is not a difference in physics.

Step 3: Conclusion The relation is $v \tan \theta = c$. Option (B) is correct, and printed option (C) is equivalent and also correct.

Quick Tip to Solve: Use $\tan(\theta) = c/v$ directly, and check whether another option merely swaps the two sides.

6. A biconvex lens of focal length f is cut into two identical plano-convex lenses. The focal length of each part will be:

[1 mark]

(A) f

(B) $f/2$

(C) $2f$

(D) $4f$

Correct Answer: (C) $2f$

Solution:

Concept:

- The thin-lens maker formula in air is $1/f = (\mu - 1)(1/R_1 - 1/R_2)$, where μ is the lens refractive index.
- An identical pair of plano-convex halves is obtained by cutting the symmetric biconvex lens through its middle perpendicular to the principal axis.

Step 1: Write the power of the original lens Let the magnitude of each curved surface radius be R .

For light travelling left to right, the symmetric biconvex lens has $R_1 = R$ and $R_2 = -R$. Therefore

$$\frac{1}{f} = (\mu - 1) \left(\frac{1}{R} + \frac{1}{R} \right) = \frac{2(\mu - 1)}{R}.$$

Step 2: Calculate the power of either half Each plano-convex half retains one curved surface and has one plane surface, whose radius is infinite. If its focal length is f_h , then

$$\frac{1}{f_h} = \frac{\mu - 1}{R} = \frac{1}{2f}, \quad f_h = 2f.$$

Cutting in this way halves the refracting power and doubles the focal length. Options (A), (B), and (D) would respectively leave the power unchanged, double it, or reduce it to one quarter; none matches the lens-maker result.

Step 3: Conclusion The focal length of each plano-convex part is $2f$. Hence the correct option is (C).

Quick Tip to Solve: When one curved surface becomes plane, halve the lens power and double its focal length.

7. The phase difference between the current and the voltage in series LCR circuit at resonance is: [1 mark]

- (A) π
- (B) $\pi/2$
- (C) $\pi/3$
- (D) zero

Correct Answer: (D) zero

Solution:

Concept:

- For a series LCR circuit, $X_L = \omega L$ and $X_C = 1/(\omega C)$ are the inductive and capacitive reactances.
- The phase angle satisfies $\tan \phi = (X_L - X_C)/R$; resonance occurs when $X_L = X_C$.

Step 1: Apply the resonance condition At resonance the inductive and capacitive voltage contributions cancel because their magnitudes are equal and their phases are opposite. Thus the net reactance vanishes and the impedance becomes purely resistive: $Z = R$.

Step 2: Determine and check the phase angle Substituting $X_L - X_C = 0$ gives $\tan \phi = 0$. For the physical phase angle of a passive series circuit with positive resistance, $\phi = 0$: the supply voltage and current are in phase. A phase difference of π would make them oppositely directed, while $\pi/2$ or $\pi/3$ would imply a reactive phase shift. Hence the nonzero angles in (A), (B), and (C) do not describe resonance.

Step 3: Conclusion The phase difference at resonance is zero. Hence the correct option is (D).

Quick Tip to Solve: At series resonance, cancel X_L against X_C before deciding the phase: only resistance remains.

8. Photons of frequency ν are incident on the surfaces of two metals A and B of threshold frequencies $3\nu/4$ and $2\nu/3$, respectively. The ratio of maximum kinetic energy of electrons emitted from A to that from B is [1 mark]

- (A) 2 : 3
(B) 4 : 3
(C) 3 : 4
(D) 3 : 2

Correct Answer: (C) 3 : 4

Solution:

Concept:

- Einstein's photoelectric equation is $K_{\max} = h\nu - h\nu_0$, where h is Planck's constant, ν is incident frequency and ν_0 is the metal's threshold frequency.
- The work function is $\phi = h\nu_0$; the photon energy remaining after overcoming it becomes the electron's maximum kinetic energy.

Step 1: Calculate the energies for the printed thresholds The official paper gives $\nu_{0A} = 3\nu/4$ and $\nu_{0B} = 2\nu/3$. Both are below the incident frequency, so both metals emit photoelectrons. Thus

$$K_A = h\nu - h\frac{3\nu}{4} = \frac{h\nu}{4}, \quad K_B = h\nu - h\frac{2\nu}{3} = \frac{h\nu}{3}.$$

Step 2: Compare the energies and test the options Dividing cancels the common photon-energy factor:

$$\frac{K_A}{K_B} = \frac{h\nu/4}{h\nu/3} = \frac{3}{4}.$$

The greater threshold of A leaves less kinetic energy, so the ratio must be less than one. This rules out (B) and (D); (A) gives an incorrect numerical ratio because it does not subtract each work function from the incident photon energy.

Step 3: Conclusion The required ratio is $K_A : K_B = 3 : 4$. Hence, the correct option is (C).

Quick Tip to Solve: Subtract each threshold frequency from the incident frequency before taking the ratio; do not divide the threshold frequencies themselves.

9. The electric flux through a closed Gaussian surface depends upon [1 mark]

- (A) Net charge enclosed and permittivity of the medium
(B) Net charge enclosed, permittivity of the medium and the size of the Gaussian surface
(C) Net charge enclosed only
(D) Permittivity of the medium only

Correct Answer: (A) Net charge enclosed and permittivity of the medium

Solution:

Concept:

- Gauss's law gives the net electric flux through a closed surface: $\Phi_E = \oint \vec{E} \cdot d\vec{S} = Q_{\text{enc}}/\epsilon$.
- Here Q_{enc} is the algebraic net charge enclosed and ϵ is the permittivity of the uniform medium.

Step 1: Identify the quantities in Gauss law The right-hand side contains the enclosed charge and the permittivity. Doubling the enclosed charge doubles the flux; increasing permittivity for the same charge reduces it. Thus both quantities matter, not just the charge or just the medium.

Step 2: Check the role of the Gaussian surface Changing the size or shape of a closed surface while keeping the same net enclosed charge does not change the total flux. The field strength and area elements can change, but their integrated product remains Q_{enc}/ϵ . Option (B) incorrectly includes surface size, while (C) and (D) each omit a necessary quantity.

Step 3: Conclusion The flux depends upon the net charge enclosed and the permittivity of the medium. Hence, the correct option is (A).

Quick Tip to Solve: For a closed surface write $\Phi_E = Q_{\text{enc}}/\epsilon$; neither its radius nor its shape enters the net-flux result.

10. A charge particle after being accelerated through a potential difference V enters in a uniform magnetic field and moves in a circle of radius r . If V is doubled, the radius of the circle will become

[1 mark]

- (A) $2r$
- (B) $\sqrt{2}r$
- (C) $4r$
- (D) $r/\sqrt{2}$

Correct Answer: (B) $\sqrt{2}r$

Solution:

Concept:

- For a particle of mass m and charge magnitude $|q|$ accelerated from rest through a potential difference V , $\frac{1}{2}mv^2 = |q|V$.
- In a perpendicular uniform magnetic field B , magnetic force supplies centripetal force: $|q|vB = mv^2/r$.

Step 1: Express the radius in terms of voltage From the energy gained, $v = \sqrt{2|q|V/m}$. The magnetic-force equation gives

$$r = \frac{mv}{|q|B} = \frac{1}{B} \sqrt{\frac{2mV}{|q|}}.$$

For the same particle and unchanged magnetic field, the radius therefore scales as $\sqrt{V}$.

Step 2: Apply the doubled potential difference Let the new radius be r' . Then

$$\frac{r'}{r} = \sqrt{\frac{2V}{V}} = \sqrt{2}.$$

Option (A) assumes radius is directly proportional to voltage; (C) gives an even larger incorrect factor. Option (D) predicts a smaller radius although the particle's speed and momentum have increased.

Step 3: Conclusion The new radius is $r' = \sqrt{2}r$. Hence, the correct option is (B).

Quick Tip to Solve: At fixed B , radius is proportional to momentum and hence to $\sqrt{V}$; doubling voltage multiplies radius by $\sqrt{2}$.

11. A point charge is placed at the centre of a hollow conducting sphere of internal radius r and outer radius $2r$. The ratio of the surface charge density of the inner surface to that of the outer surface will be _____ . [1 mark]

Solution:

Concept:

- In electrostatic equilibrium the electric field inside conducting material is zero; an internal point charge q induces charge $-q$ on the cavity surface.
- For the initially neutral conducting sphere, charge conservation leaves $+q$ on its outer surface. Spherical symmetry makes each surface charge density uniform.

Step 1: Find the induced charges Take a Gaussian surface within the metal surrounding the cavity. Its zero flux requires zero enclosed charge, so $q + Q_{\text{in}} = 0$, giving $Q_{\text{in}} = -q$. As the conductor has no stated initial charge, the usual neutral-conductor assumption gives $Q_{\text{out}} = +q$.

Step 2: Divide each charge by its surface area The inner radius is r , whereas the outer radius is $2r$. Hence

$$\sigma_{\text{in}} = \frac{-q}{4\pi r^2}, \quad \sigma_{\text{out}} = \frac{q}{4\pi (2r)^2} = \frac{q}{16\pi r^2}.$$

Dividing the signed densities gives $\sigma_{\text{in}}/\sigma_{\text{out}} = -4$; their magnitudes are in ratio 4 : 1.

Step 3: Conclusion The signed ratio is $\sigma_{\text{in}} : \sigma_{\text{out}} = -4 : 1$, or $\sigma_{\text{in}}/\sigma_{\text{out}} = -4$.

Quick Tip to Solve: Equal and opposite induced charges occupy areas in ratio 1 : 4; invert the area ratio for density and retain the negative sign.

12. The _____ , a property of materials C, Si and Ge depends upon the energy gap between their conduction and valence bands. [1 mark]

Solution:

Concept:

- Electrical conductivity describes how readily a material conducts electric current; it depends on the availability of mobile charge carriers.
- The forbidden energy gap E_g separates the valence band from the conduction band, and affects how easily electrons can reach conducting states.

Step 1: Connect the band gap with mobile carriers For an electron in the valence band to contribute as a conduction electron, it must acquire enough energy to cross the forbidden gap. A smaller gap makes thermal excitation easier and produces more electron-hole pairs at a given temperature.

Step 2: Identify the material property Carbon in its diamond form has a large band gap and very few thermally excited carriers, so it acts as an insulator. Silicon and germanium have much smaller gaps and behave as semiconductors. Their different ease of conduction is the property described by the blank. The band gap influences carrier concentration and therefore conductivity.

Step 3: Conclusion The blank is **electrical conductivity**.

Quick Tip to Solve: When a band-gap question asks how readily C, Si and Ge conduct, name conductivity and link a smaller gap with easier carrier excitation.

13. The ability of a junction diode to _____ an alternating voltage, is based on the fact that it allows current to pass only when it is forward biased. [1 mark]

Solution:

Concept:

- A junction diode conducts appreciable current in forward bias and only a very small leakage current in reverse bias under normal operation.
- Rectification converts an alternating input into a unidirectional, generally pulsating output by permitting current mainly in one direction.

Step 1: Examine the alternating input During the half-cycle that forward biases the diode, its depletion barrier is reduced and current flows through the load. In the opposite half-cycle the diode is reverse biased, its barrier increases and the current is approximately blocked.

Step 2: Name the resulting operation Because one polarity of the alternating signal is transmitted and the other is suppressed, the load current has only one direction. This operation is called rectification, and the required verb in the sentence is 'rectify'. A single diode gives half-wave rectification; the output is pulsating and does not become perfectly steady merely by using the diode.

Step 3: Conclusion The junction diode can **rectify** an alternating voltage.

Quick Tip to Solve: Forward conduction and reverse blocking select the conducting half-cycle: the process is rectification, so fill the verb blank with 'rectify'.

14. The physical quantity having SI unit $\text{N C}^{-1} \text{m}$ is _____. [1 mark]

Solution:

Concept:

- Electric potential is work done per unit test charge, so its SI unit is J C^{-1} , called the volt (V).
- Since work has unit $\text{J} = \text{N m}$, the volt may also be written as N m C^{-1} .

Step 1: Rearrange the printed unit The stated unit is $\text{N C}^{-1} \text{m}$. Multiplication of units is commutative, so

$$\text{N C}^{-1} \text{m} = \frac{\text{N m}}{\text{C}} = \frac{\text{J}}{\text{C}} = \text{V}.$$

Thus it represents energy or work per unit electric charge.

Step 2: Match the unit to the physical quantity Electric potential at a point is the work required per unit positive test charge to bring it from the reference point. Potential difference has the same unit. Electric field alone has unit N C^{-1} ; the additional metre in the printed unit changes the dimensions to those of potential, rather than electric field.

Step 3: Conclusion The physical quantity is **electric potential** (or potential difference), measured in volts.

Quick Tip to Solve: Combine N m into joules first; J/C immediately identifies electric potential.

15. A copper wire of non-uniform area of cross-section is connected to a d.c. battery. The physical quantity which remains constant along the wire is _____ . [1 mark]

Solution:

Concept:

- In a steady d.c. circuit, conservation of charge requires the same current through every cross-section of an unbranched wire.
- Current is $I = neAv_d$, where n is carrier number density, e is electron charge magnitude, A is cross-sectional area and v_d is drift speed.

Step 1: Apply conservation of charge Consider a short portion of the wire. If its incoming and outgoing currents differed, charge would continuously accumulate in that portion. This cannot occur in the steady state established by the d.c. battery, so the current entering equals the current leaving.

Step 2: Distinguish current from current density Although the current is unchanged, the current density $J = I/A$ and drift speed $v_d = I/(neA)$ are larger in the narrower portions of the copper wire. Thus non-uniform area changes these local quantities without changing the charge per unit time flowing along the wire.

Step 3: Conclusion The physical quantity that remains constant is the electric current I .

Quick Tip to Solve: For a steady current in one continuous wire, keep I constant and let J and v_d vary inversely with area.

16. Write the conditions on path difference under which (i) constructive (ii) destructive interference occur in Young's double slit experiment. [1 mark]

Solution:

Concept:

- For coherent slits oscillating in phase, the phase difference at the observation point is $\phi = 2\pi\Delta/\lambda$, where Δ is path difference and λ is wavelength.
- Constructive interference occurs when the waves arrive in phase; destructive interference occurs when they arrive in opposite phase.

Step 1: State the condition for constructive interference For reinforcement, the phase difference must be an integral multiple of 2π . Therefore the path difference must be $\Delta = n\lambda$, where n is an integer. For a non-negative path difference, the values are $0, \lambda, 2\lambda, \dots$

Step 2: State the condition for destructive interference For cancellation, the phase difference must be an odd multiple of π . The corresponding non-negative path differences are $\Delta = (2n + 1)\lambda/2$, with $n = 0, 1, 2, \dots$, giving $\lambda/2, 3\lambda/2, 5\lambda/2, \dots$

Step 3: Conclusion (i) Constructive: $\Delta = n\lambda$. (ii) Destructive: $\Delta = (2n + 1)\lambda/2$, for non-negative orders $n = 0, 1, 2, \dots$

Quick Tip to Solve: Whole wavelengths produce bright fringes; odd half-wavelengths produce dark fringes in the usual in-phase double-slit setup.

17. Plot a graph showing variation of induced e.m.f. with the rate of change of current flowing through a coil. [1 mark]

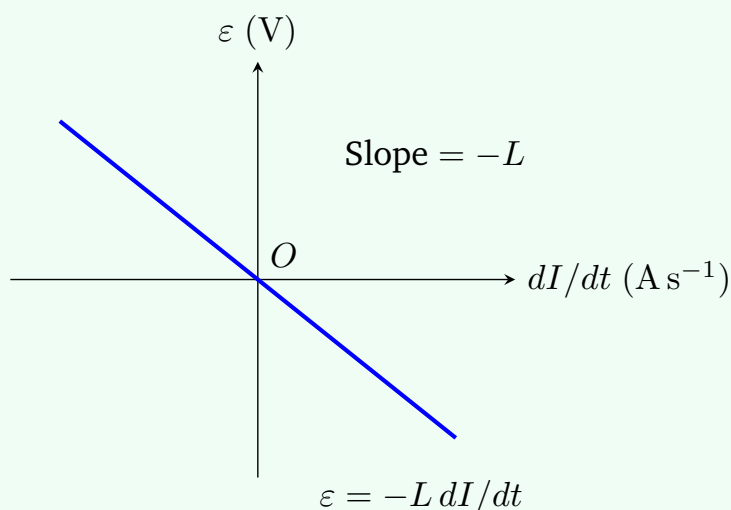
Solution:

Concept:

- Self-induced emf obeys $\varepsilon = -L dI/dt$, where L is the constant self-inductance and dI/dt is the rate of change of current.
- The minus sign expresses Lenz's law: induced emf opposes the change in current that produces it.

Step 1: Identify the straight-line relation Take $x = dI/dt$ on the horizontal axis and the signed induced emf ε on the vertical axis. The equation becomes $\varepsilon = -Lx$, so the graph is a straight line passing through the origin with slope $-L$.

Step 2: Show the sign and intercept When the current is constant, $dI/dt = 0$ and no emf is induced. Increasing current gives a negative signed emf, whereas decreasing current gives a positive signed emf. These points lie on the downward-sloping line shown. If only magnitudes are plotted, $|\varepsilon| = L|dI/dt|$ is instead a line of positive slope L in the first quadrant.



Signed induced emf versus rate of change of current

Step 3: Conclusion The signed-emf graph is a straight line through the origin with slope $-L$; the magnitude-versus-magnitude graph has slope L .

Quick Tip to Solve: Label whether emf is signed or a magnitude: the slope is $-L$ for signed quantities and $+L$ for magnitudes.

17. A series combination of an inductor (L), capacitor (C) and a resistor (R) is connected across an ac source of emf of peak value E_0 and angular frequency (ω). Plot a graph to show variation of impedance of the circuit with angular frequency (ω). [1 mark]

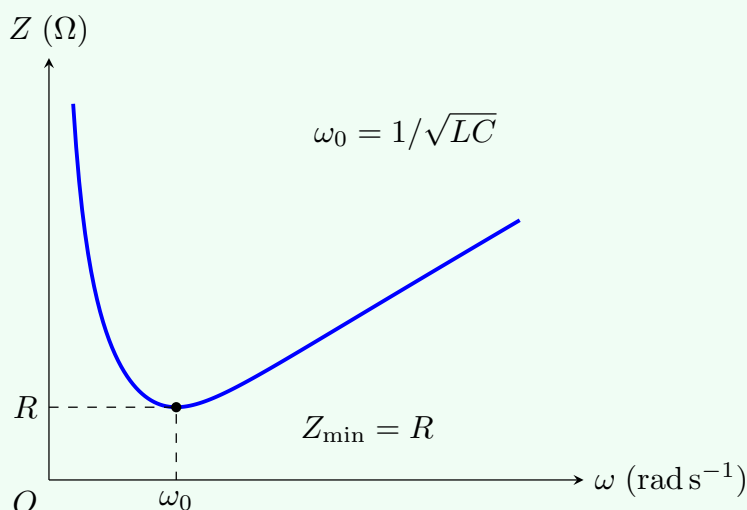
Solution:

Concept:

- A series LCR circuit has impedance $Z = \sqrt{R^2 + (\omega L - 1/(\omega C))^2}$, where R is resistance and ω is angular frequency.
- At resonance, inductive and capacitive reactances cancel: $\omega_0 L = 1/(\omega_0 C)$, giving $\omega_0 = 1/\sqrt{LC}$.

Step 1: Locate the minimum impedance Since the squared reactance term cannot be negative, $Z \geq R$. At $\omega = \omega_0$, this term vanishes and the impedance reaches its minimum, $Z_{\min} = R$. The peak source emf E_0 determines the current amplitude but does not enter the impedance formula.

Step 2: Sketch the behaviour on either side of resonance At very low positive frequency, the capacitive reactance $1/(\omega C)$ is large, so Z tends to infinity as $\omega \rightarrow 0^+$. Increasing frequency lowers Z until resonance. Beyond resonance, the inductive reactance dominates and Z rises again, approaching ωL at high frequency. The curve therefore has a rounded minimum above the horizontal axis.



Impedance versus angular frequency; the minimum is R

Step 3: Conclusion The impedance decreases from a large value, reaches R at $\omega_0 = 1/\sqrt{LC}$, and then increases, as shown.

Quick Tip to Solve: Mark the bottom of an LCR impedance curve at (ω_0, R) : resonance removes reactance but leaves the resistance.

18. Depict the fields diagram of an electromagnetic wave propagating along positive X -axis with its electric field along Y -axis. [1 mark]

Solution:

Concept:

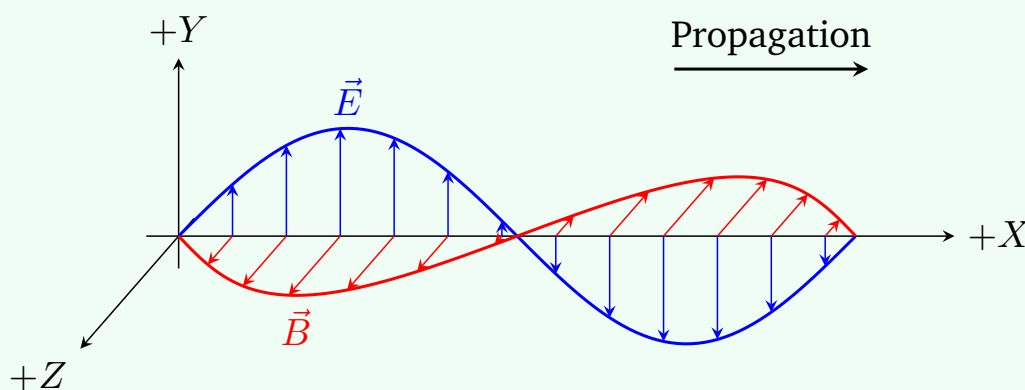
- An electromagnetic wave is transverse: the electric field $\vec{E}$, magnetic field $\vec{B}$, and propagation direction are mutually perpendicular.
- Propagation is along $\vec{E} \times \vec{B}$. In vacuum the fields are in phase and their amplitudes satisfy $E_0 = cB_0$.

Step 1: Determine the magnetic-field axis The wave travels along $+X$ and the electric field oscillates along Y . Since $\hat{y} \times \hat{z} = \hat{x}$, the magnetic field oscillates along Z . When $\vec{E}$ points along $+Y$, $\vec{B}$ points along $+Z$; both reverse together in the next half-cycle.

Step 2: Sketch the in-phase transverse fields A suitable representation is

$$\vec{E} = E_0 \sin(kx - \omega t) \hat{y}, \quad \vec{B} = B_0 \sin(kx - \omega t) \hat{z}.$$

The curves below show a snapshot: both fields have zero crossings and extrema at the same positions. Their amplitudes are drawn on separate scales.

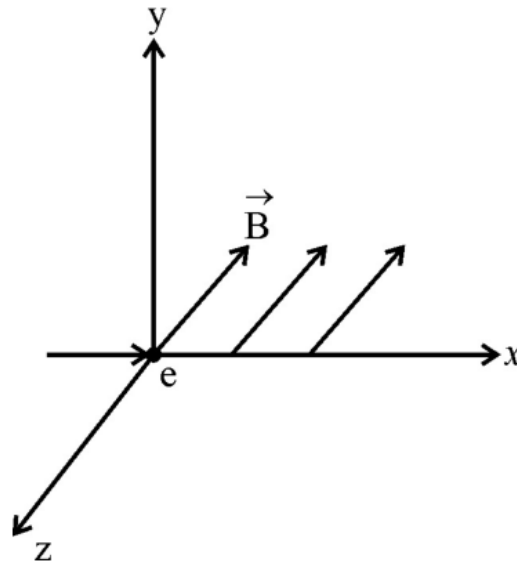


In-phase transverse fields; amplitudes drawn on separate scales.

Step 3: Conclusion The diagram has $\vec{E} \parallel Y$, $\vec{B} \parallel Z$, and propagation along $+X$, with $\vec{E} \times \vec{B} \parallel +X$.

Quick Tip to Solve: Use $\hat{y} \times \hat{z} = \hat{x}$ to fix the magnetic-field axis before sketching the wave.

19. An electron moves along $+x$ direction. It enters into a region of uniform magnetic field B directed along $-z$ direction as shown in fig. Draw the shape of trajectory followed by the electron after entering the field. [1 mark]



Solution:

Concept:

- The magnetic force is $\vec{F} = q(\vec{v} \times \vec{B})$. For an electron, $q = -e$, so its force is opposite to $\vec{v} \times \vec{B}$.
- When velocity is perpendicular to a uniform magnetic field, the magnetic force provides centripetal force; the speed is constant and the path is circular.

Step 1: Determine the initial force Here $\vec{v} = v\hat{x}$ and $\vec{B} = -B\hat{z}$. Therefore

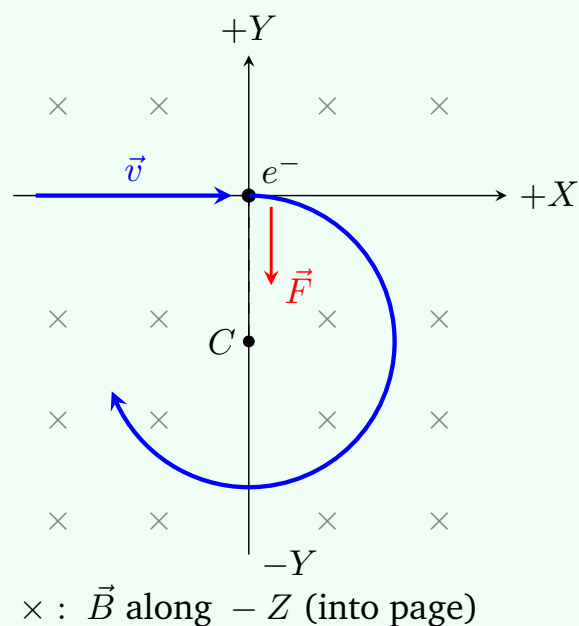
$$\vec{v} \times \vec{B} = +vB\hat{y}, \quad \vec{F} = -evB\hat{y}.$$

Thus the electron initially bends towards $-Y$, below the positive X -axis.

Step 2: Draw the trajectory in the perpendicular plane The force remains perpendicular to the velocity and does no work. Hence

$$evB = \frac{mv^2}{r}, \quad r = \frac{mv}{eB}.$$

The trajectory is an arc of a circle in the XY -plane, curving clockwise when viewed from $+Z$. Its initial tangent is along $+X$; the centre of curvature lies on the negative Y -axis.



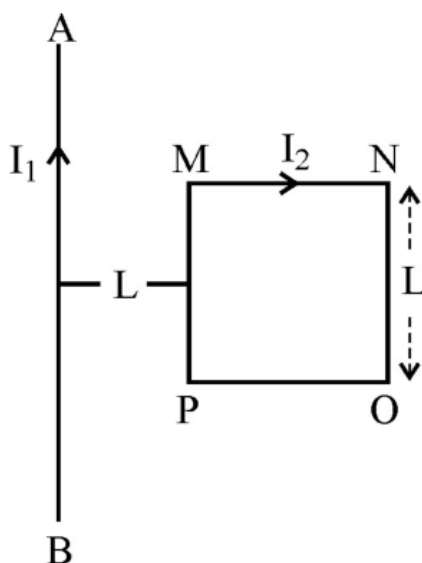
View from $+Z$: the electron bends downwards in the XY -plane.

Step 3: Conclusion The electron follows a circular arc bending initially towards $-Y$, clockwise as viewed from $+Z$.

Quick Tip to Solve: Find $\vec{v} \times \vec{B}$ first, then reverse its direction for the electron's negative charge.

OR

19. A square shaped current carrying loop $MNOP$ is placed near a straight long current carrying wire AB as shown in the fig. The wire and the loop lie in the same plane. If the loop experiences a net force F towards the wire, find the magnitude of the force on the side NO of the loop. [1 mark]



Solution:

Concept:

- A long straight wire carrying current I_1 produces magnetic field $B = \mu_0 I_1 / (2\pi r)$ at distance r .
- The force on a parallel side of length L carrying current I_2 is $I_2 L B$. Parallel currents attract and antiparallel currents repel.

Step 1: Compare the forces on the two vertical sides The figure gives both the loop side and its distance from the wire as L . Hence the near side PM is at distance L and the far side NO is at $2L$. Current in PM is upward, parallel to the wire, so its force is towards the wire. Current in NO is downward, so its force is away from the wire. Their magnitudes satisfy

$$F_{PM} = \frac{\mu_0 I_1 I_2 L}{2\pi L}, \quad F_{NO} = \frac{\mu_0 I_1 I_2 L}{2\pi(2L)} = \frac{F_{PM}}{2}.$$

Step 2: Relate the resultant to the far-side force Forces on the horizontal sides MN and OP are equal and opposite: their corresponding elements are at identical distances from the wire and carry opposite currents. They cancel. Therefore the given net force towards the wire is

$$F = F_{PM} - F_{NO} = 2F_{NO} - F_{NO}.$$

Step 3: Conclusion The magnitude of the force on side NO is $\boxed{F}$; this side's force acts away from the wire.

Quick Tip to Solve: Read both labelled distances: the far side is at $2L$, so its force is half the near-side force.

20. Define the term 'current sensitivity' of a moving coil galvanometer.

[1 mark]

Solution:

Concept:

- Current sensitivity measures the angular deflection produced per unit current in a moving coil galvanometer.
- In a radial magnetic field, the magnetic torque is $NIAB$ and the restoring torque is $k\theta$, where k is the torsional constant.

Step 1: State the defining ratio If a current I produces an equilibrium angular deflection θ , the current sensitivity is

$$S_I = \frac{\theta}{I}.$$

Thus it gives the deflection for unit current; a larger value means a larger deflection for the same small current.

Step 2: Express the ratio through instrument parameters At equilibrium $NIAB = k\theta$, hence $S_I = NBA/k$. Here N is the number of turns, A the coil area and B the radial magnetic-field magnitude. When deflection is measured in radians, the SI unit is rad A^{-1} .

Step 3: Conclusion Current sensitivity is the angular deflection of a galvanometer per unit current through it: $S_I = \theta/I$.

Quick Tip to Solve: For current sensitivity put current in the denominator: deflection divided by current, not voltage.

Section B (14 marks)

21. (a) Define one Becquerel.

(b) A radioactive substance disintegrates into two types of daughter nuclei, one type with disintegration constant λ_1 and the other type with disintegration constant λ_2 . Determine the half-life of the radioactive substance. [2 marks]

Solution:

Concept:

- Activity is the number of nuclear disintegrations per unit time. Its SI unit is the becquerel, with $1 \text{ Bq} = 1 \text{ s}^{-1}$.
- Independent alternative decay channels remove nuclei from the same parent population, so their decay constants add.
- For effective decay constant λ , the parent population is $N(t) = N_0 e^{-\lambda t}$; half-life is the time at which $N = N_0/2$.

Step 1: (a) Define one becquerel One becquerel is the activity of a radioactive sample in which one nuclear disintegration occurs per second. It measures the rate of decay events, rather than the number of nuclei present or the energy released in an event.

Step 2: (b) Add the rates of the two decay channels Let N be the number of undecayed parent nuclei at time t . The rates of removal through the two channels are $\lambda_1 N$ and $\lambda_2 N$. Since either channel destroys a parent nucleus, the total rate is

$$-\frac{dN}{dt} = (\lambda_1 + \lambda_2)N.$$

Integrating with $N(0) = N_0$ gives $N = N_0 e^{-(\lambda_1 + \lambda_2)t}$.

Step 3: Apply the half-life condition At $t = T_{1/2}$, the number of remaining parent nuclei is half the initial number. Thus

$$\frac{1}{2} = e^{-(\lambda_1 + \lambda_2)T_{1/2}}, \quad (\lambda_1 + \lambda_2)T_{1/2} = \ln 2.$$

Both constants have units of inverse time, so this expression yields a half-life in the corresponding time unit.

Step 4: Conclusion (a) 1 Bq means one nuclear disintegration per second. (b) The half-life of the parent substance is $T_{1/2} = \ln 2 / (\lambda_1 + \lambda_2)$.

Quick Tip to Solve: For two alternative decay channels of the same parent, add the decay constants before applying $T_{1/2} = \ln 2 / \lambda$.

22. In a single slit diffraction experiment, the width of the slit is increased. How will the (i) size and (ii) intensity of central bright band be affected ? Justify your answer. [2 marks]

Solution:

Concept:

- For a slit of width a , the first diffraction minima satisfy $a \sin \theta = \lambda$, where λ is the wavelength.
- For a distant screen at distance D , the central maximum has width $w \simeq 2D\lambda/a$. At its centre, contributions from the whole slit arrive in phase.

Step 1: (i) Effect on the size of the central band The central bright band lies between the first minima on the two sides of the centre. For small diffraction angles, their positions are $y = \pm D\lambda/a$, hence

$$w = \frac{2D\lambda}{a}.$$

Keeping the wavelength and screen distance fixed, increasing a decreases w . The angular width, approximately $2\lambda/a$, decreases as well. Thus the central bright band becomes narrower.

Step 2: (ii) Effect on the intensity of the central band With the same incident illumination, a wider slit admits more light. At the centre the path difference between contributions from all parts of the slit is zero, so these contributions add constructively. The resultant amplitude is proportional to a ; therefore the peak intensity is proportional to a^2 . The central maximum consequently becomes brighter while its width decreases. This comparison assumes that the incident intensity remains unchanged.

Step 3: Conclusion On increasing the slit width, (i) the central bright band becomes narrower and (ii) its central intensity increases: $w \propto 1/a$ and $I_{\text{centre}} \propto a^2$.

Quick Tip to Solve: Locate the first minima to find the width; add in-phase amplitudes at the centre to compare brightness.

23. In case of photo electric effect experiment, explain the following facts, giving reasons.

(a) The wave theory of light could not explain the existence of the threshold frequency.

(b) The photo electric current increases with increase of intensity of incident light.

[2 marks]

Solution:

Concept:

- In classical wave theory, radiation transfers energy continuously; its intensity determines the energy supplied per unit area per unit time.
- In the photon model, each photon has energy $h\nu$. Emission requires $h\nu \geq \phi$, where ϕ is the metal's work function and $\nu_0 = \phi/h$ is its threshold frequency.

Step 1: (a) Why wave theory fails to explain a threshold Classical wave theory allows an electron to accumulate energy continuously from the incident light. It would therefore predict emission at any frequency if the intensity were sufficient or the electron absorbed energy for long enough. It cannot account for a sharp minimum frequency below which no emission occurs, however intense the light. The photon picture explains this: one photon transfers its energy to one electron, and if $h\nu < \phi$, that energy cannot free the electron. Increasing intensity increases the number of such insufficient-energy photons, not the energy of an individual photon.

Step 2: (b) Why greater intensity produces greater current For a fixed frequency above the threshold,

increasing intensity increases the number of photons incident per second because each photon still has the same energy $h\nu$. Consequently more electrons are emitted per second. Under the same collection conditions, the photoelectric current $I = eN$, where N is the number of electrons collected per second, increases. The maximum kinetic energy $K_{\max} = h\nu - \phi$ does not increase merely by increasing intensity.

Step 3: Conclusion (a) Continuous energy absorption in wave theory gives no threshold frequency; photon energy $h\nu$ must exceed the work function. (b) Above threshold, higher intensity supplies more photons and hence a larger photoelectric current.

Quick Tip to Solve: Separate energy per photon, set by frequency, from photons per second, set by intensity at fixed frequency.

24. Gamma rays and radio waves travel with the same velocity in free space. Distinguish between them in terms of their origin and the main application. [2 marks]

Solution:

Concept:

- All electromagnetic waves propagate in free space with speed $c = 1/\sqrt{\mu_0\epsilon_0}$. Their common speed does not imply the same origin or photon energy.
- Gamma rays originate in nuclear transitions, whereas radio waves are commonly generated by accelerating charges in electrical circuits and antennas.

Step 1: Gamma rays: origin and main application Gamma rays are emitted when an excited atomic nucleus changes to a lower-energy nuclear state, as in radioactive decay. Their very high frequency gives them high photon energy and penetrating, ionising power. A main application is radiotherapy: a controlled gamma-ray beam is used to destroy malignant cells in the treatment of cancer. Their origin is therefore associated with changes within the nucleus, rather than ordinary oscillating currents in an antenna.

Step 2: Radio waves: origin and main application Radio waves are produced by charges accelerating or oscillating in conducting circuits, such as the high-frequency alternating current in a transmitting antenna. A main application is wireless communication, including radio and television broadcasting. Information is impressed on a radio-frequency carrier and recovered at the receiver. Compared with gamma rays, radio waves have much lower frequencies and longer wavelengths, although both travel at the same speed c in vacuum.

Step 3: Conclusion Gamma rays arise from nuclear transitions and are used in cancer radiotherapy. Radio waves arise from oscillating or accelerating charges in antennas and are used for wireless communication and broadcasting.

Quick Tip to Solve: Link nuclear origin to gamma-ray cancer treatment, and an oscillating antenna current to radio communication.

25. Use Bohr's model of hydrogen atom to obtain the relationship between the angular momentum and the magnetic moment of the revolving electron. [2 marks]

Solution:

Concept:

- An electron of mass m_e moving with speed v in a circular Bohr orbit of radius r has angular momentum magnitude $L = m_e v r$.
- A current loop has magnetic moment magnitude $\mu = IA$. The electron's charge is $-e$, where $e > 0$, so conventional current is opposite to its motion.

Step 1: Calculate the orbital magnetic moment The time taken by the electron to complete one revolution is $T = 2\pi r/v$. The magnitude of the equivalent circulating current is therefore

$$I = \frac{e}{T} = \frac{ev}{2\pi r}.$$

The area enclosed by the circular orbit is $A = \pi r^2$. Thus the magnitude of its magnetic moment is

$$\mu = IA = \frac{ev}{2\pi r} \pi r^2 = \frac{evr}{2}.$$

Step 2: Relate magnetic moment to angular momentum Substituting $vr = L/m_e$ gives $\mu = eL/(2m_e)$. The angular momentum points along the normal obtained by the right-hand rule for the electron's motion; the magnetic moment points along the normal for conventional current. Because the electron is negatively charged, these two vectors are antiparallel. Consequently the vector relation is

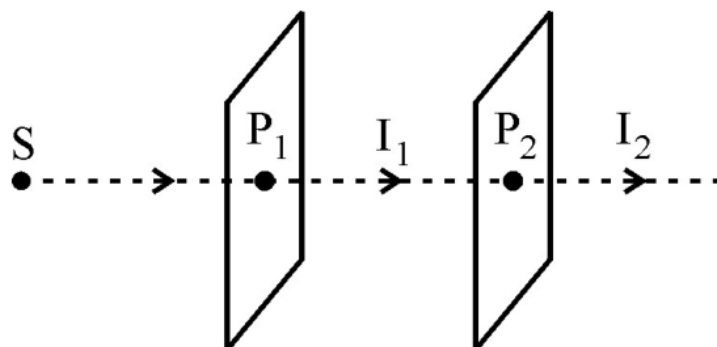
$$\boldsymbol{\mu} = -\frac{e}{2m_e} \mathbf{L}.$$

Bohr's quantisation $L = nh/(2\pi) = n\hbar$ further yields $\mu = ne\hbar/(2m_e) = n\mu_B$, where $\mu_B = e\hbar/(2m_e)$ is the Bohr magneton. This is the orbital magnetic moment in the Bohr model.

Step 3: Conclusion The revolving electron obeys $\boldsymbol{\mu} = -(e/2m_e)\mathbf{L}$; in magnitude, $\mu/L = e/(2m_e)$, and its magnetic moment is opposite to its angular momentum.

Quick Tip to Solve: Use charge per orbital period to get the current; insert the negative sign only in the vector relation.

26. Light from a sodium lamp (S) passes through two polaroid sheets P_1 and P_2 as shown in fig. What will be the effect on the intensity of the light transmitted (i) by P_1 and (ii) by P_2 on rotating polaroid P_1 about the direction of propagation of light? Justify your answer in both cases. [2 marks]



Solution:

Concept:

- Unpolarised light of intensity I_0 passing through an ideal polaroid emerges plane polarised with intensity $I_1 = I_0/2$, independent of the orientation of its transmission axis.
- Malus's law for an analyser is $I_2 = I_1 \cos^2 \theta$, where θ is the angle between the transmission axes of the two polaroids.

Step 1: (i) Intensity transmitted by the rotating first polaroid Light from the sodium lamp is unpolarised: its electric-field directions are distributed equally in the plane perpendicular to propagation. The first sheet P_1 transmits the component along its own transmission axis. The average squared projection is one half for every orientation, so

$$I_1 = \frac{I_0}{2}.$$

Rotating P_1 about the direction of propagation rotates the direction of the emerging polarisation, but does not change its intensity. This assumes ideal sheets and an unchanged lamp intensity.

Step 2: (ii) Intensity transmitted by the fixed second polaroid The light reaching P_2 is plane polarised along the instantaneous axis of P_1 . Although I_1 remains constant, rotation of P_1 changes its angle θ with the fixed axis of P_2 . By Malus's law,

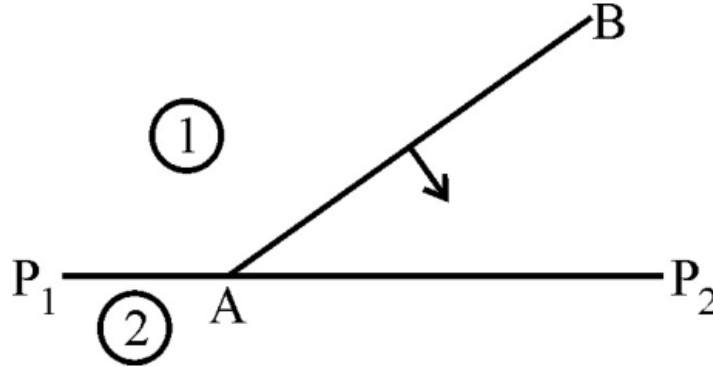
$$I_2 = I_1 \cos^2 \theta = \frac{I_0}{2} \cos^2 \theta.$$

It is maximum, $I_0/2$, for parallel axes and zero for perpendicular axes. During a complete rotation these maxima and minima occur alternately, with the intensity pattern repeating after each 180° rotation.

Step 3: Conclusion When P_1 is rotated, (i) its transmitted intensity remains $I_0/2$; (ii) the intensity transmitted by P_2 varies between zero and $I_0/2$, according to $I_2 = (I_0/2) \cos^2 \theta$.

Quick Tip to Solve: The first sheet sees unpolarised light; the second sees a polarisation direction that rotates with P_1 .

26. Define the term 'wave front of light'. A plane wave front AB propagating from denser medium (1) into a rarer medium (2) is incident on the surface P_1P_2 separating the two media as shown in fig. Using Huygen's principle, draw the secondary wavelets and obtain the refracted wave front in the diagram. [2 marks]



Solution:

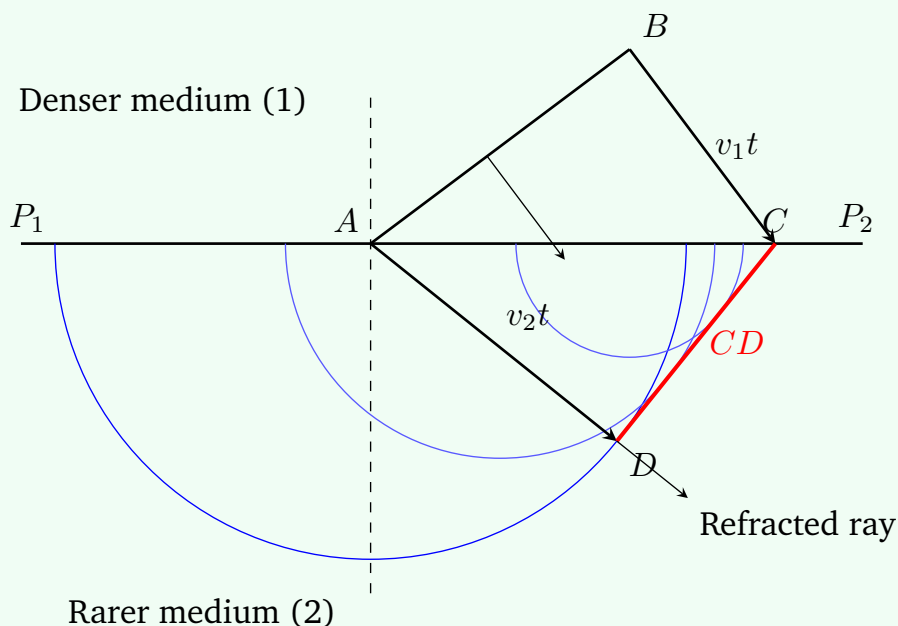
Concept:

- A wavefront is the locus of all points of a light wave that vibrate in the same phase at a given instant. In an isotropic medium the rays are perpendicular to the wavefront.
- Huygens' principle: each point reached by a wave acts as a source of secondary wavelets; their forward common tangent gives the new wavefront. If the speeds are v_1 and v_2 , passage from denser to rarer medium means $v_2 > v_1$.

Step 1: Advance the incident wavefront to the interface In the given orientation, point A of AB reaches P_1P_2 first. Draw BC perpendicular to AB , with C on the interface. Let t be the time for the disturbance at B to travel to C ; then $BC = v_1 t$. During this interval the secondary wavelet from A spreads into medium (2) through distance $v_2 t$.

Step 2: Construct the forward tangent to the secondary wavelets Draw a semicircular wavelet centred at A in medium (2), with radius $AD = v_2 t$. Draw the tangent from C to this wavelet, touching it at D . Secondary wavelets from the intermediate points of the interface have smaller radii because those points are reached later; they have the same forward tangent CD . Since $AD \perp CD$, the refracted ray along AD is normal to the refracted wavefront. Here $AD > BC$, so the ray bends away from the normal. Let i and r be the incident and refracted ray angles to the normal. In the right triangles,

$$\sin i = \frac{BC}{AC}, \quad \sin r = \frac{AD}{AC}, \quad \frac{\sin i}{\sin r} = \frac{v_1}{v_2} < 1.$$



Huygens construction: secondary wavelets in the rarer medium and their common tangent CD .

Step 3: Conclusion The wavefront joins points in the same phase. The common forward tangent CD is the required refracted plane wavefront; the refracted ray makes $r > i$ with the normal in the rarer medium.

Quick Tip to Solve: Start the largest wavelet at the point that reaches the interface first; draw its tangent from the point reached last, and make the radius larger in the faster medium.

27. Derive the expression for the torque acting on an electric dipole, when it is held in a uniform electric field. Identify the orientation of the dipole in the electric field, in which it attains a stable equilibrium. [2 marks]

Solution:

Concept:

- An electric dipole consists of charges $+q$ and $-q$, separated by $2a$. Its dipole moment $\vec{p}$ points from the negative to the positive charge and has magnitude $p = 2aq$.
- Equal and opposite parallel forces with distinct lines of action form a couple. Torque equals either force multiplied by the perpendicular distance between their lines of action.

Step 1: Find the force couple on the dipole Hold the dipole with its axis at angle θ to the uniform field $\vec{E}$. The positive charge experiences $+q\vec{E}$ and the negative charge experiences $-q\vec{E}$. Their net force is zero, but their lines of action are separated by perpendicular distance $2a \sin \theta$. Therefore the torque magnitude is

$$\tau = (qE)(2a \sin \theta) = pE \sin \theta, \quad \vec{\tau} = \vec{p} \times \vec{E}.$$

Its direction follows the right-hand rule, and its rotational effect is to reduce the angle between $\vec{p}$ and $\vec{E}$.

Step 2: Test the equilibrium orientation for stability Torque vanishes for $\theta = 0$ and $\theta = \pi$. At $\theta = 0$, a small angular displacement produces a restoring torque towards alignment. Equivalently, the potential energy $U = -pE \cos \theta$ is minimum, $U = -pE$, in this orientation. At $\theta = \pi$, the energy is maximum

and a small displacement carries the dipole further away, so that orientation is unstable.

Step 3: Conclusion The dipole torque is $\vec{\tau} = \vec{p} \times \vec{E}$, of magnitude $pE \sin \theta$. Stable equilibrium occurs when $\vec{p}$ is parallel to $\vec{E}$, with the positive charge towards the field direction.

Quick Tip to Solve: Zero torque alone does not prove stability: compare the aligned and antiparallel dipoles using $U = -pE \cos \theta$; the energy minimum is stable.

OR

27. Obtain the expression for the energy stored in a capacitor connected across a dc battery. Hence define energy density of the capacitor. [2 marks]

Solution:

Concept:

- For a capacitor of capacitance C , the instantaneous potential difference when its charge is q is $v = q/C$. Moving a further small charge dq onto it requires work $dW = v dq$.
- Energy density is the electric-field energy stored per unit volume. For a parallel-plate capacitor, neglecting edge effects, the field occupies volume Ad , where A is plate area and d the separation.

Step 1: Integrate the work of charging the capacitor Start with an uncharged capacitor. As charge accumulates, its potential difference rises from zero to the final battery voltage V . The energy accumulated in its electric field is the work needed to transfer charge against this rising potential difference:

$$U = \int_0^Q \frac{q}{C} dq = \frac{Q^2}{2C}.$$

Since $Q = CV$, the equivalent forms are

$$U = \frac{1}{2}QV = \frac{1}{2}CV^2.$$

In direct charging by an ideal constant-voltage battery, the battery supplies QV ; only half is stored in the capacitor, while the rest is dissipated during the charging transient. Thus the stored energy is not QV .

Step 2: Divide the stored energy by the field volume For parallel plates with a uniform dielectric of permittivity ϵ , use $C = \epsilon A/d$ and $V = Ed$. Then

$$u = \frac{U}{Ad} = \frac{\frac{1}{2}(\epsilon A/d)(Ed)^2}{Ad} = \frac{1}{2}\epsilon E^2.$$

In air or vacuum, $\epsilon \simeq \epsilon_0$. Energy density measures the energy in each unit volume of the capacitor's electric field and is measured in J m^{-3} .

Step 3: Conclusion A capacitor at battery voltage V stores $U = \frac{1}{2}CV^2 = \frac{1}{2}QV$. Its energy density is energy per unit field volume, with $u = \frac{1}{2}\epsilon_0 E^2$ for an air or vacuum parallel-plate capacitor.

Quick Tip to Solve: Integrate using the capacitor's rising voltage q/C , then divide by Ad ; substituting $C = \epsilon_0 A/d$ and $V = Ed$ cancels the plate dimensions.

Section C (21 marks)

28. (a) Two point charges q_1 and q_2 are kept at a distance of r_{12} in air. Deduce the expression for the electrostatic potential energy of this system.

(b) If an external electric field ($\vec{E}$) is applied on the system, write the expression for the total energy of this system. [3 marks]

Solution:

Concept:

- Electrostatic potential energy equals the work done by an external agent in assembling charges slowly from infinite separation, with no change in kinetic energy. The reference interaction energy is zero at infinite separation.
- The potential of a point charge q_1 in air at distance r is $V_1(r) = q_1/(4\pi\epsilon_0 r)$. A charge q at external potential V_{ext} has energy qV_{ext} , using a consistent potential reference.

Step 1: Assemble the two charges from infinity (a) First place q_1 at $\vec{r}_1$. No work against another charge's field is needed because q_2 is still at infinity. Now bring q_2 slowly to $\vec{r}_2$, a distance r_{12} from q_1 . The potential there due to q_1 is

$$V_1(\vec{r}_2) = \frac{1}{4\pi\epsilon_0} \frac{q_1}{r_{12}}.$$

The work done by the external agent is q_2 times this potential, giving

$$U_{12} = q_2 V_1(\vec{r}_2) = \frac{q_1 q_2}{4\pi\epsilon_0 r_{12}}.$$

This is a single pair interaction, so it must not be counted again as $q_1 V_2$. It is positive for like charges, for which assembly requires positive work, and negative for unlike charges, for which the external agent removes energy during assembly.

Step 2: Add the interactions with the applied field (b) Let $V_{\text{ext}}(\vec{r})$ be the potential due only to the sources of the applied field, with $\vec{E} = -\nabla V_{\text{ext}}$. The energy of each charge in this field must be added to their mutual interaction energy:

$$U_{\text{total}} = q_1 V_{\text{ext}}(\vec{r}_1) + q_2 V_{\text{ext}}(\vec{r}_2) + \frac{q_1 q_2}{4\pi\epsilon_0 r_{12}}.$$

For a uniform applied field, choose the reference $V_{\text{ext}}(\vec{0}) = 0$; then $V_{\text{ext}}(\vec{r}) = -\vec{E} \cdot \vec{r}$. A uniform field cannot have its potential set to zero at infinity, so a finite reference is used for the external terms. The formula excludes the fixed sources' own energy and point-charge self-energies.

Step 3: Conclusion (a) The two-charge interaction energy in air is $U_{12} = q_1 q_2 / (4\pi\epsilon_0 r_{12})$. (b) In an external field, add $q_1 V_{\text{ext}}(\vec{r}_1) + q_2 V_{\text{ext}}(\vec{r}_2)$ to this pair energy, with the external potential measured from the chosen reference.

Quick Tip to Solve: Count the mutual pair once, then add one external-potential term for each charge; use potentials due only to the applied field in those extra terms.

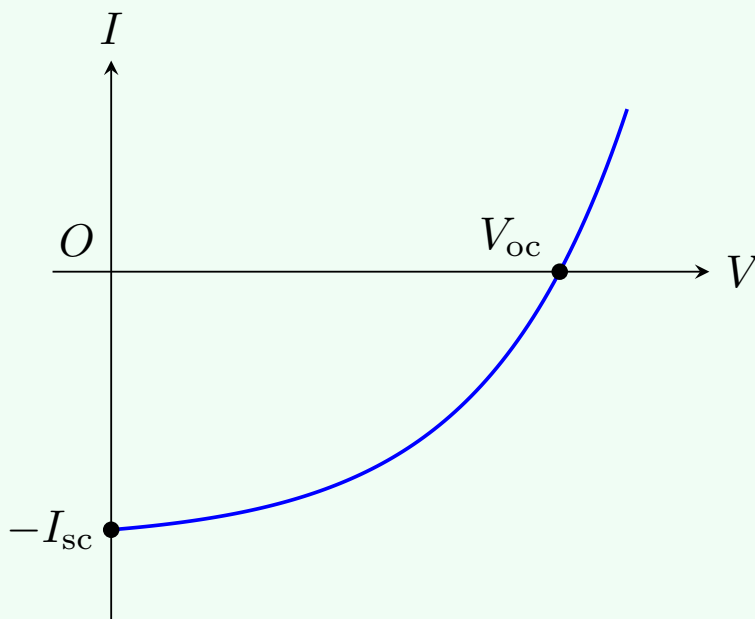
29. What is a solar cell ? Draw its V - I characteristics. Explain the three processes involved in its working. [3 marks]

Solution:

Concept:

- A solar cell is a large-area semiconductor p - n junction that converts incident light energy directly into electrical energy by the photovoltaic effect, without an external bias supply.
- A photon with energy $h\nu \geq E_g$, where E_g is the semiconductor band gap, can generate an electron-hole pair. The built-in electric field of the depletion layer separates the carriers.
- With forward diode current taken positive, the illuminated junction obeys $I = I_0[\exp(qV/kT) - 1] - I_{ph}$. Here I_{ph} is the light-generated current; electrical power is delivered in the fourth quadrant.

Step 1: State the device and interpret its characteristic The exposed junction receives light through a thin surface layer and a grid contact. Its illuminated V - I characteristic crosses the current axis at $I = -I_{sc}$, the short-circuit current, and the voltage axis at $V = V_{oc}$, the open-circuit voltage. Between these intercepts, $V > 0$ and $I < 0$: current flows out of the positive terminal into a load, so the cell supplies power. Neither the open-circuit point nor the short-circuit point supplies power because one of V and I is zero. A working load selects a point between them.



Illuminated solar-cell characteristic; positive current is the forward diode direction.

Step 2: Generation of charge carriers Incident photons of sufficient energy are absorbed in and near the depletion region and produce electron-hole pairs. Light creates the mobile carriers; it does not simply heat the junction to produce the voltage. Carriers generated close enough to the junction can reach its depletion layer before recombining.

Step 3: Separation by the junction field The built-in electric field points from the positively charged donor ions on the n side towards the negatively charged acceptor ions on the p side. It drives electrons towards the n side and holes towards the p side. This separates the photogenerated charges and makes the p contact positive relative to the n contact.

Step 4: Collection at the contacts The metal contacts collect electrons at the n side and holes at the p side. On connecting a load, electrons pass through the external circuit from the n contact to the p contact; conventional current through the load flows in the opposite direction. Continuous

illumination maintains generation, separation and collection, thereby sustaining the photocurrent.

Step 5: Conclusion A solar cell is an unbiased photovoltaic p - n junction. Its three working processes are generation, separation and collection of electron-hole pairs; its power-generating characteristic lies between $(0, -I_{sc})$ and $(V_{oc}, 0)$.

Quick Tip to Solve: Label both axis intercepts and show the power-delivery curve in the fourth quadrant when forward diode current is positive.

OR

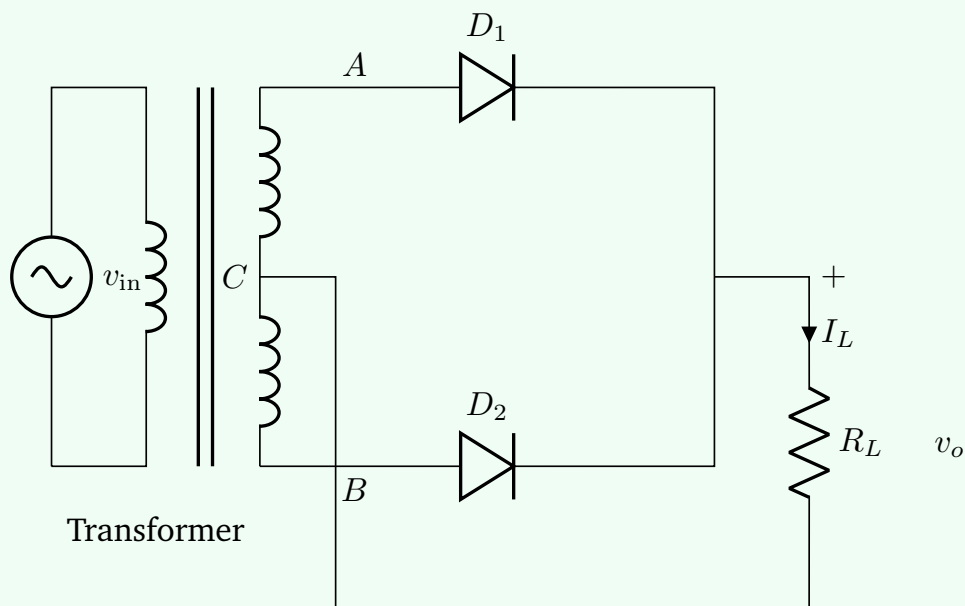
29. Draw the circuit diagram of a full wave rectifier. Explain its working showing its input and output waveforms. [3 marks]

Solution:

Concept:

- A full wave rectifier uses both half-cycles of an alternating voltage to produce a unidirectional, pulsating current through a load resistance R_L .
- In the centre-tapped arrangement, two diodes D_1, D_2 conduct alternately. The secondary ends A, B have equal voltages of opposite phase relative to the centre tap C .
- For ideal diodes and a sinusoidal half-secondary voltage $v_{AC} = V_m \sin \omega t$, the load voltage is $v_o = V_m |\sin \omega t|$; the output frequency is twice the input frequency.

Step 1: Connect the full wave rectifier The circuit uses a transformer with a centre-tapped secondary. Connect end A to the anode of D_1 and end B to the anode of D_2 . Join the cathodes to the upper, positive terminal of R_L , and connect the lower load terminal to centre tap C . The transformer primary receives the alternating input. The diodes therefore offer alternate conducting paths while sharing the same load-current direction.



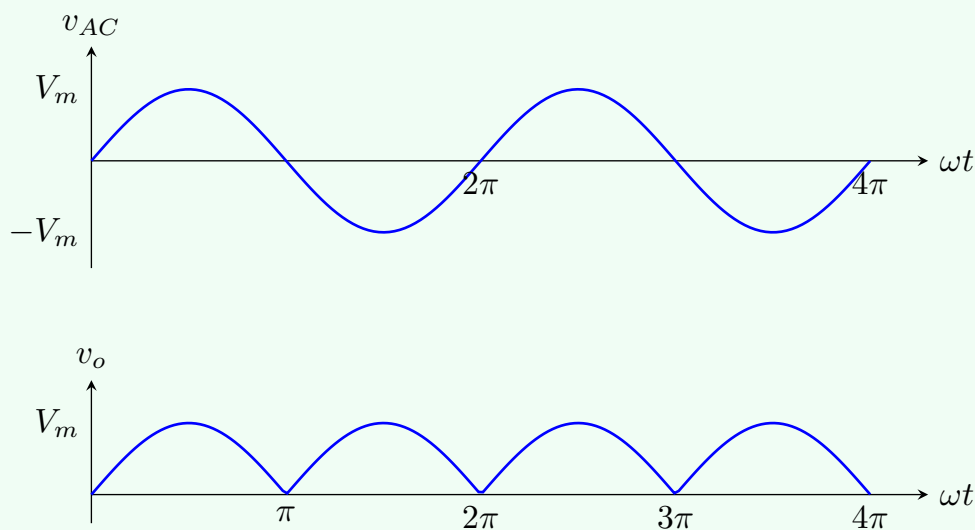
Centre-tapped full wave rectifier; the joined cathodes form the positive output.

Step 2: Explain the first half-cycle When A is positive and B is negative relative to C , D_1 is forward biased and D_2 is reverse biased. Current flows from A , through D_1 , down through R_L , and back to C .

through the upper half of the secondary winding. The upper terminal of the load is positive.

Step 3: Explain the second half-cycle In the next half-cycle, B becomes positive and A becomes negative relative to C . Now D_2 conducts and D_1 blocks. Current flows from B , through D_2 , down through the same load, and back to C through the lower half of the secondary. Thus reversal of the input does not reverse the load current.

Step 4: Compare the input and output waveforms The input waveform alternates above and below zero. The negative input half-cycles appear as positive pulses across the load, so there are two output pulses in each input period T . Consequently the ripple period is $T/2$ and its frequency is $2f$, where $f = 1/T$. The shown output assumes ideal diodes; practical forward drops reduce the pulse heights. Rectification alone does not give a constant voltage: a filter is needed to smooth this pulsating output.



Sinusoidal input and full wave rectified output for ideal diodes.

Step 5: Conclusion The two diodes conduct on alternate half-cycles, but current through R_L always flows in the same direction. The resulting full wave output is $V_m |\sin \omega t|$ for ideal diodes and has twice the input frequency.

Quick Tip to Solve: Trace both conducting paths through the load: both must enter its positive terminal, even though different secondary ends supply them.

30. (a) Define internal resistance of a cell.

(b) A cell of emf E and internal resistance r is connected across a variable resistor R . Plot the shape of graphs showing variation of terminal voltage V with (i) R and (ii) circuit current I . [3 marks]

Solution:

Concept:

- Internal resistance r is the resistance offered inside a cell by its electrolyte and electrodes to the passage of current.
- For a cell delivering current I through external resistance R , $I = E/(R + r)$ and $V = E - Ir = IR$, where E is its emf and V its terminal voltage.

Step 1: (a) Define internal resistance Internal resistance is the opposition to current within the cell itself, due to the electrolyte and electrodes. When current flows, a potential drop Ir occurs inside the

cell, so its terminal voltage is smaller than its emf. Its SI unit is the ohm, Ω .

Step 2: (b)(i) Variation with external resistance From $E = I(R + r)$,

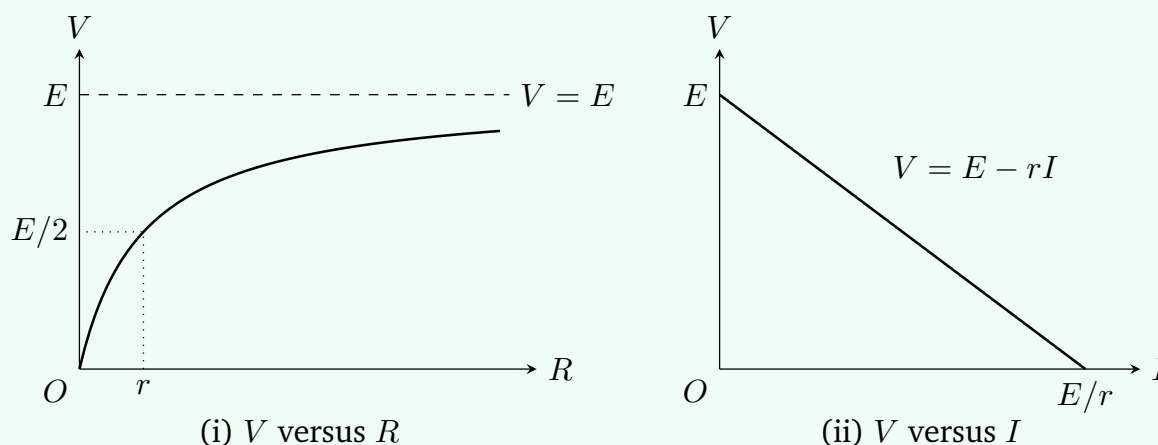
$$V = IR = \frac{ER}{R + r}.$$

At $R = 0$, the terminals are short-circuited and $V = 0$. Increasing R decreases the current and the internal voltage drop. Thus V increases, but its slope decreases: $dV/dR = Er/(R + r)^2 > 0$. For very large R , V approaches E ; the horizontal line $V = E$ is an asymptote. At $R = r$, $V = E/2$, providing a useful point on the curve.

Step 3: (b)(ii) Variation with circuit current The terminal voltage during discharge obeys

$$V = E - rI.$$

This is a straight line with vertical intercept E and slope $-r$. For an open circuit, $I = 0$ and $V = E$. For a short circuit, $V = 0$ and $I = E/r$. The relevant range for a nonnegative external resistance therefore runs from $(0, E)$ to $(E/r, 0)$. The sketches show the intercepts and limiting behaviour; the curved graph never crosses its emf asymptote for a finite positive resistance.



Terminal voltage versus external resistance and discharge current, for $r > 0$.

Step 4: Conclusion Internal resistance opposes current inside the cell. The V -versus- R graph rises from the origin towards $V = E$; the V -versus- I graph is a decreasing straight line with intercepts E and E/r .

Quick Tip to Solve: Use $V = IR = E - Ir$: eliminate I for the resistance graph and retain I for the current graph.

31. Calculate the de-Broglie wavelength associated with the electron revolving in the first excited state of hydrogen atom. The ground state energy of the hydrogen atom is -13.6 eV. [3 marks]

Solution:

Concept:

- Hydrogen has energy levels $E_n = -13.6/n^2$ eV; its first excited state has principal quantum number $n = 2$.
- In a Coulomb-bound Bohr orbit the kinetic energy is $K = -E_n$. For a nonrelativistic electron of

mass m_e , momentum is $p = \sqrt{2m_e K}$ and de-Broglie wavelength is $\lambda = h/p$.

Step 1: Identify the energy of the first excited state The ground state is $n = 1$, so the first excited state is $n = 2$, not $n = 1$. Using the supplied ground-state energy,

$$E_2 = \frac{-13.6}{2^2} = -3.4 \text{ eV}.$$

This is the total orbital energy. To calculate momentum we need the positive kinetic energy rather than the magnitude of an excitation-energy difference.

Step 2: Find the electron's kinetic energy and momentum For the electrostatic orbit, force balance gives $mv^2 = ke^2/r$, so $K = ke^2/(2r)$ and potential energy $U = -ke^2/r = -2K$. Consequently $E = K + U = -K$. Thus

$$K = 3.4 \text{ eV} = 3.4(1.602 \times 10^{-19}) = 5.447 \times 10^{-19} \text{ J}.$$

Taking $m_e = 9.109 \times 10^{-31} \text{ kg}$,

$$p = \sqrt{2m_e K} = 9.96 \times 10^{-25} \text{ kg m s}^{-1}.$$

This kinetic energy is very small compared with the electron's rest energy, so the nonrelativistic expression is appropriate.

Step 3: Apply the de-Broglie relation With Planck's constant $h = 6.626 \times 10^{-34} \text{ J s}$,

$$\lambda = \frac{h}{p} = \frac{6.626 \times 10^{-34}}{9.96 \times 10^{-25}} = 6.65 \times 10^{-10} \text{ m}.$$

As a consistency check, Bohr's standing-wave condition is $2\pi r_n = n\lambda_n$. Since r_n is proportional to n^2 , the wavelength is proportional to n , so the first-excited-state wavelength is twice the ground-state value.

Step 4: Conclusion The wavelength in the first excited state is

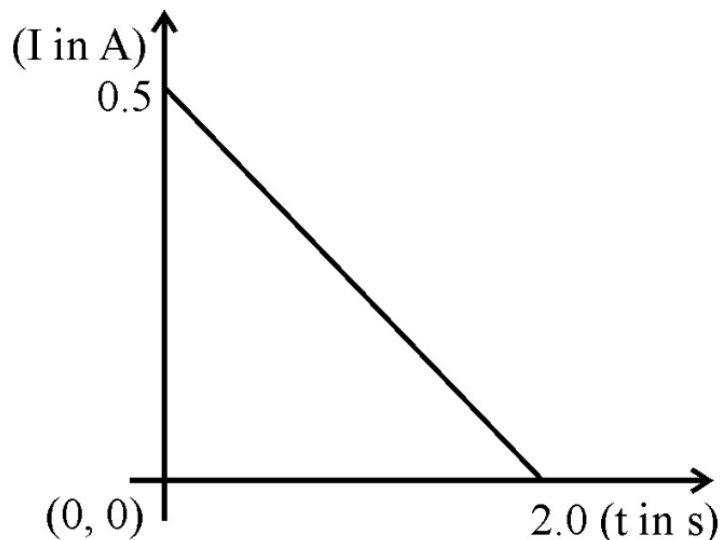
$$\lambda \simeq 6.65 \times 10^{-10} \text{ m} = 0.665 \text{ nm}.$$

Quick Tip to Solve: For a hydrogen orbit use $K = |E_n|$ in $\lambda = h/\sqrt{2m_e K}$; first excited means $n = 2$.

32. When a conducting loop of resistance $10\ \Omega$ and area $10\ \text{cm}^2$ is removed from an external magnetic field acting normally, the variation of induced current I in the loop with time is shown in the figure. Find the

- total charge passed through the loop.
- change in magnetic flux through the loop.
- magnitude of the field applied.

[3 marks]



Solution:

Concept:

- Charge passing through a loop is $Q = \int I dt$, the area under its current-time graph.
- Faraday's law gives $\varepsilon = -d\Phi/dt$. With constant resistance R , $I = \varepsilon/R$, so the magnitude of the flux change is $|\Delta\Phi| = RQ$ when the induced current keeps one direction.
- For a uniform magnetic field normal to a single loop of area A , $\Phi = BA$. Complete removal makes the final external flux zero.

Step 1: (a) Calculate the total charge The printed graph starts at $I = 0.5\text{ A}$ when $t = 0$, then decreases linearly to zero at $t = 2.0\text{ s}$. Current is positive throughout this interval, so total charge is the triangular area under the graph:

$$Q = \frac{1}{2} \times (2.0\text{ s}) \times (0.5\text{ A}) = 0.50\text{ C}.$$

Equivalently the average current is $(0.5 + 0)/2 = 0.25\text{ A}$, maintained for 2.0 s . Using the initial current throughout would incorrectly double the charge.

Step 2: (b) Relate the charge to the change in flux Integrating Faraday's law and Ohm's law over the removal interval gives

$$R \int I dt = - \int d\Phi = -\Delta\Phi.$$

Thus the magnitude of the flux change is

$$|\Delta\Phi| = RQ = (10\ \Omega)(0.50\text{ C}) = 5.0\text{ Wb}.$$

The loop loses its initial magnetic flux during removal. Taking the initial field direction as the positive area-normal direction, $\Phi_i > 0$ and $\Phi_f = 0$, so $\Delta\Phi = \Phi_f - \Phi_i$ is negative. This explicitly distinguishes

the signed change from its magnitude.

Step 3: (c) Calculate the applied magnetic field The field is normal to the loop, so no projection factor is needed. Convert its area before substituting:

$$A = 10 \text{ cm}^2 = 10 \times 10^{-4} \text{ m}^2 = 1.0 \times 10^{-3} \text{ m}^2.$$

Since the final flux is zero, $BA = |\Delta\Phi|$. Therefore

$$B = \frac{|\Delta\Phi|}{A} = \frac{5.0}{1.0 \times 10^{-3}} \text{ T}.$$

Squaring the centimetre-to-metre conversion is essential; cm^2 is an area unit, not a length unit.

Step 4: Conclusion (a) $Q = 0.50 \text{ C}$. (b) The flux decreases by 5.0 Wb , i.e. $\Delta\Phi = -5.0 \text{ Wb}$ for a normal along the initial field. (c) $B = 5.0 \times 10^3 \text{ T}$.

Quick Tip to Solve: Read the triangle's area first, then use $|\Delta\Phi| = RQ$ and $B = RQ/A$, converting cm^2 to m^2 .

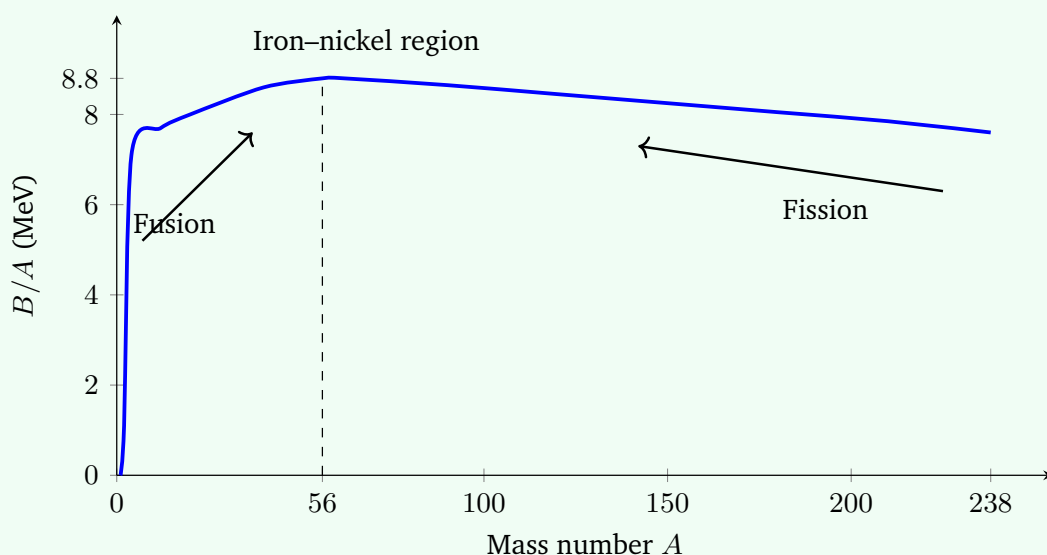
33. Draw the curve showing the variation of binding energy per nucleon with the mass number of nuclei. Using it explain the fusion of nuclei lying on ascending part and fission of nuclei lying on descending part of this curve. [3 marks]

Solution:

Concept:

- Binding energy B is the energy required to separate a nucleus into its constituent nucleons. Binding energy per nucleon, B/A , measures the average binding of each nucleon, where A is the mass number.
- A greater B/A generally means a more tightly bound nucleus. In an energy-releasing nuclear reaction, the total binding energy of the products exceeds that of the reactants; the increase appears as released energy, $Q = \Delta B = \Delta mc^2$.

Step 1: Draw and interpret the binding-energy curve Plot mass number A on the horizontal axis and binding energy per nucleon B/A , in MeV, on the vertical axis. The curve rises rapidly for light nuclei, has a broad maximum of about 8.8 MeV near the iron–nickel region ($A \approx 56$ –62), and then decreases slowly towards heavy nuclei. Thus nuclei in the middle of the curve are, on average, more tightly bound than very light or very heavy nuclei. The sketch shows the overall trend; small variations among individual light nuclei are not represented.



Schematic binding energy per nucleon curve; both processes move towards the broad maximum.

Step 2: Explain fusion on the ascending part When suitable light nuclei combine to form a heavier nucleus nearer the maximum, the product has a larger binding energy per nucleon. Since nucleon number is conserved, this means the product has a greater total binding energy than the initial nuclei. Its mass is correspondingly smaller than their combined initial mass, and the mass decrease is released as energy. For example, hydrogen isotopes can fuse to form a more tightly bound helium nucleus. Energy release follows the movement towards the maximum; it is not a claim that every possible fusion reaction releases energy.

Step 3: Explain fission on the descending part A sufficiently heavy nucleus can split into two medium-mass fragments, usually with some free neutrons. The fragments lie closer to the broad maximum and are more tightly bound per nucleon than the parent. The total binding energy therefore increases, with the decrease in total rest mass appearing mainly as kinetic energy of the fragments and neutrons. Thus fission of a heavy nucleus, such as uranium, can release energy because it produces nuclei nearer the maximum of the curve.

Step 4: Conclusion Both fusion of suitable light nuclei and fission of suitable heavy nuclei release energy by producing more tightly bound products closer to the peak of the binding-energy-per-nucleon curve.

Quick Tip to Solve: For either process, point the direction of change towards the peak: light nuclei move right by fusion and heavy nuclei move left by fission.

34. An optical instrument uses a lens of 100 D for the objective lens and 50 D for its eye piece. When the tube length is kept at 20 cm, the final image is formed at infinity.

(a) Identify the optical instrument.

(b) Calculate the magnification produced by the instrument.

[3 marks]

Solution:

Concept:

- Lens power and focal length are related by $P = 1/f$, with P in dioptres and f in metres. A compound microscope uses a short-focal-length objective to form a magnified real intermediate image and an eyepiece to magnify that image angularly.
- For normal adjustment (final image at infinity), the standard compound-microscope formula is $M \simeq (L/f_o)(D/f_e)$. Here L is the tube length used in the long-tube approximation, f_o and f_e are objective and eyepiece focal lengths, and $D = 25$ cm is the conventional least distance of distinct vision.

Step 1: (a) Identify the instrument using its focal lengths Convert the powers to focal lengths:

$$f_o = \frac{1}{100} \text{ m} = 1 \text{ cm},$$
$$f_e = \frac{1}{50} \text{ m} = 2 \text{ cm}.$$

Both lenses have short positive focal lengths, with the objective having the shorter focal length. This is the arrangement of a compound microscope. Its objective produces the real enlarged intermediate image of a nearby small object, which acts as the object for the eyepiece. For the stated final image at infinity, this intermediate image is at the first focal plane of the eyepiece, so the eye receives parallel rays.

Step 2: (b) Calculate the magnifying power in normal adjustment Use all lengths in centimetres. The objective's approximate linear magnification in magnitude is L/f_o , and the eyepiece's angular magnification for an image at infinity is D/f_e . Thus

$$|m_o| \simeq \frac{20}{1} = 20,$$
$$M_e = \frac{25}{2} = 12.5,$$
$$M \simeq 20 \times 12.5 = 250.$$

Magnifying power is a ratio and has no unit. The objective inverts the image, so a signed magnification may be written as approximately -250 ; the requested magnifying power is its magnitude. Since the final image is at infinity, the eyepiece factor is D/f_e , rather than $1 + D/f_e$, which applies to the final image at the near point. The value uses the usual CBSE long-tube approximation implicit in the stated tube length.

Step 3: Conclusion (a) The instrument is a compound microscope. (b) Its magnifying power in normal adjustment is approximately 250 ; the final image is inverted relative to the object.

Quick Tip to Solve: Convert dioptres to centimetres using $f(\text{cm}) = 100/P$, then use D/f_e when the final image is at infinity.

Section D (15 marks)

35. (a) Write two important characteristics of equipotential surfaces.

(b) A thin circular ring of radius r is charged uniformly so that its linear charge density becomes λ . Derive an expression for the electric field at a point P at a distance x from it along the axis of the ring. Hence, prove that at large distances ($x \gg r$), the ring behaves as a point charge. [5 marks]

Solution:

Concept:

- An equipotential surface has the same electric potential at every point; the work done in moving a test charge along it is $W = q\Delta V = 0$.
- The electrostatic field obeys $\vec{E} = -\nabla V$, so it is normal to an equipotential surface wherever the field is nonzero.
- Coulomb's law and superposition give $dE = (4\pi\epsilon_0)^{-1}dq/R^2$. A uniformly charged ring has total charge $Q = 2\pi r\lambda$, where r is its radius and λ its linear charge density.

Step 1: State the two characteristics in part (a) First, moving a charge between any two points on an equipotential surface requires no work against the electrostatic force, because their potential difference is zero. This statement applies to any path confined to that surface.

Second, electric field lines meet an equipotential surface at right angles. If the field had a tangential component, a small displacement along the surface would give $dV = -\vec{E} \cdot d\vec{l} \neq 0$, contradicting constant potential. At a point where the field vanishes, there is no definite field direction to specify.

Step 2: Set up the ring geometry for part (b) Let O be the ring's centre and let P lie on its axis with $OP = x$. For a small charge element $dq = \lambda dl$, its distance from P is the same as that of every other ring element:

$$R = \sqrt{r^2 + x^2}, \quad \cos \theta = \frac{x}{\sqrt{r^2 + x^2}}.$$

Here θ is the angle between the element's field at P and the axis. Take the positive axial direction from O towards P , with $x > 0$.

Step 3: Cancel transverse components and integrate the axial field Every element has a diametrically opposite partner. Their transverse field components at P cancel, while their axial components add. Thus the net field is purely axial. The signed axial component from dq is

$$dE_x = \frac{1}{4\pi\epsilon_0} \frac{dq}{r^2 + x^2} \cos \theta = \frac{1}{4\pi\epsilon_0} \frac{x dq}{(r^2 + x^2)^{3/2}}.$$

The geometrical factor is constant around the ring, so

$$E_x = \frac{1}{4\pi\epsilon_0} \frac{x}{(r^2 + x^2)^{3/2}} \oint dq = \frac{1}{4\pi\epsilon_0} \frac{2\pi r\lambda x}{(r^2 + x^2)^{3/2}}.$$

For positive charge the field points away from the centre on this side of the ring; negative charge reverses it. The formula also gives zero field at the centre, consistent with symmetry.

Step 4: Take the large-distance limit Write the result in terms of the ring's total charge, retaining the radius correction explicitly:

$$E_x = \frac{1}{4\pi\epsilon_0} \frac{Q}{x^2} \left(1 + \frac{r^2}{x^2}\right)^{-3/2}.$$

When $x \gg r$, the ratio r^2/x^2 is negligible and the bracket tends to unity. The remaining field is

precisely the inverse-square field of a point charge Q placed at O ; the finite size of the ring no longer affects the leading field.

Step 5: Conclusion Equipotential surfaces require no work for motion along them and are perpendicular to the nonzero electric field. The ring's axial field is

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{2\pi r \lambda x}{(r^2 + x^2)^{3/2}} \hat{x}.$$

For $x \gg r$, this becomes $\vec{E} \simeq (4\pi\epsilon_0)^{-1}(Q/x^2)\hat{x}$, with $Q = 2\pi r\lambda$, proving the point-charge behaviour.

Quick Tip to Solve: Pair diametrically opposite ring elements before integrating: only the axial component survives. Replace the ring by its total charge only after taking $x \gg r$.

OR

35. (a) State Gauss's law on electrostatics and derive an expression for the electric field due to a long straight thin uniformly charged wire (linear charge density λ) at a point lying at a distance r from the wire.

(b) The magnitude of electric field (in NC^{-1}) in a region varies with the distance r (in m) as

$$E = 10r + 5$$

By how much does the electric potential increase in moving from point at $r = 1$ m to a point at $r = 10$ m. [5 marks]

Solution:

Concept:

- Gauss's law states that the total electric flux through a closed surface is $\oint_S \vec{E} \cdot d\vec{A} = Q_{\text{enclosed}}/\epsilon_0$; $d\vec{A}$ points outward.
- For an effectively infinite uniform line charge, cylindrical symmetry makes the field radial, with a magnitude depending only on perpendicular distance from the line.
- Electric potential difference is $V_B - V_A = -\int_A^B \vec{E} \cdot d\vec{l}$. A field magnitude alone determines the size of the potential change along a field line; its sign also requires the field direction.

Step 1: State Gauss's law and choose the Gaussian surface In part (a), Gauss's law relates the electric flux through any closed surface to its net enclosed charge:

$$\oint_S \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enclosed}}}{\epsilon_0}.$$

Take a cylindrical Gaussian surface of radius r and length L , coaxial with the uniformly charged thin wire. A long wire is treated as infinite when the observation point is sufficiently far from its ends that end effects can be neglected. The enclosed charge is $Q_{\text{enclosed}} = \lambda L$.

Step 2: Evaluate the flux and obtain the wire's field Cylindrical symmetry gives a radial field of constant magnitude on the curved surface. For positive λ , it is parallel to the outward area vector there. On both circular ends, the area vector is parallel to the wire, whereas the field is perpendicular to the wire; hence the flux through the ends is zero. Gauss's law therefore gives

$$E(2\pi rL) + 0 + 0 = \frac{\lambda L}{\epsilon_0}, \quad E = \frac{\lambda}{2\pi\epsilon_0 r} \quad (\lambda > 0).$$

More generally, the vector expression is $\vec{E} = \lambda \hat{r} / (2\pi\epsilon_0 r)$, with $\hat{r}$ directed away from the wire. Its magnitude is $|\lambda| / (2\pi\epsilon_0 r)$. A negatively charged wire therefore produces an inward field. The cancellation of L shows that the chosen cylinder length cannot affect the answer.

Step 3: Integrate the field in part (b) The field in part (b) is a separate field specified by $E(r) = 10r + 5$, with r in metres and E in N C^{-1} . It is not the inverse-distance field derived in part (a). The magnitude of the potential change along the specified radial path is

$$\begin{aligned} |\Delta V| &= \int_1^{10} (10r + 5) dr \\ &= [5r^2 + 5r]_1^{10} \\ &= (500 + 50) - (5 + 5) \\ &= 540 \text{ V}. \end{aligned}$$

Integration of field in N C^{-1} over distance in metres gives $\text{N m C}^{-1} = \text{V}$.

Step 4: Interpret the printed word increase with the field sign Potential increases when moving against the electric field. Thus the increase requested in the paper is 540 V if the field is directed towards decreasing r , so that $\vec{E} \cdot d\vec{l} = -(10r + 5) dr$.

The question explicitly supplies only the magnitude, without specifying direction. If instead $\vec{E} = (10r + 5)\hat{r}$, the signed difference is

$$V(10) - V(1) = - \int_1^{10} (10r + 5) dr = -540 \text{ V},$$

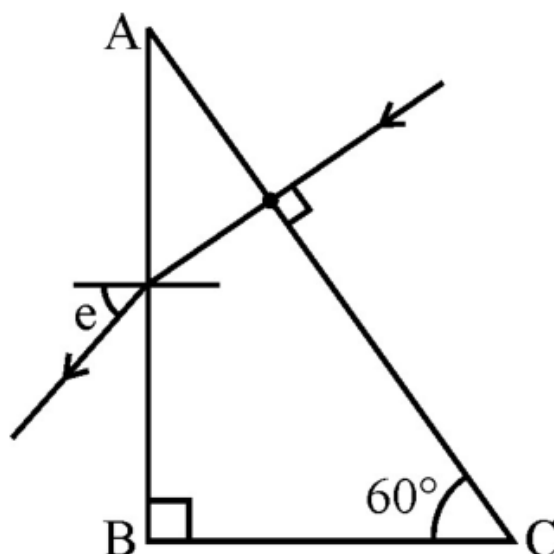
a decrease of 540 V. Consequently the magnitude is fixed, but a positive increase requires motion opposite to the field; the magnitude formula alone does not establish that direction.

Step 5: Conclusion Gauss's law gives $\vec{E} = \lambda \hat{r} / (2\pi\epsilon_0 r)$ for a long uniformly charged wire, neglecting end effects. In part (b), the potential change has magnitude 540 V: the requested increase is 540 V for motion against the field. For a field along increasing r , the signed change is -540 V ; the printed magnitude alone leaves this sign unspecified.

Quick Tip to Solve: Use a coaxial cylinder for the wire and discard end-cap flux. For the separate numerical field, integrate first, then use the field direction to decide whether potential rises or falls.

36. (a) Define the term ‘focal length of a mirror’. With the help of a ray diagram, obtain the relation between its focal length and radius of curvature.

(b) Calculate the angle of emergence (e) of the ray of light incident normally on the face AC of a glass prism ABC of refractive index $\sqrt{3}$. How will the angle of emergence change qualitatively, if the ray of light emerges from the prism into a liquid of refractive index 1.3 instead of air ? [5 marks]

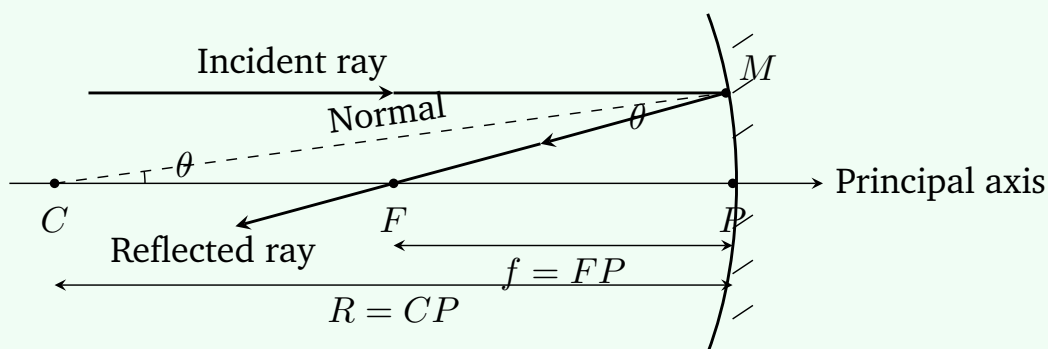


Solution:

Concept:

- The focal length of a spherical mirror is the distance between its pole P and principal focus F , measured along the principal axis. The principal focus of a concave mirror is the point where rays parallel and close to the principal axis converge after reflection.
- The radius drawn from the centre of curvature C to a point on a spherical mirror is normal to its surface. The law of reflection gives equal angles of incidence and reflection. The relation $f = R/2$ applies to paraxial rays of a mirror with a small aperture.
- At an interface, Snell's law is $n_1 \sin i = n_2 \sin r$, where each angle is measured from the normal and n_1, n_2 are the refractive indices of the incident and refracting media.

Step 1: (a) Construct the reflected ray and identify equal angles Consider a concave spherical mirror with pole P , centre of curvature C and principal focus F . Let a ray parallel to the principal axis strike the mirror at M , close to P . Join CM , the normal at M . The reflected ray meets the axis at F . If $\angle MCP = \theta$, the incident ray, being parallel to the axis, makes an angle θ with the normal. Therefore, by the law of reflection, $\angle CMF = \theta$. In triangle CMF , $\angle MCF = \angle CMF$, and consequently the opposite sides are equal: $CF = FM$.



Paraxial ray construction for a concave mirror; CM is the normal and MF is the reflected ray.

Step 2: (a) Apply the paraxial approximation For rays close to the axis, the height of M and its displacement from the pole are small, so $FM \simeq FP$. Thus $CF \simeq FP$. Since $CP = CF + FP$,

$$R \simeq 2f, \quad f = \frac{R}{2} \quad (\text{paraxial approximation}).$$

Here the distances in the geometrical construction are magnitudes. Under the Cartesian sign convention, both R and f are negative for this concave mirror, so the same signed relation holds. A convex mirror obeys the same relation using the virtual focus and centre behind the mirror. Rays far from the axis need not converge at this same point; the small-aperture assumption is essential.

Step 3: (b) Determine incidence at the emerging face From the printed prism, $\angle B = 90^\circ$ and $\angle C = 60^\circ$, so $\angle A = 30^\circ$. The ray is incident normally on face AC , hence its first angle of refraction is zero and it continues undeviated inside the glass. The normals to faces AC and AB make an acute angle of 30° . Therefore the internal angle of incidence at face AB is $i = 30^\circ$. The emergence angle e is measured from the horizontal normal to AB , as shown in the question.

Step 4: (b) Calculate emergence into air and compare with liquid For glass to air, Snell's law gives

$$\sqrt{3} \sin 30^\circ = \sin e = \frac{\sqrt{3}}{2}, \quad e = 60^\circ.$$

Since $\sin e < 1$, the ray emerges and there is no total internal reflection. If the outside medium is a liquid of refractive index 1.3, the internal direction and incidence angle remain unchanged, but

$$\sin e_{\text{liq}} = \frac{\sqrt{3} \sin 30^\circ}{1.3} = \frac{\sqrt{3}}{2.6} \simeq 0.6662.$$

Thus $e_{\text{liq}} \simeq 41.8^\circ$, which is smaller than 60° . Qualitatively, the reduced refractive-index contrast means less bending away from the normal, so the angle of emergence decreases.

Step 5: Conclusion (a) The focal length is the pole-to-principal-focus distance; for a spherical mirror and paraxial rays, $f = R/2$. (b) The angle of emergence into air is 60° . It decreases when the ray emerges into the liquid (approximately 41.8°).

Quick Tip to Solve: First find the prism's apex angle from its other two angles. Normal entry gives zero first refraction, so the incidence at the second face equals the apex angle.

36. (a) Define the term ‘resolving power of a telescope’. How will the resolving power be effected with the increase in

(i) Wavelength of light used.

(ii) Diameter of the objective lens.

Justify your answers.

(b) A screen is placed 80 cm from an object. The image of the object on the screen is formed by a convex lens placed between them at two different locations separated by a distance 20 cm. Determine the focal length of the lens. [5 marks]

Solution:

Concept:

- Resolving power is the reciprocal of the smallest angular separation $\theta_{\min}$ between two distant point objects whose images can just be distinguished.
- For a circular telescope objective of diameter D , Rayleigh’s criterion gives $\theta_{\min} = 1.22\lambda/D$, in radians, where λ is the wavelength. Thus resolving power is $D/(1.22\lambda)$.
- In the lens displacement method, a fixed object-screen separation S and a separation d between the two sharp-image lens positions give $f = (S^2 - d^2)/(4S)$.

Step 1: (a) Define resolving power and explain its wavelength dependence A telescope resolves two distant objects when their diffraction images can be distinguished as separate. At the limiting separation, the principal maximum of one diffraction pattern coincides with the first minimum of the other. For a circular objective,

$$\theta_{\min} = \frac{1.22\lambda}{D}, \quad \mathcal{R} = \frac{1}{\theta_{\min}} = \frac{D}{1.22\lambda}.$$

(i) At fixed objective diameter, increasing the wavelength increases the diffraction spread and hence increases the minimum angular separation. Therefore the resolving power decreases, in inverse proportion to λ . For example, doubling the wavelength halves the resolving power.

Step 2: (a) Explain the effect of objective diameter (ii) At fixed wavelength, increasing D makes the diffraction pattern narrower. Since $\theta_{\min} \propto 1/D$, the telescope can distinguish objects with smaller angular separation. Its resolving power therefore increases directly in proportion to the objective diameter. Doubling the diameter doubles $\mathcal{R}$. This concerns separation of close images, rather than simply enlargement of an image.

Step 3: (b) Relate the two lens positions to the conjugate distances Let a and b be the positive object-lens and lens-screen distances for one sharp image. Here $a + b = S = 80$ cm. With the Cartesian signs $u = -a$ and $v = b$, the thin-lens formula becomes

$$\frac{1}{f} = \frac{1}{b} + \frac{1}{a}, \quad f = \frac{ab}{a+b}.$$

The other lens position interchanges a and b , so its displacement is $d = |b - a| = 20$ cm. Taking $b > a$,

$$a = \frac{S - d}{2} = 30 \text{ cm}, \quad b = \frac{S + d}{2} = 50 \text{ cm}.$$

Step 4: (b) Substitute in the focal-length relation Multiplying the two conjugate distances gives

$ab = (S^2 - d^2)/4$. Thus

$$f = \frac{S^2 - d^2}{4S} = \frac{80^2 - 20^2}{4 \times 80} \text{ cm} = \frac{6000}{320} \text{ cm} = 18.75 \text{ cm}.$$

As a check, the distances 30 cm and 50 cm add to the given screen separation, and their difference is the given lens displacement. Also $S > 4f$, consistent with two distinct sharp-image positions for a convex lens.

Step 5: Conclusion (a) Resolving power is $1/\theta_{\min} = D/(1.22\lambda)$. It decreases when the wavelength increases and increases when the objective diameter increases. (b) The lens focal length is 18.75 cm.

Quick Tip to Solve: Use positive conjugate distances: their sum is the object-screen separation and their difference is the lens displacement; this gives the focal length without a sign error.

37. (a) Show that an ideal inductor does not dissipate power in an ac circuit.

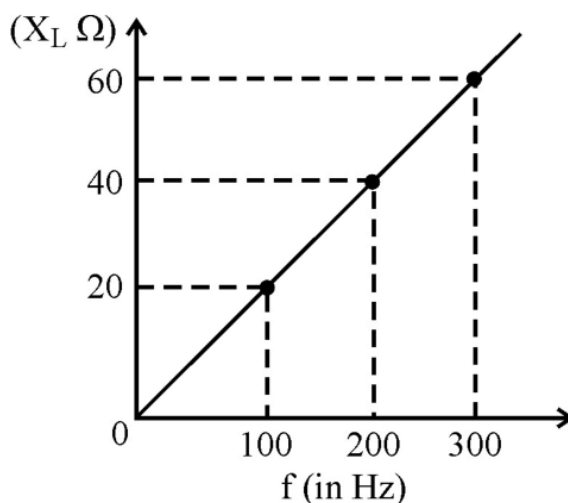
(b) The variation of inductive reactance (X_L) of an inductor with the frequency (f) of the ac source of 100 V and variable frequency is shown in the fig.

(i) Calculate the self-inductance of the inductor.

(ii) When this inductor is used in series with a capacitor of unknown value and a resistor of 10Ω at 300 s^{-1} , maximum power dissipation occurs in the circuit. Calculate the capacitance of the capacitor.

[5 marks]

the fig.



Solution:

Concept:

- For an ideal inductor, $v = L di/dt$. The current lags the voltage by $\pi/2$, so its average power is $V_{\text{rms}} I_{\text{rms}} \cos(\pi/2) = 0$.
- Inductive reactance is $X_L = \omega L = 2\pi f L$, where f is frequency in hertz. The slope of an X_L -versus-

f graph is $2\pi L$.

- In a series RLC circuit, $Z^2 = R^2 + (\omega L - 1/(\omega C))^2$. For fixed rms voltage, average power $P = V_{\text{rms}}^2 R/Z^2$ is maximum at resonance, where $X_L = X_C$.

Step 1: (a) Obtain the phase relation for an ideal inductor Write the applied voltage as $v = V_m \sin \omega t$, with V_m its peak value. For steady sinusoidal current through an ideal inductor,

$$L \frac{di}{dt} = V_m \sin \omega t, \quad i = -\frac{V_m}{\omega L} \cos \omega t = I_m \sin(\omega t - \pi/2),$$

where $I_m = V_m/(\omega L)$. Thus the current lags the applied voltage by a quarter of a cycle. The ideal inductor has no resistance; its magnetic field stores energy during part of the cycle and returns it to the source during another part.

Step 2: (a) Average the instantaneous power over a cycle The instantaneous power delivered to the inductor is

$$p(t) = vi = -V_m I_m \sin \omega t \cos \omega t = -\frac{V_m I_m}{2} \sin 2\omega t.$$

Over a full period $T = 2\pi/\omega$, the positive and negative portions cancel:

$$P_{\text{av}} = \frac{1}{T} \int_0^T p(t) dt = 0.$$

Equivalently, $P_{\text{av}} = V_{\text{rms}} I_{\text{rms}} \cos \phi$ with $\phi = \pi/2$. Hence an ideal inductor dissipates no average power, even though instantaneous energy exchange occurs. A real coil can dissipate power in its winding resistance, which is excluded here.

Step 3: (b)(i) Read the graph and calculate the self-inductance The plotted straight line passes through $(f, X_L) = (100 \text{ Hz}, 20 \Omega)$, $(200 \text{ Hz}, 40 \Omega)$, and $(300 \text{ Hz}, 60 \Omega)$. Using the last point,

$$L = \frac{X_L}{2\pi f} = \frac{60}{2\pi \times 300} \text{ H} = \frac{1}{10\pi} \text{ H} \simeq 0.0318 \text{ H}.$$

The other points yield the same value, confirming that the graph slope represents constant self-inductance. The source voltage does not enter this determination.

Step 4: (b)(ii) Use maximum power to identify resonance With a fixed source voltage and $R = 10 \Omega$, the series circuit dissipates power only in the resistor. The expression

$$P = \frac{V_{\text{rms}}^2 R}{R^2 + (X_L - X_C)^2}$$

is largest when the denominator is smallest, namely when $X_L - X_C = 0$. Therefore the specified maximum occurs at series resonance. The stated 300 s^{-1} is the frequency $f = 300 \text{ Hz}$, consistently with the horizontal axis of the graph; consequently $\omega = 2\pi f = 600\pi \text{ rad s}^{-1}$. At this frequency the graph gives $X_L = 60 \Omega$.

Step 5: (b)(ii) Calculate the resonant capacitance At resonance $X_C = 1/(2\pi fC) = X_L$. Hence

$$C = \frac{1}{2\pi f X_L} = \frac{1}{2\pi \times 300 \times 60} \text{ F} = 8.84 \times 10^{-6} \text{ F} = 8.84 \mu\text{F}.$$

The same value follows from $C = 1/(\omega^2 L)$. The given resistance and source voltage set the power attained at resonance; the resonance capacitance itself depends on the inductance and the specified frequency.

Step 6: Conclusion (a) The ideal inductor's cycle-average power is zero.

(b)(i) The self-inductance is $L = 0.0318 \text{ H}$, or 31.8 mH.

(b)(ii) Maximum power indicates series resonance, giving $C \simeq 8.84 \mu\text{F}$.

Quick Tip to Solve: At the resonant frequency read reactance directly from the graph and use $C = 1/(2\pi f X_L)$; keep ordinary frequency distinct from angular frequency.

OR

37. (a) A conductor of length l is rotated about one of its ends at a constant angular speed ω in a plane perpendicular to a uniform magnetic field B . Plot graphs to show variations of the emf induced across the ends of the conductor with (i) angular speed ω and (ii) length of the conductor l .

(b) Two concentric circular loops of radius 1 cm and 20 cm are placed coaxially.

(i) Find mutual inductance of the arrangement.

(ii) If the current passed through the outer loop is changed at a rate of 5 A/ms, find the emf induced in the inner loop. Assume the magnetic field on the inner loop to be uniform. [5 marks]

Solution:

Concept:

- Motional emf across a short element of length dr moving perpendicular to a uniform field is $d\varepsilon = Bv dr$, where v is its speed and B is the magnetic flux density.
- For rotation about one end with angular speed ω , an element at distance r from the pivot has speed $v = \omega r$.
- Mutual inductance M relates the flux through the inner loop to the current I in the outer loop: $\Phi = MI$. Faraday's law gives $\varepsilon = -M dI/dt$.

Step 1: (a) Derive the emf of the rotating conductor Take an element dr at distance r from the fixed end. Its velocity is tangential to the circular path and is perpendicular to the magnetic field. Therefore the motional emf magnitude of this element is $B\omega r dr$. Integrating from the pivot to the free end gives

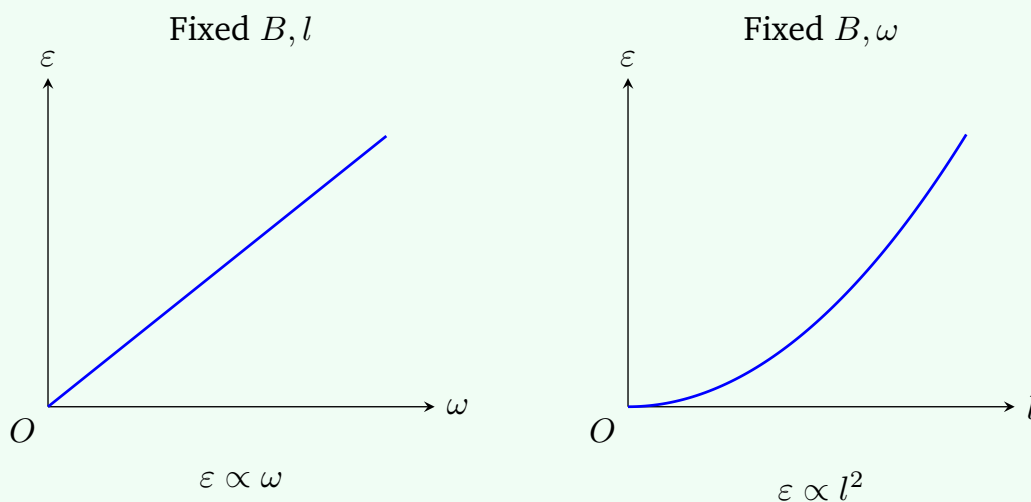
$$\varepsilon = \int_0^l B\omega r dr = \frac{1}{2} B\omega l^2.$$

Here ε denotes the magnitude across the two ends; the polarity depends on the directions of the magnetic field and rotation. The factor 1/2 appears because the speed rises from zero at the pivot to

ωl at the free end; the whole rod does not move at its tip speed.

Step 2: (a)(i) Plot emf against angular speed Keep B and l fixed. Then $\varepsilon = (Bl^2/2)\omega$, so the graph is a straight line through the origin, with positive constant slope $Bl^2/2$. Doubling the angular speed doubles the emf magnitude. The first panel below shows the required dependence for non-negative angular speed.

Step 3: (a)(ii) Plot emf against conductor length Keep B and ω fixed. Then $\varepsilon = (B\omega/2)l^2$, so the graph is an upward-opening parabola through the origin in the physical region $l \geq 0$. Its initial slope is zero, and its slope increases with length. Doubling the rod length makes the induced emf four times as large. The second panel shows this quadratic dependence.



Emf magnitude versus angular speed (fixed B, l) and length (fixed B, ω); axes show qualitative dependence.

Step 4: (b)(i) Find the mutual inductance Let the inner radius be $a = 1 \text{ cm} = 0.01 \text{ m}$, and the outer radius be $R = 20 \text{ cm} = 0.20 \text{ m}$. For current I in the single outer circular loop, the field near its centre is $B_c = \mu_0 I / (2R)$. Using the stated uniform-field approximation across the small inner loop, its flux is

$$\Phi = B_c \pi a^2 = \frac{\mu_0 I \pi a^2}{2R}.$$

Hence the mutual inductance magnitude is

$$\begin{aligned} M &= \frac{\Phi}{I} = \frac{\mu_0 \pi a^2}{2R} \\ &= \frac{(4\pi \times 10^{-7})\pi(0.01)^2}{2(0.20)} \text{ H} \\ &= \pi^2 \times 10^{-10} \text{ H}. \end{aligned}$$

Both loops are single turns, and the linked area is the area of the smaller loop. The small ratio $a/R = 0.05$ is consistent with using the centre field over that area.

Step 5: (b)(ii) Use the current-change rate to find the induced emf Convert the given rate carefully:

$5 \text{ A/ms} = 5 \times 10^3 \text{ A/s}$. Apply Faraday's law to the inner loop:

$$|\varepsilon_{\text{inner}}| = M \left| \frac{dI}{dt} \right| = (\pi^2 \times 10^{-10})(5 \times 10^3) \text{ V} = 5\pi^2 \times 10^{-7} \text{ V}.$$

Lenz's law fixes the polarity: the induced current, if the inner loop is closed, produces a field opposing the change in the outer loop's flux. Since no current or winding orientation is specified, the numerical result is the magnitude.

Step 6: Conclusion The rotating rod has $\varepsilon = \frac{1}{2}B\omega l^2$: the ε - ω graph is a straight line through the origin, and the ε - l graph is an upward-opening parabola. The mutual inductance is $M \approx 9.87 \times 10^{-10} \text{ H}$, and the induced emf magnitude is $4.93 \times 10^{-6} \text{ V} = 4.93 \mu\text{V}$.

Quick Tip to Solve: Integrate the rod's varying speed; for the loops use the outer radius in the centre-field formula and the inner radius in the flux area, then convert A/ms to A/s.