

CBSE Class 12 Physics Question Paper 2020 with Solution PDF-Set 55/5/3



Question Paper with Solutions

Conducted by the Central Board of Secondary Education (CBSE)

Time Allowed: 3 hours

Maximum Marks: 70

General Instructions

- (i) This question paper comprises **four** sections – A, B, C and D.
- (ii) There are **37** questions in the question paper. **All** questions are compulsory.
- (iii) Section A : Q. no. **1 to 20** are very short-answer type questions carrying **1** mark each.
- (iv) Section B : Q. no. **21 to 27** are short-answer type questions carrying **2** marks each.
- (v) Section C : Q. no. **28 to 34** are long-answer type questions carrying **3** marks each.
- (vi) Section D : Q. no. **35 to 37** are also long answer type questions carrying **5** marks each.
- (vii) There is no overall choice in the question paper. However, an internal choice has been provided in **two** questions of **one** mark, **two** questions of **two** marks, **one** question of **three** marks and all the **three** questions of **five** marks. You have to attempt **only one** of the choices in such questions.
- (viii) However, separate instructions are given with each section and question, wherever necessary.
- (ix) Use of calculators and log tables is not permitted.

You may use the following values of physical constants wherever necessary.

$$c = 3 \times 10^8 \text{ m/s}$$

$$h = 6.63 \times 10^{-34} \text{ Js}$$

$$e = 1.6 \times 10^{-19} \text{ C}$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ T m A}^{-1}$$

$$\epsilon_0 = 8.854 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$$

$$\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$$

$$\text{Mass of electron } (m_e) = 9.1 \times 10^{-31} \text{ kg}$$

$$\text{Mass of neutron} = 1.675 \times 10^{-27} \text{ kg}$$

$$\text{Mass of proton} = 1.673 \times 10^{-27} \text{ kg}$$

$$\text{Avogadro's number} = 6.023 \times 10^{23} \text{ per gram mole}$$

$$\text{Boltzmann constant} = 1.38 \times 10^{-23} \text{ JK}^{-1}$$

Section A (20 marks)

Note : Select the most appropriate option from those given below each question.

1. A biconcave lens of power P vertically splits into two identical plano concave parts. The power of each part will be [1 mark]

- (A) $2P$
- (B) $P/2$
- (C) P
- (D) $P/\sqrt{2}$

Correct Answer: (B) $P/2$

Solution:

Concept:

- Lens power follows the lens-maker formula $P = (n - 1)(1/R_1 - 1/R_2)$, where n is the refractive index relative to the surroundings.
- A plane surface has infinite radius and hence zero curvature.

Step 1: Express the original power For the identical curved surfaces of a symmetric biconcave lens, take $R_1 = -R$ and $R_2 = +R$. Thus $P = -2(n - 1)/R$. The negative sign indicates that the lens is diverging.

Step 2: Evaluate one plano-concave part Each identical part retains one concave surface and gains a plane surface. Its power is therefore $P_{\text{part}} = -(n - 1)/R = P/2$. Doubling the power, keeping it unchanged or dividing it by $\sqrt{2}$ would not account for removing one of the two equal curvature contributions.

Step 3: Conclusion The power of each part is $P/2$. Hence, the correct option is (B).

Quick Tip to Solve: For identical plano-concave halves, retain one of the two equal curvature contributions to lens power.

2. The power factor of a series LCR circuit at resonance will be:

[1 mark]

- (A) 1
- (B) 0
- (C) $1/2$
- (D) $1/\sqrt{2}$

Correct Answer: (A) 1

Solution:

Concept:

- The power factor is $\cos \phi = R/Z$, where R is resistance and Z is impedance.
- At resonance, inductive reactance $X_L = \omega L$ equals capacitive reactance $X_C = 1/(\omega C)$.

Step 1: Apply the resonance condition The net reactance is $X_L - X_C = 0$. Consequently, the series impedance is $Z = \sqrt{R^2 + (X_L - X_C)^2} = R$, so voltage and current are in phase.

Step 2: Determine the power factor Therefore $\cos \phi = R/R = 1$. A zero power factor would require

purely reactive behaviour; the fractional values $1/2$ and $1/\sqrt{2}$ would imply a nonzero phase angle. None describes this resistive resonant circuit.

Step 3: Conclusion The power factor at resonance is unity. Hence, the correct option is (A).

Quick Tip to Solve: Cancel the equal inductive and capacitive reactances at resonance before calculating the phase angle.

3. If photons of frequency ν are incident on the surfaces of metals A & B of threshold frequencies $\nu/2$ and $\nu/3$ respectively, the ratio of the maximum kinetic energy of electrons emitted from A to that from B is [1 mark]

- (A) 2 : 3
- (B) 3 : 4
- (C) 1 : 3
- (D) $\sqrt{3} : \sqrt{2}$

Correct Answer: (B) 3 : 4

Solution:

Concept:

- Einstein's photoelectric equation is $K_{\max} = h(\nu - \nu_0)$, where h is Planck's constant and ν_0 is threshold frequency.
- The work function is $h\nu_0$; subtract it from the same incident photon energy for each metal.

Step 1: Calculate the two kinetic energies For metal A , $K_A = h(\nu - \nu/2) = h\nu/2$. For metal B , $K_B = h(\nu - \nu/3) = 2h\nu/3$. Both are positive because the incident frequency exceeds both thresholds.

Step 2: Form the required ratio

$$\frac{K_A}{K_B} = \frac{h\nu/2}{2h\nu/3} = \frac{3}{4}.$$

Thus the ratio is neither the threshold ratio 2 : 3, nor 1 : 3. The printed square-root ratio in option (D) does not follow from the kinetic-energy equation.

Step 3: Conclusion The maximum kinetic-energy ratio is 3 : 4. Hence, the correct option is (B).

Quick Tip to Solve: Subtract each threshold from the incident frequency before taking the ratio; threshold ratios are not energy ratios.

4. The electric flux through a closed Gaussian surface depends upon

[1 mark]

- (A) Net charge enclosed and permittivity of the medium
- (B) Net charge enclosed, permittivity of the medium and the size of the Gaussian surface
- (C) Net charge enclosed only
- (D) Permittivity of the medium only

Correct Answer: (A) Net charge enclosed and permittivity of the medium

Solution:

Concept:

- Gauss's law gives the net electric flux $\Phi_E = \oint \mathbf{E} \cdot d\mathbf{A} = Q_{\text{enc}}/\varepsilon$ in the stated uniform medium.
- Here Q_{enc} is algebraic net enclosed charge and ε is the permittivity of the medium.

Step 1: Identify the quantities in Gauss's law The expression $Q_{\text{enc}}/\varepsilon$ contains both enclosed net charge and medium permittivity. Changing either while holding the other fixed changes the electric flux through the closed surface.

Step 2: Check the surface-size alternative For the same enclosed charge and medium, enlarging or deforming the Gaussian surface does not change its net flux. Thus option (B) wrongly includes size. Options (C) and (D) each omit one relevant quantity; neither gives the complete dependence asked here.

Step 3: Conclusion Flux depends on net enclosed charge and the permittivity of the medium. Hence, the correct option is (A).

Quick Tip to Solve: Write the complete Gauss-law fraction first: surface size never appears in the expression for net flux.

5. A charge particle after being accelerated through a potential difference V enters in a uniform magnetic field and moves in a circle of radius r . If V is doubled, the radius of the circle will become

[1 mark]

- (A) $2r$
- (B) $\sqrt{2}r$
- (C) $4r$
- (D) $r/\sqrt{2}$

Correct Answer: (B) $\sqrt{2}r$

Solution:

Concept:

- Acceleration from rest through potential difference V gives $\frac{1}{2}mv^2 = |q|V$.
- For circular motion perpendicular to a uniform magnetic field B , $r = mv/(|q|B)$.

Step 1: Relate radius to accelerating voltage Combining the energy and magnetic-force relations gives

$$r = \frac{\sqrt{2m|q|V}}{|q|B}.$$

For the same particle and magnetic field, mass, charge magnitude and field strength remain fixed. Hence r varies as $\sqrt{V}$.

Step 2: Double the potential difference With V replaced by $2V$, the new radius satisfies $r'/r = \sqrt{2V/V} = \sqrt{2}$. Radius therefore increases by $\sqrt{2}$, not by 2 or 4; a decrease to $r/\sqrt{2}$ would contradict the increased speed.

Step 3: Conclusion The new radius is $\sqrt{2}r$. Hence, the correct option is (B).

Quick Tip to Solve: Acceleration voltage determines kinetic energy, so speed and magnetic radius scale with its square root.

6. The wavelength and intensity of light emitted by a LED depend upon

[1 mark]

- (A) forward bias and energy gap of the semiconductor
- (B) energy gap of the semiconductor and reverse bias
- (C) energy gap only
- (D) forward bias only

Correct Answer: (A) forward bias and energy gap of the semiconductor

Solution:

Concept:

- In a forward-biased LED, electron-hole recombination releases photons with energy approximately equal to the semiconductor band gap E_g .
- The photon relation $E_g \simeq hc/\lambda$ determines wavelength, while forward current controls the recombination rate and light intensity.

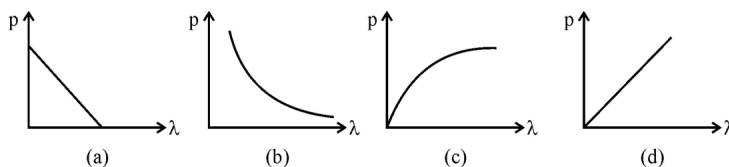
Step 1: Identify the wavelength dependence The semiconductor energy gap fixes the characteristic emitted photon energy. Since $\lambda \simeq hc/E_g$, a larger band gap corresponds to a shorter wavelength. Forward bias alone cannot specify the emission colour.

Step 2: Identify the intensity dependence Forward bias allows carrier injection; the resulting forward current changes the number of recombinations per second and therefore the light intensity. Ordinary reverse bias does not provide the intended LED emission. Energy gap alone omits intensity control, while forward bias alone omits wavelength control. Option (A) includes both required factors.

Step 3: Conclusion The relevant factors are forward bias and the semiconductor energy gap. Hence, the correct option is (A).

Quick Tip to Solve: Associate LED colour with band gap and brightness with forward current; choose the option containing both.

7. The graph showing the correct variation of linear momentum (p) of a charge particle with its de-Broglie wavelength (λ) is — [1 mark]



- (A) (a)
(B) (b)
(C) (c)
(D) (d)

Correct Answer: (B) (b)

Solution:

Concept:

- The de Broglie relation is $\lambda = h/p$, where p is momentum magnitude and h is Planck's constant.
- Thus $p = h/\lambda$ and the product $p\lambda$ remains constant.

Step 1: Find the shape and trend As wavelength increases, momentum decreases inversely. The curve is a rectangular hyperbola in the positive quadrant; it approaches the coordinate axes without reaching them at finite positive values.

Step 2: Compare all four printed graphs Graph (b) shows this decreasing curved dependence. Graph (a) is a straight line with constant negative slope and incorrectly reaches zero momentum at a finite wavelength. Graphs (c) and (d) increase with wavelength, contradicting $p = h/\lambda$. Only graph (b) has the required inverse variation.

Step 3: Conclusion The correct graph is (b). Hence, the correct option is (B).

Quick Tip to Solve: Check that doubling wavelength halves momentum; a decreasing straight line does not satisfy this test.

8. The selectivity of a series LCR a.c. circuit is large, when [1 mark]

- (A) L is large and R is large
(B) L is small and R is small
(C) L is large and R is small
(D) $L = R$

Correct Answer: (C) L is large and R is small

Solution:

Concept:

- Selectivity describes how sharply a series LCR circuit responds around its resonant angular frequency ω_0 .
- The resonance bandwidth is $\Delta\omega = R/L$, and the quality factor is $Q = \omega_0 L/R$.

Step 1: Connect selectivity to bandwidth A selective circuit admits a narrow range of frequencies around resonance. Decreasing R/L narrows the bandwidth, producing a sharper resonance curve.

Large resistance instead broadens the resonance.

Step 2: Choose the required parameter combination A large inductance together with a small resistance makes R/L small, giving high selectivity. Making both large or both small does not guarantee a small ratio. The statement $L = R$ is not an appropriate physical criterion because inductance and resistance have different units.

Step 3: Conclusion Selectivity is large when L is large and R is small. Hence, the correct option is (C).

Quick Tip to Solve: Use bandwidth R/L : increasing L and decreasing R together make the resonance narrower.

9. Photo diodes are used to detect

[1 mark]

- (A) radio waves
- (B) gamma rays
- (C) IR rays
- (D) optical signals

Correct Answer: (D) optical signals

Solution:

Concept:

- A photodiode converts incident light into photocurrent.
- It is normally operated in reverse bias; incident photons generate electron-hole pairs in and near the depletion region.

Step 1: Identify the detection mechanism Absorption of incident light produces mobile carriers, which the junction field separates. The reverse photocurrent therefore follows variations in the optical input and provides an electrical representation of a light signal.

Step 2: Check all options Radio waves are normally detected using antennas, and gamma rays require suitable radiation detectors. Selected photodiodes may respond to infrared wavelengths, but IR describes only a restricted spectral range. Optical signals is the general textbook use asked here, including optical communication and light detection.

Step 3: Conclusion Photodiodes detect optical signals. The correct option is (D).

Quick Tip to Solve: Associate reverse-biased photodiodes with light converted into electrical signals.

10. The relationship between Brewster angle θ and the speed of light v in the denser medium is

[1 mark]

(A) $v \tan \theta = c$

(B) $c \tan \theta = v$

(C) $v \sin \theta = c$

(D) $c \sin \theta = v$

Correct Answer: (A) $v \tan \theta = c$

Solution:

Concept:

- Brewster's law is $\tan \theta = \mu$ for incidence from air.
- The refractive index is $\mu = c/v$, where v is the speed in the denser medium.

Step 1: Apply Brewster law At the polarising angle the reflected beam is completely plane-polarised. Substituting the speed definition of refractive index gives

$$\tan \theta = \frac{c}{v}.$$

The ratio exceeds unity for a denser medium.

Step 2: Test the alternatives Multiplication by v yields $v \tan \theta = c$. Option (B) reverses the refractive-index ratio and would require a tangent less than unity. Option (C) requires $\sin \theta = c/v > 1$, which is impossible. Option (D) uses a sine relation appropriate to a different condition, whereas Brewster law specifically requires tangent.

Step 3: Conclusion The relation is $v \tan \theta = c$, option (A).

Quick Tip to Solve: Write tangent equals refractive index first, then insert the speed ratio.

11. The ability of a junction diode to _____ an alternating voltage, is based on the fact that it allows current to pass only when it is forward biased.

[1 mark]

Solution:

Concept:

- A junction diode conducts in forward bias and offers very high resistance in reverse bias.
- Rectification converts an alternating input into a unidirectional output.

Step 1: Follow the first half-cycle During the half-cycle that forward biases the diode, the junction barrier is reduced and an appreciable current flows through the load. The applied alternating voltage therefore gives a conducting interval.

Step 2: Follow the opposite half-cycle The other half-cycle reverse biases the diode, so load current is negligible in the ideal approximation. Current through the load thus flows only in one direction despite the alternating polarity of the input. The resulting output is pulsating and unidirectional. This process is rectification; because the blank follows the word "to", the corresponding verb is required.

Step 3: Conclusion The missing word is rectify.

Quick Tip to Solve: Use rectify after "to"; rectification is the name of the conversion process.

12. A point charge is placed at the centre of a hollow conducting sphere of internal radius r and outer radius $2r$. The ratio of the surface charge density of the inner surface to that of the outer surface will be _____ . [1 mark]

Solution:

Concept:

- The electric field within conducting material is zero in electrostatic equilibrium.
- Surface charge density is $\sigma = Q/A$, with spherical area $A = 4\pi R^2$.

Step 1: Find the induced charges Let the point charge be q . A Gaussian surface inside the metal encloses zero net charge, so the inner surface carries $-q$. For the initially neutral sphere implicit in this standard question the outer surface carries $+q$. Central placement makes the induced densities uniform by spherical symmetry.

Step 2: Compare densities The radii give

$$\sigma_{\text{in}} = \frac{-q}{4\pi r^2}, \quad \sigma_{\text{out}} = \frac{q}{16\pi r^2}.$$

The outer surface has four times the area, while the charges have equal magnitudes and opposite signs. Hence the signed ratio is -4 ; retaining the sign distinguishes density from its magnitude.

Step 3: Conclusion The ratio is $\boxed{-4 : 1}$; the magnitude ratio is $4 : 1$.

Quick Tip to Solve: Square the radii when comparing areas and retain the opposite induced-charge signs.

13. The _____ , a property of materials C, Si and Ge depends upon the energy gap between their conduction and valence bands. [1 mark]

Solution:

Concept:

- The band gap E_g separates the valence and conduction bands.
- Electrical conductivity depends on the density of mobile electrons and holes.

Step 1: Connect gap and excitation Thermal energy can excite electrons from the valence band into the conduction band, leaving holes behind. Electrons in the conduction band and holes in the valence band can respond to an applied electric field and contribute to electrical conduction.

Step 2: Identify the property At the same temperature, a smaller forbidden energy gap allows more thermally generated carriers and generally greater conductivity. The larger gap of carbon in diamond form makes it an insulator, while silicon and germanium are semiconductors. Thus the blank refers to the material property that measures its ability to conduct electric current.

Step 3: Conclusion The missing property is electrical conductivity.

Quick Tip to Solve: Connect the band gap to mobile carrier concentration and then to conductivity.

14. A copper wire of non-uniform area of cross-section is connected to a d.c. battery. The physical quantity which remains constant along the wire is _____ . [1 mark]

Solution:

Concept:

- In steady direct current, charge does not accumulate continuously in any wire segment.
- The current is $I = neAv_d$, where A is cross-sectional area and v_d is drift speed.

Step 1: Apply charge conservation Any difference between current entering and leaving a segment would cause continuing accumulation of charge. In steady state the surface charges adjust the local electric field so that the rate of charge flow is identical at all cross-sections. The current therefore has a single constant value.

Step 2: Distinguish other quantities Because A varies, the current density $J = I/A$ varies. For copper, carrier density is approximately uniform, so $v_d = I/(neA)$ also varies inversely with area. The electric field $E = \rho J$ changes as well. These local variations allow the same current to pass through both narrow and broad portions.

Step 3: Conclusion The constant quantity is electric current.

Quick Tip to Solve: Conserve current in a steady series wire; a smaller area increases current density.

15. The physical quantity having SI unit NC^{-1}m is _____ . [1 mark]

Solution:

Concept:

- Electric field has unit NC^{-1} .
- Electric potential is work per unit charge, $V = W/q$, measured in JC^{-1} .

Step 1: Simplify the specified unit Using the SI definition of work, $1\text{ J} = 1\text{ N m}$, the unit reduces to

$$\text{NC}^{-1}\text{m} = \frac{\text{J}}{\text{C}} = \text{V}.$$

Thus the extra length factor changes electric-field units into potential units.

Step 2: Identify the quantity Work done per unit charge is electric potential or potential difference. This agrees with $\Delta V = -\int \vec{E} \cdot d\vec{l}$, whose unit is electric field multiplied by length. Electric field alone would lack the metre, while potential energy has unit joule without division by charge.

Step 3: Conclusion The quantity is electric potential (or potential difference).

Quick Tip to Solve: Combine newton times metre into joule, then recognise joule per coulomb as volt.

16. Depict the fields diagram of an electromagnetic wave propagating along positive X -axis with its electric field along Y -axis. [1 mark]

Solution:

Concept:

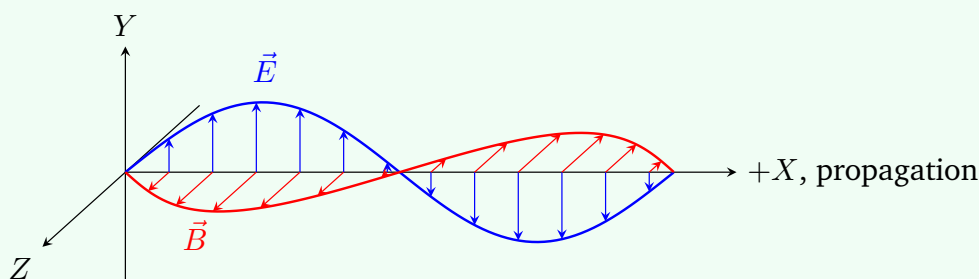
- An electromagnetic wave is transverse: $\vec{E}$, $\vec{B}$ and the propagation direction are mutually perpendicular.
- The propagation direction is along $\vec{E} \times \vec{B}$; the two fields oscillate in phase.

Step 1: Choose the magnetic-field axis For propagation along positive X and electric field along Y , the magnetic field lies along Z . When $\vec{E}$ points along $+Y$, $\vec{B}$ points along $+Z$, because $\hat{y} \times \hat{z} = \hat{x}$.

Step 2: Represent the fields At a fixed instant, both fields have sinusoidal variation along X :

$$E_y = E_0 \sin(kx - \omega t), \quad B_z = B_0 \sin(kx - \omega t).$$

Their maxima and zeros coincide, and $E_0 = cB_0$ in vacuum. The sketch uses separately scaled amplitudes.



Mutually perpendicular, in-phase electric and magnetic fields; amplitudes separately scaled.

Step 3: Conclusion The required field diagram has $\vec{E} \parallel Y$, $\vec{B} \parallel Z$, and propagation along $+X$.

Quick Tip to Solve: Use the right-hand rule on $\vec{E} \times \vec{B}$ to fix the magnetic-field direction.

17. Write the conditions on path difference under which (i) constructive (ii) destructive interference occur in Young's double slit experiment. [1 mark]

Solution:

Concept:

- For coherent waves of wavelength λ , a path difference Δ gives phase difference $\phi = 2\pi\Delta/\lambda$.
- In Young's experiment the two slits act as coherent sources with the same initial phase.

Step 1: Constructive interference The two disturbances arrive in phase when $\phi = 2n\pi$. Their displacements reinforce, producing a bright fringe. Thus the path difference is an integral multiple of the wavelength: $\Delta = n\lambda$.

Step 2: Destructive interference The disturbances arrive in opposite phase when $\phi = (2n + 1)\pi$. For

equal amplitudes they cancel, producing a dark fringe. Thus $\Delta = (2n + 1)\lambda/2$, an odd multiple of half a wavelength. Here n is an integer; for nonnegative path-difference magnitudes use $n = 0, 1, 2, \dots$

Step 3: Conclusion (i) Constructive: $\Delta = n\lambda$. (ii) Destructive: $\Delta = (2n + 1)\lambda/2$.

Quick Tip to Solve: Whole wavelengths give bright fringes; odd half-wavelengths give dark fringes.

18. Plot a graph showing variation of induced e.m.f. with the rate of change of current flowing through a coil. [1 mark]

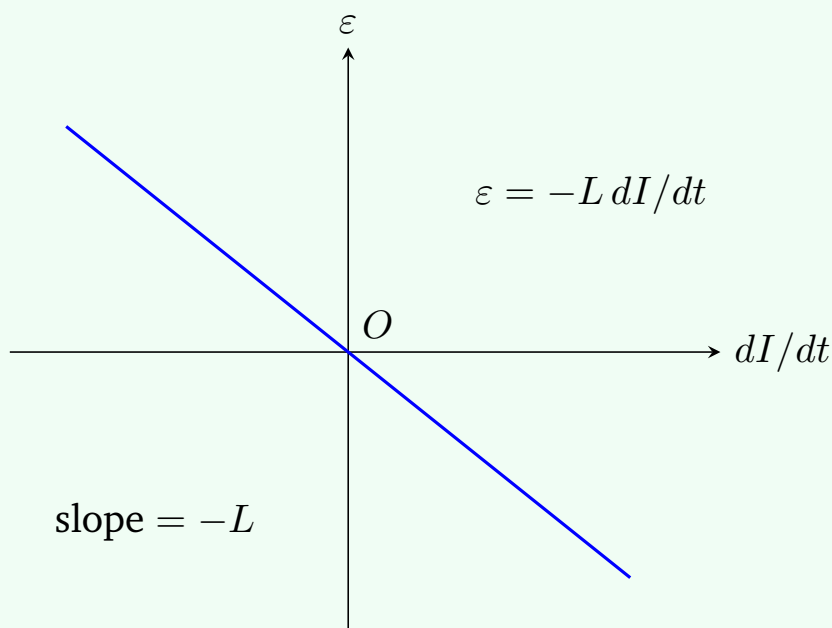
Solution:

Concept:

- Self-induction produces an emf $\varepsilon = -L dI/dt$, where L is the constant self-inductance of the coil.
- The minus sign expresses Lenz's law: the induced emf opposes the change in current.

Step 1: Identify the relation Put $u = dI/dt$ on the horizontal axis and the signed induced emf ε on the vertical axis. Then $\varepsilon = -Lu$, which is a linear relation with no constant term.

Step 2: Plot the line The graph passes through the origin and has slope $-L$. For positive dI/dt , the induced emf is negative; for negative dI/dt , it is positive. If only magnitudes are plotted instead, $|\varepsilon| = L|dI/dt|$ gives a positive-slope line in the first quadrant. The figure explicitly plots signed quantities.



Signed induced emf versus rate of change of current.

Step 3: Conclusion The signed emf versus dI/dt graph is a straight line through the origin with gradient $-L$.

Quick Tip to Solve: Check the axis convention: signed emf has negative slope, while magnitudes have positive slope.

18. A series combination of an inductor (L), capacitor (C) and a resistor (R) is connected across an ac source of emf of peak value E_0 and angular frequency (ω). Plot a graph to show variation of impedance of the circuit with angular frequency (ω). [1 mark]

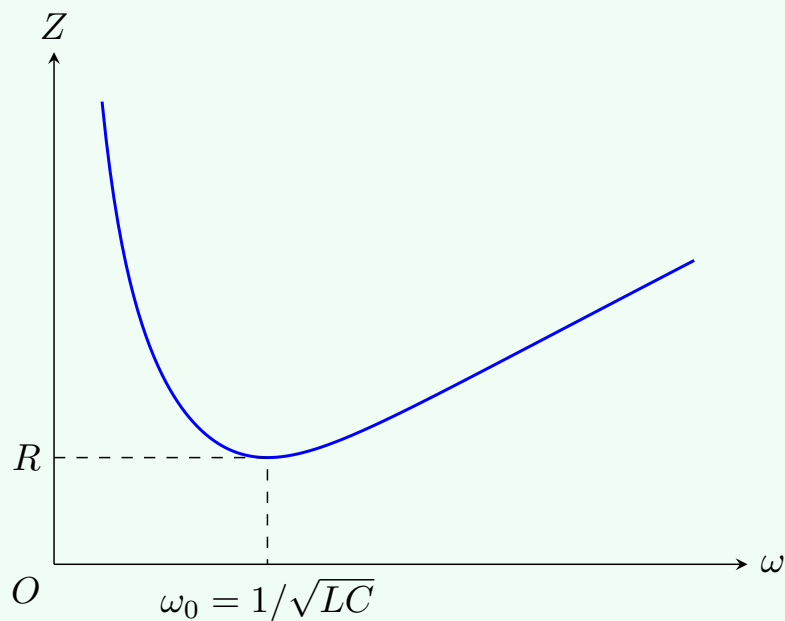
Solution:

Concept:

- The series LCR impedance is $Z = \sqrt{R^2 + (\omega L - 1/(\omega C))^2}$, where R, L, C are positive circuit constants.
- At resonance, $\omega_0 = 1/\sqrt{LC}$, the inductive and capacitive reactances cancel.

Step 1: Locate the minimum At $\omega = \omega_0$, $\omega_0 L = 1/(\omega_0 C)$, so the reactance term vanishes and the impedance is minimum: $Z_{\min} = R$. The source peak emf E_0 does not affect this impedance-frequency relation.

Step 2: Sketch the frequency dependence As $\omega \rightarrow 0^+$, the capacitive reactance becomes very large and $Z \rightarrow \infty$. Increasing frequency reduces Z to R at resonance. Beyond resonance the inductive reactance dominates and Z rises; as $\omega \rightarrow \infty$, $Z \sim \omega L$. The minimum is rounded for nonzero R .



Impedance minimum at the resonance angular frequency.

Step 3: Conclusion The graph falls from large impedance to $Z = R$ at $\omega_0 = 1/\sqrt{LC}$, and then rises.

Quick Tip to Solve: Mark the minimum above the frequency axis at R , not at zero impedance.

19. Define the term 'current sensitivity' of a moving coil galvanometer.

[1 mark]

Solution:

Concept:

- Current sensitivity is the angular deflection produced per unit current through the galvanometer coil.
- In a radial field, magnetic torque is $NIAB$; restoring torque is $k\theta$.

Step 1: State the definition If a current I produces equilibrium angular deflection θ , the current sensitivity is $S_I = \theta/I$. Its SI unit is rad A^{-1} . It describes how much the pointer turns for a given small current.

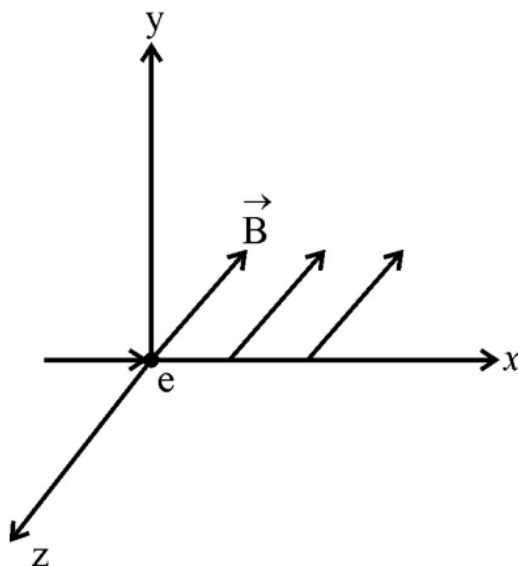
Step 2: Relate it to the instrument At equilibrium, $NIAB = k\theta$, giving $S_I = NAB/k$. Here N is the number of turns, A the coil area, B the magnetic field, and k the torsional constant of the suspension. Higher NAB or smaller k increases the deflection per unit current; this is distinct from deflection per unit voltage.

Step 3: Conclusion $S_I = \theta/I = NAB/k$: angular deflection per unit current in the moving coil galvanometer.

Quick Tip to Solve: For current sensitivity divide deflection by current; use voltage only for voltage sensitivity.

20. An electron moves along $+x$ direction. It enters into a region of uniform magnetic field B directed along $-z$ direction as shown in fig. Draw the shape of trajectory followed by the electron after entering the field.

[1 mark]



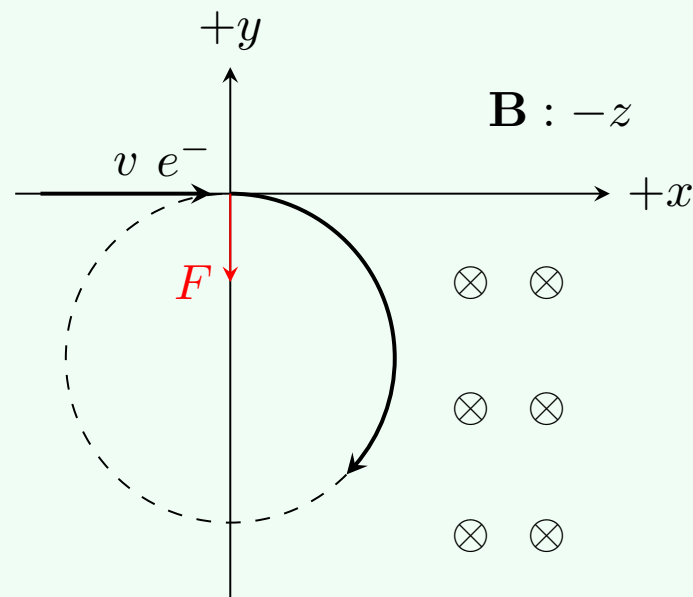
Solution:

Concept:

- The magnetic force is $\mathbf{F} = q(\mathbf{v} \times \mathbf{B})$; an electron has charge $q = -e$.
- A force perpendicular to velocity changes its direction, not speed, and acts as the centripetal force.

Step 1: Determine the initial bending Here $\mathbf{v}$ is along $+x$ and $\mathbf{B}$ along $-z$. Thus $\mathbf{v} \times \mathbf{B}$ points along $+y$. Multiplication by the negative electron charge reverses this direction, so its initial force is along $-y$.

Step 2: Find the shape and plane Since the velocity is perpendicular to the magnetic field, the electron has no motion parallel to the field. It follows a circular arc in the xy -plane, initially curving downwards. In the uniform field, $evB = mv^2/r$, giving $r = mv/(eB)$. The speed stays constant because the magnetic force does no work.



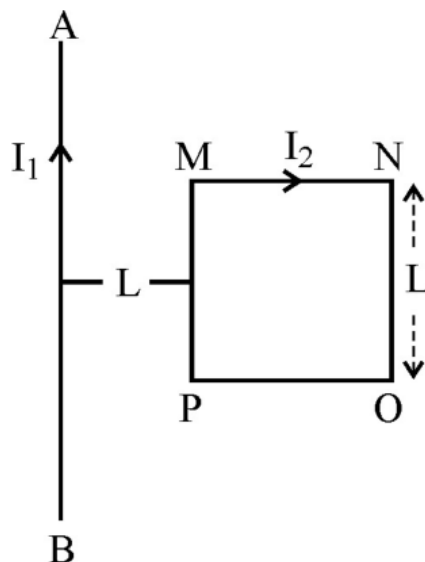
Electron path in the xy -plane; the field is into the page.

Step 3: Conclusion The trajectory is circular in the xy -plane, bending towards $-y$ on entry (clockwise when viewed from $+z$).

Quick Tip to Solve: Apply the right-hand rule first, then reverse the force direction for an electron.

OR

20. A square shaped current carrying loop $MNOP$ is placed near a straight long current carrying wire AB as shown in the fig. The wire and the loop lie in the same plane. If the loop experiences a net force F towards the wire, find the magnitude of the force on the side NO of the loop. [1 mark]



Solution:**Concept:**

- The field at distance r from the long wire is $B = \mu_0 I_1 / (2\pi r)$.
- For a parallel side of length L , $F_{\text{side}} = I_2 L B$; parallel currents attract and antiparallel currents repel.

Step 1: Compare the forces on the vertical sides The side MP is at distance L , with current upwards, parallel to I_1 . It is attracted with force $F_{MP} = \mu_0 I_1 I_2 / (2\pi)$. Side NO is at distance $2L$, with downward current, and is repelled with $F_{NO} = \mu_0 I_1 I_2 / (4\pi)$. Hence $F_{MP} = 2F_{NO}$.

Step 2: Use the given net force The horizontal sides MN and OP have opposite currents and equal field distributions; their equal, opposite vertical forces cancel. Therefore the force towards the wire is $F = F_{MP} - F_{NO} = 2F_{NO} - F_{NO} = F_{NO}$.

Step 3: Conclusion The magnitude of force on side NO is $[F]$, directed away from the straight wire.

Quick Tip to Solve: When the far side is twice as far from the wire, its force is half the near-side force.

Section B (14 marks)

21. Write shortcomings of Rutherford atomic model. Explain how these were overcome by the postulates of Bohr's atomic model. [2 marks]

Solution:**Concept:**

- An orbiting electron is an accelerated charge; classical electrodynamics predicts continuous radiation.
- Bohr's allowed stationary states have definite energies and angular momentum $mvr = nh/(2\pi)$, where $n = 1, 2, \dots$

Step 1: Shortcomings of the Rutherford model An electron revolving around the nucleus has centripetal acceleration. According to classical theory it should continuously radiate energy, lose orbital energy, and spiral into the nucleus. Thus Rutherford's model cannot explain the observed stability of atoms. Also, a continuously changing orbit would produce continuously changing frequencies, so the model cannot explain the discrete line spectrum of hydrogen or specify its allowed energy levels.

Step 2: Bohr postulates resolving these shortcomings Bohr postulated that electrons can occupy only certain stationary orbits and do not radiate while remaining in one of them. These allowed orbits satisfy $mvr = nh/(2\pi)$ and have discrete energies E_n ; the lowest allowed state prevents continuous collapse. Radiation is emitted or absorbed only when an electron changes between allowed states, with $h\nu = |E_i - E_f|$. Discrete energy differences therefore yield discrete spectral lines. These are new postulates, rather than consequences of classical radiation theory.

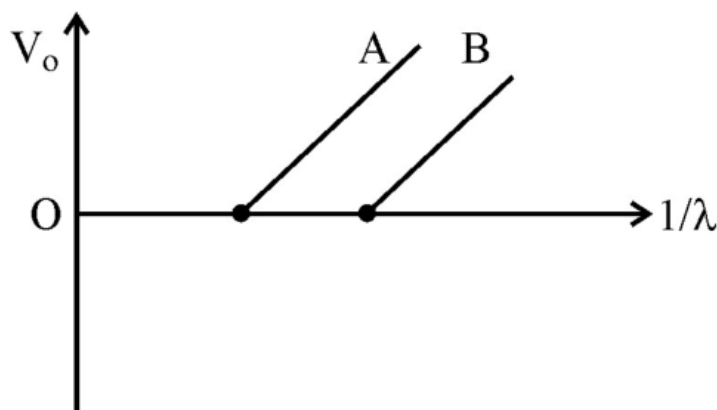
Step 3: Conclusion Stationary non-radiating states explain atomic stability; transitions between quantised energy levels explain the hydrogen line spectrum.

Quick Tip to Solve: Pair each Rutherford difficulty with its remedy: stability with stationary states, spectra with energy-level transitions.

22. Figure shows the stopping potential (V_0) for the photo electron versus $(1/\lambda)$ graph, for two metals A and B, λ being the wavelength of incident light.

(a) How is the value of Planck's constant determined from the graph ?

(b) If the distance between the light source and the surface of metal A is increased, how will the stopping potential for the electrons emitted from it be effected ? Justify your answer. [2 marks]



Solution:

Concept:

- Einstein's photoelectric equation is $eV_0 = hc/\lambda - \phi$, with ϕ the work function.
- The stopping potential measures maximum photoelectron kinetic energy: $K_{\max} = eV_0$.

Step 1: Determine Planck's constant from the slope Put $x = 1/\lambda$. Then $V_0 = (hc/e)x - \phi/e$, which has the straight-line form $y = mx + b$. Take two points well separated on either metal's line and calculate $m = \Delta V_0 / \Delta(1/\lambda)$. Thus

$$h = \frac{e}{c} \frac{\Delta V_0}{\Delta(1/\lambda)}.$$

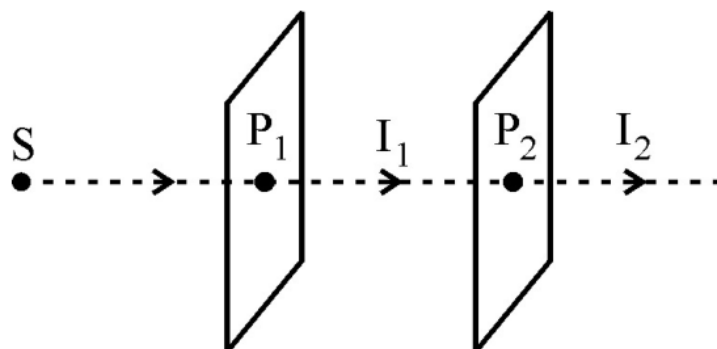
Both metals give the same slope because h , c and e are universal constants. Their different work functions change the intercepts, not this slope. Use inverse wavelength in m^{-1} to obtain h in J s.

Step 2: Effect of increasing the source distance Increasing distance reduces incident intensity, so fewer photons reach unit area per second and the photoelectric current decreases. However, the wavelength and therefore each photon's energy hc/λ stay the same. Metal A's work function also stays fixed. Hence the maximum kinetic energy, and the stopping potential needed to stop the fastest electrons, remain unchanged.

Step 3: Conclusion (a) $h = (e/c) \Delta V_0 / \Delta(1/\lambda)$. (b) The stopping potential for metal A remains unchanged at fixed wavelength.

Quick Tip to Solve: Read the horizontal axis before using a slope: for inverse wavelength the slope is hc/e , not h/e .

23. Light from a sodium lamp S passes through two polaroid sheets P_1 and P_2 as shown in fig. What will be the effect on the intensity of the light transmitted (i) by P_1 and (ii) by P_2 on rotating polaroid P_1 about the direction of propagation of light? Justify your answer in both cases. [2 marks]



Solution:

Concept:

- For unpolarised incident light of intensity I_0 , an ideal first polaroid transmits $I_1 = I_0/2$.
- Malus's law gives $I_2 = I_1 \cos^2 \theta$, where θ is the angle between the transmission axes.

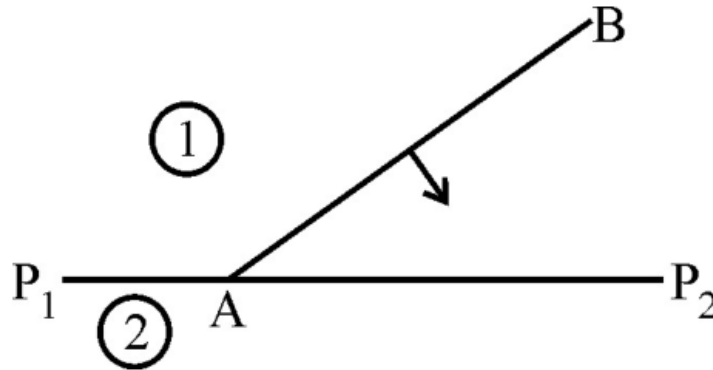
Step 1: Intensity transmitted by the rotating first polaroid The sodium lamp supplies unpolarised light: its electric-field directions are distributed equally in the transverse plane. Rotating P_1 about the light's direction changes which transverse component it selects, but the mean squared component is unchanged. Thus $I_1 = I_0/2$ stays constant throughout the rotation. The direction of polarisation emerging from P_1 , however, rotates along with its transmission axis.

Step 2: Intensity transmitted by the fixed second polaroid The light incident on P_2 is plane polarised along the changing axis of P_1 . Since P_2 remains fixed, rotation of P_1 changes their relative angle θ . Only the electric-field component along P_2 passes, so $I_2 = (I_0/2) \cos^2 \theta$. It is maximum when the axes are parallel and zero when they are perpendicular. A full rotation produces repeated maxima and minima; it does not cause a monotonic fall in intensity.

Step 3: Conclusion (i) The intensity through P_1 remains constant. (ii) The intensity through P_2 varies according to $I_2 = (I_0/2) \cos^2 \theta$.

Quick Tip to Solve: Rotating the first polaroid leaves its unpolarised-light throughput constant but changes the relative axis angle.

23. Define the term 'wave front of light'. A plane wave front AB propagating from denser medium (1) into a rarer medium (2) is incident on the surface P_1P_2 separating the two media as shown in fig. Using Huygen's principle, draw the secondary wavelets and obtain the refracted wave front in the diagram. [2 marks]



Solution:

Concept:

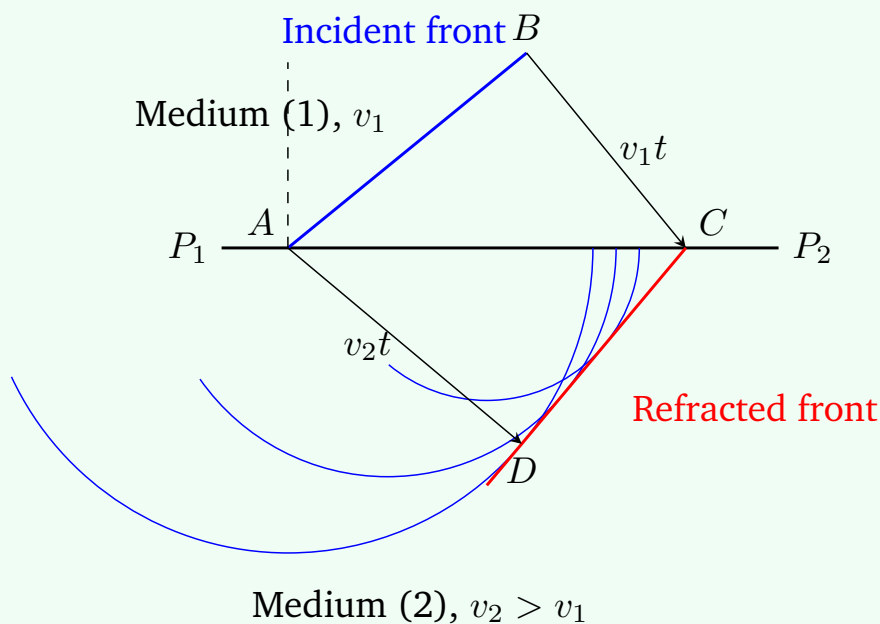
- A wavefront is a surface joining points of a wave that are in the same phase at a given instant.
- By Huygens' principle every point on a wavefront emits secondary wavelets; their common forward tangent is the new wavefront. Light has speeds v_1 and v_2 in media (1) and (2), respectively, with $v_2 > v_1$.

Step 1: Follow the last point of the incident front Let A be the first point of the incident plane wavefront AB to reach the interface. While the disturbance from B travels along its incident ray to C on the interface in time t , it covers $BC = v_1 t$. The ray BC is perpendicular to AB . During this same interval, the secondary wavelet from A expands into medium (2) with radius $AD = v_2 t$.

Step 2: Draw the common tangent in medium (2) Draw the forward semicircular wavelet centred at A below the interface, of radius $v_2 t$, and the wavelets from intervening interface points with their appropriate shorter travel times. The common tangent CD from C touches the wavelet from A at D , so $AD \perp CD$. It is the refracted plane wavefront. With incidence angle i and refraction angle r ,

$$\frac{\sin i}{\sin r} = \frac{BC/AC}{AD/AC} = \frac{v_1}{v_2}.$$

Therefore $r > i$, for refraction to the rarer medium, provided the incidence angle is below the critical angle as required by this construction.

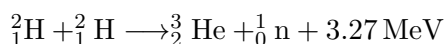


Huygens construction: secondary wavelets and refracted wavefront CD

Step 3: Conclusion The required refracted wavefront is CD , the forward common tangent to the secondary wavelets in medium (2); the refracted rays are perpendicular to CD and bend away from the normal.

Quick Tip to Solve: Use the same time interval for both media: draw $BC = v_1 t$, then the larger radius $AD = v_2 t$, and finally its tangent from C .

24. Calculate for how many years will the fusion of 2.0 kg deuterium keep 800 W electric lamp glowing. Take the fusion reaction as



[2 marks]

Solution:

Concept:

- Each deuterium nucleus has molar mass approximately 2.0 g mol^{-1} . The number of nuclei is $N = (m/M)N_A$, where $N_A = 6.02 \times 10^{23} \text{ mol}^{-1}$.
- Each fusion reaction consumes two deuterium nuclei and releases 3.27 MeV. Energy and operating time satisfy $E = Pt$, with P the lamp power.

Step 1: Count the fusion reactions The available deuterium mass is $2.0 \text{ kg} = 2000 \text{ g}$. Its amount and the number of pairwise fusion reactions are

$$n = \frac{2000}{2.0} = 1000 \text{ mol}, \quad N_{\text{reaction}} = \frac{1000(6.02 \times 10^{23})}{2} = 3.01 \times 10^{26}.$$

The factor of two is essential: the reaction consumes a pair of deuterium nuclei, rather than one nucleus.

Step 2: Convert the energy and divide by lamp power Using $1 \text{ MeV} = 1.6 \times 10^{-13} \text{ J}$, the energy released per reaction is

$$Q = 3.27(1.6 \times 10^{-13}) = 5.232 \times 10^{-13} \text{ J}.$$

Assuming complete fusion and conversion of the released energy to electrical energy,

$$E = (3.01 \times 10^{26})(5.232 \times 10^{-13}) = 1.575 \times 10^{14} \text{ J.}$$

Hence

$$t = \frac{1.575 \times 10^{14}}{800} = 1.969 \times 10^{11} \text{ s.}$$

Taking one year as $365 \times 24 \times 3600 = 3.1536 \times 10^7 \text{ s}$, convert seconds to years.

Step 3: Conclusion The lamp can glow for

$$\frac{1.969 \times 10^{11}}{3.1536 \times 10^7} \simeq 6.24 \times 10^3 \text{ years.}$$

This is the ideal duration with complete energy conversion.

Quick Tip to Solve: Divide the number of deuterium nuclei by two before multiplying by the reaction energy, then use joules divided by watts to obtain seconds.

25. In a single slit diffraction experiment, the width of the slit is decreased. How will the (i) size (ii) intensity of the central bright band be affected. Justify your answer. [2 marks]

Solution:

Concept:

- For a slit of width a , the first diffraction minima satisfy $a \sin \theta = \lambda$, where λ is the wavelength.
- The central maximum extends between the first minima on its two sides. Its width on a distant screen is $W = 2D\lambda/a$, where D is the slit-screen distance and small angles are assumed.

Step 1: Determine the change in central-band size The printed question decreases the slit width. For fixed wavelength and screen distance,

$$W = \frac{2D\lambda}{a}.$$

Decreasing a increases W ; equivalently, $\sin \theta = \lambda/a$ increases and the first minima move farther from the central direction. Thus the central bright band spreads over a greater angular and linear width.

Step 2: Determine the change in central intensity At the centre, all secondary wavelets from the slit arrive in phase. With unchanged incident illumination and screen geometry, the resultant amplitude is proportional to the slit width, because a narrower slit admits a smaller contributing wavefront. Since intensity is proportional to the square of amplitude,

$$I_{\text{central}} \propto a^2.$$

Thus a decrease in slit width reduces the central peak intensity. The wider pattern does not imply a brighter centre: the transmitted light decreases and is spread over a broader pattern.

Step 3: Conclusion (i) The central bright band becomes wider. (ii) Its central intensity decreases, for the same incident illumination, wavelength and screen distance.

Quick Tip to Solve: Remember the opposite dependences: central width varies as $1/a$, while central peak intensity varies as a^2 under fixed illumination.

26. Derive the expression for the torque acting on an electric dipole, when it is held in a uniform electric field. Identify the orientation of the dipole in the electric field, in which it attains a stable equilibrium. [2 marks]

Solution:

Concept:

- An electric dipole consists of charges $+q$ and $-q$ separated by $2l$. Its moment is $\vec{p} = q(2\vec{l})$, directed from the negative charge to the positive charge.
- In a uniform electric field $\vec{E}$, the equal opposite forces on the charges form a couple. The potential energy is $U = -\vec{p} \cdot \vec{E}$.

Step 1: Calculate the moment of the force couple Let the dipole moment make angle θ with the uniform field. The positive charge experiences $+q\vec{E}$ and the negative charge experiences $-q\vec{E}$. Their resultant force is zero, but their lines of action have perpendicular separation $2l \sin \theta$. Therefore the magnitude of the torque is

$$\tau = qE(2l \sin \theta) = pE \sin \theta.$$

Equivalently, each force has lever arm $l \sin \theta$ about the midpoint, so the two moments add. The vector expression is $\vec{\tau} = \vec{p} \times \vec{E}$, whose direction is given by the right-hand rule.

Step 2: Identify the stable equilibrium Torque vanishes for $\theta = 0$ and $\theta = \pi$. The potential energy

$$U = -pE \cos \theta$$

has its minimum $-pE$ at $\theta = 0$, where $\vec{p}$ is parallel to $\vec{E}$. A small displacement from this position produces a restoring torque that tends to realign the dipole with the field. At $\theta = \pi$, energy is a maximum and a slight displacement makes the dipole turn farther away from that position, so this equilibrium is unstable.

Step 3: Conclusion The torque is $\vec{\tau} = \vec{p} \times \vec{E}$, with magnitude $\tau = pE \sin \theta$. Stable equilibrium occurs when $\vec{p}$ is parallel to $\vec{E}$, at $\theta = 0$.

Quick Tip to Solve: Zero torque alone does not establish stability: compare $U = -pE \cos \theta$ at the two zero-torque orientations and select its minimum.

OR

26. Obtain the expression for the energy stored in a capacitor connected across a dc battery. Hence define energy density of the capacitor. [2 marks]

Solution:

Concept:

- A capacitor of capacitance C carrying charge q has instantaneous potential difference $v = q/C$.
- Electrostatic energy equals the work done in transferring charge between the plates. Energy density is energy per unit volume of the electric field.

Step 1: Calculate the charging work At an intermediate stage of charging, the charge is q , so transferring a further small charge dq requires work $dW = v dq = (q/C) dq$. Adding the work from

zero charge to the final charge Q ,

$$U = \int_0^Q \frac{q}{C} dq = \frac{Q^2}{2C}.$$

The voltage rises during charging, which is why the stored energy is half of QV .

Step 2: Express the energy and its density For a battery voltage V , the final charge is $Q = CV$, giving

$$U = \frac{1}{2}CV^2 = \frac{1}{2}QV.$$

For a parallel-plate capacitor in vacuum, with plate area A , separation d , and negligible fringing, $C = \epsilon_0 A/d$ and $E = V/d$. The field occupies volume Ad , so

$$u = \frac{U}{Ad} = \frac{\frac{1}{2}(\epsilon_0 A/d)V^2}{Ad} = \frac{1}{2}\epsilon_0 E^2.$$

Its SI unit is J m^{-3} .

Step 3: Conclusion The stored energy is $U = \frac{1}{2}CV^2 = \frac{Q^2}{2C}$. Energy density means energy stored per unit volume; in vacuum, $u = \frac{1}{2}\epsilon_0 E^2$.

Quick Tip to Solve: Integrate the changing capacitor voltage q/C ; using the final voltage throughout doubles the stored energy.

27. Gamma rays and radio waves travel with the same velocity in free space. Distinguish between them in terms of their origin and the main application. [2 marks]

Solution:

Concept:

- All electromagnetic waves travel at $c = 1/\sqrt{\mu_0\epsilon_0}$ in vacuum, independent of their frequency.
- Origin and application distinguish different regions of the electromagnetic spectrum.

Step 1: Gamma rays: origin and application Gamma rays originate in transitions of an excited atomic nucleus to lower nuclear energy levels, often following radioactive decay. They have very high photon energies and strong penetrating power. A main application is radiotherapy: a controlled gamma-ray beam is used to destroy cancerous cells. Their nuclear origin distinguishes them from radio waves produced by oscillating charges in an aerial.

Step 2: Radio waves: origin and application Radio waves originate from accelerated or oscillating electric charges, for example alternating currents in a transmitting antenna. A main application is radio and television communication: information is carried by modulating a radio-frequency wave and recovered at a receiver. Their much lower frequency means a much lower photon energy than gamma rays, although their free-space speed is identical.

Step 3: Conclusion Gamma rays: nuclear transitions; cancer treatment by radiotherapy. Radio waves: oscillating charges in antennas; radio and television communication. Both travel at c in free space.

Quick Tip to Solve: Associate gamma radiation with nuclear energy changes and radio radiation with alternating antenna currents.

Section C (21 marks)

28. (a) Define the term half-life of a radioactive substance.

(b) The half life of ${}_{92}^{238}\text{U}$ undergoing α decay is 4.5×10^9 years. Calculate the activity of 5 g sample of ${}_{92}^{238}\text{U}$. [3 marks]

Solution:

Concept:

- Half-life $T_{1/2}$ is the time required for the number of undecayed radioactive nuclei in a sample to become half its initial value.
- Activity $A = \lambda N$, where $\lambda = \ln 2/T_{1/2}$ is the decay constant and N is the number of undecayed nuclei.
- For mass m and molar mass M , $N = (m/M)N_A$, where $N_A = 6.022 \times 10^{23} \text{ mol}^{-1}$.

Step 1: (a) Define half-life The half-life of a radioactive substance is the time interval in which half the initially present radioactive nuclei decay. Thus, if the initial number is N_0 , then $N = N_0/2$ after one half-life. Since activity is proportional to the undecayed population, the activity also falls to half its initial value in that same time.

Step 2: (b) Find the number of uranium nuclei The printed sample mass is 5 g. Taking the molar mass of uranium-238 as 238 g mol^{-1} ,

$$N = \frac{5}{238} (6.022 \times 10^{23}) = 1.265 \times 10^{22}.$$

The sample is taken to consist of uranium-238 nuclei; the alpha-decay mode does not change the formula for the parent activity.

Step 3: Convert half-life and calculate activity Using $1 \text{ year} = 365 \times 24 \times 3600 \text{ s}$,

$$\begin{aligned} T_{1/2} &= 4.5 \times 10^9 (3.1536 \times 10^7) \\ &= 1.41912 \times 10^{17} \text{ s}, \\ \lambda &= \frac{0.693}{1.41912 \times 10^{17}} \\ &= 4.883 \times 10^{-18} \text{ s}^{-1}. \end{aligned}$$

Therefore the instantaneous decay rate is

$$A = \lambda N = (4.883 \times 10^{-18})(1.265 \times 10^{22}) = 6.18 \times 10^4 \text{ s}^{-1}.$$

One decay per second is one becquerel; this is the sample activity, not the number of nuclei present.

Step 4: Conclusion Half-life is the time for half the radioactive nuclei to decay. The activity of the 5 g uranium-238 sample is approximately $6.2 \times 10^4 \text{ Bq}$.

Quick Tip to Solve: Convert the half-life from years to seconds before using $A = (\ln 2/T_{1/2})(m/M)N_A$.

29. Explain the formation of potential barrier and depletion region in a p-n junction diode. What is effect of applying forward bias on the width of depletion region ? [3 marks]

Solution:

Concept:

- Carrier diffusion across a p-n junction follows the concentration gradient: electrons diffuse from n to p, while holes diffuse from p to n.
- Immobile ionised donor and acceptor atoms form a space-charge region and an electric field that opposes further majority-carrier diffusion.
- Forward bias connects the p side to the positive battery terminal and the n side to the negative terminal, reducing the junction barrier.

Step 1: Explain formation of the depletion region When p-type and n-type semiconductors are joined, the n side has a much higher electron concentration and the p side has a much higher hole concentration. Electrons near the junction diffuse from n to p and recombine with holes; holes diffuse from p to n and recombine with electrons. Consequently, a thin region on either side of the junction becomes depleted of mobile majority charge carriers. The uncovered positive donor ions on the n side and negative acceptor ions on the p side remain fixed in the crystal lattice. This region is called the depletion region.

Step 2: Explain formation of the potential barrier The fixed positive and negative ions establish an electric field directed from the n side towards the p side. The field opposes the diffusion of majority carriers: it pushes electrons towards the n side and holes towards the p side. The associated potential difference across the depletion region is the potential barrier. As the barrier develops, diffusion is increasingly opposed; at equilibrium, field-driven drift balances diffusion, producing zero net current through the unbiased junction.

Step 3: Describe the effect of forward bias In forward bias the p side is connected to the positive terminal and the n side to the negative terminal of the battery. The applied field opposes the junction field and lowers the effective potential barrier. Majority carriers are pushed towards the junction and neutralise part of its exposed ionic charge. Hence the width of the depletion region decreases, making majority-carrier passage across the junction easier.

Step 4: Conclusion Diffusion and recombination leave fixed ions that create the depletion region and potential barrier. Applying forward bias reduces the barrier and decreases the width of the depletion region.

Quick Tip to Solve: Remember the fixed-ion signs: n-side donors are positive, p-side acceptors negative; forward bias narrows their region.

29. What is photo diode ? Briefly explain its working and draw its V - I characteristics.

[3 marks]

Solution:

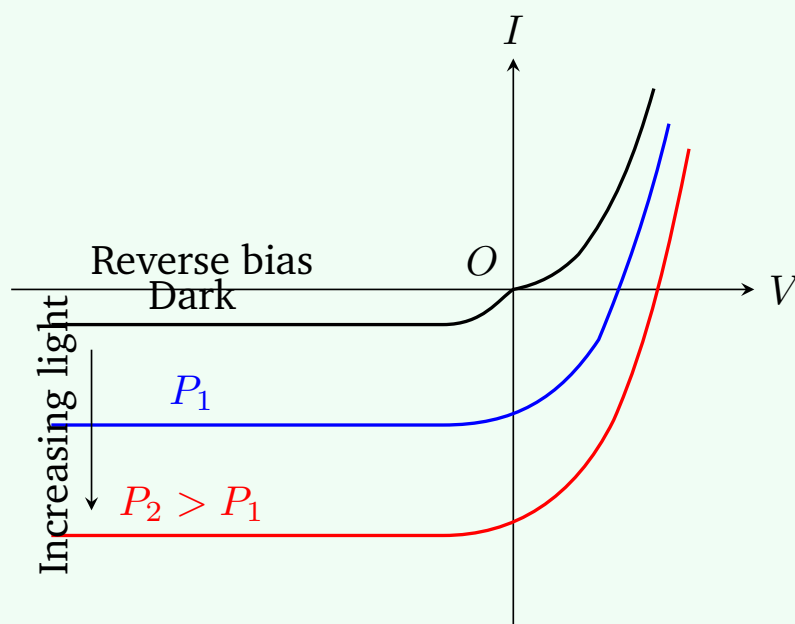
Concept:

- A photodiode is a light-sensitive semiconductor p-n junction designed to convert incident light into an electrical signal. It is normally operated under reverse bias.
- A photon with energy $h\nu \geq E_g$, where E_g is the semiconductor band gap, can generate an electron-hole pair near the junction.
- The depletion-region electric field separates these carriers. At a fixed reverse voltage, the magnitude of photocurrent increases with illumination intensity.

Step 1: State the construction and operating condition A photodiode has a transparent window through which light reaches the junction region. The p side is connected to the negative terminal and the n side to the positive terminal of the supply, so the junction is reverse biased. This widens the depletion region and establishes an electric field that collects the photo-generated carriers. In darkness, a small reverse current, called dark current, flows due to thermally generated minority carriers.

Step 2: Explain how light produces the signal When suitable photons are absorbed in or near the depletion region, electron-hole pairs are created. The junction field drives electrons towards the n side and holes towards the p side, preventing immediate recombination and producing a reverse photocurrent in the external circuit. More incident photons per unit time generate more carriers, so at the same reverse voltage the photocurrent magnitude increases approximately in proportion to the light intensity. The current therefore provides a measure of the incident illumination.

Step 3: Interpret the V - I characteristics With the conventional diode sign convention, reverse voltage and reverse current are both negative. The illuminated characteristics therefore lie in the third quadrant. Away from zero bias and before breakdown, the reverse current is nearly independent of reverse voltage but depends strongly on light intensity. The sketch shows dark current and two illuminated curves; $P_2 > P_1$ denotes higher incident optical power and a larger magnitude of reverse current. The forward branch has the usual diode rise, and the illuminated curves intersect the current axis below zero because short-circuit photocurrent can flow even at zero applied voltage.



Photodiode V-I characteristics; illumination increases from darkness to P_1 to P_2 . Breakdown is omitted.

Step 4: Conclusion A photodiode detects light through the reverse current caused by photo-generated electron-hole pairs. Greater illumination produces a greater reverse-current magnitude, as shown by the ordered characteristics.

Quick Tip to Solve: Draw the reverse characteristics in the third quadrant and place the stronger-illumination curve farther below the voltage axis.

30. Calculate the de-Broglie wavelength associated with the electron in the 2nd excited state of hydrogen atom. The ground state energy of the hydrogen atom is 13.6 eV. [3 marks]

Solution:

Concept:

- The second excited state has principal quantum number $n = 3$; the ground state is $n = 1$.
- For hydrogen, $E_n = -13.6/n^2$ eV. The printed positive value 13.6 eV represents the magnitude of the ground-state binding energy.
- In a Coulomb orbit the kinetic energy is $K = -E_n$, and the de-Broglie wavelength is $\lambda = h/\sqrt{2m_e K}$.

Step 1: Find the kinetic energy in the specified state The first excited state is $n = 2$, so the second excited state asked in the paper is $n = 3$. Its total energy is negative because the electron is bound:

$$E_3 = -\frac{13.6}{9} \text{ eV} = -1.511 \text{ eV}.$$

The Coulomb potential satisfies $U = -2K$, so $E = K + U = -K$. Consequently the kinetic energy is positive:

$$K = 1.511 \text{ eV} = 1.511(1.602 \times 10^{-19}) \text{ J} = 2.421 \times 10^{-19} \text{ J}.$$

Step 2: Use the de-Broglie relation With $m_e = 9.11 \times 10^{-31} \text{ kg}$ and $h = 6.626 \times 10^{-34} \text{ Js}$, the

nonrelativistic relation $p = \sqrt{2m_e K}$ gives

$$p = \sqrt{2(9.11 \times 10^{-31})(2.421 \times 10^{-19})} = 6.642 \times 10^{-25} \text{ kg m s}^{-1}.$$

Hence,

$$\lambda = \frac{6.626 \times 10^{-34}}{6.642 \times 10^{-25}} = 9.98 \times 10^{-10} \text{ m}.$$

The standing-wave relation also checks this result: $2\pi r_n = n\lambda$ and $r_n = n^2 a_0$ imply $\lambda = 2\pi n a_0$, giving the same wavelength for $n = 3$.

Step 3: Conclusion For the printed second excited state, $\lambda \approx 9.98 \times 10^{-10} \text{ m} = 0.998 \text{ nm}$.

Quick Tip to Solve: Second excited means $n=3$; convert the positive kinetic energy from eV to joules before taking its square root.

31. (a) Differentiate between electrical resistance and resistivity of a conductor.

(b) Two metallic rods, each of length L , area of cross A_1 and A_2 , having resistivities ρ_1 and ρ_2 are connected in parallel across a d.c. battery. Obtain the expression for the effective resistivity of this combination. [3 marks]

Solution:

Concept:

- Electrical resistance is $R = V/I$, the opposition of a particular conductor to current. For a uniform rod, $R = \rho L/A$.
- Resistivity $\rho = RA/L$ characterises the material at a specified temperature; it is independent of the dimensions of a homogeneous sample.
- Parallel rods have a common potential difference and their conductances add: $1/R_{\text{eq}} = 1/R_1 + 1/R_2$.

Step 1: (a) Distinguish resistance from resistivity Resistance is a property of the complete conductor: it depends on its material, length, cross-sectional area and temperature. Its SI unit is the ohm, Ω . For fixed material and temperature, increasing length increases resistance while increasing area decreases it, as $R = \rho L/A$. Resistivity is a property of the material at a given temperature. Its SI unit is $\Omega \text{ m}$. Changing the length or area of a uniform sample changes its resistance but does not change its resistivity. Both resistance and resistivity can vary with temperature; independence from dimensions does not mean independence from temperature.

Step 2: (b) Find the equivalent resistance of the rods Since both rods have length L , their resistances are

$$R_1 = \frac{\rho_1 L}{A_1}, \quad R_2 = \frac{\rho_2 L}{A_2}.$$

They are connected across the same battery, so their currents add at the common voltage. Thus,

$$\frac{1}{R_{\text{eq}}} = \frac{A_1}{\rho_1 L} + \frac{A_2}{\rho_2 L}, \quad R_{\text{eq}} = \frac{L\rho_1\rho_2}{\rho_2 A_1 + \rho_1 A_2}.$$

Step 3: Define the effective resistivity of the composite Replace the parallel combination by one equivalent conductor of the same length L and total conducting cross-sectional area $A_1 + A_2$. By

definition,

$$R_{\text{eq}} = \frac{\rho_{\text{eff}} L}{A_1 + A_2}.$$

Equating this expression to the calculated resistance yields

$$\rho_{\text{eff}} = \frac{\rho_1 \rho_2 (A_1 + A_2)}{\rho_2 A_1 + \rho_1 A_2}.$$

As a consistency check, when the materials have identical resistivity, the effective resistivity equals that common value, regardless of the two areas.

Step 4: Conclusion Resistance depends on sample dimensions and has unit Ω ; resistivity is a material property at fixed temperature and has unit $\Omega \text{ m}$. The combination has $\rho_{\text{eff}} = \frac{\rho_1 \rho_2 (A_1 + A_2)}{\rho_2 A_1 + \rho_1 A_2}$.

Quick Tip to Solve: First add the parallel conductances, then use the total area $A_1 + A_2$ to convert equivalent resistance into effective resistivity.

- 32. (a)** Two point charges $+Q_1$ and $-Q_2$ are placed r distance apart. Obtain the expression for the amount of work done to place a third charge Q_3 at the midpoint of the line joining the two charges. **(b)** At what distance from charge $+Q_1$ on the line joining the two charges (in terms of Q_1 , Q_2 and r) will this work done be zero. [3 marks]

Solution:

Concept:

- Electric potential adds algebraically: $V = k \sum_i q_i / d_i$, where $k = 1/(4\pi\epsilon_0)$, q_i is a source charge and d_i is its distance from the point.
- The external work in slowly bringing Q_3 from infinity is $W = Q_3 V$, with source charges fixed and potential at infinity taken as zero.

Step 1: (a) Potential at the midpoint Both source charges are at distance $r/2$. The negative charge contributes negative potential, giving

$$V_{\text{mid}} = k \left(\frac{Q_1}{r/2} - \frac{Q_2}{r/2} \right) = \frac{2k(Q_1 - Q_2)}{r}.$$

Potential is a scalar, so the signed contributions add without resolving directions.

Step 2: (a) Work done to place the third charge The work equals the additional interaction energy of the third charge with the two source charges:

$$W = Q_3 V_{\text{mid}} = \frac{2kQ_3(Q_1 - Q_2)}{r}.$$

The existing interaction energy of the fixed source charges does not change. A negative result means the electric field supplies energy and the external agent removes it.

Step 3: (b) Zero-work point on the joining segment Let its distance from $+Q_1$ towards $-Q_2$ be x , so

its distance from the negative charge is $r - x$. For nonzero Q_3 , zero work requires zero potential:

$$k \left(\frac{Q_1}{x} - \frac{Q_2}{r-x} \right) = 0,$$

$$Q_1(r-x) = Q_2x,$$

$$x = \frac{Q_1r}{Q_1 + Q_2}.$$

Here Q_1, Q_2 are positive magnitudes, so this point lies between the charges. Equal magnitudes give the midpoint, consistent with part (a).

Step 4: Conclusion $W = \frac{Q_3(Q_1 - Q_2)}{2\pi\epsilon_0 r}$; the zero-work point on the joining segment is $x = \frac{Q_1r}{Q_1 + Q_2}$ from $+Q_1$ towards $-Q_2$.

Quick Tip to Solve: Zero placement work requires zero scalar potential; do not set the vector electric field to zero.

33. An optical instrument uses an objective lens of power 100 D and an eyepiece of power 40 D. The final image is formed at infinity when the tube length of the instrument is kept at 20 cm.

(a) Identify the optical instrument.

(b) Calculate the angular magnification produced by the instrument.

[3 marks]

Solution:

Concept:

- Lens power P in dioptres and focal length f in metres obey $f = 1/P$. A compound microscope uses a short-focal-length objective and a longer-focal-length eyepiece.
- For normal adjustment, the final image is at infinity. The standard microscope magnifying-power formula is $M \simeq (L/f_o)(D/f_e)$, where L is tube length and $D = 25$ cm is the least distance of distinct vision.

Step 1: Find focal lengths and identify the instrument The given powers yield

$$f_o = \frac{1}{100} \text{ m} = 1 \text{ cm}, \quad f_e = \frac{1}{40} \text{ m} = 2.5 \text{ cm}.$$

The objective has a much shorter focal length than the eyepiece, the arrangement characteristic of a compound microscope. The objective forms a real enlarged intermediate image and the eyepiece views it as a magnifier.

Step 2: Use the infinity-image condition For the final image at infinity, the intermediate image lies in the first focal plane of the eyepiece. Thus its angular magnification has magnitude $m_e = D/f_e = 25/2.5 = 10$. The additional factor $1 + D/f_e$ applies when the final image is at the near point, so it does not apply here.

Step 3: Calculate the total angular magnification Using the usual CBSE tube-length approximation, the objective magnification has magnitude $m_o \simeq L/f_o = 20/1 = 20$. Therefore

$$|M| \simeq \frac{L}{f_o} \frac{D}{f_e} = \frac{20}{1} \times \frac{25}{2.5} = 200.$$

This standard expression assumes the object is close to the objective focus and replaces the objective

image distance by the tube length. The final image remains inverted relative to the object, so a signed magnification would be negative.

Step 4: Conclusion (a) The instrument is a compound microscope. (b) Its angular magnification in normal adjustment is approximately 200 in magnitude; the final image is inverted.

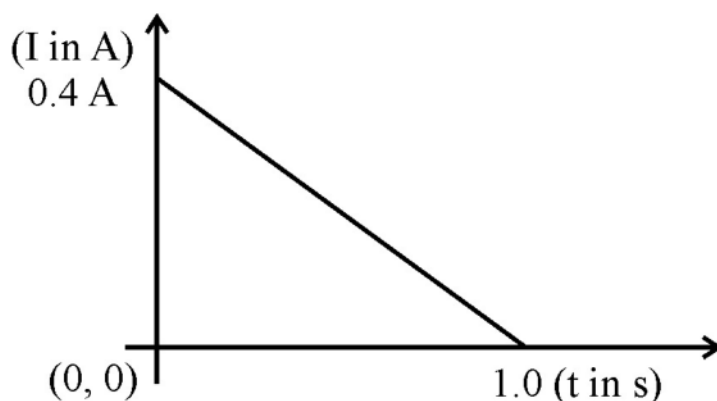
Quick Tip to Solve: Convert powers to focal lengths first, then use D/f_e for an infinity image and keep all lengths in the same unit.

34. When a conducting loop of resistance $10\ \Omega$ and area $10\ \text{cm}^2$ is removed from an external magnetic field acting normally, the variation of induced current in the loop with time is shown in the figure.

Find the

- total charge passed through the loop.
- change in magnetic flux through the loop.
- magnitude of the magnetic field applied.

[3 marks]



Solution:

Concept:

- The charge transported is $q = \int I dt$, equal to the area beneath the current-time graph. Here I is current and t is time.
- Faraday's law and Ohm's law give $IR = -d\Phi/dt$, so $|\Delta\Phi| = Rq$ for a current of one sign. Normal magnetic flux through a single loop is $\Phi = BA$.

Step 1: (i) Integrate the current-time graph The graph shows a straight decrease from $0.4\ \text{A}$ at $t = 0$ to zero at $t = 1.0\ \text{s}$. The area under it is a triangle, and hence

$$q = \frac{1}{2} \times 1.0\ \text{s} \times 0.4\ \text{A} = 0.20\ \text{C}.$$

This uses the whole changing current, rather than treating its initial value as constant throughout the interval.

Step 2: (ii) Relate charge to flux change Integrating the induced emf over the removal interval gives

$$|\Delta\Phi| = \int |\varepsilon| dt = R \int I dt = 10 \times 0.20 = 2.0\ \text{Wb}.$$

On removal the final flux is zero, so the flux decreases by this amount. If the initial applied field defines the positive area normal, the signed change is $\Delta\Phi = \Phi_f - \Phi_i = -2.0\ \text{Wb}$.

Step 3: (iii) Calculate the original magnetic field The field is normal to the loop, so the initial flux magnitude is BA , without a further cosine factor. The loop area in SI units is

$$A = 10 \text{ cm}^2 = 10 \times 10^{-4} \text{ m}^2 = 10^{-3} \text{ m}^2.$$

Since removal reduces the flux to zero,

$$B = \frac{|\Delta\Phi|}{A} = \frac{2.0}{10^{-3}} = 2.0 \times 10^3 \text{ T}.$$

Although large, this value follows directly from the printed graph and area.

Step 4: Conclusion (i) $q = 0.20 \text{ C}$. (ii) The magnetic flux decreases by 2.0 Wb . (iii) $B = 2.0 \times 10^3 \text{ T}$.

Quick Tip to Solve: Find the triangular graph area first, multiply it by resistance for flux change, and convert square centimetres with 10^{-4} .

Section D (15 marks)

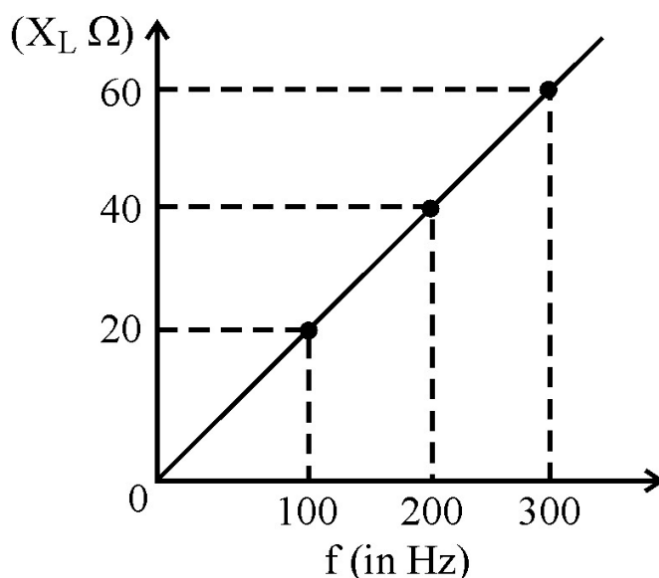
35. (a) Show that an ideal inductor does not dissipate power in an ac circuit.

(b) The variation of inductive reactance (X_L) of an inductor with the frequency (f) of the ac source of 100 V and variable frequency is shown in the fig.

(i) Calculate the self-inductance of the inductor.

(ii) When this inductor is used in series with a capacitor of unknown value and a resistor of $10\ \Omega$ at 300 s^{-1} , maximum power dissipation occurs in the circuit. Calculate the capacitance of the capacitor.

[5 marks]



Solution:

Concept:

- An ideal inductor has no resistance. Its current lags its applied sinusoidal voltage by $\pi/2$, so its cycle-averaged power is $P_{av} = V_{rms} I_{rms} \cos \phi$.
- Inductive reactance is $X_L = \omega L = 2\pi f L$, where L is self-inductance and f is the frequency in hertz. The slope of an X_L -versus- f graph is $2\pi L$.
- In a series LCR circuit, $Z^2 = R^2 + (\omega L - 1/(\omega C))^2$. For a fixed rms supply voltage, the average power $V_{rms}^2 R/Z^2$ is greatest at resonance, where $X_L = X_C$.

Step 1: (a) Obtain the current and instantaneous power Let the applied voltage be $v = V_m \sin \omega t$. Since $v = L di/dt$, the steady alternating current is

$$i = -\frac{V_m}{\omega L} \cos \omega t = I_m \sin(\omega t - \pi/2), \quad I_m = \frac{V_m}{\omega L}.$$

Thus the instantaneous power delivered to the inductor is

$$p = vi = -V_m I_m \sin \omega t \cos \omega t = -\frac{V_m I_m}{2} \sin 2\omega t.$$

The power changes sign during a cycle: the inductor absorbs energy while its magnetic field grows and returns energy to the source while the field decreases.

Step 2: (a) Average the power over a complete cycle For period $T = 2\pi/\omega$, the integral of $\sin 2\omega t$ over a complete period vanishes. Hence

$$P_{av} = \frac{1}{T} \int_0^T p \, dt = 0.$$

Equivalently, $P_{av} = V_{rms} I_{rms} \cos(\pi/2) = 0$. The stored energy $U = \frac{1}{2} Li^2$ is exchanged reversibly with the source. There is no resistance to convert electrical energy into heat, so an ideal inductor does not dissipate power even though its instantaneous power is generally nonzero.

Step 3: (b)(i) Read the graph and calculate self-inductance The straight line passes through the origin and the point $f = 300 \text{ Hz}$, $X_L = 60 \Omega$. Therefore

$$\begin{aligned} \frac{X_L}{f} &= \frac{60}{300} = 0.20 \Omega \text{ Hz}^{-1}, \\ L &= \frac{X_L}{2\pi f} = \frac{60}{2\pi(300)} = \frac{1}{10\pi} \text{ H} \simeq 3.18 \times 10^{-2} \text{ H}. \end{aligned}$$

The other plotted points, (100 Hz, 20 Ω) and (200 Hz, 40 Ω), give the same ratio and confirm this value.

Step 4: (b)(ii) Apply the condition for maximum power The stated 300 s^{-1} is the source frequency $f = 300 \text{ Hz}$, consistent with the horizontal axis; the corresponding angular frequency is $\omega = 2\pi(300) \text{ rad s}^{-1}$. With $R = 10 \Omega$, the power is greatest when the net reactance vanishes. Thus

$$\begin{aligned} X_C &= X_L = 60 \Omega, \\ \frac{1}{2\pi f C} &= 60 \Omega, \\ C &= \frac{1}{2\pi(300)(60)} \text{ F} \\ &= 8.84 \times 10^{-6} \text{ F} = 8.84 \mu\text{F}. \end{aligned}$$

At this frequency $Z = R$, so the current is in phase with the applied voltage. The resistor dissipates the power; the ideal inductor and capacitor only store and return energy. The supplied voltage and resistance determine the maximum power, but they do not alter the resonance condition used to obtain C .

Step 5: Conclusion (a) An ideal inductor has zero average power dissipation. (b)(i) Its self-inductance is $L = 3.18 \times 10^{-2} \text{ H}$ (about 31.8 mH). (b)(ii) The required capacitance is $C = 8.84 \mu\text{F}$.

Quick Tip to Solve: Read $X_L = 60 \Omega$ directly at 300 Hz; at resonance use that same value for X_C . Convert frequency to $2\pi f$ before substituting.

35. (a) A conductor of length l is rotated about one of its ends at a constant angular speed ω in a plane perpendicular to a uniform magnetic field B . Plot graphs to show variations of the emf induced across the ends of the conductor with (i) angular speed ω and (ii) length of the conductor l .

(b) Two concentric circular loops of radius 1 cm and 20 cm are placed coaxially.

(i) Find mutual inductance of the arrangement.

(ii) If the current passed through the outer loop is changed at a rate of 5 A/ms, find the emf induced in the inner loop. Assume the magnetic field on the inner loop to be uniform. [5 marks]

Solution:

Concept:

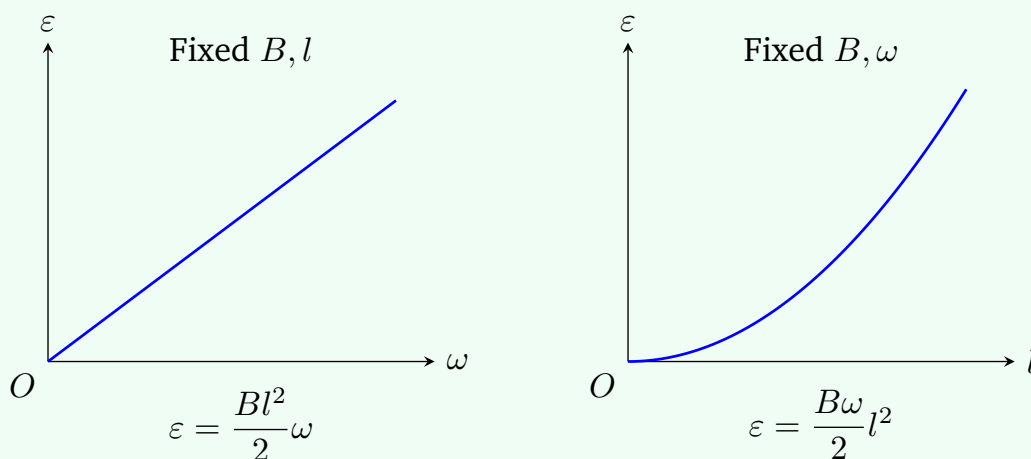
- A rod element at distance x from its rotation axis moves at speed $v = \omega x$. For motion perpendicular to a uniform magnetic field, the magnitude of its motional emf is $d\varepsilon = Bv dx$.
- Mutual inductance is flux through one loop per unit current in the other: $M = \Phi_{\text{inner}}/I_{\text{outer}}$. Each loop here has one turn.
- The magnetic field at the centre of a circular loop of radius R is $B = \mu_0 I/(2R)$. The question permits treating this field as uniform over the much smaller inner loop.
- Faraday's law gives $\varepsilon_{\text{inner}} = -M dI_{\text{outer}}/dt$; the minus sign expresses Lenz's law.

Step 1: (a) Obtain the emf of the rotating conductor Take an element dx at distance x from the fixed end. Its velocity is tangential, while the magnetic field is perpendicular to the plane of rotation, so the magnetic force on charges acts along the conductor. Integrating the magnitude of the elemental emf from the axis to the free end gives

$$\varepsilon = \int_0^l B\omega x dx = \frac{1}{2}B\omega l^2.$$

Thus the result involves the square of the length; using the tip speed for the whole rod would incorrectly give twice the emf. The polarity depends on the directions of rotation and field, and the requested graphs show the emf magnitude.

Step 2: (a)(i) and (ii) Plot the two variations With B and l fixed, $\varepsilon = (Bl^2/2)\omega$, so the emf versus angular speed is a straight line through the origin with slope $Bl^2/2$. With B and ω fixed, $\varepsilon = (B\omega/2)l^2$, so the emf versus length is an upward-curving parabola through the origin, for physical lengths $l \geq 0$. Its slope increases with length; doubling l multiplies the emf by four.



Emf magnitude versus angular speed at fixed length, and versus length at fixed angular speed; the scales are schematic.

Step 3: (b)(i) Calculate the mutual inductance Let the inner radius be $r = 1 \text{ cm} = 0.010 \text{ m}$ and the outer radius be $R = 20 \text{ cm} = 0.20 \text{ m}$. If current I flows in the outer loop, the approximately uniform field through the inner loop is $\mu_0 I / (2R)$. The flux is therefore

$$\Phi_{\text{inner}} = \frac{\mu_0 I}{2R} \pi r^2, \quad M = \frac{\mu_0 \pi r^2}{2R}.$$

Substituting $\mu_0 = 4\pi \times 10^{-7} \text{ H m}^{-1}$,

$$M = \frac{(4\pi \times 10^{-7})\pi(0.010)^2}{2(0.20)} = \pi^2 \times 10^{-10} \text{ H} \simeq 9.87 \times 10^{-10} \text{ H}.$$

The small-loop approximation is justified here by $r/R = 0.05$, and is explicitly stipulated in the question.

Step 4: (b)(ii) Find the emf induced in the inner loop Convert the current-change rate to SI units before applying Faraday's law:

$$\left| \frac{dI}{dt} \right| = 5 \text{ A/ms} = 5 \times 10^3 \text{ A s}^{-1}.$$

Hence the induced-emf magnitude is

$$|\varepsilon_{\text{inner}}| = M \left| \frac{dI}{dt} \right| = (9.87 \times 10^{-10})(5 \times 10^3) = 4.93 \times 10^{-6} \text{ V}.$$

Its direction opposes the change of magnetic flux. A definite terminal polarity cannot be assigned without specifying the direction of the current and the loop orientation.

Step 5: Conclusion (a) $\varepsilon = \frac{1}{2} B \omega l^2$: the ε - ω graph is a straight line through the origin, and the ε - l graph is a parabola for $l \geq 0$. (b)(i) $M = 9.87 \times 10^{-10} \text{ H}$; (ii) the induced emf has magnitude $4.93 \mu\text{V}$, with direction given by Lenz's law.

Quick Tip to Solve: Integrate the rod's local speed rather than using its tip speed everywhere; for the loops, use the outer radius in the field formula and the inner radius in the flux area.

36. (a) Write two important characteristics of equipotential surfaces.

(b) A thin circular ring of radius r is charged uniformly so that its linear charge density becomes λ . Derive an expression for the electric field at a point P at a distance x from it along the axis of the ring. Hence, prove that at large distances ($x \gg r$), the ring behaves as a point charge. [5 marks]

Solution:

Concept:

- An equipotential surface has the same electric potential at every point. Since work is related to potential difference by $W = q\Delta V$, motion on such a surface requires no electrostatic work.
- Coulomb's law gives the field of a small charge element: $dE = \frac{1}{4\pi\epsilon_0} \frac{dq}{R^2}$. Uniform circular symmetry cancels components perpendicular to the ring axis.
- For ring radius r , linear charge density λ , and axial distance x , the total charge is $Q = 2\pi r\lambda$ and the distance of every element from the observation point is $R = \sqrt{x^2 + r^2}$.

Step 1: (a) Characteristics of equipotential surfaces First, no work is done by the electrostatic field in moving a test charge between any two points on an equipotential surface: their potentials are equal, so $\Delta V = 0$ and the work is zero. Second, the electric field is perpendicular to an equipotential surface at every point where the field is nonzero. If it had a tangential component, displacement along the surface would involve nonzero work and change the potential, contradicting the definition. Thus field lines meet equipotential surfaces normally.

Step 2: (b) Resolve the field of a ring element Take a small element of length dl carrying charge $dq = \lambda dl$. At the axial point P , its field magnitude is $dE = (4\pi\epsilon_0)^{-1} dq / (x^2 + r^2)$. Let θ be the angle between this field and the axis. Geometry gives $\cos \theta = x / \sqrt{x^2 + r^2}$. For every element, the diametrically opposite element has an equal and opposite transverse field component, while their axial components add. Therefore only the axial component survives:

$$dE_x = dE \cos \theta = \frac{1}{4\pi\epsilon_0} \frac{x dq}{(x^2 + r^2)^{3/2}}.$$

Step 3: Integrate over the uniformly charged ring The factors x and $(x^2 + r^2)^{3/2}$ are constant for all ring elements. Hence

$$\begin{aligned} E_x &= \frac{1}{4\pi\epsilon_0} \frac{x}{(x^2 + r^2)^{3/2}} \int dq \\ &= \frac{1}{4\pi\epsilon_0} \frac{(2\pi r\lambda)x}{(x^2 + r^2)^{3/2}}. \end{aligned}$$

For a positively charged ring and $x > 0$, the field points along the axis away from the ring; reversing the sign of λ reverses its direction. At the centre, $x = 0$, the symmetry result gives zero field.

Step 4: Obtain the far-distance approximation Writing the exact expression in terms of total charge gives

$$E_x = \frac{Q}{4\pi\epsilon_0 x^2} \left(1 + \frac{r^2}{x^2} \right)^{-3/2}, \quad x > 0.$$

When $x \gg r$, the ratio r^2/x^2 is negligible, so the bracket approaches unity. The resulting inverse-square

field is precisely the field of a point charge Q placed at the centre of the ring.

Step 5: Conclusion Equipotential surfaces require zero work for motion along them and are normal to the electric field. The axial ring field is $E_x = \frac{1}{4\pi\epsilon_0} \frac{2\pi r \lambda x}{(x^2 + r^2)^{3/2}}$. For $x \gg r$, it becomes $E_x \simeq \frac{Q}{4\pi\epsilon_0 x^2}$, where $Q = 2\pi r \lambda$, proving point-charge behaviour.

Quick Tip to Solve: Cancel transverse components first; integrate only the axial component using the same element-to-point distance for the entire ring.

OR

36. (a) State Gauss's law on electrostatics and derive an expression for the electric field due to a long straight thin uniformly charged wire (linear charge density λ) at a point lying at a distance r from the wire.

(b) The magnitude of electric field (in NC^{-1}) in a region varies with the distance r (in m) as

$$E = 10r + 5$$

By how much does the electric potential increase in moving from point at $r = 1$ m to a point at $r = 10$ m.

[5 marks]

Solution:

Concept:

- Gauss's law states that the total electric flux through any closed surface equals the charge enclosed divided by the permittivity of free space: $\oint_S \vec{E} \cdot d\vec{A} = Q_{\text{enc}}/\epsilon_0$.
- Cylindrical symmetry of a very long uniformly charged wire makes its electric field radial, with magnitude depending only on perpendicular distance r . End effects are neglected.
- Potential difference is determined by $V_B - V_A = - \int_A^B \vec{E} \cdot d\vec{l}$. A scalar field magnitude alone does not specify the sign of its radial component.

Step 1: (a) Choose a cylindrical Gaussian surface Consider a long straight wire of uniform linear charge density λ , and a closed cylinder coaxial with it, of radius r and length L . Rotational symmetry implies that the electric field is perpendicular to the wire and is radial. Translation symmetry along a sufficiently long wire implies that its magnitude is independent of the axial position. Thus its magnitude is constant over the curved surface. The field is parallel to both plane end faces, whose area vectors lie along the wire, so each end face contributes zero flux.

Step 2: Apply Gauss's law to derive the wire field The curved surface area is $2\pi rL$, and the enclosed charge is $Q_{\text{enc}} = \lambda L$. With E_r denoting the signed outward radial component, Gauss's law gives

$$E_r(2\pi rL) = \frac{\lambda L}{\epsilon_0}, \quad E_r = \frac{\lambda}{2\pi\epsilon_0 r}.$$

Consequently the magnitude is $E = |\lambda|/(2\pi\epsilon_0 r)$. For positive λ the direction is radially outward, and for negative λ it is radially inward. The length L cancels, as required for a local field independent of the arbitrary Gaussian surface length.

Step 3: (b) Integrate the specified field-distance relation For radial motion from $r = 1$ m to $r = 10$ m,

the magnitude of the potential change is

$$\begin{aligned} |\Delta V| &= \int_1^{10} (10r + 5) dr \\ &= [5r^2 + 5r]_1^{10} \\ &= (500 + 50) - (5 + 5) = 540 \text{ V.} \end{aligned}$$

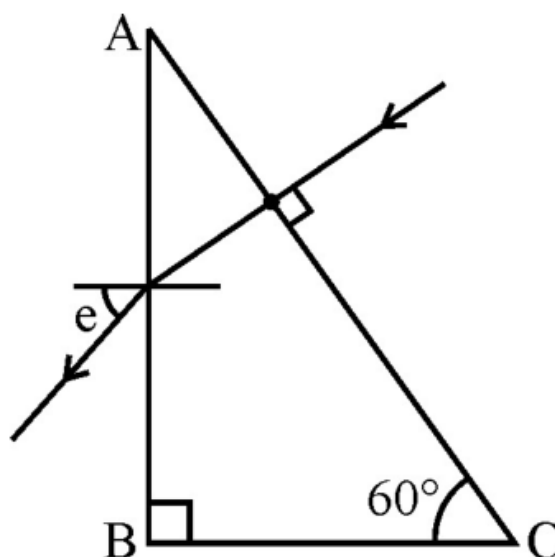
The integral has units of electric field times distance, $\text{N C}^{-1}\text{m} = \text{V}$. The printed question asks for an increase but supplies only the magnitude of the field. An increase of 540 V corresponds to the field pointing toward decreasing r , so the displacement is opposite to the field. If the field is instead assumed to point along increasing r , then $V(10) - V(1) = -540 \text{ V}$: the potential decreases by that amount. This distinction follows directly from the minus sign in the potential-field relation.

Step 4: Conclusion Gauss's law gives $\vec{E} = \frac{\lambda}{2\pi\epsilon_0 r} \hat{r}$ for the long wire. The potential-change magnitude in part (b) is 540 V; the requested increase is +540 V if the field is directed inward. For an outward radial field, the signed change is -540 V, a decrease.

Quick Tip to Solve: Integrate the field to obtain the potential-change magnitude, then use the field direction and the minus sign to decide increase or decrease.

37. (a) Define the term 'focal length of a mirror'. With the help of a ray diagram, obtain the relation between its focal length and radius of curvature.

(b) Calculate the angle of emergence (e) of the ray of light incident normally on the face AC of a glass prism ABC of refractive index $\sqrt{3}$. How will the angle of emergence change qualitatively, if the ray of light emerges from the prism into a liquid of refractive index 1.3 instead of air? [5 marks]



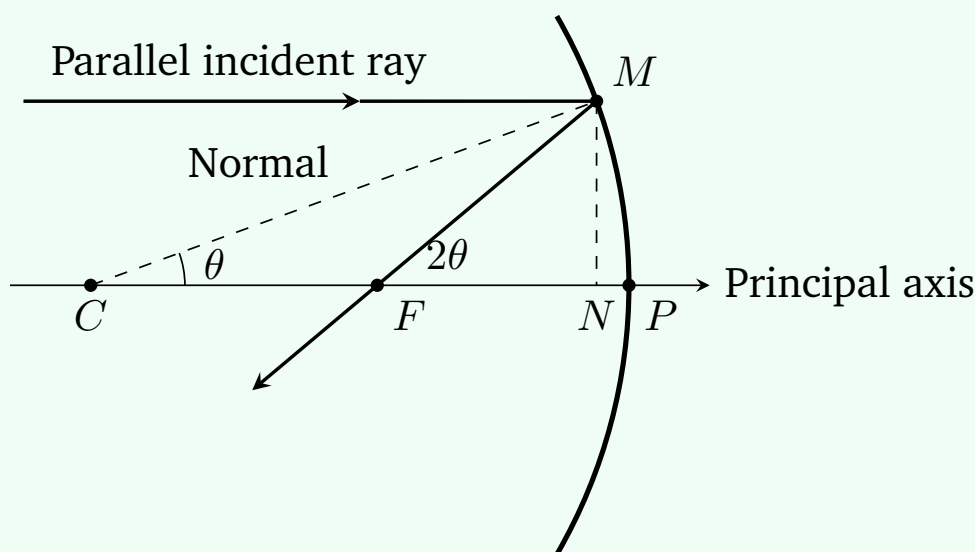
Solution:

Concept:

- The principal focus of a spherical mirror is the point on its principal axis where rays parallel and close to that axis converge after reflection (concave mirror), or appear to diverge from (convex mirror). The focal length is the directed distance from the pole P to this principal focus F .

- A normal to a spherical mirror at a point M passes through its centre of curvature C . The law of reflection gives equal angles of incidence and reflection. The relation $f = R/2$ applies to paraxial rays, for which small-angle approximations are valid.
- At a refracting interface, Snell's law is $n_1 \sin i = n_2 \sin e$, with both angles measured from the normal. Normal incidence produces no change of direction at the entry face.

Step 1: (a) Set up the reflection geometry Consider a concave spherical mirror of radius R , with pole P and centre C . A ray parallel to the principal axis strikes the mirror at M , a small height h above the axis, and is reflected through F . Join CM ; this is the normal. Let the angle between CM and the axis be θ . Since the incident ray is parallel to the axis, its angle of incidence is θ . The reflected ray also makes angle θ with CM , so it makes angle 2θ with the principal axis. The diagram illustrates the geometry; the height of the ray is exaggerated for clarity.



Concave mirror: the normal CM bisects the angle between the incident and reflected rays.

Step 2: (a) Obtain the focal-length relation for paraxial rays Let N be the perpendicular projection of M on the axis. For a ray close to the axis, N is very close to P , so $CN \simeq CP = |R|$ and $FN \simeq FP = |f|$. From the right triangles,

$$\tan \theta = \frac{MN}{CN} \simeq \frac{h}{|R|}, \quad \tan 2\theta = \frac{MN}{FN} \simeq \frac{h}{|f|}.$$

For small θ , $\tan \theta \simeq \theta$ and $\tan 2\theta \simeq 2\theta$. Thus $h/|f| \simeq 2h/|R|$. A convex mirror gives the same result using the backward extension of the reflected ray. Under the Cartesian sign convention, f and R have the same sign for either mirror, so the directed relation also follows.

Step 3: (b) Find the incidence angle at the exit face In the printed prism, $\angle B = 90^\circ$ and $\angle C = 60^\circ$, hence $\angle A = 30^\circ$. The incident ray is normal to face AC , so it enters the glass without deviation. Face AC makes 60° with the horizontal base BC ; its normal therefore makes 30° with the horizontal. The normal to exit face AB is horizontal. Consequently, the ray reaches AB at incidence angle $i = 30^\circ$. Using the prism relation gives the same result: the internal angles satisfy $r_1 + r_2 = A$, and $r_1 = 0$.

Step 4: (b) Apply Snell's law for air and liquid For emergence from glass into air, $n_g = \sqrt{3}$ and $n_a = 1$, so

$$\sin e = \frac{n_g}{n_a} \sin i = \sqrt{3} \sin 30^\circ = \frac{\sqrt{3}}{2}.$$

The physical emergence angle is between 0° and 90° . The sine obtained is less than unity, so the ray

emerges; total internal reflection does not occur. If the surrounding medium is a liquid of refractive index 1.3, the internal ray direction and i remain unchanged because entry is still normal. At the exit,

$$\sin e_\ell = \frac{\sqrt{3}}{1.3} \sin 30^\circ = \frac{\sqrt{3}}{2.6} \simeq 0.6662.$$

Since this is smaller than the air value and sine increases on this angle range, the emergence angle decreases. The higher external refractive index reduces bending away from the normal.

Step 5: Conclusion (a) Focal length is the distance from the pole to the principal focus; for a spherical mirror and paraxial rays, $f = R/2$. (b) The angle of emergence in air is $e = 60^\circ$. In the liquid it decreases (to approximately 41.8°), with all emergence angles measured from the normal to AB .

Quick Tip to Solve: At normal entry, draw the ray perpendicular to AC first; the exit incidence equals the prism's 30-degree apex angle, not the printed 60-degree base angle.

OR

37. (a) Define the term 'resolving power of a telescope'. How will the resolving power be effected with the increase in

(i) Wavelength of light used.

(ii) Diameter of the objective lens.

Justify your answers.

(b) A screen is placed 80 cm from an object. The image of the object on the screen is formed by a convex lens placed between them at two different locations separated by a distance 20 cm. Determine the focal length of the lens. [5 marks]

Solution:

Concept:

- Resolving power measures the ability to distinguish two closely spaced distant objects as separate. It is the reciprocal of their minimum resolvable angular separation, $RP = 1/\theta_{\min}$.
- For a circular telescope objective, the Rayleigh criterion gives $\theta_{\min} = 1.22\lambda/D$, where λ is the wavelength and D is the objective diameter. Thus $RP = D/(1.22\lambda)$.
- For a thin convex lens forming a real image, the positive distance magnitudes a and b obey $1/f = 1/a + 1/b$. With fixed object-screen separation L , $a + b = L$; the two lens positions interchange a and b .

Step 1: (a) Define resolving power and its diffraction limit A telescope's resolving power is the reciprocal of the smallest angular separation between two distant point objects whose images can just be distinguished. Each point object produces a diffraction pattern because the objective has a finite circular aperture. According to the Rayleigh criterion, the two images are just resolved when the central maximum of one diffraction pattern falls at the first minimum of the other. For small angles,

$$\theta_{\min} = \frac{1.22\lambda}{D}, \quad RP = \frac{1}{\theta_{\min}} = \frac{D}{1.22\lambda}.$$

Here the angular separation is measured in radians. Resolving power concerns distinguishing nearby images; it should not be confused with merely enlarging an image by increasing magnification.

Step 2: (a)(i) Increase the wavelength Keep the diameter D fixed. Increasing λ increases $\theta_{\min}$, so

the objects must have a larger angular separation to be distinguished. This happens because a longer wavelength makes the diffraction pattern wider. Since $RP \propto 1/\lambda$, the resolving power decreases. For example, doubling the wavelength doubles the limiting angular separation and halves the resolving power.

Step 3: (a)(ii) Increase the objective diameter Keep the wavelength fixed. Increasing D decreases $\theta_{\min}$ and narrows the diffraction pattern, allowing the telescope to separate objects with a smaller angular separation. Since $RP \propto D$, the resolving power increases. Doubling the diameter halves the limiting angular separation and doubles the resolving power.

Step 4: (b) Obtain the two conjugate distances Let a and b be the positive object and image distance magnitudes at one lens position. The fixed separation is $L = 80$ cm, hence $a + b = 80$ cm. The lens equation is symmetric in these two distances, so at the other position they interchange. Consequently the separation of the two lens positions is $d = |b - a| = 20$ cm. Choose the position with $b > a$; then

$$a = \frac{L - d}{2} = \frac{80 - 20}{2} = 30 \text{ cm},$$

$$b = \frac{L + d}{2} = \frac{80 + 20}{2} = 50 \text{ cm}.$$

At the second position the distances are 50 cm and 30 cm. Both positions therefore form real images on the same screen.

Step 5: (b) Apply the lens formula In the Cartesian sign convention, the first position has $u = -30$ cm and $v = +50$ cm. Therefore

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u} = \frac{1}{50} + \frac{1}{30} = \frac{80}{1500} \text{ cm}^{-1},$$

$$f = \frac{1500}{80} \text{ cm} = 18.75 \text{ cm}.$$

Equivalently, the displacement formula follows from $f = ab/(a + b)$:

$$f = \frac{L^2 - d^2}{4L} = \frac{80^2 - 20^2}{4(80)} \text{ cm}.$$

The positive focal length is consistent with the convex lens, and $L > 4f$ confirms that two distinct real-image positions are possible.

Step 6: Conclusion (a) Resolving power is $1/\theta_{\min} = D/(1.22\lambda)$. It decreases when the wavelength increases and increases when the objective diameter increases. (b) The focal length is 18.75 cm.

Quick Tip to Solve: With fixed object-screen separation, use the sum and difference to find the conjugate distances: $(L - d)/2$ and $(L + d)/2$.