

CBSE Class 12 Mathematics 2020 - Set 65-C-2 - Compartment



Question Paper

Conducted by the Central Board of Secondary Education (CBSE)

Time Allowed: 3 Hours

Maximum Marks: 80

General Instructions

- (i) This question paper comprises four sections — A, B, C and D. This question paper carries 36 questions. All questions are compulsory.
- (ii) Section A – Question nos. 1 to 20 comprises of 20 questions of 1 mark each.
- (iii) Section B – Question nos. 21 to 26 comprises of 6 questions of 2 marks each.
- (iv) Section C – Question nos. 27 to 32 comprises of 6 questions of 4 marks each.
- (v) Section D – Question nos. 33 to 36 comprises of 4 questions of 6 marks each.
- (vi) There is no overall choice in the question paper. However, an internal choice has been provided in 3 questions of one mark, 2 questions of two marks, 2 questions of four marks and 2 questions of six marks. Only one of the choices in such questions have to be attempted.
- (vii) In addition to this, separate instructions are given with each section and question, wherever necessary.
- (viii) Use of calculators is not permitted.

Section A

1. If $\vec{a} = \hat{i} + \lambda\hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + \hat{j} + \hat{k}$ and $|\vec{a} + \vec{b}| = |\vec{a}| + |\vec{b}|$, then the value of λ is

- (A) 1
- (B) -1
- (C) 2
- (D) -2

2. The order and the degree of the differential equation $\left(\frac{dy}{dx}\right)^3 + \left(\frac{d^3y}{dx^3}\right)^3 + 5x = 0$ are

- (A) 3; 6
- (B) 3; 3
- (C) 3; 9
- (D) 6; 3

3. If $\sec^{-1}\left(\frac{1+x}{1-y}\right) = a$, then $\frac{dy}{dx}$ is equal to

- (A) $\frac{x-1}{y-1}$
- (B) $\frac{x-1}{y+1}$
- (C) $\frac{y-1}{x-1}$
- (D) $\frac{y+1}{x-1}$

4. If $A = \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$, then A^2 equals

- (A) $\begin{pmatrix} 2 & -2 \\ -2 & 2 \end{pmatrix}$
- (B) $\begin{pmatrix} 2 & -2 \\ -2 & -2 \end{pmatrix}$
- (C) $\begin{pmatrix} -2 & -2 \\ -2 & 2 \end{pmatrix}$
- (D) $\begin{pmatrix} -2 & 2 \\ 2 & -2 \end{pmatrix}$

5. $\begin{vmatrix} 43 & 44 & 45 \\ 44 & 45 & 46 \\ 45 & 46 & 47 \end{vmatrix}$ equals

- (A) 0
- (B) -1
- (C) 1
- (D) 2

6. $\int \frac{e^x}{x+1} [1 + (x+1)\log(x+1)] dx$ equals

- (A) $\frac{e^x}{x+1} + c$
- (B) $e^x \frac{1}{x+1} + c$
- (C) $e^x \log(x+1) + e^x + c$
- (D) $e^x \log(x+1) + c$

7. $\int_0^{\pi/2} (\sin^{100} x - \cos^{100} x) dx$ equals

- (A) $\frac{\pi}{100}$
- (B) 0
- (C) $\frac{1}{100}$
- (D) $\frac{100}{(100)^{100}}$

8. A specific problem can be solved independently by A and B with probabilities $\frac{1}{3}$ and $\frac{1}{5}$ respectively. If both try, independently, to solve the problem, then the probability that the problem is solved is

- (A) $\frac{7}{15}$
- (B) $\frac{8}{15}$
- (C) $\frac{2}{15}$
- (D) $\frac{14}{15}$

9. The objective function of an LPP is

- (A) a constant
- (B) a linear function to be optimised
- (C) an inequality
- (D) a quadratic expression

10. If the two lines $L_1 : x = 5, \frac{y}{3-\alpha} = \frac{z}{-2}$ and $L_2 : x = 2, \frac{y}{-1} = \frac{z}{2-\alpha}$ are perpendicular, then the value of α is

- (A) $\frac{2}{3}$
- (B) 3
- (C) 4
- (D) $\frac{7}{3}$

11. The integrating factor of the differential equation $x \frac{dy}{dx} - y = \log x$ is _____.

12. Two angles of a triangle are $\cot^{-1} 2$ and $\cot^{-1} 3$. The third angle of the triangle is _____.

13. From a pack of 52 cards, 3 cards are drawn at random (without replacement). The probability that they are two red cards and one black card, is _____.

14(a). A square matrix A is said to be singular if _____.

OR

14(b). If $A = \begin{pmatrix} 3 & -5 \\ 2 & 0 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 17 \\ 0 & -10 \end{pmatrix}$, then $|AB| =$ _____.

15. If $x = e^t \sin t, y = e^t \cos t$, then the value of $\frac{dy}{dx}$ at $t = \frac{\pi}{4}$ is _____.

16. If $f(x) = \frac{1-x}{1+x}$, then find $(f \circ f)(x)$.

17(a). If $\begin{pmatrix} 4 & x+2 \\ 2x-3 & x+1 \end{pmatrix}$ is a symmetric matrix, then find the value of x .

OR

17(b). If A is a square matrix such that $A^2 = A$, then find $(2+A)^3 - 19A$.

18. The Cartesian equation of a line is $\frac{x-5}{3} = \frac{2y+4}{7} = \frac{6-z}{2}$. Write its vector equation.

19(a). If $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$ and $\vec{b} = 5\hat{i} - 3\hat{j} - 4\hat{k}$, then find the ratio $\frac{\text{projection of } \vec{a} \text{ on } \vec{b}}{\text{projection of } \vec{b} \text{ on } \vec{a}}$.

OR

19(b). Let $\hat{a}$ and $\hat{b}$ be two unit vectors. If the vectors $\vec{c} = \hat{a} + 2\hat{b}$ and $\vec{d} = 5\hat{a} - 4\hat{b}$ are perpendicular to each other, then find the angle between the vectors $\hat{a}$ and $\hat{b}$.

20. Find the particular solution of the differential equation $\frac{dy}{dx} = y \tan x$, when $y(0) = 1$.

Section B

21. For the matrix $A = \begin{pmatrix} 1 & 5 \\ 6 & 7 \end{pmatrix}$, verify that

- (i) $(A + A')$ is a symmetric matrix.
- (ii) $(A - A')$ is a skew-symmetric matrix.

22. Solve: $x \frac{dy}{dx} = y - x \cos^2 \left(\frac{y}{x} \right)$.

23. If $\vec{a}$ and $\vec{b}$ are any two non-zero vectors, then show that vector $|\vec{a}|\vec{b} + |\vec{b}|\vec{a}$ is perpendicular to vector $|\vec{a}|\vec{b} - |\vec{b}|\vec{a}$.

24. If $y = e^x + e^{-x}$, then show that $\frac{dy}{dx} = \sqrt{y^2 - 4}$.

25(a). Let W denote the set of words in the English dictionary. Define the relation R by $R = \{(x, y) \in W \times W : x \text{ and } y \text{ have at least one letter in common}\}$. Show that this relation R is reflexive and symmetric, but not transitive.

OR

25(b). Find the inverse of the function $f(x) = \left(\frac{4x}{3x + 4} \right)$.

26(a). A bag contains 19 tickets, numbered 1 to 19. A ticket is drawn at random and then another ticket is drawn without replacing the first one in the bag. Find the probability distribution of the number of even numbers on the ticket.

OR

26(b). Find the probability distribution of the number of successes in two tosses of a die, when a success is defined as "number greater than 5".

Section C

27. Prove that $\tan^{-1} \sqrt{x} = \frac{1}{2} \cos^{-1} \left(\frac{1-x}{1+x} \right)$, $x \in [0, 1]$.

28. If $\hat{a}$ and $\hat{b}$ are unit vectors inclined at an angle θ , then prove that $\sin \frac{\theta}{2} = \frac{1}{2} |\hat{a} - \hat{b}|$.

29(a). Using properties of determinants show that
$$\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = (1+a^2+b^2)^3.$$

OR

29(b). Using determinants, find the equation of the line joining A(1, 3) and B(0, 0). Also find k if a point D(k, 0) is such that the area of $\triangle ABD$ is 3 square units.

30. A bag contains 5 red and 4 black balls, a second bag contains 3 red and 6 black balls. One of the two bags is selected at random and two balls are drawn at random (without replacement), both of which are found to be red. Find the probability that these two balls are drawn from the second bag.

31(a). Find the shortest distance between the following lines and hence write whether the lines are intersecting or not: $\frac{x-1}{2} = \frac{y+1}{3} = z$, $\frac{x+1}{5} = \frac{y-2}{1}, z=2$.

OR

31(b). Find the equation of the plane through the line of intersection of the planes $\vec{r} \cdot (\hat{i} + 3\hat{j}) + 6 = 0$ and $\vec{r} \cdot (3\hat{i} - \hat{j} - 4\hat{k}) = 0$, which is at a unit distance from the origin.

32. A company produces two types of goods, A and B, that require gold and silver. Each unit of type A requires 3 g of silver and 1 g of gold, while that of type B requires 1 g of silver and 2 g of gold. The company can use at the most 9 g of silver and 8 g of gold. If each unit of type A brings a profit of Rs.120 and that of type B Rs.150, then find the number of units of each type that the company should produce to maximise profit. Formulate the above LPP and solve it graphically. Also find the maximum profit.

Section D

33. If $y = 3 \cos(\log x) + 4 \sin(\log x)$, then show that $x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + y = 0$.

34(a). Find the intervals in which the function f defined as $f(x) = \sin x + \cos x$, $0 \leq x \leq 2\pi$ is strictly increasing or strictly decreasing.

OR

34(b). Prove that the radius of the right circular cylinder of greatest curved surface area which can be inscribed in a given cone is half of that of the cone.

35. Evaluate: $\int_{\pi/6}^{\pi/3} \frac{\sin x + \cos x}{\sqrt{\sin 2x}} dx$.

36(a). If the area between the curves $x = y^2$ and $x = 4$ is divided into two equal parts by the line $x = a$, then find the value of a using integration.

OR

36(b). Find: $\int \frac{x}{(x-1)^2(x+2)} dx$.

- End of Question Paper -