

CBSE Class 12 Mathematics 2020 - Set 65-C-3 - Compartment



Question Paper

Conducted by the Central Board of Secondary Education (CBSE)

Time Allowed: 3 Hours

Maximum Marks: 80

General Instructions

- (i) This question paper comprises four sections — A, B, C and D. This question paper carries 36 questions. All questions are compulsory.
- (ii) Section A – Question nos. 1 to 20 comprises of 20 questions of 1 mark each.
- (iii) Section B – Question nos. 21 to 26 comprises of 6 questions of 2 marks each.
- (iv) Section C – Question nos. 27 to 32 comprises of 6 questions of 4 marks each.
- (v) Section D – Question nos. 33 to 36 comprises of 4 questions of 6 marks each.
- (vi) There is no overall choice in the question paper. However, an internal choice has been provided in 3 questions of one mark, 2 questions of two marks, 2 questions of four marks and 2 questions of six marks. Only one of the choices in such questions have to be attempted.
- (vii) In addition to this, separate instructions are given with each section and question, wherever necessary.
- (viii) Use of calculators is not permitted.

Section A

1. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are the position vectors of the points A(2, 3, -4), B(3, -4, -5) and C(3, 2, -3) respectively, then $|\vec{a} + \vec{b} + \vec{c}|$ is equal to

- (A) $\sqrt{113}$
- (B) $\sqrt{185}$
- (C) $\sqrt{203}$
- (D) $\sqrt{209}$

2. If $A = \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$, then A^2 equals

- (A) $\begin{pmatrix} 2 & -2 \\ -2 & 2 \end{pmatrix}$
- (B) $\begin{pmatrix} 2 & -2 \\ -2 & -2 \end{pmatrix}$
- (C) $\begin{pmatrix} -2 & -2 \\ -2 & 2 \end{pmatrix}$
- (D) $\begin{pmatrix} -2 & 2 \\ 2 & -2 \end{pmatrix}$

3. The distance between the planes $4x - 4y + 2z + 5 = 0$ and $2x - 2y + z + 6 = 0$ is

- (A) $\frac{1}{6}$
- (B) $\frac{1}{7}$
- (C) $\frac{11}{6}$
- (D) $\frac{16}{6}$

4. A specific problem can be solved independently by A and B with probabilities $\frac{1}{3}$ and $\frac{1}{5}$ respectively. If both try, independently, to solve the problem, then the probability that the problem is solved is

- (A) $\frac{7}{15}$
- (B) $\frac{8}{15}$
- (C) $\frac{2}{15}$
- (D) $\frac{14}{15}$

5. The objective function of an LPP is

- (A) a constant
- (B) a linear function to be optimised
- (C) an inequality
- (D) a quadratic expression

6. If $\sec^{-1} \left(\frac{1+x}{1-y} \right) = a$, then $\frac{dy}{dx}$ is equal to

- (A) $\frac{x-1}{y-1}$
- (B) $\frac{y-1}{x-1}$
- (C) $\frac{y-1}{x+1}$
- (D) $\frac{x+1}{y+1}$

7. If $\int \frac{\cos 8x + 1}{\tan 2x - \cot 2x} dx = \lambda \cos 8x + c$, then the value of λ is

- (A) $\frac{1}{16}$
- (B) $\frac{1}{8}$
- (C) $-\frac{1}{16}$
- (D) $-\frac{1}{8}$

8. The order and degree of the differential equation of the family of parabolas having vertex at origin and axis along positive x-axis is

- (A) 1, 1
- (B) 1, 2
- (C) 2, 1
- (D) 2, 2

9. $\int_0^1 \tan(\sin^{-1} x) dx$ equals

- (A) 2
(B) 0
(C) -1
(D) 1

10. The roots of the equation $\begin{vmatrix} x & 0 & 8 \\ 4 & 1 & 3 \\ 2 & 0 & x \end{vmatrix} = 0$ are

- (A) -4, 4
(B) 2, -4
(C) 2, 4
(D) 2, 8

11(a). A square matrix A is said to be singular if _____.

OR

11(b). If $A = \begin{pmatrix} 3 & -5 \\ 2 & 0 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 17 \\ 0 & -10 \end{pmatrix}$, then $|AB| =$ _____.

12. Two angles of a triangle are $\cot^{-1} 2$ and $\cot^{-1} 3$. The third angle of the triangle is _____.

13. A card is picked at random from a pack of 52 playing cards. Given that the picked up card is a queen, the probability of it being a queen of spades is _____.

14. The integrating factor of the differential equation $x \frac{dy}{dx} - y = \log x$ is _____.

15. If $y = \log x$, then $\frac{d^2 y}{dx^2} =$ _____.

16(a). If $\begin{pmatrix} 4 & x+2 \\ 2x-3 & x+1 \end{pmatrix}$ is a symmetric matrix, then find the value of x .

OR

16(b). If A is a square matrix such that $A^2 = A$, then find $(2 + A)^3 - 19A$.

17. Find the solution of the differential equation $\log \left(\frac{dy}{dx} \right) = ax + by$.

18(a). If $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$ and $\vec{b} = 5\hat{i} - 3\hat{j} - 4\hat{k}$, then find the ratio $\frac{\text{projection of } \vec{a} \text{ on } \vec{b}}{\text{projection of } \vec{b} \text{ on } \vec{a}}$.

OR

18(b). Let $\hat{a}$ and $\hat{b}$ be two unit vectors. If the vectors $\vec{c} = \hat{a} + 2\hat{b}$ and $\vec{d} = 5\hat{a} - 4\hat{b}$ are perpendicular to each other, then find the angle between the vectors $\hat{a}$ and $\hat{b}$.

19. If $f : \mathbb{R} \rightarrow \mathbb{R}$ be given by $f(x) = (3 - x^3)^{1/3}$, then find $(f \circ f)(x)$.

20. Find the distance of the point (a, b, c) from the x-axis.

Section B

21. If $y = \sqrt{a + \sqrt{a + x}}$, then find $\frac{dy}{dx}$.

22. Show that the three vectors $2\hat{i} - \hat{j} + \hat{k}$, $\hat{i} - 3\hat{j} - 5\hat{k}$, and $3\hat{i} - 4\hat{j} - 4\hat{k}$ form the vertices of a right-angled triangle.

23(a). A bag contains 19 tickets, numbered 1 to 19. A ticket is drawn at random and then another ticket is drawn without replacing the first one in the bag. Find the probability distribution of the number of even numbers on the ticket.

OR

23(b). Find the probability distribution of the number of successes in two tosses of a die, when a success is defined as "number greater than 5".

24(a). Let W denote the set of words in the English dictionary. Define the relation R by $R = \{(x, y) \in W \times W : x \text{ and } y \text{ have at least one letter in common}\}$. Show that this relation R is reflexive and symmetric, but not transitive.

OR

24(b). Find the inverse of the function $f(x) = \left(\frac{4x}{3x+4}\right)$.

25. For the matrix $A = \begin{pmatrix} 2 & 3 \\ -4 & -6 \end{pmatrix}$, verify the following: $A(\text{adj } A) = (\text{adj } A)A = |A|I$.

26. Solve the following homogeneous differential equation: $x \frac{dy}{dx} = x + y$.

Section C

27. Solve for x : $\sin^{-1}(1-x) - 2\sin^{-1}x = \frac{\pi}{2}$.

28. The random variable X has a probability distribution $P(X)$ of the following form, where k is some number:

$$P(X) = \begin{cases} k, & \text{if } x = 0 \\ 2k, & \text{if } x = 1 \\ 3k, & \text{if } x = 2 \\ 0, & \text{otherwise} \end{cases}$$

Find:

(i) the value of k

(ii) $P(X < 2)$, $P(X \leq 2)$, $P(X \geq 2)$

29. A company produces two types of goods, A and B, that require gold and silver. Each unit of type A requires 3 g of silver and 1 g of gold, while that of type B requires 1 g of silver and 2 g of gold. The company can use at the most 9 g of silver and 8 g of gold. If each unit of type A brings a profit of Rs.120 and that of type B Rs.150, then find the number of units of each type that the company should produce to maximise profit. Formulate the above LPP and solve it graphically. Also find the maximum profit.

30(a). Find the shortest distance between the following lines and hence write whether the lines are intersecting or not: $\frac{x-1}{2} = \frac{y+1}{3} = z$, $\frac{x+1}{5} = \frac{y-2}{1}, z = 2$.

OR

30(b). Find the equation of the plane through the line of intersection of the planes $\vec{r} \cdot (\hat{i} + 3\hat{j}) + 6 = 0$ and $\vec{r} \cdot (3\hat{i} - \hat{j} - 4\hat{k}) = 0$, which is at a unit distance from the origin.

31. Find a unit vector perpendicular to each of the vectors $(\vec{a} + \vec{b})$ and $(\vec{a} - \vec{b})$, where $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$.

32(a). Using properties of determinants show that $\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = (1+a^2+b^2)^3$.

OR

32(b). Using determinants, find the equation of the line joining A(1, 3) and B(0, 0). Also find k if a point D(k, 0) is such that the area of $\triangle ABD$ is 3 square units.

Section D

33(a). Find the intervals in which the function f defined as $f(x) = \sin x + \cos x$, $0 \leq x \leq 2\pi$ is strictly increasing or strictly decreasing.

OR

33(b). Prove that the radius of the right circular cylinder of greatest curved surface area which can be inscribed in a given cone is half of that of the cone.

34. Evaluate $\int_1^3 (x^2 + 1 + e^x) dx$ as the limit of sums.

35(a). If the area between the curves $x = y^2$ and $x = 4$ is divided into two equal parts by the line $x = a$, then find the value of a using integration.

OR

35(b). Find: $\int \frac{x}{(x-1)^2(x+2)} dx$.

36. If $y = x^{\sin x} + \sin^{-1} \sqrt{x}$, then find $\frac{dy}{dx}$.

- End of Question Paper -