

CBSE Class 12 Physics Question Paper 2021 with Solution PDF-Set 55/1/1



Question Paper with Solutions

Conducted by the Central Board of Secondary Education (CBSE)

Time Allowed: 3 hours

Maximum Marks: 70

General Instructions

- (i) This question paper comprises **five** Sections – **A, B, C, D** and **E**.
- (ii) There are **33** questions in the question paper. **All** questions are compulsory.
- (iii) **Section A** – Questions no. **1 to 14** are very short answer type questions, carrying **one** mark each.
- (iv) **Section B** – Questions no. **15** and **16** are case study based short answer type questions, carrying **four marks** each.
- (v) **Section C** – Questions no. **17 to 25** are short answer type questions, carrying **two** marks each.
- (vi) **Section D** – Questions no. **26 to 30** are long answer type questions, carrying **three** marks each.
- (vii) **Section E** – Questions no. **31 to 33** are also long answer type questions, carrying **five** marks each.
- (viii) There is no overall choice in the question paper. However, an internal choice has been provided in 4 questions of 1 mark, 3 questions of 2 marks, 2 questions of 3 marks and all the 3 questions of 5 marks. You have to attempt only one of the choices in such questions.
- (ix) In addition to this, separate instructions are given with each section and question, wherever necessary.
- (x) Use of calculators and log tables is not permitted.

You may use the following values of physical constants wherever necessary :

$$c = 3 \times 10^8 \text{ m/s}$$

$$h = 6.63 \times 10^{-34} \text{ J s}$$

$$e = 1.6 \times 10^{-19} \text{ C}$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ T m A}^{-1}$$

$$\epsilon_0 = 8.854 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$$

$$\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$$

$$\text{Mass of electron } (m_e) = 9.1 \times 10^{-31} \text{ kg}$$

$$\text{Mass of neutron} = 1.675 \times 10^{-27} \text{ kg}$$

$$\text{Mass of proton} = 1.673 \times 10^{-27} \text{ kg}$$

$$\text{Avogadro's number} = 6.023 \times 10^{23} \text{ per gram mole}$$

$$\text{Boltzmann constant} = 1.38 \times 10^{-23} \text{ J K}^{-1}$$

Section A (14 marks)

1. Two wires X and Y of the same material and of equal lengths having area of cross-section A and $2A$ respectively, are connected in parallel across an ideal battery of emf E . What is the ratio of current density (J_X/J_Y) in them? [1 mark]

Solution:

Concept:

- For a uniform wire, resistance is $R = \rho l/S$, where ρ is resistivity, l is length and S is cross-sectional area.
- Current density is $J = I/S$; parallel branches have the same potential difference.

Step 1: Compare the branch currents Both wires have the same resistivity and length. Therefore $R_X = \rho l/A$ and $R_Y = \rho l/(2A) = R_X/2$. The ideal battery applies the same emf E across each branch, giving $I_Y = 2I_X$.

Step 2: Allow for the different areas Although wire Y carries twice the current, its cross-sectional area is also twice as large. Thus

$$\frac{J_X}{J_Y} = \frac{I_X/A}{I_Y/(2A)} = \frac{2I_X}{2I_X} = 1.$$

Equivalently, each wire has $J = E/(\rho l)$.

Step 3: Conclusion The ratio is $J_X : J_Y = 1 : 1$.

Quick Tip to Solve: For equal-length wires of the same material in parallel, use $J = V/(\rho l)$; area cancels.

2(a). An object approaches a converging lens with a uniform speed of 5 m/s and stops at the focus. How will the image move with respect to the lens? Specify its nature. [1 mark]

Solution:

Concept:

- A converging lens forms a real image on the other side when the object is outside its principal focus.
- Using positive distance magnitudes $s > f$ and v , the lens equation is $1/f = 1/s + 1/v$.

Step 1: Find the image position Let s be the object distance from the lens and f its focal length. The lens equation gives $v = fs/(s - f)$. As the object approaches the focus from outside it, s decreases towards f , so the denominator decreases and the image distance grows without limit.

Step 2: Determine motion and nature For the given object motion, $ds/dt = -5 \text{ m s}^{-1}$. Differentiation gives

$$\frac{dv}{dt} = -\frac{f^2}{(s - f)^2} \frac{ds}{dt} = \frac{5f^2}{(s - f)^2} \text{ m s}^{-1}.$$

The positive image velocity increases as s falls. Before the object reaches the focus, its image is real and inverted; at the focus the emergent rays are parallel and the limiting image is at infinity.

Step 3: Conclusion The real, inverted image moves away from the lens with increasing speed and tends to infinity when the object reaches the focus.

Quick Tip to Solve: The shrinking denominator in $v = fs/(s - f)$ immediately reveals the image motion near the focus.

OR

2(b). In a simple microscope, a convex lens of focal length 5 cm is used. Calculate the magnifying power when the object is placed at the focus of the lens. [1 mark]

Solution:

Concept:

- A simple microscope produces angular magnification using a convex lens.
- For the final image at infinity, magnifying power is $M = D/f$, where $D = 25$ cm is the standard least distance of distinct vision.

Step 1: Identify the viewing condition The object is placed at the principal focus of the convex lens. The rays emerging from the lens are therefore parallel, so the final image is at infinity and the normal eye views it with relaxed accommodation. This is the normal-adjustment condition.

Step 2: Substitute the focal length Use the relaxed-eye formula, with both lengths in the same unit:

$$M = \frac{D}{f} = \frac{25 \text{ cm}}{5 \text{ cm}} = 5.$$

Magnifying power is a ratio of viewing angles, so it has no unit. The expression $1 + D/f$ applies to an image at the near point, which is not the stated condition.

Step 3: Conclusion The magnifying power is $\boxed{M = 5}$, i.e. five times.

Quick Tip to Solve: Object at focus means image at infinity: choose D/f , without the extra 1 used for near-point viewing.

3. An alternating current $I = (10 \text{ A}) \sin(100 \pi t)$ is passed through a resistor of 20Ω . What is the average power consumed by the resistor over a complete cycle? [1 mark]

Solution:

Concept:

- For a sinusoidal current, $I_{\text{rms}} = I_0/\sqrt{2}$, where I_0 is the peak current.
- In a pure resistor, average power is $P_{\text{av}} = I_{\text{rms}}^2 R$, since current and voltage are in phase.

Step 1: Read the peak current Comparing $I = (10 \text{ A}) \sin(100\pi t)$ with $I = I_0 \sin(\omega t)$, the peak current is $I_0 = 10 \text{ A}$. This coefficient is not the rms current. Thus $I_{\text{rms}} = 10/\sqrt{2} \text{ A}$.

Step 2: Average the heating power The instantaneous power is $I^2 R$. Over one complete cycle, $\langle \sin^2(\omega t) \rangle = 1/2$; hence

$$P_{\text{av}} = \frac{I_0^2 R}{2} = \frac{(10)^2 (20)}{2} \text{ W} = 1000 \text{ W}.$$

The average current itself is zero, but the positive squared current produces nonzero heating.

Step 3: Conclusion The average power consumed is $\boxed{1000 \text{ W}}$.

Quick Tip to Solve: Square the peak amplitude and divide by 2 before multiplying by resistance; do not average current first.

4(a). What oscillates in an electromagnetic wave of frequency 10 MHz?

[1 mark]

Solution:

Concept:

- An electromagnetic wave consists of time-varying electric and magnetic fields.
- These fields oscillate in phase, mutually perpendicular and perpendicular to the direction of propagation.

Step 1: Identify the oscillating quantities The electric field $\vec{E}$ and the magnetic field $\vec{B}$ vary periodically as the wave travels. A material medium is not required: it is the fields themselves, rather than material particles, whose values oscillate.

Step 2: Use the stated frequency Both fields oscillate at the wave frequency $f = 10 \text{ MHz} = 10^7 \text{ Hz}$. At a fixed point, each field completes 10^7 cycles every second. Their common time period is $T = 1/f = 10^{-7} \text{ s}$, consistent with the same periodic variation of both fields.

Step 3: Conclusion The electric and magnetic fields oscillate, each at $\boxed{10 \text{ MHz}}$.

Quick Tip to Solve: For an electromagnetic wave, name both fields; do not identify air or other medium particles as the oscillators.

OR

4(b). Name the electromagnetic radiation used for killing germs in water purifiers.

[1 mark]

Solution:

Concept:

- Ultraviolet radiation is the part of the electromagnetic spectrum beyond the violet end of visible light.
- The germicidal action of ultraviolet light is used to disinfect water by inactivating microorganisms.

Step 1: Identify the radiation Water purifiers use ultraviolet radiation for killing or inactivating germs. The germicidal ultraviolet band, commonly called UV-C, is suited to this purpose because its photons can damage the genetic material of microorganisms.

Step 2: Relate the radiation to purification Damage to microbial DNA or RNA prevents effective

reproduction and helps disinfect the water exposed to the radiation. This use is distinct from the heating action associated with infrared radiation; the required answer names the ultraviolet portion of the electromagnetic spectrum.

Step 3: Conclusion The radiation used is ultraviolet (UV) radiation.

Quick Tip to Solve: Associate germ-killing in water purifiers with ultraviolet radiation, especially the germicidal UV-C band.

5. What is the power of an equiconvex lens of refractive index n_2 dipped in a liquid of refractive index n_1 , where $n_1 < n_2$? [1 mark]

Solution:

Concept:

- For a thin lens in a surrounding liquid, $P = 1/f = (n_2/n_1 - 1)(1/R_1 - 1/R_2)$.
- Here n_2 is the lens index, n_1 is the liquid index, and the signed radii follow the Cartesian convention.

Step 1: Apply the equiconvex shape Let R be the common magnitude of the radii of curvature. For light travelling from left to right, an equiconvex lens has $R_1 = +R$ and $R_2 = -R$, because the two centres of curvature lie on opposite sides of their respective surfaces.

Step 2: Use relative refractive index The lens-maker formula requires the refractive index of the lens relative to the surrounding liquid, namely n_2/n_1 . Therefore

$$P = \left(\frac{n_2}{n_1} - 1 \right) \left(\frac{1}{R} - \frac{1}{-R} \right) = \frac{2}{R} \left(\frac{n_2}{n_1} - 1 \right).$$

Since $n_2 > n_1$, this power is positive and the lens remains converging.

Step 3: Conclusion $P = \frac{2(n_2 - n_1)}{n_1 R}$, in dioptres when R is in metres.

Quick Tip to Solve: Use the relative index n_2/n_1 , then substitute opposite signs for the two equiconvex radii.

6. Why are neutrons preferred as better projectiles in causing nuclear reaction? [1 mark]

Solution:

Concept:

- The nucleus is positively charged, so a positively charged projectile faces electrostatic repulsion.
- A neutron has zero electric charge and therefore experiences no Coulomb repulsion from the nucleus.

Step 1: Compare the interaction with the nucleus A proton or another positively charged projectile must have sufficient kinetic energy to overcome the Coulomb barrier before it can approach the nucleus closely. This repulsive barrier makes nuclear reactions with such projectiles more difficult to initiate.

Step 2: Explain the neutron advantage A neutron is electrically neutral. It can approach the nucleus without overcoming an electrostatic barrier and can come within the short range where the strong nuclear interaction acts. Consequently, even relatively low-energy neutrons can be absorbed by

suitable nuclei and induce nuclear reactions.

Step 3: Conclusion Neutrons are preferred because their neutrality allows them to enter nuclei without Coulomb repulsion, even at comparatively low energies.

Quick Tip to Solve: Link the neutron advantage directly to zero charge and absence of the nuclear Coulomb barrier.

7. If the frequency of the radiation incident on a photosensitive surface increases ($\nu > \nu_0$), how will the stopping potential change? [1 mark]

Solution:

Concept:

- Einstein's photoelectric equation is $K_{\max} = h\nu - \phi$, where h is Planck's constant, ν is frequency and $\phi = h\nu_0$ is the work function.
- The stopping-potential magnitude V_s satisfies $eV_s = K_{\max}$, where e is the positive elementary charge.

Step 1: Express the stopping potential For the same photosensitive surface, the work function and threshold frequency remain fixed. Combining the two relations gives

$$V_s = \frac{h}{e}(\nu - \nu_0).$$

Step 2: Compare the frequencies Since $\nu > \nu_0$, photoemission occurs. Increasing ν increases the maximum kinetic energy of the photoelectrons; therefore a larger retarding potential is required to stop even the fastest electrons. The increase obeys $\Delta V_s = (h/e)\Delta\nu$.

Step 3: Conclusion The magnitude of the stopping potential increases linearly with the incident frequency.

Quick Tip to Solve: Keep the metal fixed and use the slope h/e of the stopping-potential versus frequency graph.

8(a). An electron is accelerated through a potential difference of 100 V. Calculate the de Broglie wavelength associated with it. [1 mark]

Solution:

Concept:

- The de Broglie wavelength is $\lambda = h/p$, where p is the electron momentum.
- An electron accelerated from rest through a potential difference V gains kinetic energy $eV = p^2/(2m_e)$.

Step 1: Find the wavelength expression At 100 V, the electron energy is much less than its rest energy, so the nonrelativistic relation applies:

$$\lambda = \frac{h}{\sqrt{2m_e eV}}.$$

Step 2: Substitute the physical constants Use $h = 6.63 \times 10^{-34}$ J s, $m_e = 9.1 \times 10^{-31}$ kg, $e =$

$1.6 \times 10^{-19} \text{ C}$ and $V = 100 \text{ V}$. Then

$$\begin{aligned} p &= \sqrt{2(9.1 \times 10^{-31})(1.6 \times 10^{-19})(100)} \\ &= 5.40 \times 10^{-24} \text{ kg m s}^{-1}, \\ \lambda &= \frac{6.63 \times 10^{-34}}{5.40 \times 10^{-24}} \\ &= 1.23 \times 10^{-10} \text{ m}. \end{aligned}$$

Step 3: Conclusion The electron wavelength is $1.23 \times 10^{-10} \text{ m}$, or approximately 0.123 nm.

Quick Tip to Solve: For an electron accelerated through V volts, use $\lambda \simeq 12.27/\sqrt{V}$ in angstroms; here $\sqrt{100} = 10$.

OR

8(b). A proton and a deuteron are moving with the same speed. Find the ratio of the de Broglie wavelength (λ_p/λ_d) associated with them. [1 mark]

Solution:

Concept:

- For a nonrelativistic particle of mass m and speed v , $\lambda = h/(mv)$.
- A deuteron contains a proton and a neutron, so its mass is approximately twice the proton mass: $m_d \simeq 2m_p$.

Step 1: Write the ratio at equal speed The speed v is the same for the two particles. Consequently,

$$\frac{\lambda_p}{\lambda_d} = \frac{h/(m_p v)}{h/(m_d v)} = \frac{m_d}{m_p}.$$

The factors h and v cancel, leaving an inverse dependence of wavelength on mass.

Step 2: Use the mass relation With $m_d \simeq 2m_p$, the ratio becomes $\lambda_p/\lambda_d \simeq 2$. The heavier deuteron has twice the momentum at the same speed and hence half the wavelength. The square-root mass relation would apply to equal kinetic energies, which is not the condition here.

Step 3: Conclusion $\lambda_p : \lambda_d = 2 : 1$, using the standard approximation $m_d \simeq 2m_p$.

Quick Tip to Solve: At equal speed cancel v directly: wavelength ratios are inverse mass ratios, without a square root.

9. How does the energy gap of an intrinsic semiconductor change when doped with a trivalent impurity? [1 mark]

Solution:

Concept:

- The host semiconductor band gap is $E_g = E_c - E_v$, the separation of the conduction-band edge E_c and valence-band edge E_v .

- A trivalent dopant introduces an acceptor level E_A slightly above the valence band and produces a p-type semiconductor.

Step 1: Locate the impurity energy level The trivalent impurity has one fewer valence electron than the host atom. It accepts a valence electron, leaving a mobile hole. The acceptor level lies within the forbidden gap, close to E_v , so $E_A - E_v$ is small.

Step 2: Distinguish the band gap from the excitation energy In the standard dilute-doping model, the separation of the host valence and conduction bands remains essentially unchanged. However, electrons need only the much smaller energy $E_A - E_v$ to enter acceptor levels and create holes; the energy needed to generate carriers is therefore reduced. Doping introduces a level within the gap rather than moving the two main bands together.

Step 3: Conclusion The intrinsic valence-to-conduction band gap remains essentially unchanged; a trivalent dopant introduces an acceptor level just above the valence band, reducing the energy required to create holes.

Quick Tip to Solve: Draw acceptor levels just above the valence band; distinguish their small activation energy from the host band gap.

10(a). In a p-n junction under equilibrium, there is no net current. Why?

[1 mark]

Solution:

Concept:

- Majority-carrier concentration gradients drive diffusion across a p-n junction.
- The electric field in the depletion region drives a drift current opposite to the diffusion current.

Step 1: Identify the two competing currents Holes diffuse from the p-side to the n-side and electrons from the n-side to the p-side. The exposed ionised donors and acceptors establish a built-in electric field that opposes further majority-carrier diffusion and produces a drift current.

Step 2: Apply the equilibrium condition At thermal equilibrium, the opposing drift and diffusion currents are equal in magnitude. Taking signed currents,

$$I_{\text{net}} = I_{\text{diff}} + I_{\text{drift}} = 0.$$

Individual carriers still move thermally, but there is no sustained net transfer of charge through the unbiased junction.

Step 3: Conclusion There is no net current because the drift current exactly balances the diffusion current in the opposite direction.

Quick Tip to Solve: Equilibrium means cancellation of drift and diffusion currents; it does not mean that all charge carriers stop moving.

10(b). On what factor does the wavelength of the light emitted by an LED depend?

[1 mark]

Solution:

Concept:

- An LED emits photons when electrons recombine with holes near a forward-biased p-n junction.
- The photon energy is approximately the band-gap energy: $hc/\lambda \simeq E_g$, with h Planck's constant and c the speed of light.

Step 1: Relate emitted light to the band gap An electron falling from the conduction band to the valence band loses energy approximately equal to the energy gap of the semiconductor. Radiative recombination releases this energy as a photon, so

$$E_{\text{photon}} = h\nu = \frac{hc}{\lambda} \simeq E_g.$$

Step 2: Identify the material property Rearranging gives $\lambda \simeq hc/E_g$. Thus the emitted wavelength is set primarily by the semiconductor material and its band gap. A larger gap corresponds to a shorter wavelength, whereas a smaller gap corresponds to a longer wavelength.

Step 3: Conclusion The wavelength depends on the band-gap energy of the semiconductor material used in the LED: $\lambda \simeq hc/E_g$.

Quick Tip to Solve: To compare LED colours, compare the material band gaps: a higher photon energy means a shorter wavelength.

11. Two statements are given - one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer to this question from the codes (A), (B), (C) and (D) as given below:

Assertion (A): Higher the range of an ammeter, smaller is its resistance.

Reason (R): To increase the range of the ammeter, additional shunt needs to be connected across it.

[1 mark]

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
 (B) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).
 (C) Assertion (A) is true but Reason (R) is false.
 (D) Assertion (A) is false and Reason (R) is also false.

Correct Answer: (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

Solution:

Concept:

- A galvanometer has resistance G and full-scale current I_g . A low-resistance shunt S bypasses the current above I_g .
- The full-scale terminal voltage stays $I_g G$, so an ammeter of range I has effective resistance $R_A = I_g G / I$.

Step 1: Evaluate Assertion A For the same galvanometer, increasing the full-scale range I decreases

$R_A = I_g G / I$. The assertion that a higher-range ammeter has a smaller resistance is therefore true.

Step 2: Evaluate Reason R To increase the range of an existing ammeter, a suitable additional shunt is connected in parallel across its terminals. The added path carries part of the total current, allowing the total measured current to exceed the original full-scale value. Thus the reason is true.

Step 3: Check whether R explains A The extra parallel shunt reduces the equivalent resistance:

$$R_{\text{new}} = \frac{R_{\text{old}} S}{R_{\text{old}} + S} < R_{\text{old}}.$$

This directly explains the decrease in resistance as the range increases. Option B misses this causal connection, while C and D wrongly reject true statements.

Step 4: Conclusion Both A and R are true, and R correctly explains A. Hence, the correct option is (A).

Quick Tip to Solve: An added parallel shunt simultaneously increases the current range and lowers the equivalent resistance.

12. Two statements are given - one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer to this question from the codes (A), (B), (C) and (D) as given below:

Assertion (A): In most of the northern hemisphere, the north pole of the dip needle tilts upwards.

Reason (R): This gives the direction of magnetic field of the Earth at that place. [1 mark]

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
(B) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).
(C) Assertion (A) is true but Reason (R) is false.
(D) Assertion (A) is false and Reason (R) is also false.

Correct Answer: A is false; R is true as the general dip-needle principle, but false if "This" means the asserted upward direction. No unique printed option is defensible.

Solution:

Concept:

- A freely suspended dip needle aligns with the local terrestrial magnetic field in the vertical magnetic-meridian plane.
- In most of the northern hemisphere, the vertical component of the Earth's magnetic field points downward, so the north-seeking end dips below the horizontal.

Step 1: Evaluate Assertion A The north pole of the dip needle tilts downward in most of the northern hemisphere, not upward as printed. The assertion is false. Near the magnetic equator the dip approaches zero; this does not make upward tilt the usual northern-hemisphere behaviour.

Step 2: Evaluate Reason R The general physical statement that the orientation of a freely suspended dip needle gives the local magnetic-field direction is true: its north-seeking end points along the field. However, if the word "This" refers specifically to the claimed upward tilt, that alleged direction is incorrect. The wording therefore permits two interpretations of the reason.

Step 3: Compare the printed codes On the general physical reading of R, A is false and R is true; none of the printed options represents that combination. Options A and B require both statements true, C

requires A true and R false, and D requires both false. On the narrower reading that R endorses the incorrect upward direction, D can be selected, but this is not a unique interpretation of the printed wording.

Step 4: Conclusion A is false. The statement that a dip needle indicates the local field direction is true, so no printed code fits that reading. The item is ambiguous because the referent of "This" can instead make R false, giving (D).

Quick Tip to Solve: In the northern hemisphere draw the north-seeking end below the horizontal, then judge R independently of the incorrect upward tilt.

13. Two statements are given - one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer to this question from the codes (A), (B), (C) and (D) as given below:

Assertion (A): The angle of minimum deviation for a prism is lesser for red light than that for blue light.

Reason (R): The refractive index of the material of a prism for blue light is greater than that for red light. [1 mark]

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
(B) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).
(C) Assertion (A) is true but Reason (R) is false.
(D) Assertion (A) is false and Reason (R) is also false.

Correct Answer: (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

Solution:

Concept:

- For a prism of angle A_p , the refractive index is $\mu = \sin[(A_p + \delta_m)/2] / \sin(A_p/2)$, where δ_m is minimum deviation.
- In normal dispersion, blue light has a larger refractive index than red light.

Step 1: Judge the Assertion For a fixed prism angle, minimum deviation increases with refractive index. Thus red light, which is refracted less strongly than blue light, has a smaller minimum deviation. The Assertion is true.

Step 2: Judge the Reason Blue light has a shorter wavelength than red light. In ordinary prism glass under normal dispersion, $\mu_b > \mu_r$. The Reason is therefore true.

Step 3: Check the explanatory link Rearrange the prism relation as $\delta_m = 2 \sin^{-1}[\mu \sin(A_p/2)] - A_p$. Since inverse sine increases with its argument in the relevant range, $\mu_b > \mu_r$ gives $\delta_{m,b} > \delta_{m,r}$. The Reason directly explains the Assertion; option (B) incorrectly denies that link, while (C) and (D) incorrectly reject true statements.

Step 4: Conclusion Both statements are true, and the Reason correctly explains the Assertion. Hence, the correct option is (A).

Quick Tip to Solve: For the same prism, compare refractive indices first: larger μ means larger minimum deviation.

14. Two statements are given - one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer to this question from the codes (A), (B), (C) and (D) as given below:

Assertion (A): In his study of photoelectric emission, Hallwachs connected a negatively charged zinc plate to an electroscope. He found that negatively charged particles were emitted from the zinc plate under the action of visible light.

Reason (R): An uncharged zinc plate becomes positively charged when it is irradiated by visible light. [1 mark]

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

(B) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

(C) Assertion (A) is true but Reason (R) is false.

(D) Assertion (A) is false and Reason (R) is also false.

Correct Answer: (D) Assertion (A) is false and Reason (R) is also false.

Solution:

Concept:

- Photoelectric emission requires photon energy $h\nu$ at least equal to the metal work function ϕ ; the threshold frequency is $\nu_0 = \phi/h$.
- In the Hallwachs experiment, ultraviolet light ejects electrons from zinc; ordinary visible light does not provide the required photon energy.

Step 1: Judge the Assertion The zinc plate and electroscope are correctly named, but the claim of emission under visible light is false. Hallwachs used ultraviolet radiation. A negatively charged zinc plate loses electrons and discharges under suitable ultraviolet illumination, not ordinary visible illumination.

Step 2: Judge the Reason An initially uncharged zinc plate can become positively charged if ultraviolet radiation ejects electrons. The printed Reason specifies visible light, whose photons lie below the zinc emission threshold. It is therefore false.

Step 3: Check the code Both statements fail because each names the wrong incident radiation. There is no valid explanatory link to assess between them. Options (A) and (B) require both statements to be true, and option (C) requires the Assertion to be true; none fits the printed wording.

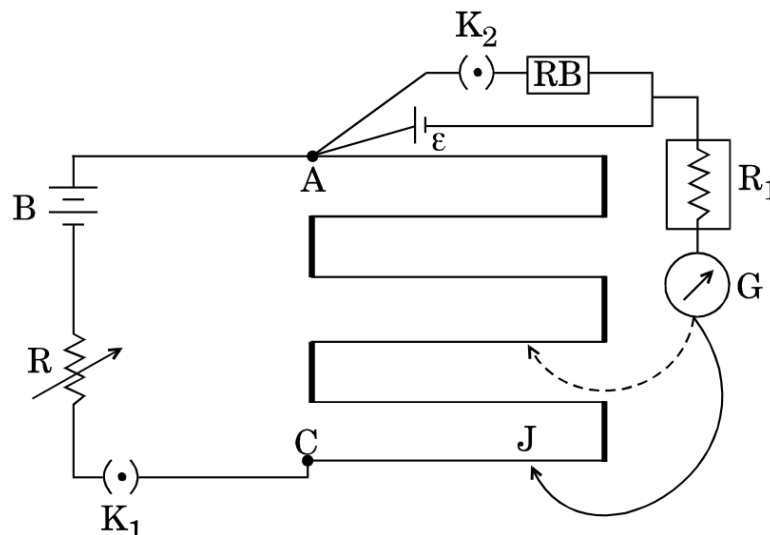
Step 4: Conclusion Both the Assertion and the Reason are false. Hence, the correct option is (D).

Quick Tip to Solve: In Hallwachs questions, check the radiation named: zinc responds photoelectrically to ultraviolet light.

Section B (8 marks)

Note : Questions number 15 and 16 are case-study based questions and are compulsory. Attempt any 4 sub-parts from each question. Each sub-part carries 1 mark.

15. A potentiometer draws a steady current from the battery (B) of emf E in the circuit as shown in the figure. The balancing length for the cell of emf ε when key K_2 is kept open, is ℓ_1 . If key K_2 is closed and a resistance S is introduced in the resistance box (RB), the balancing length becomes ℓ_2 . [4 marks]



15(i). The balancing length $\ell_1 > \ell_2$ because [1 mark]

- (A) $E > \varepsilon$.
- (B) positive terminal of the battery is connected to positive terminal of the cell.
- (C) the voltage drop across the cell is greater than that across the battery.
- (D) terminal potential difference of the cell is less than its emf.

Correct Answer: (D) terminal potential difference of the cell is less than its emf.

Solution:

Concept:

- A uniform potentiometer wire has constant potential gradient k , giving a balanced potential difference $V = k\ell$.
- For a cell of emf ε , internal resistance r and load current I , terminal voltage is $V = \varepsilon - Ir$.

Step 1: Compare open and loaded cell voltages With K_2 open, the cell supplies no load current at null balance, so the potentiometer measures its emf: $\varepsilon = k\ell_1$. Closing K_2 connects the resistance box as a load; current flows and the internal resistance causes a drop Ir . The measured voltage is then $V = k\ell_2 = \varepsilon - Ir < \varepsilon$.

Step 2: Compare the lengths and options The primary circuit current is steady, so k is unchanged. Hence a lower measured voltage requires a shorter balancing length: $\ell_2 < \ell_1$. The battery emf and

polarity conditions in (A) and (B) enable balancing but do not explain this change on loading. Option (C) is neither the relevant comparison nor generally true.

Step 3: Conclusion The loaded cell terminal potential difference is smaller than its emf. Hence, the correct option is (D).

Quick Tip to Solve: Use the same potential gradient for both balances: open cell gives emf, loaded cell gives terminal voltage.

15(ii). The high resistance R_1 in the circuit is used to

[1 mark]

- (A) keep the balance point near the mid-point of the potentiometer wire AC.
- (B) reduce the current supplied by the battery (B) to zero at balance point.
- (C) protect the galvanometer from damage due to large current.
- (D) make the galvanometer more sensitive.

Correct Answer: (C) protect the galvanometer from damage due to large current.

Solution:

Concept:

- The galvanometer detects current in the secondary branch; at balance its current is zero.
- A large series resistance limits an off-balance current by Ohm's law, $I = \Delta V / R_{\text{total}}$.

Step 1: Identify the protective resistor The printed circuit places R_1 in series with the galvanometer in the cell-and-jockey branch. Before a balance point is found, the potential difference across this branch can be appreciable. Adding high resistance keeps the resulting galvanometer current small and protects its delicate coil.

Step 2: Reject the other functions The primary rheostat R , not R_1 , controls the current through wire AC and its potential gradient, so (A) is not the role of this protective resistor. At null balance only the galvanometer branch has zero current; the battery still supplies the primary wire, rejecting (B). A protective series resistance does not increase the intrinsic sensitivity of the galvanometer, rejecting (D).

Step 3: Conclusion The high resistance protects the galvanometer from damage due to large current. Hence, the correct option is (C).

Quick Tip to Solve: Trace the branch: a high resistance in series with the galvanometer limits current before null balance.

15(iii). If the key K_2 is kept open and resistance R is increased, the balancing length AJ of the wire will [1 mark]

- (A) increase.
- (B) decrease.
- (C) remain unaffected.
- (D) become zero.

Correct Answer: (A) increase.

Solution:

Concept:

- The primary current is $I_p = E/(R + R_w + r_B)$, where R_w is wire resistance and r_B is the driving battery internal resistance.
- The potential gradient is $k = I_p R_w / L$; an unloaded cell balances at length $AJ = \varepsilon / k$.

Step 1: Find how the gradient changes The printed question increases the primary rheostat R . With the same battery and potentiometer wire, this increases total primary resistance and reduces I_p . Consequently the potential drop per unit length k decreases. The high protective resistance R_1 in the secondary branch is not the resistance varied here.

Step 2: Find how the balancing length changes Keeping K_2 open means the cell remains unloaded at balance and its emf ε stays fixed. Thus $AJ = \varepsilon / k$ increases when k decreases, provided a balance remains within the wire. A decrease would require a larger gradient; an unchanged length would require unchanged gradient. The length cannot become zero for a nonzero cell emf.

Step 3: Conclusion The balancing length AJ increases. Hence, the correct option is (A).

Quick Tip to Solve: Increasing the driving-circuit rheostat lowers the gradient, so the same cell needs a longer balance length.

15(iv). Two potentiometers P and Q use the same wire of lengths 5 m and 10 m respectively, between A and C. They are connected to battery (B) in the circuit separately. The potentiometer which is preferred to compare the emfs of two cells is [1 mark]

- (A) P, because potential gradient is less.
- (B) Q, because potential gradient is less.
- (C) P, because current is more.
- (D) Q, because resistance of the potentiometer is less.

Correct Answer: (B) Q, because potential gradient is less.

Solution:

Concept:

- A potentiometer is more sensitive when its potential gradient $k = V_{AC}/L$ is smaller.
- A small emf difference $\Delta\varepsilon$ produces a balancing-length difference $\Delta\ell = \Delta\varepsilon/k$.

Step 1: Compare the two wires For the same wire material and cross-section, the resistance of the 10 m wire is twice that of the 5 m wire. Connected separately to the same driving battery under the stated arrangement, the longer wire has the smaller potential gradient. In the ideal limit of the same potential across each wire, doubling length directly halves k .

Step 2: Choose the more sensitive comparison A smaller gradient gives a larger length difference for the same small difference between cell emfs. Q therefore lets the balance positions be distinguished more accurately. Option (A) wrongly assigns the smaller gradient to P. Although P can draw a greater current, that is not the preferred sensitivity criterion in (C). Option (D) wrongly states that the longer wire has less resistance.

Step 3: Conclusion Potentiometer Q is preferred because its potential gradient is less. Hence, the correct option is (B).

Quick Tip to Solve: For precise emf comparison, choose the smaller potential gradient: it spreads a small emf difference over more wire.

15(v). The reason for preferring manganin wire in a potentiometer is because

[1 mark]

- (A) of high value of its temperature coefficient of resistivity.
- (B) of its low resistivity.
- (C) of low value of its temperature coefficient of resistivity.
- (D) its resistance decreases with increase in temperature.

Correct Answer: (C) of low value of its temperature coefficient of resistivity.

Solution:

Concept:

- Wire resistivity varies approximately as $\rho(T) = \rho(T_0)[1 + \alpha(T - T_0)]$, where α is its temperature coefficient.
- A stable resistance helps maintain a stable potential gradient along the potentiometer wire.

Step 1: Identify the useful material property Current through the wire can warm it, and room temperature can also change. Manganin has a very small temperature coefficient of resistivity, so these temperature changes cause only a small fractional change in resistance. Its resistance per unit length, and hence the balancing calibration, stays nearly stable.

Step 2: Compare all options Option (C) states the required low temperature coefficient. A high coefficient in (A) would make the resistance more temperature-sensitive. Low resistivity in (B) is not the reason given; manganin is an alloy used for relatively high, stable resistivity. A general decrease of resistance with increasing temperature in (D) is not the defining property needed for a stable potentiometer.

Step 3: Conclusion Manganin is preferred for its low temperature coefficient of resistivity. Hence, the correct option is (C).

Quick Tip to Solve: For potentiometer wire, look for resistance stability with temperature rather than simply low resistance.

16. In Rutherford's nuclear model of the atom, the entire positive charge and most of the mass of the atom are concentrated in the nucleus. The electrons move in orbits around the nucleus. The nucleus is made of protons and neutrons. Because the nucleus is extremely small as compared to the atom, most of an atom is empty space. The protons and the neutrons are held together in the nucleus by very strong nuclear forces. [4 marks]

16(i). The radius R of a nucleus of mass number A is given by [1 mark]

- (A) $R = R_0 A^3$
- (B) $R = R_0 A^{1/3}$
- (C) $R = R_0^3 A$
- (D) $R = R_0^3 A^{1/3}$

Correct Answer: (B) $R = R_0 A^{1/3}$

Solution:

Concept:

- Nuclei have approximately constant density, so their volume is proportional to mass number A .
- Treating a nucleus as a sphere gives $V = (4/3)\pi R^3$; R_0 is a constant length of order 10^{-15} m.

Step 1: Relate volume to the number of nucleons A nucleus contains A nucleons with nearly equal masses. Its mass is therefore proportional to A . Since nuclear density is approximately independent of A , volume must also grow in proportion to A . Thus $R^3 \propto A$, rather than $R \propto A$ or $R \propto A^3$.

Step 2: Take the cube root and test the options Taking the cube root gives $R = R_0 A^{1/3}$, the printed option (B). Option (A) predicts a much stronger, incorrect dependence on nucleon count. In (C) and (D), R_0^3 has dimensions of volume, so multiplying it by a dimensionless power of A cannot give a radius.

Step 3: Conclusion The nuclear radius is $R = R_0 A^{1/3}$. Hence, the correct option is (B).

Quick Tip to Solve: Remember volume grows with nucleon count; the radius therefore grows with its cube root, not the count itself.

16(ii). The ratio of nuclear density of nuclei X^{27} to Y^8 is [1 mark]

- (A) 3 : 2
- (B) 27 : 8
- (C) 1 : 1
- (D) 2 : 3

Correct Answer: (C) 1 : 1

Solution:

Concept:

- Nuclear mass is approximately $M = Am_N$, where m_N is the mean nucleon mass.
- The radius law $R = R_0 A^{1/3}$ gives volume $V = (4/3)\pi R_0^3 A$.

Step 1: Derive the nuclear density Density equals mass divided by volume. Substituting the mass and radius expressions gives

$$\rho = \frac{Am_N}{(4/3)\pi R_0^3 A} = \frac{3m_N}{4\pi R_0^3}.$$

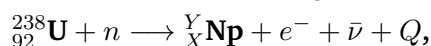
The mass number cancels; all nuclei have approximately the same nuclear density in this model.

Step 2: Compare the nuclei and alternatives For X^{27} and Y^8 , mass and nuclear volume both have ratio $27 : 8$, so their quotient has ratio $1 : 1$. The ratio $3 : 2$ in (A) is the ratio of their radii, and $27 : 8$ in (B) is their approximate mass ratio. The inverse radius ratio in (D) is also not a density ratio.

Step 3: Conclusion The ratio of nuclear densities is $\rho_X : \rho_Y = 1 : 1$. Hence, the correct option is (C).

Quick Tip to Solve: For nuclear density ratios, cancel A between mass and volume; different mass numbers do not change the result.

16(iii). In the following nuclear reaction



the values of X and Y are

[1 mark]

(A) $X = 92$; $Y = 238$

(B) $X = 92$; $Y = 239$

(C) $X = 93$; $Y = 239$

(D) $X = 93$; $Y = 238$

Correct Answer: (C) $X = 93$; $Y = 239$

Solution:

Concept:

- In a nuclear reaction, total nucleon number and electric charge are conserved. A neutron has mass number 1 and atomic number 0.
- A beta-minus particle is an electron, ${}_{-1}^0e$; an antineutrino has zero electric charge and zero nucleon number.

Step 1: Conserve the nucleon number The uranium nucleus contributes 238 nucleons and the incoming neutron contributes one. Neither the emitted electron nor the antineutrino contributes a nucleon. Therefore $238 + 1 = Y$, so $Y = 239$. Options (A) and (D), with $Y = 238$, omit the captured neutron.

Step 2: Conserve electric charge Equating the charge numbers on both sides gives $92 + 0 = X - 1 + 0$, hence $X = 93$. The electron carries away one negative elementary charge, so the daughter nucleus has one more positive charge than uranium. Option (B) conserves nucleon number but fails charge conservation; only option (C) satisfies both conditions.

Step 3: Conclusion $X = 93$ and $Y = 239$, giving ${}_{93}^{239}\text{Np}$. The correct option is (C).

Quick Tip to Solve: Balance the superscripts and subscripts separately, giving the emitted electron subscript -1.

16(iv). The saturation property of the nuclear forces is due to the fact that they are

[1 mark]

- (A) charge independent forces.
- (B) non-central forces.
- (C) spin-dependent forces.
- (D) short-range forces.

Correct Answer: (D) short-range forces.

Solution:

Concept:

- Nuclear force is effective only over distances comparable to nuclear dimensions, of the order of a few femtometres.
- Saturation means that each nucleon interacts strongly with only a limited number of nearby nucleons rather than with every nucleon in a large nucleus.

Step 1: Connect range with saturation When more nucleons are added to a nucleus, distant nucleons lie outside the effective range of the force acting on a particular nucleon. Its strong interactions remain mainly with its immediate neighbours. Thus its binding contribution does not grow in proportion to the total number of nucleons. This limited neighbourhood produces saturation.

Step 2: Check the other properties Charge independence concerns similarity of the nuclear interaction between different nucleon pairs. Non-central character concerns dependence on orientation, while spin dependence concerns the nucleons' spin states. None of these properties by itself limits how many distant nucleons can interact strongly with a given nucleon. Therefore options (A), (B) and (C) do not explain saturation.

Step 3: Conclusion Saturation is due to the short range of nuclear forces. The correct option is (D).

Quick Tip to Solve: Link saturation to a finite neighbourhood: short range limits the number of interacting nucleons.

16(v). In Geiger-Marsden scattering experiment, thin gold foil is used to scatter alpha particles because alpha particles will

[1 mark]

- (A) not suffer more than one scattering and gold nucleus is 50 times heavier than alpha particle.
- (B) not suffer more than one scattering and gold nucleus is lighter than alpha particle.
- (C) not suffer more than few scatterings and gold nucleus is 25 times heavier than alpha particle.
- (D) suffer more than one scattering and gold nucleus is 25 times heavier than alpha particle.

Correct Answer: (A) not suffer more than one scattering and gold nucleus is 50 times heavier than alpha particle.

Solution:

Concept:

- A very thin foil makes multiple scattering negligible, so an observed deflection can be attributed to a single significant encounter with a nucleus.
- The approximate nuclear mass ratio is $m_{\text{Au}}/m_{\alpha} \simeq 197/4 \simeq 49.25$, about 50. The much heavier gold nucleus recoils comparatively little.

Step 1: Explain the thinness of the foil Alpha particles must pass through only a small thickness of

material. This keeps the probability of successive significant scatterings small and allows Rutherford's single-scattering interpretation. The printed wording "not suffer more than one scattering" expresses this experimental approximation; it does not mean that repeated encounters are mathematically impossible.

Step 2: Compare the nuclear masses and options Gold has mass number approximately 197, whereas an alpha particle has mass number 4. Their ratio is close to 50, as stated in option (A). Option (B) incorrectly makes gold lighter than the alpha particle. Options (C) and (D) use the incorrect ratio 25, and their descriptions of repeated scattering do not give the intended single-scattering condition.

Step 3: Conclusion The foil gives essentially single scattering, and the gold nucleus is about 50 times heavier than an alpha particle. The correct option is (A).

Quick Tip to Solve: Check both clauses: thin foil implies single scattering, and the mass ratio is approximately 197/4.

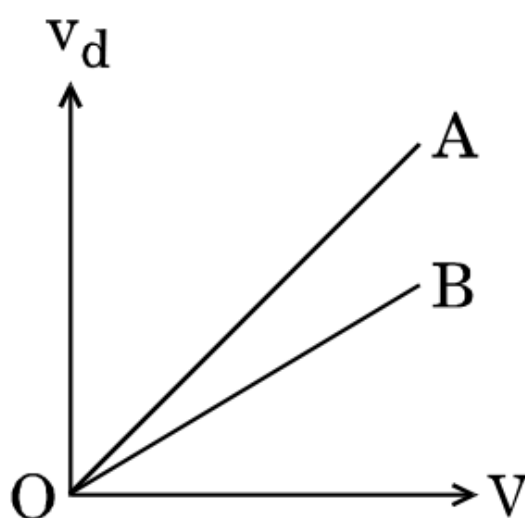
Section C (18 marks)

17. Define drift velocity of electrons in a conductor connected across a battery. Figure shows variation of the drift velocity (v_d) of electrons in two copper wires A and B of different lengths versus the potential difference (V) applied across their ends.

(i) What does the slope of the line represent?

(ii) Which one of the two wires is longer?

[2 marks]



Solution:

Concept:

- Drift velocity is the average velocity acquired by free electrons in a conductor under an applied electric field. For electrons its direction is opposite to the electric field and conventional current.
- Its magnitude is $v_d = eE\tau/m_e$, where e is the magnitude of electronic charge, m_e is electron mass and τ is the mean relaxation time. For a uniform wire of length l , $E = V/l$.

Step 1: Interpret the slope of the graph The plotted positive quantity is the magnitude of the drift

velocity. Substituting the electric field into its expression gives

$$v_d = \frac{e\tau}{m_e l} V, \quad s = \frac{\Delta v_d}{\Delta V} = \frac{e\tau}{m_e l} = \frac{\mu}{l},$$

where $\mu = e\tau/m_e$ is electron mobility. Thus the slope represents drift speed per unit applied potential difference, or mobility divided by wire length. It is not the mobility itself, since the horizontal axis is potential difference rather than electric field.

Step 2: Compare the lengths of the two wires Both wires are copper; taking them at the same temperature gives the same relaxation time and mobility. Hence their slopes are inversely proportional to their lengths. In the printed figure, line A is steeper than line B: $s_A > s_B$. Therefore $l_A < l_B$. A longer wire has a smaller electric field for the same applied voltage and consequently a smaller drift speed.

Step 3: Conclusion Drift velocity is the electrons' average velocity due to the applied field, opposite to that field. (i) The slope is $e\tau/(m_e l) = \mu/l$. (ii) Wire B is longer.

Quick Tip to Solve: For the same material and temperature, a steeper drift-speed versus voltage line means a shorter wire.

18(a). (i) The angle of dip at a location in southern India is about 18° . Would you expect a greater or smaller dip angle in Britain? Justify your answer.

(ii) "The declination in India is small, it being $0^\circ 41'$ E at Delhi and $0^\circ 58'$ W at Mumbai." What is the significance of this statement? [2 marks]

Solution:

Concept:

- The angle of dip δ is the angle made by Earth's magnetic field with the horizontal in the magnetic meridian. Its components satisfy $\tan \delta = B_V/B_H$, where B_V and B_H are the vertical and horizontal field magnitudes.
- Declination is the angle between the geographic meridian (true north-south direction) and the magnetic meridian. East or west specifies which side of true north contains magnetic north.

Step 1: Compare the dip in Britain and southern India Britain is at a higher northern magnetic latitude and is nearer to the northern magnetic polar region than southern India. Earth's field is more steeply inclined to the horizontal there, with a larger vertical-to-horizontal component ratio. Therefore its dip angle is expected to be greater than 18° . In the usual broad pattern, dip is zero at the magnetic equator and approaches 90° in magnitude at a magnetic pole.

Step 2: Interpret the quoted declinations At Delhi, magnetic north lies $0^\circ 41'$ east of geographic north. At Mumbai, it lies $0^\circ 58'$ west of geographic north. Since both deviations are less than one degree, the magnetic and geographic meridians nearly coincide at these locations. A freely suspended compass therefore indicates approximately true north, with the stated small local correction required for precise navigation. These values describe a horizontal directional deviation, not the inclination of

the field below the horizontal.

Step 3: Conclusion (i) Britain has a greater expected dip angle than 18° . (ii) The small declinations mean that magnetic north is close to true north, slightly east at Delhi and slightly west at Mumbai.

Quick Tip to Solve: Dip measures tilt from the horizontal; declination measures the compass direction's deviation from true north.

OR

18(b). The magnetic moment of a circular coil carrying current I , having N turns, each of radius r , is M . Find the magnetic moment of the same coil if it is unwound and rewound into a coil having $2N$ turns for the same current. [2 marks]

Solution:

Concept:

- The magnitude of the magnetic moment of a circular coil is $M = NIA = NI\pi r^2$, where N is the number of turns, I the current and A the area of each turn.
- Rewinding the same wire conserves its total length. For closely wound circular turns, ignoring connecting leads, this length is $L = N(2\pi r)$.

Step 1: Find the radius of each new turn Let the radius after rewinding be r' . The same wire now forms $2N$ circular turns, so conservation of wire length gives

$$N(2\pi r) = (2N)(2\pi r'), \quad r' = \frac{r}{2}.$$

Doubling the number of turns halves each turn's circumference and hence halves its radius. For the same continuous wire, the product of turn count and turn radius stays fixed. The new area of each turn is $A' = \pi(r/2)^2 = A/4$; it does not merely halve.

Step 2: Calculate the new magnetic moment The current remains I , as specified. Using the new number of turns and the reduced area,

$$M' = (2N)I\pi \left(\frac{r}{2}\right)^2 = \frac{NI\pi r^2}{2} = \frac{M}{2}.$$

Although there are twice as many turns, each encloses only one quarter of the initial area. Their combined area factor is therefore $2 \times 1/4 = 1/2$. The moment vector remains normal to the coil plane, with its sense determined by the current's circulation.

Step 3: Conclusion The magnetic moment of the rewound coil for the same current is $M' = M/2$.

Quick Tip to Solve: With fixed wire length and current, circular-coil magnetic moment varies inversely with the number of turns.

19. Two identical circular discs, one of copper and another of aluminium, are rotated about their geometrical axes with same angular speed in the same magnetic field acting perpendicular to their planes. Compare the (i) induced emf, and (ii) induced current produced in discs between its centre and edge. Justify your answers. [2 marks]

Solution:

Concept:

- A point at radius r in a disc rotating with angular speed ω has speed $v = \omega r$. In a perpendicular magnetic field B , the motional emf across a radial element is $d\varepsilon = B\omega r dr$.
- For identical geometry, effective electrical resistance is proportional to resistivity ρ . Copper has lower resistivity than aluminium, and current in a completed circuit obeys $I = \varepsilon/R$.

Step 1: Compare the centre-to-edge emfs Let the common disc radius be a . Adding the emfs of radial elements gives

$$\varepsilon = \int_0^a B\omega r dr = \frac{1}{2}B\omega a^2.$$

The two discs have the same B , ω and a ; their materials do not enter this expression. The emf magnitudes and their polarities, for the same sense of rotation, are therefore identical.

Step 2: Compare the currents for the same contact arrangement With the centre and rim connected to complete the circuit in the same way, the copper disc offers less resistance because $\rho_{\text{Cu}} < \rho_{\text{Al}}$. Therefore

$$\frac{I_{\text{Cu}}}{I_{\text{Al}}} = \frac{R_{\text{Al}}}{R_{\text{Cu}}} > 1.$$

If disc resistance alone is considered, this ratio is $\rho_{\text{Al}}/\rho_{\text{Cu}}$. Adding the same external resistance reduces the difference but leaves copper's current larger. An isolated disc develops the emf but carries no steady circuit current; the requested comparison assumes a closed centre-to-rim circuit.

Step 3: Conclusion (i) $\varepsilon_{\text{Cu}} = \varepsilon_{\text{Al}} = \frac{1}{2}B\omega a^2$. (ii) For comparable completed circuits, $I_{\text{Cu}} > I_{\text{Al}}$, because copper has the lower resistivity.

Quick Tip to Solve: Compare geometry and speed for motional emf; compare material resistivity only when calculating current.

20. (a) The electric field of an electromagnetic wave is represented as $E_x = E_0 \sin(\omega t + kz)$.

(i) In which direction is the wave propagating?

(ii) In which direction does the magnetic field oscillate?

(b) Write two characteristics of electromagnetic waves. [2 marks]

Solution:

Concept:

- A wavefront moves so that its phase remains constant. For $\omega t + kz$, with positive ω and k , constant phase gives motion toward decreasing z .
- Electromagnetic waves are transverse: $\vec{E}$, $\vec{B}$ and the propagation direction are mutually perpendicular. Energy flows along $\vec{E} \times \vec{B}$.

Step 1: Determine the direction of propagation in part (a)(i) Tracking a point of constant phase gives

$$\omega t + kz = \text{constant}, \quad \frac{dz}{dt} = -\frac{\omega}{k}.$$

Hence the wave travels along the negative z -axis. The plus sign before kz is essential: a phase $\omega t - kz$ would instead describe positive z -direction propagation.

Step 2: Determine the magnetic-field direction in part (a)(ii) The given electric field is along the x -axis. Thus the magnetic field oscillates along the y -axis. To give energy flow along $-\hat{z}$, its instantaneous sense must satisfy

$$\hat{x} \times (-\hat{y}) = -\hat{z}.$$

In vacuum, a compatible expression is

$$\vec{B} = -\frac{E_0}{c} \sin(\omega t + kz) \hat{y}.$$

Its sense reverses as the sinusoidal field reverses, while its oscillation axis remains y .

Step 3: State two characteristics for part (b) First, electromagnetic waves are transverse: both fields are perpendicular to each other and to the direction of travel. Second, they require no material medium and propagate in vacuum with speed

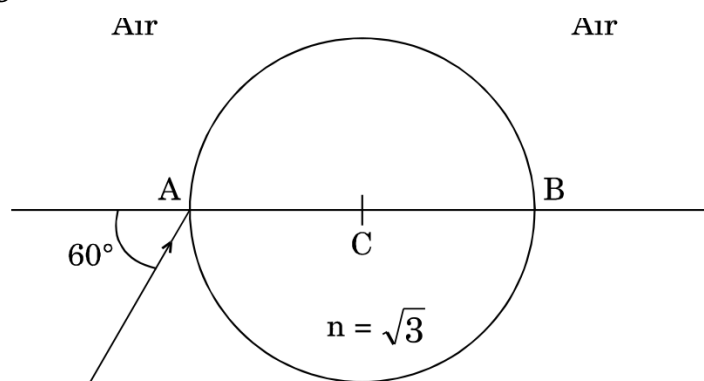
$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}} \simeq 3.0 \times 10^8 \text{ m s}^{-1}.$$

Here μ_0 and ϵ_0 are the permeability and permittivity of vacuum.

Step 4: Conclusion (a)(i) The wave propagates along $-z$. (a)(ii) The magnetic field oscillates along the y -axis, in the negative y sense when $E_x > 0$. (b) Electromagnetic waves are transverse and travel through vacuum without a material medium at speed c .

Quick Tip to Solve: Keep the phase constant to find wave direction, then use the right-hand rule for the electric-field cross magnetic-field direction.

21. A ray of light falls on a transparent sphere of $n = \sqrt{3}$ at an angle of incidence 60° with the diameter AB of the sphere having centre C. The ray emerges from the sphere parallel to the line AB. Find the angle of emergence. [2 marks]



Solution:

Concept:

- At a spherical refracting surface, the normal is the radius through the point of incidence. Snell's law is $n_1 \sin i = n_2 \sin r$, with angles measured from the normal.
- The two radii joining the centre to the ends of an internal ray chord form an isosceles triangle. The chord makes equal angles with these radii, so the internal incidence angle at emergence equals the first refraction angle.

Step 1: Apply refraction at A The ray strikes the sphere at A, the end of diameter AB. Therefore AC, lying along AB, is the normal there and the given angle is the true incidence angle $i = 60^\circ$. Taking air's refractive index as unity and the sphere's index as $n = \sqrt{3}$,

$$\sin 60^\circ = \sqrt{3} \sin r, \quad \sin r = \frac{\sqrt{3}/2}{\sqrt{3}} = \frac{1}{2}.$$

The physical refraction angle is $r = 30^\circ$.

Step 2: Apply refraction at the exit point Let P be the point where the internal ray AP meets the sphere again. Since $CA = CP$, triangle ACP is isosceles. Hence the angle between AP and the exit normal CP is also 30° ; this is the internal angle of incidence i_2 . For refraction from the sphere into air,

$$\sqrt{3} \sin 30^\circ = \sin e, \quad \sin e = \frac{\sqrt{3}}{2}.$$

The emergence angle is taken between the emerging ray and the outward normal, so it lies between 0° and 90° . Also, CP makes 60° with AB; the emerging ray consequently travels parallel to AB, consistent with the stated condition.

Step 3: Conclusion The angle of emergence, measured from the normal at the exit surface, is $e = 60^\circ$.

Quick Tip to Solve: For a sphere surrounded by the same medium on both sides, use the equal base angles of the internal chord triangle to relate entry and exit.

22(a). How would the angular width of central maximum of diffraction pattern be affected when (i) width of the slit is decreased, and (ii) monochromatic light is replaced by polychromatic light? Justify your answers. [2 marks]

Solution:

Concept:

- In single-slit diffraction, the first minima satisfy $a \sin \theta_1 = \lambda$, where a is slit width and λ is wavelength. The central maximum lies between the first minima on either side.
- Its full angular width is $\Delta\theta = 2 \sin^{-1}(\lambda/a)$, approximately $2\lambda/a$ for small diffraction angles. Each wavelength in polychromatic light forms its own diffraction pattern.

Step 1: Decrease the slit width in part (i) At fixed wavelength, decreasing a increases λ/a . Consequently the first minima move farther away from the central direction and the central maximum becomes wider. In the usual small-angle approximation,

$$\Delta\theta \simeq \frac{2\lambda}{a}.$$

Thus, for example, halving the slit width approximately doubles the angular width. This statement assumes the usual regime where the first minima exist.

Step 2: Replace monochromatic light by polychromatic light in part (ii) There is now a range of wavelengths rather than one λ . Their individual central maxima have different widths: longer wavelengths have wider maxima and shorter wavelengths have narrower maxima. For visible light,

$$\Delta\theta_{\text{red}} > \Delta\theta_{\text{violet}}.$$

All wavelengths have their central peak at $\theta = 0$, so the overlapping centre appears white when white light is used. Away from the centre, wavelength-dependent minima give coloured fringes. The superposed pattern therefore has no single wavelength-independent angular width or common first dark minimum; its central region is spread over the different component widths.

Step 3: Conclusion (i) Decreasing the slit width increases the angular width of the central maximum. (ii) Polychromatic light produces overlapping central maxima of different widths, with the longest wavelengths widest; white light gives a white centre with colour-dependent edges instead of one sharp monochromatic width.

Quick Tip to Solve: Start with twice the first-minimum angle: slit width is in the denominator, while each component wavelength is in the numerator.

OR

22(b). In interference of light, write the expression for the intensity of resultant wave if I_0 is the intensity of light wave from each slit. Hence, obtain an expression for the intensity of resultant wave if the two sources are (i) incoherent, and (ii) coherent. [2 marks]

Solution:

Concept:

- Interference comes from adding wave amplitudes before squaring. For two waves of intensities I_1, I_2 and relative phase ϕ , the resultant intensity is $I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \phi$.

- Coherent sources maintain a constant relative phase at a given observation point. For incoherent sources, the relative phase varies randomly, so the interference term averages to zero over the detector's observation time.

Step 1: Write the resultant intensity for equal slit intensities Let the electric-field amplitudes from the slits both be E_0 , so each individual intensity is $I_0 \propto E_0^2$. Vector addition of the oscillations gives a squared resultant amplitude $2E_0^2(1 + \cos \phi)$. Therefore

$$I = I_0 + I_0 + 2\sqrt{I_0 I_0} \cos \phi = 2I_0(1 + \cos \phi).$$

Here ϕ includes both the source phase difference and the phase acquired from the path difference to the observation point.

Step 2: Obtain the incoherent result in part (i) For incoherent sources, the phase difference is random and rapidly changing. Its time average satisfies $\langle \cos \phi \rangle = 0$, hence

$$\langle I \rangle = 2I_0.$$

Intensities therefore add, and there is no stable pattern of bright and dark interference fringes.

Step 3: Obtain the coherent result in part (ii) A stable phase difference preserves the interference term. Using $1 + \cos \phi = 2 \cos^2(\phi/2)$,

$$I = 4I_0 \cos^2\left(\frac{\phi}{2}\right).$$

The phase generally differs from point to point. At constructive interference $\phi = 2m\pi$, $I_{\max} = 4I_0$; at destructive interference $\phi = (2m + 1)\pi$, $I_{\min} = 0$, where m is an integer. Coherence alone does not imply that every point has maximum intensity.

Step 4: Conclusion The general equal-intensity expression is $I = 2I_0(1 + \cos \phi)$. (i) Incoherent sources give $\langle I \rangle = 2I_0$. (ii) Coherent sources give $I = 4I_0 \cos^2(\phi/2)$, ranging from zero to $4I_0$.

Quick Tip to Solve: Keep the cosine interference term for coherent sources; average it to zero for incoherent sources before reporting the measured intensity.

23. For the light of wavelength 400 nm incident on the cathode of a photocell, the stopping potential is 6 V. If the wavelength of incident light is increased to 600 nm, calculate the new stopping potential. (Take $h = 4.14 \times 10^{-15} \text{ eV} \cdot \text{s}$) [2 marks]

Solution:

Concept:

- Einstein's photoelectric equation is $eV_s = hc/\lambda - \phi$, where V_s is stopping potential and ϕ is the cathode's work function.
- For the same cathode, subtracting the two equations eliminates ϕ . A photon energy in electron-volts has the same numerical value as the stopping potential in volts after subtracting the work function in electron-volts.

Step 1: Eliminate the work function Let $\lambda_1 = 400 \text{ nm}$, $\lambda_2 = 600 \text{ nm}$ and $V_1 = 6 \text{ V}$. Using $c =$

$3.0 \times 10^8 \text{ m s}^{-1}$, the printed value of h gives $hc = 1242 \text{ eV nm}$. Subtraction gives

$$e(V_2 - V_1) = hc \left(\frac{1}{\lambda_2} - \frac{1}{\lambda_1} \right).$$

The bracket is negative because the wavelength increases, so the stopping potential must decrease.

Step 2: Substitute the printed data Express photon energies in electron-volts and potentials in volts:

$$\begin{aligned} V_2 - V_1 &= 1242 \left(\frac{1}{600} - \frac{1}{400} \right) \text{ V} \\ &= -1.035 \text{ V}, \\ V_2 &= 6 - 1.035 = 4.965 \text{ V}. \end{aligned}$$

There is a physical inconsistency in the printed data: the first photon has only 3.105 eV, whereas $eV_1 = 6 \text{ eV}$. This would require $\phi = 3.105 - 6 = -2.895 \text{ eV}$, impossible for an ordinary photocathode. The result above is the formal calculation requested using the stated values.

Step 3: Conclusion The stopping potential calculated from the printed data is

$$V_2 = 4.965 \text{ V} \approx 4.97 \text{ V}.$$

The initial data are physically inconsistent for an ordinary photocell.

Quick Tip to Solve: Subtract the two photoelectric equations first; this removes the unknown work function and fixes the sign of the voltage change.

24(a). With the help of the circuit diagram, explain the working of a diode as a half-wave rectifier.

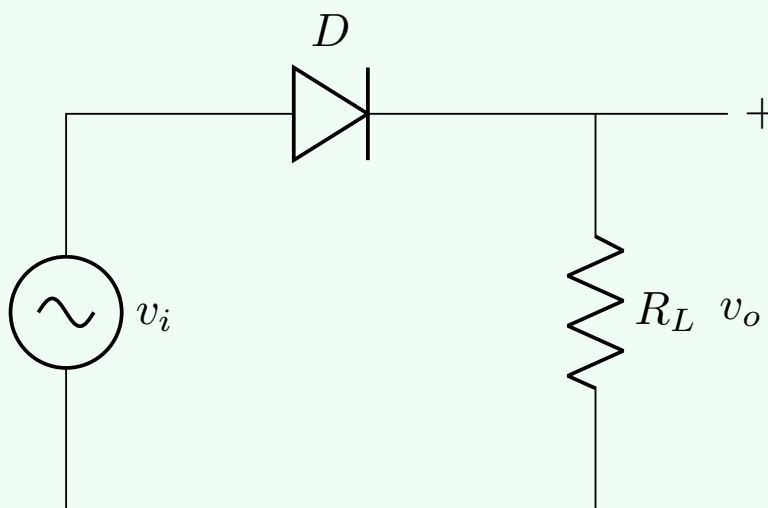
[2 marks]

Solution:

Concept:

- A junction diode conducts in forward bias and blocks current in reverse bias. The anode is connected to the p-side and the cathode to the n-side.
- Rectification converts alternating voltage into a unidirectional output. A half-wave rectifier passes only one half of each input cycle through a load resistor R_L .

Step 1: Connect the diode and load in series Connect the ac input across a series combination of diode D and load resistance R_L , as in the circuit. Take the output across the load, with its upper terminal labelled positive. The diode's anode faces the upper input terminal, so this terminal becoming positive permits conduction. The lower wire is the common return path.



Half-wave rectifier: the output is taken across the load.

Step 2: Explain the two half-cycles During the positive half-cycle, the upper input terminal is positive relative to the lower one. The diode is forward biased; current flows through D and R_L , producing a positive load voltage. For an ideal diode, $v_o = v_i$ throughout this conducting interval.

During the negative half-cycle, the diode is reverse biased and its ideal current is zero. Thus no current flows through R_L and $v_o = 0$. This repeats every cycle: the output contains positive half-sine pulses separated by zero-output intervals. It is pulsating dc, and its repetition frequency equals the input frequency.

Step 3: Conclusion The diode passes the positive half-cycles and suppresses the negative half-cycles, giving a unidirectional, pulsating voltage across R_L .

Quick Tip to Solve: Track the input polarity at the diode's anode in each half-cycle; the load receives a pulse only when the diode is forward biased.

OR

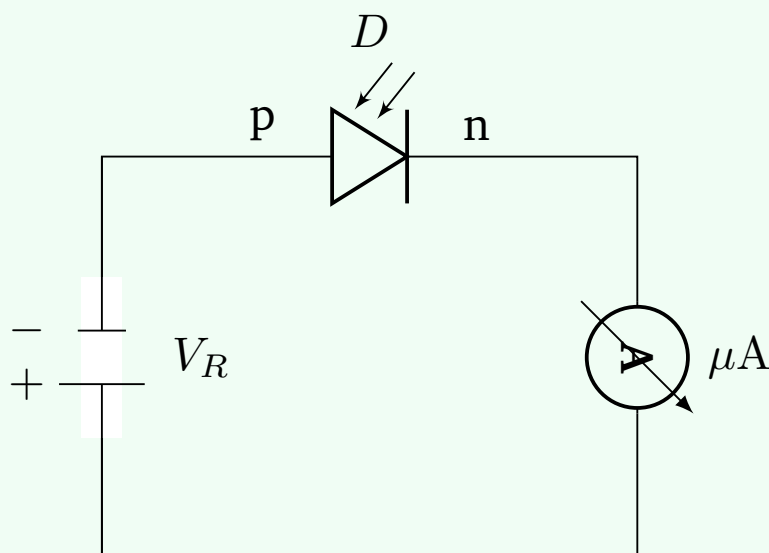
24(b). With the help of the circuit diagram, explain briefly the working of a photodiode. [2 marks]

Solution:

Concept:

- A photodiode is a p-n junction whose light-sensitive depletion region is operated under reverse bias. Its n-side (cathode) is connected toward the battery's positive terminal.
- Photons of sufficient energy create electron-hole pairs. The junction's electric field separates these carriers, producing a photocurrent that increases with illumination.

Step 1: Arrange the reverse-biased circuit Connect the photodiode as shown, with the n-side toward the positive supply and the p-side toward the negative terminal. The microammeter measures the small reverse current. The incoming arrows indicate light falling on the junction; arrows pointing away would instead denote a light-emitting diode. In darkness only a small reverse saturation current, called dark current, flows.



Reverse-biased photodiode with a microammeter measuring the reverse current.

Step 2: Explain the photocurrent When radiation with photon energy at least the band gap falls on the depletion region, it creates additional electron-hole pairs. The electric field across the reverse-biased junction sweeps electrons toward the n-side and holes toward the p-side before they recombine. This produces an additional reverse current in the external circuit.

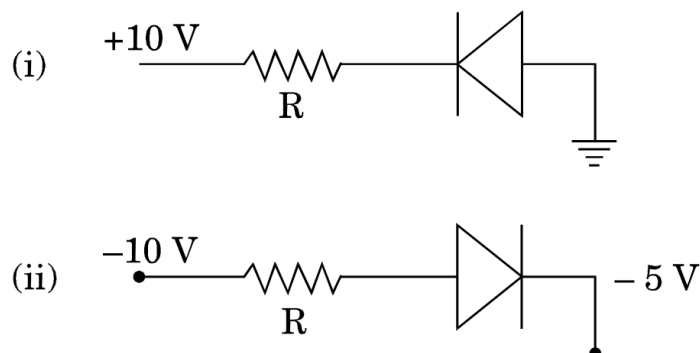
Under fixed reverse bias, increasing the incident light intensity creates more pairs per unit time and therefore increases the photocurrent approximately in proportion to intensity. The measured reverse current consists of the small dark current plus this light-generated current, so the circuit acts as a light detector.

Step 3: Conclusion The reverse-biased photodiode converts incident light into a measurable reverse photocurrent, whose magnitude increases with the illumination intensity.

Quick Tip to Solve: In a photodiode circuit, draw light arrows toward the junction and connect its cathode toward the positive supply to show reverse bias.

25. (a) State whether the given ideal diodes are forward or reverse biased:

(b) The current in the forward bias is known to be more ($\sim \text{mA}$) than the current in the reverse bias ($\sim \mu\text{A}$). What is the reason then to operate the photodiodes in reverse bias? [2 marks]



Solution:

Concept:

- A diode is forward biased when its anode (p-side) is at a higher potential than its cathode (n-side). The straight bar in the symbol identifies the cathode.
- Photodiodes detect the light-induced change in current. Sensitivity depends on that change relative to the background current, rather than on the absolute current being large.

Step 1: Identify the bias of both diodes In circuit (i), the bar is on the left, so the cathode is connected through R to $+10 \text{ V}$, while the anode is grounded at 0 V . Thus the anode is less positive than the cathode and the diode is reverse biased. With an ideal reverse-biased diode there is no current and therefore no voltage drop across R .

In circuit (ii), the bar is on the right. The anode is connected through R to -10 V , and the cathode is at -5 V . Since $-10 < -5$, the anode is again at the lower potential. The anode-to-cathode voltage is -5 V , so this diode too is reverse biased.

Step 2: Explain why the photodiode uses reverse bias Under reverse bias the dark current is small. Illumination generates electron-hole pairs in and near the depletion region, and the junction field separates them to add a photocurrent. The change caused by light is therefore readily measurable against the small dark current and varies approximately in proportion to light intensity.

In forward bias the large majority-carrier current would mask the much smaller change due to illumination. Reverse bias also widens the depletion region and assists rapid collection of the photogenerated carriers. A larger total forward current is consequently not an advantage for detecting light.

Step 3: Conclusion Both (i) and (ii) are reverse biased. Photodiodes use reverse bias because the small background current makes the light-generated current change much easier to detect.

Quick Tip to Solve: Locate the cathode bar first, then compare anode and cathode potentials algebraically; a negative potential can still be higher than another negative potential.

Section D (15 marks)

26(a). Two concentric circular coils X and Y of radii r_1 and r_2 ($r_1 > r_2$) having N_1 and N_2 turns respectively are placed coaxially with centres coinciding. Obtain an expression for (i) the mutual inductance for the arrangement, and (ii) the magnetic flux linked with coil Y when current I flows through coil X. [3 marks]

Solution:

Concept:

- Mutual inductance M relates the total magnetic flux linkage of coil Y to the current in coil X: $N_2\Phi_Y = MI$, where Φ_Y is the flux through one turn of Y.
- At the centre of an N_1 -turn circular coil of radius r_1 , the magnetic field is $B_X = \mu_0 N_1 I / (2r_1)$, normal to its plane. Here μ_0 is the permeability of free space.
- The usual school-level expression treats this central field as uniform over the smaller coil, whose area is $A_Y = \pi r_2^2$. This approximation is most accurate when $r_2 \ll r_1$.

Step 1: Find the magnetic field produced by coil X Current I flows through each of the N_1 turns of the outer coil X. Their fields add at the common centre, giving

$$B_X = \frac{\mu_0 N_1 I}{2r_1}.$$

The smaller coil Y is coaxial with X, so its area vector is parallel to this field. Thus the angle entering the flux formula is zero. Using the central-field approximation, the flux through each turn of Y is

$$\Phi_Y = B_X \pi r_2^2 = \frac{\mu_0 N_1 I \pi r_2^2}{2r_1}.$$

Step 2: Obtain the mutual inductance and total flux linkage Coil Y has N_2 turns, so all its turns together have flux linkage

$$N_2\Phi_Y = \frac{\mu_0 N_1 N_2 I \pi r_2^2}{2r_1}.$$

Dividing this linkage by the current in X gives

$$M = \frac{N_2\Phi_Y}{I} = \frac{\mu_0 N_1 N_2 \pi r_2^2}{2r_1}.$$

The current cancels: for this fixed geometry in air, mutual inductance depends on the radii and numbers of turns, not on the chosen current. Strictly, the field varies across a finite inner coil; $r_1 > r_2$ alone does not guarantee uniformity. The expression above is the conventional centre-field approximation used for this arrangement.

Step 3: Conclusion (i) $M \simeq \frac{\mu_0 N_1 N_2 \pi r_2^2}{2r_1}$. (ii) The total flux linked with coil Y is $N_2\Phi_Y = MI \simeq \frac{\mu_0 N_1 N_2 I \pi r_2^2}{2r_1}$; the flux per turn is $\Phi_Y \simeq \frac{\mu_0 N_1 I \pi r_2^2}{2r_1}$. These use the central-field approximation.

Quick Tip to Solve: Use the outer radius in the field denominator, the inner radius in the flux area, and multiply by both turn counts only for total flux linkage.

26(b). What are eddy currents? Why does the pendulum plate with holes or slots reduce electromagnetic damping? How are the eddy currents minimised in the core of a transformer? Why is the electromagnetic braking effect smooth in a train? [3 marks]

Solution:

Concept:

- Eddy currents are closed circulating currents induced within the bulk of a conductor when its magnetic flux changes; they obey Faraday's law and Lenz's law.
- The magnetic effect of an induced current opposes the change producing it. Electrical resistance dissipates energy as heat, with power $P = I^2 R$.
- Interrupting conducting loops or increasing their resistance reduces eddy currents and the resulting magnetic damping or energy loss.

Step 1: Define eddy currents and explain the slotted pendulum When a conducting plate moves into or out of a magnetic field, the magnetic flux through possible conducting loops in the plate changes. Induced currents circulate in these loops inside the metal; these are eddy currents. By Lenz's law their interaction with the applied field opposes the plate's motion, converting mechanical energy into heat and damping the oscillation. Holes, and especially slots that cut across the current paths, interrupt large loops and constrain current to smaller, more resistive paths. The resulting eddy currents and opposing magnetic force are smaller, so the pendulum experiences less electromagnetic damping.

Step 2: Explain how transformer cores minimise eddy currents The alternating flux in a transformer can induce unwanted circulating currents in its conducting iron core. Instead of a solid block, the core is made from thin laminations insulated from one another. The insulation prevents large current loops from passing between sheets, while each thin sheet restricts the available loop area and raises the effective resistance of the current path. Consequently the eddy currents are much smaller and the associated heating and energy loss are reduced. The laminations are arranged so that the useful magnetic flux travels along the sheets.

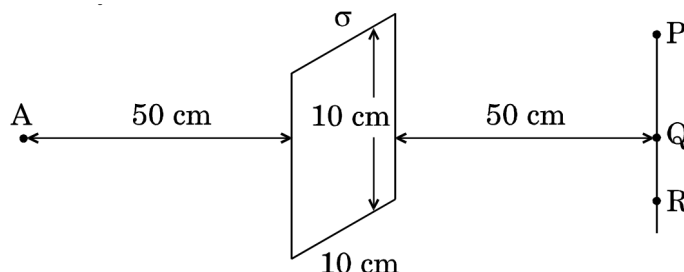
Step 3: Explain smooth electromagnetic braking in a train As a conducting brake disc or rail moves relative to a magnetic field, eddy currents are induced in the conductor. Their magnetic interaction produces a force opposing the relative motion, so the train's kinetic energy is dissipated as heat. This braking force acts continuously through the magnetic interaction, without rubbing brake surfaces or repeated mechanical impacts. It therefore gives smooth braking. As the speed falls, the induced emf and eddy-current braking force also diminish; ordinary eddy-current braking alone does not hold a stationary train.

Step 4: Conclusion Eddy currents are induced circulating currents inside a conductor. Slots or holes reduce pendulum damping by disrupting their paths; insulated core laminations suppress transformer eddy currents; and the continuous magnetic force opposing a train's motion makes electromagnetic braking smooth.

Quick Tip to Solve: For each application, trace the same chain: changing flux, induced closed currents, Lenz-law opposition, then either disrupt loops or use the braking force.

27(a). A uniformly charged large plane sheet has charge density $\sigma = \left(\frac{1}{18\pi}\right) \times 10^{-15} \text{ C/m}^2$. Find the electric field at point A which is 50 cm from the sheet.

Consider a straight line with three points P, Q and R, placed 50 cm from the charged sheet on the right side as shown in the figure. At which of these points, does the magnitude of the electric field due to the sheet remain the same as that at point A and why? [3 marks total]



Solution:

Concept:

- For a uniformly charged large plane sheet, Gauss's law gives $E = \sigma/(2\epsilon_0)$, where σ is surface charge density and ϵ_0 is the permittivity of free space.
- The ideal large-sheet field is uniform, normal to the sheet and independent of distance. A positive sheet produces outward fields of equal magnitude on both sides.

Step 1: Apply Gauss's law Choose a cylindrical Gaussian pillbox with its flat faces parallel to the sheet, each of area S . The curved surface contributes no flux because the field is parallel to it. The flux through the two faces is $2ES$, while the enclosed charge is σS . Thus $2ES = \sigma S/\epsilon_0$, giving $E = \sigma/(2\epsilon_0)$.

Step 2: Substitute the density and compare points Using $1/(4\pi\epsilon_0) = 9 \times 10^9 \text{ N m}^2/\text{C}^2$,

$$E = 2\pi(9 \times 10^9) \left(\frac{1}{18\pi} \times 10^{-15} \right) = 10^{-6} \text{ N/C}.$$

The distance 50 cm does not enter this expression. Points P, Q and R are all on the right of the same uniformly charged sheet, so every one has the same field magnitude as A. At A the field points away from the sheet towards the left; at P, Q and R it points towards the right. The stated large-sheet approximation neglects edge effects.

Step 3: Conclusion $E_A = 1.0 \times 10^{-6} \text{ N/C}$. The magnitudes at P, Q, R are all equal to that at A, because the large-sheet field is uniform.

Quick Tip to Solve: For a large plane sheet use $\sigma/(2\epsilon_0)$, with no inverse-square distance factor.

27(b). Two small identical conducting spheres carrying charge $10\ \mu\text{C}$ and $-20\ \mu\text{C}$ when separated by a distance of r , experience a force F each. If they are brought in contact and then separated to a distance of $\frac{r}{2}$, what is the new force between them in terms of F ?

Solution:

Concept:

- Coulomb's law gives the force magnitude $F = k|q_1q_2|/r^2$, where $k = 1/(4\pi\epsilon_0)$ and r is the centre-to-centre separation.
- On contact, identical conducting spheres share their conserved total charge equally. Like charges repel, while unlike charges attract.

Step 1: Find the charge after contact Initially the charges are $q_1 = 10\ \mu\text{C}$ and $q_2 = -20\ \mu\text{C}$. Their total is $-10\ \mu\text{C}$. Because the conducting spheres are identical, each has the same capacitance; equal potentials after contact therefore give equal charges. Consequently

$$q'_1 = q'_2 = \frac{10 - 20}{2} \mu\text{C} = -5\ \mu\text{C}.$$

Charge has been redistributed, not created or destroyed.

Step 2: Compare the new force with the original The original magnitude is $F = k(200 \times 10^{-12})/r^2$, and it is attractive because the original charges have opposite signs. After separation to $r/2$,

$$F' = \frac{k(25 \times 10^{-12})}{(r/2)^2} = \frac{k(100 \times 10^{-12})}{r^2} = \frac{F}{2}.$$

Reducing the distance contributes a factor of four, while the magnitude of the charge product changes by a factor of $25/200 = 1/8$. Their product is $1/2$. Both final charges are negative, so the new force is repulsive.

Step 3: Conclusion The new force has magnitude $\boxed{F/2}$ on each sphere and is $\boxed{\text{repulsive}}$.

Quick Tip to Solve: Compute the conserved charge first, then compare charge-product and distance-squared ratios separately.

28(a)(i). In an LCR series circuit connected to an ac source, the voltage and the current are in the same phase. If the capacitor is filled with a dielectric, will the current lead or lag behind or remain in phase with the voltage? Explain.

[3 marks total]

Solution:

Concept:

- In a series LCR circuit, $X_L = \omega L$, $X_C = 1/(\omega C)$ and $\tan \phi = (X_L - X_C)/R$, where ϕ is the voltage phase relative to current.
- At resonance $X_L = X_C$; inserting a dielectric of relative permittivity $K > 1$ increases capacitance to KC .

Step 1: Use the initial phase condition The given in-phase voltage and current mean that the original circuit is resonant at the fixed applied frequency: $\omega L = 1/(\omega C)$. The inductive and capacitive voltage phasors cancel, leaving only the resistive voltage.

Step 2: Determine the change in reactance With the dielectric, $X'_C = 1/(\omega KC) = X_C/K < X_L$,

while X_L remains unchanged. Hence the net reactance is positive and the circuit becomes inductive. The applied voltage now leads the current; it cannot stay in phase because the original cancellation has been lost.

Step 3: Conclusion The current lags behind the voltage, since increasing capacitance makes $X_L > X'_C$.

Quick Tip to Solve: At fixed frequency, more capacitance means less capacitive reactance; compare it with the unchanged inductive reactance.

28(a)(ii). In the circuit, why is the rms value of net voltage not equal to the sum of voltage drops across individual elements?

Solution:

Concept:

- Rms voltages in an ac circuit add as phasors because their sinusoidal variations generally have different phases.
- In series LCR, $V_R = IR$ is in phase with current, $V_L = IX_L$ leads by 90° , and $V_C = IX_C$ lags by 90° .

Step 1: Compare the voltage phases Although the same current flows through all three components, their voltage maxima occur at different times. In particular, V_L and V_C are in opposite directions in a phasor diagram and therefore partly cancel instead of reinforcing each other.

Step 2: Add the phasors The resultant reactive voltage $V_L - V_C$ is perpendicular to V_R . Therefore the applied rms voltage is

$$V = \sqrt{V_R^2 + (V_L - V_C)^2},$$

rather than $V_R + V_L + V_C$. Instantaneous voltages obey Kirchhoff's law, but taking rms values removes the signs and phase information needed for an arithmetic sum.

Step 3: Conclusion The rms drops cannot be added arithmetically because they differ in phase;

$$V = \sqrt{V_R^2 + (V_L - V_C)^2}.$$

Quick Tip to Solve: Add ac voltage phasors before taking their magnitude; rms values alone do not encode relative phase.

28(a)(iii). Draw a graph showing variation of the impedance of the circuit with the frequency of the applied voltage.

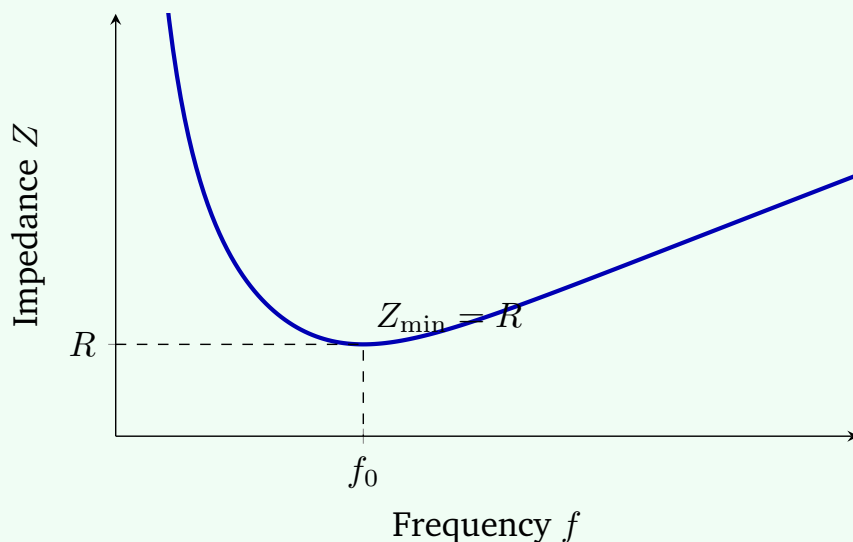
Solution:

Concept:

- The impedance is $Z = \sqrt{R^2 + (2\pi fL - 1/(2\pi fC))^2}$, where f is source frequency.
- Resonance occurs at $f_0 = 1/(2\pi\sqrt{LC})$, giving the minimum impedance $Z_{\min} = R$.

Step 1: Identify the minimum As frequency increases from a low value, the large capacitive reactance decreases. At $f = f_0$, the two reactances are equal and cancel, so only the resistance contributes to impedance. The curve reaches R , not zero, for a circuit with nonzero resistance.

Step 2: Describe the two branches Below resonance the circuit is capacitive and Z decreases towards R . Above resonance it becomes inductive, and Z increases as $2\pi fL$ grows. In the limiting cases $f \rightarrow 0$ and $f \rightarrow \infty$, impedance becomes large. The labelled curve shows these features; its two branches need not be symmetric.



Impedance of a series LCR circuit against source frequency

Step 3: Conclusion The impedance-frequency graph has its minimum $Z = R$ at $f = f_0$, with impedance increasing on either side.

Quick Tip to Solve: Mark a nonzero minimum R at resonance; a series LCR impedance graph is not a current-resonance peak.

OR

28(b)(i). An LCR series circuit is connected to an ac source. If the angular resonant frequency of the circuit is ω_0 , will the current lead or lag behind or be in phase with the voltage when $\omega < \omega_0$ and why? [3 marks total]

Solution:

Concept:

- Series LCR resonance satisfies $\omega_0^2 = 1/(LC)$, where L is inductance and C is capacitance.
- For $X_C > X_L$, the net reactance $X_L - X_C$ is negative and current leads the applied voltage.

Step 1: Compare reactances below resonance If $\omega < \omega_0$, then $\omega^2 LC < 1$. Rearranging gives $\omega L < 1/(\omega C)$, that is, $X_L < X_C$. Thus capacitive reactance dominates even though an inductor is present.

Step 2: Infer the phase relation Using $\tan \phi = (X_L - X_C)/R$, the voltage phase relative to current is negative. Accordingly voltage lags current, or equivalently current leads voltage. Exact phase equality is possible only at resonance, when the net reactance vanishes.

Step 3: Conclusion For $\omega < \omega_0$, the current leads the voltage, because $X_C > X_L$.

Quick Tip to Solve: Compare the ratio $X_L/X_C = \omega^2 LC$ with one to decide the phase immediately.

28(b)(ii). We cannot step up a dc voltage using a transformer. Why?

Solution:

Concept:

- A transformer operates by mutual induction: $\varepsilon_s = -N_s d\Phi/dt$, where N_s is secondary turns and Φ is core flux per turn.
- Continuous secondary emf requires changing magnetic flux; steady direct current produces steady core flux.

Step 1: Examine the effect of steady dc After the brief switching transient, direct current through the primary is unidirectional and steady. The flux it establishes in the common magnetic core is then constant, so $d\Phi/dt = 0$. There is therefore no sustained induced emf in the secondary.

Step 2: Apply the induction condition Increasing the secondary turn count cannot compensate for a zero rate of flux change: $\varepsilon_s = -N_s(0) = 0$. A brief induced pulse may occur when the dc supply is switched on or off, but it is not a continuous stepped-up output. The usual turns-ratio voltage relation presupposes alternating flux.

Step 3: Conclusion A transformer cannot step up steady dc because $d\Phi/dt = 0$, so there is no sustained secondary induced emf.

Quick Tip to Solve: Ask whether the core flux changes with time before using any transformer turns-ratio formula.

28(b)(iii). On what principle does a metal detector work?

Solution:

Concept:

- A metal detector uses electromagnetic induction: changing magnetic flux induces emf and currents in nearby conducting metal.
- The induced eddy currents create a magnetic field that alters the detecting coil's electromagnetic response.

Step 1: Produce a changing magnetic field Alternating current in the detector's coil produces a changing magnetic field. If a metal object is brought nearby, the changing flux through conducting paths in the object induces circulating eddy currents according to Faraday's law.

Step 2: Detect the response of the metal By Lenz's law, these currents produce their own magnetic field opposing the flux change that caused them. This field changes the coil's effective impedance or the signal in a receiver coil. The detector circuitry senses that change and gives an indication. Merely placing metal in a constant field would not provide the same sustained induction signal.

Step 3: Conclusion A metal detector works on $\boxed{\text{electromagnetic induction}}$, detecting the response of induced eddy currents in metal.

Quick Tip to Solve: Remember the chain: alternating coil field, induced eddy currents, secondary magnetic field, detectable signal change.

29(a). Explain the formation of the fringes due to diffraction at a single slit, when path difference of light waves from the ends of the slit on reaching a point on the screen is (i) λ , and (ii) $\frac{3\lambda}{2}$.
[3 marks total]

Solution:

Concept:

- For slit width a , the path difference of waves from the two ends is $\Delta = a \sin \theta$, where θ is observation angle.
- Each point across the slit acts as a coherent secondary source; waves differing in path by $\lambda/2$ cancel pairwise when their amplitudes are equal.

Step 1: Case (i): end-to-end difference equals one wavelength When $a \sin \theta = \lambda$, divide the slit into two equal halves. For every source point in the upper half, the corresponding point in the lower half is separated by $a/2$. Their path difference is $(a/2) \sin \theta = \lambda/2$, so they have opposite phase. All such pairs cancel, giving zero resultant amplitude: the first dark fringe.

Step 2: Case (ii): end-to-end difference equals three half-wavelengths For $a \sin \theta = 3\lambda/2$, divide the slit into three equal strips. Corresponding sources in adjacent strips have path difference $(a/3) \sin \theta = \lambda/2$. Contributions from two strips cancel, leaving a nonzero contribution from the third. The intensity is therefore weak and bright, in the region of the first secondary maximum. In the usual elementary description, $3\lambda/2$ is the approximate condition for that maximum; the exact maximum is slightly displaced. For $\beta = \pi a \sin \theta / \lambda = 3\pi/2$, $I/I_0 = (\sin \beta / \beta)^2 = 4/(9\pi^2)$, much smaller than the central intensity.

Step 3: Conclusion $\Delta = \lambda$ gives the first dark fringe; $\Delta = 3\lambda/2$ gives a weak bright fringe near the first secondary maximum.

Quick Tip to Solve: Pair slit strips whose path differences are half a wavelength; total cancellation gives a minimum, while a residual strip gives weak light.

29(b). Show the intensity distribution in the fringes due to diffraction at a single slit.

Solution:

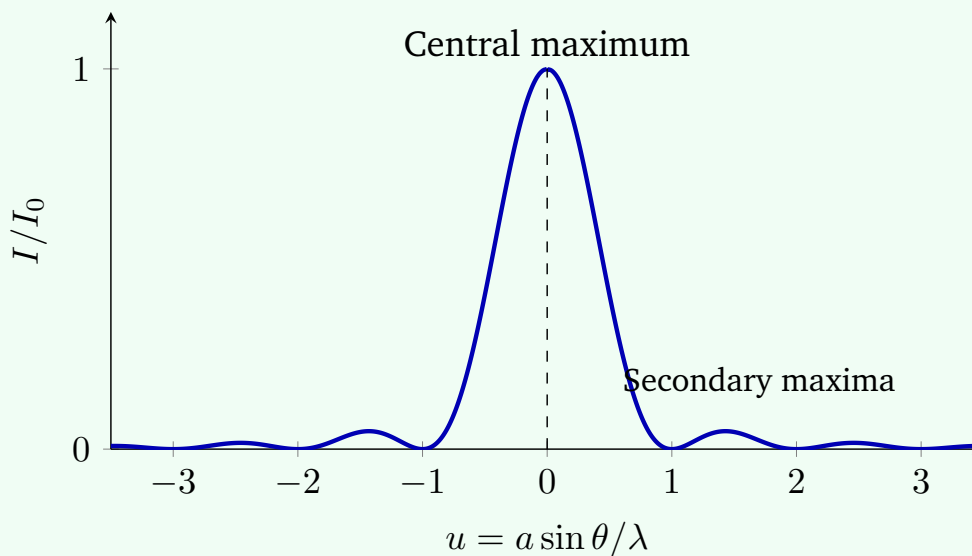
Concept:

- The single-slit diffraction intensity is $I(\theta) = I_0(\sin \beta / \beta)^2$, with $\beta = \pi a \sin \theta / \lambda$. Here a is slit width and I_0 is central intensity.
- Minima occur at $a \sin \theta = m\lambda$, with $m = \pm 1, \pm 2, \dots$; secondary maxima lie between adjacent minima.

Step 1: Locate the central maximum and minima At $\theta = 0$, the limit $\sin \beta / \beta = 1$ gives the strongest maximum, $I = I_0$. The first minima flank it at $a \sin \theta = \pm \lambda$. Subsequent minima occur at nonzero integral multiples of λ . Thus the central maximum spans two adjacent first-minimum intervals and is twice as wide as each secondary fringe in the small-angle approximation.

Step 2: Show the intensity profile The graph is symmetric about the central maximum. On either side, small secondary maxima occur between zero-intensity minima, with their heights diminishing away from the centre. The horizontal coordinate $u = a \sin \theta / \lambda$ makes the minima appear at nonzero integers. On a distant screen at distance D , $\sin \theta \simeq y/D$, so these minima are at $y = m\lambda D/a$ and the central width is $2\lambda D/a$. Exact secondary maxima satisfy $\tan \beta = \beta$; their decreasing heights follow

the plotted intensity formula.



Single-slit diffraction intensity with minima at nonzero integer u

Step 3: Conclusion The distribution has a broad, intense central maximum and weaker secondary maxima, separated by dark minima at $a \sin \theta = m\lambda$.

Quick Tip to Solve: Make the central lobe twice as wide as the side lobes, and draw side maxima decreasing in height.

30(a). In a Geiger-Marsden experiment, find the distance of closest approach to the gold nucleus (mass no. = 79) of a 7.7 MeV α -particle before it comes momentarily to rest and reverses its direction. Why is it different from actual radius of gold nucleus? [3 marks total]

Solution:

Concept:

- In a head-on approach to a stationary nucleus, an alpha particle's initial kinetic energy becomes electrostatic potential energy at the turning point: $K = k(2e)(Ze)/r_{\min}$, where $k = 1/(4\pi\epsilon_0)$.
- Nuclear charge is set by atomic number Z , not mass number. Gold has $Z = 79$, so its nuclear charge is $+79e$. The printed phrase "mass no. = 79" must be understood as the charge-number value for this Coulomb calculation.

Step 1: Apply conservation of energy at closest approach The particle comes momentarily to rest and reverses along its incident line, so the collision is head-on. Taking potential energy to be zero far from the nucleus and neglecting recoil of the much heavier gold nucleus,

$$K = \frac{k(2e)(79e)}{r_{\min}}, \quad r_{\min} = \frac{158ke^2}{K}.$$

Convert the incident energy to SI units:

$$K = 7.7 \times 10^6 (1.602 \times 10^{-19}) = 1.23354 \times 10^{-12} \text{ J}.$$

Step 2: Calculate the turning distance and interpret it Using $k = 8.99 \times 10^9 \text{ N m}^2\text{C}^{-2}$ and $e =$

$$1.602 \times 10^{-19} \text{ C},$$

$$r_{\min} = \frac{158(8.99 \times 10^9)(1.602 \times 10^{-19})^2}{1.23354 \times 10^{-12}} = 2.96 \times 10^{-14} \text{ m}.$$

This is the distance between the particle and nuclear centres at which Coulomb repulsion has exhausted the initial kinetic energy. It is not a measurement of the nuclear surface. The particle turns back before reaching that surface; the actual gold nuclear radius is smaller. Increasing the incident energy would reduce this turning distance, whereas the nuclear radius does not depend on the incident energy.

Step 3: Conclusion The distance of closest approach is $r_{\min} \simeq 2.96 \times 10^{-14} \text{ m}$. It exceeds the actual nuclear radius because electrostatic repulsion stops the alpha particle outside the nucleus.

Quick Tip to Solve: For a head-on alpha particle use nuclear charge Ze and set its kinetic energy equal to $2kZe^2/r$; do not substitute mass number for charge number.

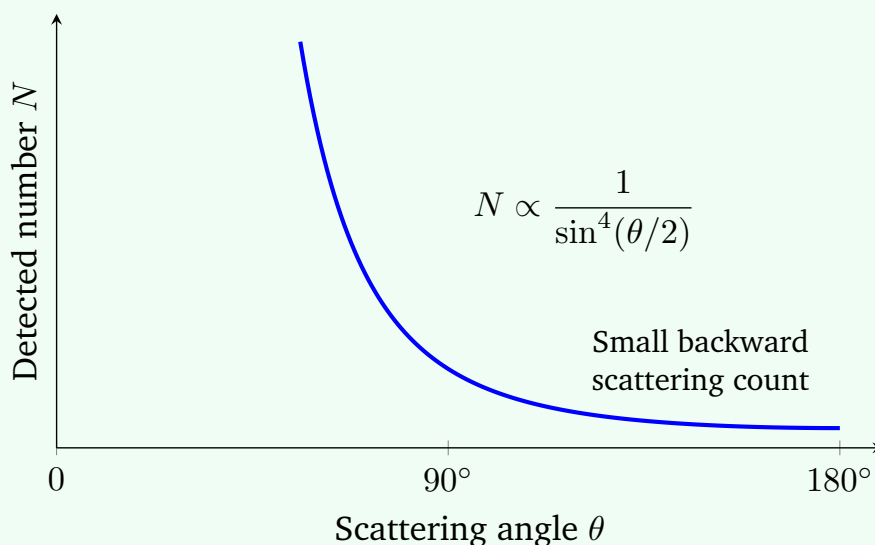
30(b). Plot a graph between number of scattered α -particles detected in gold foil experiment and angle of scattering. What is the main assumption in plotting this graph?

Solution:

Concept:

- Rutherford scattering describes the deflection of an alpha particle by electrostatic repulsion from a small, positively charged nucleus.
- For fixed incident energy and equal detector solid angles, the number detected varies as $N(\theta) = C/\sin^4(\theta/2)$, where θ is the scattering angle and C contains the incident flux, exposure and target factors.

Step 1: Draw the angular distribution Plot scattering angle on the horizontal axis and the detected number of alpha particles on the vertical axis. Most particles are detected close to the forward direction: the curve is very high for small angles, falls rapidly as θ increases, and approaches a small but nonzero value at 180° . For example, relative to the value at 180° , the Rutherford expression gives $N(90^\circ)/N(180^\circ) = 4$. The sketch is schematic; the ideal point-charge law diverges as θ approaches zero, where the unscattered incident beam dominates a real measurement.



Schematic angular distribution for equal detector solid angles; the narrow forward beam is outside the plotted curve.

Step 2: State the assumption behind the graph The main experimental assumption is single scattering: the gold foil is sufficiently thin that each appreciably deflected alpha particle is scattered primarily by one nucleus, rather than accumulating successive deflections from many nuclei. The Rutherford model also treats the positive nuclear charge as concentrated in a very small region and uses Coulomb repulsion; electron scattering and recoil of the much heavier nucleus are neglected. To compare detector counts at different angles, incident energy, flux, observation time and the detector's solid-angle acceptance must be kept the same.

Step 3: Conclusion The detected number decreases steeply with angle according to $N(\theta) \propto \sin^{-4}(\theta/2)$. The essential assumption is single Coulomb scattering in a sufficiently thin foil.

Quick Tip to Solve: Draw a sharply falling curve with a small backward-scattering tail, and state that a thin foil permits single scattering by one nucleus.

Section E (15 marks)

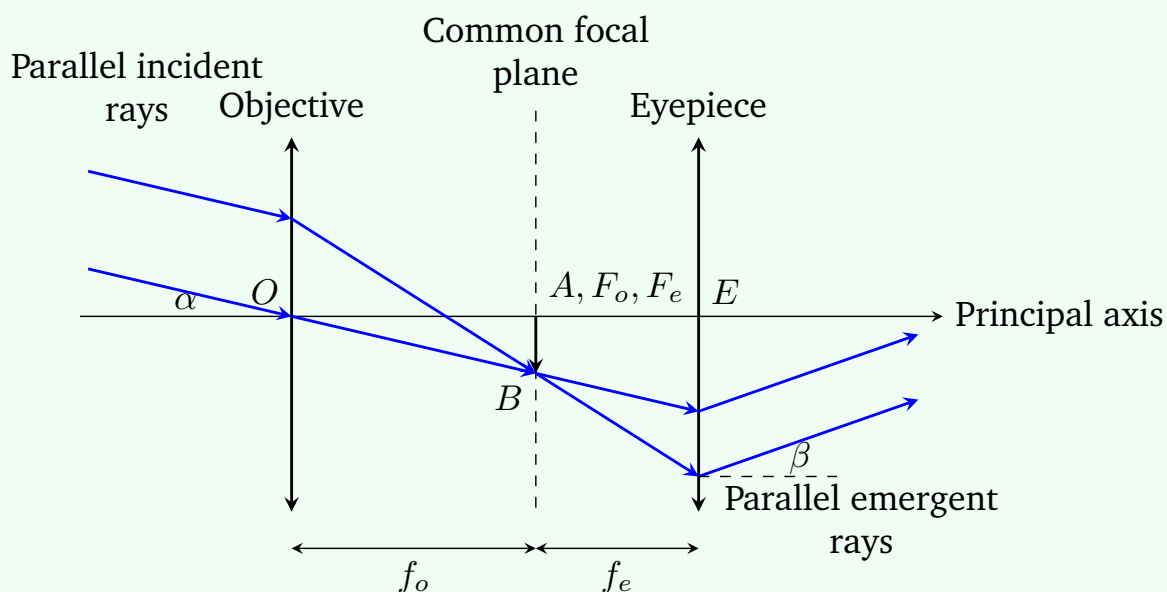
- 31(a).** (i) Draw a ray diagram of an astronomical refracting telescope in normal adjustment. Obtain an expression for its magnifying power. How can we increase the magnifying power of the telescope?
 (ii) A beam of light converges at a point P. A lens is placed in the path of the beam at a distance of 25 cm from P. The final image is formed at infinity. Calculate the power of the lens. [5 marks]

Solution:

Concept:

- An astronomical refracting telescope uses a convex objective of large focal length f_o and a convex eyepiece of smaller focal length f_e . The objective forms a real, inverted intermediate image of a distant object.
- In normal adjustment the intermediate image lies in the common focal plane of both lenses. The eyepiece sends out parallel rays, placing the final image at infinity for relaxed-eye viewing; lens separation is $L = f_o + f_e$.
- Angular magnifying power is $M = \beta/\alpha$, where α is the angular size seen directly and β is the angular size seen through the telescope. The negative sign of M denotes inversion.
- With light travelling from left to right, Cartesian distances to the right are positive. A converging incident beam has a virtual object to the right of a lens. The thin-lens formula is $1/f = 1/v - 1/u$, and power is $P = 1/f$ with f in metres.

Step 1: (i) Ray diagram and image formation in normal adjustment Parallel rays from a distant object enter the objective at a small angle α to the principal axis. The objective brings them to the inverted real image AB in its rear focal plane. This plane coincides with the front focal plane of the eyepiece, so the diverging rays from B emerge as a parallel bundle after refraction by the eyepiece. The final image is virtual, inverted and at infinity. In the diagram, A lies on the principal axis and B lies below it. Thus the objective-eyepiece separation is $f_o + f_e$.



Astronomical telescope in normal adjustment: the objective's rear focal plane coincides with the eyepiece's front focal plane; rays from B emerge parallel.

Step 2: (i) Derive the angular magnifying power Let the magnitude of the intermediate-image height be $h = AB$. For the small angles subtended by a distant astronomical object, $\tan \alpha \simeq \alpha$ and $\tan \beta \simeq \beta$. The objective gives $|\alpha| \simeq h/f_o$, while viewing the same intermediate image through the eyepiece gives $|\beta| \simeq h/f_e$. Therefore

$$|M| = \frac{|\beta|}{|\alpha|} = \frac{h/f_e}{h/f_o} = \frac{f_o}{f_e}.$$

The image is inverted relative to the distant object, so the signed angular magnification is $M = -f_o/f_e$. This expression applies specifically to normal adjustment, when the final rays are parallel.

Step 3: (i) Increase the magnifying power Increase the objective focal length f_o , or decrease the eyepiece focal length f_e , so that their ratio increases. A longer-focal-length objective also increases the tube length required for normal adjustment. Increasing the objective aperture helps collect more light and improve resolution, but aperture does not appear in $|M| = f_o/f_e$, so it alone does not increase the angular magnifying power in this expression.

Step 4: (ii) Identify the virtual object and apply the lens formula Without the inserted lens the incident beam would meet at P , which is 25 cm beyond the lens in the direction of propagation. Thus P is a virtual object for the lens and $u = +25$ cm. The final image is at infinity, so $1/v = 0$. Substitution gives

$$\begin{aligned} \frac{1}{f} &= \frac{1}{v} - \frac{1}{u} = 0 - \frac{1}{25 \text{ cm}}, \\ f &= -25 \text{ cm} = -0.25 \text{ m}, \\ P &= \frac{1}{-0.25 \text{ m}} = -4.0 \text{ D}. \end{aligned}$$

Physically, a diverging lens removes the convergence of the incoming beam, leaving an emergent parallel beam. This also checks the negative focal length and power.

Step 5: Conclusion (i) For an astronomical telescope in normal adjustment, $L = f_o + f_e$ and $M = -f_o/f_e$, with magnitude f_o/f_e . A larger f_o or a smaller f_e increases magnifying power. (ii) The required lens has focal length -25 cm and power $\boxed{-4.0 \text{ D}}$; it is a diverging lens.

Quick Tip to Solve: For a beam already converging toward a point beyond the lens, use a positive virtual-object distance; making the beam parallel requires a negative-power lens.

OR

31(b). (i) A coin is placed inside a denser medium. Why does it appear to be raised? Obtain an expression for the height through which the object appears to be raised in terms of refractive index of the medium and real depth.

(ii) A compound microscope consists of an objective lens of focal length 2 cm and an eyepiece of focal length 6.25 cm separated by a distance of 15 cm. How far from the objective should an object be placed in order to obtain the final image at the least distance of distinct vision (25 cm)? Calculate the magnifying power of the microscope. [5 marks]

Solution:

Concept:

- Refraction from a denser medium into air bends a ray away from the normal. The backward extensions of the emergent rays locate a virtual image nearer the surface.
- For viewing close to the normal through a plane surface, refractive index relative to air is $\mu = d/d'$, where d is real depth and d' is apparent depth.
- The thin-lens formula is $1/f = 1/v - 1/u$. Distances are measured from each lens, positive along incident light; a virtual final image on the incident side has negative v .
- For a compound microscope with final image at the near point D , the magnitude of magnifying power is $M = |v_o/u_o|(1 + D/f_e)$, with subscripts o and e referring to objective and eyepiece.

Step 1: (i) Explain the raised appearance and derive apparent depth Let the coin be at O , a real depth d below a horizontal plane surface. A ray from it meets the surface at lateral distance x from the normal through O . Write i for its angle of incidence inside the medium and r for its refracted angle in air. The ray bends away from the normal, so $r > i$. Its backward extension meets the normal at the virtual image I , of depth d' . Geometry gives

$$\tan i = \frac{x}{d}, \quad \tan r = \frac{x}{d'}.$$

Snell's law is $\mu \sin i = \sin r$. For paraxial viewing, $\sin i \simeq \tan i$ and $\sin r \simeq \tan r$, hence

$$\mu = \frac{\sin r}{\sin i} \simeq \frac{\tan r}{\tan i} = \frac{d}{d'}, \quad d' = \frac{d}{\mu}.$$

Since $\mu > 1$, the virtual image lies above the coin and closer to the surface. The upward displacement is

$$h = d - d' = d \left(1 - \frac{1}{\mu} \right).$$

This standard depth relation assumes a plane interface, air above the medium, and small viewing angles.

Step 2: (ii) Find the intermediate image position using the eyepiece Here $f_o = 2$ cm, $f_e = 6.25$ cm,

lens separation $L = 15$ cm, and $D = 25$ cm. The final image is virtual at the near point, so for the eyepiece $v_e = -25$ cm. Apply the lens formula:

$$\frac{1}{u_e} = \frac{1}{v_e} - \frac{1}{f_e} = -\frac{1}{25} - \frac{1}{6.25} = -\frac{1}{5},$$

$$u_e = -5 \text{ cm.}$$

Thus the real intermediate image lies 5 cm before the eyepiece. It must therefore be $v_o = L - 5 = 10$ cm to the right of the objective.

Step 3: Find the object distance from the objective For the objective, the intermediate image is real, giving $v_o = +10$ cm. Therefore

$$\frac{1}{u_o} = \frac{1}{v_o} - \frac{1}{f_o} = \frac{1}{10} - \frac{1}{2} = -\frac{1}{2},$$

$$u_o = -2.5 \text{ cm.}$$

The negative sign places the object on the incident side of the objective. It is just beyond the objective's focal point, as required for an enlarged real intermediate image. Its objective magnification is $m_o = v_o/u_o = 10/(-2.5) = -4$; the intermediate image is inverted.

Step 4: Calculate the magnifying power at the near point The angular magnification of the eyepiece for final image at D is

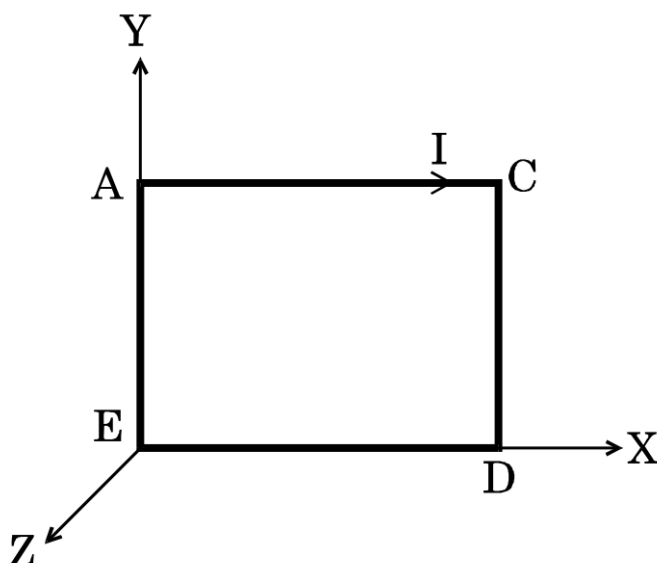
$$M_e = 1 + \frac{D}{f_e} = 1 + \frac{25}{6.25} = 5.$$

Multiplying by the objective magnification gives the signed microscope magnification $m_o M_e = (-4)(5) = -20$. Its magnitude is 20; the minus sign indicates that the final image is inverted relative to the original object. The exact objective distances calculated from the stated lens separation are used here.

Step 5: Conclusion (i) The coin appears raised because of refraction at the denser-medium-to-air surface; for near-normal viewing its rise is $h = d(1 - 1/\mu)$. (ii) Place the object **2.5 cm** in front of the objective. The magnifying power is **20** in magnitude, or -20 with image orientation indicated.

Quick Tip to Solve: Start at the eyepiece with the virtual near-point image distance negative, then subtract its object distance magnitude from the lens separation to obtain the objective image distance.

- 32(a).** (i) An α -particle, a deuteron and a proton enter into a uniform magnetic field normally with the same kinetic energy and describe circular paths. Find the ratio of radii of their paths.
- (ii) Give the direction of magnetic field acting on the current carrying coil ACDE shown in the figure so that the coil is in unstable equilibrium.
- (iii) Why do we use a low resistance ammeter in a circuit to measure current? [5 marks]



Solution:

Concept:

- For motion perpendicular to a uniform magnetic field, the magnetic force supplies centripetal force: $qvB = mv^2/r$, hence $r = mv/(qB)$, where q is the positive charge magnitude.
- With kinetic energy $K = mv^2/2$, momentum is $p = \sqrt{2mK}$, so $r = \sqrt{2mK}/(qB)$. At equal K and B , radii are proportional to $\sqrt{m}/q$.
- A current loop has magnetic moment $\mu = IA\hat{n}$, directed by the right-hand rule. Its torque is $\tau = \mu \times B$, and potential energy is $U = -\mu \cdot B$.
- An ammeter is connected in series, so its resistance adds to the circuit resistance. Small ammeter resistance keeps the current close to its value before insertion.

Step 1: (i) Derive the dependence of radius on mass and charge Since each particle enters normally to the field, its speed is entirely perpendicular to B , and its trajectory is circular. The magnetic force does no work, so its kinetic energy stays constant. Combining the centripetal-force equation with the given equal kinetic energies gives

$$qvB = \frac{mv^2}{r}, \quad r = \frac{mv}{qB} = \frac{\sqrt{2mK}}{qB}.$$

Thus one must account for both the mass and the charge. Equal kinetic energy does not imply equal speed or equal momentum.

Step 2: Substitute the masses and charges of the three particles Let the proton mass be m_p and its charge be e . To the usual nuclear-mass approximation, an alpha particle has $m_\alpha = 4m_p$, $q_\alpha = 2e$; a

deuteron has $m_d = 2m_p$, $q_d = e$; and a proton has m_p , $q_p = e$. Therefore

$$\begin{aligned} r_\alpha : r_d : r_p &= \frac{\sqrt{4m_p}}{2e} : \frac{\sqrt{2m_p}}{e} : \frac{\sqrt{m_p}}{e} \\ &= 1 : \sqrt{2} : 1. \end{aligned}$$

The alpha particle's increased momentum is exactly offset by its doubled charge, whereas the deuteron's charge remains the same as the proton's and its radius is larger.

Step 3: (ii) Determine the magnetic moment of coil ACDE The figure shows current from A to C along the upper side. It therefore circulates A to C to D to E to A. With $+X$ to the right and $+Y$ upward in the coil's plane, this circulation is clockwise as viewed from the $+Z$ side. Curling the right-hand fingers along the current gives the magnetic moment along $-Z$, into the plane of the coil. The drawn $+Z$ axis points outward from this plane.

Step 4: Choose the field for unstable equilibrium Equilibrium requires zero torque, so the field must be parallel or antiparallel to the magnetic moment. Parallel alignment gives minimum energy and is stable. For unstable equilibrium, the moment must be antiparallel to the field, with angle $\theta = 180^\circ$. Here this requires $\mathbf{B}$ along $+Z$, out of the coil's plane. Indeed,

$$U = -\mu B \cos \theta, \quad U(180^\circ) = +\mu B.$$

This is the maximum potential energy: a small rotation lowers the energy and the ensuing torque turns the coil farther away from its original orientation, rather than restoring it.

Step 5: (iii) Explain why an ammeter must have low resistance Let the source emf be $\mathcal{E}$ and let the total circuit resistance before inserting the meter be R , including any source resistance. The original current is $I_0 = \mathcal{E}/R$. An ammeter of resistance R_A , inserted in series, changes it to

$$I = \frac{\mathcal{E}}{R + R_A}, \quad \frac{I}{I_0} = \frac{R}{R + R_A}.$$

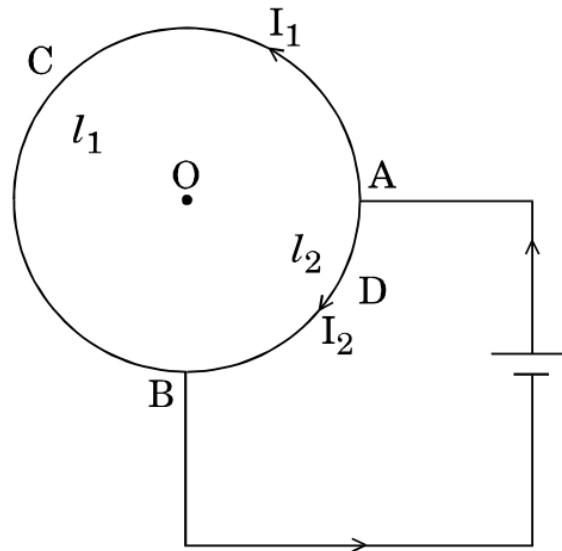
If $R_A \ll R$, this ratio is close to unity. Consequently, the meter produces negligible extra voltage drop and measures a current very close to the undisturbed circuit current. A large resistance would reduce the current being measured and introduce a significant measurement error. An ideal ammeter has zero resistance; a practical one has very small resistance.

Step 6: Conclusion (i) The radii are $r_\alpha : r_d : r_p = 1 : \sqrt{2} : 1$. (ii) For unstable equilibrium, apply the magnetic field along the $+Z$ direction, opposite to the coil's magnetic moment. (iii) A low-resistance series ammeter causes negligible change in the circuit current.

Quick Tip to Solve: For equal kinetic energies compare $\sqrt{m}/q$, then use the right-hand rule to find the coil's magnetic moment before reversing it for unstable equilibrium.

32(b). (i) Draw a diagram to show the magnetic field lines produced by two parallel straight wires carrying currents in the same direction. Obtain an expression for the force per unit length between these wires and hence define SI unit of current.

(ii) The figure shows a circular loop connected to a battery. The arc ACB of length ℓ_1 carries a current I_1 and arc ADB of length ℓ_2 carries a current I_2 . Show that the net magnetic field at the centre of the loop is zero. [5 marks]



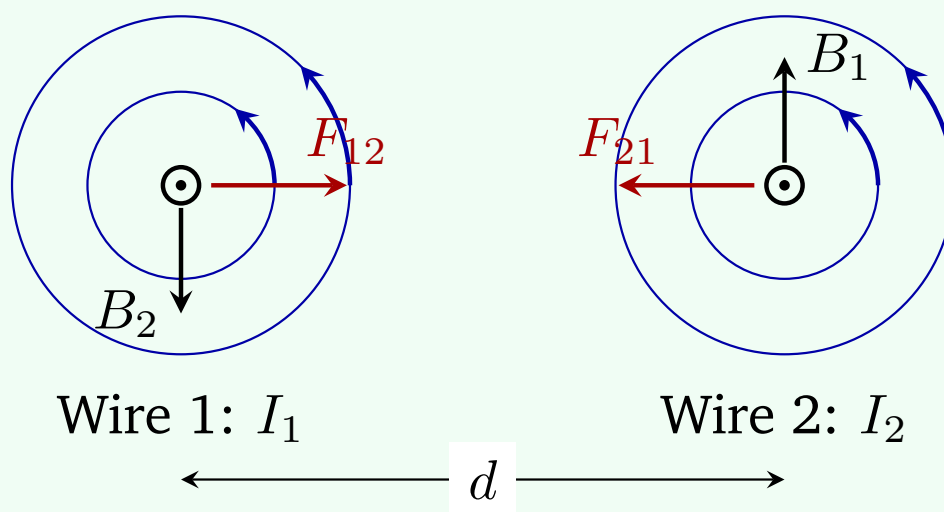
Solution:

Concept:

- A long straight wire carrying current I produces circular magnetic field lines. At perpendicular distance d in vacuum, $B = \mu_0 I / (2\pi d)$; the right-hand thumb rule gives the field direction.
- The magnetic force on a straight segment of length L is $\vec{F} = I\vec{L} \times \vec{B}$. For perpendicular current and field, $F = ILB$.
- For a uniform wire, resistance is proportional to length: $R = \rho\ell/S$, where ρ is resistivity and S is cross-sectional area. Parallel branches have the same potential difference.
- A circular arc of radius r , angle θ in radians and current I produces $B = \mu_0 I\theta / (4\pi r)$ at its centre; its direction follows the right-hand rule.

Step 1: (i) Show the field directions and the attraction View the two parallel wires end-on, with both currents coming out of the page (dots). Each current produces anticlockwise circular field lines. At wire 2, the field due to wire 1 points upwards in the page; the force on wire 2 points towards wire 1. Similarly, the force on wire 1 points towards wire 2. Thus currents in the same direction attract. The fields of the wires oppose one another in the region between them and reinforce outside them. The circles in the sketch show each wire's own field, used to determine the force on the other wire.

⊙: current out of the page



End-on view: both currents emerge from the page; circular fields are anticlockwise and forces point inward.

Step 2: (i) Derive the force per unit length Let the long straight wires carry currents I_1 and I_2 , and let their perpendicular separation be d . The magnetic field produced by wire 1 at wire 2 is

$$B_1 = \frac{\mu_0 I_1}{2\pi d}.$$

This field is perpendicular to the current in wire 2. Therefore, for a length L of wire 2,

$$F_{21} = I_2 L B_1 = \frac{\mu_0 I_1 I_2 L}{2\pi d}, \quad \frac{F}{L} = \frac{\mu_0 I_1 I_2}{2\pi d}.$$

Interchanging the wire labels gives the same magnitude for the force on wire 1. The two forces have opposite directions and are attractive. The expression assumes wires sufficiently long that end effects can be neglected and vacuum between them.

Step 3: (i) State the force-based definition of the ampere Put $I_1 = I_2 = 1 \text{ A}$, $d = 1 \text{ m}$, and use $\mu_0 = 4\pi \times 10^{-7} \text{ N A}^{-2}$ in the force-based SI convention used in this question. Then

$$\frac{F}{L} = \frac{4\pi \times 10^{-7}}{2\pi} = 2 \times 10^{-7} \text{ N m}^{-1}.$$

Accordingly, one ampere is the constant current which, maintained in each of two straight parallel conductors of infinite length and negligible circular cross-section, placed one metre apart in vacuum, produces a force of $2 \times 10^{-7} \text{ N}$ per metre between them.

Step 4: (ii) Relate the currents in the two arcs The two arcs ACB and ADB are parallel branches between A and B. For the uniform circular wire, their resistances are $R_1 = \rho \ell_1 / S$ and $R_2 = \rho \ell_2 / S$. Since both branches have the same potential difference V_{AB} ,

$$I_1 R_1 = I_2 R_2 = V_{AB} \Rightarrow I_1 \ell_1 = I_2 \ell_2.$$

The longer branch therefore carries the smaller current. The currents are not generally equal, since the two arcs need not have equal lengths. If their angles at O are θ_1 and θ_2 , then $\ell_1 = r\theta_1$ and $\ell_2 = r\theta_2$, so $I_1 \theta_1 = I_2 \theta_2$.

Step 5: (ii) Add the magnetic fields at the centre In the printed figure, current I_1 travels from A to B along ACB anticlockwise, so its field at O is out of the page. Current I_2 travels from A to B along ADB clockwise, so its field is into the page. The field magnitudes are

$$B_1 = \frac{\mu_0 I_1 \theta_1}{4\pi r} = \frac{\mu_0 I_1 \ell_1}{4\pi r^2}, \quad B_2 = \frac{\mu_0 I_2 \ell_2}{4\pi r^2}.$$

Taking out of the page as positive and using the current-length relation,

$$B_{\text{loop}} = B_1 - B_2 = \frac{\mu_0}{4\pi r^2} (I_1 \ell_1 - I_2 \ell_2) = 0.$$

Thus the two arc contributions cancel exactly at O. This result concerns the circular loop and follows from its uniform resistance per unit length.

Step 6: Conclusion The parallel wires attract with force per unit length $F/L = \mu_0 I_1 I_2 / (2\pi d)$, giving the force-based definition of one ampere as stated above. For the circular loop, $I_1 \ell_1 = I_2 \ell_2$; the two arcs produce equal and opposite fields, so $B_{\text{loop}}(O) = 0$.

Quick Tip to Solve: For a uniform loop split into parallel arcs, first use equal branch voltages to prove $I\ell$ is the same in each arc; then compare their centre fields, which are proportional to $I\ell$.

33(a). (i) (A) Why does the electric field inside a dielectric slab decrease when kept in an external electric field?

(B) Derive an expression for the capacitance of a parallel plate capacitor filled with a medium of dielectric constant K .

(ii) A charge $q = 2 \mu\text{C}$ is placed at the centre of a sphere of radius 20 cm. What is the amount of work done in moving $4 \mu\text{C}$ from one point to another point on its surface?

(iii) Write a relation for polarisation $\vec{P}$ of a dielectric material in the presence of an external electric field.

[5 marks]

Solution:

Concept:

- Polarisation is the electric dipole moment per unit volume of a dielectric. Bound surface charges produce an induced field opposing the applied field.
- Capacitance is $C = Q/V$. For parallel plates of area A , separation d and negligible edge effects, the vacuum field is $E_0 = \sigma/\epsilon_0$, where $\sigma = Q/A$.
- In a homogeneous linear dielectric, $\epsilon = K\epsilon_0$ and $\vec{P} = \epsilon_0 \chi_e \vec{E}$, where $\chi_e = K - 1$ and $\vec{E}$ is the resultant field inside the material.
- Work done by an external agent in moving a test charge q_t slowly is $W = q_t(V_B - V_A)$. The potential of a point charge at distance r is $V = q/(4\pi\epsilon_0 r)$.

Step 1: Explain the decrease in field in part (i)(A) When the dielectric slab is placed in the external field, the positive and negative bound charges shift slightly relative to one another. In a polar dielectric, permanent molecular dipoles also tend to align with the field. The resulting polarisation produces bound charges on the opposite faces of the slab. Their electric field points opposite to the externally applied field within the slab. Therefore the net internal field is reduced: in magnitude, $E = E_0 - E_p$, where E_p is the opposing polarisation field. It is not generally zero, since a dielectric has bound charges rather than freely mobile charges as in a conductor.

Step 2: Derive the dielectric-filled capacitance in part (i)(B) Consider two large parallel plates carrying charges $+Q$ and $-Q$. Fill the entire gap of thickness d with the dielectric of relative permittivity K . Neglect fringing. With the free charge held fixed, polarisation reduces the field by the factor K :

$$E = \frac{E_0}{K} = \frac{\sigma}{K\epsilon_0} = \frac{Q}{K\epsilon_0 A}.$$

The field is uniform, so the potential difference between the plates is

$$V = Ed = \frac{Qd}{K\epsilon_0 A}.$$

Applying the definition of capacitance gives

$$C = \frac{Q}{V} = \frac{K\epsilon_0 A}{d} = KC_0, \quad C_0 = \frac{\epsilon_0 A}{d}.$$

Thus, for a given free plate charge, the dielectric lowers the voltage and increases capacitance. The expression assumes complete filling of the gap.

Step 3: Compare the surface potentials in part (ii) Both the initial point A and final point B lie at the same distance $R = 20 \text{ cm} = 0.20 \text{ m}$ from the central charge $q = 2 \times 10^{-6} \text{ C}$. Hence

$$V_A = V_B = \frac{1}{4\pi\epsilon_0} \frac{q}{R} = \frac{(9 \times 10^9)(2 \times 10^{-6})}{0.20} = 9.0 \times 10^4 \text{ V}.$$

For the transported charge $q_t = 4 \times 10^{-6} \text{ C}$, the work is

$$W = q_t(V_B - V_A) = (4 \times 10^{-6})(0) = 0 \text{ J}.$$

The sphere's surface is equipotential. Accordingly, both the electrostatic work and the quasistatic external work vanish for this displacement.

Step 4: State the polarisation relation in part (iii) For a linear isotropic dielectric the polarisation vector is parallel to the electric field in the dielectric:

$$\vec{P} = \epsilon_0 \chi_e \vec{E} = \epsilon_0 (K - 1) \vec{E}.$$

Here $\vec{P}$ is the dipole moment per unit volume, measured in C m^{-2} , and χ_e is the electric susceptibility. This uses the resultant internal field, which includes the opposing field produced by polarisation.

Step 5: Conclusion The field decreases because polarisation creates an opposing field. The fully filled capacitor has $C = K\epsilon_0 A/d$. The work between any two points on the sphere is 0 J, and $\vec{P} = \epsilon_0 \chi_e \vec{E}$.

Quick Tip to Solve: For a central point charge, compare the two radii before calculating work: equal radii imply equal potentials and zero work, regardless of the charge moved.

33(b). (i) Obtain an expression for the potential energy of an electric dipole placed in a uniform electric field.

(ii) Three capacitors of capacitance C_1 , C_2 and C_3 are connected in series to a source of V volt. Show that the total energy stored in the combination of capacitors is equal to sum of the energy stored in individual capacitors.

(iii) A capacitor of capacitance C is connected across a battery. After charging, the battery is disconnected and the separation between the plates is doubled. How will (i) the capacitance of the capacitor, and (ii) the electric field between the plates be affected? Justify your answer. [5 marks]

Solution:

Concept:

- A dipole of moment $\vec{p}$ in a uniform field $\vec{E}$ experiences torque $\vec{\tau} = \vec{p} \times \vec{E}$. Its potential energy is defined relative to a chosen orientation.
- A capacitor stores energy $U = \frac{1}{2}CV^2 = Q^2/(2C)$. Series capacitors carry equal charge magnitudes, while their voltages add.
- For parallel plates, $C = \epsilon_0 A/d$ and $E = \sigma/\epsilon_0 = Q/(\epsilon_0 A)$, neglecting edge effects. Disconnecting the battery fixes the charge on an isolated capacitor.

Step 1: Derive the dipole potential energy in part (i) Let θ be the angle between $\vec{p}$ and $\vec{E}$, with $p = q\ell$ for charges separated by distance ℓ . The electric torque tends to decrease θ , so its component for increasing θ is $-pE \sin \theta$. To rotate the dipole slowly, an external agent supplies the balancing torque $+pE \sin \theta$. For a small positive angular displacement,

$$dW_{\text{ext}} = pE \sin \theta d\theta = dU.$$

Integrating from a reference angle θ_0 to θ ,

$$\begin{aligned} U(\theta) - U(\theta_0) &= \int_{\theta_0}^{\theta} pE \sin \theta' d\theta' \\ &= pE(\cos \theta_0 - \cos \theta). \end{aligned}$$

Choose zero energy at $\theta_0 = \pi/2$. Since $\cos(\pi/2) = 0$,

$$U(\theta) = -pE \cos \theta = -\vec{p} \cdot \vec{E}.$$

The aligned orientation has minimum energy $-pE$, and the opposite orientation has maximum energy $+pE$. These limits also verify the sign of the result.

Step 2: Relate the series charge and voltage in part (ii) Let C_s be the equivalent series capacitance. Because the junctions between initially uncharged capacitors remain electrically isolated, each capacitor carries the same charge magnitude Q . Their voltage drops are $V_1 = Q/C_1$, $V_2 = Q/C_2$ and $V_3 = Q/C_3$. Thus,

$$V = Q \left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \right), \quad \frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}.$$

Also $Q = C_s V$. Equal charge, rather than equal voltage, is the defining constraint for this series combination.

Step 3: Add the individual stored energies The energy in each capacitor is $U_i = Q^2/(2C_i)$. Therefore,

$$\begin{aligned}U_1 + U_2 + U_3 &= \frac{Q^2}{2} \left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \right) \\&= \frac{Q^2}{2C_s} = \frac{1}{2} C_s V^2.\end{aligned}$$

The last expression is exactly the total energy stored by the equivalent capacitor across the source voltage V . Hence the combination's energy equals the sum of the energies of the individual capacitors; no assumption that their capacitances are equal is needed.

Step 4: Apply the disconnected-battery condition in part (iii) Let the original area and separation be A and d , with $C = \epsilon_0 A/d$. Once the battery is disconnected, no charge can enter or leave the plates; consequently Q and $\sigma = Q/A$ remain unchanged. Doubling the separation gives

$$C' = \frac{\epsilon_0 A}{2d} = \frac{C}{2}.$$

Meanwhile the field depends on charge per unit plate area:

$$E' = \frac{\sigma}{\epsilon_0} = \frac{Q}{\epsilon_0 A} = E.$$

Thus capacitance is halved, but the field is unchanged under the parallel-plate approximation. As a check, $V' = Q/C' = 2V$ and $E' = V'/(2d) = V/d$. The voltage is allowed to increase because the battery is no longer connected.

Step 5: Conclusion With energy zero at perpendicular orientation, $U = -\vec{p} \cdot \vec{E}$. In series, $U_{\text{total}} = U_1 + U_2 + U_3 = \frac{1}{2} C_s V^2$. After disconnecting the battery and doubling separation, $C' = C/2$ and $E' = E$.

Quick Tip to Solve: Identify the constraint first: series capacitors share charge, and a disconnected capacitor keeps its charge. Use $Q^2/(2C)$ for series energies and $E = Q/(\epsilon_0 A)$ for the isolated plates.