CBSE Class 12 Mathematics 2021 - Set 65/1/1 - Compartment / Improvement (2020-21)

Question Paper

Conducted by the Central Board of Secondary Education (CBSE)

Time Allowed: 3 Hours **Maximum Marks: 80**

General Instructions

(i) This question paper contains two parts - A and B. Each part is compulsory. Part A carries 24 marks and Part B carries 56 marks.

(ii) Part A has objective type questions and Part B has descriptive type questions.

(iii) Part A consists of Sections I and II. Section I comprises 16 very short answer questions of 1 mark each, with choices in 5 questions.

(iv) Section II contains 2 case studies. Each comprises 5 multiple choice questions of 1 mark each; attempt any 4 of the 5 in each case study. Both case studies are compulsory.

(v) Part B consists of Sections III, IV and V.

(vi) Section III comprises 10 questions of 2 marks each.

(vii) Section IV comprises 7 questions of 3 marks each.

(viii) Section V comprises 3 questions of 5 marks each.

(ix) Internal choice is provided in 3 questions of Section III, 2 questions of Section IV and all 3 questions of Section V. Attempt only one alternative in each such question.

Part A - Section I

1. If A is a square matrix of order 3 such that $A(\text{adj } A) = \begin{bmatrix} -2 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -2 \end{bmatrix}$, then find $|A|$.

2(a). Find the order of the matrix A such that $\begin{bmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{bmatrix} A = \begin{bmatrix} -1 & -8 \\ 1 & -2 \\ 9 & 22 \end{bmatrix}$.

OR

2(b). If $B = \begin{bmatrix} 1 & -5 \\ 0 & -3 \end{bmatrix}$ and $A + 2B = \begin{bmatrix} 0 & 4 \\ -7 & 5 \end{bmatrix}$, find the matrix A.

3. Write the smallest reflexive relation on set $A = \{a, b, c\}$.

4(a). Find $\int e^x \left(\log \sqrt{x} + \frac{1}{2x} \right) dx$.

OR

4(b). Find $\int e^{2 \log x} \, dx$.

5(a). Find the angle between the vectors $\hat{i} - \hat{j}$ and $\hat{j} - \hat{k}$.

OR

5(b). Write the projection of the vector $\vec{r} = 3\hat{i} - 4\hat{j} + 12\hat{k}$ on (i) x-axis, and (ii) y-axis.

6. If $\vec{a} = \alpha\hat{i} + 3\hat{j} - 6\hat{k}$ and $\vec{b} = 2\hat{i} - \hat{j} - \beta\hat{k}$, find the value of α and β so that $\vec{a}$ and $\vec{b}$ may be collinear.

7. If $f = \{(1, 2), (2, 4), (3, 1), (4, k)\}$ is a one-one function from set A to A, where $A = \{1, 2, 3, 4\}$, then find the value of k.

8(a). Check whether the relation R defined on the set $\{1, 2, 3, 4\}$ as $R = \{(a, b) : b = a + 1\}$ is transitive. Justify your answer.

OR

8(b). If the relation R on the set $A = \{x : 0 \leq x \leq 12\}$ given by $R = \{(a, b) : a = b\}$ is an equivalence relation, then find the set of all elements related to 1.

9. If $A = \begin{bmatrix} 1 & 0 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 2 \\ 5 \\ 6 \end{bmatrix}$, find AB.

10(a). Write the order and degree of the differential equation $\frac{d^2y}{dx^2} + 3\left(\frac{dy}{dx}\right)^2 = x^2 \log\left(\frac{d^2y}{dx^2}\right)$.

OR

10(b). Find the general solution of the differential equation $\frac{dy}{dx} = a$, where a is an arbitrary constant.

11. Show that the function $f(x) = \frac{3}{x} + 7$ is strictly decreasing for $x \in \mathbb{R} - \{0\}$.

12. Find the magnitude of vector $\vec{a}$ given by $\vec{a} = (\hat{i} + 3\hat{j} - 2\hat{k}) \times (-\hat{i} + 3\hat{k})$.

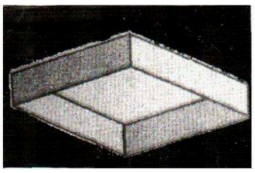
13. Write the equation of the plane that cuts the coordinate axes at $(2, 0, 0)$, $(0, 4, 0)$ and $(0, 0, 7)$.

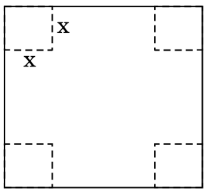
14. Find the distance between the two parallel planes $3x + 5y + 7z = 3$ and $9x + 15y + 21z = 12$.

15. If A and B are two independent events and $P(A) = \frac{1}{3}$ and $P(B) = \frac{1}{2}$, find $P(\overline{A} \mid \overline{B})$.

16. A coin is tossed once. If head comes up, a die is thrown, but if tail comes up, the coin is tossed again. Find the probability of obtaining head and number 6.

17. A factory makes an open cardboard box for a jewellery shop from a square sheet of side 18 cm by cutting off squares from each corner and folding up the flaps.





Based on the above information, answer any four of the following five questions, if x is the length of each square cut from corners. $4\times1=4$

(i) The volume of the open box is:

(A) $4x(x^2-18x+81)$

(B) $2x(2x^2+36x+162)$

(C) $2x(2x^2+36x-162)$

(D) $4x(x^2+18x+81)$

(ii) The condition for the volume (V) to be maximum is:

(A) $\frac{dV}{dx}=0$ and $\frac{d^2V}{dx^2}<0$

(B) $\frac{dV}{dx}=0$ and $\frac{d^2V}{dx^2}>0$

17. (continued)

(C) $\frac{dV}{dx}>0$ and $\frac{d^2V}{dx^2}=0$

(D) $\frac{dV}{dx}<0$ and $\frac{d^2V}{dx^2}=0$

(iii) What should be the side of square to be cut off so that the volume is maximum?

(A) 6 cm

(B) 9 cm

(C) 3 cm

(D) 4 cm

(iv) Maximum volume of the open box is:

(A) 423 cm^3

(B) 432 cm^3

(C) 400 cm^3

(D) 216 cm^3

(v) The total area of the removed squares is:

(A) 324 cm^2

(B) 144 cm^2

(C) 36 cm^2

(D) 64 cm^2

18. In answering a multiple choice test for class XII, a student either knows or guesses or copies the answer to a multiple choice question with four choices. The probability that he makes a guess is $\frac{1}{3}$ and the probability that he copies the answer is $\frac{1}{6}$. The probability that his answer is correct given that he copied is $\frac{1}{8}$. Let E_1, E_2, E_3 be the events that the student guesses, copies or knows the answer respectively and A is the event that the student answers correctly. Based on the above information, answer any four of the following five questions. $4 \times 1 = 4$

(i) What is the probability that the student knows the answer?

(A) 1

(B) $\frac{1}{2}$

(C) $\frac{2}{3}$

(D) $\frac{1}{4}$

(ii) What is the probability that he answers correctly given that he knew the answer?

(A) 1

(B) 0

(C) $\frac{1}{4}$

(D) $\frac{1}{8}$

18. (continued)

(iii) What is the probability that he answers correctly given that he had made a guess?

(A) $\frac{1}{4}$

(B) 0

(C) 1

(D) $\frac{1}{8}$

(iv) What is the probability that he knew the answer to the question, given that he answered it correctly?

(A) $\frac{24}{29}$

(B) $\frac{4}{29}$

(C) $\frac{1}{29}$

(D) $\frac{3}{29}$

(v) $\sum_{k=1}^{3} P(E_k \mid A)$ is:

(A) 0

(B) $\frac{1}{3}$

(C) 1

(D) $\frac{11}{8}$

Part B - Section III

19. A random variable X has the probability distribution:

X	0	1	2	3	4
$P(X)$	0	K	$4K$	$3K$	$2K$

Find the value of K and $P(X \leq 2)$.

20. Simplify $\sec^{-1}\left(\frac{1}{2x^2-1}\right)$, $0 < x < \frac{1}{\sqrt{2}}$.

21. If the matrix $A = \begin{bmatrix} 0 & 6-5x \\ x^2 & x+3 \end{bmatrix}$ is symmetric, find the values of x.

22(a). Find the relationship between a and b so that the function f defined by $f(x) = \begin{cases} ax+1 & \text{if } x \leq 3 \\ bx+3 & \text{if } x > 3 \end{cases}$ is continuous at $x = 3$.

OR

22(b). Check the differentiability of $f(x) = |x-3|$ at $x = 3$.

23. Find $\int \frac{x^2+2}{x^2+1}\,dx$.

24(a). Evaluate $\int_{-1}^{1} \frac{|x|}{x}\,dx$.

OR

24(b). Evaluate $\int_0^{\pi/2} \log\left(\frac{4+3\sin x}{4+3\cos x}\right)\,dx$.

25. Find the integrating factor of $x\frac{dy}{dx} + (1 + x\cot x)y = x$.

26. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are three mutually perpendicular unit vectors, find the value of $|\vec{a}+2\vec{b}+3\vec{c}|$.

27. If the sides AB and BC of a parallelogram $ABCD$ are represented as vectors $\overrightarrow{AB} = 2\hat{i} + 4\hat{j} - 5\hat{k}$ and $\overrightarrow{BC} = \hat{i} + 2\hat{j} + 3\hat{k}$, then find the unit vector along diagonal AC.

28(a). Using integration, find the area bounded by the curve $y^2 = 4x$, y-axis and $y = 3$.

OR

28(b). Using integration, find the area of the region bounded by the line $2y = -x + 8$, x-axis, $x = 2$ and $x = 4$.

Part B - Section IV

29. Show that the function $f : \mathbb{R} - \{-1\} \to \mathbb{R} - \{1\}$ given by $f(x) = \frac{x}{x+1}$ is bijective.

30(a). If $x = a\cos\theta + b\sin\theta$, $y = a\sin\theta - b\cos\theta$, then show that $\frac{dy}{dx} = -\frac{x}{y}$ and hence show that $y^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = 0$.

OR

30(b). If $e^{y-x} = y^x$, prove that $\frac{dy}{dx} = \frac{y(1+\log y)}{x\log y}$.

31. Differentiate $\sin^2 x$ w.r.t. $e^{\cos x}$.

32(a). Find the equation of the normal to the curve $y^2 = 4ax$ at $\left(\frac{a}{m^2}, \frac{2a}{m}\right)$.

OR

32(b). Find the equation of the tangent to the curve $y(1+x^2) = 2 - x$, where it crosses x-axis.

33. Find $\int \frac{x^2}{(x-1)(x+1)^2}\,dx$.

34. If the solution of the differential equation $\frac{dy}{dx} = \frac{2xy - y^2}{2x^2}$ is $\frac{ax}{y} = b\log|x| + C$, find the value of a and b.

35. Using integration, find the area bounded by the circle $x^2 + y^2 = 9$.

Part B - Section V

36(a). If $A = \begin{bmatrix} 3 & 4 & 2 \\ 0 & 2 & -3 \\ 1 & -2 & 6 \end{bmatrix}$, find A^{-1}. Hence, solve the following system of equations: $3x + 4y + 2z = 8$, $2y - 3z = 3$, $x - 2y + 6z = -2$.

OR

36(b). If $A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$, find $(AB)^{-1}$.

37(a). Find the shortest distance between the following lines: $\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}$ and $\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$.

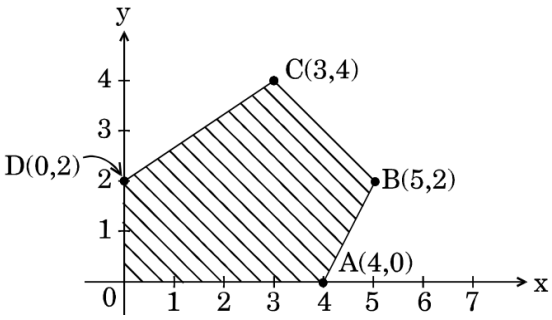
OR

37(b). Find the distance of the point $(-1, -5, -10)$ from the point of intersection of the line $\vec{r} = 2\hat{i} - \hat{j} + 2\hat{k} + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k})$ and the plane $\vec{r} \cdot (\hat{i} - \hat{j} + \hat{k}) = 5$.

38(a). Solve the following linear programming problem graphically: Maximise $z = 3x + 9y$ subject to $x + 3y \leq 60$, $x + y \geq 10$, $x \leq y$, $x, y \geq 0$.

OR

38(b). The corner points of the feasible region determined by the system of linear inequations are as shown below:



Answer each of the following:

(i) Let $z = 13x - 15y$ be the objective function. Find the maximum and minimum values of z and also the corresponding points at which the maximum and minimum values occur.

(ii) Let $z = kx + y$ be the objective function. Find k, if the value of z at A is the same as the value of z at B.

- End of Question Paper -