

CBSE Class 12 Physics Question Paper 2022 with Solution PDF-Set 55/4/3

Question Paper with Solutions

Conducted by the Central Board of Secondary Education (CBSE)



Time Allowed: 2 hours

Maximum Marks: 35

General Instructions

- (i) This question paper contains **12** questions. **All** questions are compulsory.
- (ii) This question paper is divided into **three** sections – **Section A, B, and C.**
- (iii) **Section A** – Questions no. **1 to 3** are of **2** marks each.
- (iv) **Section B** – Questions no. **4 to 11** are of **3** marks each.
- (v) **Section C** – Question no. **12** is a Case Study-Based Question of **5** marks.
- (vi) There is no overall choice in the question paper. However, internal choice has been provided in some of the questions. Attempt any one of the alternatives in such questions.
- (vii) Use of log tables is permitted, if necessary, but use of calculator is **not** permitted.

$$c = 3 \times 10^8 \text{ m/s}$$

$$h = 6.63 \times 10^{-34} \text{ J s}$$

$$e = 1.6 \times 10^{-19} \text{ C}$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ T m A}^{-1}$$

$$\epsilon_0 = 8.854 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$$

$$\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$$

$$\text{Mass of electron } (m_e) = 9.1 \times 10^{-31} \text{ kg}$$

$$\text{Mass of neutron} = 1.675 \times 10^{-27} \text{ kg}$$

$$\text{Mass of proton} = 1.673 \times 10^{-27} \text{ kg}$$

$$\text{Avogadro's number} = 6.023 \times 10^{23} \text{ per gram mole}$$

$$\text{Boltzmann constant} = 1.38 \times 10^{-23} \text{ J K}^{-1}$$

Section A (6 marks)

1(a). Name the spectral series for a hydrogen atom which lies in the visible region. Find the ratio of the maximum to the minimum wavelengths of this series. [2 marks]

Solution:

Concept:

- The Balmer series consists of hydrogen emission transitions from levels $n = 3, 4, 5, \dots$ to the fixed lower level $n = 2$; its familiar spectral lines lie in the visible region.
- The Rydberg formula is $1/\lambda = R_H(1/2^2 - 1/n^2)$, where R_H is the hydrogen Rydberg constant and n is the initial principal quantum number.

Step 1: Identify the longest wavelength Name the visible hydrogen series as the Balmer series. Since photon energy is hc/λ , the smallest energy difference gives the longest wavelength. Among transitions ending at $n = 2$, this is the transition from $n = 3$ to $n = 2$:

$$\frac{1}{\lambda_{\max}} = R_H \left(\frac{1}{4} - \frac{1}{9} \right) = \frac{5R_H}{36}, \quad \lambda_{\max} = \frac{36}{5R_H}.$$

Step 2: Find the series limit and form the ratio The shortest wavelength is the series limit, corresponding to the greatest possible energy difference with the lower level still fixed at $n = 2$. Thus take the initial level to $n \rightarrow \infty$, so that $1/n^2 \rightarrow 0$:

$$\frac{1}{\lambda_{\min}} = \frac{R_H}{4}, \quad \lambda_{\min} = \frac{4}{R_H}.$$

Divide the longest wavelength by this limiting wavelength; the Rydberg constant cancels:

$$\frac{\lambda_{\max}}{\lambda_{\min}} = \frac{36/(5R_H)}{4/R_H} = \frac{9}{5}.$$

Step 3: Conclusion The series is the Balmer series, and the required ratio is $\lambda_{\max} : \lambda_{\min} = 9 : 5$.

Quick Tip to Solve: For any hydrogen series, use the first transition for the longest wavelength and the infinite initial level for the shortest; keep the final level fixed.

OR

1(b). What are matter waves? A proton and an alpha particle are accelerated through the same potential difference. Find the ratio of the de Broglie wavelength associated with the proton to that with the alpha particle. [2 marks]

Solution:

Concept:

- Matter waves are waves associated with moving material particles. Their de Broglie wavelength is $\lambda = h/p$, where h is Planck's constant and p is the particle momentum.
- A particle of mass m and charge q , accelerated from rest through a potential difference V , gains kinetic energy qV . In the non-relativistic treatment, $p = \sqrt{2mqV}$.

Step 1: Express the wavelength after acceleration Matter waves describe the wave nature of a moving particle; they are associated with the particle rather than being electromagnetic waves. For acceleration from rest, equate the electrical work to the kinetic energy:

$$qV = \frac{p^2}{2m} \Rightarrow p = \sqrt{2mqV}, \quad \lambda = \frac{h}{\sqrt{2mqV}}.$$

Thus equal accelerating potentials do not mean equal momenta: the wavelength depends on the product of mass and charge as well as the potential.

Step 2: Substitute the proton and alpha-particle properties A proton has charge e and mass m_p . An alpha particle has charge $2e$ and mass approximately $4m_p$. Both are accelerated through the same V , so

$$\lambda_p = \frac{h}{\sqrt{2m_p e V}}, \quad \lambda_\alpha = \frac{h}{\sqrt{2(4m_p)(2e)V}}.$$

Taking their ratio cancels the common factors h , e , m_p and V :

$$\frac{\lambda_p}{\lambda_\alpha} = \sqrt{\frac{(4m_p)(2e)}{m_p e}} = \sqrt{8} = 2\sqrt{2}.$$

Step 3: Conclusion Matter waves are the waves associated with moving material particles, with $\lambda = h/p$. The required wavelength ratio is $\lambda_p : \lambda_\alpha = 2\sqrt{2} : 1$.

Quick Tip to Solve: At the same accelerating potential, compare mq , not mass alone: the alpha particle contributes a factor of four in mass and two in charge.

2. What is meant by energy band gap in a solid? Draw the energy band diagrams for a conductor, an insulator and a semiconductor. [2 marks]

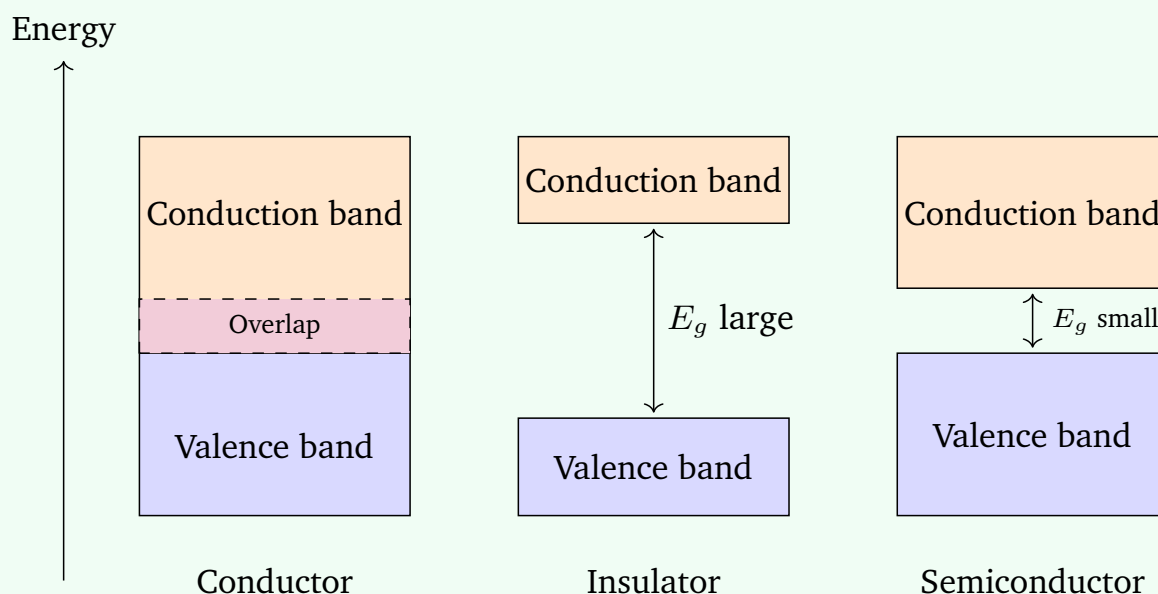
Solution:

Concept:

- In a solid, closely spaced allowed electron energies form bands. The valence band contains the valence electrons, while electrons in the conduction band can move through the solid and carry current.
- The energy band gap is the forbidden energy interval between the top of the valence band and the bottom of the conduction band: $E_g = E_C - E_V$. No allowed electron energy states exist in this interval.

Step 1: Compare the allowed bands For a conductor, the valence and conduction bands overlap (or an allowed band is partly filled), so electrons can access nearby empty states without overcoming a forbidden gap. An insulator has a large gap, so ordinary thermal energy cannot promote appreciable numbers of electrons to the conduction band. A semiconductor has a small gap; thermal excitation can promote some electrons to the conduction band, leaving holes in the valence band.

Step 2: Represent the band gaps In the diagrams, energy increases upwards. The shaded rectangles show allowed bands. The conductor has overlapping bands and hence no forbidden separation, the insulator has a wide forbidden interval, and the semiconductor has a narrow forbidden interval. These relative gap sizes explain their different electrical conductivities.



Energy bands for a conductor, an insulator and a semiconductor (schematic).

Step 3: Conclusion The energy band gap E_g is the forbidden interval between the valence and conduction bands. It is absent between overlapping bands in a conductor, large in an insulator and small in a semiconductor, as shown.

Quick Tip to Solve: Draw energy upwards and compare the separation of the two bands: overlap, wide gap, then narrow gap.

3. Define barrier potential. Why does the thickness of the depletion layer in a p-n junction diode vary with increase in reverse bias? [2 marks]

Solution:

Concept:

- Initial diffusion of electrons from the n-region and holes from the p-region leaves immobile positive donor ions on the n-side and negative acceptor ions on the p-side. This region is depleted of mobile majority carriers.
- The fixed ions establish an electric field from the n-side to the p-side and a potential difference across the depletion layer which opposes further majority-carrier diffusion.

Step 1: Define the barrier potential Barrier potential is the potential difference established across the depletion region of a p-n junction due to the uncovered, immobile ions. Its associated electric field opposes the diffusion of majority carriers across the junction. At equilibrium, this field produces a drift current that balances diffusion current, preventing a net current through the unbiased junction.

Step 2: Explain the effect of increasing reverse bias In reverse bias, the p-side is connected to the negative terminal and the n-side to the positive terminal. The applied field reinforces the junction field. Holes on the p-side and electrons on the n-side are pulled away from the junction, exposing additional immobile ions on both sides. Consequently the carrier-depleted region extends farther into both regions: its thickness increases as the reverse-bias voltage increases. The total potential barrier also increases, further suppressing majority-carrier diffusion.

Step 3: Conclusion Barrier potential is the built-in potential difference across the depletion layer

that opposes majority-carrier diffusion. Increasing reverse bias withdraws majority carriers from the junction and exposes more fixed ions, so the depletion layer becomes wider.

Quick Tip to Solve: For reverse bias, remember p to negative and n to positive: both majority carriers are pulled away, widening the depletion layer.

Section B (24 marks)

4. Answer the following, giving reason:

- (a) The resistance of a p - n junction is low when it is forward biased and is high when it is reversed biased.
- (b) Doping of intrinsic semiconductors is a necessity for making electronic devices.
- (c) Photodiodes are operated in reverse bias. [3 marks]

Solution:

Concept:

- The depletion region and potential barrier of a p - n junction oppose diffusion of majority carriers. Bias changes this barrier and the junction current.
- Doping adds suitable impurities to control carrier concentration and conductivity; absorbed photons generate electron-hole pairs in a photodiode.

Step 1: (a) Compare forward and reverse bias In forward bias, the p -side is connected to the positive terminal and the n -side to the negative terminal. The applied field opposes the junction field, lowering the potential barrier and narrowing the depletion layer. Majority carriers then cross the junction readily, producing a large current for a small voltage; the resistance is low. Reverse bias increases the barrier and widens the depletion region. Majority-carrier flow is suppressed and only a small minority-carrier current flows before breakdown, giving high resistance.

Step 2: (b) Explain the necessity of doping An intrinsic semiconductor has a relatively small concentration of thermally generated electrons and holes. Its conductivity is low and strongly temperature dependent. Controlled addition of pentavalent impurities produces n -type material with electrons as majority carriers; trivalent impurities produce p -type material with holes as majority carriers. Doping therefore provides useful, controllable carrier concentrations and conductivity. It also enables p - n junctions to be formed, as required for diodes and transistors.

Step 3: (c) Explain the operating bias of a photodiode Light produces electron-hole pairs in and near the depletion region. Under reverse bias, the strong junction electric field separates these carriers and sweeps them to opposite sides, producing a photocurrent. The dark reverse current is small, so the light-induced change is readily detected; the photocurrent is approximately proportional to incident light intensity. Reverse bias also widens the carrier collection region. A large forward majority-carrier current would obscure the small light-induced change.

Step 4: Conclusion Forward bias reduces the barrier and resistance; reverse bias increases them. Doping controls the carrier concentration needed for electronic devices. A photodiode uses reverse bias to detect the small, light-dependent current sensitively.

Quick Tip to Solve: Relate bias to barrier height, doping to carrier concentration, and photodetection to separation of light-generated carriers above a small dark current.

5. (a) Differentiate between nuclear fission and nuclear fusion.

(b) Deuterium undergoes fusion as per the reaction: ${}^2_1\text{H} + {}^2_1\text{H} \rightarrow {}^3_2\text{He} + {}^1_0\text{n} + 3.27 \text{ MeV}$. Find the duration for which an electric bulb of 500 W can be kept glowing by the fusion of 100 g of deuterium. [3 marks]

Solution:

Concept:

- Nuclear fission splits a heavy nucleus into lighter nuclei, whereas nuclear fusion combines light nuclei into a heavier nucleus. Both can release energy through an increase in binding energy per nucleon.
- The number of nuclei in mass m is $N = (m/M)N_A$, where M is the molar mass and N_A is Avogadro's constant. For constant power P , the duration is $t = E/P$.

Step 1: (a) Distinguish fission from fusion In fission, a heavy nucleus, such as uranium, splits into two lighter nuclei with release of energy and usually neutrons. Neutron-induced fission can sustain a chain reaction and is used in nuclear reactors. In fusion, two light nuclei combine to produce a heavier nucleus, sometimes with another emitted particle, and release energy. Fusion requires extremely high temperature, so the nuclei can overcome their electrostatic repulsion; it is the energy source of stars. Thus the processes differ in the direction of the nuclear change and their operating conditions.

Step 2: (b) Count the available fusion reactions Take the molar mass of deuterium as $M = 2 \text{ g mol}^{-1}$. The given mass contains

$$N = \frac{100}{2}(6.022 \times 10^{23}) = 3.011 \times 10^{25}$$

deuterium nuclei. The printed reaction consumes two deuterium nuclei per event, so the number of events is $N/2 = 1.5055 \times 10^{25}$, not N .

Step 3: Convert the released energy and calculate the duration Energy released per event is

$$Q = 3.27 \times 10^6(1.602 \times 10^{-19}) = 5.23854 \times 10^{-13} \text{ J.}$$

Assuming all the deuterium undergoes this reaction and the released energy is fully available to supply the bulb,

$$\begin{aligned} E &= (1.5055 \times 10^{25})(5.23854 \times 10^{-13}) \\ &= 7.8866 \times 10^{12} \text{ J,} \\ t &= \frac{E}{500} = 1.5773 \times 10^{10} \text{ s.} \end{aligned}$$

Using $365 \times 24 \times 3600 \text{ s}$ per year, this corresponds to approximately 500 years. The ideal conversion assumption is needed because no efficiency is specified.

Step 4: Conclusion Fission splits heavy nuclei, while fusion combines light nuclei. The 500 W bulb can ideally glow for $1.58 \times 10^{10} \text{ s}$, or approximately 500 years, using the stated fusion of 100 g of deuterium.

Quick Tip to Solve: Divide the number of deuterium nuclei by two before multiplying by 3.27 MeV: each fusion event consumes a pair of nuclei.

6. Briefly explain how bright and dark fringes are formed on a screen due to the diffraction at a single slit. Hence, explain why the intensity at the bright fringes decreases sharply as their order (n) increases. [3 marks]

Solution:

Concept:

- By the Huygens principle, each point across a slit of width a acts as a coherent secondary source. Their waves superpose at an observation angle θ .
- The path difference between the extreme rays is $a \sin \theta$. For wavelength λ , define $\beta = \pi a \sin \theta / \lambda$; the intensity is $I = I_0 (\sin \beta / \beta)^2$, where I_0 is the central intensity.

Step 1: Formation of dark fringes At $a \sin \theta = \lambda$, divide the slit into two equal halves. Each source in one half pairs with a source in the other half whose path difference is $\lambda/2$. The paired waves arrive in opposite phase and cancel. More generally, for $a \sin \theta = n\lambda$, with $n = 1, 2, 3, \dots$, division into $2n$ equal strips allows complete pairwise cancellation. Thus minima occur on both sides of the centre at $a \sin \theta = \pm n\lambda$.

Step 2: Formation of bright fringes At $\theta = 0$, all secondary waves have equal paths and arrive in phase, producing the strongest central maximum. Between successive minima the cancellation is incomplete and secondary bright maxima occur. Their positions are approximately $a \sin \theta \simeq (n + \frac{1}{2})\lambda$, for $n = 1, 2, \dots$; the exact noncentral maxima satisfy $\tan \beta = \beta$. The central bright fringe lies between the first minima and is twice as wide as a secondary fringe in the small-angle approximation.

Step 3: Why higher-order maxima are weak As the angle increases, the phase differences across the slit grow and an increasing proportion of contributions cancel. Near a secondary maximum, $|\sin \beta|$ is approximately unity while $\beta \simeq (n + \frac{1}{2})\pi$. Hence

$$\frac{I_n}{I_0} \simeq \frac{1}{\pi^2(n + \frac{1}{2})^2}.$$

Thus intensity falls approximately as the inverse square of the order. The first and second secondary maxima have only about 4.7% and 1.6% of the central intensity, respectively.

Step 4: Conclusion Dark fringes result from complete destructive interference, while bright fringes result from constructive or incomplete destructive interference. The secondary maxima become sharply weaker with increasing order because cancellation increases and their intensity envelope falls as $1/\beta^2$.

Quick Tip to Solve: Use $a \sin \theta = n\lambda$ for minima; the half-integer relation for secondary maxima is only approximate.

7(a). (i) Draw a labelled ray diagram showing the formation of the image at infinity by an astronomical telescope.

(ii) A telescope consists of an objective of focal length 150 cm and an eyepiece of focal length 6.0 cm. If the final image is formed at infinity, then calculate:

(I) the length of the tube in this adjustment, and

(II) the magnification produced.

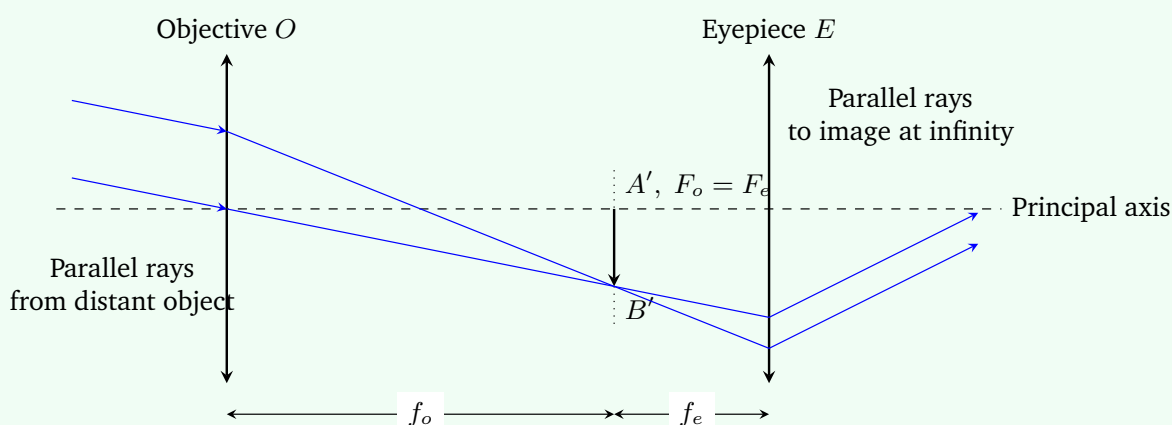
[3 marks]

Solution:

Concept:

- In normal adjustment, an astronomical telescope forms its final image at infinity. The objective forms a real, inverted intermediate image of a distant object in its focal plane, which coincides with the first focal plane of the eyepiece.
- For objective and eyepiece focal lengths f_o and f_e , the lens separation is $L = f_o + f_e$ and the signed angular magnification is $M = -f_o/f_e$. The negative sign denotes inversion.

Step 1: (i) Ray diagram for normal adjustment Parallel rays from a distant object point enter the objective O and converge at B' , forming the inverted intermediate image $A'B'$. This image is at the common focal plane $F_o = F_e$. Since it lies in the first focal plane of eyepiece E , the eyepiece sends the rays out as a parallel bundle. The eye therefore sees the final virtual image at infinity and views it without accommodation. In the diagram the incident and emergent bundles represent one off-axis object point; the dashed line is the principal axis.



Astronomical telescope in normal adjustment (schematic, not to scale)

Step 2: (ii)(I) Length of the tube The objective image is f_o behind the objective and f_e in front of the eyepiece. Thus the distance between the two lenses is their focal-length sum:

$$L = f_o + f_e = 150 + 6.0 = 156 \text{ cm} = 1.56 \text{ m}.$$

This is the tube length for the specified adjustment with the final image at infinity.

Step 3: (ii)(II) Angular magnification If the intermediate image has height magnitude h , the small angles subtended at the objective and eyepiece have magnitudes h/f_o and h/f_e , respectively. Their ratio gives the magnitude of angular magnification, and inversion supplies its negative sign:

$$M = -\frac{f_o}{f_e} = -\frac{150}{6.0} = -25.$$

Angular magnification is dimensionless. Hence the telescope enlarges the angular size by a factor of

25, with the image inverted relative to the distant object.

Step 4: Conclusion In normal adjustment the common objective-image and eyepiece focal plane produces parallel emergent rays. The tube length is 156 cm, and the angular magnification is -25 (magnitude 25).

Quick Tip to Solve: For a telescope with the final image at infinity, add the focal lengths for tube length and divide objective focal length by eyepiece focal length for magnification magnitude.

OR

7(b). (i) Draw a labelled ray diagram showing the formation of the image at least distance of distinct vision by a compound microscope.

(ii) A small object is placed at a distance of 3.0 cm from a magnifier of focal length 4.0 cm. Find:

(I) the position of the image formed, and

(II) the linear magnification produced.

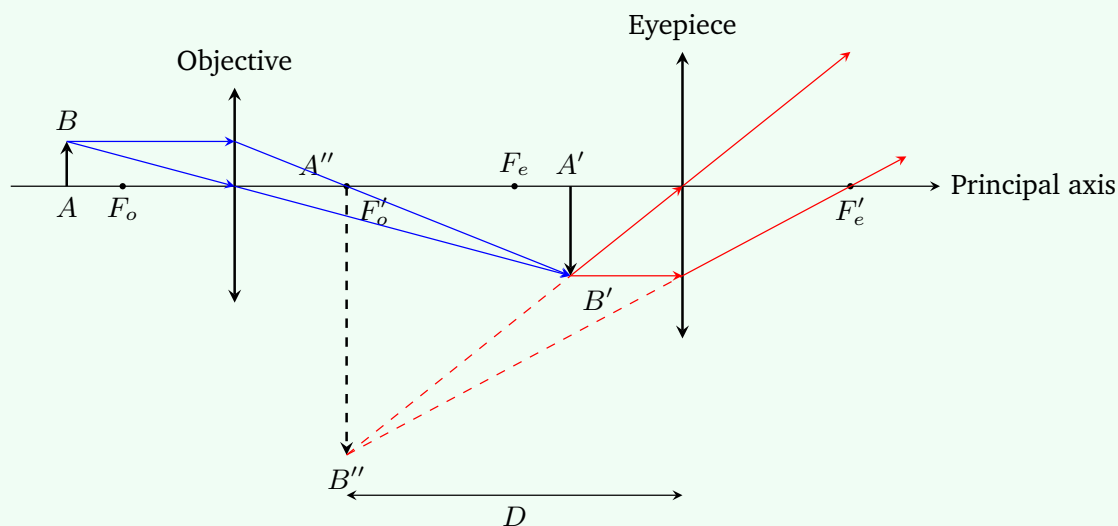
[3 marks]

Solution:

Concept:

- A compound microscope uses a short-focal-length objective to produce a real, inverted, enlarged intermediate image. The eyepiece acts as a magnifier for this image.
- For the final image at the near point, the intermediate image is within the eyepiece's focal length. The final image is virtual, enlarged and inverted relative to the original object, at distance D from the eyepiece.
- With Cartesian signs, a convex lens obeys $1/f = 1/v - 1/u$, and its linear magnification is $m = v/u$. Here u and v are signed object and image distances.

Step 1: (i) Construct the compound-microscope image Place the object AB just beyond the objective focus F_o . A ray from B parallel to the axis passes through F'_o after refraction; a ray through the objective's optical centre travels undeviated. They meet at B' , giving the real, inverted intermediate image $A'B'$. Position this image between the eyepiece and its first focus F_e . The rays emerging from the eyepiece diverge; their backward extensions meet at B'' . Thus $A''B''$ is the virtual final image at the least distance of distinct vision. The dashed lines below are backward extensions, not actual light paths; light travels from left to right.



Compound microscope adjusted for a final virtual image at the near point (schematic, not to scale).

Step 2: (ii)(I) Find the image position for the magnifier The numerical data refer to the separate simple magnifier. Its object is on the incident side, so $u = -3.0$ cm, while $f = +4.0$ cm. Applying the lens formula,

$$\frac{1}{v} = \frac{1}{f} + \frac{1}{u} = \frac{1}{4.0} - \frac{1}{3.0} = -\frac{1}{12} \text{ cm}^{-1},$$

$$v = -12 \text{ cm}.$$

The negative image distance means that the image lies on the object's side of the lens. This is consistent with an object inside the focal length of a convex lens: the emerging rays diverge and produce a virtual image.

Step 3: (ii)(II) Calculate the linear magnification Using signed distances,

$$m = \frac{v}{u} = \frac{-12}{-3.0} = +4.$$

The positive sign indicates an erect image, and the magnitude shows that its height is four times the object's height. This is linear magnification, so the near-point formula for angular magnifying power is not used for this numerical part.

Step 4: Conclusion The microscope diagram shows a virtual final image at the near point. For the separate magnifier, the image is 12 cm from the lens on the object's side ($v = -12$ cm), and its linear magnification is $m = +4$: virtual, erect and four times as tall.

Quick Tip to Solve: When the object is inside a convex lens's focal length, expect a negative image distance and positive linear magnification; use this to check the signs.

8. A proton is accelerated through a potential difference V . After acceleration, the de Broglie wavelength associated with it is λ . If the proton is replaced by an alpha particle, then find the de Broglie wavelength associated with it if it were accelerated through the same potential difference V . What will be the momentum of the alpha particle? [3 marks]

Solution:

Concept:

- A particle accelerated from rest through a potential difference V gains kinetic energy $K = qV$, where q is its positive charge.
- In the non-relativistic treatment, $K = p^2/(2m)$, so $p = \sqrt{2mqV}$. The de Broglie relation is $\lambda = h/p$, where h is Planck's constant.
- An alpha particle has charge $q_\alpha = 2e$ and mass $m_\alpha \simeq 4m_p$, with e the proton charge and m_p the proton mass.

Step 1: Express the proton's momentum using its wavelength For the proton, the energy gained from the accelerating field is eV . Hence

$$\frac{p_p^2}{2m_p} = eV, \quad p_p = \sqrt{2m_p eV} = \frac{h}{\lambda}.$$

The supplied wavelength λ therefore fixes the proton's momentum without requiring a numerical value of the potential difference.

Step 2: Compare the alpha particle's momentum with the proton's At the same potential difference the alpha particle gains $2eV$, twice the proton's kinetic energy. Its mass is approximately four times as large, so

$$p_\alpha = \sqrt{2(4m_p)(2e)V} = \sqrt{16m_p eV},$$

$$\frac{p_\alpha}{p_p} = \sqrt{\frac{m_\alpha q_\alpha}{m_p e}} = \sqrt{4 \times 2} = 2\sqrt{2}.$$

Both the mass and the charge must be included. Equal accelerating voltage does not mean equal kinetic energy for particles with different charges, and momentum depends on the square root of the mass-energy product.

Step 3: Obtain the wavelength and momentum of the alpha particle The de Broglie wavelength is inversely proportional to momentum. Consequently,

$$\lambda_\alpha = \frac{h}{p_\alpha} = \frac{h}{2\sqrt{2} p_p} = \frac{\lambda}{2\sqrt{2}}.$$

Substituting $p_p = h/\lambda$ into the momentum ratio gives

$$p_\alpha = \frac{2\sqrt{2} h}{\lambda} = 4\sqrt{m_p eV}.$$

The wavelength has dimensions of length, and h/λ has dimensions of momentum, providing a direct dimensional check of both expressions.

Step 4: Conclusion The alpha particle has de Broglie wavelength $\lambda_\alpha = \lambda/(2\sqrt{2})$ and momentum $p_\alpha = 2\sqrt{2} h/\lambda$. Equivalently, its momentum is $4\sqrt{m_p eV}$, in kg m s^{-1} when SI quantities are used.

Quick Tip to Solve: At fixed accelerating voltage use $\lambda \propto 1/\sqrt{mq}$: multiply the mass ratio by the charge ratio before taking the square root.

9. (a) In Geiger-Marsden experiment, calculate the distance of closest approach for an alpha particle with energy 2.56×10^{-12} J. Consider that the particle approaches the gold nucleus ($Z = 79$) in head-on position.

(b) If the above experiment is repeated with a proton of the same energy, then what will be the value of the distance of closest approach? [3 marks]

Solution:

Concept:

- In a head-on encounter, the incident particle slows down because of electrostatic repulsion from the positive gold nucleus. At the turning point its kinetic energy is zero.
- Taking the nucleus as stationary and the potential energy at infinity as zero, energy conservation gives $K = kqZe/r_0$, where r_0 is the distance between centres at closest approach, q is the projectile charge, and $k = 1/(4\pi\epsilon_0)$.

Step 1: (a) Apply energy conservation to the alpha particle The alpha particle has charge $q_\alpha = 2e$, while the gold nucleus has charge $79e$. At closest approach the initial kinetic energy becomes electrostatic potential energy:

$$K = \frac{k(2e)(79e)}{r_\alpha}, \quad r_\alpha = \frac{2k(79)e^2}{K}.$$

Substitute $k = 9.0 \times 10^9 \text{ N m}^2\text{C}^{-2}$, $e = 1.6 \times 10^{-19} \text{ C}$, and $K = 2.56 \times 10^{-12} \text{ J}$:

$$r_\alpha = \frac{2(9.0 \times 10^9)(79)(1.6 \times 10^{-19})^2}{2.56 \times 10^{-12}}.$$

The numerator is $3.64032 \times 10^{-26} \text{ J m}$; dividing by the given energy yields a length in metres. The turning-point condition uses the whole incident energy, since the collision is head-on and there is no transverse motion.

Step 2: (b) Replace the alpha particle by a proton A proton carries charge $+e$, half the alpha-particle charge. The nuclear charge and incident kinetic energy remain the same. Thus

$$r_p = \frac{k(79)e^2}{K} = \frac{r_\alpha}{2}.$$

The proton can approach more closely because its repulsive potential energy at a given separation is smaller. The projectile mass does not appear in this comparison: equal kinetic energies, rather than

equal incident speeds, are specified.

Step 3: Conclusion (a) $r_\alpha = 1.422 \times 10^{-14} \text{ m} \approx 1.42 \times 10^{-14} \text{ m}$. (b) $r_p = 7.11 \times 10^{-15} \text{ m}$, half the alpha-particle value.

Quick Tip to Solve: At fixed incident energy and nuclear charge, closest approach is proportional to projectile charge; replacing $2e$ by e halves the distance.

10(a). Identify electromagnetic waves which:

(i) are used in radar system.

(ii) affect a photographic plate.

(iii) are used in surgery.

Write their frequency range.

[3 marks]

Solution:

Concept:

- Electromagnetic waves are classified by frequency or wavelength. In vacuum, $\nu = c/\lambda$, where $c = 3.0 \times 10^8 \text{ m s}^{-1}$. Shorter wavelengths correspond to higher frequencies.
- Frequency boundaries between neighbouring regions are approximate and may overlap. The ranges below are conventional school-level spectrum ranges, rather than the frequency of one particular device.

Step 1: (i) Waves used in radar Radar uses microwaves. Their short wavelength allows a narrow, directed beam, and the reflected signal can be detected to locate an object. Using the approximate microwave wavelength band 0.1 m to 10^{-3} m , the corresponding lower and upper frequencies are

$$\nu_{\min} = \frac{3.0 \times 10^8}{0.1} = 3.0 \times 10^9 \text{ Hz}, \quad \nu_{\max} = \frac{3.0 \times 10^8}{10^{-3}} = 3.0 \times 10^{11} \text{ Hz}.$$

Step 2: (ii) Waves affecting a photographic plate Visible light affects a photographic plate through a photochemical reaction in its light-sensitive coating. The visible wavelength interval is approximately 700 nm to 400 nm , giving

$$4.3 \times 10^{14} \text{ Hz} \lesssim \nu \lesssim 7.5 \times 10^{14} \text{ Hz}.$$

This is often rounded to 4×10^{14} to $8 \times 10^{14} \text{ Hz}$. This property is not exclusive to visible light: suitably sensitive photographic plates also detect ultraviolet radiation and X-rays. Visible light is a valid identification for the stated property.

Step 3: (iii) Waves used in surgery Ultraviolet rays are used in precise eye surgery, such as LASIK. Their short wavelength allows a finely focused beam. Taking the approximate ultraviolet wavelength interval as 400 nm to 1 nm , its frequency range is

$$7.5 \times 10^{14} \text{ Hz} \lesssim \nu \lesssim 3.0 \times 10^{17} \text{ Hz}.$$

These are the frequencies of the ultraviolet region as a whole; a surgical laser operates at a selected

wavelength within the region.

Step 4: Conclusion (i) Microwaves: approximately 3×10^9 to 3×10^{11} Hz. (ii) Visible light: approximately 4.3×10^{14} to 7.5×10^{14} Hz. (iii) Ultraviolet rays: approximately 7.5×10^{14} to 3×10^{17} Hz.

Quick Tip to Solve: Recall radar as microwaves and precision eye surgery as ultraviolet; convert a wavelength band with $\nu = c/\lambda$, reversing the order of its endpoints.

OR

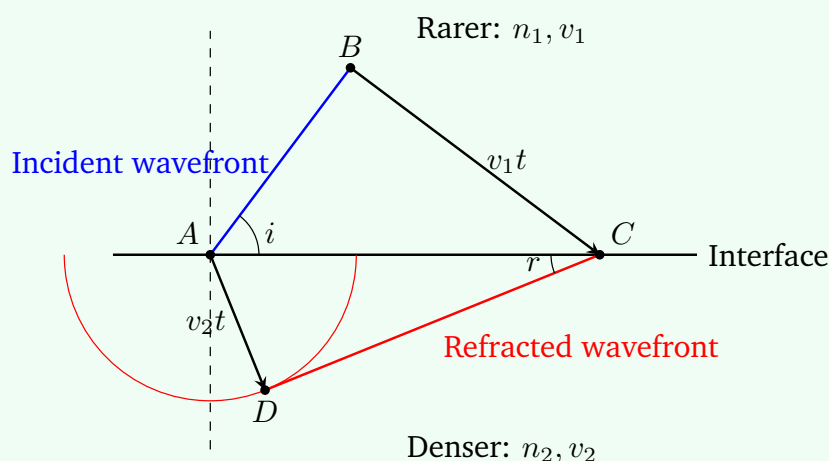
10(b). A plane wavefront is propagating from a rarer into a denser medium. Use Huygens principle to show the refracted wavefront and verify Snell's law. [3 marks]

Solution:

Concept:

- Every point on a wavefront emits secondary wavelets; the forward common tangent to them forms the new wavefront.
- For refractive index n , wave speed is $v = c/n$. Denser medium means $n_2 > n_1$, hence $v_2 < v_1$.

Step 1: Construct the secondary wavelet Let AB be an incident plane wavefront in medium 1. Point A reaches the plane interface first, while B is still in the rarer medium. In time t , point B travels along its ray to C on the interface, so $BC = v_1 t$. During the same interval, the secondary wavelet from A in medium 2 grows to radius $AD = v_2 t$. Draw from C the forward tangent CD to this wavelet. Thus $AD \perp CD$ and $AB \perp BC$. The tangent CD is the refracted plane wavefront.



Huygens construction: the denser-medium wavelet has radius AD .

Step 2: Relate the wavefront geometry to the ray angles Let i and r be the incident and refracted ray angles with the normal. The angle between AB and the interface AC is also i , because the ray and normal are perpendicular to these respective lines. Similarly the angle between CD and AC is r . In

the two right triangles,

$$\sin i = \frac{BC}{AC} = \frac{v_1 t}{AC}, \quad \sin r = \frac{AD}{AC} = \frac{v_2 t}{AC}.$$

The shared length AC and elapsed time cancel on taking their ratio.

Step 3: Verify the refractive-index relation Therefore

$$\frac{\sin i}{\sin r} = \frac{v_1}{v_2} = \frac{c/n_1}{c/n_2} = \frac{n_2}{n_1}.$$

Since the secondary wavelet in the denser medium has the smaller radius, $r < i$: the ray bends toward the normal. The wavefront correspondingly makes a smaller angle with the interface. The construction uses the same time interval on both sides; using unequal times would not describe a single new wavefront.

Step 4: Conclusion The forward tangent CD is the refracted wavefront, and $n_1 \sin i = n_2 \sin r$ verifies Snell's law.

Quick Tip to Solve: Use the common hypotenuse AC in both right triangles; their sine ratio immediately equals v_1/v_2 .

11. A diver looking up through water ($\mu = \frac{4}{3}$) sees the outside world contained in a circular area on the surface of water. If the diver's eyes are $\sqrt{7}$ m below the surface of water, then calculate the area of the circle. [3 marks]

Solution:

Concept:

- The limiting ray in water makes the critical angle C with the normal: $\sin C = 1/\mu$, taking the refractive index of air as unity.
- For eye depth h , the limiting cone intersects the horizontal surface in a circle of radius $r = h \tan C$. Its area is $A = \pi r^2$.

Step 1: Identify the boundary of the visible world Light from the outside world enters water from air. As the angle of incidence in air approaches 90° , its refracted angle in water approaches the critical angle C . By reversibility, the same ray from the diver toward air would emerge grazing the surface. Rays at larger water angles undergo total internal reflection and cannot connect the eye to the outside world. Rotational symmetry about the vertical through the eye makes this boundary a circle.

Step 2: Calculate the critical-angle geometry The printed refractive index is $\mu = 4/3$. Thus

$$\sin C = \frac{3}{4}, \quad \cos C = \sqrt{1 - \frac{9}{16}} = \frac{\sqrt{7}}{4}, \quad \tan C = \frac{3}{\sqrt{7}}.$$

The angle is measured from the vertical normal, so the right triangle has the depth as its adjacent side and surface radius as its opposite side. Hence

$$r = h \tan C = \sqrt{7} \times \frac{3}{\sqrt{7}} = 3 \text{ m}.$$

Step 3: Obtain the surface area Using the radius, rather than the diameter, in the circle-area formula

gives

$$A = \pi r^2 = \pi(3\text{ m})^2 = 9\pi\text{ m}^2.$$

This area describes the window through which the complete outside hemisphere is compressed into the divers' field of view; it is not the area of a hemispherical surface.

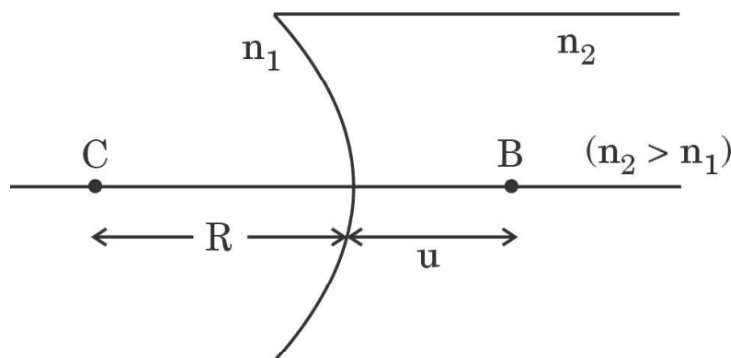
Step 4: Conclusion The circular area is $9\pi\text{ m}^2$, approximately 28.3 m^2 .

Quick Tip to Solve: Use $r = h/\sqrt{\mu^2 - 1}$; with $h = \sqrt{7}$ and $\mu = 4/3$, the radicals cancel.

Section C (5 marks)

12. Two transparent media of refractive indices n_1 and n_2 are separated by a spherical transparent surface. The rays of light incident on the surface get refracted into the medium on the other side. The laws of refraction are valid at each point of the spherical surface. A lens is a transparent optical medium bounded by two surfaces, at least one of which should be spherical. The focal length of a lens is determined by the radii of curvature (R_1 and R_2) of its two surfaces and the refractive index (n) of the medium of the lens with respect to the surrounding medium. Depending on R_1 and R_2 , a lens behaves as a diverging or a converging lens. The ability of a lens to diverge or converge a beam of light incident on it defines its power. [5 marks]

12(a). An object is placed at the point B as shown in the figure. The object distance (u) and the image distance (v) are related as: [1 mark]



(A) $\frac{1}{v} - \frac{1}{u} = \left(\frac{n_2 - n_1}{n_1}\right) \frac{1}{R}$

(B) $\frac{1}{v} - \frac{1}{u} = \left(\frac{n_1 - n_2}{n_2}\right) \frac{1}{R}$

(C) $\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$

(D) $\frac{n_1}{v} - \frac{n_2}{u} = \frac{n_1 - n_2}{R}$

Correct Answer: (D) $\frac{n_1}{v} - \frac{n_2}{u} = \frac{n_1 - n_2}{R}$

Solution:

Concept:

- For paraxial refraction, $n_{\text{out}}/v - n_{\text{in}}/u = (n_{\text{out}} - n_{\text{in}})/R$, with all distances measured from the pole using one Cartesian sign convention.
- The object location determines the incident medium; the larger refractive index alone does not determine which index multiplies $1/u$.

Step 1: Identify the incident and refracted media The figure places object B on the right of the surface, in medium n_2 . Its light travels from right to left into medium n_1 . Consequently $n_{\text{in}} = n_2$ and $n_{\text{out}} = n_1$. Choose the positive axis along this incident light direction; the object distance is then negative and the centre C , on the left, has positive radius.

Step 2: Compare each printed formula Substitution gives

$$\frac{n_1}{v} - \frac{n_2}{u} = \frac{n_1 - n_2}{R}.$$

Printed option (D) has exactly these coefficients. Option (C) interchanges the incident and outgoing indices and describes incidence from the opposite medium. Options (A) and (B) omit the required unequal coefficients of the object and image terms; dividing the correct equation by either index does not make both coefficients unity.

Step 3: Conclusion The correct relation is $\boxed{\frac{n_1}{v} - \frac{n_2}{u} = \frac{n_1 - n_2}{R}}$. Hence, the correct option is (D).

Quick Tip to Solve: Locate the object first: the index of its medium belongs with u , and the other index belongs with v .

12(b). A point object is placed in air at a distance R in front of a convex spherical refracting surface of radius of curvature R . If the medium on the other side of the surface is glass, then the image is: [1 mark]

- (A) real and formed in glass.
- (B) real and formed in air.
- (C) virtual and formed in glass.
- (D) virtual and formed in air.

Correct Answer: (D) virtual and formed in air.

Solution:

Concept:

- The spherical-refraction equation is $n_g/v - n_a/u = (n_g - n_a)/R$. For an object in air, $n_a = 1$.
- Take positive distances along the incident ray toward the glass. A convex surface has its centre in glass, so its radius is positive.

Step 1: Apply the Cartesian signs The object lies a distance R in front of the surface, so $u = -R$, while the convex surface has radius $+R$. For ordinary glass, use $n_g = 3/2$. The refraction equation becomes

$$\frac{3}{2v} + \frac{1}{R} = \frac{1}{2R}.$$

Keeping the negative sign of the object distance is essential: replacing u by $+R$ would predict the wrong image side.

Step 2: Locate and classify the image Rearranging yields

$$\frac{3}{2v} = -\frac{1}{2R}, \quad v = -3R.$$

The negative image distance places the image on the incident side, in air. Refracted rays in glass diverge, and their backward extensions intersect there, so the image is virtual. Both real-image options fail, while option (C) incorrectly puts the virtual image in glass. More generally, $v = n_g R / (n_g - 2)$, negative for ordinary glass with $1 < n_g < 2$.

Step 3: Conclusion The image is virtual and formed in air. Hence, the correct option is (D).

Quick Tip to Solve: For this geometry, $n_g/v = (n_g - 2)/R$; ordinary glass makes the right side negative.

12(c). An object is kept at $2F$ in front of an equiconvex lens. The image formed is:

[1 mark]

- (A) real and of the size of the object.
- (B) virtual and of the size of the object.
- (C) real and enlarged.
- (D) virtual and diminished.

Correct Answer: (A) real and of the size of the object.

Solution:

Concept:

- A convex lens in air is converging, with positive focal length f . The Cartesian lens formula is $1/f = 1/v - 1/u$, where u and v are signed object and image distances.
- Linear magnification is $m = v/u$; its magnitude gives the image-to-object size ratio, while a negative sign denotes inversion.

Step 1: Locate the image using the lens formula The object is at twice the focal distance on the incident side, so $u = -2f$. Substituting gives

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{2f} \Rightarrow \frac{1}{v} = \frac{1}{2f}, \quad v = +2f.$$

The positive image distance places the image on the other side of the lens, where the refracted rays actually meet; therefore it is real.

Step 2: Check the size and eliminate alternatives The magnification is $m = (2f)/(-2f) = -1$. Thus the image is inverted and equal in size to the object. Both virtual-image choices fail because rays actually converge, and the enlarged-image choice fails because $|m| = 1$, rather than greater than one.

Step 3: Conclusion The image is real, inverted and the same size as the object. Hence, the correct option is (A).

Quick Tip to Solve: For a convex lens, an object at $2F$ gives an image at the opposite $2F$, with unit magnitude of magnification.

12(d). A thin converging lens of focal length 10 cm and a thin diverging lens of focal length 20 cm are placed coaxially in contact. The power of the combination is: [1 mark]

- (A) -5 D
- (B) $+5\text{ D}$
- (C) $+15\text{ D}$
- (D) -15 D

Correct Answer: (B) $+5\text{ D}$

Solution:

Concept:

- Lens power is $P = 1/f$, with f in metres and P in dioptries (D). A converging lens has positive power and a diverging lens has negative power.
- For thin lenses placed coaxially in contact, the equivalent power is $P = P_1 + P_2$.

Step 1: Assign signs and convert focal lengths The converging lens has $f_1 = +10\text{ cm} = +0.10\text{ m}$, while the diverging lens has $f_2 = -20\text{ cm} = -0.20\text{ m}$. Therefore

$$P_1 = \frac{1}{0.10} = +10\text{ D}, \quad P_2 = \frac{1}{-0.20} = -5\text{ D}.$$

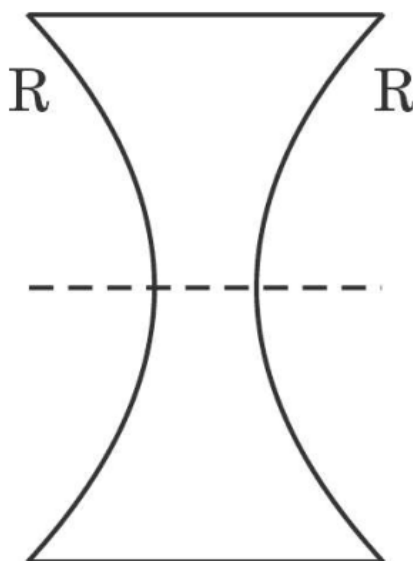
The negative sign for the second focal length is essential: its action opposes convergence.

Step 2: Add the powers and compare all choices As the lenses are in contact, $P = 10 - 5 = +5\text{ D}$. The positive sign means the combination is converging, with focal length $+0.20\text{ m}$. The -5 D choice reverses the net sign; $+15\text{ D}$ incorrectly treats both lenses as converging; and -15 D has both an incorrect magnitude and sign.

Step 3: Conclusion The power of the combination is $+5\text{ D}$. Hence, the correct option is (B).

Quick Tip to Solve: Convert centimetres to metres and attach the diverging lens's minus sign before adding reciprocal focal lengths.

12(e). An equiconcave lens of focal length f is cut into two identical parts along the dotted line as shown in the figure. The focal length of each part will be: [1 mark]



- (A) $\frac{f}{4}$
- (B) $\frac{f}{2}$
- (C) f
- (D) $2f$

Correct Answer: (C) f

Solution:

Concept:

- The thin-lens maker formula is $1/f = (n - 1)(1/R_1 - 1/R_2)$, where n is the lens refractive index relative to its surroundings and R_1, R_2 are the signed surface radii.
- Focal length depends on refractive index and surface curvatures. Reducing the aperture while retaining both curved surfaces does not change their optical power.

Step 1: Identify the direction of the cut The dotted line in the printed figure runs horizontally along the principal axis, dividing the lens into upper and lower halves. Each half still has portions of both original concave refracting surfaces, with the same radii of curvature. The flat cut face lies along the direction of the paraxial beam; it does not replace either original refracting surface.

Step 2: Apply the lens-maker relation to each half Since n , R_1 , and R_2 are unchanged, the reciprocal focal length of each part remains

$$\frac{1}{f_{\text{part}}} = (n - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right) = \frac{1}{f}.$$

The smaller aperture transmits fewer rays but does not alter the focal length. Choices $f/4$ and $f/2$ would require increased power, while $2f$ would require halved power; none follows from this cut.

The $2f$ result instead applies when an equiconcave lens is split perpendicular to its principal axis into plano-concave lenses.

Step 3: Conclusion The focal length of each part remains f . Hence, the correct option is (C).

Quick Tip to Solve: First compare the cut with the principal axis: a parallel cut reduces aperture while leaving both surface curvatures unchanged.