

CBSE Class 12 Mathematics Question Paper 2022 with Solution PDF-Set 65-2-1

Question Paper with Solutions

Conducted by the Central Board of Secondary Education (CBSE)



Time Allowed: 2 hours

Maximum Marks: 40

General Instructions

- (i) This question paper contains **three** sections – **Section A, B** and **C**.
- (ii) Each section is **compulsory**.
- (iii) **Section A** has **6** short answer type I questions of **2** marks each.
- (iv) **Section B** has **4** short answer type II questions of **3** marks each.
- (v) **Section C** has **4** long answer type questions of **4** marks each.
- (vi) There is an internal choice in some questions.
- (vii) Question no. **14** is a case-study based question with **2** sub-parts of **2** marks each.

Section A (12 marks)

Questions number 1 to 6 carry 2 marks each.

1. Find the product of the order and the degree of the differential equation $\left[\frac{d}{dx}(xy^2) \right] \frac{dy}{dx} + y = 0$. [2 marks]

Solution:

Concept:

- The order is the order of the highest derivative appearing when all differentiations in the equation have been carried out.
- The degree is the highest power of the highest-order derivative when the equation is polynomial in its derivatives. Its coefficients may involve x and y .

Step 1: Apply the product rule The quantity xy^2 is a product, with y depending on x . Thus

$$\frac{d}{dx}(xy^2) = y^2 + 2xy \frac{dy}{dx}.$$

Substituting this into the printed equation and multiplying by the outside factor gives

$$2xy \left(\frac{dy}{dx} \right)^2 + y^2 \frac{dy}{dx} + y = 0.$$

Step 2: Identify order and degree The only derivative occurring is dy/dx ; there is no second or higher derivative. Hence the order is 1. The equation is already polynomial in its derivatives, and the highest power of dy/dx is 2. Its degree is therefore 2. The factor $2xy$ is a coefficient, so it does not contribute to the degree. Order and degree describe the equation as given, rather than special values of a particular solution.

Step 3: Conclusion The required product is $1 \times 2 = \boxed{2}$.

Quick Tip to Solve: Expand the product derivative before counting powers: the outside factor of dy/dx makes the resulting equation quadratic.

- 2(a). Find: $\int \frac{\sin 3x}{\sin x} dx$. [2 marks]

Solution:

Concept:

- The triple-angle identity is $\sin 3x = 3 \sin x - 4 \sin^3 x$. It reduces the quotient to elementary trigonometric terms.
- Use $\sin^2 x = (1 - \cos 2x)/2$, and integrate $\cos 2x$ as $\frac{1}{2} \sin 2x$.

Quick Tip to Solve: Use the triple-angle identity first, then convert $\sin^2 x$ into a double-angle cosine so that both terms integrate immediately.

Step 1: Simplify the quotient On any interval where $\sin x \neq 0$, the triple-angle identity gives

$$\frac{\sin 3x}{\sin x} = 3 - 4 \sin^2 x.$$

Using the double-angle identity, this becomes

$$3 - 4 \left(\frac{1 - \cos 2x}{2} \right) = 1 + 2 \cos 2x.$$

This avoids attempting a direct substitution with the numerator, whose angle differs from the denominator.

Step 2: Integrate the elementary expression Integrating term by term,

$$\int (1 + 2 \cos 2x) dx = x + 2 \left(\frac{\sin 2x}{2} \right) + C.$$

The factor $1/2$ appears because the inner angle is $2x$. As a check, differentiating $x + \sin 2x$ gives $1 + 2 \cos 2x$, which is precisely the simplified integrand. The arbitrary constant is required because this is an indefinite integral.

Step 3: Conclusion Thus, $\int \frac{\sin 3x}{\sin x} dx = \boxed{x + \sin 2x + C}$.

Quick Tip to Solve: Use the triple-angle identity first, then convert $\sin^2 x$ into a double-angle cosine so that both terms integrate immediately.

OR

2(b). Evaluate: $\int_0^{\frac{1}{2} \log 3} \frac{e^x}{e^{2x} + 1} dx.$

[2 marks]

Solution:

Concept:

- For a substitution $t = e^x$, the differential is $dt = e^x dx$. The limits must be transformed along with the integrand.
- The standard integral is $\int dt/(1 + t^2) = \tan^{-1} t + C$, with the principal inverse tangent taking values between $-\pi/2$ and $\pi/2$.

Step 1: Change variable and limits Let $t = e^x$. Then $dt = e^x dx$ and $e^{2x} = t^2$, so the numerator is exactly the differential needed. At $x = 0$, $t = 1$. At $x = \frac{1}{2} \log 3$, $t = e^{(\log 3)/2} = \sqrt{3}$. Here $\log$ denotes the natural logarithm. Hence the definite integral becomes

$$I = \int_1^{\sqrt{3}} \frac{dt}{1 + t^2}.$$

Step 2: Use the inverse-tangent antiderivative Applying the standard antiderivative with the new

Quick Tip to Solve: Choose $t = e^x$ because its differential matches the numerator; convert the upper limit to $\sqrt{3}$ before evaluating inverse tangents.

limits gives

$$I = [\tan^{-1} t]_1^{\sqrt{3}} = \tan^{-1} \sqrt{3} - \tan^{-1} 1.$$

Both inputs are positive, so the principal values lie in the first quadrant: $\tan^{-1} \sqrt{3} = \pi/3$ and $\tan^{-1} 1 = \pi/4$. Their difference is positive, consistent with the positive original integrand and increasing limits.

Step 3: Conclusion Therefore, $I = \frac{\pi}{3} - \frac{\pi}{4} = \boxed{\frac{\pi}{12}}$.

Quick Tip to Solve: Choose $t = e^x$ because its differential matches the numerator; convert the upper limit to $\sqrt{3}$ before evaluating inverse tangents.

3. $\vec{a}$ and $\vec{b}$ are two unit vectors such that $|2\vec{a} + 3\vec{b}| = |3\vec{a} - 2\vec{b}|$. Find the angle between $\vec{a}$ and $\vec{b}$.

[2 marks]

Solution:

Concept:

- For any vectors, $|\vec{u} + \vec{v}|^2 = |\vec{u}|^2 + |\vec{v}|^2 + 2\vec{u} \cdot \vec{v}$. Squaring equal magnitudes lets us use this dot-product identity.
- For unit vectors, $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos \theta = \cos \theta$, where the angle between them lies in $0 \leq \theta \leq \pi$.

Step 1: Expand the squared magnitudes Both magnitudes are nonnegative, so their equality implies equality of their squares. On expanding by the dot product, we obtain

$$4|\vec{a}|^2 + 9|\vec{b}|^2 + 12\vec{a} \cdot \vec{b} = 9|\vec{a}|^2 + 4|\vec{b}|^2 - 12\vec{a} \cdot \vec{b}.$$

The positive cross term on the left and negative cross term on the right follow from the signs of the vector coefficients.

Step 2: Use the unit-vector condition As $|\vec{a}| = |\vec{b}| = 1$, the constant terms on each side sum to 13. Hence

$$13 + 12\vec{a} \cdot \vec{b} = 13 - 12\vec{a} \cdot \vec{b} \implies 24\vec{a} \cdot \vec{b} = 0.$$

Thus $\cos \theta = \vec{a} \cdot \vec{b} = 0$. In the permitted interval for the angle between vectors, this gives $\theta = \pi/2$.

Step 3: Conclusion The required angle is $\boxed{90^\circ}$.

Quick Tip to Solve: Square the given magnitudes first; the constant terms cancel because both vectors have unit length.

4. A pair of dice is thrown. It is given that the sum of numbers appearing on both dice is an even number. Find the probability that the number appearing on at least one die is 3. [2 marks]

Solution:

Concept:

- Conditional probability is $P(A | E) = P(A \cap E) / P(E)$, provided $P(E) > 0$. Here E is the event that the dice sum is even and A is the event that at least one die shows 3.
- For two fair dice, the 36 ordered pairs are equally likely. An even sum requires both numbers to have the same parity.

Step 1: Count the outcomes with an even sum Each die has three odd faces, 1, 3, 5, and three even faces, 2, 4, 6. The sum is even when both dice show odd numbers or both show even numbers. Therefore

$$n(E) = 3 \times 3 + 3 \times 3 = 18, \quad P(E) = \frac{18}{36}.$$

The order matters: for example, (1, 3) and (3, 1) are distinct outcomes.

Step 2: Count the favourable outcomes within the condition If a die shows 3 and the sum is even, the other die must also show an odd number. The outcomes in $A \cap E$ are

$$(3, 1), (3, 3), (3, 5), (1, 3), (5, 3).$$

There are five such outcomes, with (3, 3) counted only once. Consequently,

$$P(A | E) = \frac{P(A \cap E)}{P(E)} = \frac{5/36}{18/36} = \frac{5}{18}.$$

Step 3: Conclusion The required conditional probability is $\boxed{\frac{5}{18}}$.

Quick Tip to Solve: Restrict the sample space to the 18 even-sum outcomes, and count (3,3) only once.

5. Probabilities of A and B solving a specific problem are $\frac{2}{3}$ and $\frac{3}{5}$ respectively. If both of them try independently to solve the problem, then find the probability that the problem is solved. [2 marks]

Solution:

Concept:

- Let S_A and S_B denote the events that A and B solve the problem. The problem is solved when at least one succeeds, so the required event is $S_A \cup S_B$.
- Independent attempts give independent failures. Hence $P(S_A^c \cap S_B^c) = P(S_A^c)P(S_B^c)$, and the probability of at least one success is one minus the probability of both failures.

Step 1: Calculate each person's failure probability From the given success probabilities,

$$P(S_A) = \frac{2}{3}, \quad P(S_B) = \frac{3}{5}.$$

Quick Tip to Solve: For independent attempts, multiply the failure probabilities and subtract their product from one.

Taking complementary events gives

$$P(S_A^c) = 1 - \frac{2}{3} = \frac{1}{3}, \quad P(S_B^c) = 1 - \frac{3}{5} = \frac{2}{5}.$$

Both persons attempt the problem; it remains unsolved only when both attempts fail.

Step 2: Apply independence and take the complement Since the attempts are independent, the probability that neither person solves the problem is

$$P(S_A^c \cap S_B^c) = \frac{1}{3} \times \frac{2}{5} = \frac{2}{15}.$$

Thus

$$P(S_A \cup S_B) = 1 - \frac{2}{15} = \frac{13}{15}.$$

As a check, the union formula yields $2/3 + 3/5 - (2/3)(3/5) = 13/15$; subtracting the intersection prevents counting a joint success twice.

Step 3: Conclusion The probability that the problem is solved is $\boxed{\frac{13}{15}}$.

Quick Tip to Solve: For independent attempts, multiply the failure probabilities and subtract their product from one.

6. Write the cartesian equation of the line PQ passing through points $P(2, 2, 1)$ and $Q(5, 1, -2)$. Hence, find the y -coordinate of the point on the line PQ whose z -coordinate is -2 . [2 marks]

Solution:

Concept:

- A direction vector of a line through two distinct points is obtained by subtracting their position vectors: $\vec{PQ} = \vec{Q} - \vec{P}$.
- Through (x_0, y_0, z_0) , with nonzero direction ratios a, b, c , the cartesian equation is $\frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c}$. Their common value is a parameter t .

Step 1: Calculate the direction ratios and write the line Subtract the coordinates of $P(2, 2, 1)$ from those of $Q(5, 1, -2)$:

$$\vec{PQ} = (5 - 2)\hat{i} + (1 - 2)\hat{j} + (-2 - 1)\hat{k} = 3\hat{i} - \hat{j} - 3\hat{k}.$$

Thus the direction ratios are $3, -1, -3$. Using P as the point on the line gives

$$\frac{x - 2}{3} = \frac{y - 2}{-1} = \frac{z - 1}{-3}.$$

Step 2: Apply the specified coordinate Denote the common ratio by t . Then $x = 2 + 3t$, $y = 2 - t$ and

Quick Tip to Solve: When a prescribed coordinate matches an endpoint, check that endpoint after finding the line parameter; here $z = -2$ identifies Q .

$z = 1 - 3t$. The condition $z = -2$ gives

$$1 - 3t = -2 \implies t = 1, \quad y = 2 - 1 = 1.$$

At this parameter, $x = 5$, so the point is exactly Q . Since the coefficient of t in z is nonzero, this parameter and the required point are unique.

Step 3: Conclusion The cartesian equation is $\frac{x-2}{3} = \frac{y-2}{-1} = \frac{z-1}{-3}$, and the required y -coordinate is $\boxed{1}$.

Quick Tip to Solve: When a prescribed coordinate matches an endpoint, check that endpoint after finding the line parameter; here $z = -2$ identifies Q .

Section B (12 marks)

Questions number 7 to 10 carry 3 marks each.

7. $ABCD$ is a parallelogram such that $\overrightarrow{AC} = \hat{i} + \hat{j}$ and $\overrightarrow{BD} = 2\hat{i} + \hat{j} + \hat{k}$. Find $\overrightarrow{AB}$ and $\overrightarrow{AD}$. Also, find the area of the parallelogram $ABCD$. [3 marks]

Solution:

Concept:

- For adjacent side vectors $\vec{a} = \overrightarrow{AB}$ and $\vec{b} = \overrightarrow{AD}$ of a parallelogram, the directed diagonals are $\overrightarrow{AC} = \vec{a} + \vec{b}$ and $\overrightarrow{BD} = \vec{b} - \vec{a}$.
- The area of the parallelogram is $|\vec{a} \times \vec{b}|$. The cross product magnitude equals the product of adjacent side lengths and the sine of their included angle.

Step 1: Express the adjacent sides using the directed diagonals Write $\vec{u} = \overrightarrow{AC} = \hat{i} + \hat{j}$ and $\vec{v} = \overrightarrow{BD} = 2\hat{i} + \hat{j} + \hat{k}$. Since opposite sides have equal vectors, travelling from A to C gives $\vec{u} = \vec{a} + \vec{b}$. Travelling from B to D via A gives $\vec{v} = -\vec{a} + \vec{b}$. Subtracting and adding these equations respectively gives

$$\begin{aligned}\vec{a} &= \frac{\vec{u} - \vec{v}}{2} = -\frac{1}{2}\hat{i} - \frac{1}{2}\hat{k}, \\ \vec{b} &= \frac{\vec{u} + \vec{v}}{2} = \frac{3}{2}\hat{i} + \hat{j} + \frac{1}{2}\hat{k}.\end{aligned}$$

As a sign check, their sum is $\hat{i} + \hat{j}$ and $\vec{b} - \vec{a} = 2\hat{i} + \hat{j} + \hat{k}$, agreeing with both printed diagonals.

Step 2: Evaluate the cross product and its magnitude The required area uses the adjacent sides, so calculate

$$\begin{aligned}\vec{a} \times \vec{b} &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -\frac{1}{2} & 0 & -\frac{1}{2} \\ \frac{3}{2} & 1 & \frac{1}{2} \end{vmatrix} \\ &= \frac{1}{2}\hat{i} - \frac{1}{2}\hat{j} - \frac{1}{2}\hat{k}.\end{aligned}$$

Quick Tip to Solve: Keep the direction BD fixed: it is $AD - AB$, so subtract BD from AC to find AB ; adding finds AD .

Its magnitude is

$$|\vec{a} \times \vec{b}| = \sqrt{\frac{1}{4} + \frac{1}{4} + \frac{1}{4}} = \frac{\sqrt{3}}{2}.$$

Alternatively, $\vec{a} \times \vec{b} = \frac{1}{2}(\vec{u} \times \vec{v})$, which checks the factor of one-half when using diagonals for area.

Step 3: Conclusion $\vec{AB} = -\frac{1}{2}\hat{i} - \frac{1}{2}\hat{k}$, $\vec{AD} = \frac{3}{2}\hat{i} + \hat{j} + \frac{1}{2}\hat{k}$, and the area is $\frac{\sqrt{3}}{2}$ square units.

Quick Tip to Solve: Keep the direction BD fixed: it is $AD - AB$, so subtract BD from AC to find AB ; adding finds AD .

8(a). Evaluate: $\int_0^1 \tan^{-1} x \, dx$

[3 marks]

Solution:

Concept:

- Integration by parts states that $\int u \, dv = uv - \int v \, du$. Choose the inverse trigonometric factor as u , since differentiation turns it into a rational function.
- For real x , $\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$. Also, $\tan^{-1} 1 = \pi/4$ and $\tan^{-1} 0 = 0$, with angles in radians.

Step 1: Apply integration by parts Put $I = \int_0^1 \tan^{-1} x \, dx$. Take $u = \tan^{-1} x$ and $dv = dx$, so that $du = dx/(1+x^2)$ and $v = x$. Hence

$$I = [x \tan^{-1} x]_0^1 - \int_0^1 \frac{x}{1+x^2} \, dx.$$

The boundary term is $1 \cdot \pi/4 - 0 \cdot 0 = \pi/4$. The original integrand is continuous on the entire interval, so integration by parts applies without an improper limit.

Step 2: Evaluate the remaining rational integral The numerator is half the derivative of the denominator. Using $t = 1 + x^2$, we have $dt = 2x \, dx$. The limits change from $x = 0, 1$ to $t = 1, 2$, respectively. Thus

$$\int_0^1 \frac{x}{1+x^2} \, dx = \frac{1}{2} \int_1^2 \frac{dt}{t} = \frac{1}{2} [\log t]_1^2 = \frac{1}{2} \log 2.$$

Here $\log$ denotes the natural logarithm. Since $1+x^2 > 0$, no absolute-value complication arises. As a check, differentiating $x \tan^{-1} x - \frac{1}{2} \log(1+x^2)$ cancels the two rational terms and gives $\tan^{-1} x$.

Step 3: Conclusion Therefore, $\int_0^1 \tan^{-1} x \, dx = \frac{\pi}{4} - \frac{1}{2} \log 2$.

Quick Tip to Solve: Differentiate the inverse tangent in integration by parts; the remaining numerator x matches the derivative of $1+x^2$.

8(b). Find: $\int \frac{2x}{x^2 + 3x + 2} dx$

[3 marks]

Solution:**Concept:**

- A proper rational function with distinct linear denominator factors can be expressed as partial fractions. Integrate each term using $\int dx/(x+a) = \log|x+a| + C$.
- An indefinite integral represents a family of antiderivatives. Its constant C is arbitrary, and the formula applies on any interval that avoids the zeros of the original denominator.

Step 1: Factor and resolve into partial fractions Factor the denominator as $x^2 + 3x + 2 = (x+1)(x+2)$. Since the numerator has lower degree, write

$$\frac{2x}{(x+1)(x+2)} = \frac{A}{x+1} + \frac{B}{x+2}.$$

Clearing the denominator gives the polynomial identity $2x = A(x+2) + B(x+1)$. Comparing the coefficient of x and the constant term yields $A + B = 2$ and $2A + B = 0$. Subtracting these equations gives $A = -2$; substituting back gives $B = 4$. Therefore the integrand equals $-2/(x+1) + 4/(x+2)$ wherever it is defined.

Step 2: Integrate and verify the antiderivative Applying the logarithmic integration formula term by term,

$$\begin{aligned} \int \frac{2x}{x^2 + 3x + 2} dx &= -2 \int \frac{dx}{x+1} + 4 \int \frac{dx}{x+2} \\ &= -2 \log|x+1| + 4 \log|x+2| + C. \end{aligned}$$

To verify, differentiation gives

$$-\frac{2}{x+1} + \frac{4}{x+2} = \frac{-2(x+2) + 4(x+1)}{(x+1)(x+2)} = \frac{2x}{x^2 + 3x + 2}.$$

The absolute values preserve this real-valued formula on all intervals avoiding $x = -1$ and $x = -2$; different intervals may have different constants of integration.

Step 3: Conclusion The required integral is $\boxed{4 \log|x+2| - 2 \log|x+1| + C}$, for $x \neq -1, -2$, where $\log$ is the natural logarithm.

Quick Tip to Solve: After factoring, check both the x coefficient and the constant term: the two partial-fraction numerators must combine to exactly $2x$.

9. Find the particular solution of the differential equation $(y + 3x^2)\frac{dx}{dy} = x$, given that $y = 1$ when $x = 1$. [3 marks]

Solution:

Concept:

- A first-order linear equation in y has the form $\frac{dy}{dx} + P(x)y = Q(x)$. Its integrating factor is $I(x) = e^{\int P(x) dx}$.
- After multiplication by the integrating factor, the left side becomes $\frac{d}{dx}(Iy)$. The initial condition fixes the arbitrary constant in the general solution.

Step 1: Rewrite the equation with x as the independent variable The given equation is $(y + 3x^2)\frac{dx}{dy} = x$. Near the prescribed point $(1, 1)$, both x and $y + 3x^2$ are nonzero, so we may write

$$\frac{dy}{dx} = \frac{y + 3x^2}{x}, \quad \frac{dy}{dx} - \frac{y}{x} = 3x.$$

This is linear in y , with $P(x) = -1/x$ and $Q(x) = 3x$. Since the initial point has $x = 1$, we work on an interval with $x > 0$.

Step 2: Apply the integrating factor and integrate The integrating factor is

$$I(x) = e^{-\int dx/x} = e^{-\log x} = \frac{1}{x}.$$

Multiplying the linear equation by $1/x$ gives

$$\frac{1}{x} \frac{dy}{dx} - \frac{y}{x^2} = 3 \implies \frac{d}{dx} \left(\frac{y}{x} \right) = 3.$$

Integration with respect to x yields $y/x = 3x + C$, or $y = 3x^2 + Cx$, where C is a constant.

Step 3: Use the given condition and check the result Substituting $x = 1$ and $y = 1$ into the general solution gives $1 = 3 + C$, hence $C = -2$. For this value, $dy/dx = 6x - 2$. Also, $y + 3x^2 = 6x^2 - 2x = x(6x - 2)$; therefore the original equation holds wherever the local inverse derivative $dx/dy = 1/(6x - 2)$ is defined. At the initial point, $dy/dx = 4$, and the condition $y(1) = 1$ is satisfied.

Step 4: Conclusion The required particular solution is $y = 3x^2 - 2x$, locally about $(1, 1)$.

Quick Tip to Solve: When an equation contains dx/dy but its coefficients are simple in x , invert the derivative and check whether the resulting equation is linear in y .

10(a). Find the equation of the plane passing through points $(2, 1, 0)$, $(3, -2, -2)$ and $(1, 1, -7)$. Also, obtain its distance from the origin. [3 marks]

Solution:

Concept:

- A plane through a point A , with nonzero normal vector $\mathbf{n}$, has equation $\mathbf{n} \cdot (\mathbf{r} - \mathbf{a}) = 0$, where $\mathbf{a}$ is the position vector of A .
- For three noncollinear points A, B, C , the vector $\overrightarrow{AB} \times \overrightarrow{AC}$ is perpendicular to their plane. The distance of (x_0, y_0, z_0) from $ax + by + cz + d = 0$ is $\frac{|ax_0 + by_0 + cz_0 + d|}{\sqrt{a^2 + b^2 + c^2}}$.

Step 1: Form two vectors lying in the plane Let $A = (2, 1, 0)$, $B = (3, -2, -2)$ and $C = (1, 1, -7)$. Subtracting the coordinates of A gives

$$\overrightarrow{AB} = (1, -3, -2), \quad \overrightarrow{AC} = (-1, 0, -7).$$

These vectors are not parallel, so the three points determine a unique plane. A normal vector can be found by taking their cross product.

Step 2: Calculate the normal vector and plane equation The cross product is

$$\begin{aligned} \overrightarrow{AB} \times \overrightarrow{AC} &= ((-3)(-7) - (-2)(0), (-2)(-1) - (1)(-7), (1)(0) - (-3)(-1)) \\ &= (21, 9, -3) = 3(7, 3, -1). \end{aligned}$$

Thus we may use the simpler normal vector $(7, 3, -1)$. Passing the plane through A gives

$$7(x - 2) + 3(y - 1) - (z - 0) = 0, \quad 7x + 3y - z - 17 = 0.$$

As a check, substituting B gives $21 - 6 + 2 - 17 = 0$, and substituting C gives $7 + 3 + 7 - 17 = 0$. Thus all three given points lie on this plane.

Step 3: Find the perpendicular distance from the origin For the plane obtained above, $a = 7$, $b = 3$, $c = -1$ and $d = -17$. Substituting the origin $(0, 0, 0)$ in the point-to-plane distance formula yields

$$D = \frac{|7(0) + 3(0) - 0 - 17|}{\sqrt{7^2 + 3^2 + (-1)^2}} = \frac{17}{\sqrt{59}}.$$

The absolute value ensures that the perpendicular distance is positive, irrespective of the chosen orientation of the normal vector.

Step 4: Conclusion The required plane is $7x + 3y - z - 17 = 0$, and its distance from the origin is $\frac{17}{\sqrt{59}}$ units.

Quick Tip to Solve: Divide a cross-product normal by any common factor before writing the plane equation; this makes the distance calculation shorter without changing the plane.

10(b). Find the distance between the lines $x = \frac{y-1}{2} = \frac{z-2}{3}$ and $x+1 = \frac{y+2}{2} = \frac{z-1}{3}$. [3 marks]

Solution:

Concept:

- For parallel lines through points A and B , with common direction vector $\vec{d}$, the distance is $D = \frac{|\vec{AB} \times \vec{d}|}{|\vec{d}|}$.
- This formula gives the length of the component of $\vec{AB}$ perpendicular to the lines; its component along $\vec{d}$ does not contribute to their separation.

Step 1: Identify points and the common direction Put each symmetric expression equal to a parameter. For the first line,

$$x = t, \quad y = 1 + 2t, \quad z = 2 + 3t.$$

It therefore passes through $A = (0, 1, 2)$ and has direction vector $\vec{d} = (1, 2, 3)$. For the second line,

$$x = -1 + s, \quad y = -2 + 2s, \quad z = 1 + 3s.$$

Hence it passes through $B = (-1, -2, 1)$ with the same direction vector. The lines are parallel, so the parallel-line distance formula applies. Also,

$$\vec{AB} = B - A = (-1, -3, -1).$$

This vector is not a scalar multiple of $\vec{d}$, so the lines are distinct rather than coincident.

Step 2: Calculate the perpendicular separation Evaluate the cross product using the chosen point on each line:

$$\begin{aligned} \vec{AB} \times \vec{d} &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & -3 & -1 \\ 1 & 2 & 3 \end{vmatrix} \\ &= (-9 + 2)\hat{i} - (-3 + 1)\hat{j} + (-2 + 3)\hat{k} \\ &= -7\hat{i} + 2\hat{j} + \hat{k}. \end{aligned}$$

Thus $|\vec{AB} \times \vec{d}| = \sqrt{49 + 4 + 1} = \sqrt{54}$, whereas $|\vec{d}| = \sqrt{1 + 4 + 9} = \sqrt{14}$. Substituting into the distance formula gives

$$D = \frac{\sqrt{54}}{\sqrt{14}} = \sqrt{\frac{27}{7}} = \frac{3\sqrt{21}}{7}.$$

As a check, the squared perpendicular component is $|\vec{AB}|^2 - (\vec{AB} \cdot \vec{d})^2 / |\vec{d}|^2 = 11 - 100/14 = 27/7$, agreeing with this calculation.

Step 3: Conclusion The distance between the two parallel lines is $\boxed{\frac{3\sqrt{21}}{7} \text{ units}}.$

Quick Tip to Solve: Read the direction ratios first: both lines have ratios 1 : 2 : 3, so use the parallel-line cross-product formula immediately.

Section C (16 marks)

Questions number 11 to 14 carry 4 marks each.

11. Find the distance of the point $(-1, -5, -10)$ from the point of intersection of the line $\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{12}$ and the plane $x - y + z = 5$. [4 marks]

Solution:

Concept:

- For a line in symmetric form, set all three ratios equal to a common parameter t . This gives the coordinates of every point on the line and lets us impose the equation of a plane.
- If $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ are two points in space, their distance is

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}.$$

Step 1: Express a general point on the line Let the common value of the three ratios be t :

$$\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{12} = t.$$

Multiplying each ratio by its denominator gives

$$x = 2 + 3t, \quad y = -1 + 4t, \quad z = 2 + 12t.$$

Thus a general point on the given line is $Q(2 + 3t, -1 + 4t, 2 + 12t)$.

Step 2: Locate the intersection with the plane At the intersection, the coordinates of Q must also satisfy $x - y + z = 5$. Substitute the parameter expressions:

$$\begin{aligned}(2 + 3t) - (-1 + 4t) + (2 + 12t) &= 5, \\ 5 + 11t &= 5, \\ 11t &= 0, \\ t &= 0.\end{aligned}$$

The coefficient of t is nonzero, so this determines a unique intersection. Substituting $t = 0$ gives $Q = (2, -1, 2)$. Check: $2 - (-1) + 2 = 5$, and all three line ratios equal zero.

Step 3: Calculate the distance from the given point The given point is $P = (-1, -5, -10)$. Its coordinate differences from the intersection $Q = (2, -1, 2)$ are

$$2 - (-1) = 3, \quad -1 - (-5) = 4, \quad 2 - (-10) = 12.$$

Quick Tip to Solve: First test the line's base point $(2, -1, 2)$ in the plane: it already satisfies $x - y + z = 5$, so the intersection has $t = 0$. Then use the coordinate differences 3, 4, 12.

Apply the distance formula to these two points:

$$\begin{aligned}PQ &= \sqrt{3^2 + 4^2 + 12^2} \\&= \sqrt{9 + 16 + 144} \\&= \sqrt{169}.\end{aligned}$$

Step 4: Conclusion The point of intersection is $(2, -1, 2)$, and the required distance from $(-1, -5, -10)$ is 13 units.

Quick Tip to Solve: First test the line's base point $(2, -1, 2)$ in the plane: it already satisfies $x - y + z = 5$, so the intersection has $t = 0$. Then use the coordinate differences 3, 4, 12.

12. Evaluate: $\int_0^1 x(1-x)^n dx$

[4 marks]

Solution:

Concept:

- A substitution in a definite integral changes both the variable and the limits. Here, setting $t = 1 - x$ turns the factor $(1 - x)^n$ into a simple power of t .
- The power rule gives $\int t^a dt = t^{a+1}/(a+1)$ for $a \neq -1$. At an endpoint where $t = 0$, the integral of t^a converges when $a > -1$.

Step 1: Substitute and transform the limits Write $I = \int_0^1 x(1-x)^n dx$. Take $t = 1 - x$, so that $x = 1 - t$ and $dx = -dt$. When $x = 0$, $t = 1$; when $x = 1$, $t = 0$. Consequently,

$$\begin{aligned}I &= \int_1^0 (1-t)t^n (-dt) \\&= \int_0^1 (1-t)t^n dt.\end{aligned}$$

Reversing the limits removes the minus sign. For real n , the required convergence condition is $n > -1$; in particular, it holds for every nonnegative integer n .

Step 2: Expand and integrate the two powers Distribute the factor $1 - t$ before integrating. This avoids expanding a potentially large integer power in the original variable:

$$\begin{aligned}I &= \int_0^1 (t^n - t^{n+1}) dt \\&= \left[\frac{t^{n+1}}{n+1} - \frac{t^{n+2}}{n+2} \right]_0^1.\end{aligned}$$

Since $n > -1$, both exponents $n+1$ and $n+2$ are positive. Thus both terms at the lower limit tend to zero, and substitution of the upper limit gives $I = 1/(n+1) - 1/(n+2)$. If n is not an integer, these endpoint statements are understood as limits from within the interval.

Quick Tip to Solve: Put $t = 1 - x$ and reverse the limits immediately; the integrand becomes just $t^n - t^{n+1}$, giving two direct power-rule integrals.

Step 3: Simplify the endpoint expression Use the common denominator $(n+1)(n+2)$:

$$\frac{1}{n+1} - \frac{1}{n+2} = \frac{(n+2) - (n+1)}{(n+1)(n+2)}.$$

The numerator reduces to 1. As a check, at $n = 0$ the original integral becomes $\int_0^1 x \, dx = 1/2$, which agrees with the expression obtained above. For real $n \leq -1$, the original integrand has a nonintegrable endpoint singularity at $x = 1$, so the definite integral diverges.

Step 4: Conclusion $\int_0^1 x(1-x)^n \, dx = \frac{1}{(n+1)(n+2)}$, for $n > -1$, including all nonnegative integer values of n .

Quick Tip to Solve: Put $t = 1 - x$ and reverse the limits immediately; the integrand becomes just $t^n - t^{n+1}$, giving two direct power-rule integrals.

13(a). Using integration, find the area of the smaller region enclosed by the curve $4x^2 + 4y^2 = 9$ and the line $2x + 2y = 3$. [4 marks]

Solution:

Concept:

- The curve $x^2 + y^2 = \frac{9}{4}$ is a circle centred at the origin with radius $r = \frac{3}{2}$. Its upper semicircle is $y = \sqrt{\frac{9}{4} - x^2}$.
- The area between an upper curve $y = f(x)$ and a lower curve $y = g(x)$, from $x = a$ to $x = b$, is $\int_a^b [f(x) - g(x)] \, dx$. The limits must be the abscissae of their intersection points.

Step 1: Locate the intersections and select the smaller region Dividing the given equations by 4 and 2, respectively, gives

$$x^2 + y^2 = \frac{9}{4}, \quad y = \frac{3}{2} - x.$$

Substitute the line into the circle:

$$\begin{aligned} x^2 + \left(\frac{3}{2} - x\right)^2 &= \frac{9}{4}, \\ 2x^2 - 3x &= 0, \\ x(2x - 3) &= 0. \end{aligned}$$

Thus the endpoints of the chord are $(0, \frac{3}{2})$ and $(\frac{3}{2}, 0)$. The line cuts off the circular segment in the first quadrant. On $0 < x < \frac{3}{2}$, both curve heights are positive and

$$\left(\frac{9}{4} - x^2\right) - \left(\frac{3}{2} - x\right)^2 = x(3 - 2x) > 0,$$

so the upper arc lies above the line. This segment is contained in one quadrant of the circle, hence its area is less than half the circle and it is the required smaller region.

Step 2: Set up and evaluate the area integral Integrating the vertical difference between the arc and

Quick Tip to Solve: Divide both equations first: the chord then joins the two positive intercepts of the circle. Integrate the upper semicircle minus the chord between those intercepts; the quadrant-minus-triangle calculation checks the result.

the chord gives

$$A = \int_0^{3/2} \left[\sqrt{\frac{9}{4} - x^2} - \left(\frac{3}{2} - x \right) \right] dx.$$

Using the standard circle integral with $r = \frac{3}{2}$,

$$\begin{aligned} A &= \left[\frac{x}{2} \sqrt{\frac{9}{4} - x^2} + \frac{9}{8} \sin^{-1} \left(\frac{2x}{3} \right) - \frac{3x}{2} + \frac{x^2}{2} \right]_0^{3/2} \\ &= \frac{9}{8} \cdot \frac{\pi}{2} - \frac{9}{4} + \frac{9}{8} \\ &= \frac{9\pi}{16} - \frac{9}{8}. \end{aligned}$$

Here $\sin^{-1}(1) = \frac{\pi}{2}$, and the square-root term vanishes at both endpoints.

Step 3: Conclusion The area of the smaller enclosed region is $\boxed{\frac{9}{16}(\pi - 2) \text{ square units}}$.

Quick Tip to Solve: Divide both equations first: the chord then joins the two positive intercepts of the circle. Integrate the upper semicircle minus the chord between those intercepts; the quadrant-minus-triangle calculation checks the result.

OR

13(b). If the area of the region bounded by the curve $y^2 = 4ax$ and the line $x = 4a$ is $\frac{256}{3}$ sq. units, then using integration, find the value of a , where $a > 0$. [4 marks]

Solution:

Concept:

- The area between two curves is obtained by integrating the nonnegative difference of their ordinates over the interval bounded by their intersections.
- The parabola $y^2 = 4ax$, with $a > 0$, opens to the right and has the two branches $y = 2\sqrt{ax}$ and $y = -2\sqrt{ax}$. These branches are symmetric about the x -axis.
- For a vertical strip at x , its height is the upper ordinate minus the lower ordinate. The bounded region extends from the vertex $x = 0$ to the given line $x = 4a$.

Step 1: Find the intersections and limits Substituting $x = 4a$ in the parabola gives

$$y^2 = 4a(4a) = 16a^2.$$

Since $a > 0$, the intersections are $(4a, 4a)$ and $(4a, -4a)$. The parabola has vertex $(0, 0)$, so vertical strips cover the enclosed region for $0 \leq x \leq 4a$.

Step 2: Express the entire enclosed area as an integral Let the required area be A . The strip height is

$$2\sqrt{ax} - (-2\sqrt{ax}) = 4\sqrt{ax}.$$

Quick Tip to Solve: Use the difference of the two parabola branches, $4\sqrt{ax}$, so that the integral counts both symmetric halves of the bounded region.

Consequently,

$$A = \int_0^{4a} 4\sqrt{ax} \, dx = 4\sqrt{a} \int_0^{4a} x^{1/2} \, dx.$$

This includes both the upper and lower halves of the region.

Step 3: Evaluate the integral and determine the positive parameter Using the power rule,

$$\begin{aligned} A &= 4\sqrt{a} \left[\frac{2}{3} x^{3/2} \right]_0^{4a} \\ &= \frac{8\sqrt{a}}{3} (4a)^{3/2} \\ &= \frac{8\sqrt{a}}{3} \cdot 8a^{3/2} = \frac{64a^2}{3}. \end{aligned}$$

Equating this to the area in the question gives

$$\frac{64a^2}{3} = \frac{256}{3} \implies a^2 = 4.$$

The algebraic possibilities are $a = 2$ and $a = -2$. The latter does not satisfy the printed condition $a > 0$.

Step 4: Conclusion Hence, the required value is $\boxed{a = 2}$.

Quick Tip to Solve: Use the difference of the two parabola branches, $4\sqrt{ax}$, so that the integral counts both symmetric halves of the bounded region.

14. Case-Study Based Question

At the start of a cricket match, a coin is tossed and the team winning the toss has the opportunity to choose to bat or bowl. Such a coin is unbiased with equal probabilities of getting head and tail.

Based on the above information, answer the following questions:

[4 marks]

14(a). If such a coin is tossed 2 times, then find the probability distribution of number of tails.

[2 marks]

Solution:

Concept:

- An unbiased coin has $P(H) = P(T) = \frac{1}{2}$, where H denotes a head and T denotes a tail. Successive tosses are independent.
- The probability distribution of a random variable lists every possible value and its probability. Here X denotes the number of tails in two tosses, so $X \in \{0, 1, 2\}$.

Step 1: List the equally likely outcomes The ordered sample space for two tosses is $S = \{HH, HT, TH, TT\}$. Independence gives probability $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$ for each outcome. The outcomes HT and TH must be counted separately because they correspond to different orders of the two tosses.

Step 2: Associate the number of tails with each outcome The outcome HH has no tail, HT and TH

Quick Tip to Solve: For two fair tosses, group the four ordered outcomes by tail count; one tail occurs in two different orders.

each have one tail, and TT has two tails. Hence

$$P(X = 0) = \frac{1}{4}, \quad P(X = 1) = \frac{2}{4} = \frac{1}{2}, \quad P(X = 2) = \frac{1}{4}.$$

These are all the possible values of X . The probabilities are non-negative and their sum is $\frac{1}{4} + \frac{1}{2} + \frac{1}{4} = 1$, as required for a probability distribution.

Step 3: Conclusion The required probability distribution is

x	0	1	2
$P(X = x)$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

Quick Tip to Solve: For two fair tosses, group the four ordered outcomes by tail count; one tail occurs in two different orders.

14(b). Find the probability of getting at least one head in three tosses of such a coin.

[2 marks]

Solution:

Concept:

- For an unbiased coin, $P(H) = P(T) = \frac{1}{2}$. Probabilities multiply for specified outcomes on independent tosses.
- The complement rule states $P(A) = 1 - P(A^c)$. The complement of getting at least one head is getting no heads, which means all three tosses are tails.

Step 1: Identify the event and its complement Let A be the event of obtaining at least one head in three tosses. This includes one, two or three heads. It does not mean exactly one head. There are $2^3 = 8$ equally likely ordered outcomes in three independent tosses. The only outcome outside A is TTT , because every other outcome contains a head.

Step 2: Compute the complementary probability By independence, the probability that each of the three tosses gives a tail is

$$P(A^c) = P(TTT) = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}.$$

The event A and its complement are disjoint and together exhaust the sample space. Therefore

$$P(A) = 1 - P(A^c) = 1 - \frac{1}{8} = \frac{7}{8}.$$

Equivalently, seven of the eight equally likely outcomes contain at least one head, giving the same value.

Step 3: Conclusion The probability of getting at least one head in three tosses is $\boxed{\frac{7}{8}}$.

Quick Tip to Solve: For at least one head, subtract the probability of all tails from one instead of counting the separate head cases.