

Question Paper

Conducted by the Central Board of Secondary Education (CBSE)



Time Allowed: 3 Hours

Maximum Marks: 80

General Instructions

- (i) This question paper contains **38 questions**. All questions are compulsory.
- (ii) Section A: Questions 1 to 18 are multiple choice questions and Questions 19 and 20 are Assertion-Reason questions. Each carries 1 mark.
- (iii) Section B: Questions 21 to 25 carry 2 marks each.
- (iv) Section C: Questions 26 to 31 carry 3 marks each.
- (v) Section D: Questions 32 to 35 carry 5 marks each.
- (vi) Section E: Questions 36 to 38 carry 4 marks each. These are case study questions.
- (vii) Attempt one alternative where an internal choice is provided. Follow the instructions within each question.
- (viii) Use of calculators is not allowed.

Section A

1. $\int \frac{1 + \tan x}{1 - \tan x} dx$ is equal to:

- (A) $\sec^2\left(\frac{\pi}{4} + x\right) + C$ (B) $\sec^2\left(\frac{\pi}{4} - x\right) + C$
 (C) $\log\left|\sec\left(\frac{\pi}{4} + x\right)\right| + C$ (D) $\log\left|\sec\left(\frac{\pi}{4} - x\right)\right| + C$

2. $\int_0^{\pi/6} \sec^2\left(x - \frac{\pi}{6}\right) dx$ is equal to:

- (A) $\frac{1}{\sqrt{3}}$ (B) $-\frac{1}{\sqrt{3}}$
 (C) $\sqrt{3}$ (D) $-\sqrt{3}$

3. The sum of the order and the degree of the differential equation $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^3 = \sin y$ is:

- (A) 5 (B) 2
 (C) 3 (D) 4

4. The value of p for which the vectors $2\hat{i} + p\hat{j} + \hat{k}$ and $-4\hat{i} - 6\hat{j} + 26\hat{k}$ are perpendicular to each other, is:

- (A) 3 (B) -3
 (C) $-\frac{17}{3}$ (D) $\frac{17}{3}$

5. If the vector $\hat{i} - b\hat{j} + \hat{k}$ is equally inclined to the coordinate axes, then the value of b is:

- (A) -1 (B) 1
 (C) $-\frac{1}{\sqrt{3}}$ (D) $-\frac{1}{3}$

6. If $\vec{a} + \vec{b} = \hat{i}$ and $\vec{a} = 2\hat{i} - 2\hat{j} + 2\hat{k}$, then $|\vec{b}|$ equals:

- (A) $\sqrt{14}$ (B) 3
 (C) $\sqrt{12}$ (D) $\sqrt{17}$

7. Direction cosines of a line perpendicular to both x -axis and z -axis are:

- (A) 1, 0, 1 (B) 1, 1, 1
 (C) 0, 0, 1 (D) 0, 1, 0

8. If $P\left(\frac{A}{B}\right) = 0.3$, $P(A) = 0.4$ and $P(B) = 0.8$, then $P\left(\frac{B}{A}\right)$ is equal to:

- (A) 0.6 (B) 0.3
 (C) 0.06 (D) 0.4

9. For what value of k may the function $f(x) = \begin{cases} \frac{k(3x^2 - 5x)}{\sin x}, & x < 0 \\ \cos x, & x > 0 \end{cases}$ become continuous?

- (A) 0 (B) 1
 (C) $-\frac{1}{5}$ (D) no value

10. If $A = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ and $(3I + 4A)(3I - 4A) = x^2I$, then the value(s) of x is/are:

- (A) $\pm\sqrt{7}$ (B) 0
 (C) ± 5 (D) 25

11. The general solution of the differential equation $x dy - (1 + x^2) dx = dx$ is:

- (A) $y = 2x + \frac{x^3}{3} + C$ (B) $y = 2 \log x + \frac{x^3}{3} + C$
 (C) $y = \frac{x^2}{2} + C$ (D) $y = 2 \log x + \frac{x^2}{2} + C$

12. If $f(x) = a(x - \cos x)$ is strictly decreasing in $\mathbb{R}$, then a belongs to:

- (A) $\{0\}$ (B) $(0, \infty)$
 (C) $(-\infty, 0)$ (D) $(-\infty, \infty)$

13. The corner points of the feasible region in the graphical representation of a linear programming problem are $(2, 72)$, $(15, 20)$ and $(40, 15)$. If $z = 18x + 9y$ be the objective function, then:

- | | |
|--|--|
| (A) z is maximum at $(2, 72)$, minimum at $(15, 20)$ | (B) z is maximum at $(15, 20)$, minimum at $(40, 15)$ |
| (C) z is maximum at $(40, 15)$, minimum at $(15, 20)$ | (D) z is maximum at $(40, 15)$, minimum at $(2, 72)$ |

14. The number of corner points of the feasible region determined by the constraints $x - y \geq 0$, $2y \leq x + 2$, $x \geq 0$, $y \geq 0$ is:

- | | |
|-------|-------|
| (A) 2 | (B) 3 |
| (C) 4 | (D) 5 |

15. If for a square matrix A , $A^2 - 3A + I = O$ and $A^{-1} = xA + yI$, then the value of $x + y$ is:

- | | |
|--------|--------|
| (A) -2 | (B) 2 |
| (C) 3 | (D) -3 |

16. If $\left| \frac{A^{-1}}{2} \right| = \frac{1}{k|A|}$, where A is a 3×3 matrix, then the value of k is:

- | | |
|-------------------|-------------------|
| (A) $\frac{1}{8}$ | (B) 8 |
| (C) 2 | (D) $\frac{1}{2}$ |

17. Let A be a 3×3 matrix such that $|\text{adj } A| = 64$. Then $|A|$ is equal to:

- | | |
|------------|-------------|
| (A) 8 only | (B) -8 only |
| (C) 64 | (D) 8 or -8 |

18. If $A = \begin{pmatrix} 3 & 4 \\ 5 & 2 \end{pmatrix}$ and $2A + B$ is a null matrix, then B is equal to:

- | | |
|---|---|
| (A) $\begin{pmatrix} 6 & 8 \\ 10 & 4 \end{pmatrix}$ | (B) $\begin{pmatrix} -6 & -8 \\ -10 & -4 \end{pmatrix}$ |
| (C) $\begin{pmatrix} 5 & 8 \\ 10 & 3 \end{pmatrix}$ | (D) $\begin{pmatrix} -5 & -8 \\ -10 & -3 \end{pmatrix}$ |

19. **Assertion (A):** The equation of the line passing through the points $(1, 2, 3)$ and $(3, -1, 3)$ is $\frac{x-3}{2} = \frac{y+1}{3} = \frac{z-3}{0}$.

Reason (R): The equation of the line passing through the points (x_1, y_1, z_1) and (x_2, y_2, z_2) is $\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$.

- | | |
|--|---|
| (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A). | (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A). |
| (C) Assertion (A) is true, but Reason (R) is false. | (D) Assertion (A) is false, but Reason (R) is true. |

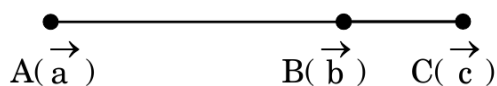
20. **Assertion (A):** The number of onto functions from a set P containing 5 elements to a set Q containing 2 elements is 30.

Reason (R): Number of onto functions from a set containing m elements to a set containing n elements is n^m .

- | | |
|--|---|
| (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A). | (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A). |
| (C) Assertion (A) is true and Reason (R) is false. | (D) Assertion (A) is false and Reason (R) is true. |

Section B

21. (a) Position vectors of the points A, B and C as shown in the figure below are $\vec{a}$, $\vec{b}$ and $\vec{c}$ respectively.



If $\vec{AC} = \frac{5}{4}\vec{AB}$, express $\vec{c}$ in terms of $\vec{a}$ and $\vec{b}$.

OR

(b) Check whether the lines given by equations $x = 2\lambda + 2$, $y = 7\lambda + 1$, $z = -3\lambda - 3$ and $x = -\mu - 2$, $y = 2\mu + 8$, $z = 4\mu + 5$ are perpendicular to each other or not.

22. If $y = (x + \sqrt{x^2 - 1})^2$, then show that $(x^2 - 1) \left(\frac{dy}{dx} \right)^2 = 4y^2$.

23. Find the sub-intervals in which $f(x) = \log(2+x) - \frac{x}{2+x}$, $x > -2$ is increasing or decreasing.

24. (a) A function $f : A \rightarrow B$ defined as $f(x) = 2x$ is both one-one and onto. If $A = \{1, 2, 3, 4\}$, then find the set B .

OR

(b) Evaluate:

$$\sin^{-1}\left(\sin \frac{3\pi}{4}\right) + \cos^{-1}\left(\cos \frac{3\pi}{4}\right) + \tan^{-1}(1)$$

25. For two non-zero vectors $\vec{a}$ and $\vec{b}$, if $|\vec{a} - \vec{b}| = |\vec{a} + \vec{b}|$, then find the angle between $\vec{a}$ and $\vec{b}$.

Section C

26. (a) Find the general solution of the differential equation: $\frac{dx}{dy} = \frac{e^{x/y} \left(\frac{x}{y} - 1 \right)}{1 + e^{x/y}}$.

OR

(b) Find the particular solution of the differential equation $\frac{dy}{dx} + \cot x \cdot y = \cos^2 x$, given that when $x = \frac{\pi}{2}$, $y = 0$.

27. Solve the following linear programming problem graphically: Maximize $P = 100x + 5y$ subject to the constraints

$$\begin{aligned} x + y &\leq 300, \\ 3x + y &\leq 600, \\ y &\leq x + 200, \\ x, y &\geq 0. \end{aligned}$$

28. (a) The probability distribution of a random variable X is given below:

X	1	2	3
$P(X)$	$\frac{k}{2}$	$\frac{k}{3}$	$\frac{k}{6}$

- (i) Find the value of k . (ii) Find $P(1 \leq X < 3)$.
(iii) Find $E(X)$, the mean of X .

OR

(b) A and B are independent events such that $P(A \cap \bar{B}) = \frac{1}{4}$ and $P(\bar{A} \cap B) = \frac{1}{6}$. Find $P(A)$ and $P(B)$.

29. (a) Evaluate: $\int_0^{\pi/2} e^x \sin x \, dx$.

OR

(b) Find: $\int \frac{1}{\cos(x-a) \cos(x-b)} \, dx$.

30. Evaluate: $\int_0^{\pi/4} |\sin x - \cos x| \, dx$.

31. Find: $\int \frac{1}{\sqrt{x}(\sqrt{x}+1)(\sqrt{x}+2)} \, dx$.

Section D

32. (a) Find the vector and the Cartesian equations of a line passing through the point $(1, 2, -4)$ and parallel to the line joining the points $A(3, 3, -5)$ and $B(1, 0, -11)$. Hence, find the distance between the two lines.

OR

(b) Find the equations of the line passing through the points $A(1, 2, 3)$ and $B(3, 5, 9)$. Hence, find the coordinates of the points on this line which are at a distance of 14 units from point B .

33. Find the area of the region $\{(x, y) : x^2 + y^2 \leq 1 \leq x + y\}$, using integration.

34. A relation R is defined on a set of real numbers $\mathbb{R}$ as $R = \{(x, y) : x \cdot y \text{ is an irrational number}\}$. Check whether R is reflexive, symmetric and transitive or not.

35. (a) If $A = \begin{pmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{pmatrix}$ and $B^{-1} = \begin{pmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{pmatrix}$, find $(AB)^{-1}$.

OR

(b) Solve the following system of equations by matrix method:

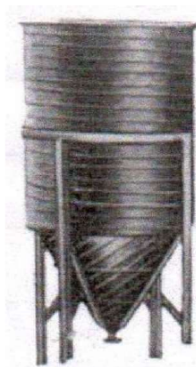
$$x + 2y + 3z = 6$$

$$2x - y + z = 2$$

$$3x + 2y - 2z = 3$$

Section E

36. A tank, as shown in the figure below, formed using a combination of a cylinder and a cone, offers better drainage as compared to a flat bottomed tank.



A tap is connected to such a tank whose conical part is full of water. Water is dripping out from a tap at the bottom at the uniform rate of $2 \text{ cm}^3/\text{s}$. The semi-vertical angle of the conical tank is 45° .

On the basis of given information, answer the following questions.

36(i). Find the volume of water in the tank in terms of its radius r .

36(ii). Find rate of change of radius at an instant when $r = 2\sqrt{2} \text{ cm}$.

36(iii). (a) Find the rate at which the wet surface of the conical tank is decreasing at an instant when radius $r = 2\sqrt{2} \text{ cm}$.

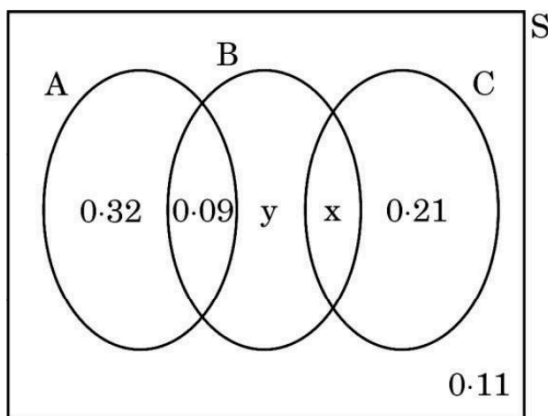
OR

(b) Find the rate of change of height h at an instant when slant height is 4 cm .

37. There are different types of Yoga which involve the usage of different poses of Yoga Asanas, Meditation and Pranayam as shown in the figure below: Anusara Yoga, Kundalini Yoga, Vinyasa Yoga, Bikram Yoga, Hatha Yoga.



The Venn diagram below represents the probabilities of three different types of Yoga, A , B and C performed by the people of a society. Further, it is given that probability of a member performing type C Yoga is 0.44.



On the basis of the above information, answer the following questions.

37(i). Find the value of x .

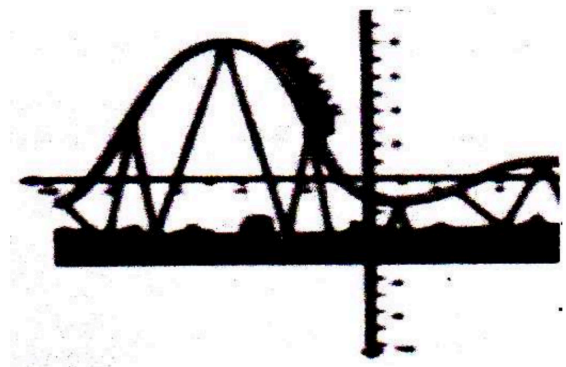
37(ii). Find the value of y .

37(iii). (a) Find $P\left(\frac{C}{B}\right)$.

OR

(b) Find the probability that a randomly selected person of the society does Yoga of type A or B but not C .

38. The equation of the path traced by a roller-coaster is given by the polynomial $f(x) = a(x + 9)(x + 1)(x - 3)$. If the roller-coaster crosses y -axis at a point $(0, -1)$, answer the following.



38(i). Find the value of ' a '.

38(ii). Find $f'(x)$ at $x = 1$.

- End of Question Paper -