

Question Paper

Conducted by the Central Board of Secondary Education (CBSE)



Time Allowed: 3 Hours

Maximum Marks: 80

General Instructions

- (i) This question paper contains **38 questions**. All questions are compulsory.
- (ii) Section A: Questions 1 to 18 are multiple choice questions and Questions 19 and 20 are Assertion-Reason questions. Each carries 1 mark.
- (iii) Section B: Questions 21 to 25 carry 2 marks each.
- (iv) Section C: Questions 26 to 31 carry 3 marks each.
- (v) Section D: Questions 32 to 35 carry 5 marks each.
- (vi) Section E: Questions 36 to 38 carry 4 marks each. These are case study questions.
- (vii) Attempt one alternative where an internal choice is provided. Follow the instructions within each question.
- (viii) Use of calculators is not allowed.

Section A

1. If $\vec{a} + \vec{b} = \hat{i}$ and $\vec{a} = 2\hat{i} - 2\hat{j} + 2\hat{k}$, then $|\vec{b}|$ equals:

- (A) $\sqrt{14}$ (B) 3
(C) $\sqrt{12}$ (D) $\sqrt{17}$

2. The direction ratios of a line parallel to z -axis are:

- (A) $\langle 1, 1, 0 \rangle$ (B) $\langle 1, 1, 1 \rangle$
(C) $\langle 0, 0, 0 \rangle$ (D) $\langle 0, 0, 1 \rangle$

3. Ashima can hit a target 2 out of 3 times. She tried to hit the target twice. The probability that she missed the target exactly once is:

- (A) $\frac{2}{3}$ (B) $\frac{4}{9}$
(C) $\frac{1}{3}$ (D) $\frac{1}{9}$

4. The function $f(x) = |x| - x$ is:

- (A) continuous but not differentiable at $x = 0$. (B) continuous and differentiable at $x = 0$.
(C) neither continuous nor differentiable at $x = 0$. (D) differentiable but not continuous at $x = 0$.

5. If $A = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ and $(3I + 4A)(3I - 4A) = x^2I$, then the value(s) of x is/are:

- (A) $\pm\sqrt{7}$ (B) 0
(C) ± 5 (D) 25

6. The general solution of the differential equation $x dy - (1 + x^2) dx = dx$ is:

- (A) $y = 2x + \frac{x^3}{3} + C$ (B) $y = 2 \log x + \frac{x^3}{3} + C$
(C) $y = \frac{x^2}{2} + C$ (D) $y = 2 \log x + \frac{x^2}{2} + C$

7. If $f(x) = a(x - \cos x)$ is strictly decreasing in $\mathbb{R}$, then a belongs to:

- (A) $\{0\}$ (B) $(0, \infty)$
(C) $(-\infty, 0)$ (D) $(-\infty, \infty)$

8. The corner points of the feasible region in the graphical representation of a linear programming problem are $(2, 72)$, $(15, 20)$ and $(40, 15)$. If $z = 18x + 9y$ be the objective function, then:

- (A) z is maximum at $(2, 72)$, minimum at $(15, 20)$ (B) z is maximum at $(15, 20)$, minimum at $(40, 15)$
(C) z is maximum at $(40, 15)$, minimum at $(15, 20)$ (D) z is maximum at $(40, 15)$, minimum at $(2, 72)$

9. The number of corner points of the feasible region determined by the constraints $x - y \geq 0$, $2y \leq x + 2$, $x \geq 0$, $y \geq 0$ is:

- (A) 2 (B) 3
(C) 4 (D) 5

10. If for a square matrix A , $A^2 - 3A + I = O$ and $A^{-1} = xA + yI$, then the value of $x + y$ is:

- (A) -2 (B) 2
(C) 3 (D) -3

11. If $\begin{pmatrix} x & 2 \\ 3 & x-1 \end{pmatrix}$ is a singular matrix, then the product of all possible values of x is:

- (A) 6 (B) -6
(C) 0 (D) -7

12. Let A be a 3×3 matrix such that $|\text{adj } A| = 64$. Then $|A|$ is equal to:

- (A) 8 only (B) -8 only
(C) 64 (D) 8 or -8

13. If $A = \begin{pmatrix} 3 & 4 \\ 5 & 2 \end{pmatrix}$ and $2A + B$ is a null matrix, then B is equal to:

- (A) $\begin{pmatrix} 6 & 8 \\ 10 & 4 \end{pmatrix}$ (B) $\begin{pmatrix} -6 & -8 \\ -10 & -4 \end{pmatrix}$
(C) $\begin{pmatrix} 5 & 8 \\ 10 & 3 \end{pmatrix}$ (D) $\begin{pmatrix} -5 & -8 \\ -10 & -3 \end{pmatrix}$

14. The primitive of $\frac{2}{1 + \cos 2x}$ is:

- (A) $\sec^2 x$ (B) $2 \sec^2 x \tan x$
(C) $\tan x$ (D) $-\cot x$

15. $\int_0^{\pi/6} \sec^2 \left(x - \frac{\pi}{6} \right) dx$ is equal to:

- (A) $\frac{1}{\sqrt{3}}$ (B) $-\frac{1}{\sqrt{3}}$
(C) $\sqrt{3}$ (D) $-\sqrt{3}$

16. The sum of the order and the degree of the differential equation $\frac{d^2 y}{dx^2} + \left(\frac{dy}{dx} \right)^3 = \sin y$ is:

- (A) 5 (B) 2
(C) 3 (D) 4

17. The value of p for which the vectors $2\hat{i} + p\hat{j} + \hat{k}$ and $-4\hat{i} - 6\hat{j} + 26\hat{k}$ are perpendicular to each other, is:

- (A) 3 (B) -3
(C) $-\frac{17}{3}$ (D) $\frac{17}{3}$

18. For what value of λ , the projection of vector $\hat{i} + \lambda\hat{j}$ on vector $\hat{i} - \hat{j}$ is $\sqrt{2}$?

- (A) -1 (B) 1
(C) 0 (D) 3

19. **Assertion (A):** The range of the function $f(x) = 2 \sin^{-1} x + \frac{3\pi}{2}$, where $x \in [-1, 1]$, is $\left[\frac{\pi}{2}, \frac{5\pi}{2} \right]$.

Reason (R): The range of the principal value branch of $\sin^{-1}(x)$ is $[0, \pi]$.

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
(C) Assertion (A) is true, but Reason (R) is false.
(D) Assertion (A) is false, but Reason (R) is true.

20. **Assertion (A):** The equation of the line passing through the points $(1, 2, 3)$ and $(3, -1, 3)$ is $\frac{x-3}{2} = \frac{y+1}{3} = \frac{z-3}{0}$.

Reason (R): The equation of the line passing through the points (x_1, y_1, z_1) , (x_2, y_2, z_2) is given by $\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$.

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
(C) Assertion (A) is true, but Reason (R) is false.
(D) Assertion (A) is false, but Reason (R) is true.

Section B

21. If the product of two positive numbers is 9, find the numbers so that the sum of their squares is minimum.

22. (a) A function $f : A \rightarrow B$ defined as $f(x) = 2x$ is both one-one and onto. If $A = \{1, 2, 3, 4\}$, then find the set B .

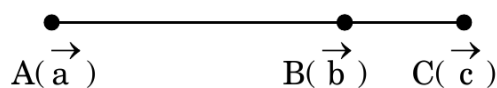
OR

(b) Evaluate:

$$\sin^{-1}\left(\sin \frac{3\pi}{4}\right) + \cos^{-1}\left(\cos \frac{3\pi}{4}\right) + \tan^{-1}(1)$$

23. Find all the vectors of magnitude $3\sqrt{3}$ which are collinear to vector $\hat{i} + \hat{j} + \hat{k}$.

24. (a) Position vectors of the points A, B and C as shown in the figure below are $\vec{a}$, $\vec{b}$ and $\vec{c}$ respectively.



If $\vec{AC} = \frac{5}{4}\vec{AB}$, express $\vec{c}$ in terms of $\vec{a}$ and $\vec{b}$.

OR

(b) Check whether the lines given by equations $x = 2\lambda + 2$, $y = 7\lambda + 1$, $z = -3\lambda - 3$ and $x = -\mu - 2$, $y = 2\mu + 8$, $z = 4\mu + 5$ are perpendicular to each other or not.

25. If $x = \sqrt{a^{\tan^{-1}t}}$, $y = \sqrt{a^{\cot^{-1}t}}$, then show that $x \frac{dy}{dx} + y = 0$.

Section C

26. (a) Evaluate: $\int_0^{\pi/2} e^x \sin x \, dx$.

OR

(b) Find: $\int \frac{1}{\cos(x-a)\cos(x-b)} \, dx$.

27. Evaluate: $\int_0^{\pi/2} [\log(\sin x) - \log(2 \cos x)] \, dx$.

28. Find: $\int \frac{x^2}{(x^2+4)(x^2+9)} \, dx$.

29. (a) Find the general solution of the differential equation: $\frac{dy}{dx} - \frac{2y}{x} = \sin \frac{1}{x}$.

OR

(b) Find the particular solution of the differential equation $\frac{dy}{dx} = \sin(x+y) + \sin(x-y)$, given that $y = 0$ when $x = \frac{\pi}{4}$.

30. Solve the following linear programming problem graphically: Maximize $z = 600x + 400y$ subject to the constraints

$$\begin{aligned} x + 2y &\leq 12, \\ 2x + y &\leq 12, \\ x + 1.25y &\geq 5, \\ x, y &\geq 0. \end{aligned}$$

31. (a) The probability distribution of a random variable X is given below:

X	1	2	3
$P(X)$	$\frac{k}{2}$	$\frac{k}{3}$	$\frac{k}{6}$

- (i) Find the value of k . (ii) Find $P(1 \leq X < 3)$.
 (iii) Find $E(X)$, the mean of X .

OR

(b) A and B are independent events such that $P(A \cap \bar{B}) = \frac{1}{4}$ and $P(\bar{A} \cap B) = \frac{1}{6}$. Find $P(A)$ and $P(B)$.

Section D

32. (a) If $A = \begin{pmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{pmatrix}$ and $B^{-1} = \begin{pmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{pmatrix}$, find $(AB)^{-1}$.

OR

(b) Solve the following system of equations by matrix method:

$$x + 2y + 3z = 6$$

$$2x - y + z = 2$$

$$3x + 2y - 2z = 3$$

33. (a) Find the vector and the Cartesian equations of a line passing through the point $(1, 2, -4)$ and parallel to the line joining the points $A(3, 3, -5)$ and $B(1, 0, -11)$. Hence, find the distance between the two lines.

OR

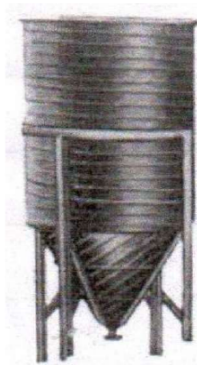
(b) Find the equations of the line passing through the points $A(1, 2, 3)$ and $B(3, 5, 9)$. Hence, find the coordinates of the points on this line which are at a distance of 14 units from point B .

34. Find the area of the region bounded by the lines $y = 4x + 5$, $x + y = 5$ and $4y = x + 5$, using integration.

35. A relation R is defined on a set of real numbers $\mathbb{R}$ as $R = \{(x, y) : x \cdot y \text{ is an irrational number}\}$. Check whether R is reflexive, symmetric and transitive or not.

Section E

36. A tank, as shown in the figure below, formed using a combination of a cylinder and a cone, offers better drainage as compared to a flat bottomed tank.



A tap is connected to such a tank whose conical part is full of water. Water is dripping out from a tap at the bottom at the uniform rate of $2 \text{ cm}^3/\text{s}$. The semi-vertical angle of the conical tank is 45° .

On the basis of given information, answer the following questions.

36(i). Find the volume of water in the tank in terms of its radius r .

36(ii). Find rate of change of radius at an instant when $r = 2\sqrt{2} \text{ cm}$.

36(iii). (a) Find the rate at which the wet surface of the conical tank is decreasing at an instant when radius $r = 2\sqrt{2} \text{ cm}$.

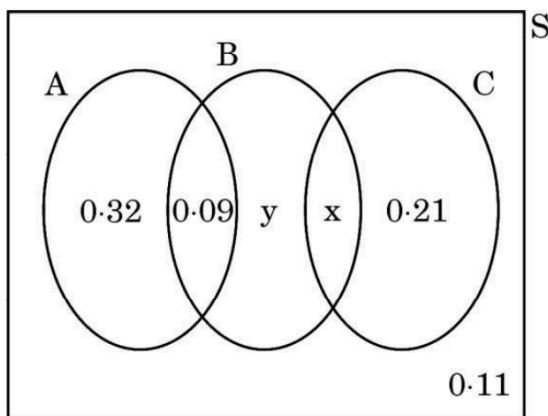
OR

(b) Find the rate of change of height h at an instant when slant height is 4 cm .

37. There are different types of Yoga which involve the usage of different poses of Yoga Asanas, Meditation and Pranayam as shown in the figure below: Anusara Yoga, Kundalini Yoga, Vinyasa Yoga, Bikram Yoga, Hatha Yoga.



The Venn diagram below represents the probabilities of three different types of Yoga, A , B and C performed by the people of a society. Further, it is given that probability of a member performing type C Yoga is 0.44.



On the basis of the above information, answer the following questions.

37(i). Find the value of x .

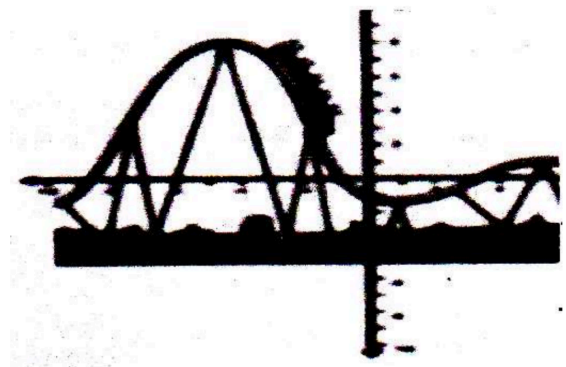
37(ii). Find the value of y .

37(iii). (a) Find $P\left(\frac{C}{B}\right)$.

OR

(b) Find the probability that a randomly selected person of the society does Yoga of type A or B but not C .

38. The equation of the path traced by a roller-coaster is given by the polynomial $f(x) = a(x + 9)(x + 1)(x - 3)$. If the roller-coaster crosses y -axis at a point $(0, -1)$, answer the following.



38(i). Find the value of ' a '.

38(ii). Find $f'(x)$ at $x = 1$.

- End of Question Paper -