

Question Paper

Conducted by the Central Board of Secondary Education (CBSE)



Time Allowed: 3 Hours

Maximum Marks: 80

General Instructions

- (i) This question paper contains **38 questions**. All questions are compulsory.
- (ii) Section A: Questions 1 to 18 are multiple choice questions and Questions 19 and 20 are Assertion-Reason questions. Each carries 1 mark.
- (iii) Section B: Questions 21 to 25 carry 2 marks each.
- (iv) Section C: Questions 26 to 31 carry 3 marks each.
- (v) Section D: Questions 32 to 35 carry 5 marks each.
- (vi) Section E: Questions 36 to 38 carry 4 marks each. These are case study questions.
- (vii) Attempt one alternative where an internal choice is provided. Follow the instructions within each question.
- (viii) Use of calculators is not allowed.

Section A

1. If $A = \begin{pmatrix} 0 & 1 & 3 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$, then A^{2023} is equal to

- (A) $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ (B) $\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$
 (C) $\begin{pmatrix} 0 & 0 & 2023 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ (D) $\begin{pmatrix} 0 & 1 & 2023 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$

2. If $\begin{pmatrix} 2 & 0 \\ 5 & 4 \end{pmatrix} = P + Q$, where P is a symmetric and Q is a skew symmetric matrix, then Q is equal to

- (A) $\begin{pmatrix} 2 & 5/2 \\ 5/2 & 4 \end{pmatrix}$ (B) $\begin{pmatrix} 0 & -5/2 \\ 5/2 & 0 \end{pmatrix}$
 (C) $\begin{pmatrix} 0 & 5/2 \\ -5/2 & 0 \end{pmatrix}$ (D) $\begin{pmatrix} 2 & 5/2 \\ 5/2 & 4 \end{pmatrix}$

3. If $\begin{pmatrix} 1 & 2 & 1 \\ 2 & 3 & 1 \\ 3 & a & 1 \end{pmatrix}$ is non-singular matrix and $a \in A$, then the set A is

- (A) $\mathbb{R}$ (B) $\{0\}$
 (C) $\{4\}$ (D) $\mathbb{R} - \{4\}$

4. If $|A| = |kA|$, where A is a square matrix of order 2, then sum of all possible values of k is

- (A) 1 (B) -1
 (C) 2 (D) 0

5. If $\frac{d}{dx}[f(x)] = ax + b$ and $f(0) = 0$, then $f(x)$ is equal to

- (A) $a + b$ (B) $\frac{ax^2}{2} + bx$
 (C) $\frac{ax^2}{2} + bx + c$ (D) b

6. Degree of the differential equation $\sin x + \cos\left(\frac{dy}{dx}\right) = y^2$ is

- (A) 2 (B) 1
 (C) not defined (D) 0

7. The integrating factor of the differential equation $(1 - y^2)\frac{dx}{dy} + yx = ay$, $(-1 < y < 1)$ is

- (A) $\frac{1}{y^2 - 1}$ (B) $\frac{1}{\sqrt{y^2 - 1}}$
 (C) $\frac{1}{1 - y^2}$ (D) $\frac{1}{\sqrt{1 - y^2}}$

8. Unit vector along $\overrightarrow{PQ}$, where coordinates of P and Q respectively are $(2, 1, -1)$ and $(4, 4, -7)$, is

- (A) $2\hat{i} + 3\hat{j} - 6\hat{k}$ (B) $-2\hat{i} - 3\hat{j} + 6\hat{k}$
 (C) $\frac{-2\hat{i}}{7} - \frac{3\hat{j}}{7} + \frac{6\hat{k}}{7}$ (D) $\frac{2\hat{i}}{7} + \frac{3\hat{j}}{7} - \frac{6\hat{k}}{7}$

9. Position vector of the mid-point of line segment AB is $3\hat{i} + 2\hat{j} - 3\hat{k}$. If position vector of the point A is $2\hat{i} + 3\hat{j} - 4\hat{k}$, then position vector of the point B is

- (A) $\frac{5\hat{i}}{2} + \frac{5\hat{j}}{2} - \frac{7\hat{k}}{2}$ (B) $4\hat{i} + \hat{j} - 2\hat{k}$
 (C) $5\hat{i} + 5\hat{j} - 7\hat{k}$ (D) $\frac{\hat{i}}{2} - \frac{\hat{j}}{2} + \frac{\hat{k}}{2}$

10. Projection of vector $2\hat{i} + 3\hat{j}$ on the vector $3\hat{i} - 2\hat{j}$ is

- (A) 0 (B) 12
 (C) $\frac{12}{\sqrt{13}}$ (D) $\frac{-12}{\sqrt{13}}$

11. Equation of a line passing through point $(1, 1, 1)$ and parallel to z -axis is

- (A) $\frac{x}{1} = \frac{y}{1} = \frac{z}{1}$ (B) $\frac{x-1}{\frac{1}{z-1}} = \frac{y-1}{1} =$
 (C) $\frac{x}{0} = \frac{y}{0} = \frac{z-1}{1}$ (D) $\frac{x-1}{\frac{0}{z-1}} = \frac{y-1}{0} =$
 $\frac{1}{1}$

12. If the sum of numbers obtained on throwing a pair of dice is 9, then the probability that number obtained on one of the dice is 4, is

- (A) $\frac{1}{9}$ (B) $\frac{4}{9}$
(C) $\frac{1}{18}$ (D) $\frac{1}{2}$

13. Anti-derivative of $\frac{\tan x - 1}{\tan x + 1}$ with respect to x is

- (A) $\sec^2\left(\frac{\pi}{4} - x\right) + c$
(B) $-\sec^2\left(\frac{\pi}{4} - x\right) + c$
(C) $\log\left|\sec\left(\frac{\pi}{4} - x\right)\right| + c$
(D) $-\log\left|\sec\left(\frac{\pi}{4} - x\right)\right| + c$

14. If (a, b) , (c, d) and (e, f) are the vertices of $\triangle ABC$ and Δ denotes the area of $\triangle ABC$, then

$\begin{vmatrix} a & c & e \\ b & d & f \\ 1 & 1 & 1 \end{vmatrix}^2$ is equal to

- (A) $2\Delta^2$ (B) $4\Delta^2$
(C) 2Δ (D) 4Δ

15. The function $f(x) = x|x|$ is

- (A) continuous and differentiable at $x = 0$.
(B) continuous but not differentiable at $x = 0$.
(C) differentiable but not continuous at $x = 0$.
(D) neither differentiable nor continuous at $x = 0$.

16. If $\tan\left(\frac{x+y}{x-y}\right) = k$, then $\frac{dy}{dx}$ is equal to

- (A) $\frac{-y}{x}$ (B) $\frac{y}{x}$
(C) $\sec^2\left(\frac{y}{x}\right)$ (D) $\sec^2\left(\frac{x}{y}\right)$

17. The objective function $Z = ax + by$ of an LPP has maximum value 42 at $(4, 6)$ and minimum value 19 at $(3, 2)$. Which of the following is true?

- (A) $a = 9, b = 1$ (B) $a = 5, b = 2$
(C) $a = 3, b = 5$ (D) $a = 5, b = 3$

18. The corner points of the feasible region of a linear programming problem are $(0, 4)$, $(8, 0)$ and $\left(\frac{20}{3}, \frac{4}{3}\right)$. If $Z = 30x + 24y$ is the objective function, then (maximum value of Z - minimum value of Z) is equal to

- (A) 40 (B) 96
(C) 120 (D) 136

19. Assertion (A): Maximum value of $(\cos^{-1} x)^2$ is π^2 .

Reason (R): Range of the principal value branch of $\cos^{-1} x$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

Choose the correct option.

- (A) Both (A) and (R) are true and (R) is the correct explanation of (A).
(B) Both (A) and (R) are true, but (R) is not the correct explanation of (A).
(C) (A) is true, but (R) is false.
(D) (A) is false, but (R) is true.

20. Assertion (A): If a line makes angles α, β, γ with positive direction of the coordinate axes, then $\sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma = 2$.

Reason (R): The sum of squares of the direction cosines of a line is 1.

Choose the correct option.

- (A) Both (A) and (R) are true and (R) is the correct explanation of (A).
(B) Both (A) and (R) are true, but (R) is not the correct explanation of (A).
(C) (A) is true, but (R) is false.
(D) (A) is false, but (R) is true.

Section B

21. (a) Evaluate $\sin^{-1}\left(\sin \frac{3\pi}{4}\right) + \cos^{-1}(\cos \pi) + \tan^{-1}(1)$.

OR

(b) Draw the graph of $\cos^{-1} x$, where $x \in [-1, 0]$. Also, write its range.

22. A particle moves along the curve $3y = ax^3 + 1$ such that at a point with x -coordinate 1, y -coordinate is changing twice as fast as x -coordinate. Find the value of a .

23. If $\vec{a}, \vec{b}, \vec{c}$ are three non-zero unequal vectors such that $\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c}$, then find the angle between $\vec{a}$ and $\vec{b} - \vec{c}$.

24. Find the coordinates of points on line $\frac{x}{1} = \frac{y-1}{2} = \frac{z+1}{2}$ which are at a distance of $\sqrt{11}$ units from origin.

25. (a) If $y = \sqrt{ax+b}$, prove that $y \left(\frac{d^2y}{dx^2} \right) + \left(\frac{dy}{dx} \right)^2 = 0$.

OR

(b) If $f(x) = \begin{cases} ax+b & ; 0 < x \leq 1 \\ 2x^2-x & ; 1 < x < 2 \end{cases}$ is a differentiable function in $(0, 2)$, then find the values of a and b .

Section C

26. (a) Evaluate $\int_0^{\pi/4} \log(1 + \tan x) dx$.

OR

(b) Find $\int \frac{dx}{\sqrt{\sin^3 x \cos(x - \alpha)}}$.

27. Find $\int e^{\cot^{-1} x} \cdot \left(\frac{1 - x + x^2}{1 + x^2} \right) dx$.

28. Evaluate $\int_{\log \sqrt{2}}^{\log \sqrt{3}} \frac{1}{(e^x + e^{-x})(e^x - e^{-x})} dx$.

29. (a) Find the general solution of the differential equation $(xy - x^2) dy = y^2 dx$.

OR

(b) Find the general solution of the differential equation $(x^2 + 1) \frac{dy}{dx} + 2xy = \sqrt{x^2 + 4}$.

30. (a) Two balls are drawn at random one by one with replacement from an urn containing equal number of red balls and green balls. Find the probability distribution of number of red balls. Also, find the mean of the random variable.

OR

(b) A and B throw a die alternately till one of them gets a '6' and wins the game. Find their respective probabilities of winning, if A starts the game first.

31. Solve the following linear programming problem graphically:

Minimize $Z = 5x + 10y$

subject to constraints: $x + 2y \leq 120$, $x + y \geq 60$,
 $x - 2y \geq 0$, $x \geq 0$, $y \geq 0$.

32. (a) If $A = \begin{pmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 2 & 0 \\ -2 & -1 & -2 \\ 0 & -1 & 1 \end{pmatrix}$, then find AB and use it to

solve the following system of equations:

$$x - 2y = 3$$

$$2x - y - z = 2$$

$$-2y + z = 3$$

OR

(b) If $f(\alpha) = \begin{pmatrix} \cos \alpha & -\sin \alpha & 0 \\ \sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 1 \end{pmatrix}$, prove that $f(\alpha) \cdot f(\beta) = f(\alpha + \beta)$.

33. (a) Find the equations of the diagonals of the parallelogram $PQRS$ whose vertices are $P(4, 2, -6)$, $Q(5, -3, 1)$, $R(12, 4, 5)$ and $S(11, 9, -2)$. Use these equations to find the point of intersection of diagonals.

OR

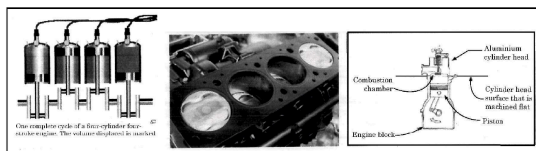
(b) A line l passes through point $(-1, 3, -2)$ and is perpendicular to both the lines $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ and $\frac{x+2}{-3} = \frac{y-1}{2} = \frac{z+1}{5}$. Find the vector equation of the line l . Hence, obtain its distance from origin.

34. Using integration, find the area of region bounded by line $y = \sqrt{3}x$, the curve $y = \sqrt{4-x^2}$ and y -axis in first quadrant.

35. A function $f : [-4, 4] \rightarrow [0, 4]$ is given by $f(x) = \sqrt{16-x^2}$. Show that f is an onto function but not a one-one function. Further, find all possible values of a for which $f(a) = \sqrt{7}$.

Section E

36. Engine displacement is the measure of the cylinder volume swept by all the pistons of a piston engine. The piston moves inside the cylinder bore.



The cylinder bore in the form of circular cylinder open at the top is to be made from a metal sheet of area $75\pi \text{ cm}^2$.

Based on the above information, answer the following questions:

(i) If the radius of cylinder is r cm and height is h cm, then write the volume V of cylinder in terms of radius r .

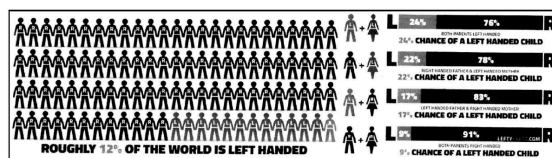
(ii) Find $\frac{dV}{dr}$.

(iii) (a) Find the radius of cylinder when its volume is maximum.

OR

(iii) (b) For maximum volume, $h > r$. State true or false and justify.

37. Recent studies suggest that roughly 12% of the world population is left handed.



Depending upon the parents, the chances of having a left handed child are as follows:

A: When both father and mother are left handed: Chances of left handed child is 24%.

B: When father is right handed and mother is left handed: Chances of left handed child is 22%.

C: When father is left handed and mother is right handed: Chances of left handed child is 17%.

D: When both father and mother are right handed: Chances of left handed child is 9%.

Assuming that $P(A) = P(B) = P(C) = P(D) = \frac{1}{4}$ and L denotes the event that child is left handed.

Based on the above information, answer the following questions:

(i) Find $P(L/C)$.

(ii) Find $P(L/A)$.

(iii) (a) Find $P(A/L)$.

OR

(iii) (b) Find the probability that a randomly selected child is left handed given that exactly one of the parents is left handed.

38. The use of electric vehicles is increasing every year and estimated electric vehicles in use at any time t is given by the function V :

$$V(t) = \frac{1}{5}t^5 - \frac{5}{2}t^4 + 25t - 2$$

where t represents the time and $t = 1, 2, 3, \dots$ corresponds to year 2001, 2002, 2003, \ldots respectively.

Based on the above information, answer the following questions:

- (i) Can the above function be used to estimate number of vehicles in the year 2000? Justify.
 - (ii) Prove that the function $V(t)$ is an increasing function.
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- End of Question Paper -