

Question Paper

Conducted by the Central Board of Secondary Education (CBSE)



Time Allowed: 3 Hours

Maximum Marks: 80

General Instructions

- (i) This question paper contains **38 questions**. All questions are compulsory.
- (ii) Section A: Questions 1 to 18 are multiple choice questions and Questions 19 and 20 are Assertion-Reason questions. Each carries 1 mark.
- (iii) Section B: Questions 21 to 25 carry 2 marks each.
- (iv) Section C: Questions 26 to 31 carry 3 marks each.
- (v) Section D: Questions 32 to 35 carry 5 marks each.
- (vi) Section E: Questions 36 to 38 carry 4 marks each. These are case study questions.
- (vii) Attempt one alternative where an internal choice is provided. Follow the instructions within each question.
- (viii) Use of calculators is not allowed.

Section A

1. If $A = \begin{pmatrix} 1 & 4 & x \\ z & 2 & y \\ -3 & -1 & 3 \end{pmatrix}$ is a symmetric matrix, then the value of $x + y + z$ is:

- (A) 10 (B) 6
(C) 8 (D) 0

2. Let A be a skew-symmetric matrix of order 3. If $|A| = x$, then $(2023)^x$ is equal to:

- (A) $\frac{1}{2023}$ (B) 2023
(C) $(2023)^2$ (D) 1

3. $\int_0^2 \sqrt{4-x^2} dx$ equals:

- (A) $2 \log 2$ (B) $-2 \log 2$
(C) $\frac{\pi}{2}$ (D) π

4. The solution of the differential equation $\frac{dx}{x} + \frac{dy}{y} = 0$ is:

- (A) $\frac{1}{x} + \frac{1}{y} = C$ (B) $\log x - \log y = C$
(C) $xy = C$ (D) $x + y = C$

5. What is the product of the order and degree of the differential equation $\frac{d^2y}{dx^2} \sin y + \left(\frac{dy}{dx}\right)^3 \cos y = \sqrt{y}$?

- (A) 3 (B) 2
(C) 6 (D) not defined

6. The direction cosines of vector $\vec{BA}$, where coordinates of A and B are $(1, 2, -1)$ and $(3, 4, 0)$ respectively, are:

- (A) $-2, -2, -1$ (B) $-\frac{2}{3}, -\frac{2}{3}, -\frac{1}{3}$
(C) $2, 2, 1$ (D) $\frac{2}{3}, \frac{2}{3}, \frac{1}{3}$

7. $\vec{a}$ and $\vec{b}$ are two non-zero vectors such that the projection of $\vec{a}$ on $\vec{b}$ is 0. The angle between $\vec{a}$ and $\vec{b}$ is:

- (A) $\frac{\pi}{2}$ (B) π
(C) $\frac{\pi}{4}$ (D) 0

8. In $\triangle ABC$, $\vec{AB} = \hat{i} + \hat{j} + 2\hat{k}$ and $\vec{AC} = 3\hat{i} - \hat{j} + 4\hat{k}$. If D is mid-point of BC , then vector $\vec{AD}$ is equal to:

- (A) $4\hat{i} + 6\hat{k}$ (B) $2\hat{i} - 2\hat{j} + 2\hat{k}$
(C) $\hat{i} - \hat{j} + \hat{k}$ (D) $2\hat{i} + 3\hat{k}$

9. If the point $P(a, b, 0)$ lies on the line $\frac{x+1}{2} = \frac{y+2}{3} = \frac{z+3}{4}$, then (a, b) is:

- (A) $(1, 2)$ (B) $\left(\frac{1}{2}, \frac{2}{3}\right)$
(C) $\left(\frac{1}{2}, \frac{1}{4}\right)$ (D) $(0, 0)$

10. For any two events A and B , if $P(\bar{A}) = \frac{1}{2}$, $P(\bar{B}) = \frac{2}{3}$ and $P(A \cap B) = \frac{1}{4}$, then $P\left(\frac{A}{B}\right)$ equals:

- (A) $\frac{3}{8}$ (B) $\frac{8}{9}$
(C) $\frac{1}{8}$ (D) $\frac{1}{4}$

11. The value of k for which function $f(x) = \begin{cases} x^2, & x \geq 0 \\ kx, & x < 0 \end{cases}$ is differentiable at $x = 0$ is:

- (A) 1 (B) 2
(C) any real number (D) 0

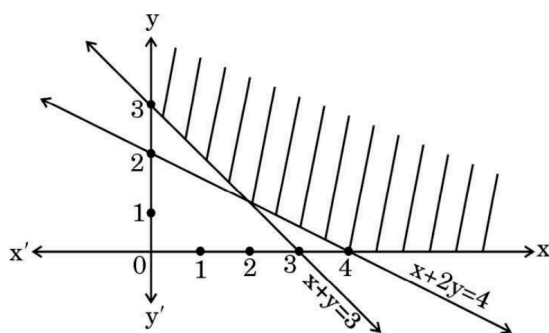
12. If $y = \frac{\cos x - \sin x}{\cos x + \sin x}$, then $\frac{dy}{dx}$ is:

- (A) $-\sec^2\left(\frac{\pi}{4} - x\right)$ (B) $\sec^2\left(\frac{\pi}{4} - x\right)$
(C) $\log \left| \sec\left(\frac{\pi}{4} - x\right) \right|$ (D) $-\log \left| \sec\left(\frac{\pi}{4} - x\right) \right|$

13. The number of feasible solutions of the linear programming problem given as: Maximize $z = 15x + 30y$ subject to constraints: $3x + y \leq 12$, $x + 2y \leq 10$, $x \geq 0$, $y \geq 0$ is:

- (A) 1 (B) 2
(C) 3 (D) infinite

14. The feasible region of a linear programming problem is shown in the figure below:



Which of the following are the possible constraints?

- (A) $x + 2y \geq 4$, $x + y \leq 3$, $x \geq 0$, $y \geq 0$ (B) $x + 2y \leq 4$, $x + y \leq 3$, $x \geq 0$, $y \geq 0$
(C) $x + 2y \geq 4$, $x + y \geq 3$, $x \geq 0$, $y \geq 0$ (D) $x + 2y \geq 4$, $x + y \geq 3$, $x \leq 0$, $y \leq 0$

15. If $A = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}$, then $B'A'$ is equal to:

- (A) $\begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}$ (B) $\begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}$
(C) $\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$ (D) $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

16. If $A \cdot (\text{adj } A) = \begin{pmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{pmatrix}$, then the value of $|A| + |\text{adj } A|$ is equal to:

- (A) 12 (B) 9
(C) 3 (D) 27

17. A and B are skew-symmetric matrices of same order. AB is symmetric, if:

- (A) $AB = O$ (B) $AB = -BA$
(C) $AB = BA$ (D) $BA = O$

18. For what value of $x \in \left[0, \frac{\pi}{2}\right]$, is $A + A' = \sqrt{3}I$, where $A = \begin{pmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{pmatrix}$?

- (A) $\frac{\pi}{3}$ (B) $\frac{\pi}{6}$
(C) 0 (D) $\frac{\pi}{2}$

19. **Assertion (A):** A line through the points $(4, 7, 8)$ and $(2, 3, 4)$ is parallel to a line through the points $(-1, -2, 1)$ and $(1, 2, 5)$.

Reason (R): Lines $\vec{r} = \vec{a}_1 + \lambda \vec{b}_1$ and $\vec{r} = \vec{a}_2 + \mu \vec{b}_2$ are parallel if $\vec{b}_1 \cdot \vec{b}_2 = 0$.

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
(C) Assertion (A) is true and Reason (R) is false.
(D) Assertion (A) is false and Reason (R) is true.

20. **Assertion (A):** Range of $[\sin^{-1} x + 2 \cos^{-1} x]$ is $[0, \pi]$.

Reason (R): Principal value branch of $\sin^{-1} x$ has range $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
(C) Assertion (A) is true and Reason (R) is false.
(D) Assertion (A) is false and Reason (R) is true.

Section B

21. Consider the statement: “There exists at least one value of $b \in \mathbb{R}$ for which $f(x) = \frac{b}{x}$, $b \neq 0$, $\mathbb{R} - \{0\}$ is strictly increasing in $\mathbb{R} - \{0\}$.” State True or False. Justify.

22. (a) Evaluate: $3 \sin^{-1} \left(\frac{1}{\sqrt{2}} \right) + 2 \cos^{-1} \left(\frac{\sqrt{3}}{2} \right) + \cos^{-1}(0)$.

OR

(b) Draw the graph of $f(x) = \sin^{-1} x$, $x \in \left[-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right]$. Also, write range of $f(x)$.

23. (a) If $y = x^{\frac{1}{x}}$, then find $\frac{dy}{dx}$ at $x = 1$.

OR

(b) If $x = a \sin 2t$, $y = a(\cos 2t + \log \tan t)$, then find $\frac{dy}{dx}$.

24. If $\vec{r} = 3\hat{i} - 2\hat{j} + 6\hat{k}$, find the value of $(\vec{r} \times \hat{j}) \cdot (\vec{r} \times \hat{k}) - 12$.

25. Find the value of p , so that lines $\frac{x-1}{-2} = \frac{y-4}{3p} = \frac{z-3}{4}$ and $\frac{x-2}{4p} = \frac{y-5}{2} = \frac{1-z}{7}$ are perpendicular to each other.

Section C

26. Find: $\int \frac{e^x}{\sqrt{e^{2x} - 4e^x - 5}} dx.$

27. (a) Find: $\int \frac{\cos x}{\sin 3x} dx.$

OR

(b) Find: $\int x^2 \log(x^2 + 1) dx.$

28. Solve the following linear programming problem graphically: Maximize $z = 3x + 9y$ subject to the constraints $x + y \geq 10$, $x + 3y \leq 60$, $x \leq y$, $x \geq 0$, $y \geq 0$.

29. (a) A pair of dice is thrown simultaneously. If X denotes the absolute difference of numbers obtained on the pair of dice, then find the probability distribution of X .

OR

(b) There are two coins. One of them is a biased coin such that $P(\text{head}) : P(\text{tail})$ is $1 : 3$ and the other coin is a fair coin. A coin is selected at random and tossed once. If the coin showed head, then find the probability that it is a biased coin.

30. (a) Find the general solution of the differential equation: $\frac{d}{dx}(xy^2) = 2y(1 + x^2).$

OR

(b) Solve the following differential equation:
 $xe^{\frac{y}{x}} - y + x \frac{dy}{dx} = 0.$

31. Evaluate: $\int_{-\pi/2}^{\pi/2} \frac{\sin^{100} x}{\sin^{100} x + \cos^{100} x} dx.$

32. (a) If $A = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{pmatrix}$, then show that $A^3 - 6A^2 + 7A + 2I = O$.

OR

(b) If $A = \begin{pmatrix} 3 & 2 \\ 5 & -7 \end{pmatrix}$, then find A^{-1} and use it to solve the following system of equations: $3x + 5y = 11$, $2x - 7y = -3$.

33. (a) Find the value of b so that the lines $\frac{x-1}{2} = \frac{y-b}{3} = \frac{z-3}{4}$ and $\frac{x-4}{5} = \frac{y-1}{2} = z$ are intersecting lines. Also, find the point of intersection of these given lines.

OR

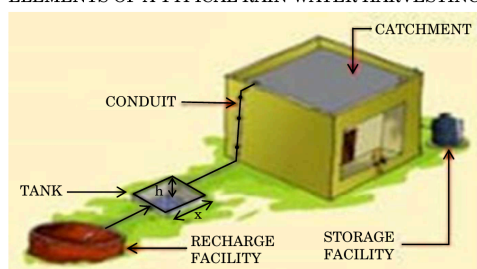
(b) Find the equations of all the sides of the parallelogram $ABCD$ whose vertices are $A(4, 7, 8)$, $B(2, 3, 4)$, $C(-1, -2, 1)$ and $D(1, 2, 5)$. Also, find the coordinates of the foot of the perpendicular from A to CD .

34. Prove that a function $f : [0, \infty) \rightarrow [-5, \infty)$ defined as $f(x) = 4x^2 + 4x - 5$ is both one-one and onto.

35. The area of the region bounded by the line $y = mx$ ($m > 0$), the curve $x^2 + y^2 = 4$ and the x -axis in the first quadrant is $\frac{\pi}{2}$ units. Using integration, find the value of m .

36. In order to set up a rain water harvesting system, a tank to collect rain water is to be dug. The tank should have a square base and a capacity of 250 m^3 . The cost of land is Rs. 5,000 per square metre and cost of digging increases with depth and for the whole tank, it is Rs. $40,000 h^2$, where h is the depth of the tank in metres. x is the side of the square base of the tank in metres.

ELEMENTS OF A TYPICAL RAIN WATER HARVESTING SYSTEM



Based on the above information, answer the following questions:

(i) Find the total cost C of digging the tank in terms of x .

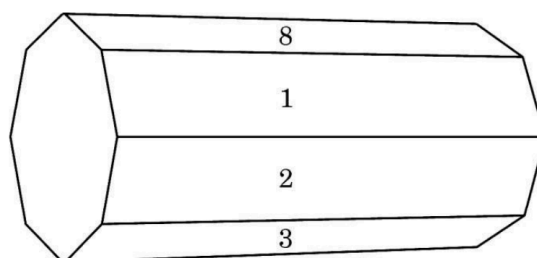
(ii) Find $\frac{dC}{dx}$.

(iii) (a) Find the value of x for which cost C is minimum.

OR

(iii) (b) Check whether the cost function $C(x)$ expressed in terms of x is increasing or not, where $x > 0$.

37. An octagonal prism is a three-dimensional polyhedron bounded by two octagonal bases and eight rectangular side faces. It has 24 edges and 16 vertices.



The prism is rolled along the rectangular faces and number on the bottom face (touching the ground) is noted. Let X denote the number obtained on the bottom face and the following table gives the probability distribution of X .

$X :$	1	2	3	4	5	6	7	8
$P(X) :$	p	$2p$	$2p$	p	$2p$	p^2	$2p^2$	$7p^2 + p$

Based on the above information, answer the following questions:

(i) Find the value of p .

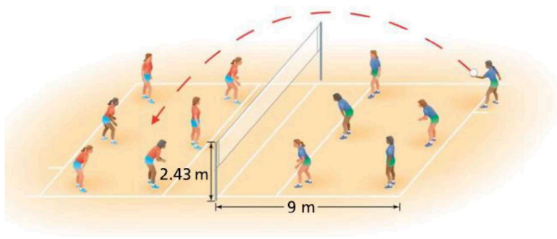
(ii) Find $P(X > 6)$.

(iii) (a) Find $P(X = 3m)$, where m is a natural number.

OR

(iii) (b) Find the mean $E(X)$.

38. A volleyball player serves the ball which takes a parabolic path given by the equation $h(t) = -\frac{7}{2}t^2 + \frac{13}{2}t + 1$, where $h(t)$ is the height of ball at any time t (in seconds), ($t \geq 0$).



Based on the above information, answer the following questions:

- (i) Is $h(t)$ a continuous function? Justify.
- (ii) Find the time at which the height of the ball is maximum.

- End of Question Paper -