

CBSE Class 12 Physics Question Paper 2023 with Solution PDF-Set 55/3/1



Question Paper with Solutions

Conducted by the Central Board of Secondary Education (CBSE)

Time Allowed: 3 hours

Maximum Marks: 70

General Instructions

- (i) This question paper contains **35** questions. **All** questions are **compulsory**.
- (ii) This question paper is divided into **five** Sections – **A, B, C, D** and **E**.
- (iii) In **Section A** – Questions no. **1 to 18** are Multiple Choice (MCQ) type questions, carrying **1** mark each.
- (iv) In **Section B** – Questions no. **19 to 25** are Very Short Answer (VSA) type questions, carrying **2** marks each.
- (v) In **Section C** – Questions no. **26 to 30** are Short Answer (SA) type questions, carrying **3** marks each.
- (vi) In **Section D** – Questions no. **31 to 33** are Long Answer (LA) type questions carrying **5** marks each.
- (vii) In **Section E** – Questions no. **34** and **35** are case-based questions carrying **4** marks each.
- (viii) There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 2 questions in Section C, 3 questions in Section D and 2 questions in Section E.
- (ix) Use of calculators is **not** allowed.

Use the following values of physical constants, if required :

$$c = 3 \times 10^8 \text{ m/s}$$

$$h = 6.63 \times 10^{-34} \text{ Js}$$

$$e = 1.6 \times 10^{-19} \text{ C}$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ T m A}^{-1}$$

$$\epsilon_0 = 8.854 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$$

$$\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$$

$$\text{Mass of electron } (m_e) = 9.1 \times 10^{-31} \text{ kg}$$

$$\text{Mass of neutron} = 1.675 \times 10^{-27} \text{ kg}$$

$$\text{Mass of proton} = 1.673 \times 10^{-27} \text{ kg}$$

$$\text{Avogadro's number} = 6.023 \times 10^{23} \text{ per gram mole}$$

$$\text{Boltzmann constant} = 1.38 \times 10^{-23} \text{ JK}^{-1}$$

Section A (18 marks)

1. An electron experiences a force $(1.6 \times 10^{-16} \text{ N}) \hat{i}$ in an electric field $\vec{E}$. The electric field $\vec{E}$ is: [1 mark]

- (A) $(1.0 \times 10^3 \text{ N/C}) \hat{i}$
- (B) $-(1.0 \times 10^3 \text{ N/C}) \hat{i}$
- (C) $(1.0 \times 10^{-3} \text{ N/C}) \hat{i}$
- (D) $-(1.0 \times 10^{-3} \text{ N/C}) \hat{i}$

Correct Answer: (B) $-(1.0 \times 10^3 \text{ N/C}) \hat{i}$

Solution:

Concept:

- Electric force and electric field are related by $\vec{F} = q\vec{E}$, where q is the signed charge.
- An electron carries charge $q = -1.6 \times 10^{-19} \text{ C}$; its force is opposite to the electric field.

Step 1: Divide the force by the signed charge The force points along positive x , but the electron charge is negative. Therefore

$$\vec{E} = \frac{\vec{F}}{q} = \frac{1.6 \times 10^{-16}}{-1.6 \times 10^{-19}} \hat{i} \text{ N/C.}$$

Step 2: Check the direction and power of ten The magnitude is $10^{-16-(-19)} = 10^3 \text{ N/C}$, and the direction is $-\hat{i}$. Option (A) ignores the negative charge. Options (C) and (D) use the inverse power of ten and so have the wrong magnitude.

Step 3: Conclusion $\vec{E} = -(1.0 \times 10^3 \text{ N/C}) \hat{i}$. Hence, the correct option is (B).

Quick Tip to Solve: For an electron, first reverse the force direction, then divide its magnitude by $1.6 \times 10^{-19} \text{ C}$.

2. Which one of the following is not a scalar quantity? [1 mark]

- (A) Electric field
- (B) Voltage
- (C) Resistivity
- (D) Power

Correct Answer: (A) Electric field

Solution:

Concept:

- A scalar is specified by magnitude (and sign where applicable), whereas a vector also has a direction and follows vector addition.
- Electric field is force per unit positive test charge: $\vec{E} = \vec{F}/q_0$.

Step 1: Identify the quantity that needs a direction Force is a vector and the test charge is a scalar. Dividing force by charge leaves a vector. At a point, the electric field must specify both its strength and the direction in which it pushes a positive test charge.

Step 2: Check the remaining quantities Voltage is work per unit charge, and power is energy transferred per unit time; both are scalars. Resistivity is also a scalar material property in the ordinary isotropic conductor model used here. A positive or negative voltage does not make voltage a vector,

since its sign is not a spatial direction.

Step 3: Conclusion Electric field is not a scalar quantity. Hence, the correct option is (A).

Quick Tip to Solve: Ask whether a spatial direction must accompany the value: an electric field needs one, while voltage, resistivity and power do not.

3. The current density due to drift of electrons in a conductor is given by (symbols have their usual meanings): [1 mark]

(A) $neAv_d$

(B) $\frac{nAv_d}{e}$

(C) $\frac{nv_d}{eA}$

(D) nev_d

Correct Answer: (D) nev_d

Solution:

Concept:

- The current in a conductor is $I = neAv_d$, where n is the free-electron number density, e the charge magnitude, A the cross-sectional area and v_d the drift-speed magnitude.
- Current density is current per unit cross-sectional area: $J = I/A$. Conventional current is opposite to electron drift.

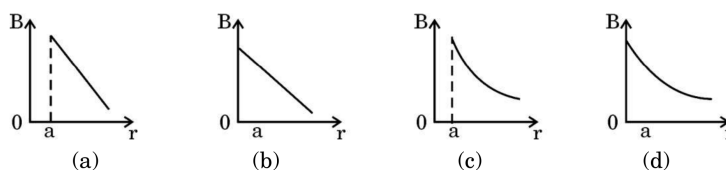
Step 1: Count the charge passing through a cross-section In time Δt , electrons in a cylinder of length $v_d\Delta t$ cross the section. Their number is $nAv_d\Delta t$, so the charge magnitude is $neAv_d\Delta t$, giving $I = neAv_d$.

Step 2: Divide by the cross-sectional area Thus $J = I/A = nev_d$, with units $A\ m^{-2}$. Option (A) is current, not current density. In options (B) and (C), charge appears in the denominator, so neither has the required units or dependence on charge magnitude.

Step 3: Conclusion The magnitude of current density is $J = nev_d$. Hence, the correct option is (D).

Quick Tip to Solve: Start with $I = neAv_d$, then cancel the area to find current density; in vector form electron drift gives $\vec{J} = -ne\vec{v}_d$.

4. Which of the following graphs correctly represents the variation of the magnitude of the magnetic field outside a straight infinite current carrying wire of radius a , as a function of distance r from the centre of the wire? [1 mark]



- (A) Graph (a)
(B) Graph (b)
(C) Graph (c)
(D) Graph (d)

Correct Answer: (C) Graph (c)

Solution:

Concept:

- For a long straight wire carrying current I , Ampere's law outside the wire gives $B(2\pi r) = \mu_0 I$.
- Outside a wire of radius a , $B = \mu_0 I / (2\pi r)$ applies for $r \geq a$; the field varies inversely with distance from the centre.

Step 1: Identify the shape of the field curve Since the entire current is enclosed by an external circular Amperian path, $B \propto 1/r$. Thus the field decreases along a curved, flattening graph and approaches zero at large distance. It does not decrease linearly, ruling out graphs (A) and (B).

Step 2: Check where the outside region begins At the surface, $r = a$, the field is finite: $B(a) = \mu_0 I / (2\pi a)$. Graph (C) starts its decreasing inverse-distance curve at a , as required. Graph (D) extends that curve inward to the centre, where the external-field expression does not apply.

Step 3: Conclusion Graph (C) correctly shows the inverse-distance variation in the outside region. Hence, the correct option is (C).

Quick Tip to Solve: For an outside-wire graph, check both the $1/r$ shape and its domain: the curve must begin at the wire surface $r = a$.

5. A particle of mass m and charge q moving with a uniform velocity $\vec{v} = v_{0x} \hat{i} + v_{0y} \hat{j}$ enters a region with a magnetic field $\vec{B} = B_0 \hat{j}$. After some time, an electric field $\vec{E} = E_0 \hat{j}$ is also switched on in the region. The resulting path described by the particle will be: [1 mark]

- (A) a circle in x - z plane
(B) a parabola in x - y plane
(C) a helix with constant pitch
(D) a helix with increasing pitch

Correct Answer: (D) a helix with increasing pitch

Solution:

Concept:

- The magnetic force $q\vec{v} \times \vec{B}$ bends the velocity perpendicular to $\vec{B}$, producing circular motion, while leaving the parallel component unchanged.

- An electric field parallel to $\vec{B}$ produces axial acceleration $a_y = qE_0/m$, without changing the transverse cyclotron period $T = 2\pi m/(|q|B_0)$.

Step 1: Resolve the motion along and across the field Here the common field axis is y . The initial x -component produces circular motion in the x - z plane, while v_{0y} carries the particle along the axis. Before the electric field is switched on, their combination is a helix of constant pitch.

Step 2: Determine how the electric field changes the pitch After switching on $\vec{E}$, the axial velocity is $v_y = v_{0y} + qE_0t/m$. Under the usual intended convention that the electric force accelerates the particle along its axial motion, the axial distance covered in each fixed cyclotron period increases. The transverse radius stays fixed. A circle omits the axial motion, a parabola omits the magnetic rotation, and constant pitch omits the axial acceleration. If the signed charge makes the electric force oppose the initial axial velocity, the pitch initially decreases until axial reversal; the printed increasing-pitch option assumes the accelerating case.

Step 3: Conclusion The intended motion is a helix with increasing pitch. Hence, the correct option is (D).

Quick Tip to Solve: Split velocity into components parallel and perpendicular to $\vec{B}$: a parallel electric field changes pitch, while the magnetic field preserves the transverse speed.

6. An inductor, a capacitor and a resistor are connected in series across an ac source of voltage. If the frequency of the source is decreased gradually, the reactance of: [1 mark]

- (A) both the inductor and the capacitor decreases.
- (B) inductor decreases and the capacitor increases.
- (C) both the inductor and the capacitor increases.
- (D) inductor increases and the capacitor decreases.

Correct Answer: (B) inductor decreases and the capacitor increases.

Solution:

Concept:

- Inductive reactance is $X_L = \omega L = 2\pi fL$, where f is the source frequency and L the inductance.
- Capacitive reactance is $X_C = 1/(\omega C) = 1/(2\pi fC)$, where C is the capacitance. Both reactances are measured in ohms.

Step 1: Examine the inductive reactance For a fixed inductor, X_L is directly proportional to frequency. Reducing the frequency therefore reduces the inductive reactance; for example, halving f halves X_L .

Step 2: Examine the capacitive reactance For a fixed capacitor, X_C is inversely proportional to frequency. Reducing the frequency increases its reactance; halving f doubles X_C . Thus the two reactances change in opposite ways, ruling out (A) and (C); option (D) reverses their dependencies. The series resistor does not alter these individual reactance formulae.

Step 3: Conclusion Inductive reactance decreases and capacitive reactance increases. Hence, the correct option is (B).

Quick Tip to Solve: Write f in the numerator for X_L and in the denominator for X_C ; a frequency decrease then gives the two trends immediately.

7. The electromagnetic radiations used to kill germs in water purifiers are called:

[1 mark]

- (A) Infrared waves
- (B) X-rays
- (C) Gamma rays
- (D) Ultraviolet rays

Correct Answer: (D) Ultraviolet rays

Solution:

Concept:

- Different regions of the electromagnetic spectrum have characteristic applications determined by their interaction with matter.
- Ultraviolet radiation has a germicidal action: it damages the genetic material of microorganisms and prevents their multiplication.

Step 1: Identify the germicidal radiation A UV water purifier exposes water to ultraviolet radiation to inactivate bacteria and other microorganisms. This use relies on the biological effect of ultraviolet radiation, rather than simply heating the water.

Step 2: Distinguish the other applications Infrared waves are associated mainly with heating. X-rays are commonly associated with medical imaging, and gamma rays with uses such as radiotherapy and irradiation sterilisation. Although sufficiently energetic radiation can also damage microorganisms, the radiation conventionally used in the water purifiers asked about here is ultraviolet.

Step 3: Conclusion The radiations are ultraviolet rays. Hence, the correct option is (D).

Quick Tip to Solve: Associate UV with germicidal water purification and infrared with heating; the device's purpose identifies the spectrum band.

8. In the wave picture of light, the intensity I of light is related to the amplitude A of the wave as:

[1 mark]

- (A) $I \propto \sqrt{A}$
- (B) $I \propto A$
- (C) $I \propto A^2$
- (D) $I \propto \frac{1}{A^2}$

Correct Answer: (C) $I \propto A^2$

Solution:

Concept:

- Intensity is the average energy transported by a wave per unit area per unit time.
- The energy density of an electromagnetic wave is proportional to the square of its field amplitude. For a sinusoidal wave in vacuum, $I = \frac{1}{2}\epsilon_0 c E_0^2$, where E_0 is the electric-field amplitude.

Step 1: Relate the energy flow to amplitude If the wave amplitude is denoted by A , its average energy flow has the form $I = kA^2$, with k fixed for the same medium and amplitude convention. The square arises because the electromagnetic energy density is quadratic in the fields.

Step 2: Test the proposed proportionalities Doubling amplitude therefore makes intensity four times as large. Neither $I \propto A$ nor $I \propto \sqrt{A}$ gives this factor, ruling out (B) and (A). The inverse-square

expression (D) would incorrectly make a larger-amplitude wave less intense.

Step 3: Conclusion The required relation is $I \propto A^2$. Hence, the correct option is (C).

Quick Tip to Solve: Compare two waves using $I_2/I_1 = (A_2/A_1)^2$; square the amplitude ratio to obtain the intensity ratio.

9. In a single-slit diffraction experiment, the width of the slit is halved. The width of the central maximum, in the diffraction pattern, will become: [1 mark]

- (A) half
- (B) twice
- (C) four times
- (D) one-fourth

Correct Answer: (B) twice

Solution:

Concept:

- In single-slit diffraction, the first minima satisfy $a \sin \theta = \pm \lambda$, where a is the slit width and λ is the wavelength.
- For small diffraction angles, the central maximum has width $W = 2D\lambda/a$ on a screen a distance D away.

Step 1: Identify the inverse dependence The central maximum lies between the first minima on either side of the centre. With the wavelength and screen distance unchanged, its width is inversely proportional to the slit width: $W \propto 1/a$.

Step 2: Compare the two widths When $a' = a/2$, the ratio is

$$\frac{W'}{W} = \frac{a}{a'} = \frac{a}{a/2} = 2.$$

Thus halving the slit increases the spread. Options (A) and (D) incorrectly decrease the width; option (C) would require the slit to be reduced to one-fourth.

Step 3: Conclusion The width of the central maximum becomes twice its original value. Hence, the correct option is (B).

Quick Tip to Solve: At fixed wavelength and screen distance, multiply slit width and central-maximum width: their product stays constant.

10. A graph is plotted between the stopping potential (on y -axis) and the frequency of incident radiation (on x -axis) for a metal. The product of the slope of the straight line obtained and the magnitude of charge on an electron is equal to: [1 mark]

- (A) h
- (B) $\frac{h}{c}$
- (C) $\frac{2h}{c}$
- (D) $\frac{h}{2c}$

Correct Answer: (A) h

Solution:

Concept:

- Einstein's photoelectric equation is $K_{\max} = h\nu - \phi$, where h is Planck's constant, ν is incident frequency and ϕ is the metal's work function.
- The stopping potential V_s satisfies $eV_s = K_{\max}$, with e the positive magnitude of electron charge.

Step 1: Express stopping potential as a straight line Combining the two relations gives

$$V_s = \frac{h}{e}\nu - \frac{\phi}{e}.$$

Comparing with $y = mx + b$, the slope of stopping potential versus frequency is $m = h/e$. The work function affects the intercept and does not affect the slope.

Step 2: Multiply the slope by electron charge The requested product is

$$me = \left(\frac{h}{e}\right)e = h.$$

The printed alternatives (B), (C) and (D) all contain a factor involving the speed of light c . Such a factor does not occur when the horizontal axis is frequency; it would enter when frequency is replaced using wavelength.

Step 3: Conclusion The product is Planck's constant h . Hence, the correct option is (A).

Quick Tip to Solve: Read both graph axes before identifying a slope: for stopping potential versus frequency, slope times e gives h .

11. Light of frequency 6.4×10^{14} Hz is incident on a metal of work function 2.14 eV. The maximum kinetic energy of the emitted electrons is about: [1 mark]

- (A) 0.25 eV
- (B) 0.51 eV
- (C) 1.02 eV
- (D) 0.10 eV

Correct Answer: (B) 0.51 eV

Solution:

Concept:

- A photon has energy $E = h\nu$, where h is Planck's constant and ν is its frequency.
- The maximum photoelectron kinetic energy is $K_{\max} = h\nu - \phi$, where ϕ is the work function. Use the same energy unit for both terms; $1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$.

Step 1: Calculate the incident photon energy Using $h = 6.63 \times 10^{-34} \text{ J s}$ and the printed frequency,

$$E = (6.63 \times 10^{-34})(6.4 \times 10^{14}) = 4.2432 \times 10^{-19} \text{ J}.$$

In electron-volts this is

$$E = \frac{4.2432 \times 10^{-19}}{1.6 \times 10^{-19}} = 2.652 \text{ eV}.$$

Step 2: Subtract the work function The energy required to release an electron is 2.14 eV. The remaining maximum energy is

$$K_{\max} = 2.652 - 2.14 = 0.512 \text{ eV}.$$

This is positive, so emission is possible. The values 0.25 eV, 1.02 eV and 0.10 eV do not equal the photon energy minus the work function; no extra factor of two belongs in Einstein's equation.

Step 3: Conclusion The maximum kinetic energy is approximately 0.51 eV. Hence, the correct option is (B).

Quick Tip to Solve: Convert the photon energy to electron-volts before subtracting a work function quoted in electron-volts.

12. The ratio of maximum frequency and minimum frequency of light emitted in Balmer series of hydrogen spectrum, in Bohr's model, is: [1 mark]

- (A) $\frac{11}{9}$
(B) $\frac{9}{5}$
(C) $\frac{11}{7}$
(D) $\frac{16}{7}$

Correct Answer: (B) $\frac{9}{5}$

Solution:

Concept:

- In the Balmer series, an electron falls from an initial hydrogen level $n \geq 3$ to the final level $n_f = 2$.
- Bohr's model gives the emitted frequency $\nu = cR_H(1/2^2 - 1/n^2)$, where c is the speed of light and R_H is the Rydberg constant for hydrogen.

Step 1: Identify the limiting transitions The smallest energy drop is from $n = 3$ to $n = 2$; it gives the minimum frequency. The largest frequency is the series limit, approached as the initial level tends to infinity:

$$\nu_{\min} = cR_H \left(\frac{1}{4} - \frac{1}{9} \right) = \frac{5cR_H}{36}, \quad \nu_{\max} = \frac{cR_H}{4}.$$

Step 2: Divide maximum frequency by minimum frequency The common factor cR_H cancels:

$$\frac{\nu_{\max}}{\nu_{\min}} = \frac{1/4}{5/36} = \frac{9}{5}.$$

Fractions $11/9$, $11/7$ and $16/7$ do not follow from the Balmer endpoints. In particular, changing the final level would describe a different spectral series.

Step 3: Conclusion The required ratio is $\frac{9}{5}$. Hence, the correct option is (B).

Quick Tip to Solve: For a series ending at level m , minimum frequency starts at $m + 1$; maximum frequency is the limit from infinity.

13. At a certain temperature in an intrinsic semiconductor, the electrons and holes concentration is $1.5 \times 10^{16} \text{ m}^{-3}$. When it is doped with a trivalent dopant, hole concentration increases to $4.5 \times 10^{22} \text{ m}^{-3}$. In the doped semiconductor, the concentration of electrons (n_e) will be: [1 mark]

- (A) $3 \times 10^6 \text{ m}^{-3}$
- (B) $5 \times 10^7 \text{ m}^{-3}$
- (C) $5 \times 10^9 \text{ m}^{-3}$
- (D) $6.75 \times 10^{38} \text{ m}^{-3}$

Correct Answer: (C) $5 \times 10^9 \text{ m}^{-3}$

Solution:

Concept:

- In an intrinsic semiconductor, the electron and hole concentrations are equal: $n_e = n_h = n_i$, where n_i is the intrinsic carrier concentration.
- At a fixed temperature and thermal equilibrium, the mass-action law is $n_e n_h = n_i^2$. A trivalent acceptor increases hole concentration, making electrons the minority carriers.

Step 1: Use the equilibrium carrier product The original concentration of each carrier type is $n_i = 1.5 \times 10^{16} \text{ m}^{-3}$. Therefore the product in the doped semiconductor is

$$n_e n_h = (1.5 \times 10^{16})^2 = 2.25 \times 10^{32} \text{ m}^{-6}.$$

The given hole concentration is $n_h = 4.5 \times 10^{22} \text{ m}^{-3}$.

Step 2: Calculate the minority electron concentration Dividing the carrier product by the hole concentration gives

$$n_e = \frac{2.25 \times 10^{32}}{4.5 \times 10^{22}} = 0.5 \times 10^{10} = 5 \times 10^9 \text{ m}^{-3}.$$

Options (A) and (B) do not preserve this product. Option (D) multiplies the given concentrations instead of dividing the squared intrinsic concentration by the hole concentration, and incorrectly predicts an enormous minority population.

Step 3: Conclusion The electron concentration is $5 \times 10^9 \text{ m}^{-3}$. Hence, the correct option is (C).

Quick Tip to Solve: Square the intrinsic concentration first, then divide by the majority concentration; retain the exponents through the division.

14. If a p-n junction diode is reverse biased:

[1 mark]

- (A) the potential barrier is lowered.
- (B) the potential barrier remains unaffected.
- (C) the potential barrier is raised.
- (D) the current is mainly due to majority carriers.

Correct Answer: (C) the potential barrier is raised.

Solution:

Concept:

- The depletion region of a p-n junction contains fixed ions and has a built-in electric field that opposes further diffusion of majority carriers.

- Reverse bias connects the p-side to the negative terminal and the n-side to the positive terminal of the supply, reinforcing the junction barrier.

Step 1: Examine the depletion region under reverse bias The negative terminal attracts holes away from the junction on the p-side, while the positive terminal draws electrons away on the n-side. The depletion region widens and the applied reverse potential adds to the built-in potential barrier.

Step 2: Test the printed statements A lowered barrier, as stated in (A), describes forward bias. Statement (B) is incorrect because the external reverse voltage changes the barrier. The barrier is raised, as in (C). Below breakdown the small reverse current is mainly due to minority carriers swept across the junction, so the majority-carrier claim in (D) is incorrect.

Step 3: Conclusion Reverse bias raises the potential barrier of the p-n junction. Hence, the correct option is (C).

Quick Tip to Solve: Reverse bias draws majority carriers away from the junction: remember wider depletion region, higher barrier and minority reverse current.

15. A voltage signal is described by: $v = V_0$ for $0 \leq t \leq \frac{T}{2}$ and $v = 0$ for $\frac{T}{2} \leq t \leq T$ for a cycle. Its rms value is: [1 mark]

- (A) $\frac{V_0}{\sqrt{2}}$
 (B) V_0
 (C) $\frac{V_0}{2}$
 (D) $\sqrt{2} V_0$

Correct Answer: (A) $\frac{V_0}{\sqrt{2}}$

Solution:

Concept:

- The root-mean-square voltage is $V_{\text{rms}} = \sqrt{\frac{1}{T} \int_0^T v^2(t) dt}$, where T is the full cycle duration.
- RMS voltage is the equivalent constant voltage producing the same average heating in a resistor; it averages the square before taking the square root.

Step 1: Average the squared voltage over the cycle The signal has squared value V_0^2 during the first half-cycle and zero during the second. Consequently,

$$\langle v^2 \rangle = \frac{1}{T} \left[V_0^2 \frac{T}{2} + 0 \frac{T}{2} \right] = \frac{V_0^2}{2}.$$

The value assigned at the single switching instant does not affect this time average.

Step 2: Take the square root and compare alternatives For the positive pulse amplitude V_0 ,

$$V_{\text{rms}} = \sqrt{\frac{V_0^2}{2}} = \frac{V_0}{\sqrt{2}}.$$

Option (B) would apply if the pulse stayed on throughout the cycle. Option (C) is the ordinary mean voltage, not the RMS voltage. Option (D) exceeds the peak magnitude, which is impossible for this signal.

Step 3: Conclusion The RMS voltage is $\frac{V_0}{\sqrt{2}}$. Hence, the correct option is (A).

Quick Tip to Solve: For a pulse of amplitude V_0 present for fraction d of each cycle, use $V_{\text{rms}} = V_0\sqrt{d}$.

16. Assertion (A): The internal resistance of a cell is constant.

Reason (R): Ionic concentration of the electrolyte remains same during use of a cell.

[1 mark]

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
- (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
- (C) Assertion (A) is true, but Reason (R) is false.
- (D) Assertion (A) is false and Reason (R) is also false.

Correct Answer: (D) Assertion (A) is false and Reason (R) is also false.

Solution:

Concept:

- A cell's internal resistance is opposition to current within the cell, including conduction through its electrolyte.
- Internal resistance depends on electrolyte conductivity, temperature, electrode geometry and the cell's condition; it need not stay fixed throughout use.

Step 1: Evaluate the assertion The statement that the internal resistance of a cell is constant is false. A constant internal resistance is sometimes an approximation in a short circuit calculation, but it is not a universal property. Changes in electrolyte composition and the state of the electrodes during discharge can change the resistance.

Step 2: Evaluate the reason The statement that ionic concentration remains the same during use is also false. Chemical reactions accompany the delivery of current: reacting species are consumed and products form, changing the electrolyte's ionic composition and concentration as the cell is used.

Step 3: Check the explanation and select the code Since the reason itself is false, it cannot explain the false assertion as a true physical principle. Options (A), (B) and (C) all require the assertion to be true and are excluded. The supplied code for both statements being false is (D).

Step 4: Conclusion Both the assertion and the reason are false. Hence, the correct option is (D).

Quick Tip to Solve: Distinguish the constant internal resistance assumed in a circuit model from the changing electrolyte of a cell during prolonged use.

17. Assertion (A): When radius of a circular loop carrying a steady current is doubled, its magnetic moment becomes four times.

Reason (R): The magnetic moment of a circular loop carrying a steady current is proportional to the area of the loop. [1 mark]

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
(C) Assertion (A) is true, but Reason (R) is false.
(D) Assertion (A) is false and Reason (R) is also false.

Correct Answer: (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

Solution:

Concept:

- The magnetic dipole moment of a single-turn current loop is $m = IA$, where I is the steady current and A is the enclosed area.
- For a circular loop of radius r , $A = \pi r^2$, so $m = I\pi r^2$.

Step 1: Evaluate the assertion Keep the current unchanged and replace the radius by $2r$. The new moment is

$$m' = I\pi(2r)^2 = 4I\pi r^2 = 4m.$$

Therefore, the assertion that the magnetic moment becomes four times is true.

Step 2: Evaluate the reason The expression $m = IA$ shows that, for a fixed steady current, magnetic moment is directly proportional to loop area. Thus the reason is also true.

Step 3: Check the explanatory link Doubling the radius quadruples the area, and proportionality to area therefore quadruples the moment. The reason directly explains the assertion. Option (B) wrongly denies this link, while (C) and (D) wrongly declare one or both statements false.

Step 4: Conclusion Both Assertion (A) and Reason (R) are true, and R correctly explains A. Hence, the correct option is (A).

Quick Tip to Solve: For a circular loop at constant current, use $m'/m = (r'/r)^2$; the moment follows area, not circumference.

18. Assertion (A): The nucleus 7_3X is more stable than the nucleus 4_3Y .

Reason (R): 7_3X contains more number of protons.

[1 mark]

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
(C) Assertion (A) is true, but Reason (R) is false.
(D) Assertion (A) is false and Reason (R) is also false.

Correct Answer: (C) Assertion (A) is true, but Reason (R) is false.

Solution:

Concept:

- In A_ZX , the proton number is Z , the nucleon number is A , and the neutron number is $N = A - Z$.
- For light nuclei, a neutron-to-proton ratio near unity favours stability; a large neutron deficiency makes a nucleus unstable.

Step 1: Evaluate the assertion For the two printed nuclei, count protons and neutrons separately:

$$\begin{aligned} {}^7_3X : Z = 3, \quad N = 7 - 3 = 4, \\ {}^4_3Y : Z = 3, \quad N = 4 - 3 = 1. \end{aligned}$$

The first is the stable lithium-7 nucleus, with four neutrons and three protons. The second is extremely neutron deficient and unstable. Thus the assertion that the first nucleus is more stable is true.

Step 2: Evaluate the reason Both nuclei have the same atomic number, $Z = 3$, and therefore both contain three protons. The greater mass number of 7_3X comes from additional neutrons, not additional protons. The reason is false.

Step 3: Check the explanatory link A false claim about proton number cannot explain the stability comparison. Options (A) and (B) incorrectly treat the reason as true; option (D) incorrectly treats the assertion as false.

Step 4: Conclusion The assertion is true, but the reason is false. Hence, the correct option is (C).

Quick Tip to Solve: Read the lower index for protons and subtract it from the upper index for neutrons before comparing nuclear stability.

Section B (14 marks)

19. A wire of length l is in the form of a circular loop A of one turn. This loop is reshaped into loop B of three turns. Find the ratio of the magnetic fields at the centres of loop A and loop B for the same current through them.

[2 marks]

Solution:

Concept:

- The magnetic field at the centre of a circular coil of N closely wound turns, each of radius r , carrying current I , is $B = \mu_0 NI / (2r)$.
- The fixed wire length is the sum of the circumferences of all turns: $l = N(2\pi r)$. Increasing the number of turns therefore reduces each turn's radius.

Step 1: Relate the radii to the fixed wire length Let the radii of loops A and B be r_A and r_B , respectively. Neglect the small lengths of connecting ends and the thickness of the wire. For one turn and three turns of the same wire,

$$l = 2\pi r_A = 3(2\pi r_B), \quad r_A = 3r_B.$$

Hence each turn in B has one-third the radius of the original single-turn loop.

Step 2: Compare the central fields The field of each turn adds in the same direction at the common centre. The current is identical in both arrangements, so

$$B_A = \frac{\mu_0 I}{2r_A}, \quad B_B = \frac{3\mu_0 I}{2r_B}.$$

Forming the ratio cancels the current and permeability:

$$\frac{B_A}{B_B} = \frac{r_B}{3r_A} = \frac{r_B}{3(3r_B)} = \frac{1}{9}.$$

Step 3: Conclusion The required ratio is $B_A : B_B = 1 : 9$. The three-turn coil produces nine times the field of the original loop.

Quick Tip to Solve: With fixed wire length and fixed current, $r \propto 1/N$, so the central field scales as N^2 .

20. What is meant by the term ‘displacement current’? Briefly explain how this current is different from a conduction current. [2 marks]

Solution:

Concept:

- Maxwell introduced displacement current to account for the magnetic field associated with a time-varying electric field.
- In vacuum, displacement current through a surface is $I_d = \varepsilon_0 d\Phi_E/dt$, where $\Phi_E = \int \vec{E} \cdot d\vec{A}$ is electric flux through that surface.

Step 1: Define displacement current Displacement current is the effective current associated with changing electric flux. It contributes to the magnetic field just as conduction current does in the Ampere-Maxwell law:

$$\oint \vec{B} \cdot d\vec{l} = \mu_0(I_c + I_d).$$

For example, while a capacitor is charging, the electric field between its plates changes. Hence there is a displacement current in the gap, even if the gap is vacuum and no charges travel from one plate to the other through it.

Step 2: Distinguish it from conduction current Conduction current I_c is caused by the actual directed motion of free charge carriers, such as electrons drifting along a metallic wire. Displacement current arises from a changing electric field and does not require free charges to cross the region. It can therefore exist in vacuum or an insulating capacitor gap. In a charging circuit, conduction current flows in the wires and an equal displacement current occurs between the plates, maintaining continuity of the total current.

Step 3: Conclusion Displacement current is $I_d = \varepsilon_0 d\Phi_E/dt$. Its origin is time-varying electric flux,

whereas conduction current originates in the transport of free charge.

Quick Tip to Solve: Use a charging capacitor as the comparison: charge flows through the wires, while changing electric flux supplies current continuity across the gap.

21(a). State Huygens' principle. How did Huygens explain the absence of the backwave? [2 marks]

Solution:

Concept:

- A wavefront is a surface joining points of the same phase. Rays are normal to the wavefront in an isotropic medium.
- Huygens' construction obtains a later wavefront as the forward envelope of secondary wavelets emitted by points on the original wavefront.

Step 1: State Huygens' principle Every point on a given wavefront acts as a source of secondary spherical wavelets. These wavelets spread through the medium with the speed of light in that medium. After a time interval Δt , each wavelet has radius $v\Delta t$, where v is the wave speed. The common forward tangent surface, or envelope, of these wavelets gives the new position of the wavefront.

Step 2: Explain the absence of a backwave If all directions were assigned equal amplitude, the construction would also permit a backward envelope. To exclude it, Huygens assumed that the secondary wavelets have maximum amplitude in the forward direction and zero amplitude in the backward direction. Thus their backward envelope does not represent a wave carrying energy. This directional assumption selects only the forward envelope as the physical new wavefront; the elementary geometrical construction alone does not establish it.

Step 3: Conclusion Every wavefront point produces secondary wavelets, whose forward envelope forms the next wavefront. The absence of a backwave is accounted for by assuming zero secondary-wavelet amplitude in the backward direction.

Quick Tip to Solve: State both parts of the principle: secondary sources and their forward envelope; then explicitly say that backward amplitude is taken to be zero.

OR

21(b). Use Huygens' principle to show reflection/refraction of a plane wave by (i) concave mirror, and (ii) a convex lens. [2 marks]

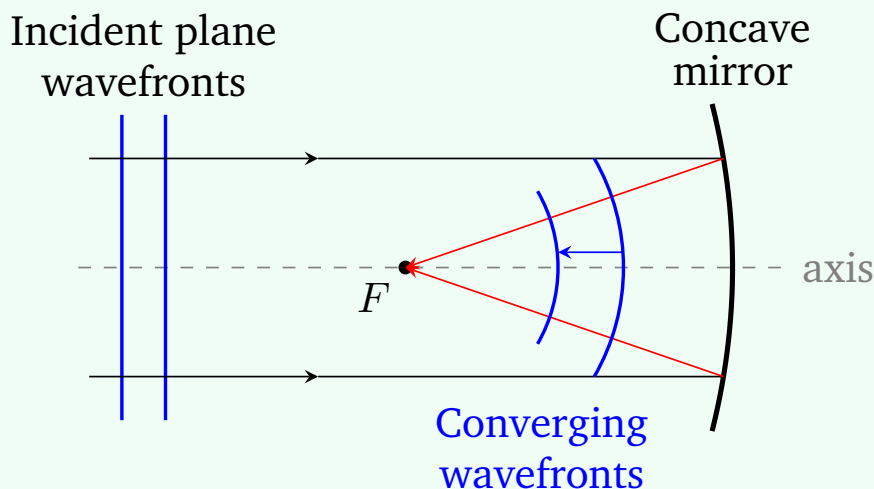
Solution:

Concept:

- Each point of a wavefront produces secondary wavelets. Their forward common envelope is the next wavefront, and rays are perpendicular to it.
- A plane wavefront represents parallel rays. A converging spherical wavefront has its centre at the point towards which its rays travel.

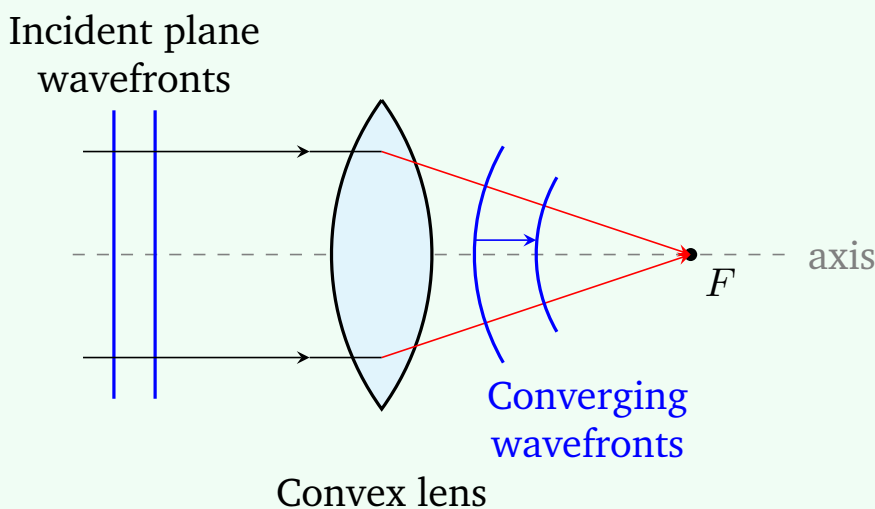
Step 1: Construct reflection at a concave mirror Let a plane wavefront fall normally on the principal axis of a concave mirror. Its outer portions meet the mirror before its central portion. The illuminated points of the mirror emit reflected secondary wavelets; those excited earlier have expanded farther

at a given instant. The common reflected envelope is a converging spherical wavefront, in the paraxial approximation. Its centre is the principal focus F , in front of the mirror. Successive reflected wavefronts contract towards F , and their normals give the reflected rays.



Concave mirror: blue arcs are reflected wavefronts converging towards F ; arrows show ray directions.

Step 2: Construct refraction through a convex lens A plane wavefront incident along the principal axis traverses a greater thickness of glass near the centre of a convex lens than at its edges. Since light travels more slowly in glass, the central portion is delayed more. Constructing the envelope of the secondary wavelets emerging from the lens therefore gives a converging spherical wavefront. Its centre is the second principal focus F , on the other side of the lens. The normals to this envelope meet at F ; the sketches show this for paraxial rays.



Convex lens: blue arcs are transmitted wavefronts converging towards the focus F .

Step 3: Conclusion A concave mirror converts an incident plane wavefront into a reflected converging spherical wavefront. A convex lens produces a transmitted converging spherical wavefront. Both converge at their respective principal foci.

Quick Tip to Solve: Draw wavefronts perpendicular to rays: after either converging optical element, successive spherical wavefronts shrink towards its focus.

22. The refractive indices of two media A and B are 2 and $\sqrt{2}$ respectively. What is the critical angle for their interface? [2 marks]

Solution:

Concept:

- The critical angle C is the angle of incidence in the optically denser medium for which the refracted ray in the rarer medium makes 90° with the normal.
- Snell's law is $n_1 \sin i = n_2 \sin r$, where n_1, n_2 are the refractive indices and i, r are measured from the normal.

Step 1: Identify the direction of incidence The indices are $n_A = 2$ and $n_B = \sqrt{2}$. Since $2 > \sqrt{2}$, medium A is optically denser than medium B . The critical angle therefore refers to light travelling from A towards B . Light travelling in the opposite direction cannot undergo total internal reflection at this interface.

Step 2: Apply Snell's law at grazing refraction Set the refracted angle equal to 90° and the incident angle equal to C :

$$n_A \sin C = n_B \sin 90^\circ,$$
$$\sin C = \frac{n_B}{n_A} = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}.$$

We take the physically relevant acute angle of incidence; the sine ratio must be less than one.

Step 3: Conclusion The critical angle is $C = 45^\circ$, measured in medium A from the normal to the interface.

Quick Tip to Solve: Put the smaller refractive index above the larger one in $\sin C = n_{\text{rarer}}/n_{\text{denser}}$; a ratio above one signals that the direction was reversed.

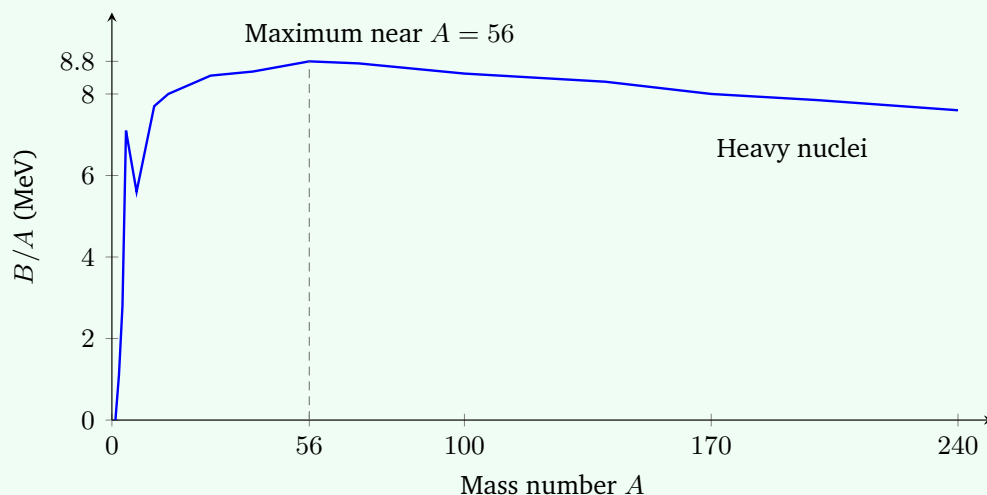
23(a). Draw a graph showing the variation of binding energy per nucleon as a function of mass number A . The binding energy per nucleon for heavy nuclei ($A > 170$) decreases with the increase in mass number. Explain. [2 marks]

Solution:

Concept:

- Binding energy per nucleon is B/A , where B is the total binding energy of a nucleus and A is its mass number. It measures the average energy needed to remove a nucleon.
- Nuclear attraction is short-ranged and saturates: each nucleon interacts strongly mainly with its near neighbours. Electrostatic repulsion acts between every pair of protons over a longer range.

Step 1: Sketch the binding-energy curve Plot mass number A horizontally and B/A in MeV vertically. The curve rises rapidly for light nuclei, with some fluctuations, and reaches a broad maximum of about 8.8 MeV near $A \simeq 56$. It then decreases slowly towards about 7.6 MeV for the heaviest nuclei. The schematic graph shows this overall trend.



Schematic variation of binding energy per nucleon with mass number.

Step 2: Explain the decrease for heavy nuclei When a heavy nucleus grows, its nucleons do not gain strong attraction from all the added nucleons because the nuclear force has a short range. The attractive binding contribution per nucleon is therefore approximately saturated. However, the increasing number of protons raises the long-range Coulomb repulsion. This repulsion reduces the net binding energy per nucleon, explaining the declining curve for $A > 170$.

Step 3: Conclusion The graph has a broad maximum near the iron region and declines for heavy nuclei because increasing proton repulsion competes with the saturated short-range nuclear attraction.

Quick Tip to Solve: Label both axes and the maximum near $A = 56$; explain the heavy-nucleus decline using nuclear-force saturation together with increasing Coulomb repulsion.

OR

23(b). Using Bohr's postulates, obtain the expression for radius of n^{th} stable orbit in a hydrogen atom. [2 marks]

Solution:

Concept:

- In Bohr's model, an electron can revolve in certain stationary circular orbits without radiating energy. Its angular momentum is quantised: $m_e v_n r_n = n\hbar$, where $\hbar = h/(2\pi)$ and $n = 1, 2, 3, \dots$
- For hydrogen, the Coulomb attraction between the proton and electron supplies the centripetal force. Here e is the magnitude of electron charge, m_e its mass, and ϵ_0 the vacuum permittivity.

Step 1: Write the radial force equation Treat the much heavier proton as fixed. For an electron of speed v_n in a circular orbit of radius r_n , the inward electrostatic force equals the required centripetal force:

$$\frac{e^2}{4\pi\epsilon_0 r_n^2} = \frac{m_e v_n^2}{r_n}.$$

Multiplying through by r_n^2 gives $m_e v_n^2 r_n = e^2/(4\pi\epsilon_0)$.

Step 2: Use angular-momentum quantisation Bohr's condition gives $v_n = n\hbar/(m_e r_n)$. Substitute this

into the force equation to eliminate the speed:

$$m_e \left(\frac{n\hbar}{m_e r_n} \right)^2 r_n = \frac{e^2}{4\pi\epsilon_0},$$
$$\frac{n^2 \hbar^2}{m_e r_n} = \frac{e^2}{4\pi\epsilon_0}.$$

Thus the permitted radii increase as the square of the positive integer n , rather than forming a continuous range.

Step 3: Conclusion The radius of the n^{th} stable orbit in hydrogen is

$$r_n = \frac{4\pi\epsilon_0 \hbar^2}{m_e e^2} n^2 = \frac{\epsilon_0 h^2}{\pi m_e e^2} n^2 = a_0 n^2,$$

where $a_0 \simeq 5.29 \times 10^{-11} \text{ m}$ is the Bohr radius.

Quick Tip to Solve: First express the speed from $m_e v r = n\hbar$, then substitute into the Coulomb force equation; this makes the n^2 dependence immediate.

24. Explain the roles of diffusion current and drift current in the formation of the depletion layer in a p-n junction diode. [2 marks]

Solution:

Concept:

- Diffusion is the movement of mobile carriers from higher to lower concentration. Across a newly formed p-n junction, electrons diffuse from the n-side to the p-side and holes diffuse in the opposite direction.
- Drift is carrier motion driven by an electric field. The uncovered fixed ions near a junction establish an electric field and a potential barrier that oppose further majority-carrier diffusion.

Step 1: Explain how diffusion creates the depletion layer The concentration gradient drives majority electrons and holes across the junction, where they recombine. The two carrier flows produce conventional diffusion current from p to n. Electrons leaving the n-side uncover immobile positively charged donor ions; holes leaving the p-side uncover immobile negatively charged acceptor ions. The region around the junction consequently becomes depleted of mobile majority carriers and is called the depletion layer. The ions remain fixed in the crystal lattice.

Step 2: Explain how drift establishes equilibrium The fixed positive ions on the n-side and negative ions on the p-side produce an electric field directed from n to p. The associated barrier opposes further majority-carrier diffusion. This field also sweeps thermally generated minority electrons from p to n and minority holes from n to p, producing conventional drift current from n to p, opposite to the diffusion current. As the depletion layer forms, the growing barrier reduces diffusion until the two currents balance.

Step 3: Conclusion Diffusion and recombination form the depletion layer; its electric field produces an opposing drift current. In an unbiased junction at thermal equilibrium, $I_{\text{diffusion}} = I_{\text{drift}}$ in magnitude, so the net current is zero and the depletion width is stable.

Quick Tip to Solve: Track the fixed ions first: n-side positive, p-side negative. Their field points n to p, which is also the direction of conventional drift current.

25. Explain the property of a p-n junction which makes it suitable for rectifying alternating voltages. Differentiate between a half-wave and a full-wave rectifier. [2 marks]

Solution:

Concept:

- Rectification converts an alternating voltage into a unidirectional, pulsating output using the asymmetric conduction of a p-n junction diode.
- A diode offers low resistance in forward bias and very high resistance in reverse bias, provided reverse breakdown is avoided.

Step 1: Explain the rectifying property Forward bias reduces the junction potential barrier and narrows the depletion layer, so appreciable current flows. Reverse bias increases the barrier and widens the depletion layer, so only a negligible reverse current flows. The junction therefore conducts effectively in one direction. Connecting it suitably in an alternating-voltage circuit allows current through the load in a fixed direction.

Step 2: Compare the two rectifiers A half-wave rectifier uses only one half-cycle of the input: the diode conducts during the forward-biased half-cycle and blocks the other half-cycle, when the load output is approximately zero. There is one output pulse per input cycle, so an input frequency f produces ripple frequency f .

A full-wave rectifier uses both half-cycles. Different diodes conduct in successive half-cycles while keeping the load current in the same direction. There are two output pulses per input cycle, giving ripple frequency $2f$. Its output is smoother and has a larger average value for the same peak load voltage under ideal conditions.

Step 3: Conclusion The p-n junction's low forward resistance and high reverse resistance make it a rectifier. A half-wave rectifier uses one half-cycle; a full-wave rectifier uses both, with unidirectional load current in each case.

Quick Tip to Solve: Count output pulses in one input cycle: one means half-wave and ripple f ; two mean full-wave and ripple $2f$.

Section C (15 marks)

26. A potential difference V is applied across a conductor of length l and uniform cross-section area A . How will the (i) electric field E , (ii) drift velocity v_d , and (iii) current density j be affected when (a) V is doubled and (b) l is halved (keeping other factors constant)? [3 marks]

Solution:

Concept:

- In a uniform conductor, the magnitude of the electric field is $E = V/l$, where V is the applied potential difference and l is the conductor's length.
- The electron drift speed is $v_d = e\tau E/m$, where e is the magnitude of electron charge, m its mass and τ the relaxation time. Electron drift is opposite to the field.
- The current-density magnitude is $j = nev_d = \sigma E$, where n is the free-electron number density and $\sigma = ne^2\tau/m$ is conductivity. At unchanged temperature and material, n , τ and σ stay constant.

Step 1: Express all quantities in terms of voltage and length The initial magnitudes are

$$E = \frac{V}{l}, \quad v_d = \frac{e\tau V}{m}, \quad j = \frac{ne^2\tau V}{ml}.$$

Thus all three are directly proportional to V/l . Although $I = jA$, the fixed cross-section A does not enter these three expressions explicitly. We treat the two stated changes separately, starting from the same original conductor.

Step 2: Double the potential difference in part (a) With $V' = 2V$ and $l' = l$,

$$E' = \frac{2V}{l} = 2E, \quad v'_d = \frac{e\tau(2E)}{m} = 2v_d, \quad j' = \sigma(2E) = 2j.$$

A doubled field doubles the force on each electron and hence its average drift speed for the same relaxation time. The carrier density is unchanged, so the current density doubles as well.

Step 3: Halve the conductor length in part (b) Here keep the original potential difference V fixed and take $l' = l/2$. Therefore,

$$E' = \frac{V}{l/2} = 2E, \quad v'_d = 2v_d, \quad j' = 2j.$$

The same potential drop now occurs across half the distance. Equivalently, $R = \rho l/A$ is halved, so $I = V/R$ doubles; since A is unchanged, $j = I/A$ doubles. This independently checks the result.

Step 4: Conclusion In each separate case, (a) doubling V or (b) halving l , the electric-field magnitude, drift speed and current-density magnitude all become twice their initial values. Their directions remain unchanged.

Quick Tip to Solve: First compare V/l before and after the change: at fixed material and temperature, E , v_d and j all scale by exactly that factor.

27. What is meant by the term ‘mutual inductance’ of a pair of coils? Obtain an expression for the mutual inductance of two long coaxial solenoids, each of length l but having different number of turns N_1 and N_2 and radii r_1 and r_2 ($r_2 > r_1$). [3 marks]

Solution:

Concept:

- Mutual inductance M is the magnetic flux linkage with one coil per unit current in the other coil: $N_2\Phi_{21} = MI_1$. Here Φ_{21} is the flux per turn of coil 2 due to current I_1 in coil 1.
- A changing current in coil 1 induces emf $\varepsilon_2 = -M dI_1/dt$ in coil 2. The minus sign expresses Lenz’s law; the SI unit of M is henry (H).
- For a long air-core solenoid of length l , N turns and current I , the internal field is $B = \mu_0 NI/l$, while the field outside is negligible. End effects are neglected.

Step 1: State the physical meaning of mutual inductance Mutual inductance measures the magnetic coupling between two coils. A pair has mutual inductance 1 H if a current changing at 1 A s^{-1} in either coil induces an emf of magnitude 1 V in the other. For fixed geometry in a linear medium, the flux linkage is proportional to the current producing it.

Step 2: Find the flux through the smaller solenoid Let solenoid 1 be the inner solenoid of radius r_1 and N_1 turns, and solenoid 2 the outer solenoid of radius $r_2 > r_1$ and N_2 turns. They have the same length l and share a common axis. Pass current I_2 through the outer solenoid. The field over the inner solenoid is

$$B_2 = \mu_0 \frac{N_2}{l} I_2.$$

The area of each inner turn is $A_1 = \pi r_1^2$, with its normal parallel to the field. Its flux is therefore

$$\Phi_{12} = B_2 A_1 = \mu_0 \frac{N_2 I_2}{l} \pi r_1^2.$$

Step 3: Calculate the total flux linkage There are N_1 inner turns, so their total flux linkage is

$$N_1 \Phi_{12} = \frac{\mu_0 N_1 N_2 \pi r_1^2}{l} I_2.$$

Divide by I_2 according to the definition of mutual inductance. Interchanging the coils gives the same result: the inner solenoid’s field occupies only its own cross-sectional area, so the linked area remains πr_1^2 , even for the larger outer turns. The larger radius r_2 therefore does not determine the common flux area.

Step 4: Conclusion For the two long air-core coaxial solenoids,

$$M_{12} = M_{21} = \frac{\mu_0 N_1 N_2 \pi r_1^2}{l}.$$

Mutual inductance is the flux linkage in either solenoid per unit current in the other. For a uniform magnetic medium, replace μ_0 by its permeability μ .

Quick Tip to Solve: Use the smaller solenoid’s area for the common magnetic flux, then multiply by both turn counts and divide by the common length.

28(a). An ac source $V = V_m \sin \omega t$ is connected across an ideal capacitor. Derive the expression for the (i) current flowing in the circuit, and (ii) reactance of the capacitor. Plot a graph of current i versus ωt . [3 marks]

Solution:

Concept:

- For an ideal capacitor of capacitance C , the charge is $q = Cv$ and the current is $i = dq/dt$, where v is the instantaneous applied voltage.
- Capacitive reactance is the ratio of voltage amplitude to current amplitude: $X_C = V_m/I_m$.

Step 1: Differentiate the capacitor charge With peak voltage V_m , angular frequency ω , and constant capacitance,

$$q(t) = CV_m \sin \omega t,$$

$$i(t) = \frac{dq}{dt} = \omega CV_m \cos \omega t.$$

Defining the peak current as $I_m = \omega CV_m$, we can write

$$i(t) = I_m \cos \omega t = I_m \sin \left(\omega t + \frac{\pi}{2} \right).$$

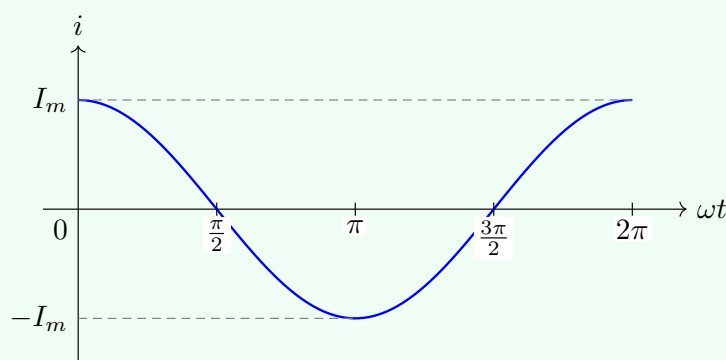
The current leads the applied voltage by $\pi/2$, or one quarter-cycle.

Step 2: Obtain the capacitive reactance Divide the voltage amplitude by the current amplitude found above:

$$X_C = \frac{V_m}{I_m} = \frac{V_m}{\omega CV_m} = \frac{1}{\omega C} = \frac{1}{2\pi f C}.$$

Here f is the frequency in hertz. Reactance, measured in ohms, decreases when frequency or capacitance increases.

Step 3: Plot the current over one cycle The current is a cosine wave. It starts at $+I_m$ when $\omega t = 0$, vanishes at $\pi/2$, reaches $-I_m$ at π , vanishes again at $3\pi/2$, and returns to $+I_m$ at 2π .



Current through the ideal capacitor as a function of phase.

Step 4: Conclusion The current is $i = \omega CV_m \cos \omega t$, leading voltage by $\pi/2$, and the capacitive reactance is $X_C = 1/(\omega C)$.

Quick Tip to Solve: Differentiate the sine voltage to get a cosine current; start the current graph at its positive maximum, not at zero.

28(b). A series combination of an inductor L , a capacitor C and a resistor R is connected across an ac source of voltage in a circuit. Obtain an expression for the average power consumed by the circuit. Find power factor for (i) purely inductive circuit, and (ii) purely resistive circuit. [3 marks]

Solution:

Concept:

- Instantaneous electric power is $p = vi$. Average power is obtained by averaging this product over one complete ac cycle.
- For a series LCR circuit, $Z = \sqrt{R^2 + (X_L - X_C)^2}$, with $X_L = \omega L$, $X_C = 1/(\omega C)$, and power factor $\cos \phi = R/Z$.

Step 1: Express voltage and current with their phase difference Let the applied voltage be $v = V_m \sin \omega t$ and let the steady-state current be $i = I_m \sin(\omega t - \phi)$. The signed phase angle satisfies $\tan \phi = (X_L - X_C)/R$; a negative value represents a current that leads the voltage. Multiplying the instantaneous voltage and current gives

$$p = V_m I_m \sin \omega t \sin(\omega t - \phi).$$

Use $2 \sin A \sin B = \cos(A - B) - \cos(A + B)$ to separate the constant and oscillating contributions:

$$p = \frac{V_m I_m}{2} [\cos \phi - \cos(2\omega t - \phi)].$$

Step 2: Average the power over a complete cycle The average of $\cos(2\omega t - \phi)$ over a source cycle is zero. Hence

$$P_{\text{av}} = \frac{V_m I_m}{2} \cos \phi = V_{\text{rms}} I_{\text{rms}} \cos \phi,$$

because $V_{\text{rms}} = V_m/\sqrt{2}$ and $I_{\text{rms}} = I_m/\sqrt{2}$. With $V_{\text{rms}} = I_{\text{rms}} Z$ and $\cos \phi = R/Z$, this also gives

$$P_{\text{av}} = I_{\text{rms}}^2 R.$$

Only the resistance dissipates energy on average. Ideal inductors and capacitors store energy during part of a cycle and return it during another part.

Step 3: Evaluate the two limiting power factors (i) In a purely inductive circuit, current lags voltage by $\phi = \pi/2$. Therefore $\cos \phi = 0$, so average power consumption is zero even though current flows. (ii) In a purely resistive circuit, current and voltage are in phase, so $\phi = 0$ and $\cos \phi = 1$. Its average power is $V_{\text{rms}} I_{\text{rms}}$.

Step 4: Conclusion The average power is $P_{\text{av}} = V_{\text{rms}} I_{\text{rms}} \cos \phi = I_{\text{rms}}^2 R$. The power factor is zero for a purely inductive circuit and unity for a purely resistive circuit.

Quick Tip to Solve: Average the product of voltage and current, then use rms values; the factor of one-half is already included in their product.

29. Calculate the wavelength of de Broglie waves associated with a proton having $\left(\frac{500}{1.673}\right)$ eV energy. How will the wavelength be affected for an alpha particle having the same energy? [3 marks]

Solution:

Concept:

- A particle of momentum p has de Broglie wavelength $\lambda = h/p$, where h is Planck's constant. For non-relativistic kinetic energy K , $p = \sqrt{2mK}$.
- At the same kinetic energy, $\lambda \propto 1/\sqrt{m}$. An alpha particle has approximately four times the mass of a proton, so its wavelength can be found by taking a mass ratio.

Step 1: Convert the proton's kinetic energy into joules The printed energy is $K = (500/1.673)$ eV, approximately 299 eV. Using $1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$,

$$K = \frac{500}{1.673} \times 1.6 \times 10^{-19} \text{ J}.$$

This is much smaller than the proton rest energy, so the non-relativistic relation $K = p^2/(2m_p)$ applies. Take $m_p = 1.673 \times 10^{-27} \text{ kg}$ and $h = 6.63 \times 10^{-34} \text{ J s}$.

Step 2: Calculate momentum and wavelength Substitute the mass and energy before taking the square root:

$$\begin{aligned} p_p &= \sqrt{2(1.673 \times 10^{-27}) \frac{500}{1.673} (1.6 \times 10^{-19})} \\ &= \sqrt{1.6 \times 10^{-43}} \text{ kg m s}^{-1} \\ &= 4.0 \times 10^{-22} \text{ kg m s}^{-1}. \end{aligned}$$

The factor 1.673 cancels exactly in this substitution. Therefore

$$\lambda_p = \frac{6.63 \times 10^{-34}}{4.0 \times 10^{-22}} = 1.6575 \times 10^{-12} \text{ m}.$$

Step 3: Compare the alpha particle at the same energy Since $m_\alpha \simeq 4m_p$ and the kinetic energy is unchanged,

$$\frac{\lambda_\alpha}{\lambda_p} = \sqrt{\frac{m_p K}{m_\alpha K}} = \frac{1}{2}.$$

Thus the alpha particle has twice the proton's momentum and half its wavelength. Its charge does not enter this comparison because equal kinetic energies are specified, rather than acceleration through the same potential difference.

Step 4: Conclusion The proton wavelength is $\lambda_p \simeq 1.66 \times 10^{-12} \text{ m}$. For an alpha particle at the same kinetic energy, the wavelength is halved: $\lambda_\alpha \simeq 8.29 \times 10^{-13} \text{ m}$.

Quick Tip to Solve: At equal kinetic energy use the square root of the inverse mass ratio; cancel the printed factor 1.673 before evaluating the proton momentum.

30(a). (i) Prove that the nuclear density is same for all nuclei.

(ii) Draw a plot of potential energy of a pair of nucleons as a function of their separation. Draw two inferences from this plot. [3 marks]

Solution:

Concept:

- The radius of a nucleus of mass number A is $R = R_0 A^{1/3}$, where $R_0 \simeq 1.2 \times 10^{-15}$ m. Its mass is approximately $M = Am_N$, with m_N the mean nucleon mass.
- For a central interaction, the radial force is $F_r = -dU/dr$. A negative potential well represents binding; the slope of the potential determines whether the force is attractive or repulsive.

Step 1: (i) Calculate nuclear density Treat the nucleus as a sphere. Its volume is

$$V = \frac{4\pi R^3}{3} = \frac{4\pi}{3} R_0^3 A.$$

Hence its mean density is

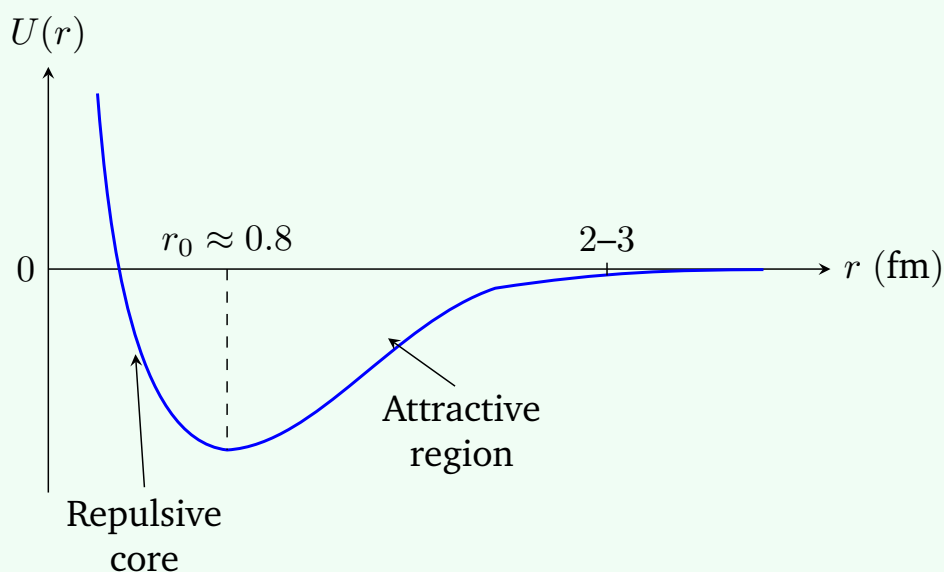
$$\rho = \frac{M}{V} = \frac{Am_N}{(4\pi/3)R_0^3 A} = \frac{3m_N}{4\pi R_0^3}.$$

The mass number cancels because both mass and volume are proportional to A . Using $m_N \simeq 1.67 \times 10^{-27}$ kg, this gives $\rho \simeq 2.3 \times 10^{17}$ kg m⁻³. Thus, within the usual nuclear-radius model, all nuclei have approximately the same mean density.

Step 2: (ii) Read the nucleon potential-energy curve Take $U = 0$ at a large separation. The schematic curve rises sharply at very small r , has a negative minimum near $r_0 \simeq 0.8$ fm, and approaches zero beyond a few femtometres. Here 1 fm = 10⁻¹⁵ m.

First, for $r < r_0$, the slope is negative, so $F_r = -dU/dr > 0$: the force is strongly repulsive. This repulsive core prevents nucleons from collapsing onto one another. For separations just beyond r_0 , the slope is positive and the force is attractive, accounting for nuclear binding.

Second, the curve becomes nearly flat at zero for separations of a few femtometres. Therefore $dU/dr \simeq 0$ there: the nuclear force is of very short range and becomes negligible outside that range. These statements describe the nuclear interaction, with any proton-proton Coulomb contribution omitted.



Schematic nuclear potential: a steep repulsive core and a short-range attractive well.

Step 3: Conclusion Nuclear density is $\rho = 3m_N/(4\pi R_0^3)$, independent of A . The potential curve shows a repulsive core and an attractive binding region, with the nuclear force negligible beyond a few femtometres.

Quick Tip to Solve: Cube $R_0 A^{1/3}$ before calculating density; read attraction from the positive slope of $U(r)$, using $F_r = -dU/dr$.

OR

30(b). (i) Draw a graph to show the variation of the number of scattered particles detected (N) in Geiger-Marsden experiment as a function of scattering angle (θ).

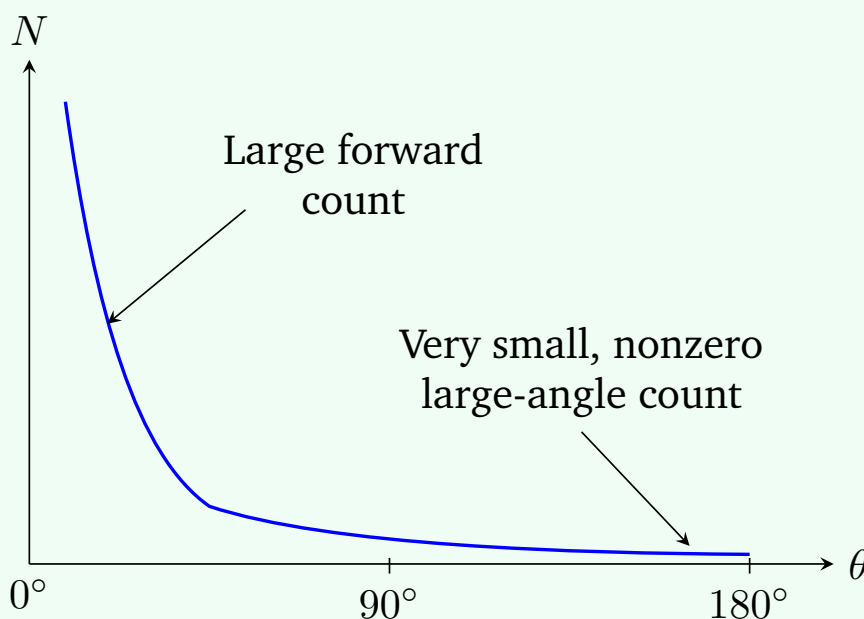
(ii) Discuss briefly two conclusions that can be drawn from this graph and how they lead to the discovery of nucleus in an atom. [3 marks]

Solution:

Concept:

- In the Geiger-Marsden experiment, a narrow beam of energetic α -particles strikes a thin metal foil. A movable detector records the number N scattered through angle θ in a fixed time.
- Coulomb repulsion between the positive α -particle and the concentrated positive nuclear charge produces the scattering. Under identical detector conditions, Rutherford scattering gives $N \propto 1/\sin^4(\theta/2)$.

Step 1: (i) Plot the variation of the detected number Put scattering angle θ on the horizontal axis and detected number N on the vertical axis. The count is very large close to the forward direction, decreases steeply as θ increases, and remains small but nonzero at large angles, including nearly backward scattering. The sketch is qualitative; it does not give a finite value of the ideal scattering formula at exactly $\theta = 0$, where the direct beam is present.



Qualitative scattering count: many forward events and very few large-angle events.

Step 2: (ii) Infer the large empty region in an atom The large count near the forward direction shows that most incident α -particles pass through the foil without appreciable deflection. Consequently,

most trajectories do not encounter a region of strong repulsive force. Most of the atomic volume is therefore empty space; the positive charge cannot occupy the entire atomic volume with a high density.

Step 3: (ii) Infer a small, massive, positively charged nucleus A very small fraction of particles suffer large deflections, and some return almost towards the source. Such strong repulsion requires a highly concentrated positive charge. The scattering centre must also be massive compared with an α -particle, so that it can reverse the particle's motion without itself recoiling excessively. Electrons are too light to produce this effect.

Since these close encounters are rare, the strongly repelling region occupies only a tiny part of the atom. Rutherford therefore proposed a very small central nucleus containing the whole positive charge and almost the entire atomic mass, with electrons occupying the surrounding region. Together, the dominant forward count and rare backward count establish this nuclear picture.

Step 4: Conclusion The N -versus- θ curve falls steeply with scattering angle. Most atoms consist mainly of empty space, while their positive charge and almost all their mass reside in a tiny central nucleus.

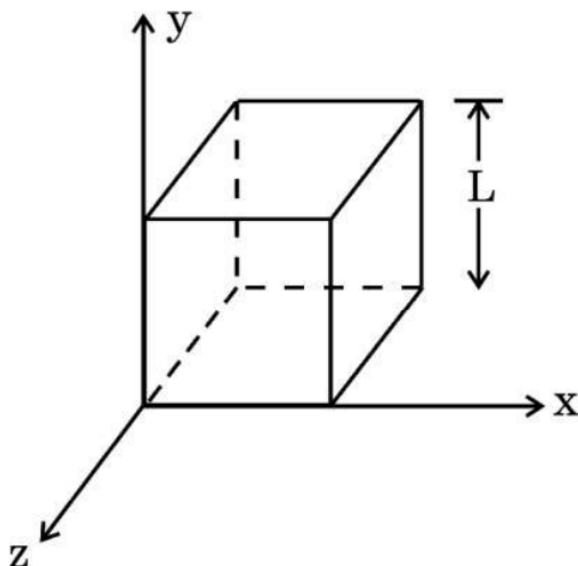
Quick Tip to Solve: Link the large forward count to empty space and the rare backward events to a tiny, dense, positively charged nucleus.

Section D (15 marks)

31(a). (i) Define electric flux and write its SI unit.

(ii) Use Gauss' law to obtain the expression for the electric field due to a uniformly charged infinite plane sheet.

(iii) A cube of side L is kept in space, as shown in the figure. An electric field $\vec{E} = (Ax + B)\hat{i}$ N/C exists in the region. Find the net charge enclosed by the cube. [5 marks]



Solution:

Concept:

- Electric flux measures the electric field passing through an oriented surface: $\Phi_E = \int_S \vec{E} \cdot d\vec{S}$, where $d\vec{S}$ is normal to the surface.

- Gauss' law states $\oint_S \vec{E} \cdot d\vec{S} = Q_{\text{enc}}/\epsilon_0$, where Q_{enc} is the net charge enclosed and ϵ_0 is the permittivity of free space.
- Surface charge density is $\sigma = Q/S$. Symmetry makes the field of an infinite uniformly charged plane normal to the plane, with equal magnitudes on its two sides.

Step 1: (i) Define electric flux and its unit Electric flux through a surface is the surface integral of the scalar product of electric field and the directed area element. For a uniform field through a plane area S ,

$$\Phi_E = ES \cos \theta,$$

where θ is the angle between the field and the chosen area normal, not the plane itself. Since electric field has SI unit N/C, the SI unit of flux is $\text{N m}^2/\text{C}$, equivalently V m . Flux is a signed scalar.

Step 2: (ii) Choose a Gaussian pillbox across the sheet Let the sheet have uniform surface charge density σ . Translational symmetry along the plane rules out a preferred tangential direction, and reflection symmetry gives equal field magnitudes on the two sides. For $\sigma > 0$, the field points normally away from each side. Take a short cylindrical Gaussian surface with its axis perpendicular to the sheet and each flat face of area S , one on either side. The field is parallel to the curved surface, so its flux there is zero. On both flat faces the field and outward area vector are parallel, giving flux ES through each face.

Step 3: Apply Gauss' law to obtain the sheet field The pillbox encloses charge σS . Hence,

$$ES + ES = \frac{\sigma S}{\epsilon_0}, \quad E = \frac{\sigma}{2\epsilon_0} \quad (\sigma > 0).$$

In general the magnitude is $|\sigma|/(2\epsilon_0)$. The direction is normal to the plane, away from a positive sheet and towards a negative sheet. The result is independent of distance because the sheet is infinite; there are no edge effects.

Step 4: (iii) Evaluate flux through the cube The cube extends from $x = 0$ to $x = L$. Its four faces normal to the y - or z -axis have zero flux because the field is along $\hat{i}$. On the face at $x = L$, the outward normal is $+\hat{i}$; on the face at $x = 0$, it is $-\hat{i}$. Their area is L^2 , so

$$\begin{aligned}\Phi_{x=L} &= (AL + B)L^2, \\ \Phi_{x=0} &= -BL^2, \\ \Phi_{\text{net}} &= (AL + B)L^2 - BL^2 = AL^3.\end{aligned}$$

Applying Gauss' law gives $Q_{\text{enc}} = \epsilon_0 \Phi_{\text{net}}$. The constant field term B cancels because it enters and leaves the cube equally. The sign of the enclosed charge is the sign of A .

Step 5: Conclusion (i) $\Phi_E = \int_S \vec{E} \cdot d\vec{S}$, with SI unit $\text{N m}^2/\text{C}$. (ii) An infinite sheet produces field magnitude $E = |\sigma|/(2\epsilon_0)$, normal to its plane. (iii) The net charge enclosed by the cube is

$$Q_{\text{enc}} = \epsilon_0 AL^3.$$

Quick Tip to Solve: For a field only along x , subtract the fields on the two opposite x -faces and multiply by their area. A spatially constant term such as B contributes no net closed-surface flux.

31(b). (i) Define electric potential at a point and write its SI unit.

(ii) Two capacitors are connected in series. Derive an expression of the equivalent capacitance of the combination.

(iii) Two point charges $+q$ and $-q$ are located at points $(3a, 0)$ and $(0, 4a)$ respectively in x - y plane. A third charge Q is kept at the origin. Find the value of Q , in terms of q and a , so that the electrostatic potential energy of the system is zero. [5 marks]

Solution:

Concept:

- Electric potential is work done by an external agent per unit positive test charge in bringing it slowly from infinity to the point, with potential at infinity taken as zero.
- Capacitance is $C = q_c/V$, where q_c is the magnitude of charge on either plate and V is the potential difference. Series capacitors carry equal charge magnitudes and their potential differences add.
- Electrostatic potential energy of point charges is the sum over distinct pairs: $U = \sum_{i < j} kq_i q_j / r_{ij}$, with $k = 1/(4\pi\epsilon_0)$. Each pair is counted once, with its charge signs retained.

Step 1: (i) Define electric potential and its unit Electric potential at a point is the external work done per unit positive test charge in bringing that charge from infinity to the point without acceleration, so that its kinetic energy does not change. Thus,

$$V = \frac{W_{\text{ext}}}{q_0},$$

where q_0 is sufficiently small not to disturb the source charges. Its SI unit is volt: $1 \text{ V} = 1 \text{ J/C}$. This uses the reference $V(\infty) = 0$.

Step 2: (ii) Relate charge and voltage in series capacitors Let two initially uncharged capacitors of capacitances C_1 and C_2 be joined in series across a potential difference V . The isolated conducting link between them remains electrically neutral, so opposite charges induced on its two ends have equal magnitude. Consequently each capacitor stores the same charge magnitude q_c . Their individual potential differences are

$$V_1 = \frac{q_c}{C_1}, \quad V_2 = \frac{q_c}{C_2}.$$

The total applied voltage is the sum of these drops, $V = V_1 + V_2$; the voltages need not be equal when the capacitances differ.

Step 3: Derive the equivalent capacitance The equivalent capacitor must store the same terminal charge q_c under the same total voltage V . Writing its capacitance as C_{eq} ,

$$\begin{aligned} \frac{q_c}{C_{\text{eq}}} &= \frac{q_c}{C_1} + \frac{q_c}{C_2}, \\ \frac{1}{C_{\text{eq}}} &= \frac{1}{C_1} + \frac{1}{C_2}, \\ C_{\text{eq}} &= \frac{C_1 C_2}{C_1 + C_2}. \end{aligned}$$

For positive capacitances the equivalent capacitance is smaller than either individual capacitance, as expected for a series combination.

Step 4: (iii) Determine the three pair separations The charge $+q$ is at $(3a, 0)$, $-q$ is at $(0, 4a)$, and Q

is at the origin. Taking $a > 0$, the distances from Q are $3a$ and $4a$. The distance between $+q$ and $-q$ follows from the right triangle:

$$r_{+-} = \sqrt{(3a)^2 + (4a)^2} = 5a.$$

Include the interaction of the two fixed charges as well as the two interactions involving Q ; the question asks for the energy of the entire system.

Step 5: Impose zero total potential energy Using the three distinct charge pairs,

$$\begin{aligned} U &= k \left[-\frac{q^2}{5a} + \frac{qQ}{3a} - \frac{qQ}{4a} \right] \\ &= \frac{k}{a} \left[-\frac{q^2}{5} + \frac{qQ}{12} \right]. \end{aligned}$$

Hence $U = 0$ requires

$$\frac{qQ}{12} = \frac{q^2}{5}, \quad Q = \frac{12q}{5}.$$

The required charge is positive: its net positive interaction energy with the fixed pair cancels their negative mutual energy. All three distances scale with a , so the common factor $1/a$ cancels.

Step 6: Conclusion (i) Electric potential is external work per unit positive test charge, with SI unit volt. (ii) $C_{\text{eq}} = C_1 C_2 / (C_1 + C_2)$. (iii) Zero total electrostatic potential energy requires $Q = +12q/5$, independent of a .

Quick Tip to Solve: For the three-charge system, first recognize the $3a$ - $4a$ - $5a$ triangle, then write all three pair energies once. Zero total energy includes the fixed pair's negative energy.

32(a). (i) Write the principle and explain the working of a moving coil galvanometer. A galvanometer as such cannot be used to measure the current in a circuit. Why?

(ii) Why is the magnetic field made radial in a moving coil galvanometer? How is it achieved?

[5 marks]

Solution:

Concept:

- A current-carrying coil in a magnetic field experiences a torque $\tau_m = NIAB \sin \alpha$, where N is the number of turns, I the current, A the area of each turn, and α the angle between its normal and the field B .
- A suspension or spring provides a restoring torque $\tau_r = k\theta$, where k is the torsional constant and θ is the angular deflection.
- In a radial magnetic field, the coil's plane remains parallel to the field throughout its rotation, so $\alpha = 90^\circ$ and the magnetic torque is $NIAB$.

Step 1: (i) State the principle and describe the arrangement A moving coil galvanometer works on the principle that a current-carrying coil placed in a magnetic field experiences a torque. A light rectangular coil of many turns of insulated fine wire is mounted so that it can rotate between the concave pole pieces of a strong permanent magnet. A cylindrical soft-iron core lies inside the coil. The suspension and spring, or suitable spiral springs, carry current to the coil and provide the restoring torque. A pointer or mirror indicates its angular deflection.

Step 2: (i) Balance the magnetic and restoring torques When a current flows through the coil, the

magnetic forces produce a couple and turn it. As it turns, the suspension or spring develops an opposing torque. At its final equilibrium position in the radial field,

$$NIAB = k\theta \Rightarrow I = \frac{k}{NAB}\theta.$$

Since N, A, B and k are fixed for the instrument, its angular deflection is directly proportional to the current. Reversing the current reverses the magnetic torque and therefore the direction of deflection. Damping helps the pointer settle without prolonged oscillations.

Step 3: (i) Explain why it is not directly an ammeter The galvanometer is a sensitive detector intended for small currents, and its coil has appreciable resistance. If inserted directly in series, this resistance changes the current that is to be measured. Moreover, an ordinary circuit current may exceed its small full-scale current and damage the fine coil. For measuring a larger current, a suitable low resistance shunt is connected in parallel with the galvanometer, carrying most of the current and making the combination's effective resistance small.

Step 4: (ii) Explain the purpose and production of a radial field In a uniform field of fixed direction, the factor $\sin \alpha$ generally changes as the coil rotates. A radial field keeps the field parallel to the coil's plane at every working position, and hence perpendicular to its normal. Thus $\sin \alpha = 1$: the torque is maximum for a given current and does not acquire an additional angular dependence. The resulting relation $\theta = (NAB/k)I$ gives a uniform current scale. A radial field is obtained by using concave cylindrical pole faces of the permanent magnet together with a cylindrical soft-iron core inside the coil; the core also strengthens the field.

Step 5: Conclusion The galvanometer converts current into a proportional deflection, $\theta = NABI/k$, by balancing magnetic and spring torques. Its resistance and small current capacity prevent its direct use as an ammeter. Concave pole pieces and a soft-iron core provide the radial field needed for maximum torque and a linear scale.

Quick Tip to Solve: Distinguish the two angles: the field is parallel to the coil's plane but perpendicular to its normal. Therefore the torque factor is $\sin 90^\circ = 1$, not zero.

OR

32(b). (i) Derive an expression for magnetic field on the axis of a current-carrying circular loop.

(ii) Write any two points of difference between a diamagnetic and a paramagnetic substance.

[5 marks]

Solution:

Concept:

- Biot–Savart law gives $d\vec{B} = \frac{\mu_0 I}{4\pi} \frac{d\vec{l} \times \vec{r}}{r^3}$, where $d\vec{l}$ points along the current and $\vec{r}$ points from the current element to the observation point.
- For a circular loop, symmetry cancels magnetic-field components perpendicular to its axis, while all axial components add.
- Magnetic susceptibility χ_m relates magnetisation to the applied magnetic field: $\vec{M} = \chi_m \vec{H}$. Its sign distinguishes diamagnetic and paramagnetic responses.

Step 1: (i) Define the geometry and find the field of one element Consider a circular loop of radius

R , carrying a steady current I , with centre O . Let P be a point on its axis at distance x from O . The distance from every current element to P is

$$r = \sqrt{R^2 + x^2}.$$

A tangent element $d\vec{l}$ is perpendicular both to the radius drawn to that element and to the loop's axis. It is therefore perpendicular to their resultant $\vec{r}$. Biot–Savart law consequently gives the magnitude

$$dB = \frac{\mu_0 I dl}{4\pi r^2}.$$

Step 2: (i) Resolve along the axis and use symmetry Let α be the angle between $\vec{r}$ and the axis. The right triangle formed by the radius, the axial distance and r gives $\sin \alpha = R/r$. Since $d\vec{B}$ is perpendicular to $\vec{r}$ in this plane, its axial component is

$$dB_{\text{axis}} = dB \sin \alpha = \frac{\mu_0 IR dl}{4\pi r^3}.$$

For each element, the diametrically opposite element contributes an equal axial component and an opposite component perpendicular to the axis. Thus the transverse components cancel pairwise, leaving only the axial field.

Step 3: (i) Integrate around the loop and state the direction The quantities R and r are the same for every element, and the total length of the loop is $\oint dl = 2\pi R$. Hence

$$\begin{aligned} B(x) &= \frac{\mu_0 IR}{4\pi(R^2 + x^2)^{3/2}} \oint dl \\ &= \frac{\mu_0 IR^2}{2(R^2 + x^2)^{3/2}}. \end{aligned}$$

The direction is along the loop's axis, given by the right-hand thumb rule: curl the fingers along the current and the thumb gives the axial field direction. At the centre, $x = 0$, the expression reduces to $B(0) = \mu_0 I / (2R)$. For N identical closely wound turns, multiply the result by N .

Step 4: (ii) Give two differences between the magnetic substances **First difference – response to a non-uniform field:** A diamagnetic substance is weakly repelled by a magnet and tends to move from a stronger field region to a weaker field region. A paramagnetic substance is weakly attracted and tends to move from a weaker field region to a stronger field region.

Second difference – susceptibility and magnetisation: A diamagnetic substance has a small negative susceptibility, $\chi_m < 0$; its induced magnetisation opposes the applied field. A paramagnetic substance has a small positive susceptibility, $\chi_m > 0$; its magnetisation is along the applied field. For example, bismuth is diamagnetic and aluminium is paramagnetic.

Step 5: Conclusion The field of a single circular current loop on its axis is $B(x) = \mu_0 IR^2 / [2(R^2 + x^2)^{3/2}]$, directed by the right-hand rule. Diamagnets are weakly repelled and have negative susceptibility; paramagnets are weakly attracted and have positive susceptibility.

Quick Tip to Solve: In the axial-field derivation, retain the projection factor R/r ; omitting it gives the wrong power of $R^2 + x^2$. Check your result by setting $x = 0$.

33(a). (i) Draw a ray diagram showing the formation of a real image of an object placed at a distance ' u ' in front of a concave mirror of radius of curvature ' R '. Hence, obtain the relation for the image distance ' v ' in terms of u and R .

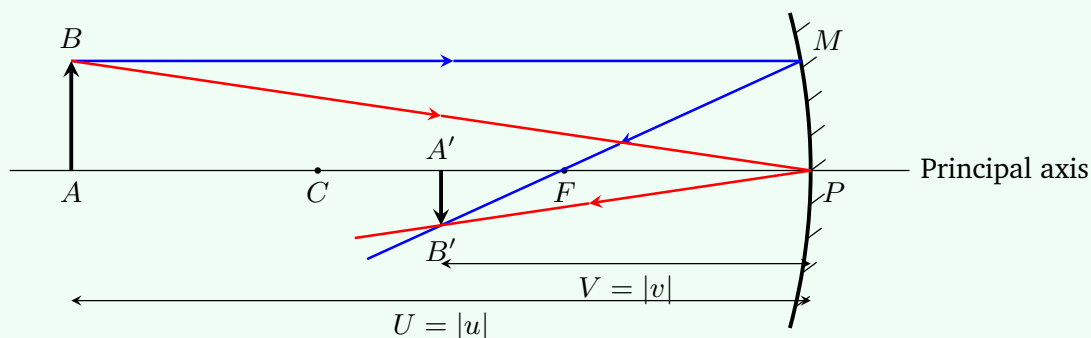
(ii) A 1.8 m tall person stands in front of a convex lens of focal length 1 m, at a distance of 5 m. Find the position and height of the image formed. [5 marks]

Solution:

Concept:

- A concave mirror reflects an incident ray parallel to the principal axis through its principal focus F . At the pole P , the angles of incidence and reflection with the principal axis are equal.
- For a spherical mirror of small aperture, paraxial rays obey $f = R/2$. All signed distances are measured from the pole, positive along the incident light; a concave mirror therefore has $R < 0$.
- For a thin lens, $1/f = 1/v - 1/u$, and transverse magnification is $m = h_i/h_o = v/u$. Here u, v, f are signed object, image and focal distances, while h_o, h_i are signed heights.

Step 1: (i) Construct the real-image ray diagram Take an upright object AB beyond the centre of curvature C of a concave mirror. Draw a ray from its tip B parallel to the principal axis; after reflection at M , it passes through F . Draw a second ray from B to the pole P ; reflection at equal angles sends it below the axis. Their intersection is B' , so $A'B'$ is a real inverted image between C and F . The construction uses the small-aperture approximation, so the axial displacement of M from the pole plane is negligible.



Paraxial ray construction: object beyond C , real inverted image between C and F .

Step 2: (i) Use similar triangles to obtain the mirror relation First use positive geometric lengths $U = AP$, $V = A'P$, $F_0 = FP$, $h = AB$ and $H = A'B'$. The right triangles ABP and $A'B'P$ are similar because the pole ray makes equal angles with the axis. Thus

$$\frac{H}{h} = \frac{V}{U}.$$

For the reflected parallel ray, the triangles with vertices F, P, M and F, A', B' are similar in the paraxial approximation. Since $PM \simeq h$ and $FA' = V - F_0$,

$$\frac{H}{h} = \frac{V - F_0}{F_0}.$$

Equating these expressions and dividing by V gives

$$\frac{V}{U} = \frac{V}{F_0} - 1,$$

$$\frac{1}{U} + \frac{1}{V} = \frac{1}{F_0} = \frac{2}{|R|}.$$

With incident light travelling to the right, the real object, real image and centre of curvature all lie to the left of P . Hence $u = -U$, $v = -V$ and $R = -|R|$, so

$$\frac{1}{u} + \frac{1}{v} = \frac{2}{R},$$

$$\frac{1}{v} = \frac{2u - R}{Ru}, \quad v = \frac{Ru}{2u - R}.$$

The algebraic relation applies throughout paraxial mirror imaging with the Cartesian sign convention, even though the sketch illustrates an object beyond C .

Step 3: (ii) Apply the lens formula The convex lens has $f = +1$ m. The person is a real object on the incident side, so $u = -5$ m and $h_o = +1.8$ m. Substitution into the thin-lens equation gives

$$\frac{1}{v} = \frac{1}{f} + \frac{1}{u} = 1 - \frac{1}{5} = \frac{4}{5} \text{ m}^{-1},$$

$$v = \frac{5}{4} \text{ m} = +1.25 \text{ m}.$$

The positive image distance places the image on the opposite side of the lens from the person. It lies between f and $2f$, as expected for an object farther than $2f$.

Step 4: (ii) Calculate the image height Use lens magnification with the same signed distances:

$$m = \frac{v}{u} = \frac{1.25}{-5} = -0.25,$$

$$h_i = mh_o = (-0.25)(1.8) = -0.45 \text{ m}.$$

The negative height denotes inversion relative to the upright person; the magnitude is one-quarter of the person's height, consistent with the image being diminished.

Step 5: Conclusion (i) The mirror relation is $v = Ru/(2u - R)$, using signed Cartesian distances. (ii) The image is real and inverted, 1.25 m from the convex lens on the opposite side, and 0.45 m tall (signed height -0.45 m).

Quick Tip to Solve: Keep the two magnification conventions distinct: a lens uses v/u , whereas a mirror uses $-v/u$. Here the lens image must lie between f and $2f$, which checks the calculated 1.25 m.

33(b). (i) Draw a ray diagram showing refraction of a ray of light through a triangular glass prism. Hence, obtain the relation for the refractive index (μ) in terms of angle of prism (A) and angle of minimum deviation (δ_m).

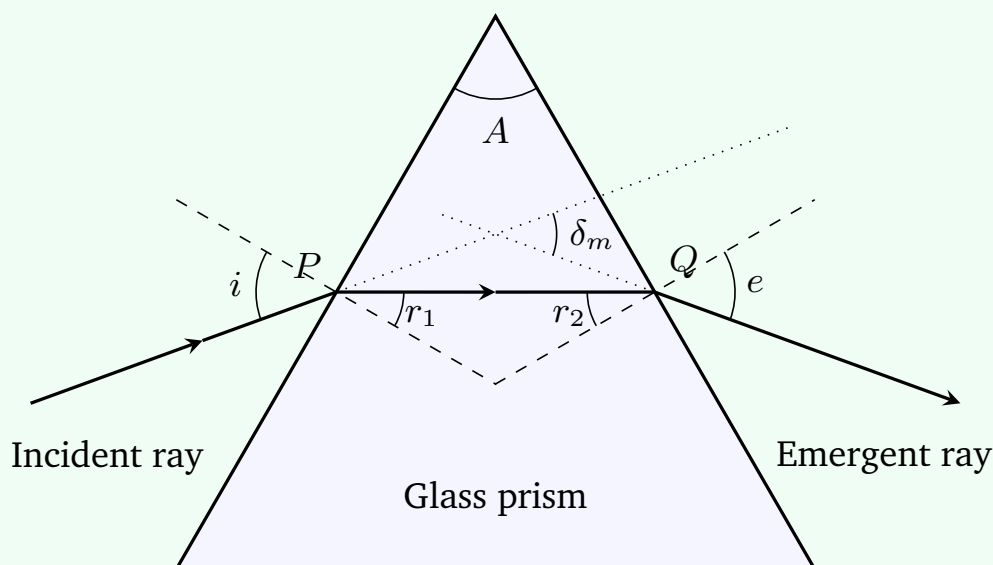
(ii) The radii of curvature of the two surfaces of a concave lens are 20 cm each. Find the refractive index of the material of the lens if its power is -5.0 D. [5 marks]

Solution:

Concept:

- Refraction at each prism face obeys Snell's law: $\mu = \sin i / \sin r_1$ for light entering glass from air. Here i is incidence and r_1 the first internal refraction angle.
- For a prism of angle A , the internal angles satisfy $r_1 + r_2 = A$; total deviation is $\delta = i + e - A$, where e is the angle of emergence.
- The thin-lens maker's formula in air is $P = 1/f = (\mu - 1)(1/R_1 - 1/R_2)$, using signed radii in metres to obtain power in dioptres.

Step 1: (i) Draw and identify the prism ray path Let a ray enter the left face at P , travel through glass to Q , and emerge into air. The dashed lines through P and Q are normals. Angles i, r_1, r_2, e are measured from the appropriate normals, not from the prism faces. On entry the ray bends towards the normal; on emergence it bends away. The dotted extensions show the angle of deviation between the initial and final directions. The illustrated symmetric path is the minimum-deviation position.



Refraction through a prism at minimum deviation; dashed lines are normals and dotted lines extend the ray directions.

Step 2: Relate the prism angles and the deviation The normals are perpendicular to the refracting faces. Geometry of the quadrilateral formed by the two faces, their normals and the internal ray gives

$$r_1 + r_2 = A.$$

The changes in ray direction at entry and exit are $i - r_1$ and $e - r_2$. Therefore

$$\delta = (i - r_1) + (e - r_2) = i + e - A.$$

This adds the deviations at both faces, so it applies to the complete passage through the prism.

Step 3: Apply the minimum-deviation condition At minimum deviation the path is symmetric by reversibility: $i = e$ and $r_1 = r_2$. Hence

$$r_1 = r_2 = \frac{A}{2}, \quad \delta_m = 2i - A, \quad i = \frac{A + \delta_m}{2}.$$

Applying Snell's law at the first air-glass interface now gives

$$\mu = \frac{\sin i}{\sin r_1} = \frac{\sin((A + \delta_m)/2)}{\sin(A/2)}.$$

The surrounding medium is air, whose refractive index is taken as unity.

Step 4: (ii) Assign signed radii and use the lens maker's formula Take light to travel from left to right and positive distances to the right. For a biconcave lens the first centre of curvature lies to the left of its surface and the second to the right. Thus

$$R_1 = -0.20 \text{ m}, \quad R_2 = +0.20 \text{ m}.$$

Substituting the given power,

$$\begin{aligned} -5.0 &= (\mu - 1) \left(\frac{1}{-0.20} - \frac{1}{+0.20} \right), \\ -5.0 &= -10(\mu - 1), \\ \mu - 1 &= 0.50. \end{aligned}$$

The negative power corresponds to a diverging concave lens, as expected; using equal unsigned radii would incorrectly give zero power.

Step 5: Conclusion (i) $\mu = \frac{\sin((A + \delta_m)/2)}{\sin(A/2)}$. (ii) The refractive index of the concave-lens material is 1.50, a dimensionless quantity.

Quick Tip to Solve: At minimum deviation use symmetry first: each internal angle is $A/2$. For a biconcave lens with light from the left, use $R_1 < 0$, $R_2 > 0$, and convert centimetres to metres before using power.

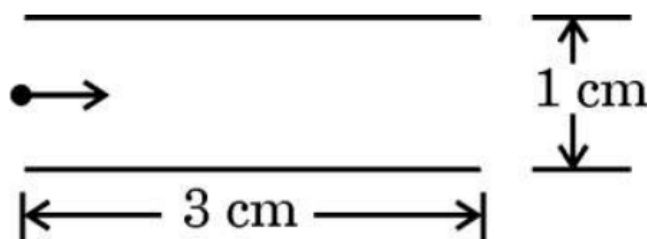
Section E (8 marks)

34(a). A beam of electrons moving horizontally with a velocity of 3×10^7 m/s enters a region between two plates as shown in the figure. A suitable potential difference is applied across the plates such that the electron beam just strikes the edge of the lower plate.

Answer the following questions based on the above:

How long does an electron take to strike the edge?

[1 mark]



Solution:

Concept:

- Between the parallel plates, the electric force is vertical, so the horizontal acceleration is zero and the horizontal speed v_x remains constant.
- Horizontal displacement obeys $L = v_x t$, where L is the plate length and t is the time spent between the plates.

Step 1: Read the relevant length from the diagram The electron enters at the left edge and just strikes the far edge of the lower plate. Its horizontal displacement is therefore the plate length, $L = 3 \text{ cm} = 3 \times 10^{-2} \text{ m}$. The 1 cm separation measures a vertical distance and is not used to determine this travel time.

Step 2: Use uniform horizontal motion The field changes only the vertical component of velocity. The horizontal component retains its entry value $3 \times 10^7 \text{ m s}^{-1}$, even though the total speed changes. Thus

$$t = \frac{L}{v_x} = \frac{3 \times 10^{-2}}{3 \times 10^7} \text{ s} = 1.0 \times 10^{-9} \text{ s}.$$

Step 3: Conclusion The electron strikes the edge after $1.0 \times 10^{-9} \text{ s}$, or 1.0 ns.

Quick Tip to Solve: Use the 3 cm horizontal plate length for the time; electric deflection does not change the horizontal velocity.

34(b). What is the shape of the path followed by the electron and why?

[1 mark]

Solution:

Concept:

- A uniform electric field exerts a constant force $\mathbf{F} = -e\mathbf{E}$ on an electron, where e is the magnitude of its charge. Its acceleration is opposite to the field.
- Uniform horizontal motion combined with constant perpendicular acceleration gives a parabolic trajectory, just as in projectile motion.

Step 1: Resolve the electron's motion Choose the entry point as the origin, with x horizontally

forward and y vertically downward. The electron enters horizontally, so its initial vertical velocity is zero. The uniform field between the plates produces constant downward acceleration $a = eE/m_e$, while the horizontal speed v_x remains constant.

Step 2: Eliminate the time The component equations are $x = v_x t$ and $y = \frac{1}{2}at^2$. Substituting $t = x/v_x$ gives

$$y = \frac{a}{2v_x^2}x^2 = \frac{eE}{2m_ev_x^2}x^2.$$

Since the coefficient is constant, the vertical displacement is proportional to the square of the horizontal displacement. The force has a fixed direction; it does not remain perpendicular to the changing velocity as would be required for circular motion.

Step 3: Conclusion The electron follows a parabolic path bending towards the lower plate, because it has constant horizontal velocity and uniform vertical acceleration.

Quick Tip to Solve: Whenever the force is constant in one direction and the initial velocity is perpendicular to it, eliminate time to obtain $y \propto x^2$.

34(c). Find the potential difference applied.

[2 marks]

Solution:

Concept:

- Between parallel plates, the uniform electric field has magnitude $E = V/d$, where d is the plate separation and V is the potential difference magnitude.
- An electron has acceleration magnitude $a = eE/m_e$, opposite to the electric field. With zero initial vertical velocity its vertical displacement is $y = at^2/2$.

Step 1: Determine the travel time and vertical displacement The horizontal speed remains $v_x = 3 \times 10^7 \text{ m s}^{-1}$ because the electric force is vertical. The plate length is $L = 3 \text{ cm} = 0.03 \text{ m}$, so

$$t = \frac{L}{v_x} = \frac{0.03}{3 \times 10^7} = 1.0 \times 10^{-9} \text{ s}.$$

The beam enters midway between plates separated by $d = 1 \text{ cm}$. Thus it falls through $y = d/2 = 5.0 \times 10^{-3} \text{ m}$ before striking the lower edge.

Step 2: Calculate the field and potential difference From the vertical motion,

$$a = \frac{2y}{t^2} = \frac{2(5.0 \times 10^{-3})}{(1.0 \times 10^{-9})^2} = 1.0 \times 10^{16} \text{ m s}^{-2}.$$

Using $m_e = 9.1 \times 10^{-31} \text{ kg}$ and $e = 1.6 \times 10^{-19} \text{ C}$,

$$E = \frac{m_e a}{e} = 5.6875 \times 10^4 \text{ V m}^{-1}, \quad V = Ed = 568.75 \text{ V}.$$

The force on the electron is downward, so the electric field is upward; the lower plate must be at the

higher potential.

Step 3: Conclusion The applied potential difference is approximately $5.69 \times 10^2 \text{ V}$, with the lower plate positive relative to the upper plate.

Quick Tip to Solve: Use half the plate separation for the electron's fall, but the full separation in $V = Ed$.

OR

34(c). Find the magnitude and direction of the magnetic field which should be created in the space between the plates so that the electron beam goes straight undeviated. [2 marks]

Solution:

Concept:

- A charged particle in crossed fields experiences $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$. It moves undeviated when the electric and magnetic forces cancel.
- With $\vec{v} \perp \vec{B}$, force balance gives $eE = evB$, or $B = E/v$. The electron's negative charge reverses the direction of $\vec{v} \times \vec{B}$ for its magnetic force.

Step 1: Find the electric field from the original deflection The electron crosses the 0.03 m plate length at $3 \times 10^7 \text{ m s}^{-1}$, taking $t = 1.0 \times 10^{-9} \text{ s}$. Its downward displacement to the lower edge is half the 0.01 m separation, namely $y = 0.005 \text{ m}$. Therefore

$$a = \frac{2y}{t^2} = 1.0 \times 10^{16} \text{ m s}^{-2}, \quad E = \frac{m_e a}{e} = 5.6875 \times 10^4 \text{ V m}^{-1}.$$

The electric force is downward, and the electric field is upward.

Step 2: Balance the forces and determine the field direction To cancel the downward electric force, the magnetic force must act upward. Its required magnitude gives

$$B = \frac{E}{v} = \frac{5.6875 \times 10^4}{3 \times 10^7} = 1.8958 \times 10^{-3} \text{ T}.$$

For velocity to the right and magnetic field out of the page, $\vec{v} \times \vec{B}$ points downward. Since the electron has negative charge, its magnetic force points upward, as required. A field into the page would instead reinforce the downward electric force.

Step 3: Conclusion Apply a magnetic field of approximately $1.90 \times 10^{-3} \text{ T}$, perpendicular to the page and directed out of the page towards the observer.

Quick Tip to Solve: First balance magnitudes using $B = E/v$; then reverse the right-hand-rule force direction because the particle is an electron.

35(a). Diffraction of light is bending of light around the corners of an object whose size is comparable with the wavelength of light. Diffraction actually defines the limits of ray optics. This limit for optical instruments is set by the wavelength of light. An experimental arrangement is set up to observe the diffraction pattern due to a single slit.

Answer the following questions based on the above:

How will the width of central maximum be affected if the wavelength of light is increased? [1 mark]

Solution:

Concept:

- For a single slit of width a , the first minima bounding the central maximum satisfy $a \sin \theta = \pm \lambda$, where λ is the wavelength.
- At small diffraction angles, a screen at distance D has central-maximum width $W = 2D\lambda/a$.

Step 1: Keep the apparatus dimensions fixed The slit width a and the slit-to-screen distance D are unchanged. The central maximum extends between the first minima on opposite sides of the straight-ahead direction.

Step 2: Compare the widths at the two wavelengths Since $W = 2D\lambda/a$, the ratio of the new width to the old width is $W'/W = \lambda'/\lambda$. Increasing the wavelength moves both first minima farther from the centre and increases their separation.

Step 3: Conclusion The central maximum becomes wider. For fixed slit width and screen distance, its width increases in direct proportion to wavelength in the small-angle approximation.

Quick Tip to Solve: Remember $W \propto \lambda/a$: increasing wavelength broadens the central maximum, while increasing slit width narrows it.

35(b). Under what condition is the first minimum obtained?

[1 mark]

Solution:

Concept:

- The path difference between rays from opposite edges of a slit of width a , observed at angle θ , is $a \sin \theta$.
- Two equal coherent contributions cancel when their path difference is $\lambda/2$, corresponding to a phase difference of π .

Step 1: Pair the two halves of the slit Divide the slit into two equal halves. Pair each point in one half with the corresponding point in the other half, a distance $a/2$ away. The path difference of each such pair is $(a/2) \sin \theta$.

Step 2: Apply destructive interference to every pair At the first minimum, $(a/2) \sin \theta = \lambda/2$. Each pair then cancels, so the resultant amplitude from the entire slit vanishes. This gives $a \sin \theta = \lambda$ on one side, with a symmetric minimum on the other side.

Step 3: Conclusion The first minima occur at $a \sin \theta = \pm \lambda$, or $|\theta| \approx \lambda/a$ for small angles. Thus the

magnitude of the path difference across the full slit is one wavelength.

Quick Tip to Solve: At the first minimum the whole slit has path difference λ ; the paired half-slit rays differ by $\lambda/2$.

35(c). Write two points of difference between interference and diffraction patterns.

[2 marks]

Solution:

Concept:

- In the standard two-slit interference pattern, light from two coherent narrow slits superposes. For equal slit intensities, the ideal bright fringes have equal intensity.
- Single-slit diffraction results from the superposition of secondary waves from different points across one slit; the central maximum and the weaker secondary maxima form its characteristic pattern.

Step 1: Compare fringe widths In two-slit interference, successive bright fringes are equally spaced and bright and dark fringes have equal widths in the small-angle approximation: $\beta = \lambda D/d$, with slit separation d and screen distance D . In single-slit diffraction, the central bright maximum is twice as wide as the secondary bright regions between successive minima. For slit width a , the central width is $2\lambda D/a$, while each secondary region has width approximately $\lambda D/a$.

Step 2: Compare the intensities of the bright fringes In ideal two-slit interference with equal narrow slits, all bright fringes have equal maximum intensity when the single-slit diffraction envelope is neglected. In single-slit diffraction, the central maximum is brightest; successive secondary maxima have progressively lower peak intensities away from the centre.

Step 3: Conclusion The two differences are: interference fringes have uniform widths and, ideally, equal peak intensities; diffraction has a central maximum twice as wide as the secondary regions and secondary maxima whose intensities decrease outwards.

Quick Tip to Solve: Compare the same two visible features: fringe width and peak intensity. State the equal-slit idealisation for interference.

OR

35(c). Two students are separated by a 7 m partition wall in a room 10 m high. If both light and sound waves can bend around obstacles, how is it that the students are unable to see each other even though they can converse easily?

[2 marks]

Solution:

Concept:

- Diffraction is appreciable when the wavelength is comparable with the relevant obstacle or opening dimension. Its angular spreading scales roughly as λ/a , where a is that dimension.
- Sound used in conversation has wavelengths of centimetres to metres, while visible light has wavelengths of approximately 4×10^{-7} to 7×10^{-7} m.

Step 1: Compare sound wavelengths with the opening A 7 m high partition in a 10 m high room leaves a 3 m space above it. Audible sound, especially its lower-frequency components with metre-scale wavelengths, undergoes appreciable diffraction around the upper edge and spreads into the region

behind the wall. Sound can therefore reach the other student even though the direct path is blocked.

Step 2: Compare light wavelengths with the same opening Visible-light wavelengths are millions of times smaller than the metre-scale wall and opening dimensions. The corresponding diffraction angle is extremely small: for a representative wavelength, $\lambda/a \sim (5 \times 10^{-7})/3 \approx 1.7 \times 10^{-7}$ rad. Light therefore does not bend sufficiently around the partition to form a visible image of the other student. The wall blocks the direct light rays needed for seeing each other.

Step 3: Conclusion Sound diffracts appreciably around the partition because its wavelength is much larger than that of light. Light diffraction is negligible on this scale, so the students can converse but cannot see each other through the opaque wall.

Quick Tip to Solve: Compare wavelength with the opening size: both waves diffract, but a larger λ/a gives much more noticeable bending.