

CBSE Class 12 Physics Question Paper 2023 with Solution PDF-Set 55/3/2



Question Paper with Solutions

Conducted by the Central Board of Secondary Education (CBSE)

Time Allowed: 3 hours

Maximum Marks: 70

General Instructions

- (i) This question paper contains **35** questions. **All** questions are **compulsory**.
- (ii) This question paper is divided into **five** Sections – **A, B, C, D** and **E**.
- (iii) In **Section A** – Questions no. **1** to **18** are Multiple Choice (MCQ) type questions, carrying **1** mark each.
- (iv) In **Section B** – Questions no. **19** to **25** are Very Short Answer (VSA) type questions, carrying **2** marks each.
- (v) In **Section C** – Questions no. **26** to **30** are Short Answer (SA) type questions, carrying **3** marks each.
- (vi) In **Section D** – Questions no. **31** to **33** are Long Answer (LA) type questions carrying **5** marks each.
- (vii) In **Section E** – Questions no. **34** and **35** are case-based questions carrying **4** marks each.
- (viii) There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 2 questions in Section C, 3 questions in Section D and 2 questions in Section E.
- (ix) Use of calculators is **not** allowed.

Use the following values of physical constants, if required :

$$c = 3 \times 10^8 \text{ m/s}$$

$$h = 6.63 \times 10^{-34} \text{ Js}$$

$$e = 1.6 \times 10^{-19} \text{ C}$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ T m A}^{-1}$$

$$\epsilon_0 = 8.854 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$$

$$\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$$

$$\text{Mass of electron } (m_e) = 9.1 \times 10^{-31} \text{ kg}$$

$$\text{Mass of neutron} = 1.675 \times 10^{-27} \text{ kg}$$

$$\text{Mass of proton} = 1.673 \times 10^{-27} \text{ kg}$$

$$\text{Avogadro's number} = 6.023 \times 10^{23} \text{ per gram mole}$$

$$\text{Boltzmann constant} = 1.38 \times 10^{-23} \text{ JK}^{-1}$$

Section A (18 marks)

1. A voltage signal is described by: $v = V_0$ for $0 \leq t \leq \frac{T}{2}$ and $v = 0$ for $\frac{T}{2} \leq t \leq T$ for a cycle. Its rms value is: [1 mark]

- (A) $\frac{V_0}{\sqrt{2}}$
(B) V_0
(C) $\frac{V_0}{2}$
(D) $\sqrt{2} V_0$

Correct Answer: (A) $\frac{V_0}{\sqrt{2}}$

Solution:

Concept:

- The root mean square voltage is $V_{\text{rms}} = \sqrt{\frac{1}{T} \int_0^T v^2 dt}$, where T is the period.
- Squaring the instantaneous voltage before averaging gives the equivalent heating effect in a resistor.

Step 1: Average the squared voltage The voltage is V_0 for half the cycle and zero for the other half. Thus only the first interval contributes:

$$\langle v^2 \rangle = \frac{V_0^2(T/2) + 0(T/2)}{T} = \frac{V_0^2}{2}.$$

Step 2: Take the square root and compare Taking the square root gives the rms value. Option (C) is the ordinary average voltage, which does not measure heating. Option (B) would apply if the voltage magnitude stayed at V_0 throughout the cycle. Option (D) exceeds the maximum voltage and cannot be its rms value.

Step 3: Conclusion $V_{\text{rms}} = V_0/\sqrt{2}$. Hence, the correct option is (A).

Quick Tip to Solve: For a pulse of amplitude V_0 present for fraction d of each cycle, use $V_{\text{rms}} = V_0\sqrt{d}$.

2. The ratio of maximum frequency and minimum frequency of light emitted in Balmer series of hydrogen spectrum, in Bohr's model, is: [1 mark]

- (A) $\frac{11}{9}$
(B) $\frac{9}{5}$
(C) $\frac{11}{7}$
(D) $\frac{16}{7}$

Correct Answer: (B) $\frac{9}{5}$

Solution:

Concept:

- In the Balmer series every radiative transition ends at $n_f = 2$, with initial quantum number $n > 2$.

- The emitted frequency is $\nu = cR(1/4 - 1/n^2)$, where R is the Rydberg constant and c the speed of light.

Step 1: Identify the limiting transitions The smallest energy drop is from $n = 3$ to $n = 2$; hence $\nu_{\min} = cR(1/4 - 1/9) = 5cR/36$. The maximum frequency is the series limit, approached as the initial level tends to infinity, so $\nu_{\max} = cR/4$.

Step 2: Form the requested ratio Both frequencies contain the same factor cR , which cancels:

$$\frac{\nu_{\max}}{\nu_{\min}} = \frac{1/4}{5/36} = \frac{9}{5}.$$

The fractions $11/9$, $11/7$ and $16/7$ do not follow from these two Balmer limits. In particular, using $n = 4$ for the lowest line would omit the allowed $3 \rightarrow 2$ transition.

Step 3: Conclusion The ratio is $9/5$. Hence, the correct option is (B).

Quick Tip to Solve: For any hydrogen series, the minimum frequency is the first line and the maximum is the series limit.

3. At a certain temperature in an intrinsic semiconductor, the electrons and holes concentration is $1.5 \times 10^{16} \text{ m}^{-3}$. When it is doped with a trivalent dopant, hole concentration increases to $4.5 \times 10^{22} \text{ m}^{-3}$. In the doped semiconductor, the concentration of electrons (n_e) will be: [1 mark]

- (A) $3 \times 10^6 \text{ m}^{-3}$
- (B) $5 \times 10^7 \text{ m}^{-3}$
- (C) $5 \times 10^9 \text{ m}^{-3}$
- (D) $6.75 \times 10^{38} \text{ m}^{-3}$

Correct Answer: (C) $5 \times 10^9 \text{ m}^{-3}$

Solution:

Concept:

- At a fixed temperature, the mass-action law is $n_e n_h = n_i^2$, where n_i is the intrinsic carrier concentration.
- A trivalent impurity makes the material p-type: holes become majority carriers and electrons become minority carriers.

Step 1: Apply the mass-action law Initially the electron and hole concentrations are each $n_i = 1.5 \times 10^{16} \text{ m}^{-3}$. After doping, $n_h = 4.5 \times 10^{22} \text{ m}^{-3}$. At the same temperature their product still equals n_i^2 , rather than their sum remaining constant.

Step 2: Evaluate the electron concentration

$$n_e = \frac{(1.5 \times 10^{16})^2}{4.5 \times 10^{22}} = \frac{2.25}{4.5} \times 10^{10} = 5 \times 10^9 \text{ m}^{-3}.$$

Options (A) and (B) have incorrect numerical powers for this quotient. Option (D) multiplies

concentrations instead of dividing and would imply an enormous majority electron population, inconsistent with p-type doping.

Step 3: Conclusion $n_e = 5 \times 10^9 \text{ m}^{-3}$. Hence, the correct option is (C).

Quick Tip to Solve: Square the intrinsic concentration first; then divide by the new majority-carrier concentration.

4. An infinitely long uniformly charged wire produces an electric field of $18 \times 10^4 \text{ N C}^{-1}$ at a distance of 1.0 cm. The linear charge density on the wire is: [1 mark]

- (A) $1.12 \times 10^{-14} \text{ C m}^{-1}$
- (B) $3.08 \times 10^{-15} \text{ C m}^{-1}$
- (C) $1.0 \times 10^{-9} \text{ C m}^{-1}$
- (D) $1.0 \times 10^{-7} \text{ C m}^{-1}$

Correct Answer: (D) $1.0 \times 10^{-7} \text{ C m}^{-1}$

Solution:

Concept:

- The field magnitude of an infinitely long uniform line charge is $E = |\lambda|/(2\pi\epsilon_0 r) = 2k|\lambda|/r$.
- Here λ is charge per unit length, r is the perpendicular distance and $k = 9 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$.

Step 1: Convert the distance and rearrange Use the printed field $E = 18 \times 10^4 \text{ N C}^{-1}$ and convert $r = 1.0 \text{ cm} = 10^{-2} \text{ m}$. Rearranging the infinite-wire formula gives $|\lambda| = Er/(2k)$. The inverse-square point-charge formula does not apply to an infinite wire.

Step 2: Substitute the values

$$|\lambda| = \frac{(18 \times 10^4)(10^{-2})}{2(9 \times 10^9)} = 1.0 \times 10^{-7} \text{ C m}^{-1}.$$

Option (C) is a factor of one hundred too small; options (A) and (B) are still much smaller and do not reproduce the given field. Since only field magnitude is specified, the calculation determines the magnitude of the linear density.

Step 3: Conclusion The linear charge density has magnitude $1.0 \times 10^{-7} \text{ C m}^{-1}$. Hence, the correct option is (D).

Quick Tip to Solve: For an infinite wire, use $\lambda = Er/(2k)$ in magnitude and convert centimetres to metres before substituting.

5. At 0 K, the resistivity of an intrinsic semiconductor is:

[1 mark]

- (A) same as that at 0°C
- (B) same as that at 300 K
- (C) zero
- (D) infinite

Correct Answer: (D) infinite

Solution:

Concept:

- An intrinsic semiconductor conducts through thermally generated electrons and holes.
- Its conductivity is $\sigma = e(n_e\mu_e + n_h\mu_h)$, and resistivity is $\rho = 1/\sigma$; μ_e and μ_h denote carrier mobilities.

Step 1: Examine the bands at absolute zero At 0 K, the valence band is completely filled and the conduction band is empty. There is no thermal energy to excite electrons across the forbidden energy gap, so no free electrons or holes are available for conduction in the ideal intrinsic material.

Step 2: Infer the resistivity With no mobile carriers, the conductivity is zero and its reciprocal tends to infinity. Option (C), zero resistivity, would describe perfect conduction, the opposite behaviour. At 0°C = 273 K and at 300 K, thermal excitation produces carriers; therefore options (A) and (B) cannot describe the absolute-zero limit.

Step 3: Conclusion The resistivity is infinite at 0 K. Hence, the correct option is (D).

Quick Tip to Solve: At absolute zero, picture a full valence band and an empty conduction band: the intrinsic semiconductor acts as an insulator.

6. Light of frequency 6.4×10^{14} Hz is incident on a metal of work function 2.14 eV. The maximum kinetic energy of the emitted electrons is about:

[1 mark]

- (A) 0.25 eV
- (B) 0.51 eV
- (C) 1.02 eV
- (D) 0.10 eV

Correct Answer: (B) 0.51 eV

Solution:

Concept:

- Einstein's photoelectric equation is $K_{\max} = h\nu - \phi$, where ϕ is the work function and $h\nu$ the incident photon energy.
- Energies must use the same unit before subtraction; $1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$.

Step 1: Calculate the photon energy For $\nu = 6.4 \times 10^{14} \text{ Hz}$, use $h = 6.63 \times 10^{-34} \text{ J s}$:

$$h\nu = 4.2432 \times 10^{-19} \text{ J} = \frac{4.2432 \times 10^{-19}}{1.6 \times 10^{-19}} \text{ eV} = 2.652 \text{ eV}.$$

Step 2: Subtract the energy needed for escape The metal uses 2.14 eV of each absorbed photon to

release the electron, leaving

$$K_{\max} = 2.652 - 2.14 = 0.512 \text{ eV}.$$

This is positive, so emission is possible. Option (A) is about half this energy and option (C) about twice it; no factor of two enters Einstein's energy balance. Option (D) also fails the direct subtraction.

Step 3: Conclusion The maximum kinetic energy is about 0.51 eV. Hence, the correct option is (B).

Quick Tip to Solve: Convert the photon energy to eV before subtracting an eV work function; never subtract joules from eV.

7. In an experiment on photoelectric effect, the intensity of incident radiation is increased, keeping the frequency ν the same. The number of photoelectrons emitted will: [1 mark]

- (A) increase
- (B) decrease
- (C) remain same
- (D) depend on frequency

Correct Answer: (A) increase

Solution:

Concept:

- At fixed frequency ν , each photon carries the same energy $h\nu$.
- Intensity is energy incident per unit area per unit time, so photon arrival rate per unit area is $I/(h\nu)$.

Step 1: Relate higher intensity to photon arrivals In the photoemission experiment the frequency is above threshold and is held fixed. Increasing intensity therefore supplies more photons to the same illuminated surface in a given time; it does not make each photon more energetic.

Step 2: Relate photon arrivals to electron emission For the same surface and observation time, more incident photons release more photoelectrons. Thus the number emitted increases. Options (B) and (C) contradict the increased photon flux. Frequency determines whether emission is possible and the maximum kinetic energy, but option (D) does not describe the effect of increasing intensity while keeping that frequency fixed.

Step 3: Conclusion The number of photoelectrons emitted increases. Hence, the correct option is (A).

Quick Tip to Solve: At fixed frequency above threshold, intensity changes the photoelectron count; frequency changes the maximum kinetic energy.

8. The SI unit of mobility of charge carriers is:

[1 mark]

- (A) $\Omega \text{ s}^{-1}$
- (B) $\text{m}^2\text{V}^{-1}\text{s}^{-1}$
- (C) $\text{m s}^{-1}\text{V}^{-1}$
- (D) $\Omega \text{ m}$

Correct Answer: (B) $\text{m}^2\text{V}^{-1}\text{s}^{-1}$

Solution:

Concept:

- Mobility is drift speed per unit electric field: $\mu = v_d/E$, where v_d denotes drift speed.
- In SI, speed has unit m s^{-1} and electric field has unit V m^{-1} .

Step 1: Divide the defining units Substitute the SI units into the definition rather than treating mobility as a speed:

$$[\mu] = \frac{\text{m s}^{-1}}{\text{V m}^{-1}} = \text{m}^2\text{V}^{-1}\text{s}^{-1}.$$

Dividing by V m^{-1} contributes one extra factor of metre to the numerator.

Step 2: Check the alternative units Option (C) misses that extra length factor and would result from dividing speed by potential difference instead of electric field. Option (D), $\Omega \text{ m}$, is the unit of resistivity. Option (A), $\Omega \text{ s}^{-1}$, also does not have the dimensions of drift speed divided by electric field.

Step 3: Conclusion The SI unit is $\text{m}^2\text{V}^{-1}\text{s}^{-1}$. Hence, the correct option is (B).

Quick Tip to Solve: Write mobility as speed divided by field and use field in volts per metre; this gives the second metre factor.

9. An inductor, a capacitor and a resistor are connected in series across an ac source of voltage. If the frequency of the source is decreased gradually, the reactance of:

[1 mark]

- (A) both the inductor and the capacitor decreases.
- (B) inductor decreases and the capacitor increases.
- (C) both the inductor and the capacitor increases.
- (D) inductor increases and the capacitor decreases.

Correct Answer: (B) inductor decreases and the capacitor increases.

Solution:

Concept:

- Inductive reactance is $X_L = 2\pi fL$, where f is the source frequency and L is the inductance.
- Capacitive reactance is $X_C = 1/(2\pi fC)$, where C is the capacitance; it varies inversely with frequency.

Step 1: Compare the frequency dependence The component values L and C remain fixed. If the frequency changes from f to kf , with $0 < k < 1$, the reactances change as

$$X'_L = kX_L, \quad X'_C = \frac{X_C}{k}.$$

Thus the two reactances respond in opposite ways to the same frequency change.

Step 2: Test the alternatives The inductor offers less reactance at lower frequency, whereas the capacitor offers more. Options (A) and (C) incorrectly make both reactances change in the same direction. Option (D) reverses the two frequency dependences. The series connection and resistance do not change these individual reactance formulae.

Step 3: Conclusion The inductive reactance decreases and the capacitive reactance increases. Hence, the correct option is (B).

Quick Tip to Solve: Remember $X_L \propto f$ but $X_C \propto 1/f$; draw opposite arrows when the frequency changes.

10. In a moving coil galvanometer, the deflecting torque τ acting on the coil is related to the current I flowing through it as: [1 mark]

(A) $\tau \propto I^3$

(B) $\tau \propto I^2$

(C) $\tau \propto I$

(D) $\tau \propto \sqrt{I}$

Correct Answer: (C) $\tau \propto I$

Solution:

Concept:

- A current-carrying coil of N turns and area A in magnetic field B experiences torque $\tau = NIAB \sin \theta$, where θ is the angle between the coil normal and the field.
- In a moving coil galvanometer, the radial magnetic field keeps the coil normal perpendicular to the field, giving $\sin \theta = 1$.

Step 1: Apply the radial-field condition The deflecting torque on the coil is therefore

$$\tau = NBAI.$$

For a given galvanometer, the number of turns, coil area and magnetic field strength are fixed. Current is the only varying factor in this expression.

Step 2: Compare the proposed powers of current Doubling the current doubles the torque. A cubic dependence would multiply it by eight, a quadratic dependence by four, and a square-root dependence by $\sqrt{2}$. Thus options (A), (B) and (D) do not follow the magnetic torque law. The restoring torque balances this deflecting torque at the final deflection.

Step 3: Conclusion The deflecting torque is directly proportional to current: $\tau \propto I$. Hence, the correct option is (C).

Quick Tip to Solve: For a moving coil galvanometer, write $\tau = NBAI$ first; every factor except current is fixed.

11. In the wave picture of light, the intensity I of light is related to the amplitude A of the wave as:

[1 mark]

- (A) $I \propto \sqrt{A}$
- (B) $I \propto A$
- (C) $I \propto A^2$
- (D) $I \propto \frac{1}{A^2}$

Correct Answer: (C) $I \propto A^2$

Solution:

Concept:

- Intensity is the time-averaged energy transported per unit area per unit time by a light wave.
- For light propagating in a fixed medium, the intensity is proportional to the square of its wave amplitude: $I = KA^2$, where K is a constant for the medium and amplitude convention.

Step 1: Relate amplitude to energy flow The electromagnetic energy density depends on the squares of the electric and magnetic fields. Time averaging a sinusoidal wave changes only the numerical factor; its dependence on the square of the field amplitude remains. Hence a wave of twice the amplitude carries four times the intensity.

Step 2: Test the given relations The square-root and linear dependences in (A) and (B) do not reproduce the energy dependence. The inverse-square relation in (D) would make a larger amplitude give a smaller intensity, which is incorrect. Do not confuse inverse-square variation with distance from a point source with the relation to amplitude here.

Step 3: Conclusion The intensity varies as the square of the amplitude, $I \propto A^2$. Hence, the correct option is (C).

Quick Tip to Solve: When amplitudes have ratio $m : n$, square that ratio to obtain the intensity ratio $m^2 : n^2$.

12. Electromagnetic waves of wavelength of the order of a few meters were first produced and detected in the laboratory by:

[1 mark]

- (A) J.C. Maxwell
- (B) J.C. Bose
- (C) H.R. Hertz
- (D) G. Marconi

Correct Answer: (C) H.R. Hertz

Solution:

Concept:

- Maxwell's electromagnetic theory predicted that changing electric and magnetic fields can propagate together as electromagnetic waves.
- Hertz supplied experimental confirmation by producing electromagnetic waves using an oscillating electric discharge and detecting them with a receiving loop.

Step 1: Identify the laboratory experiment In Hertz's experiment, a spark-gap oscillator produced rapidly varying currents and emitted electromagnetic radiation with wavelengths of the order of metres. A small spark in the gap of a nearby receiving loop demonstrated detection of the emitted

waves.

Step 2: Distinguish the scientists' contributions Maxwell predicted the waves theoretically, so option (A) is not the requested experimental discovery. J. C. Bose carried out pioneering experiments with much shorter wavelengths. Marconi developed practical wireless communication. Neither of these later achievements is the first laboratory production and detection specified here.

Step 3: Conclusion H. R. Hertz first produced and detected these electromagnetic waves in the laboratory. Hence, the correct option is (C).

Quick Tip to Solve: Associate Maxwell with prediction, Hertz with first laboratory detection, and Marconi with wireless communication.

13. The emf and internal resistance of a cell are E and r respectively. It is connected across an external resistance $R = 2r$. The potential drop across the terminals of the cell will be: [1 mark]

- (A) $\frac{E}{4}$
- (B) $\frac{E}{2}$
- (C) $\frac{2E}{3}$
- (D) $\frac{E}{3}$

Correct Answer: (C) $\frac{2E}{3}$

Solution:

Concept:

- A cell of emf E and internal resistance r supplies current $I = E/(R + r)$ through an external resistance R .
- While the cell supplies current, its terminal potential difference is $V = E - Ir = IR$; the internal resistance accounts for the lost voltage Ir .

Step 1: Find the circuit current The external resistance is $R = 2r$. Since the external and internal resistances are in series, their total is $3r$. Therefore,

$$I = \frac{E}{R + r} = \frac{E}{3r}.$$

Step 2: Calculate the external voltage drop The terminals lie across the external resistance, so

$$V = IR = \frac{E}{3r}(2r) = \frac{2E}{3}.$$

The remaining $E/3$ is lost inside the cell, so option (D) gives the internal drop rather than the terminal voltage. Options (A) and (B) do not give the voltage division corresponding to $R : r = 2 : 1$.

Step 3: Conclusion The terminal potential difference is $\frac{2E}{3}$. Hence, the correct option is (C).

Quick Tip to Solve: Use $V/E = R/(R + r)$ directly; an external-to-internal resistance ratio of $2 : 1$ gives $V/E = 2/3$.

14. In a single-slit diffraction experiment, the width of the slit is halved. The width of the central maximum, in the diffraction pattern, will become: [1 mark]

- (A) half
- (B) twice
- (C) four times
- (D) one-fourth

Correct Answer: (B) twice

Solution:

Concept:

- For a single slit of width a , the first diffraction minima satisfy $a \sin \theta = \lambda$, where λ is the wavelength.
- At small diffraction angles, the central maximum has linear width $W = 2D\lambda/a$ on a screen at distance D ; it extends between the first minima on either side.

Step 1: **Keep the wavelength and screen position fixed** Only the slit width changes in the experiment. Hence D and λ are constants, and the width of the central maximum is inversely proportional to the slit width. A narrower aperture produces greater spreading of light.

Step 2: **Replace the slit width by half its value** On putting $a' = a/2$,

$$W' = \frac{2D\lambda}{a/2} = 2\frac{2D\lambda}{a} = 2W.$$

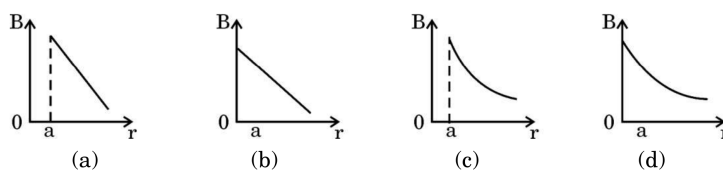
Options (A) and (D) incorrectly predict narrowing. Option (C) would require an inverse-square dependence, whereas diffraction width depends on the first inverse power of slit width.

Step 3: Conclusion The width of the central maximum becomes twice its original value. Hence, the correct option is (B).

Quick Tip to Solve: For unchanged wavelength and screen distance, use $W'a' = Wa$: halving slit width doubles diffraction width.

15. Which of the following graphs correctly represents the variation of the magnitude of the magnetic field outside a straight infinite current-carrying wire of radius a , as a function of distance r from the centre of the wire?

[1 mark]



- (A) Graph (a)
 (B) Graph (b)
 (C) Graph (c)
 (D) Graph (d)

Correct Answer: (C) Graph (c)

Solution:

Concept:

- Ampere's circuital law is $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{enc}}$, where I_{enc} is the current enclosed by the chosen path.
- Outside an infinitely long straight wire carrying current I , cylindrical symmetry gives $B = \mu_0 I / (2\pi r)$ for $r \geq a$, where a is the wire radius.

Step 1: Determine the exterior field dependence A circular path of radius $r \geq a$ encloses the entire current. The field has constant magnitude along this path and is tangential to it, so

$$B(2\pi r) = \mu_0 I, \quad B \propto \frac{1}{r}.$$

Thus the exterior graph decreases nonlinearly, becoming progressively less steep as r increases.

Step 2: Check the curve and its domain The exterior region begins at the surface $r = a$, where the field has the finite value $\mu_0 I / (2\pi a)$. Graphs (a) and (b) incorrectly show a linear decrease. Graph (d) extends the decreasing exterior form toward the centre; it does not restrict that form to the exterior region. Graph (c) begins at $r = a$ and has the required inverse-distance shape.

Step 3: Conclusion Graph (c) correctly represents $B \propto 1/r$ outside the wire. Hence, the correct option is (C).

Quick Tip to Solve: Check both shape and domain: outside a wire, the inverse-distance curve starts at $r = a$, not at its centre.

16. Assertion (A): When radius of a circular loop carrying a steady current is doubled, its magnetic moment becomes four times.

Reason (R): The magnetic moment of a circular loop carrying a steady current is proportional to the area of the loop. [1 mark]

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
(C) Assertion (A) is true, but Reason (R) is false.
(D) Assertion (A) is false and Reason (R) is also false.

Correct Answer: (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

Solution:

Concept:

- The magnitude of the magnetic dipole moment of a single current loop is $m = IS$, where I is the current and S is the enclosed area.
- For a circular loop of radius r , $S = \pi r^2$, so $m = I\pi r^2$ when the current is fixed.

Step 1: Evaluate the assertion Doubling the radius while keeping the steady current unchanged gives

$$m' = I\pi(2r)^2 = 4I\pi r^2 = 4m.$$

The magnetic moment therefore becomes four times its initial value, so the assertion is true.

Step 2: Evaluate the reason From $m = IS$, the magnetic moment is directly proportional to the enclosed area at fixed current. This is precisely the relation stated in the reason, so the reason is true.

Step 3: Test the explanatory link Doubling the radius multiplies the area by four, and direct proportionality to area then multiplies the magnetic moment by four. Thus the reason supplies the physical relation needed to explain the assertion. Option (B) incorrectly denies this link, while (C) and (D) incorrectly classify one or both statements as false.

Step 4: Conclusion Both statements are true, and the reason correctly explains the assertion. Hence, the correct option is (A).

Quick Tip to Solve: At fixed current, compare loop areas: multiplying a circular loop's radius by k multiplies its magnetic moment by k^2 .

17. Assertion (A): The nucleus 7_3X is more stable than the nucleus 4_3Y .

Reason (R): 7_3X contains more number of protons.

[1 mark]

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
- (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
- (C) Assertion (A) is true, but Reason (R) is false.
- (D) Assertion (A) is false and Reason (R) is also false.

Correct Answer: (C) Assertion (A) is true, but Reason (R) is false.

Solution:

Concept:

- In the notation A_ZX , the proton number is Z , while the neutron number is $N = A - Z$.
- Light stable nuclei generally have comparable neutron and proton numbers; a strongly proton-rich nucleus is unstable.

Step 1: Evaluate the assertion For 7_3X , $Z = 3$ and $N = 7 - 3 = 4$. For 4_3Y , $Z = 3$ and $N = 4 - 3 = 1$. The former has a much more favourable neutron-to-proton balance than the strongly proton-rich latter. Thus the assertion is true.

Step 2: Evaluate the reason Both nuclei have the same atomic number, $Z = 3$, so each contains exactly three protons. The larger mass number of X represents extra neutrons, not extra protons. The reason is false.

Step 3: Check the explanatory link A false statement about proton number cannot explain the stability difference. Options (A) and (B) incorrectly treat the reason as true; option (D) incorrectly treats the assertion as false.

Step 4: Conclusion Assertion is true, but Reason is false. Hence, the correct option is (C).

Quick Tip to Solve: Read the lower index first to count protons; subtract it from the upper index to count neutrons.

18. Assertion (A): The internal resistance of a cell is constant.

Reason (R): Ionic concentration of the electrolyte remains same during use of a cell.

[1 mark]

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
- (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
- (C) Assertion (A) is true, but Reason (R) is false.
- (D) Assertion (A) is false and Reason (R) is also false.

Correct Answer: (D) Assertion (A) is false and Reason (R) is also false.

Solution:

Concept:

- Internal resistance is the opposition to current inside a cell, including conduction through its electrolyte.

- Electrolyte conductivity depends on ionic concentration and temperature; chemical reactions during discharge change the cell's condition.

Step 1: Evaluate the assertion A cell's internal resistance is not an unchanging property. It depends on electrolyte concentration, temperature, electrode separation and immersed area. During use its chemical state changes, so its internal resistance can change. Treating it as constant in a short circuit calculation is an approximation. The assertion is false.

Step 2: Evaluate the reason Chemical reactions consume and produce species as the cell supplies current. Consequently, the ionic concentration of the electrolyte does not in general remain the same during use. The reason is also false.

Step 3: Check the explanatory link The stated constancy of ionic concentration cannot explain a constant internal resistance because both claims are false. Options (A) and (B) require a true reason, and option (C) requires a true assertion; none describes this cell.

Step 4: Conclusion Both the assertion and the reason are false. Hence, the correct option is (D).

Quick Tip to Solve: Distinguish an assumed constant internal resistance in a circuit model from the changing condition of a working cell.

Section B (14 marks)

19. How is the width of depletion layer of a p-n junction diode affected when it is (i) forward biased, and (ii) reverse biased? Justify your answers. [2 marks]

Solution:

Concept:

- The depletion layer contains exposed, immobile donor and acceptor ions after mobile electrons and holes diffuse and recombine near a p-n junction.
- Its electric field creates a potential barrier opposing further majority-carrier diffusion; an applied bias changes this barrier and the layer width.

Step 1: Apply forward bias Connect the p-side to the positive terminal and the n-side to the negative terminal. The applied field opposes the junction field and reduces the potential barrier from V_0 to approximately $V_0 - V_F$, where V_F is the forward voltage. Majority carriers move towards the junction, neutralising exposed ions near the depletion edges. Thus the depletion-layer width decreases and majority carriers cross the junction more easily.

Step 2: Apply reverse bias Connect the p-side to the negative terminal and the n-side to the positive terminal. The applied field reinforces the junction field, increasing the barrier to approximately $V_0 + V_R$, where V_R is the magnitude of the reverse voltage. Majority carriers are pulled away from the junction, exposing additional fixed ions. Therefore the depletion region widens and majority-carrier conduction is strongly suppressed.

Step 3: Conclusion Forward bias decreases the depletion-layer width by lowering the barrier; reverse bias increases the width by raising the barrier.

Quick Tip to Solve: Follow the majority carriers: towards the junction means a narrower layer, away from it means a wider layer.

20. Explain the property of a p-n junction which makes it suitable for rectifying alternating voltages. Differentiate between a half-wave and a full-wave rectifier. [2 marks]

Solution:

Concept:

- Rectification converts an alternating input into an output whose current through the load is unidirectional.
- A p-n junction offers low resistance in forward bias and very high resistance in reverse bias, provided reverse breakdown is avoided.

Step 1: Explain the rectifying property During the half-cycle that forward biases a diode, it conducts appreciable current. During the opposite half-cycle, reverse bias permits only a very small leakage current, which is neglected in the ideal model. This unequal conduction in the two directions lets the junction select or redirect alternate half-cycles of an alternating voltage.

Step 2: Describe half-wave rectification A half-wave rectifier uses only one half-cycle of each input cycle. Its load current is a pulse during the conducting half-cycle and approximately zero during the other. There is one output pulse per input cycle, so the ripple frequency equals the input frequency f .

Step 3: Describe full-wave rectification A full-wave rectifier uses both half-cycles, steering the load current in the same direction during each. There are two output pulses per input cycle, so the ripple frequency is $2f$. It provides a larger average output for the same peak load voltage and a smoother output after filtering. The unfiltered output of either type remains pulsating, rather than perfectly steady, direct voltage.

Step 4: Conclusion The junction's directional conduction enables rectification. Half-wave rectification uses one half-cycle, whereas full-wave rectification uses both and doubles the ripple frequency.

Quick Tip to Solve: Count load pulses in one AC cycle: one gives half-wave output at f , and two give full-wave output at $2f$.

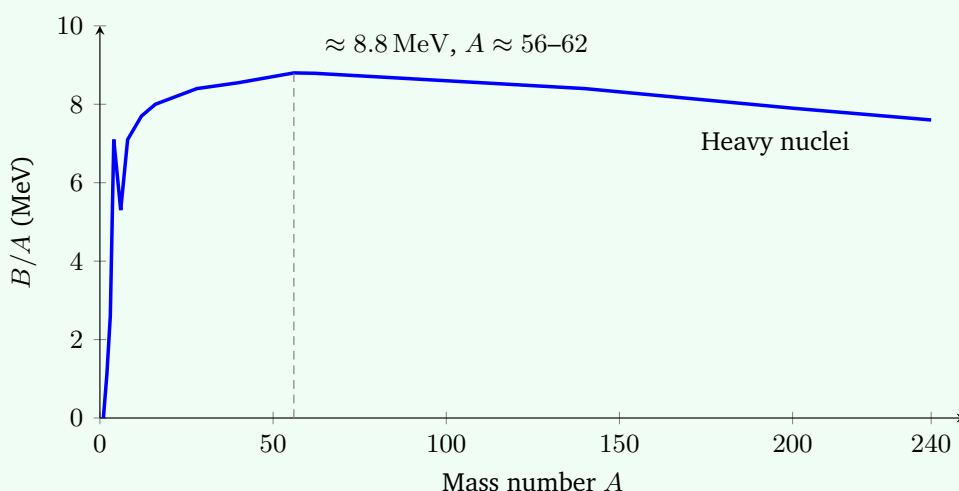
21(a). Draw a graph showing the variation of binding energy per nucleon as a function of mass number A . The binding energy per nucleon for heavy nuclei ($A > 170$) decreases with the increase in mass number. Explain. [2 marks]

Solution:

Concept:

- Binding energy per nucleon is B/A , where B is the energy required to separate the nucleus into free nucleons and A is the total nucleon number.
- The strong nuclear force is attractive, short ranged and saturating, whereas electrostatic repulsion between protons is long ranged.

Step 1: Sketch the binding-energy curve Plot mass number A on the horizontal axis and binding energy per nucleon B/A , in MeV, on the vertical axis. The curve rises rapidly for light nuclei, with irregularities for the lightest ones. It reaches a broad maximum of about 8.8 MeV near $A \approx 56-62$, then decreases slowly towards roughly 7.6 MeV for the heaviest nuclei. The sketch shows the overall trend.



Schematic variation of binding energy per nucleon with mass number.

Step 2: Explain the decline for heavy nuclei A nucleon attracts mainly its nearby neighbours because the strong force has a short range. Therefore adding more distant nucleons does not give a proportionate increase in attractive binding per nucleon: nuclear attraction saturates. As the nucleus grows, however, the number of protons increases and their long-range Coulomb repulsion becomes increasingly important. This reduces the net binding per nucleon, accounting for the gradual fall for heavy nuclei with $A > 170$.

Step 3: Conclusion The curve peaks near the iron-nickel region and falls slowly for heavy nuclei because increasing Coulomb repulsion competes with the saturating nuclear attraction.

Quick Tip to Solve: Label both axes and the broad maximum; explain the heavy-nucleus fall by contrasting short-range attraction with long-range proton repulsion.

OR

21(b). Using Bohr's postulates, obtain the expression for radius of n^{th} stable orbit in a hydrogen atom. [2 marks]

Solution:

Concept:

- In a stationary circular orbit, the electrostatic attraction between the proton and electron supplies the centripetal force. The electron does not radiate energy in an allowed orbit.
- Bohr's angular-momentum condition is $m_e v r = n \hbar = n h / (2\pi)$, where $n = 1, 2, \dots$, m_e is the electron mass and h is Planck's constant.

Step 1: Apply the force condition For the hydrogen nucleus of charge $+e$, an electron of charge $-e$ moving with speed v at radius r obeys

$$\frac{m_e v^2}{r} = \frac{e^2}{4\pi\epsilon_0 r^2}, \quad m_e v^2 r = \frac{e^2}{4\pi\epsilon_0}.$$

Here ϵ_0 is the permittivity of free space; the much heavier nucleus is treated as stationary.

Step 2: Impose angular-momentum quantisation From Bohr's condition, $v = n h / (2\pi m_e r)$. Substitut-

ing this into the force equation gives

$$\frac{n^2 h^2}{4\pi^2 m_e r} = \frac{e^2}{4\pi\epsilon_0}.$$

Thus the permitted radii are discrete, and their dependence on the principal quantum number is proportional to n^2 .

Step 3: Conclusion The radius of the n^{th} stable orbit is

$$r_n = \frac{\epsilon_0 h^2}{\pi m_e e^2} n^2 = \frac{4\pi\epsilon_0 \hbar^2}{m_e e^2} n^2 = a_0 n^2,$$

where $a_0 = 5.29 \times 10^{-11} \text{ m}$ is the Bohr radius of hydrogen.

Quick Tip to Solve: Eliminate speed using $v = nh/(2\pi m_e r)$; keep the entire denominator squared before cancelling one factor of r .

22. What is meant by the term ‘displacement current’? Briefly explain how this current is different from a conduction current. [2 marks]

Solution:

Concept:

- A time-varying electric field contributes to the magnetic field in the same way as an electric current. Maxwell called this contribution displacement current.
- In vacuum, $I_d = \epsilon_0 d\Phi_E/dt$, where $\Phi_E = \int \vec{E} \cdot d\vec{A}$ is the electric flux through the chosen surface.

Step 1: Define displacement current Displacement current is the effective current associated with changing electric flux, even in a region where no charges cross the surface. Its magnitude in vacuum is $I_d = \epsilon_0 d\Phi_E/dt$. For example, between the plates of a charging capacitor the electric field changes with time, producing a displacement current and an associated magnetic field.

Step 2: Contrast it with conduction current Conduction current is due to the actual transport of free charge carriers: $I_c = dQ/dt$. In a metal it arises from the drift of electrons. Displacement current does not require the transport of free charges across the region and can exist in vacuum or an insulating gap. Thus there is conduction current in the connecting wires of a charging capacitor, but displacement current across its ideal insulating gap. Both act as sources of magnetic field; during charging their values are equal for corresponding surfaces.

Step 3: Conclusion $I_d = \epsilon_0 d\Phi_E/dt$ describes the current due to changing electric flux. Conduction current involves free-charge flow, whereas displacement current can exist without such flow.

Quick Tip to Solve: Use a charging capacitor to distinguish the two: electrons drift in the wires, while changing electric flux supplies the current contribution across the gap.

23. Two wires of equal lengths are shaped in the form of a square loop and a circular loop. Both loops are suspended in a uniform magnetic field. Prove that for the same current, the circular loop will experience larger torque. [2 marks]

Solution:

Concept:

- The torque on a single-turn current loop in a uniform magnetic field is $\tau = IAB \sin \theta$, where A is its enclosed area and θ is the angle between its area normal and $\vec{B}$.
- For equal currents, equal magnetic fields and equal orientations, torque is proportional to enclosed area. The comparison assumes equal nonzero $\sin \theta$, or compares the maximum torques.

Step 1: Calculate both areas from the common wire length Let each wire have length L . The square has side $a = L/4$, so

$$A_s = a^2 = \frac{L^2}{16}.$$

The circle has circumference $2\pi r = L$, giving $r = L/(2\pi)$ and

$$A_c = \pi r^2 = \frac{L^2}{4\pi}.$$

Both wires form single-turn loops, so no additional turns factor is needed.

Step 2: Compare the magnetic torques With the same I , B and angle θ , the magnitudes are

$$\tau_s = \frac{IBL^2 \sin \theta}{16}, \quad \tau_c = \frac{IBL^2 \sin \theta}{4\pi}.$$

For a nonzero torque, division cancels every common factor, leaving

$$\frac{\tau_c}{\tau_s} = \frac{A_c}{A_s} = \frac{4}{\pi} \simeq 1.27 > 1.$$

If both normals are parallel to the field, both torques vanish; arbitrary unequal orientations cannot be compared using area alone.

Step 3: Conclusion At the same nonzero torque-producing orientation, the circular loop experiences $4/\pi$ times the torque on the square loop. Its maximum torque is also larger by this factor because it encloses a larger area for the same wire length.

Quick Tip to Solve: Express both areas using the same perimeter L , then cancel $IB \sin \theta$; equal wire lengths do not mean equal enclosed areas.

24(a). State Huygens' principle. How did Huygens explain the absence of the backwave? [2 marks]

Solution:

Concept:

- A wavefront is a surface of constant phase. The direction of propagation is normal to the wavefront in an isotropic medium.
- Huygens' construction locates a later wavefront using secondary wavelets emitted from the points of an earlier wavefront.

Step 1: State Huygens' principle Every point on a given wavefront acts as a source of secondary

spherical wavelets. These wavelets spread through the medium with the wave speed. After a time interval Δt , each wavelet has radius $v\Delta t$, where v is the wave speed. The forward common tangent surface, or envelope, of these wavelets gives the position of the new wavefront.

Step 2: Explain the absence of a backward wave A purely geometrical construction of spherical wavelets also permits a backward envelope. To exclude it, Huygens assumed that the amplitude of the secondary wavelets is maximum in the forward direction and zero in the backward direction. The backward envelope therefore carries no disturbance and is not a physical backwave. This directional-amplitude assumption is additional to the elementary geometrical construction; the forward envelope describes the advancing wave.

Step 3: Conclusion Secondary wavelets and their forward envelope determine the new wavefront. The backwave is excluded by taking the secondary-wavelet amplitude to be zero in the backward direction.

Quick Tip to Solve: State both parts of the principle: secondary wavelet sources and their forward envelope. Then explicitly mention zero backward amplitude.

OR

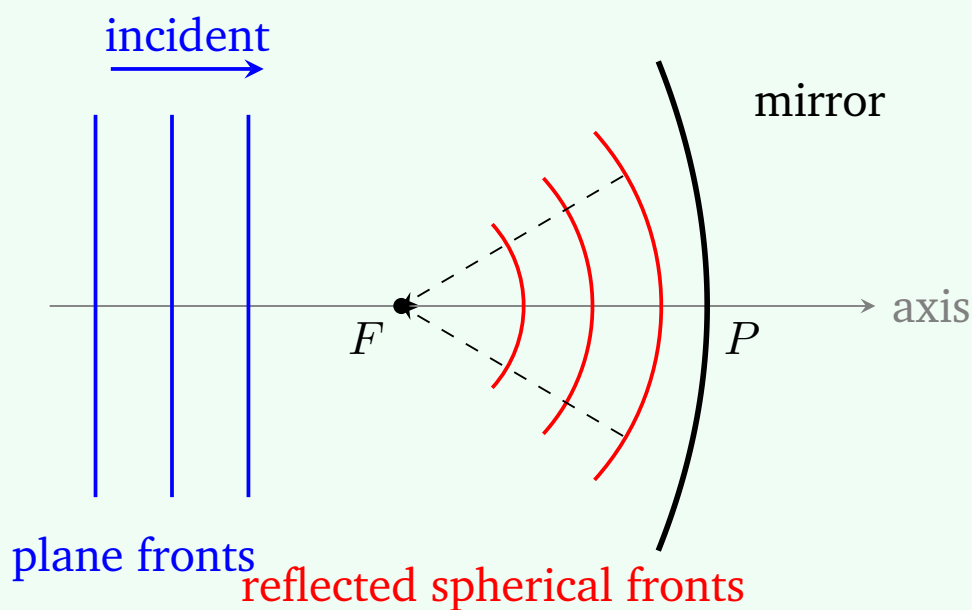
24(b). Use Huygens' principle to show reflection/refraction of a plane wave by (i) concave mirror, and (ii) a convex lens. [2 marks]

Solution:

Concept:

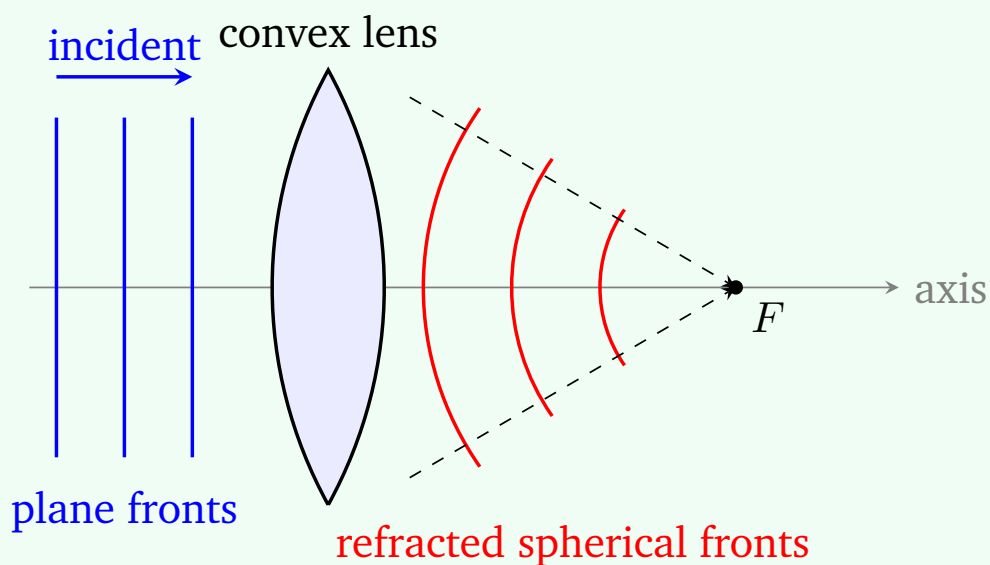
- Every point of a wavefront emits secondary wavelets; their forward common envelope gives the next wavefront. Rays are normal to the wavefront.
- A plane wave has parallel rays. A converging spherical wavefront has inward normals directed towards its centre, the focus. The constructions below use paraxial rays and a thin convex lens.

Step 1: Reflection at a concave mirror Let a plane wave travel towards a concave mirror along its principal axis; the incident plane wavefront is perpendicular to the axis. The outer portions of the wavefront meet the mirror first and begin producing reflected secondary wavelets before the central portion reaches the pole. At a later common instant, draw the reflected wavelets with radii proportional to their elapsed travel times. Their envelope is a converging spherical wavefront centred at the principal focus F , in front of the mirror. The reflected disturbance travels back towards F ; its normals meet there. For a spherical mirror this construction is paraxial, with $f \simeq R/2$.



Concave mirror: plane incident fronts become spherical reflected fronts converging towards F . Dashed lines show normals to the reflected fronts.

Step 2: Refraction through a convex lens A plane wavefront incident normally on a convex lens is delayed more near its thicker centre because light travels more slowly in glass than in air. Using the secondary wavelets from the emergent disturbance, the common envelope is therefore a converging spherical wavefront. Its centre is the principal focus F on the far side of the lens. The normals to this refracted wavefront point towards F , where the transmitted wave converges. Successive fronts approach F with decreasing radii.



Convex lens: emergent spherical wavefronts converge towards the principal focus F on the far side.

Step 3: Conclusion A concave mirror reflects a plane wave into a converging spherical wave focused on the incident side. A convex lens refracts it into a converging spherical wave focused on the opposite side.

Quick Tip to Solve: Draw the focus as the centre of curvature of the converging wavefronts, and put arrows towards it: back towards the source for the mirror, forward for the lens.

25. The refractive indices of two media A and B are 2 and $\sqrt{2}$ respectively. What is the critical angle for their interface? [2 marks]

Solution:

Concept:

- The critical angle c is the angle of incidence in the optically denser medium for which the refracted ray in the rarer medium makes 90° with the normal.
- Snell's law gives $n_d \sin c = n_r \sin 90^\circ$, so $\sin c = n_r/n_d$. Here n_d and n_r denote the denser and rarer refractive indices.

Step 1: Identify the direction of incidence Since $n_A = 2 > n_B = \sqrt{2}$, medium A is optically denser. A critical angle exists for light incident from A towards B . For incidence from B towards A , the ray bends towards the normal and cannot have a refracted angle of 90° .

Step 2: Apply Snell's law at the limiting refraction At the critical angle, the refracted ray travels along the interface. Therefore,

$$2 \sin c = \sqrt{2} \sin 90^\circ, \quad \sin c = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}.$$

Taking the inverse sine in the physical interval $0^\circ < c < 90^\circ$ selects the required incidence angle.

Step 3: Conclusion The critical angle is $c = 45^\circ$, for light travelling from medium A to medium B .

Quick Tip to Solve: Put the smaller refractive index in the numerator; a critical angle is defined for incidence from the denser medium.

Section C (15 marks)

26. (a) A particle of mass m and charge q is accelerated through a potential difference V . Plot a graph of de Broglie wavelength λ associated with it as a function of V .

(b) Calculate the energy acquired by, and de Broglie wavelength associated with, an electron accelerated through a potential difference of 400 V.

[3 marks]

Solution:

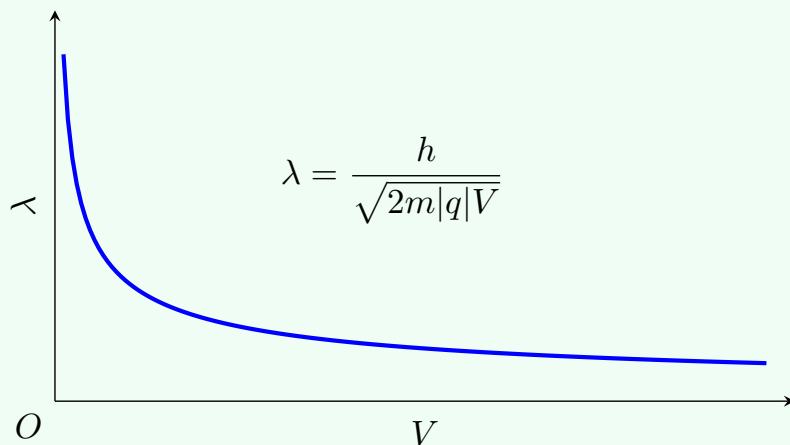
Concept:

- A particle accelerated from rest through an accelerating potential difference of magnitude V gains kinetic energy $K = |q|V$, where $|q|$ is its charge magnitude.
- In the non-relativistic approximation, $p = \sqrt{2mK}$ and the de Broglie wavelength is $\lambda = h/p$, where h is Planck's constant and m is the particle mass.

Step 1: (a) Obtain the wavelength dependence and graph Equating electric work to kinetic energy gives

$$\frac{p^2}{2m} = |q|V, \quad \lambda = \frac{h}{\sqrt{2m|q|V}}.$$

Thus $\lambda \propto V^{-1/2}$ for fixed mass and charge. The graph is a decreasing curve in the first quadrant, steep at small positive V and progressively flatter as V increases. It is not a straight line: quadrupling V halves λ . The ideal non-relativistic relation diverges as $V \rightarrow 0^+$; the curve does not pass through the origin. The plotted curve shows this functional shape over positive accelerating voltages.



Schematic de Broglie wavelength versus accelerating potential difference

Step 2: (b) Calculate the energy gained by the electron Using the magnitude of the electron charge, $e = 1.60 \times 10^{-19} \text{ C}$,

$$K = eV = (1.60 \times 10^{-19})(400) = 6.40 \times 10^{-17} \text{ J} = 400 \text{ eV}.$$

The negative sign of electron charge specifies the direction of acceleration; the kinetic energy gained is positive.

Step 3: (b) Substitute into the de Broglie relation With $h = 6.626 \times 10^{-34} \text{ J s}$ and $m_e = 9.11 \times 10^{-31} \text{ kg}$,

$$p = \sqrt{2(9.11 \times 10^{-31})(6.40 \times 10^{-17})} = 1.080 \times 10^{-23} \text{ kg m s}^{-1},$$

$$\lambda = \frac{6.626 \times 10^{-34}}{1.080 \times 10^{-23}} = 6.14 \times 10^{-11} \text{ m}.$$

The energy is much smaller than the electron rest energy, so the non-relativistic calculation used here is appropriate.

Step 4: Conclusion (a) The graph follows $\lambda \propto V^{-1/2}$. For the electron in (b),

$$K = 400 \text{ eV} = 6.40 \times 10^{-17} \text{ J},$$

$$\lambda = 6.14 \times 10^{-11} \text{ m} = 0.0614 \text{ nm}.$$

Quick Tip to Solve: For an electron accelerated from rest, use $\lambda(\text{nm}) \simeq 1.227/\sqrt{V(\text{V})}$; at 400 V, divide by 20.

27(a). An ac source $V = V_m \sin \omega t$ is connected across an ideal capacitor. Derive the expression for the (i) current flowing in the circuit, and (ii) reactance of the capacitor. Plot a graph of current i versus ωt . [3 marks]

Solution:

Concept:

- For an ideal capacitor of capacitance C , the instantaneous charge is $Q = CV$. The circuit current is the rate of charge transfer, $i = dQ/dt$.
- Capacitive reactance X_C is the ratio of voltage amplitude to current amplitude: $X_C = V_m/I_m$. It also equals the ratio of the corresponding rms values.

Step 1: (i) Express the instantaneous capacitor charge The applied voltage is $V = V_m \sin \omega t$, where V_m is its peak value and ω is the angular frequency. The ideal capacitor therefore carries charge

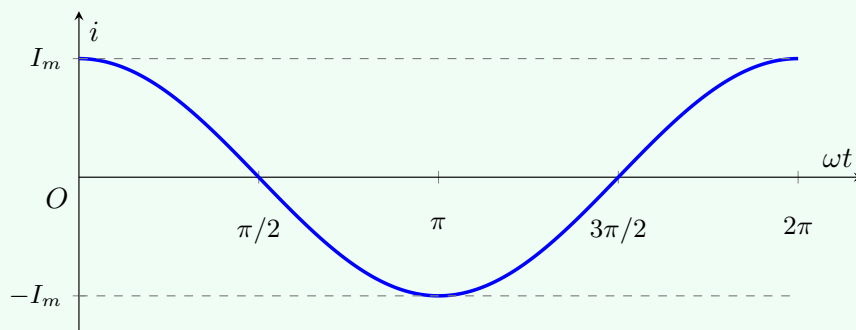
$$Q(t) = CV_m \sin \omega t.$$

Differentiating this charge gives the current in the external circuit.

Step 2: (i) Differentiate and identify the current phase

$$\begin{aligned} i(t) &= \frac{dQ}{dt} = \omega CV_m \cos \omega t \\ &= I_m \sin \left(\omega t + \frac{\pi}{2} \right), \quad I_m = \omega CV_m. \end{aligned}$$

The current leads the applied voltage by $\pi/2$, or one quarter of a cycle. The graph of i against ωt is a cosine curve: it begins at $+I_m$, crosses zero at $\pi/2$, reaches $-I_m$ at π , crosses zero again at $3\pi/2$, and returns to $+I_m$ at 2π .



Current in an ideal capacitor as a function of phase

Step 3: (ii) Derive the capacitive reactance Comparing the current amplitude with $I_m = V_m/X_C$ gives

$$X_C = \frac{V_m}{I_m} = \frac{V_m}{\omega CV_m} = \frac{1}{\omega C} = \frac{1}{2\pi f C}.$$

Here f is the frequency in hertz and X_C is measured in ohms. Increasing frequency or capacitance increases the current amplitude for a fixed voltage amplitude, and hence reduces capacitive reactance.

Step 4: Conclusion $i = \omega CV_m \cos \omega t = I_m \sin(\omega t + \pi/2)$, with $I_m = \omega CV_m$; current leads voltage by 90° . The capacitive reactance is $X_C = 1/(\omega C)$.

Quick Tip to Solve: Differentiate $Q = CV_m \sin \omega t$: sine becomes cosine, so the current graph must start at its positive maximum.

27(b). A series combination of an inductor L , a capacitor C and a resistor R is connected across an ac source of voltage in a circuit. Obtain an expression for the average power consumed by the circuit. Find power factor for (i) purely inductive circuit, and (ii) purely resistive circuit. [3 marks]

Solution:

Concept:

- Instantaneous electrical power is $p = vi$. Average power is its mean over one complete cycle of the alternating source.
- For a series LCR circuit, $Z = \sqrt{R^2 + (X_L - X_C)^2}$, where $X_L = \omega L$, $X_C = 1/(\omega C)$ and $\cos \phi = R/Z$ is the power factor.

Step 1: Express voltage, current and instantaneous power Choose the source voltage as $v = V_m \sin \omega t$. In the steady state the current is $i = I_m \sin(\omega t - \phi)$, where $\tan \phi = (X_L - X_C)/R$. The phase angle may be negative for a capacitive circuit. Multiplying voltage and current and using the product-to-sum identity gives

$$\begin{aligned} p(t) &= V_m I_m \sin \omega t \sin(\omega t - \phi) \\ &= \frac{V_m I_m}{2} [\cos \phi - \cos(2\omega t - \phi)]. \end{aligned}$$

Step 2: Average over a complete cycle The oscillatory cosine has zero average over a complete source period, whereas $\cos \phi$ is constant. Using $V_{\text{rms}} = V_m/\sqrt{2}$, $I_{\text{rms}} = I_m/\sqrt{2}$ and $V_{\text{rms}} = I_{\text{rms}} Z$,

$$\begin{aligned} P_{\text{av}} &= \frac{V_m I_m}{2} \cos \phi = V_{\text{rms}} I_{\text{rms}} \cos \phi \\ &= I_{\text{rms}}^2 Z \frac{R}{Z} = I_{\text{rms}}^2 R. \end{aligned}$$

Thus only the resistor consumes net energy; ideal inductors and capacitors store energy during part of a cycle and return it during another part.

Step 3: Evaluate the two limiting power factors (i) In a purely inductive circuit, current lags voltage by $\phi = \pi/2$. Therefore $\cos \phi = 0$, and the average power is zero despite nonzero voltage and current. (ii) In a purely resistive circuit, voltage and current are in phase, so $\phi = 0$. Therefore $\cos \phi = 1$, and $P_{\text{av}} = V_{\text{rms}} I_{\text{rms}}$.

Step 4: Conclusion The average power is $P_{\text{av}} = V_{\text{rms}} I_{\text{rms}} \cos \phi = I_{\text{rms}}^2 R$. The power factors are $\boxed{0}$ for a pure inductor and $\boxed{1}$ for a pure resistor.

Quick Tip to Solve: Average the product of voltage and current, not their separate averages: retain the constant term and drop the double-frequency cosine.

28(a). (i) Prove that the nuclear density is same for all nuclei.

(ii) Draw a plot of potential energy of a pair of nucleons as a function of their separation. Draw two inferences from this plot. [3 marks]

Solution:

Concept:

- A nucleus of mass number A has radius $R = R_0 A^{1/3}$, with $R_0 \simeq 1.2 \times 10^{-15}$ m, and approximate mass $M = Am_N$, where m_N is the mean nucleon mass.
- Nuclear density is mass divided by nuclear volume. The radial nuclear force follows from the pair potential: $F_r = -dU/dr$, with outward force taken positive.

Step 1: Prove the independence of nuclear density from mass number (i) Approximating the nucleus as a sphere, its volume is

$$V = \frac{4}{3}\pi R^3 = \frac{4}{3}\pi R_0^3 A.$$

Hence its mean density is

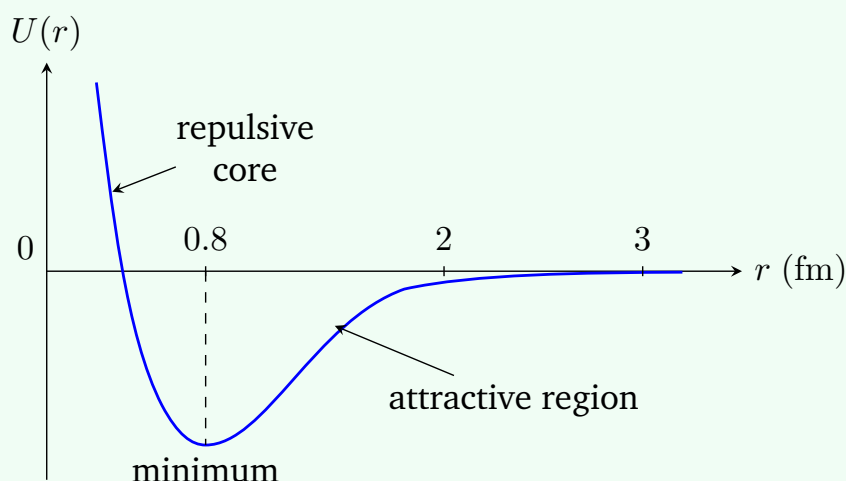
$$\rho = \frac{M}{V} = \frac{Am_N}{(4/3)\pi R_0^3 A} = \frac{3m_N}{4\pi R_0^3}.$$

The mass number cancels because both mass and volume increase in direct proportion to A . For $m_N \simeq 1.67 \times 10^{-27}$ kg,

$$\rho \simeq \frac{3(1.67 \times 10^{-27})}{4\pi(1.2 \times 10^{-15})^3} \simeq 2.3 \times 10^{17} \text{ kg m}^{-3}.$$

Thus the mean nuclear density is approximately the same for all nuclei within the empirical radius law; small binding-energy and surface corrections are neglected.

Step 2: Sketch the nucleon pair potential (ii) Set the potential energy to zero for nucleons very far apart. The qualitative nuclear potential rises steeply at very small separation, has a negative minimum at a separation of about $r_0 = 0.8$ fm, and approaches zero beyond a few femtometres. Here $1 \text{ fm} = 10^{-15} \text{ m}$; the vertical scale in the sketch is schematic.



Qualitative nuclear potential; the minimum is near 0.8 fm.

Step 3: Read two physical inferences from the plot First, the nuclear force is attractive on the outer side of the minimum: there $dU/dr > 0$, so $F_r < 0$ acts to reduce separation. Its effect is appreciable only over a few femtometres; the nearly flat potential at larger distances shows that it is a short-range force.

Second, at separations smaller than r_0 , the potential rises rapidly as the nucleons approach each other.

Here $dU/dr < 0$, hence $F_r > 0$: a strong repulsive core prevents the nucleons from collapsing into one another. The change between attraction and repulsion occurs at the potential minimum, not at the point where $U = 0$.

Step 4: Conclusion $\rho = 3m_N/(4\pi R_0^3)$ is independent of A . The pair potential exhibits short-range attraction and a strongly repulsive core at very small separation.

Quick Tip to Solve: Use the slope of the potential, not its sign, to infer the force: a rising curve means attraction and a falling curve means repulsion.

OR

28(b). (i) Draw a graph to show the variation of the number of scattered particles detected (N) in Geiger-Marsden experiment as a function of scattering angle (θ).

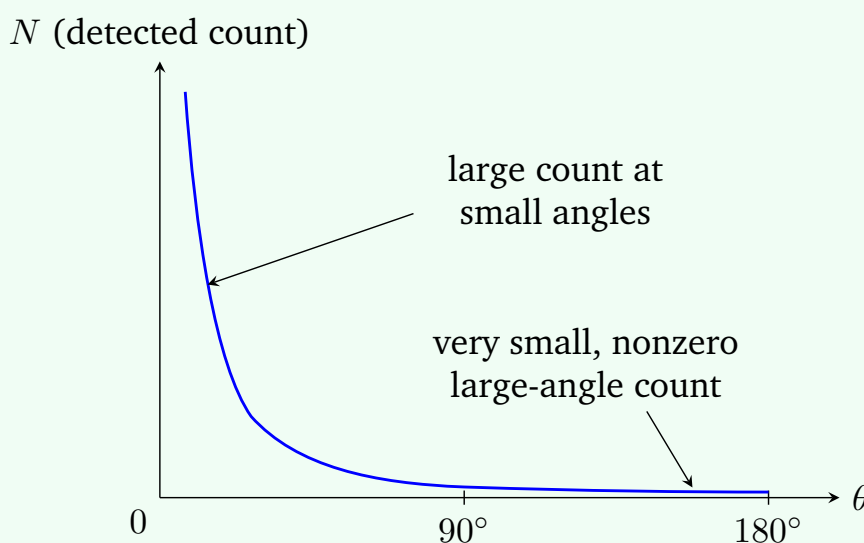
(ii) Discuss briefly two conclusions that can be drawn from this graph and how they lead to the discovery of nucleus in an atom. [3 marks]

Solution:

Concept:

- In the Geiger-Marsden experiment, a narrow beam of fast α -particles strikes a thin metal foil, and a detector counts scattered particles at different angles θ .
- For equal detector solid angle and counting time, Rutherford scattering gives $N(\theta) \propto 1/\sin^4(\theta/2)$. Large deflections arise from close encounters with a concentrated positive charge.

Step 1: Draw and interpret the angular distribution (i) Plot the number detected N vertically and the scattering angle θ horizontally. The count is very large at small angles, falls steeply as the angle increases, and remains very small but nonzero at large angles, including near 180° . Most incident particles pass through the foil with no appreciable deflection. The sketch shows the qualitative shape; the ideal scattering formula is not used at exactly $\theta = 0$, where the direct incident beam is present.



Qualitative scattered count versus angle; the large-angle count is small but nonzero.

Step 2: Infer the amount of empty space in an atom (ii) The dominance of undeflected or slightly deflected particles means that most trajectories through an atom encounter very little concentrated

matter. Therefore most of the atomic volume is empty space. If the positive charge and mass occupied the entire atom densely, the observed overwhelming transmission with tiny deflections would not occur in this manner.

Step 3: Infer a small, massive, positively charged nucleus A very small fraction of α -particles undergo large-angle scattering or even return backwards. Such a large change in momentum requires an intense repulsive electric field from a compact positive charge. Electrons are far too light to reverse the motion of these heavy, fast projectiles. Thus almost all the atomic mass and positive charge are concentrated in a tiny central region, the nucleus.

The rarity of these events means that only a very small fraction of trajectories approach this region closely: the nucleus occupies a minute fraction of the atomic volume. Together, the common small deflections and rare large deflections led to Rutherford's nuclear model of the atom.

Step 4: Conclusion The count falls rapidly with scattering angle, following $N \propto \csc^4(\theta/2)$ under equal detection conditions. The atom is mostly empty space, with almost all its mass and positive charge in a very small nucleus.

Quick Tip to Solve: Link the tall small-angle part to empty space, and the rare but nonzero backward scattering to the compact, massive positive nucleus.

29. A rectangular loop of sides 25 cm and 20 cm is lying in x - y plane. It is subjected to a magnetic field $\vec{B} = (5t^2 + 2t + 10)\hat{k}$, where B is in tesla and t is in seconds. If the resistance of the loop is $4\ \Omega$, find the emf induced and the induced current in the loop at $t = 5$ s. [3 marks]

Solution:

Concept:

- Magnetic flux through a plane loop is $\Phi_B = \vec{B} \cdot \vec{A} = BA \cos \theta$, where A is its area and θ is the angle between the field and the area normal.
- Faraday's law for a single loop is $\varepsilon = -d\Phi_B/dt$. The minus sign expresses Lenz's law: the induced current opposes the change of flux.
- Ohm's law gives the induced current as $I = \varepsilon/R$, where R is the loop resistance; current magnitude is $|\varepsilon|/R$.

Step 1: Express the flux through the loop Convert both lengths to metres before calculating the area:

$$A = (25 \times 10^{-2})(20 \times 10^{-2}) = 0.050\text{ m}^2.$$

The loop lies in the x - y plane. Choose its positive normal along $+\hat{k}$, parallel to the applied field, so $\theta = 0$ and the full field contributes to the flux:

$$\Phi_B(t) = 0.050(5t^2 + 2t + 10)\text{ Wb}.$$

Step 2: Differentiate the flux and evaluate the emf The loop has fixed area, so only the time dependence of the field contributes. The constant term 10 makes no contribution to the induced emf:

$$\varepsilon(t) = -0.050 \frac{d}{dt}(5t^2 + 2t + 10) = -0.050(10t + 2).$$

At the specified instant,

$$\varepsilon(5) = -0.050(50 + 2) = -2.60 \text{ V}.$$

This signed value uses the circulation direction associated with the chosen $+\hat{k}$ normal.

Step 3: Calculate the current and determine its direction Divide the emf magnitude by the given resistance:

$$|I| = \frac{2.60}{4} = 0.65 \text{ A}.$$

At $t = 5 \text{ s}$, $dB/dt = 52 \text{ T s}^{-1} > 0$, so the flux along $+\hat{k}$ is increasing. The induced field must point along $-\hat{k}$. By the right-hand rule the current is therefore clockwise when viewed from the positive z -axis toward the loop.

Step 4: Conclusion The induced emf has magnitude 2.60 V and the induced current has magnitude 0.65 A . The current is clockwise as viewed from the positive z -axis; with the $+\hat{k}$ normal convention, $\varepsilon = -2.60 \text{ V}$.

Quick Tip to Solve: For a fixed loop perpendicular to a changing field, use $|\varepsilon| = A|dB/dt|$: differentiate the field first, then substitute the time.

30. A potential difference 'V' is applied across a load resistor of resistance R. V and R can be varied. If the current that flows in the circuit is I, draw a plot showing the variation of power consumed by the resistor as a function of:

- (a) R, keeping V constant
- (b) I, keeping R constant
- (c) V, keeping R constant

[3 marks]

Solution:

Concept:

- Electrical power consumed by a resistor is $P = VI$, where V is the potential difference and I is the current. Power is measured in watts.
- Using Ohm's law, $V = IR$, the equivalent expressions are $P = V^2/R$ and $P = I^2R$. Select the form containing the quantity held constant and the quantity on the horizontal axis.

Step 1: Plot power against resistance at constant voltage For part (a), keep V fixed and write

$$P = \frac{V^2}{R}, \quad PR = V^2 = \text{constant}.$$

Hence P is inversely proportional to R , and the graph is a decreasing rectangular hyperbola in the positive quadrant. Doubling the resistance halves the power. For nonzero fixed voltage, the curve approaches both axes asymptotically and never passes through the origin; the ideal expression is defined for $R > 0$.

Step 2: Plot power against current at constant resistance For part (b), keep the positive resistance R fixed:

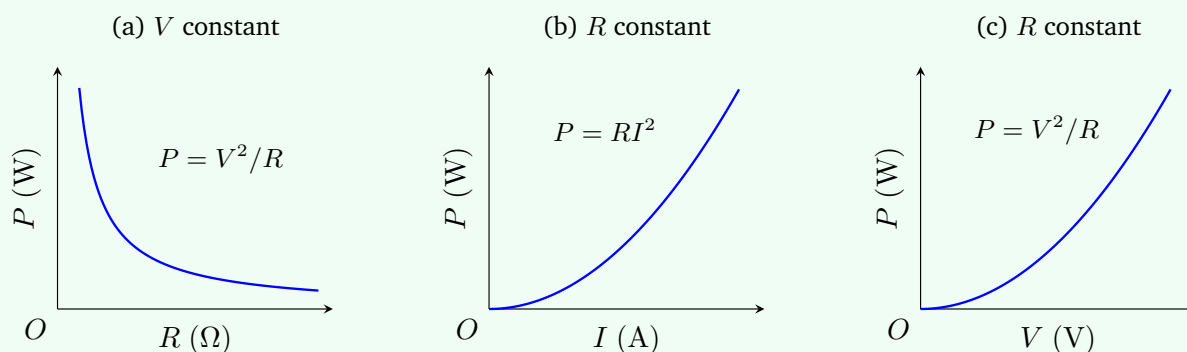
$$P = RI^2 \Rightarrow P \propto I^2.$$

The graph is an upward-opening parabola with its vertex at the origin. For positive current the power increases increasingly rapidly; doubling I makes P four times as large. The slope is zero at the origin, so the curve begins with a horizontal tangent.

Step 3: Plot power against voltage at constant resistance For part (c), the appropriate form is

$$P = \frac{1}{R}V^2 \Rightarrow P \propto V^2.$$

This also gives an upward-opening parabola through the origin with a horizontal tangent there. Doubling the voltage quadruples the power. The sketches below use nonnegative current and voltage magnitudes, as usual for a load resistor. If signed I or V is used instead, each parabolic graph extends symmetrically to negative horizontal values because power depends on the square. The plots are schematic since no numerical fixed voltage or resistance is supplied.



Schematic power variation: (a) fixed voltage; (b) and (c) fixed resistance.

Step 4: Conclusion The required plots are (a) a decreasing rectangular hyperbola, $P = V^2/R$; (b) an upward-opening parabola, $P = RI^2$; and (c) an upward-opening parabola, $P = V^2/R$ plotted against V . Both parabolic plots have their vertex at the origin.

Quick Tip to Solve: Write the fixed quantity beside each graph: fixed V gives $P \propto 1/R$, while fixed R gives square-law plots against either I or V .

Section D (15 marks)

31(a). (i) Write the principle and explain the working of a moving coil galvanometer. A galvanometer as such cannot be used to measure the current in a circuit. Why?

(ii) Why is the magnetic field made radial in a moving coil galvanometer? How is it achieved?

[5 marks]

Solution:

Concept:

- A current-carrying coil in a magnetic field experiences a torque: $\tau_m = NIAB \sin \theta$, where N is its number of turns, I the current, A the area of each turn, B the magnetic field and θ the angle between the coil's normal and the field.
- The suspension provides a restoring torque $\tau_r = k\phi$, where k is the torsional constant and ϕ is the angular deflection. The coil stops when the magnetic and restoring torques balance.
- In a radial magnetic field the coil's plane remains parallel to the local field during rotation. Thus $\theta = 90^\circ$, giving maximum torque $NIAB$ at every deflection.

Step 1: (i) State the principle and describe the arrangement A moving coil galvanometer works on the torque experienced by a current-carrying coil placed in a magnetic field. A light rectangular coil of many turns of insulated copper wire is suspended or pivoted between the concave pole pieces

of a strong permanent magnet. A soft iron cylindrical core inside the coil strengthens the field and helps make it radial. The suspension or control springs carry current to the coil and provide restoring torque; a pointer or mirror indicates its deflection.

Step 2: (i) Explain the motion and derive the current-deflection relation When current passes through the coil, magnetic forces on its opposite sides form a couple and rotate it. As it turns, the suspension twists and its restoring torque increases. In the radial field the magnetic torque is $NIAB$. At the steady deflection,

$$NIAB = k\phi,$$

$$I = \frac{k}{NAB}\phi.$$

Since k , N , A and B are fixed, deflection is proportional to current and the scale is uniform. Reversing the current reverses the torque and hence the deflection. Damping brings the coil quickly to rest at its equilibrium position.

Step 3: (i) Explain why it cannot directly serve as an ammeter A galvanometer is sensitive and can safely carry only a small current; an ordinary circuit current can produce excessive deflection and damage its delicate coil. Its appreciable resistance, if inserted in series, also changes the current being measured. To measure a larger current, it is converted into an ammeter by connecting a suitable low resistance shunt in parallel. Most of the current then bypasses the coil, and the combination has a low effective resistance.

Step 4: (ii) Explain the purpose and production of the radial field With a radial field the angle between the coil's normal and the local magnetic field remains 90° throughout the working range. The torque is therefore independent of the coil's angular position and remains maximum for a given current. This produces the linear relation $\phi \propto I$ and increases sensitivity. The field is made radial by using concave cylindrical magnetic pole faces with a coaxial cylindrical soft iron core inside the coil; the narrow annular air gap carries a strong radial field.

Step 5: Conclusion The moving coil galvanometer balances magnetic and restoring torques, giving $I = k\phi/(NAB)$. A radial field ensures maximum torque and a uniform scale. Its delicate coil and appreciable resistance require a parallel shunt for use as a practical ammeter.

Quick Tip to Solve: Keep the two angles distinct: θ is between the coil normal and the field, whereas ϕ is the pointer deflection. A radial field fixes $\theta = 90^\circ$ even while ϕ changes.

OR

- 31(b).** (i) Derive an expression for magnetic field on the axis of a current-carrying circular loop.
(ii) Write any two points of difference between a diamagnetic and a paramagnetic substance.

[5 marks]

Solution:

Concept:

- Biot–Savart law gives the field of a current element: $d\vec{B} = \frac{\mu_0 I}{4\pi} \frac{d\vec{l} \times \vec{r}}{r^3}$, where $\vec{r}$ points from the element to the observation point and μ_0 is the permeability of free space.
- On the axis of a circular loop, symmetry cancels the transverse field components of diametrically opposite elements. Their axial components have the same direction and add.

- Magnetic susceptibility χ_m relates magnetisation to the applied magnetic field intensity: $\vec{M} = \chi_m \vec{H}$. Its sign distinguishes a weak induced magnetisation opposite to the field from one along the field.

Step 1: (i) Set up the geometry of the circular loop Consider a circular loop of radius R carrying a steady current I in free space. Let O be its centre and P a point on its axis at distance x from O . Every small element dl of the loop is at the same distance from P :

$$r = \sqrt{R^2 + x^2}.$$

The tangent current element is perpendicular to both the radius of the loop and its axis. It is therefore perpendicular to the vector joining that element to P . The sine factor in the magnitude of Biot–Savart law equals unity.

Step 2: (i) Resolve the field of an element along the axis The magnitude of the field produced by the element is

$$dB = \frac{\mu_0 I dl}{4\pi(R^2 + x^2)}.$$

Let α be the angle made by $d\vec{B}$ with the axis. Because $d\vec{B}$ is perpendicular to the line joining the element to P , the right-triangle geometry gives

$$\cos \alpha = \frac{R}{r}.$$

The transverse components from opposite elements cancel, while their axial components reinforce each other. Hence the axial contribution is

$$dB_{\text{axis}} = dB \cos \alpha = \frac{\mu_0 I R dl}{4\pi(R^2 + x^2)^{3/2}}.$$

Step 3: (i) Integrate around the complete loop Since R and x are constant for every element, take these factors outside the integral and use the circumference $\oint dl = 2\pi R$:

$$\begin{aligned} B &= \frac{\mu_0 I R}{4\pi(R^2 + x^2)^{3/2}} \oint dl \\ &= \frac{\mu_0 I R}{4\pi(R^2 + x^2)^{3/2}} (2\pi R) \\ &= \frac{\mu_0 I R^2}{2(R^2 + x^2)^{3/2}}. \end{aligned}$$

The field is along the axis, with its sense given by the right-hand rule: curl the fingers in the current direction and the thumb gives the field direction. At the centre, $x = 0$, this reduces to $B = \mu_0 I / (2R)$. For N closely wound identical turns, multiply the expression by N .

Step 4: (ii) Give two differences between the substances **Response to a non-uniform magnetic field:** A diamagnetic substance is weakly repelled and moves from a stronger field region to a weaker one. A paramagnetic substance is weakly attracted and moves from a weaker field region to a stronger one.

Susceptibility and magnetisation direction: A diamagnetic substance has a small negative susceptibility, $\chi_m < 0$, so its induced magnetisation opposes the applied field. A paramagnetic substance has a small positive susceptibility, $\chi_m > 0$, so its magnetisation is along the applied field. Equivalently, their relative permeabilities are slightly below and slightly above unity, respectively, since $\mu_r = 1 + \chi_m$.

Step 5: Conclusion The magnetic field on the axis of a single circular current loop is $B =$

$\mu_0 I R^2 / [2(R^2 + x^2)^{3/2}]$, directed according to the right-hand rule. Diamagnetic substances are weakly repelled with negative susceptibility; paramagnetic substances are weakly attracted with positive susceptibility.

Quick Tip to Solve: In the axial-field derivation, use two separate facts: $d\vec{l} \perp \vec{r}$ makes the Biot–Savart sine factor one, but resolving the field still introduces the additional factor R/r .

32(a). (i) Draw a ray diagram showing the formation of a real image of an object placed at a distance u in front of a concave mirror of radius of curvature R . Hence, obtain the relation for the image distance v in terms of u and R .

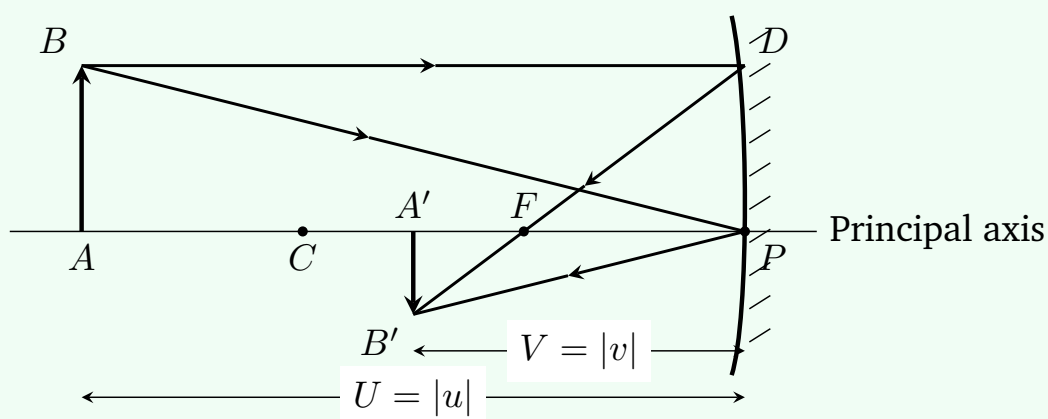
(ii) A 1.8 m tall person stands in front of a convex lens of focal length 1 m, at a distance of 5 m. Find the position and height of the image formed. [5 marks]

Solution:

Concept:

- For a paraxial ray incident parallel to a concave mirror's principal axis, the reflected ray passes through the principal focus F . The focal length is half the radius of curvature: $f = R/2$.
- Similar triangles in a mirror ray diagram relate object height, image height, object distance and image distance. Cartesian distances are measured from the pole P , positive along the direction of incident light.
- For a thin lens, $1/v - 1/u = 1/f$, and transverse magnification is $m = h_i/h_o = v/u$, where h_o and h_i are signed object and image heights.

Step 1: (i) Construct the real-image ray diagram Place the upright object AB beyond the centre of curvature C . From its top B , draw one ray parallel to the principal axis; after reflection at D , it passes through F . Draw another ray towards the pole P ; it reflects with equal angles to the principal axis. The reflected rays meet at B' below the axis, producing a real, inverted image $A'B'$ between C and F . This representative placement permits a simple geometrical derivation; the resulting mirror formula also applies to other object positions in the paraxial approximation.



Paraxial concave-mirror construction: an object beyond C gives a real, inverted image between C and F .

Step 2: Derive the distance relation from similar triangles First use positive lengths $U = AP$, $V = A'P$, $F_0 = FP$, $H = AB$ and $h = A'B'$. At the pole the incident and reflected rays make equal

angles with the axis, so the triangles ABP and $A'B'P$ give

$$\frac{h}{H} = \frac{V}{U}.$$

For a mirror of small aperture, the point D is approximately in the plane through P perpendicular to the axis. Its height is H . The triangles formed by the parallel ray's reflection through F therefore give

$$\frac{h}{H} = \frac{V - F_0}{F_0}.$$

Equating these two ratios and rearranging,

$$\begin{aligned}\frac{V}{U} &= \frac{V - F_0}{F_0}, \\ F_0 V &= UV - UF_0, \\ \frac{1}{U} + \frac{1}{V} &= \frac{1}{F_0}.\end{aligned}$$

Using Cartesian signs for this concave mirror, $u = -U$, $v = -V$, $f = -F_0$ and $R = 2f$. Thus

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f} = \frac{2}{R}.$$

Isolating the image distance gives $1/v = (2u - R)/(Ru)$. If the question's distances are instead taken as positive magnitudes, the corresponding expression is $V = rU/(2U - r)$, with $r = |R|$.

Step 3: (ii) Calculate the convex-lens image distance The person is a real object on the incident side of the lens, so $u = -5$ m. A convex lens has positive focal length, $f = +1$ m. Substitute into the lens formula:

$$\begin{aligned}\frac{1}{v} - \frac{1}{-5} &= \frac{1}{1}, \\ \frac{1}{v} &= 1 - \frac{1}{5} = \frac{4}{5} \text{ m}^{-1}.\end{aligned}$$

The positive reciprocal shows that the image forms on the other side of the lens and is real.

Step 4: Calculate the image height and interpret its sign With object height $h_o = +1.8$ m, magnification gives

$$\begin{aligned}m &= \frac{v}{u} = \frac{1.25}{-5} = -0.25, \\ h_i &= mh_o = (-0.25)(1.8) = -0.45 \text{ m}.\end{aligned}$$

The negative height denotes inversion, while $|m| = 0.25$ shows that the image is one-quarter the person's height. As an additional check, an object beyond $2f$ of a convex lens forms its real image between f and $2f$, consistent with the calculated position.

Step 5: Conclusion (i) The concave-mirror relation is $v = \frac{Ru}{2u - R}$, with Cartesian signs used consistently. (ii) The lens forms a real, inverted image 1.25 m from the lens on the opposite side, of height $h_i = -0.45$ m; its size is 45 cm.

Quick Tip to Solve: For this mirror derivation, obtain two expressions for the same height ratio before eliminating it. In the lens calculation, enter $u = -5$ m first; a negative magnification identifies the inverted image.

32(b). (i) Draw a ray diagram showing refraction of a ray of light through a triangular glass prism. Hence, obtain the relation for the refractive index (μ) in terms of angle of prism (A) and angle of minimum deviation (δ_m).

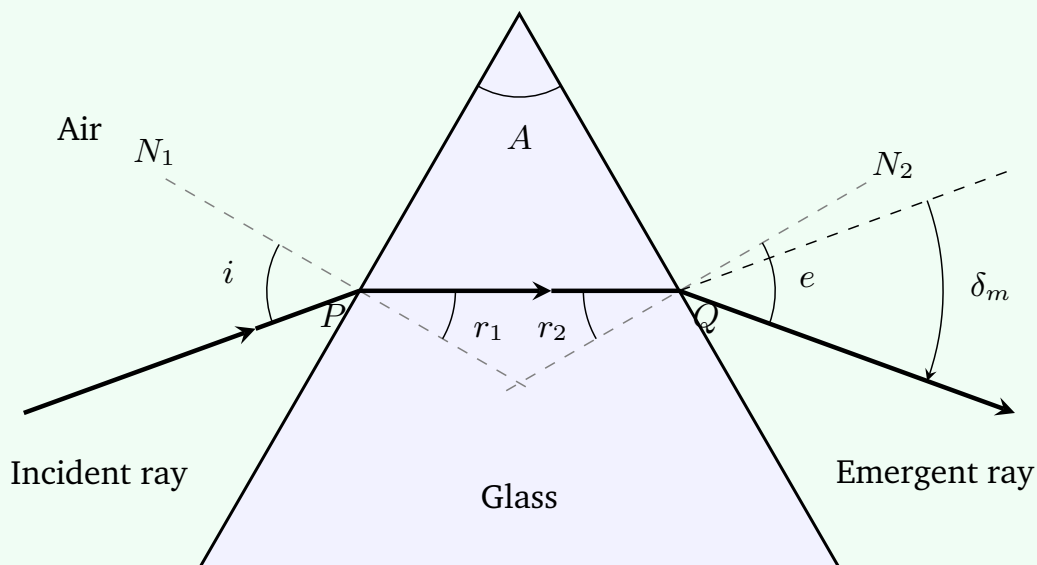
(ii) The radii of curvature of the two surfaces of a concave lens are 20 cm each. Find the refractive index of the material of the lens if its power is -5.0 D. [5 marks]

Solution:

Concept:

- For refraction from air into glass, Snell's law gives $\mu = \sin i / \sin r_1$, where i is the angle of incidence and r_1 is the first angle of refraction, both measured from the normal.
- For a prism of angle A , the internal refraction angles satisfy $r_1 + r_2 = A$. The total deviation is $\delta = i + e - A$, where e is the angle of emergence.
- The thin-lens maker's formula in air is $P = 1/f = (\mu - 1)(1/R_1 - 1/R_2)$. Power P is in dioptres when the signed radii R_1, R_2 and focal length f are in metres.

Step 1: (i) Draw the refracted path and identify its angles Let the incident ray meet the first face at P , pass through the glass along PQ , and emerge from the second face at Q . Draw normals to both faces at these points. The ray bends towards the normal on entering the glass and away from the normal on leaving it. The net deviation is towards the base of the prism. The sketch shows the symmetric path at minimum deviation; the dashed line to the right of Q is parallel to the original incident ray.



Refraction through a prism at minimum deviation; dashed lines at P, Q show normals, and the upper right dashed ray shows the incident direction.

Step 2: Derive the prism geometry and deviation equations The normals to the two refracting faces enclose an interior angle $180^\circ - A$. Applying the angle sum to the triangle formed by these normals and the internal ray gives

$$r_1 + r_2 + (180^\circ - A) = 180^\circ, \quad r_1 + r_2 = A.$$

At the first surface the ray turns through $i - r_1$, and at the second through $e - r_2$. Both turns contribute to the total deviation, so

$$\delta = (i - r_1) + (e - r_2) = i + e - A.$$

Step 3: Apply the minimum-deviation condition and Snell's law At minimum deviation the path through the prism is symmetric. Thus the angles of incidence and emergence are equal, and the two internal refraction angles are equal:

$$i = e, \quad r_1 = r_2 = \frac{A}{2}.$$

Substitution in the deviation equation gives

$$\delta_m = 2i - A, \quad i = \frac{A + \delta_m}{2}.$$

Applying Snell's law at the air-glass interface, with the refractive index of air taken as unity, therefore yields

$$\mu = \frac{\sin i}{\sin r_1} = \frac{\sin[(A + \delta_m)/2]}{\sin(A/2)}.$$

Step 4: (ii) Assign the signed radii and substitute in the lens formula Take light to travel from left to right and distances to the right as positive. For the biconcave lens, the centre of curvature of the first surface lies to its left, while that of the second surface lies to its right. Hence

$$R_1 = -20 \text{ cm} = -0.20 \text{ m}, \quad R_2 = +20 \text{ cm} = +0.20 \text{ m}.$$

Using the given power $P = -5.0 \text{ D}$,

$$\begin{aligned} -5.0 &= (\mu - 1) \left(\frac{1}{-0.20} - \frac{1}{+0.20} \right), \\ -5.0 &= -10(\mu - 1), \\ \mu - 1 &= 0.50, \quad \mu = 1.50. \end{aligned}$$

As a sign check, the negative power gives $f = 1/P = -0.20 \text{ m}$, consistent with a diverging concave lens in air.

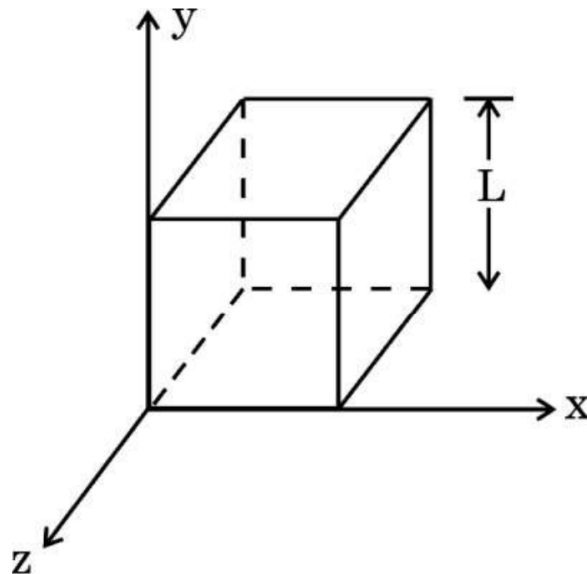
Step 5: Conclusion (i) The prism's refractive index is $\mu = \frac{\sin[(A + \delta_m)/2]}{\sin(A/2)}$. (ii) The refractive index of the lens material is 1.50, a dimensionless quantity.

Quick Tip to Solve: At minimum deviation first set $i = e$ and $r_1 = r_2 = A/2$. For a biconcave lens in air, use opposite signed radii and convert centimetres to metres before using power in dioptres.

33(a). (i) Define electric flux and write its SI unit.

(ii) Use Gauss' law to obtain the expression for the electric field due to a uniformly charged infinite plane sheet.

(iii) A cube of side L is kept in space, as shown in the figure. An electric field $\vec{E} = (Ax + B)\hat{i} \text{ N C}^{-1}$ exists in the region. Find the net charge enclosed by the cube. [5 marks]



Solution:

Concept:

- Electric flux through a surface is $\Phi_E = \int_S \vec{E} \cdot d\vec{S}$, where $d\vec{S}$ is an area vector normal to the surface. For a closed surface its direction is outward.
- Gauss' law states $\oint_S \vec{E} \cdot d\vec{S} = Q_{\text{enc}}/\epsilon_0$, where Q_{enc} is the net enclosed charge and ϵ_0 is the permittivity of free space.
- For a uniformly charged infinite plane sheet, symmetry makes the electric field perpendicular to the sheet, with equal magnitudes on its two sides. Write its surface charge density as σ .

Step 1: (i) Define flux and determine its unit Electric flux measures the electric field passing normally through a surface. For a uniform field over a plane area S ,

$$\Phi_E = \vec{E} \cdot \vec{S} = ES \cos \theta,$$

where θ is the angle between the electric field and the area normal. For a curved surface or a nonuniform field, sum the contributions using the surface integral in the concept block. Since electric field has unit N C^{-1} and area has unit m^2 , the SI unit of flux is $\text{N m}^2 \text{ C}^{-1}$, equivalently V m .

Step 2: (ii) Choose a Gaussian surface across the sheet First take a positively charged sheet. Choose a short cylindrical pillbox with its axis perpendicular to the sheet, its flat faces parallel to the sheet and on opposite sides, each of area S . The field is parallel to the curved surface, so its flux there is zero. On both flat faces the field points along the outward area vector. Thus each face contributes ES , and the enclosed charge is σS .

Step 3: Apply Gauss' law to the pillbox Summing the fluxes and substituting the enclosed charge

gives

$$\begin{aligned}ES + ES &= \frac{\sigma S}{\varepsilon_0}, \\2ES &= \frac{\sigma S}{\varepsilon_0}, \\E &= \frac{\sigma}{2\varepsilon_0}.\end{aligned}$$

The field is independent of the distance from an infinite sheet. In general its magnitude is $|\sigma|/(2\varepsilon_0)$; it points away from a positive sheet and towards a negative sheet, normally on each side.

Step 4: (iii) Calculate the outward flux through the cube The cube extends from $x = 0$ to $x = L$. Since the field has only an x -component, its dot product with the area vectors of the four faces normal to the y - and z -axes vanishes. The outward normals on the two remaining faces are $-\hat{i}$ at $x = 0$ and $+\hat{i}$ at $x = L$. Their signed fluxes are therefore

$$\begin{aligned}\Phi_{x=0} &= -BL^2, \\ \Phi_{x=L} &= (AL + B)L^2, \\ \Phi_{\text{net}} &= (AL + B)L^2 - BL^2 = AL^3.\end{aligned}$$

Gauss' law now gives $Q_{\text{enc}} = \varepsilon_0 \Phi_{\text{net}}$. The constant field term B contributes equal and opposite fluxes, so it encloses no net charge by itself.

Step 5: Conclusion (i) $\Phi_E = \int_S \vec{E} \cdot d\vec{S}$, with SI unit $\text{N m}^2 \text{C}^{-1}$. (ii) The sheet's field has magnitude $E = |\sigma|/(2\varepsilon_0)$, directed normally away from positive charge and towards negative charge. (iii) The cube encloses $Q_{\text{enc}} = \varepsilon_0 AL^3$, in coulombs when the given quantities are expressed in SI units; its sign follows that of A .

Quick Tip to Solve: For an electric field with only an x -component, subtract the left-face field from the right-face field and multiply by the face area; the uniform term cancels.

OR

33(b). (i) Define electric potential at a point and write its SI unit.

(ii) Two capacitors are connected in series. Derive an expression of the equivalent capacitance of the combination.

(iii) Two point charges $+q$ and $-q$ are located at points $(3a, 0)$ and $(0, 4a)$ respectively in x - y plane. A third charge Q is kept at the origin. Find the value of Q , in terms of q and a , so that the electrostatic potential energy of the system is zero. [5 marks]

Solution:

Concept:

- Electric potential is potential energy per unit positive test charge: $V = W_{\text{ext}}/q_0$, with zero potential chosen at infinity and the test charge moved quasistatically.
- Capacitance is $C = Q_c/V$, where Q_c is the magnitude of charge on either plate and V is the potential difference. Capacitors in series carry equal charge magnitudes and their potential differences add.
- Electrostatic potential energy of point charges is the sum over distinct pairs, $U = \sum_{i < j} kq_i q_j / r_{ij}$, where $k = 1/(4\pi\varepsilon_0)$ and r_{ij} is the pair separation.

Step 1: (i) Define electric potential and its unit Electric potential at a point is the work done by an external agent per unit positive test charge in bringing the charge from infinity to that point, without acceleration. The test charge is sufficiently small not to disturb the source-charge configuration. Taking the potential at infinity to be zero,

$$V = \frac{W_{\text{ext}}}{q_0}.$$

Its SI unit is the volt: $1 \text{ V} = 1 \text{ J C}^{-1}$. Potential is a scalar; its sign depends on the source charges and the chosen reference.

Step 2: (ii) Express the individual voltage drops Connect two initially uncharged capacitors of capacitances C_1 and C_2 in series across a potential difference V . Their common intermediate conductor remains neutral, so the facing plates at the junction acquire equal and opposite charges. Consequently the magnitude of charge on each capacitor is the same, denoted by Q_c . From the definition of capacitance,

$$V_1 = \frac{Q_c}{C_1}, \quad V_2 = \frac{Q_c}{C_2}.$$

The total potential difference between the outer terminals is $V = V_1 + V_2$.

Step 3: Derive the equivalent series capacitance Let a single equivalent capacitor C_{eq} hold charge Q_c at the same voltage V . Substituting the voltage drops gives

$$\begin{aligned} \frac{Q_c}{C_{\text{eq}}} &= \frac{Q_c}{C_1} + \frac{Q_c}{C_2}, \\ \frac{1}{C_{\text{eq}}} &= \frac{1}{C_1} + \frac{1}{C_2}, \\ C_{\text{eq}} &= \frac{C_1 C_2}{C_1 + C_2}. \end{aligned}$$

This is smaller than either individual capacitance, consistent with adding the voltage drops for a fixed charge.

Step 4: (iii) Find all three pair separations Denote the locations of $+q$, $-q$ and Q by $P = (3a, 0)$, $R = (0, 4a)$ and $O = (0, 0)$, respectively, with $a > 0$. Their separations are

$$OP = 3a, \quad OR = 4a, \quad PR = \sqrt{(3a)^2 + (4a)^2} = 5a.$$

There are three distinct pairs. The pair $+q, -q$ has negative interaction energy; the interactions involving Q must both be included with their signs.

Step 5: Set the total pair energy to zero Taking zero interaction energy at infinite separation, add each pair exactly once:

$$\begin{aligned} U &= -\frac{kq^2}{5a} + \frac{kqQ}{3a} - \frac{kqQ}{4a} \\ &= -\frac{kq^2}{5a} + \frac{kqQ}{12a}. \end{aligned}$$

For a nonzero q , imposing $U = 0$ gives

$$\frac{qQ}{12a} = \frac{q^2}{5a}, \quad Q = \frac{12q}{5}.$$

Thus the positive interaction energy of Q with the original pair compensates their negative mutual energy. The common length factor cancels, so the required charge does not depend on a .

Step 6: Conclusion (i) Electric potential is external work per unit positive test charge brought from infinity; its SI unit is $V = J C^{-1}$. (ii) $C_{eq} = C_1 C_2 / (C_1 + C_2)$. (iii) The required third charge is $Q = +12q/5$, independent of a .

Quick Tip to Solve: For three charges, write all three pair energies before simplifying. The given coordinates form a $3a-4a-5a$ triangle, so the original pair separation is $5a$.

Section E (8 marks)

34(a). Diffraction of light is bending of light around the corners of an object whose size is comparable with the wavelength of light. Diffraction actually defines the limits of ray optics. This limit for optical instruments is set by the wavelength of light. An experimental arrangement is set up to observe the diffraction pattern due to a single slit.

Answer the following questions based on the above:

How will the width of central maximum be affected if the wavelength of light is increased? [1 mark]

Solution:

Concept:

- For a slit of width a , the first diffraction minima satisfy $a \sin \theta = \pm \lambda$, where λ is the wavelength.
- At small angles, the central maximum has width $W = 2D\lambda/a$ on a screen at distance D .

Step 1: Identify the fixed quantities The experimental arrangement is unchanged, so the slit width a and the slit-to-screen distance D remain fixed. The central maximum lies between the first minima on either side of the centre.

Step 2: Compare the widths Since W is directly proportional to λ , increasing the wavelength moves both first minima farther from the centre. For two wavelengths,

$$\frac{W_2}{W_1} = \frac{\lambda_2}{\lambda_1}.$$

Step 3: Conclusion The width of the central maximum increases when the wavelength increases, for the same slit and screen arrangement.

Quick Tip to Solve: Use $W = 2D\lambda/a$: a longer wavelength broadens the central maximum when D and a are fixed.

34(b). Under what condition is the first minimum obtained?

[1 mark]

Solution:

Concept:

- In single-slit diffraction, light from different points across the slit interferes. For slit width a , the path difference between extreme rays in direction θ is $a \sin \theta$.
- Two equal contributions cancel if their path difference is $\lambda/2$, corresponding to a phase difference of π .

Step 1: Divide the slit into equal halves Pair every point in the upper half with a point a distance $a/2$

below it. For each pair, the path difference towards the same observation point is $(a/2) \sin \theta$.

Step 2: Apply the cancellation condition At the first minimum each pair cancels, requiring

$$\frac{a}{2} |\sin \theta| = \frac{\lambda}{2}.$$

Thus the extreme rays differ in path by one wavelength. At $\theta = 0$, all contributions are in phase, so the centre is a maximum, not a minimum.

Step 3: Conclusion The first minima occur when $a \sin \theta = \pm \lambda$, or approximately $\theta = \pm \lambda/a$ at small angles.

Quick Tip to Solve: For the first single-slit minimum, set the extreme-ray path difference equal to λ , not $\lambda/2$.

34(c). Write two points of difference between interference and diffraction patterns.

[2 marks]

Solution:

Concept:

- The standard interference pattern is produced by two coherent narrow slits, while the single-slit diffraction pattern results from interference of secondary wavelets across one slit.
- Compare ideal interference from equal-intensity sources with single-slit diffraction under uniform illumination. Let D be screen distance, d the separation of two slits and a the width of a single slit.

Step 1: Compare the widths of bright regions In the ideal two-slit interference pattern, bright fringes have equal widths; adjacent maxima are separated by $\beta = \lambda D/d$. In single-slit diffraction, the central maximum has width $2\lambda D/a$, while secondary bright regions have approximately width $\lambda D/a$. Thus the central diffraction maximum is twice as wide as a secondary maximum.

Step 2: Compare the intensities of maxima For two coherent sources of equal intensity, ideal interference maxima have the same intensity because the same two amplitudes add in phase at every maximum. In single-slit diffraction, the central maximum is brightest, and successive secondary maxima become progressively weaker away from the centre. Real finite-width double slits can also impose a diffraction envelope on the interference fringes; the equal-intensity comparison refers to the ideal interference pattern.

Step 3: Conclusion The two differences are: equal bright-fringe widths in ideal interference versus a diffraction central maximum of double width; and equal ideal interference maxima versus diffraction maxima of decreasing intensity away from the centre.

Quick Tip to Solve: Compare width and peak intensity: a broad, strongest central maximum is the signature of single-slit diffraction.

34(c). Two students are separated by a 7 m partition wall in a room 10 m high. If both light and sound waves can bend around obstacles, how is it that the students are unable to see each other even though they can converse easily? [2 marks]

Solution:

Concept:

- Diffraction is appreciable when the wavelength is comparable with the relevant obstacle or opening dimension. The larger the ratio λ/a , the greater the angular spreading for an opening of size a .
- Visible light has wavelengths of order 10^{-7} m, whereas sound wavelengths in ordinary speech are much longer, ranging from centimetres to metres.

Step 1: Compare the wavelengths with the opening The partition is 7 m high in a room 10 m high, leaving a 3 m gap above it. Visible wavelengths are extremely small compared with this gap and the wall dimensions. Consequently, light spreads through only a very small diffraction angle and does not appreciably bend into the region behind the wall to form a view of the other student.

Step 2: Explain why sound reaches the other student The much longer wavelengths of sound, especially its lower-frequency components, give far greater diffraction around the partition's upper edge and through the gap. Sound can therefore reach the region behind the partition where direct propagation is blocked. Reflections from room surfaces can also help sound reach the listener, but the contrast asked here follows from the much greater diffraction of sound than of visible light.

Step 3: Conclusion The students can converse because sound diffracts appreciably around the wall; they cannot see each other because the very short wavelength of visible light makes its diffraction around such a large obstacle negligible.

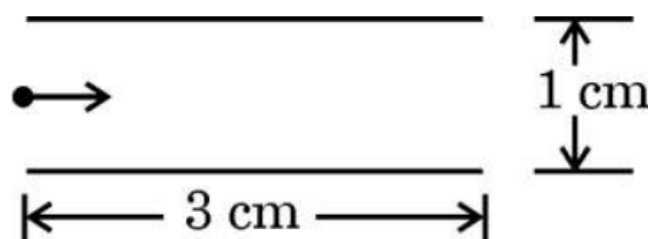
Quick Tip to Solve: Compare wavelength with the obstacle or gap size; both waves diffract, but sound's much larger wavelength makes the effect noticeable.

35(a). A beam of electrons moving horizontally with a velocity of 3×10^7 m/s enters a region between two plates as shown in the figure. A suitable potential difference is applied across the plates such that the electron beam just strikes the edge of the lower plate.

Answer the following questions based on the above:

How long does an electron take to strike the edge?

[1 mark]



Solution:

Concept:

- Between parallel plates, the uniform electric field is perpendicular to the plates. The electric force on an electron therefore changes only its vertical velocity.

- Horizontal motion remains uniform: $x = u_x t$, where u_x is the incident horizontal speed and t is the time spent between the plates.

Step 1: Read the horizontal distance The printed figure shows a plate length $L = 3 \text{ cm} = 3 \times 10^{-2} \text{ m}$. Since the beam just strikes the far edge of the lower plate, the electron travels this full horizontal distance before striking. The 1 cm plate separation is a vertical distance and is not used to find this time.

Step 2: Calculate the flight time Using the unchanged horizontal speed $u_x = 3 \times 10^7 \text{ m s}^{-1}$,

$$t = \frac{L}{u_x} = \frac{3 \times 10^{-2}}{3 \times 10^7} = 1.0 \times 10^{-9} \text{ s}.$$

Step 3: Conclusion The electron takes $1.0 \times 10^{-9} \text{ s} = 1.0 \text{ ns}$ to strike the edge of the lower plate.

Quick Tip to Solve: Find the time from plate length divided by horizontal speed; vertical acceleration does not change the horizontal flight time.

35(b). What is the shape of the path followed by the electron and why?

[1 mark]

Solution:

Concept:

- A uniform electric field exerts a constant force $\vec{F} = -e\vec{E}$ on an electron, where e is the magnitude of its charge.
- Uniform horizontal velocity together with constant perpendicular acceleration produces a parabolic trajectory, as in projectile motion.

Step 1: Resolve the motion into components Take the entry point as the origin, horizontal motion as the positive x -direction and downward motion as the positive y -direction. The field is vertical, so the horizontal acceleration is zero and $x = ut$. The electron starts with no vertical velocity and accelerates downwards towards the lower plate with constant magnitude $a = eE/m$.

Step 2: Eliminate time from the equations The downward displacement is $y = \frac{1}{2}at^2$. Substituting $t = x/u$ gives

$$y = \frac{a}{2u^2}x^2 = \frac{eE}{2mu^2}x^2.$$

This is the equation of a parabola. The electric force is constant in direction; it is not always perpendicular to the changing velocity, so the path is not circular.

Step 3: Conclusion The electron follows a parabolic path bending towards the lower plate because its horizontal velocity is constant while it has uniform vertical acceleration.

Quick Tip to Solve: Whenever one component is uniform motion and the perpendicular component is uniformly accelerated, eliminate time to obtain a parabola.

35(c). Find the potential difference applied.

[2 marks]

Solution:**Concept:**

- An electron of charge magnitude e and mass m_e has acceleration magnitude $a = eE/m_e$ in a uniform electric field E .
- Horizontal velocity remains constant. For plate length L , the transit time is $t = L/v_x$; with zero initial vertical velocity, the downward displacement is $y = at^2/2$.
- For parallel plates separated by d , the magnitude of their potential difference is $V = Ed$.

Step 1: Find the transit time and vertical displacement The beam enters midway between the plates, whose separation is $d = 1 \text{ cm} = 1.0 \times 10^{-2} \text{ m}$. To reach the lower plate it moves downward through half this separation. The horizontal electric force is zero, so

$$t = \frac{3.0 \times 10^{-2}}{3.0 \times 10^7} = 1.0 \times 10^{-9} \text{ s}, \quad y = \frac{d}{2} = 5.0 \times 10^{-3} \text{ m}.$$

Step 2: Calculate the acceleration and electric field The initial vertical velocity is zero. Using the vertical displacement during this time gives

$$a = \frac{2y}{t^2} = \frac{2(5.0 \times 10^{-3})}{(1.0 \times 10^{-9})^2} = 1.0 \times 10^{16} \text{ m s}^{-2}.$$

With $m_e = 9.1 \times 10^{-31} \text{ kg}$ and $e = 1.6 \times 10^{-19} \text{ C}$,

$$E = \frac{m_e a}{e} = \frac{(9.1 \times 10^{-31})(1.0 \times 10^{16})}{1.6 \times 10^{-19}} = 5.6875 \times 10^4 \text{ V m}^{-1}.$$

Since the negatively charged electron accelerates downward, the electric field points upward, from the positive lower plate to the negative upper plate.

Step 3: Conclusion The applied potential difference is

$$V = Ed = (5.6875 \times 10^4)(1.0 \times 10^{-2}) = 568.75 \text{ V} \approx 5.69 \times 10^2 \text{ V}.$$

The lower plate is at the higher potential.

Quick Tip to Solve: Use half the plate separation for the electron's vertical displacement, but the full separation in $V = Ed$.

OR**35(c). Find the magnitude and direction of the magnetic field which should be created in the space between the plates so that the electron beam goes straight undeviated.**

[2 marks]

Solution:**Concept:**

- In crossed electric and magnetic fields, an electron goes undeviated when its net Lorentz force vanishes: $q(\vec{E} + \vec{v} \times \vec{B}) = 0$.
- For a magnetic field perpendicular to the velocity, the magnetic force magnitude is evB . Balance

against the electric force eE therefore requires $B = E/v$.

Step 1: Determine the electric field from the original deflection The horizontal speed is $v = 3.0 \times 10^7 \text{ m s}^{-1}$ and the plate length is $L = 3.0 \times 10^{-2} \text{ m}$. Hence the transit time is $t = L/v = 1.0 \times 10^{-9} \text{ s}$. The beam enters midway between the plates, so its downward displacement at the edge is $y = 0.5 \text{ cm} = 5.0 \times 10^{-3} \text{ m}$. Thus

$$a = \frac{2y}{t^2} = 1.0 \times 10^{16} \text{ m s}^{-2}, \quad E = \frac{m_e a}{e} = 5.6875 \times 10^4 \text{ V m}^{-1}.$$

Here $m_e = 9.1 \times 10^{-31} \text{ kg}$ and $e = 1.6 \times 10^{-19} \text{ C}$. The original electric force on the electron is downward, so the electric field is upward.

Step 2: Balance the forces and determine the field direction With the applied electric field unchanged, the magnetic force must be upward to cancel the downward electric force. Equating their magnitudes gives

$$eE = evB \Rightarrow B = \frac{E}{v} = \frac{5.6875 \times 10^4}{3.0 \times 10^7} = 1.8958 \times 10^{-3} \text{ T}.$$

For velocity to the right and magnetic field out of the page, $\vec{v} \times \vec{B}$ points downward. Multiplication by the electron's negative charge reverses this direction, giving the required upward magnetic force. A field into the page would instead reinforce the downward deflection.

Step 3: Conclusion Apply a magnetic field of magnitude $B \approx 1.90 \times 10^{-3} \text{ T}$, perpendicular to the page and directed out of the page, towards the observer. The electric and magnetic forces then cancel, and the electron beam travels straight.

Quick Tip to Solve: First use $B = E/v$ for the magnitude; reverse the direction of $\vec{v} \times \vec{B}$ when finding the force on an electron.